The Sommerfeld Enhancement for Thermal Relic Dark Matter with an Excited State

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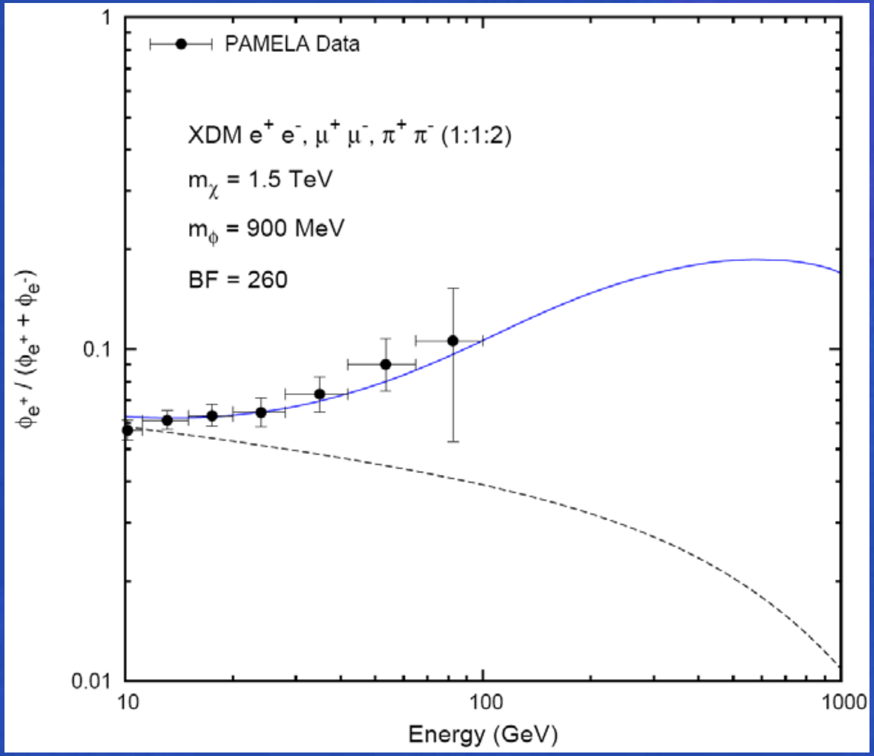
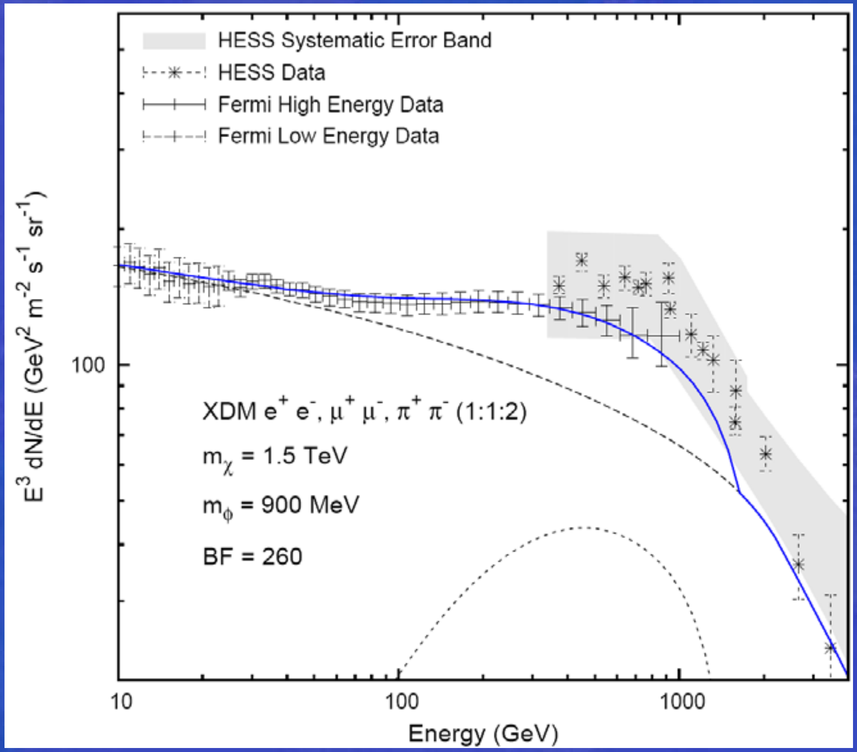
Context

- PAMELA and Fermi cosmic-ray anomalies motivate large DM annihilation cross sections; can be achieved by Sommerfeld enhancement.
- Feng, Kaplinghat & Yu (2010): claim that a boost of ~1500 is needed to obtain the cosmic-ray signals, whereas requiring the correct thermal relic density gives a maximum Sommerfeld enhancement of ~100.
- However, their work assumes:
  - Small local DM density – 0.3 GeV/cm^3 – in conflict with latest estimates (Catena & Ullio, Salucci et al, Pato et al).
  - High DM mass (~2.4 TeV) – required boost factor scales as ~m_χ.
  - 4-muon final state – large fraction of power goes into neutrinos, requires higher boosts than modes with significant electron branching ratios.
- With a local DM density of 0.43 GeV/cm^3 (Salucci et al), and annihilation to theoretically motivated final states, good fits to the data are obtained with BFs ~100-300: at most factor-of-a-few tension with maximal Sommerfeld enhancement.
- Models with nearly-degenerate excited states also have higher maximal Sommerfeld enhancement, by a factor of 2-5: removes all tension.

Example curves for PAMELA/Fermi





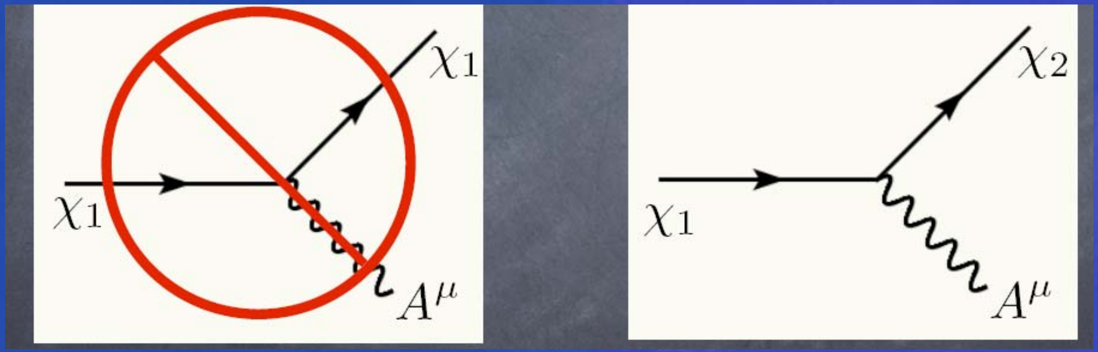
Spectral shapes look fine, but a large "boost factor" is required.

Ingredients of the models

- DM has some new U(1) gauge interaction, broken at the ~GeV scale by a dark Higgs h_D.
- Coupling to SM: U(1) gauge boson mixes kinetically with hypercharge (with a small mixing angle).
- DM annihilates to (on-shell) dark gauge bosons (there are also subdominant annihilation channels involving the dark Higgs). These in turn decay to light charged SM particles – mixture of electrons, muons, charged pions depending on gauge boson mass.
- Exchange of dark gauge bosons mediates an attractive force, giving Sommerfeld enhancement to annihilation at low velocities.

Dark matter excited states

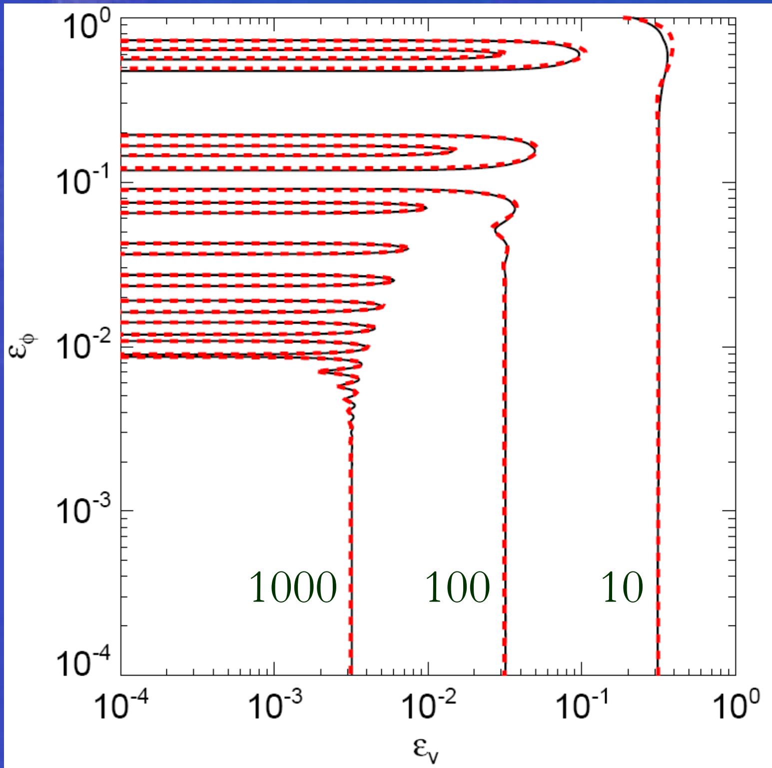
- DM is already Dirac and hence multi-component; any higher-dimension operator that gives the DM a Majorana mass will split the mass eigenstates. If the dark gauge group was non-Abelian, such splittings would be generated radiatively, but this does not occur for our simple U(1) example.
- Operator in benchmark model: y $\chi\chi h_D{}^* h_D{}^*/\Lambda$,
  Majorana mass scale ~ GeV^2/ TeV ~ MeV.
- Furthermore, the mass eigenstates are 45° rotated from the gauge eigenstates, so interactions between DM particles and the gauge boson are purely off-diagonal.



The Sommerfeld enhancement (no excited state)

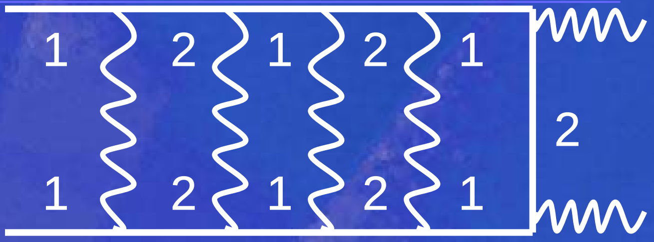
- Enhancement to annihilation due to attractive force between DM particles; scales as 1/v for,

  $m_\phi/m_\chi < v < \alpha$.
- Saturates when $m_\phi/m_\chi \sim v$.
- Resonances occur at special values of $(m_\phi/m_\chi)/\alpha$; on these resonances the enhancement scales as $1/v^2$ and saturates later.
- Effect is small at freezeout ($v \sim 0.3$), large in the present-day Galactic halo ($v \sim 5*10^{-4}$).

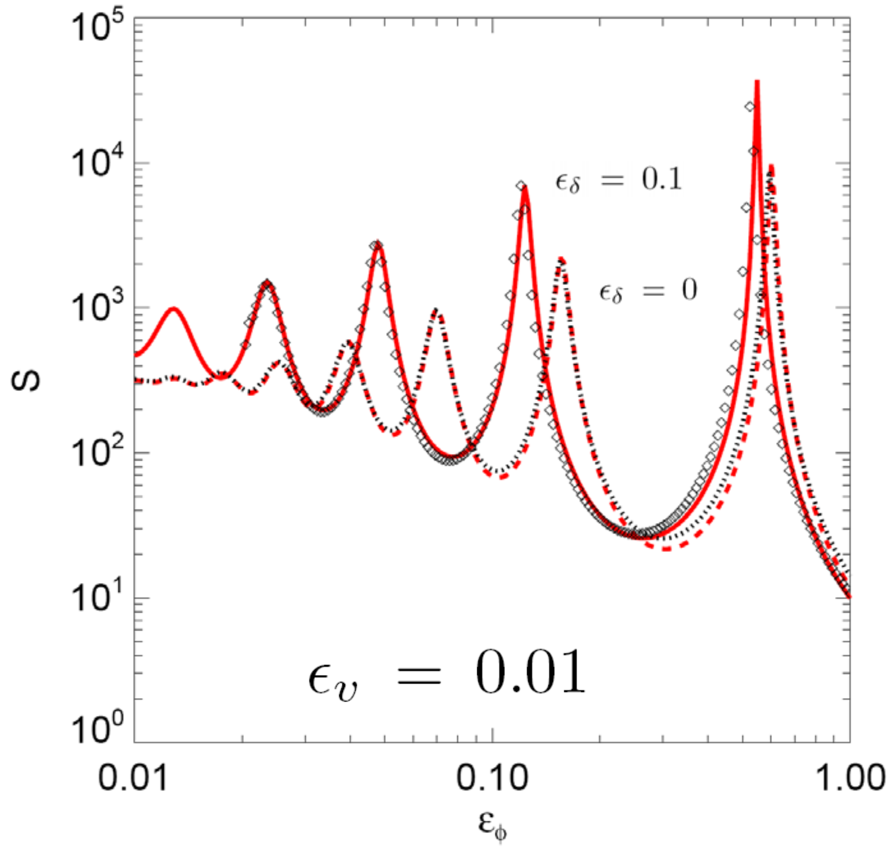


$\varepsilon_\phi = (m_\phi/m_\chi)/\alpha$, $\varepsilon_v = v/\alpha$
Contours are 10, 100, 1000.

The Sommerfeld enhancement for inelastic models



- Ladder diagrams for Sommerfeld enhancement now involve excited state, even if particles begin in ground state.
- Enhancement cuts off if $\delta > \alpha^2 m_\chi$ (potential energy of DM-DM system).
- However, if $\frac{1}{2} m_\chi v^2 < \delta < \alpha^2 m_\chi$, enhancement can actually be increased.
  - Resonances shift to lower m_ϕ.
  - Resonances increase in size (~4x).
  - Unsaturated, nonresonant enhancement increases by 2x.



Red lines: semi-analytic approximation taken from TRS 0910.5713.

Self-annihilation vs co-annihilation

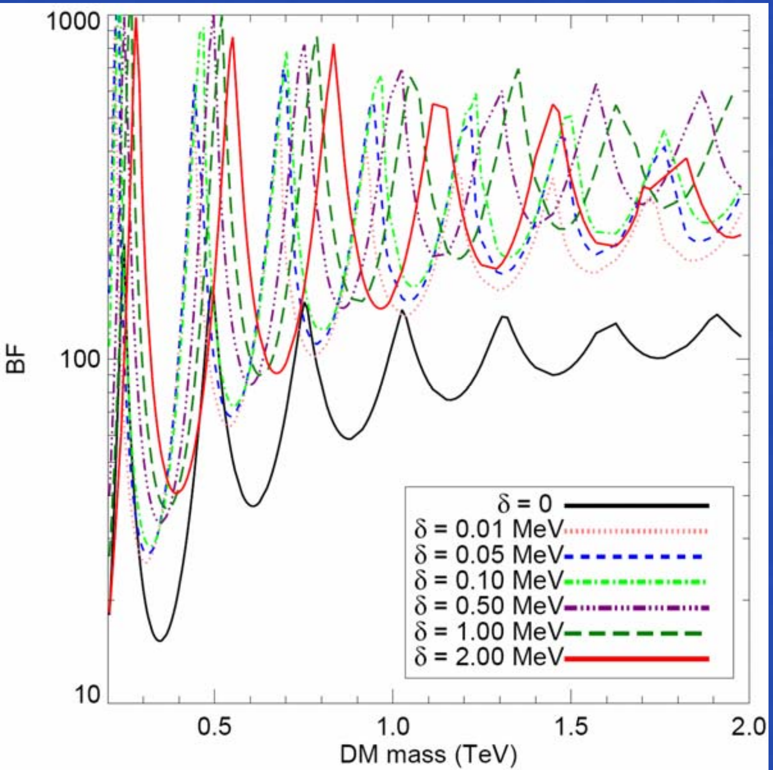
- Coannihilation and self-annihilation rates can (and generally will) differ; consequently, the rate depends on the relative population of ground and excited states, so differs in early universe (½ excited state) and present day (all ground state).
- Minimal model: the self-annihilation is stronger in s-wave (there is also a significant p-wave contribution to the self-annihilation for some parts of parameter space). Consequently, the DM annihilates more rapidly once the temperature drops below the mass splitting, independent of Sommerfeld enhancement.
- Parameterize this effect by κ, ratio of (s-wave) coannihilation to self-annihilation: if the s-wave terms dominate at freezeout, the ratio $\langle \sigma v \rangle_{present} / \langle \sigma v \rangle_{freezeout} = 2/(1+\kappa)$.
- Singly charged dark Higgs: $\kappa=1/4$, ratio = 8/5,
  doubly charged dark Higgs: $\kappa=1$, ratio = 1.

Sommerfeld enhancement and the thermal relic density

- In the presence of Sommerfeld enhancement, the standard relic density calculation (assuming $\langle \sigma v \rangle$ constant) is no longer completely correct; freezeout is delayed by rising $\langle \sigma v \rangle$, so the underlying annihilation cross section needs to be smaller (see e.g. Vogelsberger, Zavala and White 2009).
- We numerically solve the Boltzmann equation, taking Sommerfeld enhancement into account (in the two-state case, we need to solve coupled DEs for the ground- and excited-state populations, including upscattering, downscattering and decay of the excited state).
- The two-state case is more complicated, but the results are very similar to the zero-splitting case, since the relic density is largely determined by the enhancement around time of freezeout, where $T >> \delta$.
- Boost factors in the local halo where $T \sim \delta$, however, can change significantly.

Effects on local halo annihilation

- Define BF = present-day $\langle \sigma v \rangle$ / $3*10^{-26}$ cm^3/s.
- For several SM final states (m_ϕ held constant), compute BF as a function of m_χ, adjusting α_D to obtain correct relic density.



m_ϕ = 900 MeV

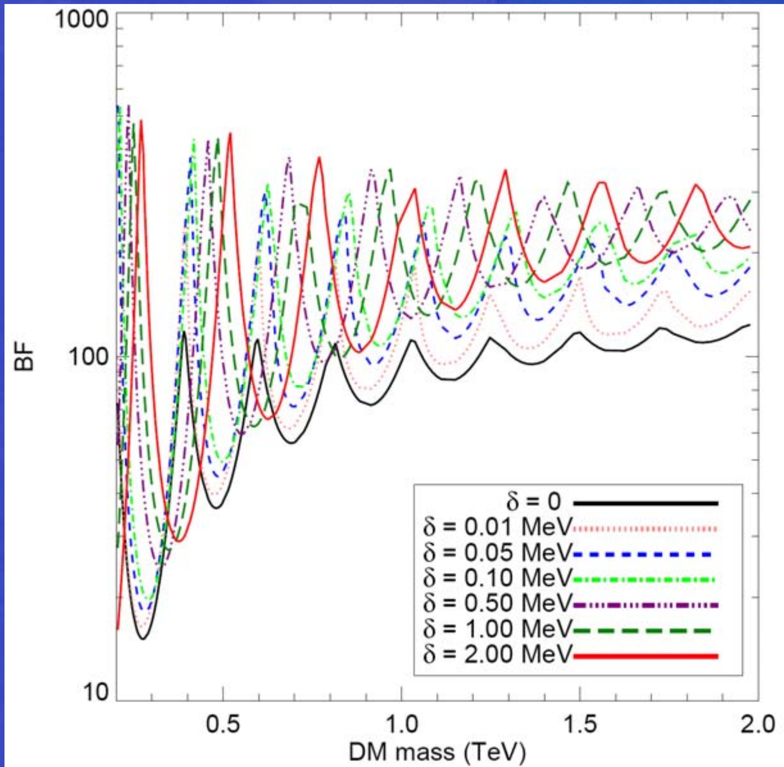
1:1:2 e:μ:π

κ = ¼

m_ϕ = 580 MeV

1:1:1 e:μ:π

κ = 1



Constraints from the cosmic microwave background

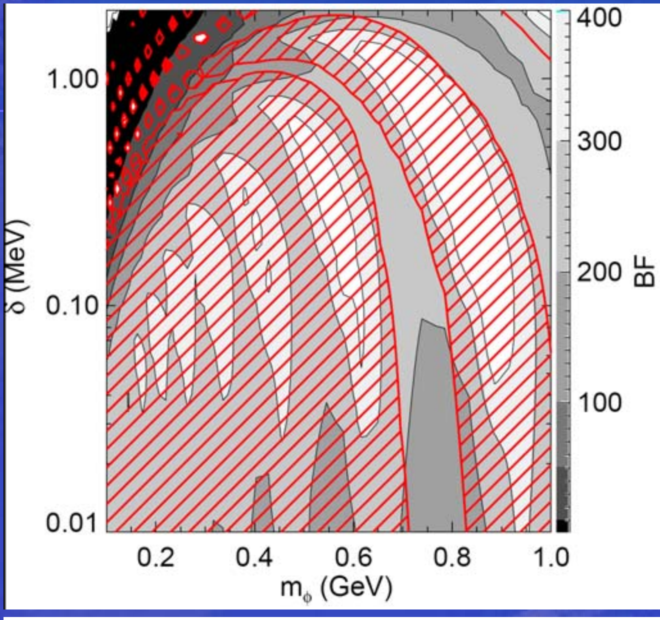
- High-energy electrons and photons injected around the redshift of last scattering give rise to a cascade of secondary photons and electrons, which modify the cosmic ionization history and hence the CMB.
- Robust constraints from WMAP5 require,

  $\langle \sigma v \rangle_{z\sim1000} < (120/f)\ (m_\chi/1\ \text{TeV})\ 3{*}10^{-26}\ \text{cm}^3/\text{s}$

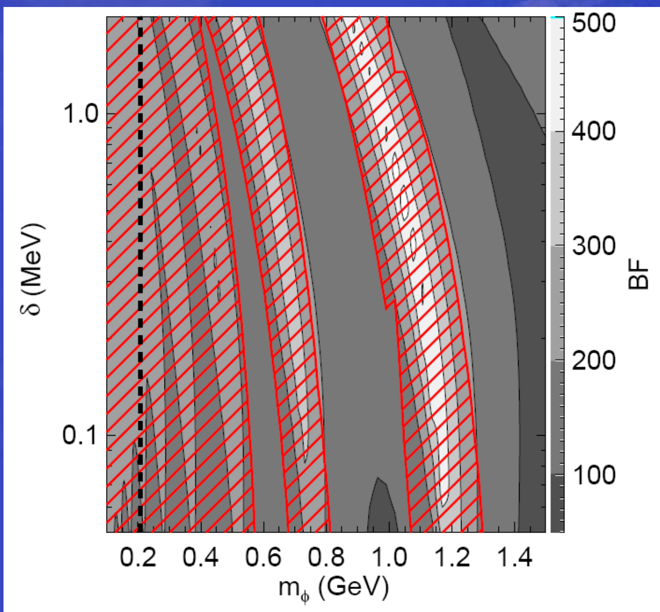
  f is an efficiency factor depending on the SM final state.

  e^+e^-: f=0.7, $\mu^+\mu^-$: f=0.24, $\pi^+\pi^-$: f=0.2

κ=1/4



κ=1



Example: effect of CMB constraints on parameter space for 1.2 TeV DM. Red-hatched = ruled out by CMB.

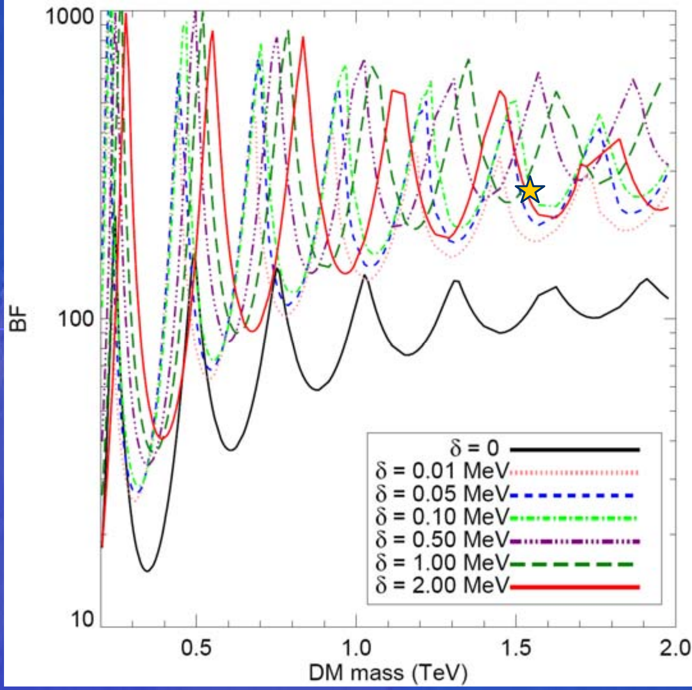
Example benchmark

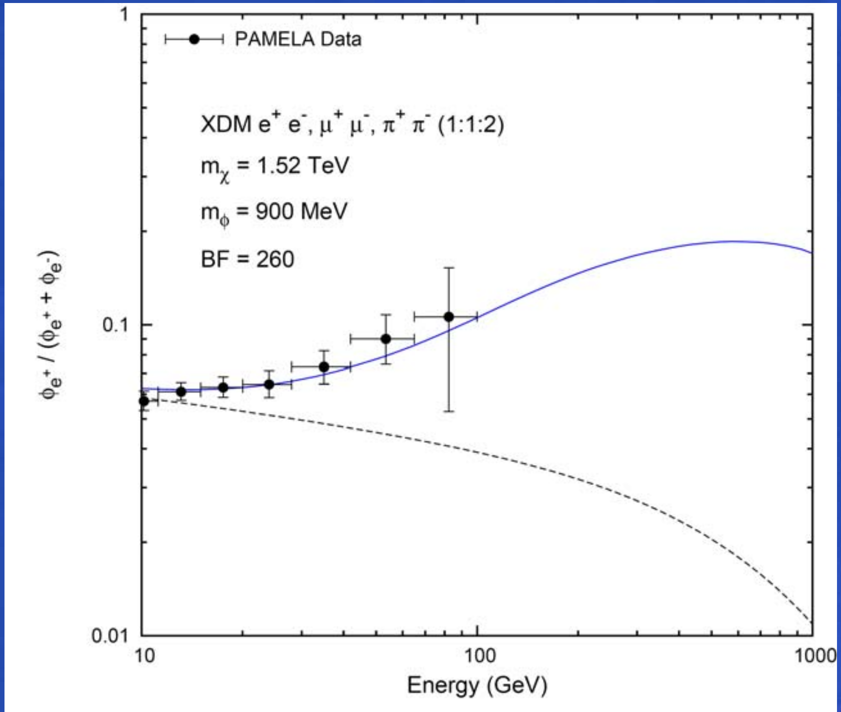
α=0.037 κ = ¼

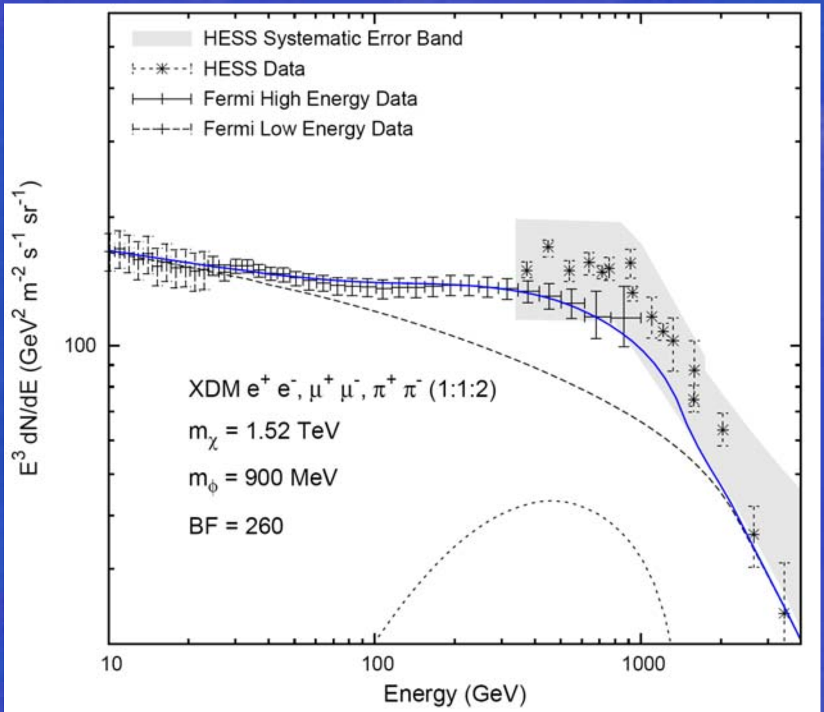
m_ϕ = 900 MeV m_χ = 1520 GeV δ = 1.1 MeV

Local BF = 260 Saturated BF = 365

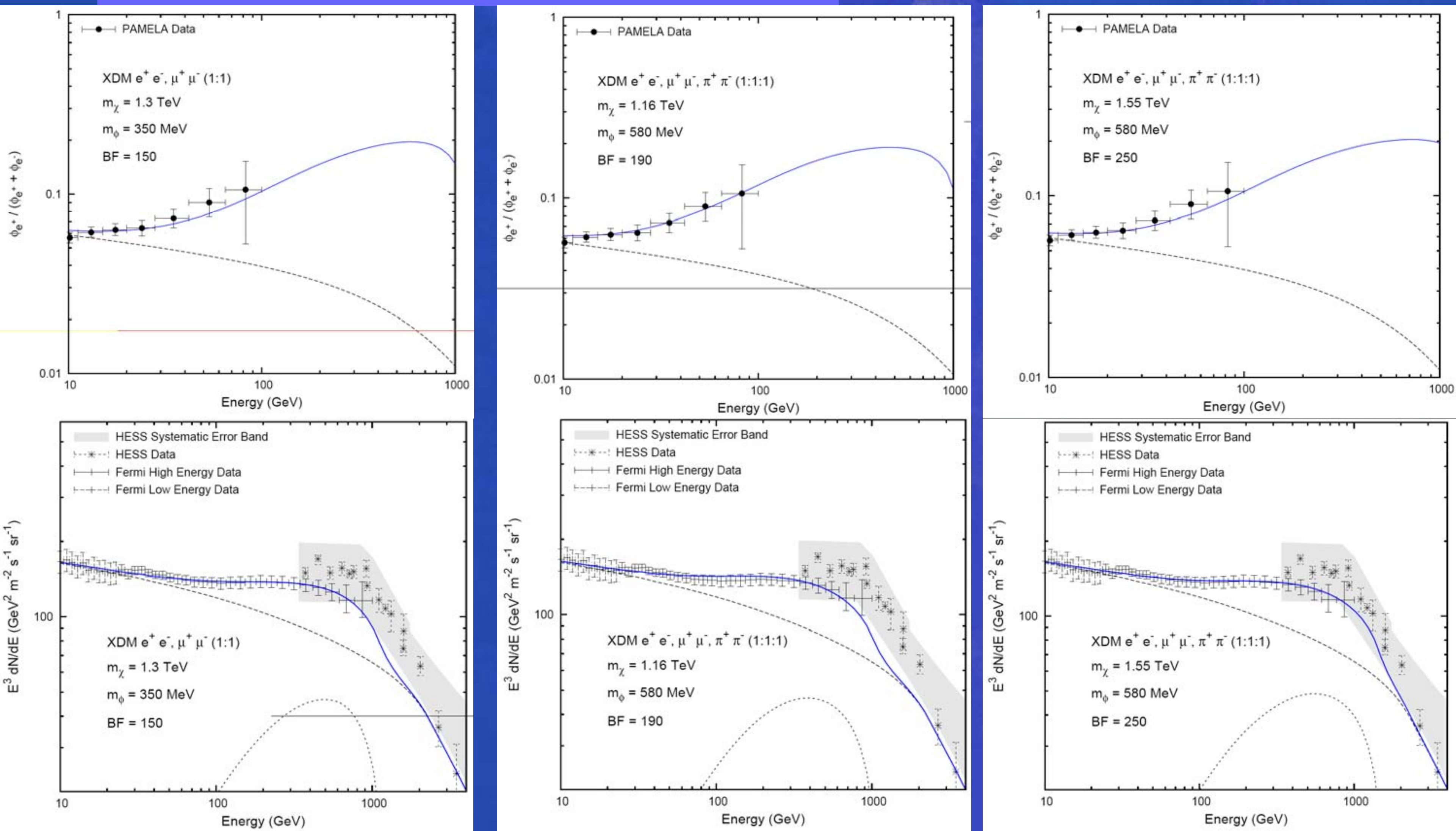
CMB limit = 545







More benchmarks at different mediator / DM masses



Conclusions

- Models of a light dark sector coupled to the Standard Model via kinetic mixing can fit the PAMELA/Fermi cosmic ray anomalies well, with required boost factors of order 100-300 and DM masses of 1-1.5 TeV, depending on the light gauge boson mass.

- These boost factors can be achieved by Sommerfeld enhancement alone, without violating constraints from the CMB, in models where the DM possesses a nearly-degenerate excited state and has the right thermal relic density, in contrast to recent claims in the literature for the elastic case.

- In purely elastic models, there is tension at the O(2) level for thermal relic DM, however, there are significant astrophysical uncertainties in the required enhancement.

BONUS SLIDES

The local dark matter density

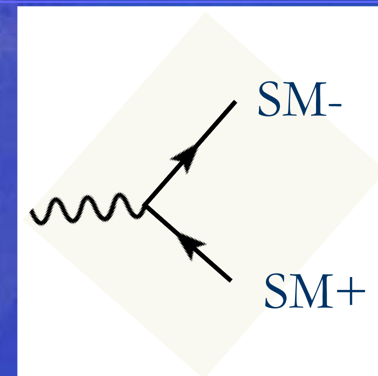
- 1980s: estimated at 0.3 GeV/cm^3, uncertain at factor of 2 level (see e.g. Gates, Gyuk and Turner 1995, and references therein).
- Recent studies:
  - Catena and Ullio (0907.0018), $\rho = 0.385 \pm 0.027$ GeV/cm^3 (Einasto profile, small modifications for other profiles).
  - Salucci et al (1003.3101), $\rho = 0.43(11)(10)$ GeV/cm^3 (no dependence on mass profile, does not rely on mass modeling of the Galaxy).
  - Pato et al (1006.1322), $\rho = 0.466 \pm 0.033(\text{stat}) \pm 0.077(\text{syst})$. Dynamical measurements assuming sphericity and ignoring presence of stellar disk systematically underestimate ρ by ~20%.

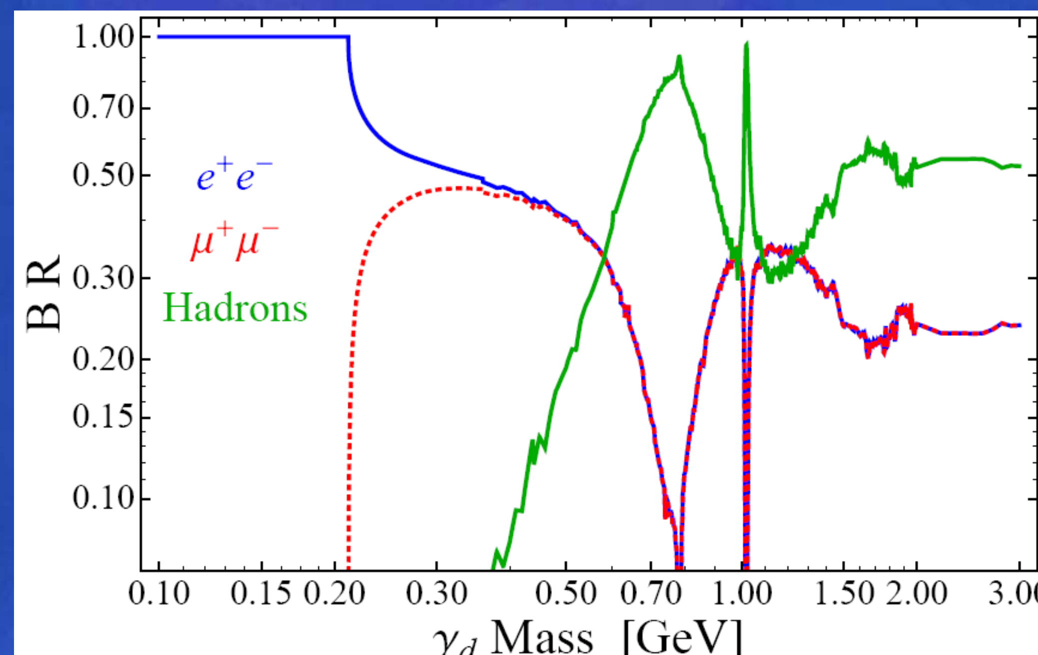
Increase in DM annihilation signal relative to $\rho = 0.3$ GeV/cm^3

1	1.6	2.1	2.5	3.4	4.4
0.30	0.38	0.43	0.47	0.55	0.63

Final SM states for DM annihilation

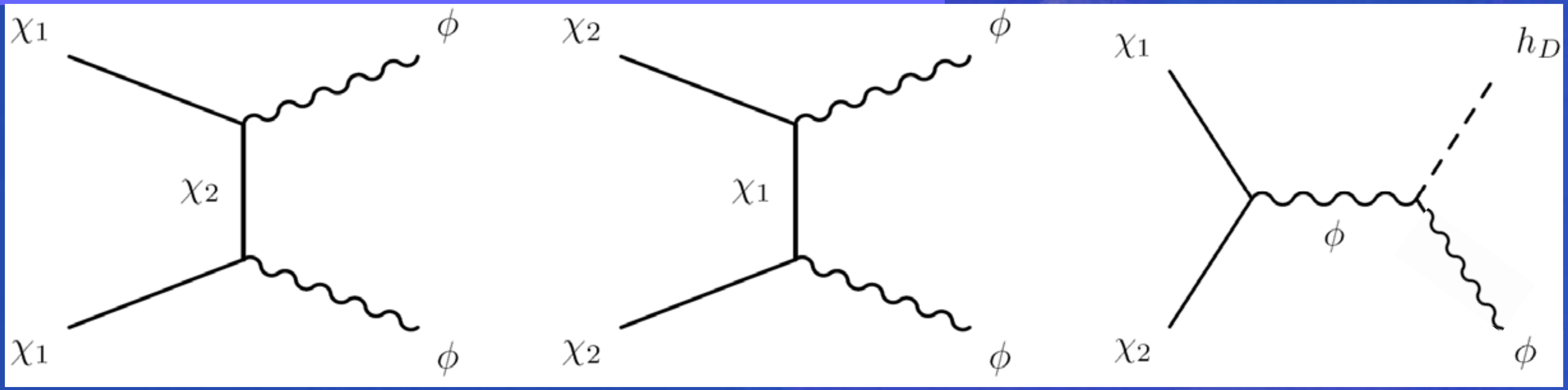
- If SM coupling is via kinetic mixing, dark gauge boson ϕ couples dominantly to charge: the coupling through the Z is suppressed by m_ϕ^4/m_Z^4.
- Thus the ϕ decays to kinematically accessible charged SM final states, depending on its mass.





Falkowski, Ruderman, Volansky and Zupan, 1002.2952

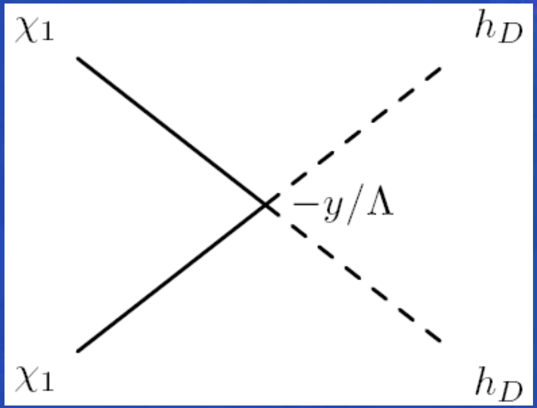
Annihilation channels in inelastic models

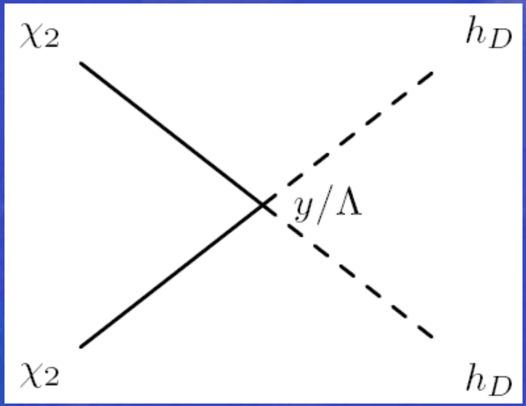


- $|11\rangle$ and $|22\rangle$ initial states: annihilate to 2ϕ, $(\sigma v_{rel})_{11} = (\sigma v_{rel})_{22} \approx \pi\alpha^2/m_\chi^2$.
  $|12\rangle$ initial state: annihilates to $\phi + h_D$, $(\sigma v_{rel})_{12} \approx \pi\alpha^2/4m_\chi^2$.
- Annihilation rate depends on relative population of ground and excited states, so differs in early universe (½ excited state) and present day (all ground state). If $(\sigma v_{rel})_{12} \approx \kappa\pi\alpha^2/m_\chi^2$, then the ratio is $2/(1+\kappa)$: in the "minimal" case of a singly charged dark Higgs, $\kappa = 1/4$, but more generally, there could be other dark-charged final states.

Annihilation from the mass splitting operator

- In this specific realization of this class of models, there is also a more model-dependent annihilation channel, from the operator generating the mass splitting,





- $(\sigma\, v)_{splitting} \sim S_{rep}\, v^2\, (m_\chi \delta / m_\phi^{\,2})^2\, (\sigma v_{rel})_{11}$
- Highly velocity suppressed (p-wave, + Sommerfeld effect *suppresses* annihilation), negligible in present day – but can be important, even dominant, at freezeout, especially for large δ + small m_ϕ.

Inelastic dark matter (iDM)

- Suppose some higher-dimension operator (e.g. of the form $\chi\chi h_D^* h_D^* / \Lambda$) gives the DM a small Majorana mass. Working in two-component notation, the mass matrix becomes,

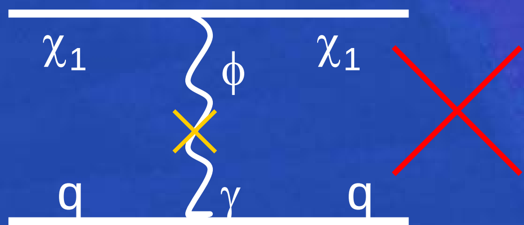
$$\mathcal{M} = \begin{pmatrix} m_M & m_\chi \\ m_\chi & m_M \end{pmatrix} \longrightarrow \begin{pmatrix} m_\chi + \delta/2 & 0 \\ 0 & -(m_\chi - \delta/2) \end{pmatrix}$$

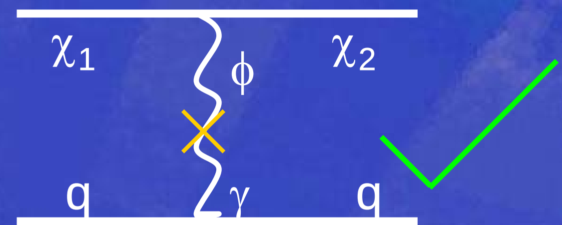
45 degree rotation $\qquad \delta = 2m_M$

- The generic scale of the splitting is, $\sim \langle h_D \rangle^2 / \Lambda \sim \mathrm{GeV}^2 / \mathrm{TeV} \sim \mathrm{MeV}$.
- The resulting split mass eigenstates have purely off-diagonal couplings to the gauge boson ϕ.

iDM in direct detection

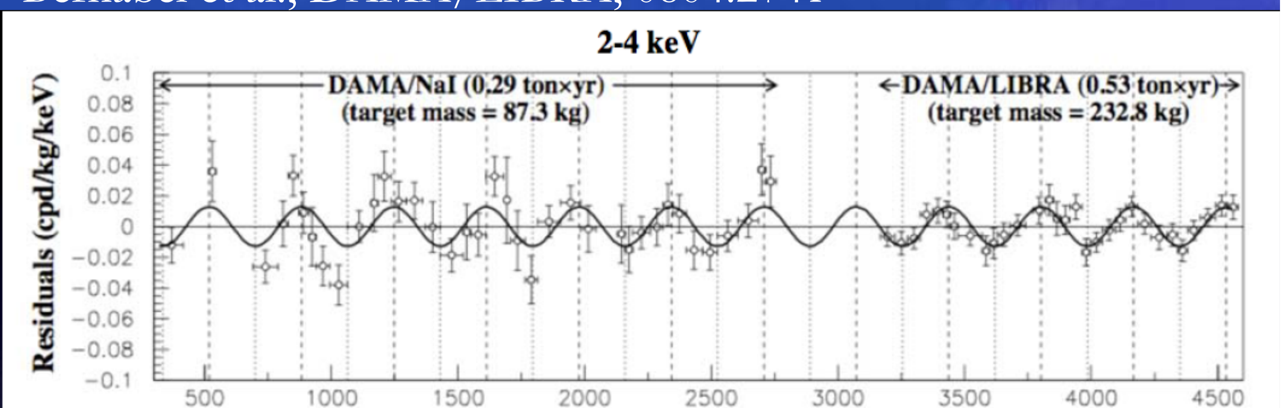
- If δ >> 100 keV (typical kinetic energy of local halo DM), direct detection signal is very small due to kinematics.





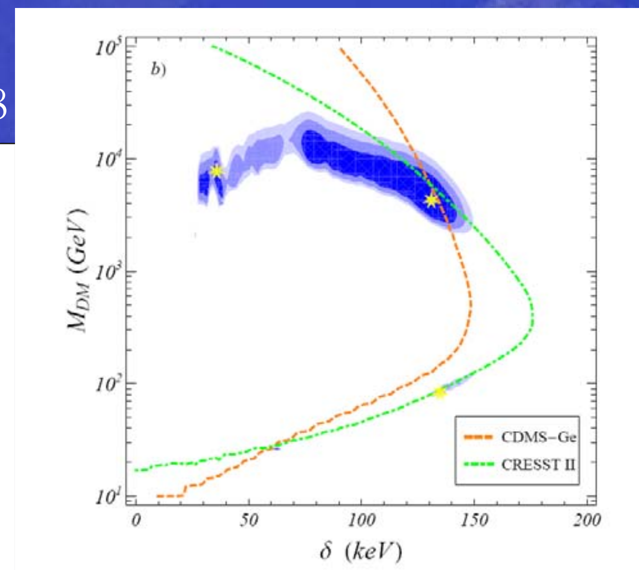
- If δ ~ 100 keV, possible to reconcile DAMA/LIBRA modulation with null results of other experiments.

Bernabei et al., DAMA/LIBRA, 0804.2741



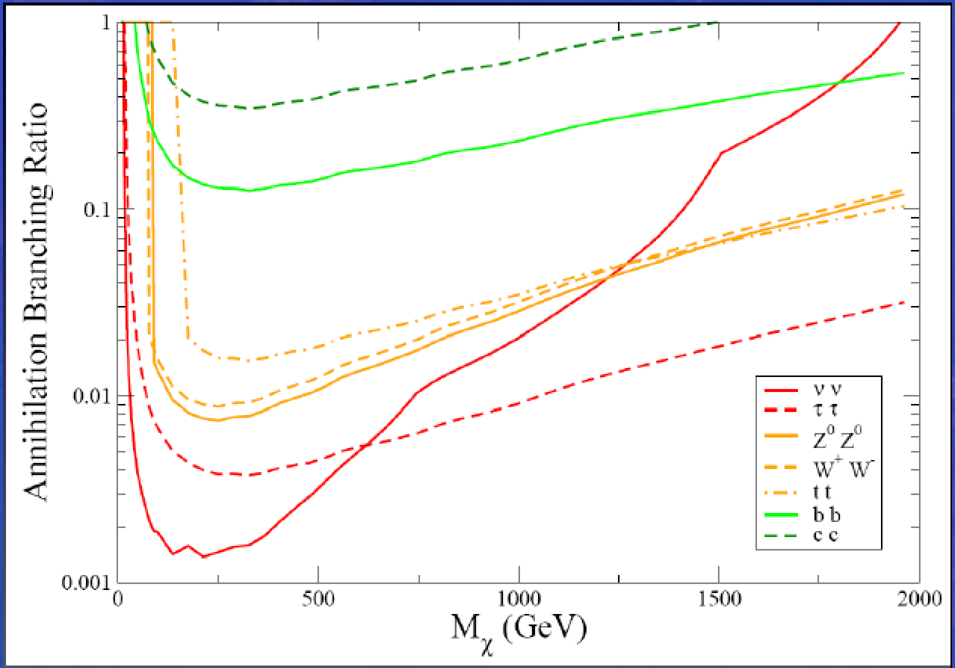
Bernabei et al., DAMA/LIBRA, 0804.2741

Ling et al, 0909.2028



iDM in indirect detection

- In iDM models that explain the DAMA/LIBRA anomaly, strong constraints from bounds on neutrinos, from DM capture + annihilation in the Sun.
- Light SM final states (electrons, muons, pions, kaons) evade these bound, so models with large annihilation branching ratios into light states are favored – leads us back toward PAMELA/Fermi cosmic ray signals!



Bounds on the branching ratio to various SM final states from SuperK

Nussinov et al 0905.1333

DM excited states in indirect detection

- At slightly larger mass splittings, ~1 MeV rather than ~100 keV, iDM-style models can explain the 511 keV excess from the inner Galaxy, observed by the INTEGRAL spectrometer.
- Spectral shape implies positrons injected at low energy – not from TeV-scale WIMP annihilation.
- Collisional excitation of DM excited state, followed by decay to ground state producing e^+e^- pair, can explain signal, if mass splitting is slightly larger than 2 m_e.

G. Weidenspointner et al.: The sky distribution of positronium continuum emission

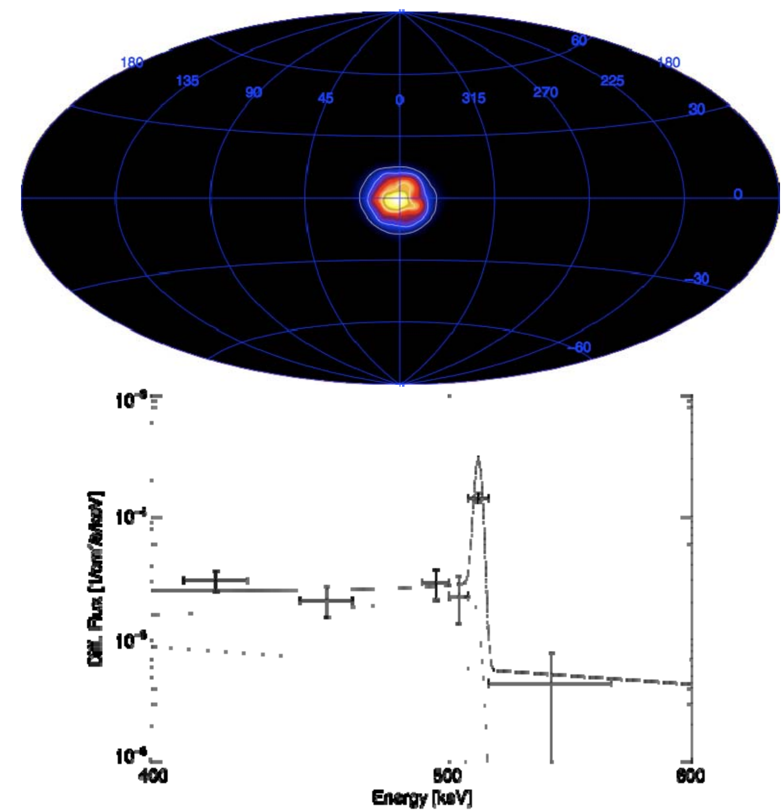


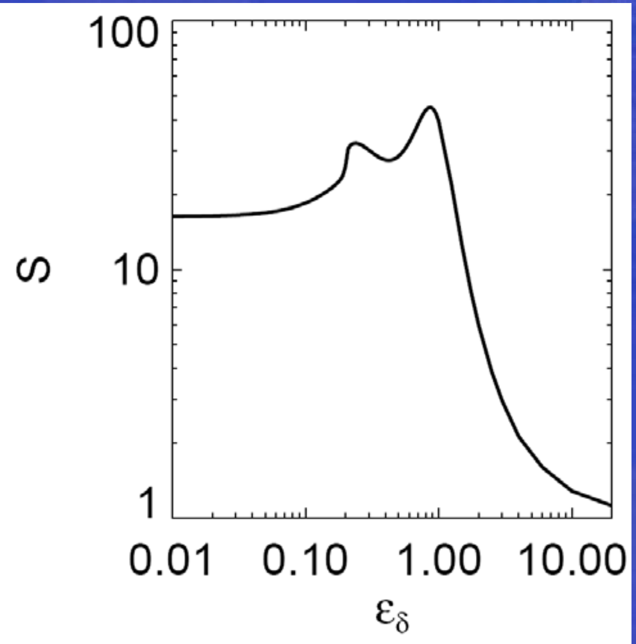
Fig. 2. A fit of the SPI result for the diffuse emission from the GC region ($|l|, |b| \leq 16°$) obtained with a spatial model consisting of an 8° *FWHM* Gaussian bulge and a CO disk. In the fit a diagonal response was assumed. The spectral components are: 511 keV line (dotted), Ps continuum (dashes), and power-law continuum (dash-dots). The summed models are indicated by the solid line. Details of the fitting procedure are given in the text.

Weidenspointner et al 06

How does inelasticity affect the Sommerfeld enhancement?

- Pure off-diagonal interaction: $|11\rangle$ and $|22\rangle$ states couple to each other, not to $|12\rangle$.

- Initial question: does the Sommerfeld enhancement turn off when kinetic energy << mass splitting?
- NO, however, it does cut off if the kinetic energy + potential energy $\sim \alpha^2 m_\chi << \delta$.



Solving for the Sommerfeld enhancement in a two-state system

- Need to solve Schrodinger equation with 2*2 matrix potential (corresponding to $|11\rangle$ and $|22\rangle$ basis states; $|12\rangle$ state is decoupled) for distortion of scattering-solution wavefunction near origin. Treat annihilation as contact interaction.

$$\psi''(r) = \begin{pmatrix} \frac{l(l+1)}{r^2} - \epsilon_v^2 & -\frac{e^{-\epsilon_\phi r}}{r} \\ -\frac{e^{-\epsilon_\phi r}}{r} & \frac{l(l+1)}{r^2} + \epsilon_\delta^2 - \epsilon_v^2 \end{pmatrix} \psi(r)$$

$$\epsilon_v = (v/c)/\alpha, \; \epsilon_\delta = \sqrt{2\delta/m_\chi}/\alpha, \; \epsilon_\phi = (m_\phi/m_\chi)/\alpha$$

- Work in dimensionless parameters, focus on s-wave case.
- 3D parameter space with sharp resonances + severe numerical instabilities in some regions of interest => parameter scans are computationally difficult.

A simple semi-analytic approximation

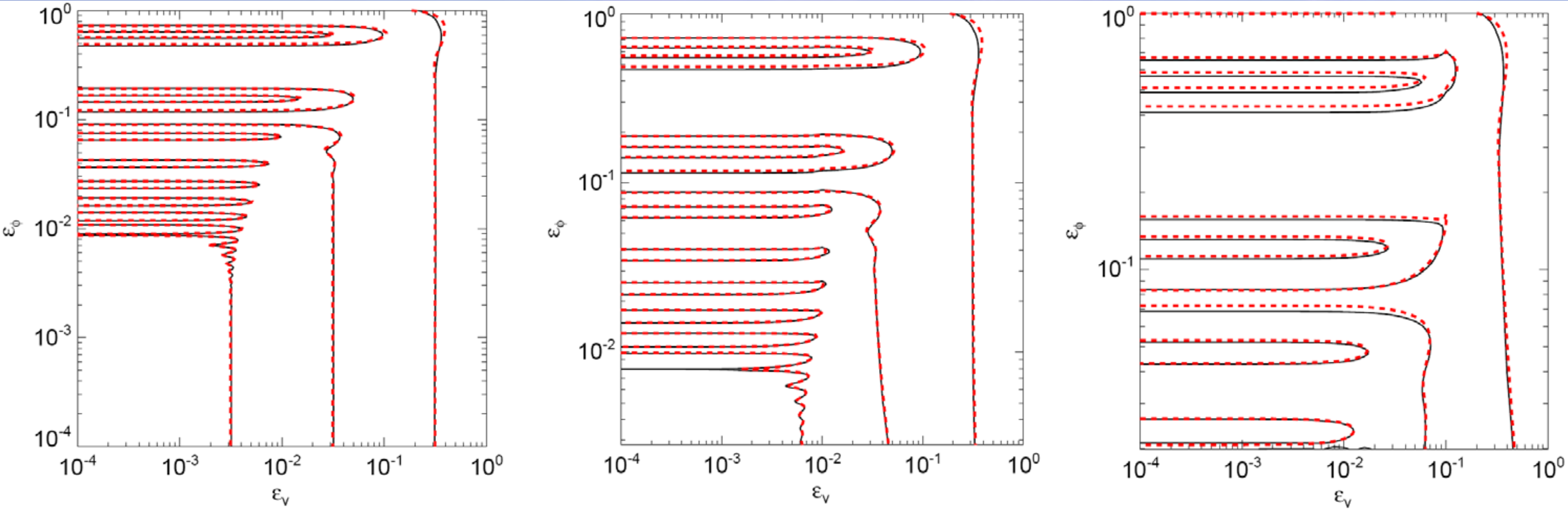
- For particles in the ground state:

$$S = \frac{2\pi}{\epsilon_v} \sinh\left(\frac{\epsilon_v \pi}{\mu}\right) \begin{cases} \frac{1}{\cosh(\epsilon_v \pi/\mu) - \cos\left(\sqrt{\epsilon_\delta^2 - \epsilon_v^2}\,\pi/\mu + 2\theta_-\right)} & \epsilon_v < \epsilon_\delta, \\ \frac{\cosh\left(\left(\epsilon_v + \sqrt{-\epsilon_\delta^2 + \epsilon_v^2}\right)\pi/2\mu\right) \operatorname{sech}\left(\left(\epsilon_v - \sqrt{-\epsilon_\delta^2 + \epsilon_v^2}\right)\pi/2\mu\right)}{\cosh\left(\left(\epsilon_v + \sqrt{-\epsilon_\delta^2 + \epsilon_v^2}\right)\pi/\mu\right) - \cos(2\theta_-)} & \epsilon_v > \epsilon_\delta. \end{cases}$$

- The angle θ_- controls the resonance locations and is given by a numerical integration, which is stable and fast to compute. The parameter μ is an analytic function of ε_ϕ and ε_δ, but generally satisfies $\mu \sim \varepsilon_\phi$.
- This approximation assumes the conditions required for large enhancement: ε_v, ε_δ, $\varepsilon_\phi < 1$. This result may also be less accurate when $\delta > \alpha\, m_\phi$.

Derived using the WKB approximation and an exact solution for a two-state system with exponential potential: details in TRS 0910.5713.

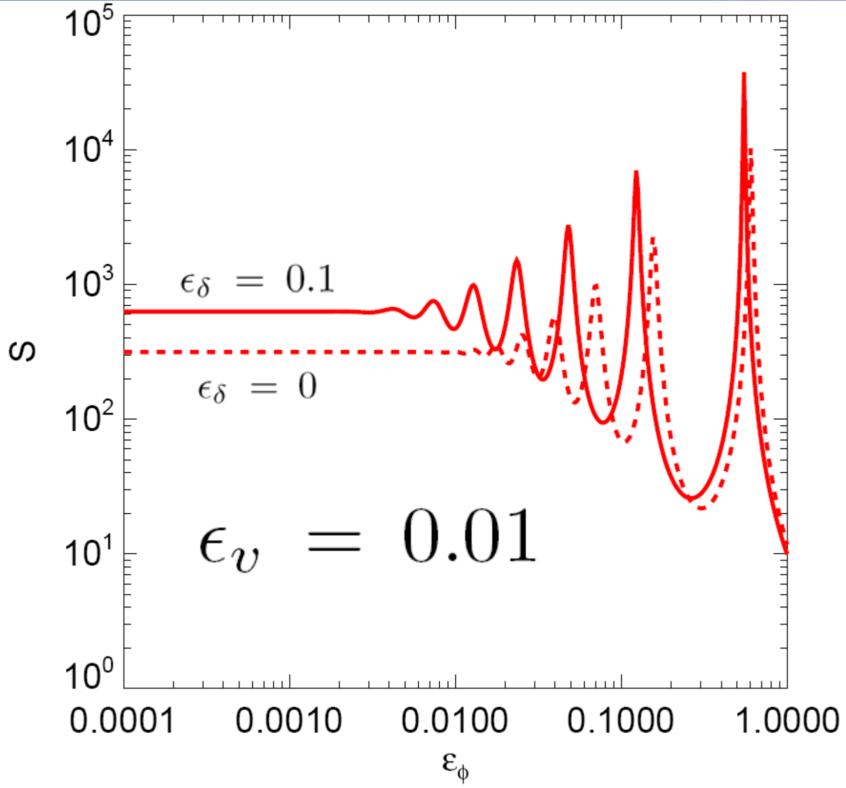
Tests of the semi-analytic solution

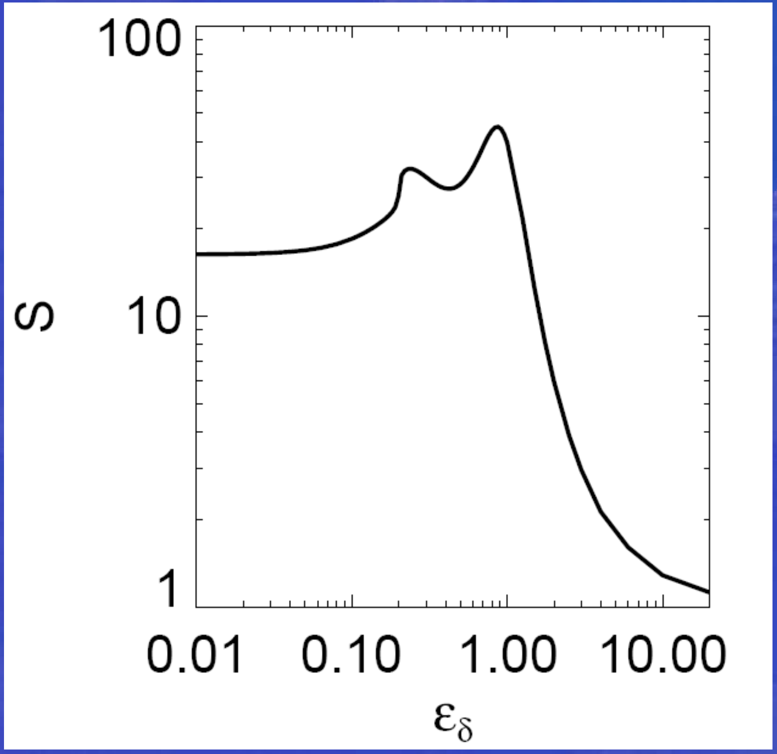


- Black lines = numerical result, red dotted lines = approximate solution. Contours at 10, 100, 1000.
- ε_δ = (left) 0, (middle) 0.01, (right) 0.1.

The $2\pi\alpha/v$ non-saturated enhancement

- When $\varepsilon_v < \varepsilon_\delta$, or $\varepsilon_v > \varepsilon_\delta >> (\mu\varepsilon_v)^{0.5}$, the non-resonant, unsaturated enhancement is given by $2\pi\alpha/v$ instead of $\pi\alpha/v$.





$\epsilon_v = 0.2, \ \epsilon_\phi = 0.1$

Why the factor of 2?

- This can be understood in the quantum mechanics picture, in terms of the evolution of the eigenstates with r.
- In the adiabatic / large δ limit, a state initially in the lower-energy eigenstate at infinity (ground state) will smoothly transform into the lower-energy eigenstate at small r, which experiences an attractive potential.
- In the diabatic / small δ limit, the small-r state corresponding to either asymptotic eigenstate will be an even mixture of attracted and repulsed components (i.e. lower- and higher-energy eigenstates).

r → ∞

$$\psi_+ \approx \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \psi_- \approx \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\lambda_+ \approx \frac{l(l+1)}{r^2} - \epsilon_v^2 + \epsilon_\delta^2 + \frac{V(r)^2}{\epsilon_\delta^2}$$

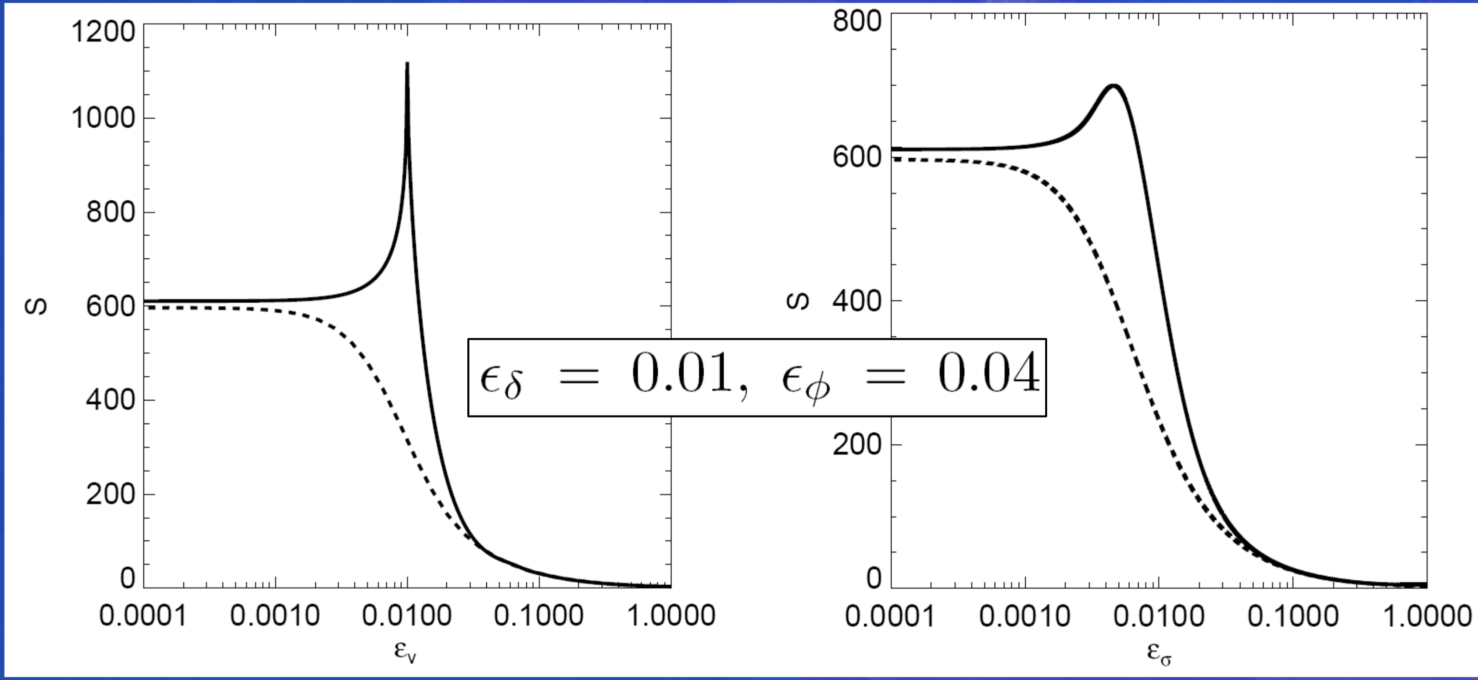
$$\lambda_- \approx \frac{l(l+1)}{r^2} - \epsilon_v^2 - \frac{V(r)^2}{\epsilon_\delta^2}$$

r → 0

$$\psi_\pm \approx \frac{1}{\sqrt{2}} \begin{pmatrix} \mp 1 \\ 1 \end{pmatrix}$$

$$\lambda_\pm \approx \frac{l(l+1)}{r^2} - \epsilon_v^2 + \frac{\epsilon_\delta^2}{2} \pm V(r)$$

Behavior near excitation threshold



- Enhancement at excitation threshold relative to saturated value can be up to a factor of 2 (in fine-tuned regions).
- Smoothing by velocity distribution of particles (MB distribution in right panel) will reduce this factor further.

Constraints on Sommerfeld models

- Summary of CMB constraint:
  $\langle \sigma v \rangle_{v \to 0} < (120/f)\ 3*10^{-26}\ cm^3/s$, i.e.
  $BF_{saturated} < 120/f$
- f determined by branching fractions to SM final states. We employ,
  e^+e^-: f = 0.7 $\mu^+\mu^-$: f=0.24 $\pi^+\pi^-$: f=0.2
- As previously, we choose the coupling α_D to obtain the correct relic density, but now also calculate the saturated boost factor and require it to obey these bounds (for the mixture of $e/\mu/\pi$ final states relevant to the chosen m_ϕ).

Limits on models fitting PAMELA/Fermi