

A Bayesian determination of the local dark matter density

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R. Catena and P. Ullio, arXiv:0907.0018 [astro-ph.CO].

To be published in JCAP

- Direct detection signals depend from dark halo properties.
- **Example** : Spin-independent dark matter-nucleus scattering.

- The expected event rate reads

$$\frac{dR}{dE_r} = \frac{\sigma_p \rho_{DM}(R_0)}{2\mu_{p,DM}^2 m_{DM}} \left\langle \int_{v_{\min}}^{\infty} \frac{f_{DM}(v, t)}{v} dv \right\rangle A^2 F^2(E_r)$$

- It crucially depends on $\rho_{DM}(R_0)$ (this talk) and $f_{DM}(\vec{v}, t)$.

- **Bayesian approach:**

R. Catena and P. Ullio, arXiv:0907.0018 [astro-ph.CO].
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- * It assumes a Galactic model (spherical dark matter profile)
- * Different observations on different scales
- * ... more in this talk ...

- **N-body approach:**

M.Pato, O.Agertz, G.Bertone, B.Moore and R.Teyssier arXiv:1006.1322 [astro-ph.CO]

- * N-body simulations
- * Aim: to estimate systematic effects
- * ... more in the previous talk ...

- **Poisson equation approach:**

P. Salucci, F. Nesti, G. Gentile and C. F. Martins, arXiv:1003.3101 [astro-ph.GA].

- * Local quantities
- * **Strategy:** $\rho_{DM}(R_0) = \frac{1}{4\pi GR_0^2} \frac{\partial}{\partial R} (R\Theta^2)_{R=R_0} - K$,
- * ... more in the next talk ...

- Maximum Likelihood approach:

M. Weber and W. de Boer, arXiv:0910.4272 [astro-ph.CO].

- * Only three free parameters (many fixed a priori)
- * For some choice of the fixed parameters (with reasonable M_{vir}) it is in agreement with other approaches

- Fisher matrix forecasts:

L. E. Strigari and R. Trotta, JCAP **0911** (2009) 019 [arXiv:0906.5361 [astro-ph.HE]].

- * Assumed a reference point in parameter space it tests the reconstruction capabilities of a future direct detection experiment accounting for astrophysical uncertainties.

- 1 The underlying Galactic Model
- 2 The experimental constraints
- 3 The method: Bayesian inference with Markov Chain Monte Carlo
- 4 Results and Conclusions

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The underlying Galactic Model

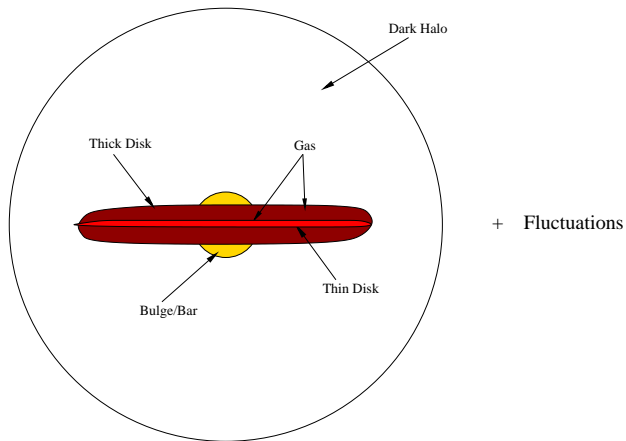


Figure: Schematic representation of the Galaxy

The underlying Galactic Model

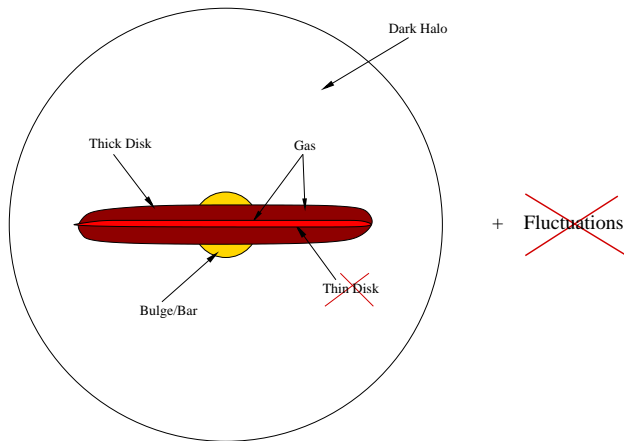


Figure: Schematic representation of the assumed Galactic model

- The stellar disk:

$$\rho_d(R, z) = \frac{\Sigma_d}{2z_d} e^{-\frac{R}{R_d}} \operatorname{sech}^2\left(\frac{z}{z_d}\right) \quad \text{with} \quad R < R_{\text{dm}}$$

H. T. Freudenreich, *Astrophys. J.* **492**, 495 (1998)

- The dust layer:

The distribution of the Interstellar Medium is assumed axisymmetric as well.

T. M. Dame, *AIP Conference Proceedings* **278** (1993) 267.

- The stellar bulge/bar:

$$\rho_{bb}(x, y, z) = \rho_{bb}(0) \left[\exp\left(-\frac{s_b^2}{2}\right) + s_a^{-1.85} \exp(-s_a) \right]$$

where

$$s_a^2 = \frac{q_a^2(x^2 + y^2) + z^2}{z_b^2} \quad s_b^2 = \left[\left(\frac{x}{x_b}\right)^2 + \left(\frac{y}{y_b}\right)^2 \right]^2 + \left(\frac{z}{z_b}\right)^4 .$$

H. Zhao, arXiv:astro-ph/9512064.

- The Dark Matter halo:

$$\rho_h(R) = \rho' f\left(\frac{R}{a_h}\right),$$

where f is the Dark Matter profile.

- M_{vir} , and c_{vir} as halo parameters:

$$\rho' = \rho'(M_{vir}, c_{vir})$$

$$a_h = a_h(M_{vir}, c_{vir})$$

- The Dark Matter profile:

$$f_E(x) = \exp \left[-\frac{2}{\alpha_E} (x^{\alpha_E} - 1) \right]$$

J.F. Navarro et al., MNRAS **349** (2004) 1039.

A.W. Graham, D. Merritt, B. Moore, J. Diemand and B. Terzic, Astron. J. **132** (2006) 2701.

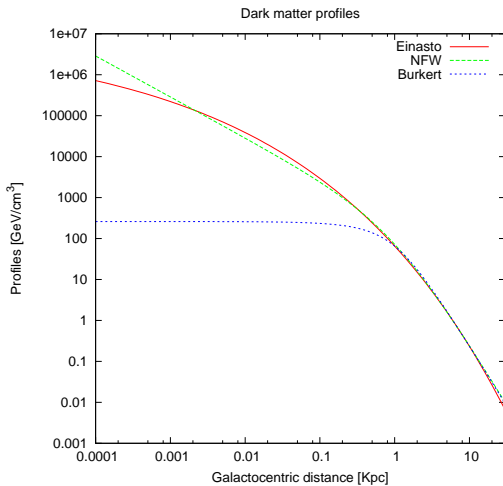
$$f_{NFW}(x) = \frac{1}{x(1+x)^2}$$

J.F. Navarro, C.S. Frenk and S.D.M. White, Astrophys. J. **462**, 563 (1996); Astrophys. J. **490**, 493 (1997).

$$f_B(x) = \frac{1}{(1+x)(1+x^2)}.$$

A. Burkert, Astrophys. J. **447** (1995) L25.

The underlying Galactic Model



Galactic components	Parameters
Disk	Σ_d
Disk	R_d
Bulge/bar	$\rho_{bb}(0)$
Halo	α_E
Halo	M_{vir}
Halo	c_{vir}
All components	R_0
All components	β_\star

Constraints:

- Oort's constants: $A - B = \frac{\Theta_0}{R_0}$; $A + B = -\frac{\partial\Theta(R_0)}{\partial R}$
- terminal velocities
- total mean surface density within $|z| < 1.1\text{kpc}$
- local disk surface mass density
- total mass inside 50 kpc and 100 kpc
- l.s.r. velocity, proper motion and parallaxes distance of high mass star forming regions in the outer Galaxy
- radial velocity dispersion of tracers from the SDSS
- stellar motions around the massive black hole in the GC
- peculiar motion of SgrA*

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- **The dataset:** population of stars with distances up to ~ 60 kpc from the Galactic center. The distances are accurate to $\sim 10\%$ and the radial velocity errors are less than 30 km s^{-1} .

- It is a strong constraint in the range $10 \text{ kpc} \lesssim R \lesssim 60 \text{ kpc}$

- **To compare the data to the predictions:** Jeans Equation

$$\sigma_r^2(r) = \frac{1}{r^{2\beta_\star} \rho_\star(r)} \int_r^\infty d\tilde{r} \tilde{r}^{2\beta_\star-1} \rho_\star(\tilde{r}) \Theta^2(\tilde{r})$$

- where β_\star is the anisotropy parameter: $\beta_\star \equiv 1 - \sigma_t^2 / \sigma_r^2$.

Parametric model
of the Galaxy

{

Frequentist approach \implies Maximum Likelihood

Bayesian approach \implies Posterior probability density

- This work \rightarrow Bayesian approach

- Target: posterior pdf (Bayes' theorem):

$$p(\eta|d) = \frac{\mathcal{L}(d|\eta)\pi(\eta)}{p(d)}; \quad d = \text{data}; \quad \eta = \text{parameters}$$

- Output: the mean and the variance with respect to $p(\eta|d)$ of functions $f(\eta)$.
- We will focus on $f = \eta$ and $f = \rho_{DM}(R_0)$.

- Monte Carlo expectation values:

$$\langle f(\eta) \rangle = \int d\eta f(\eta) p(\eta|\mathbf{d}) \approx \frac{1}{N} \sum_{t=0}^{N-1} f(\eta^{(t)}) ,$$

where $\eta^{(t)}$ was sampled from $p(\eta|\mathbf{d})$.

- Monte Carlo technics require a method to sample $\eta^{(t)} \implies$ Markov chains.

- Markov chains :

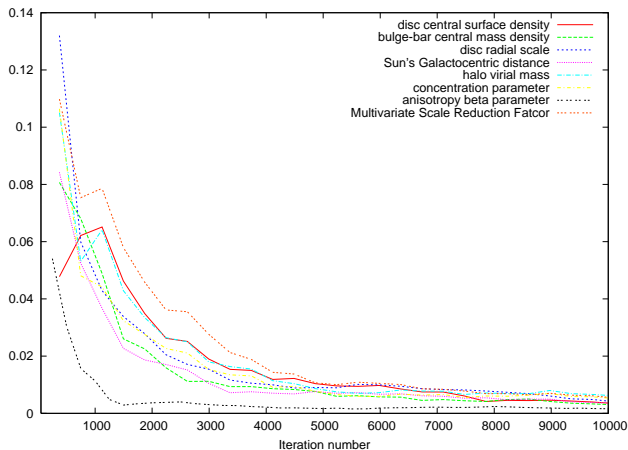
$$\left. \begin{array}{l} p(\eta^{(0)}) \\ T(\eta^{(t)}, \eta^{(t+1)}) \end{array} \right\} \implies \eta^{(t)} \text{ distributed according to } p(\eta|\mathbf{d}).$$

Convergence of the Markov chains

$R \equiv$ (Scale reduction factor).

Convergence: $R < 1.1$ and roughly constant.

1-R as a function of the iteration number:



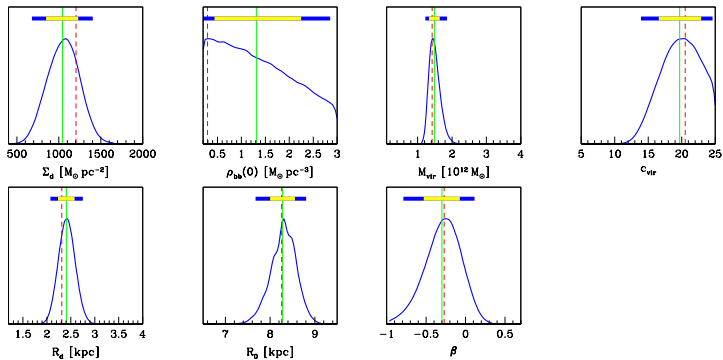


Figure: Marginal posterior pdf of the Galactic model parameters (NFW profile).

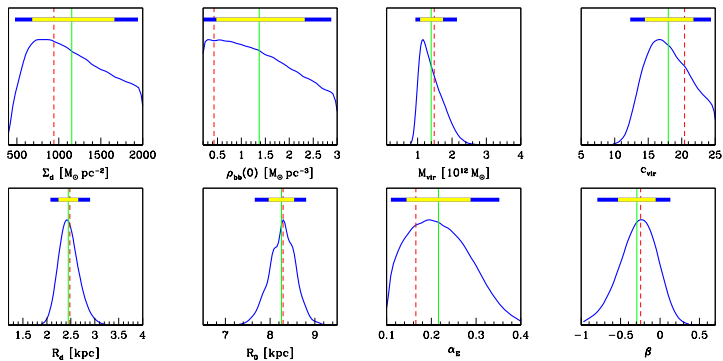


Figure: Marginal posterior pdf of the Galactic model parameters (Einasto profile).

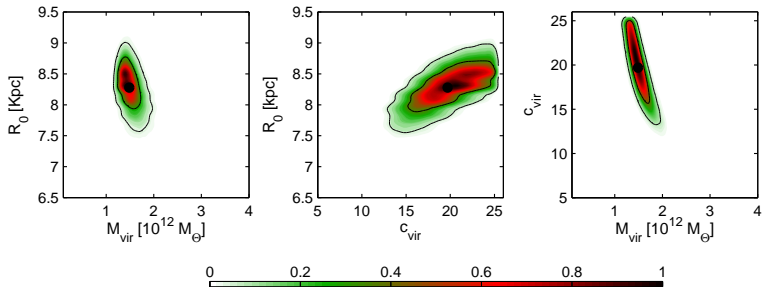


Figure: Two dimensional marginal posterior pdf in the planes spanned by combinations of the Galactic model parameters (NFW profile).

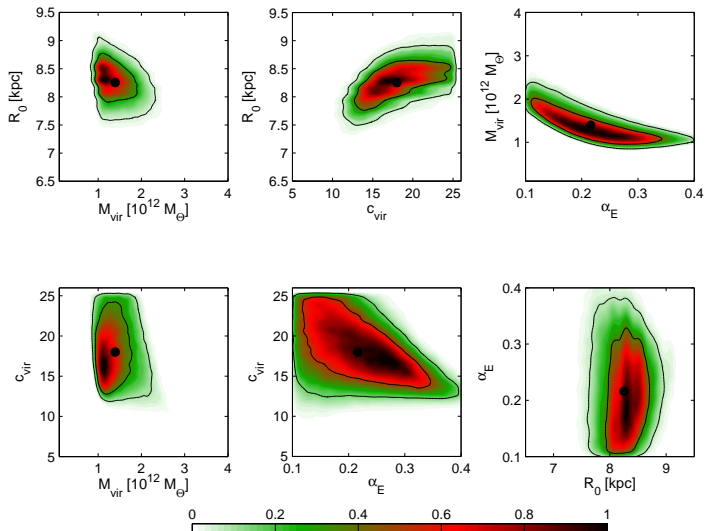


Figure: Two dimensional marginal posterior pdf in the planes spanned by combinations of the Galactic model parameters (**Einasto profile**).

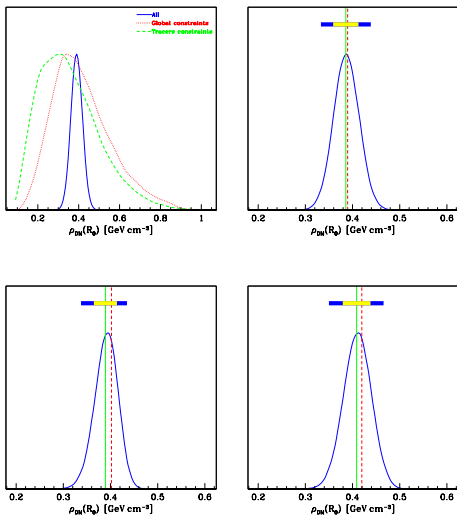


Figure: Marginal posterior pdf for the local Dark Matter density. Top left panel: Einasto profile, applying different subsets of constraints. Top right panel: Einasto profile. Bottom left panel: NFW profile. Bottom right panel: Burkert profile.

- Numerically we find:

$$\rho_{DM}(R_0) = (0.385 \pm 0.027) \text{ GeV cm}^{-3} \quad (\text{Einasto})$$

$$\rho_{DM}(R_0) = (0.389 \pm 0.025) \text{ GeV cm}^{-3} \quad (\text{NFW})$$

$$\rho_{DM}(R_0) = (0.409 \pm 0.029) \text{ GeV cm}^{-3} \quad (\text{Burkert})$$

- No strong dependences from the assumed halo profile.

- We proved that Bayesian probabilistic inference is a good method to constrain the local dark matter density.
- For a given dark matter profile, and assuming spherical symmetry, we can therefore estimate the local dark matter density with an accuracy of roughly the 7%.
- This result does not include a number of systematic uncertainties which are related to the galactic model, *e.g.*:
 - baryonic compression
 - dark disk
 - ...