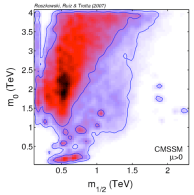
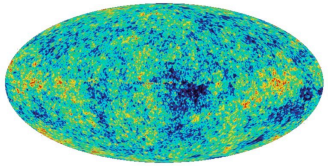
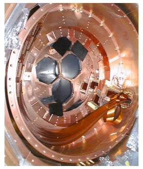
TeV Particle Astrophysics Meeting - Paris, July 2010





# Prospects for DM detection from global fits of supersymmetry parameter space

Roberto Trotta
Imperial College London, Astrophysics

Thanks to Oliver Buchmuller for providing some of the plots

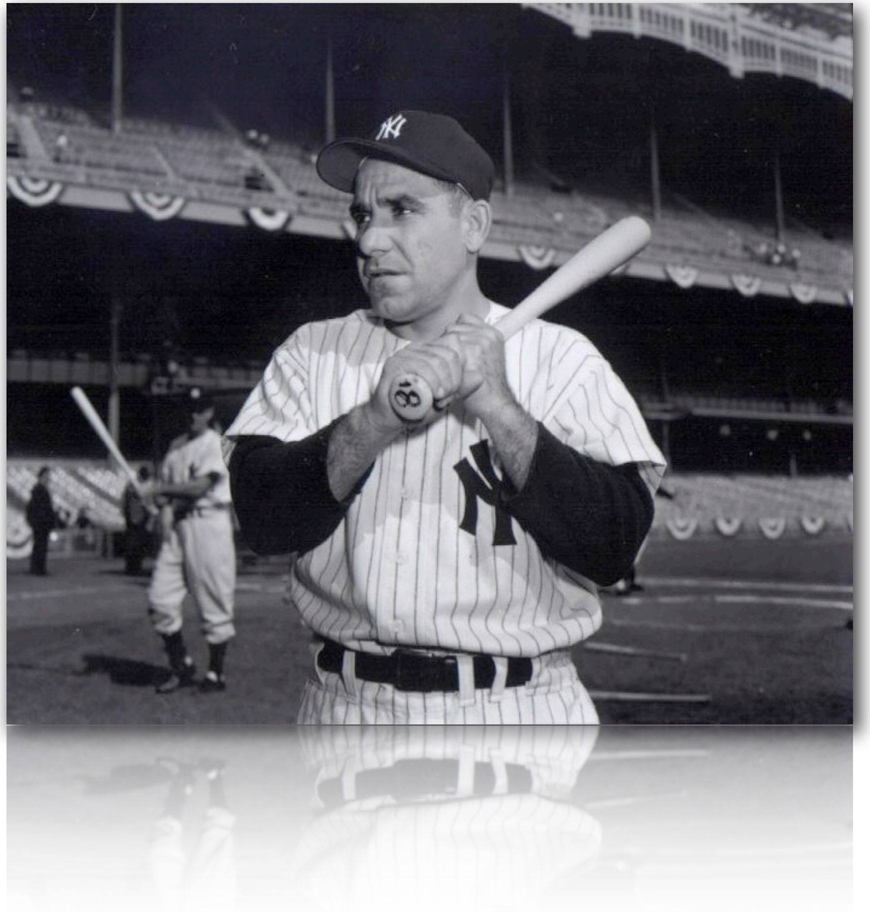
Imperial College London


1.
the need for
global scans
2.
where do we
stand today?
3.
prospects for the
LHC and direct
detection

A word about statistics:

*90% of the game is half mental.*

Yogi Berra



# Exploration with "random scans"

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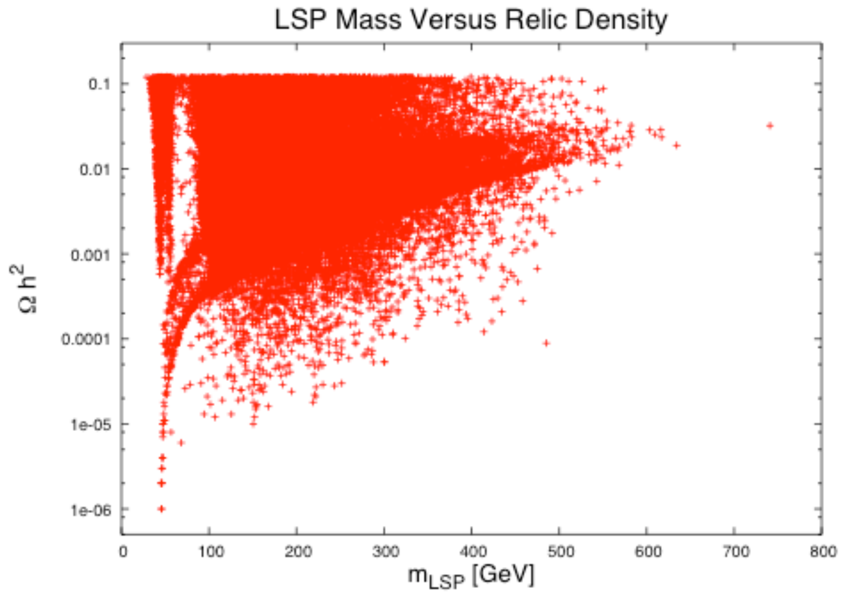
C. F. B
Hewett
Supers
Prejudi
[arXiv

check t
pMSSM

- Points accepted/rejected in a in/out fashion (e.g., 2-sigma cuts)
- No statistical measure attached to density of points: no probabilistic interpretation of results possible, although the temptation cannot be resisted...
- Inefficient in high dimensional parameters spaces (D>5)
- **HIDDEN PROBLEM:** Random scan explore only a very limited portion of the parameter space!

One recent example:
Berger et al (0812.0980)
pMSSM scans
(20 dimensions)



# Random scans explore only a small fraction of the parameter space

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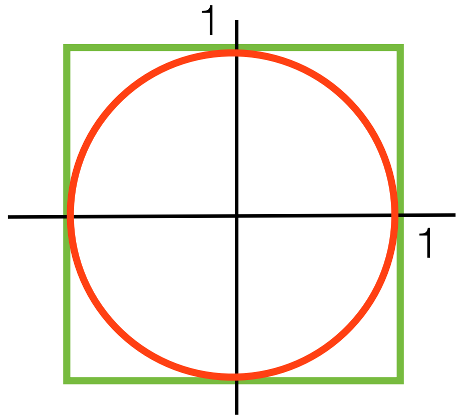
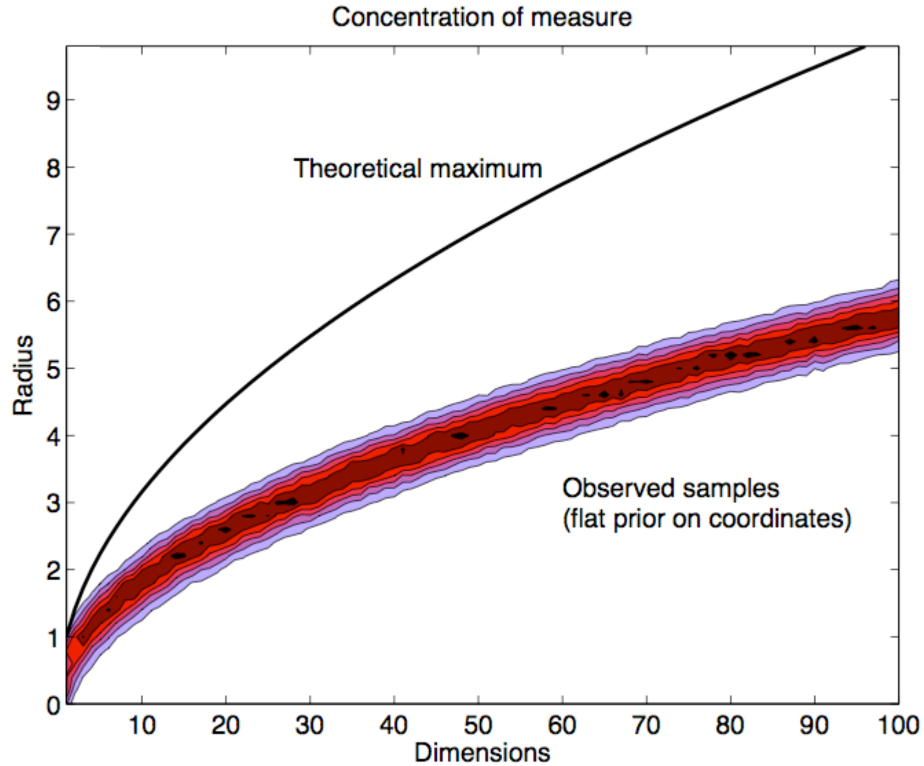
- "Random scans" of a high-dimensional parameter space only probe a very limited sub-volume: this is **the concentration of measure phenomenon.**

- **Statistical fact:** the norm of *D* draws from U[0,1] concentrates around $(D/3)^{1/2}$ with constant variance



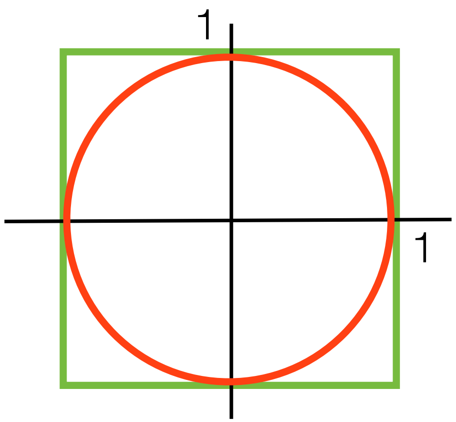



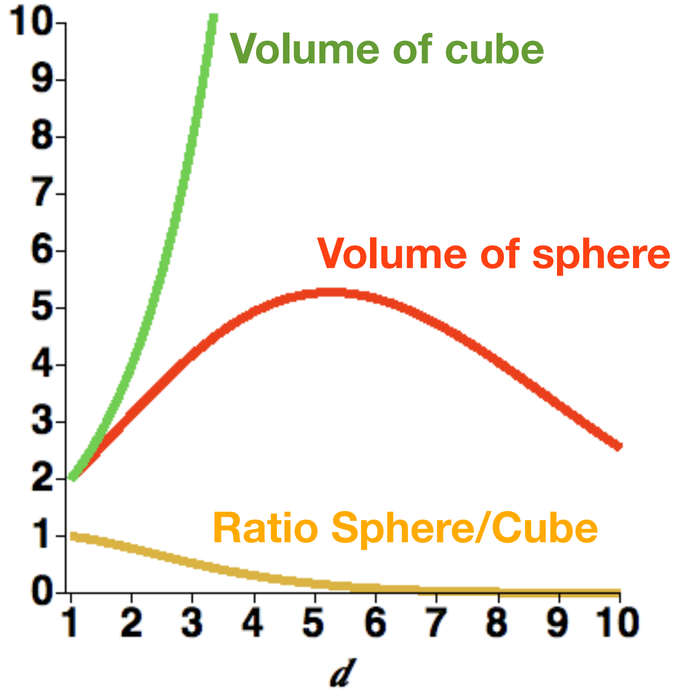
# Geometry in high-D spaces



- **Geometrical fact:** in *D* dimensions, most of the volume is near the boundary. The volume inside the spherical core of *D*-dimensional cube is negligible.

**Together, these two facts mean that random scan only explore a very small fraction of the available parameter space in high-dimesional models.**



# 2D scans

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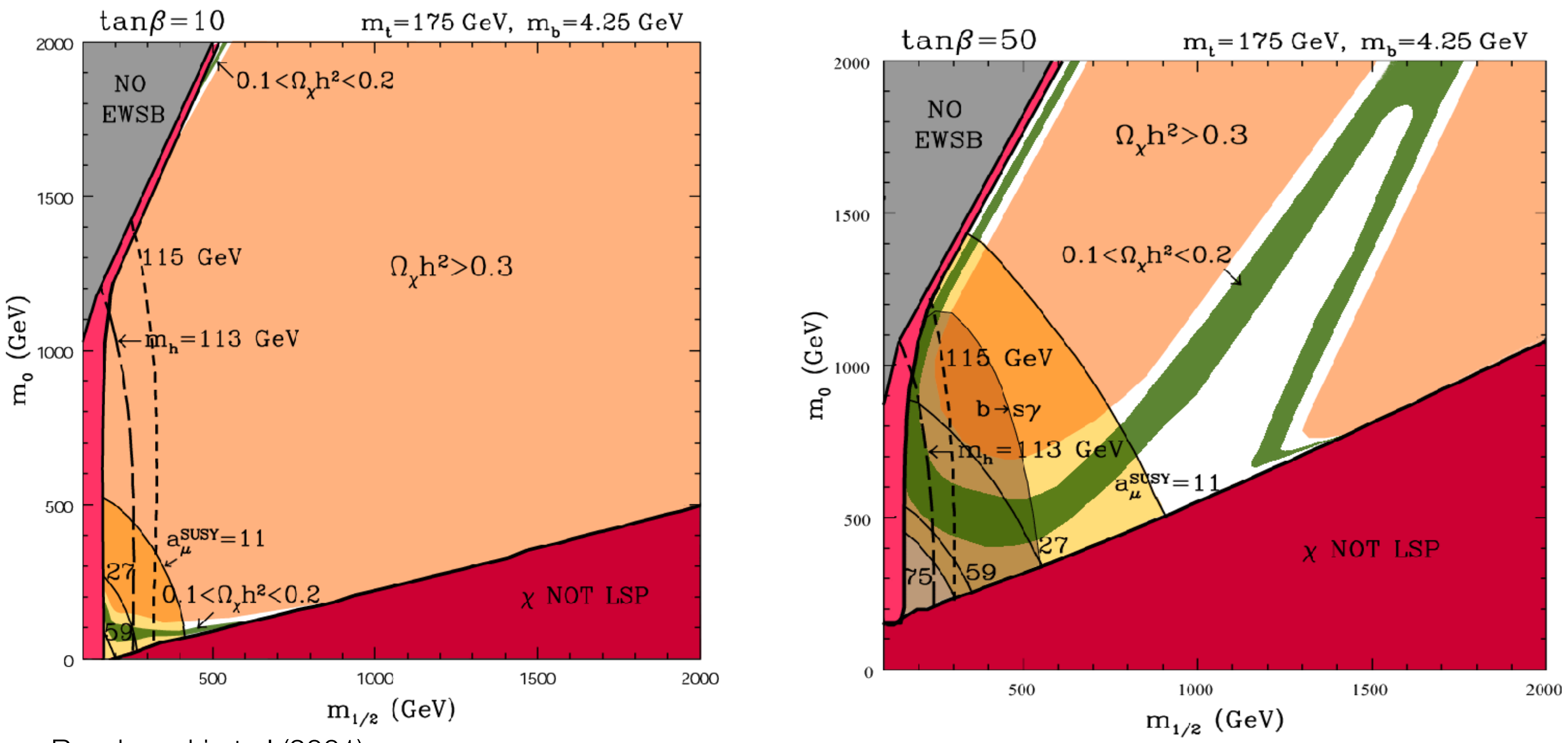
Determining constraints on SUSY models is a multi-dimensional problem. Even in one of the simplest cases, the CMSSM, there are four 4 parameters ($M_0$, $M_{1/2}$, $A_0$, $\tan\beta$) as well as SM parameters (e.g. $M_{top}$, $M_b$) The traditional strategy in the field was to carry out "2D scans" by fixing the other relevant parameters to certain values.




Roszkowski et al (2001)

# Dependency on SM (nuisance) parameters

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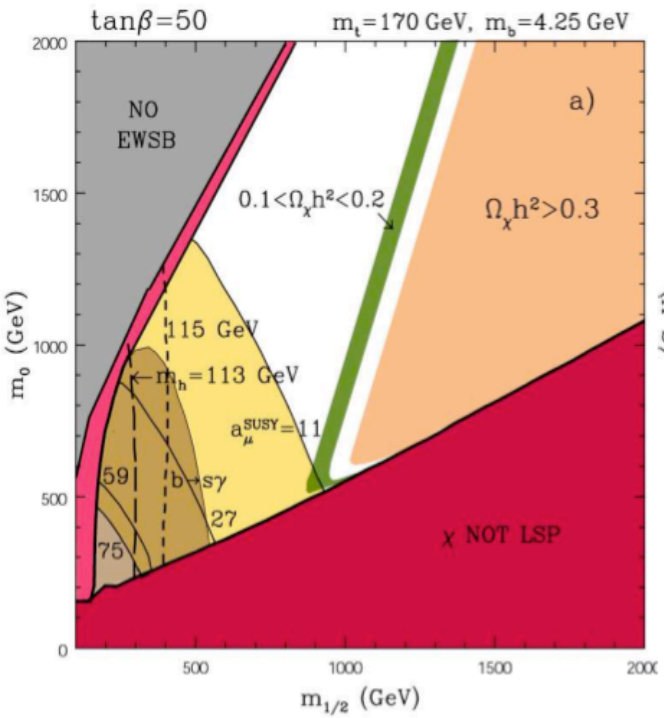
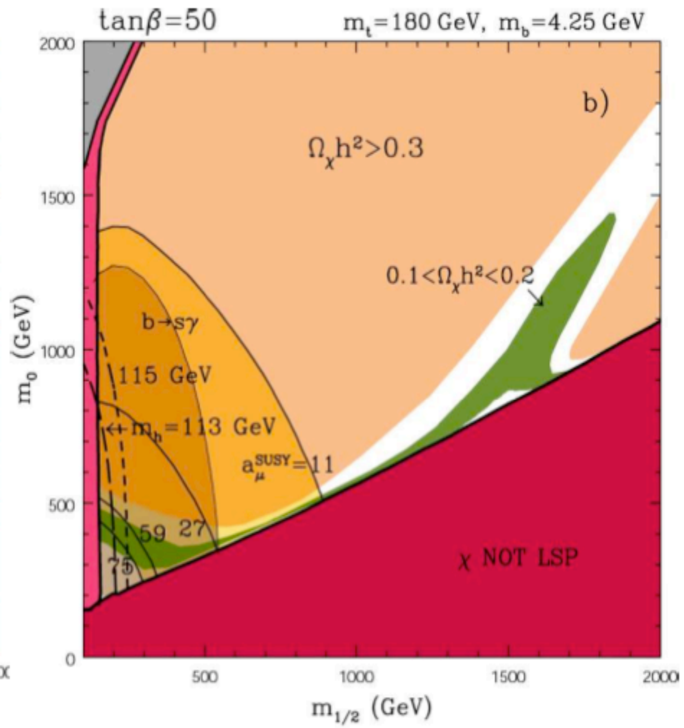
$M_{top}$=170 GeV




$M_{top}$=180 GeV

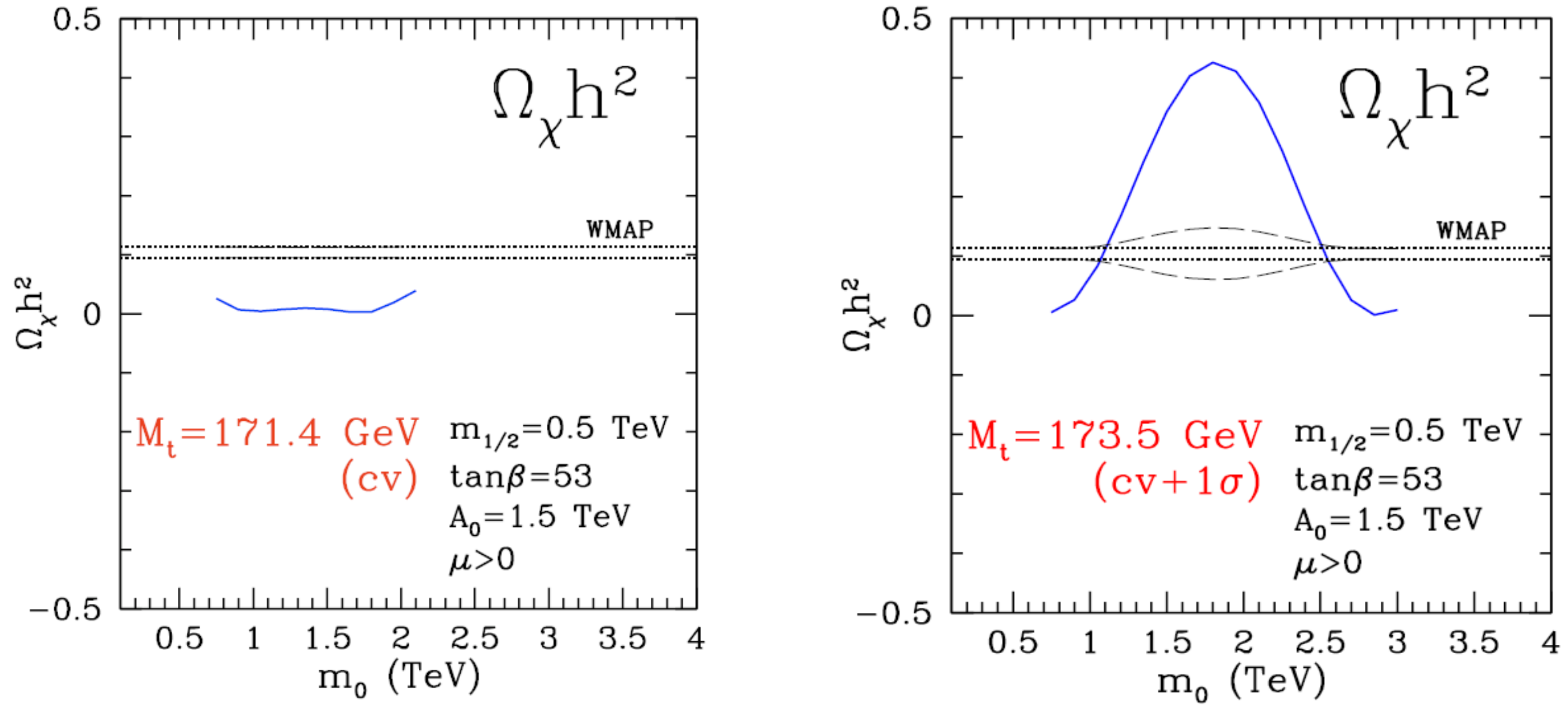



There is also a strong dependence on the important SM parameters!
(which are known only with limited accuracy)

# Impact of top mass on the relic abundance



Changing $M_{top}$ within $\pm 1\sigma$ has dramatic consequences for the predicted relic abundance: this parameter cannot be fixed to its central value.




Roszkowski et al (2007)

# Solution: global fits

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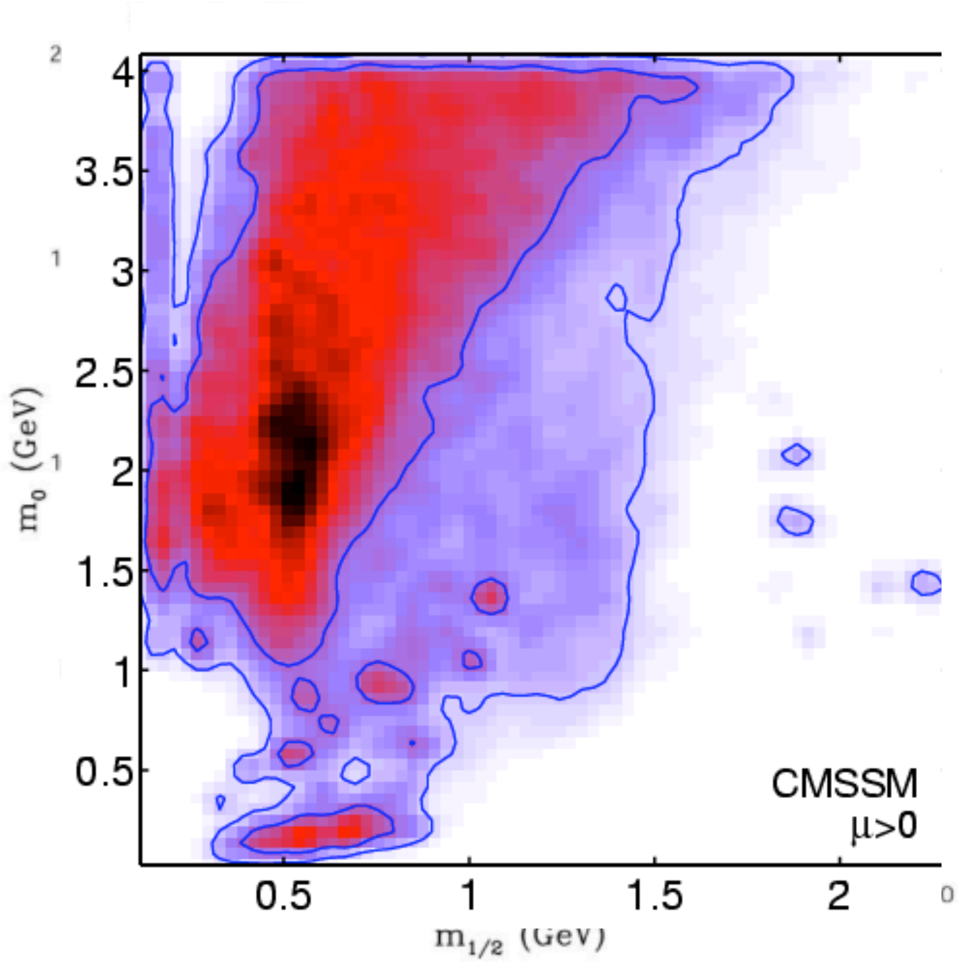
Carry out a **simultaneous fit** of all relevant SUSY and SM parameter to the experimental data/constraints.

**Marginalize (= integrate) or maximise along the hidden dimensions** to obtain results that account for the multi-dimensional nature of the problem.



# The “WMAP strips”

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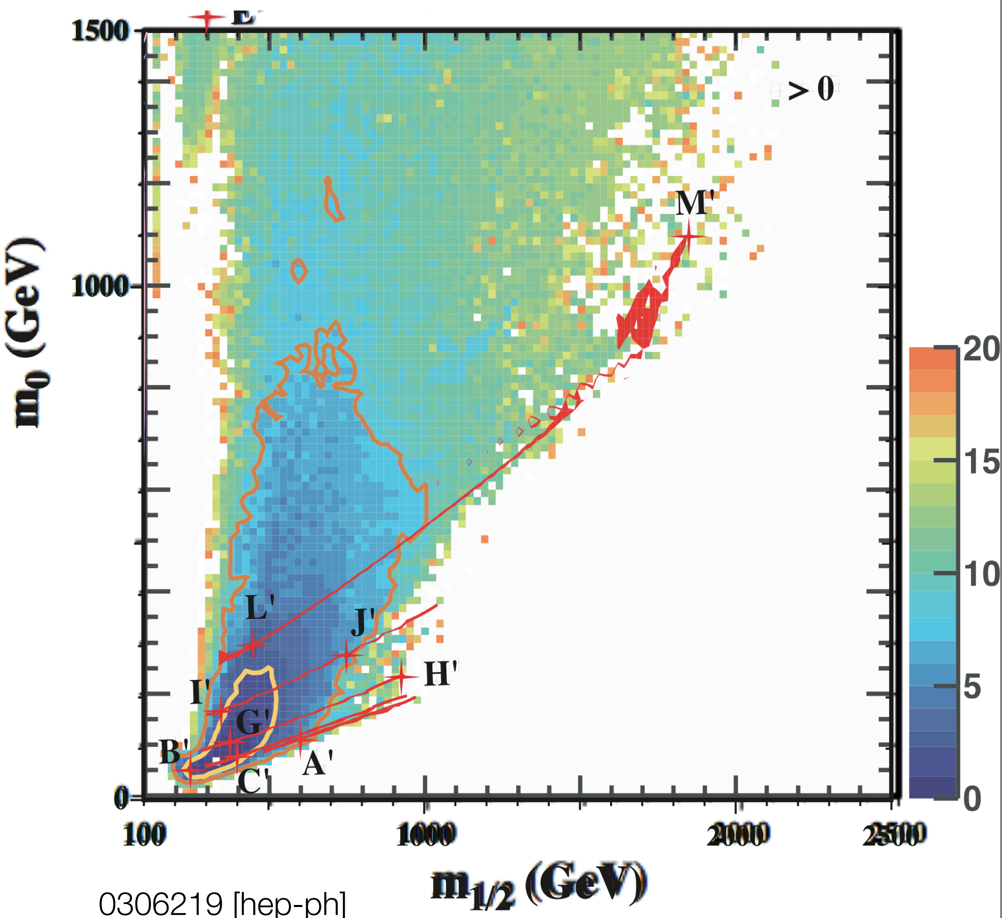
In 2D scans, enforcing the cosmological relic abundance results in narrow “allowed regions” (the “WMAP strips”), whose location changes with the value of the fixed parameters.

Once fixed parameters are included and hidden dimensions accounted for, WMAP strips widen to become “WMAP blobs”

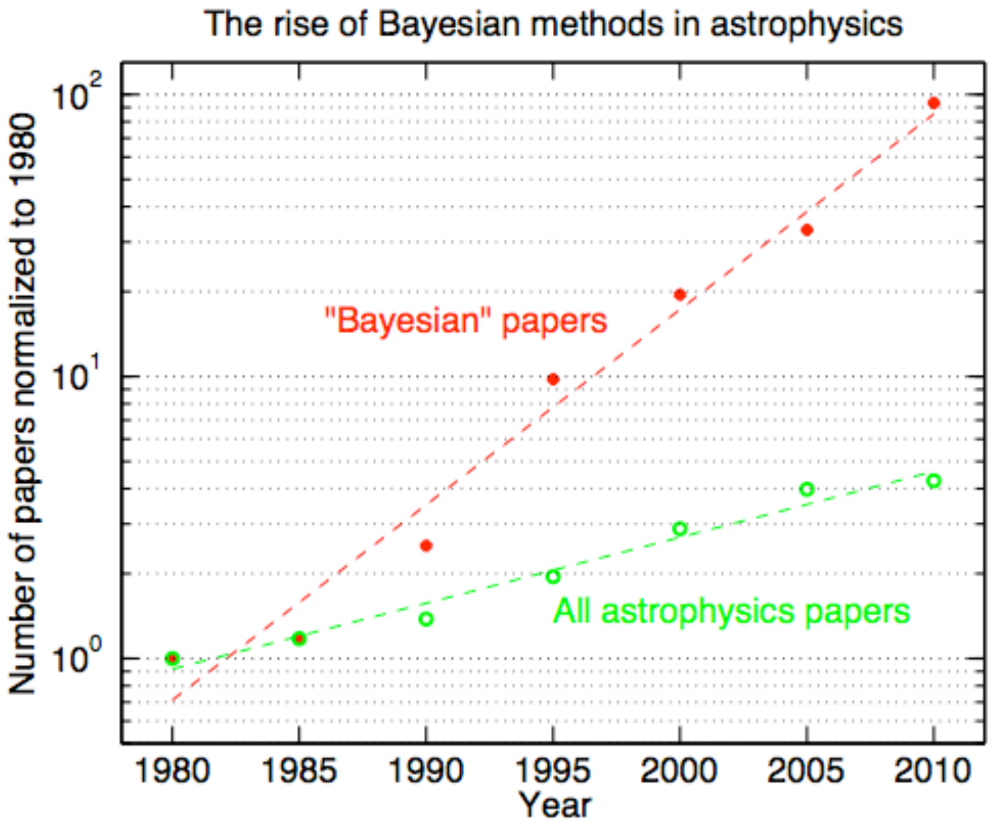
WMAP strips a few years ago




0306219 [hep-ph]

# Bayesian methods on the rise

The frequentist approach (= probability as frequency, based on the likelihood) is naturally suited to particle physics. Bayesian methods are being imported from astrophysics, where they are the norm:



# Bayes' theorem

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posterior — likelihood — prior — evidence

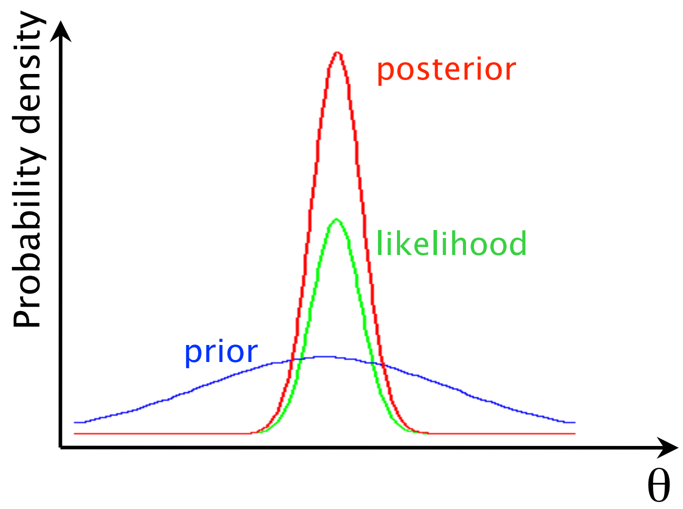
$$P(\theta|d,I) = \frac{P(d|\theta,I)P(\theta|I)}{P(d|I)}$$

**θ:** parameters
**d:** data
**I:** any other external information, or the assumed model

For parameter inference it is sufficient to consider

$$P(\theta|d,I) \propto P(d|\theta,I)P(\theta|I)$$

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$



# The matter with priors

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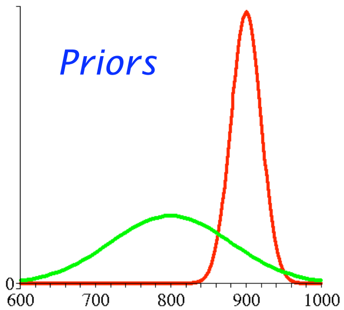
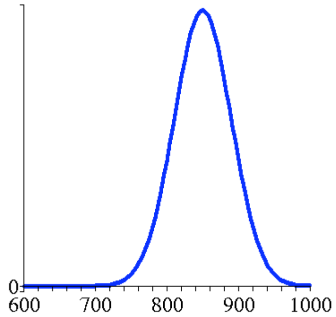
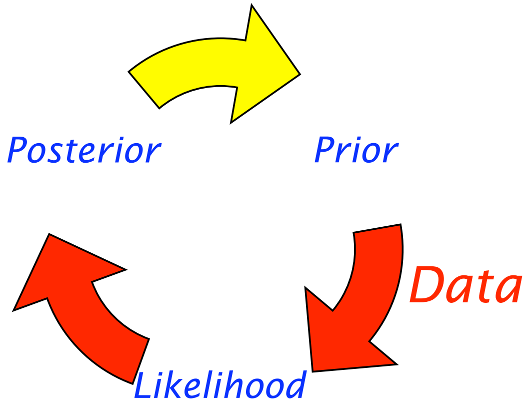
- In parameter inference, prior dependence will **in principle** vanish for strongly constraining data.

**THIS IS CURRENTLY NOT THE CASE EVEN FOR THE CMSSM!**

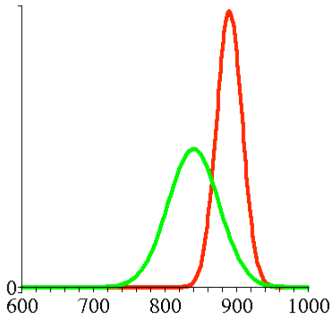



*Likelihood (1 datum)*

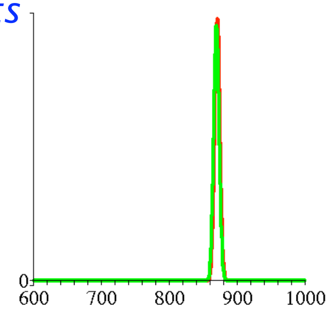



*Posterior after 1 datum*



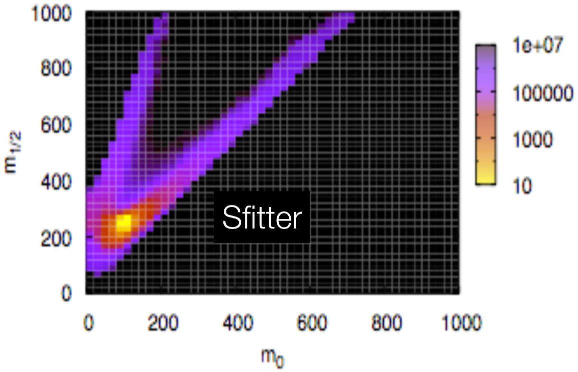

*Posterior after 100 data points*



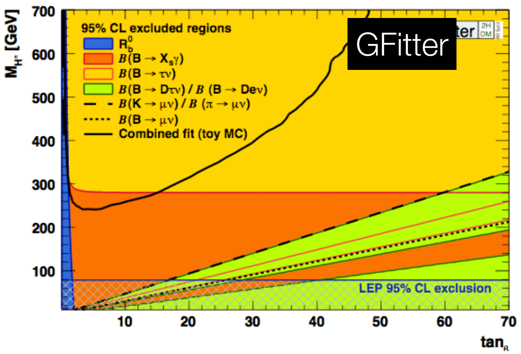
# Global fits in the LHC era

Imperial College London

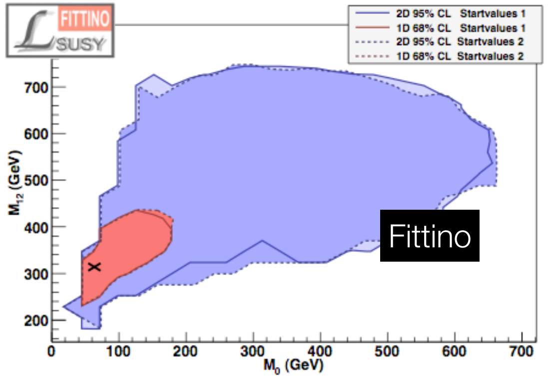
R. Lafaye , M. Rauch, T. Plehn, D. Zerwas



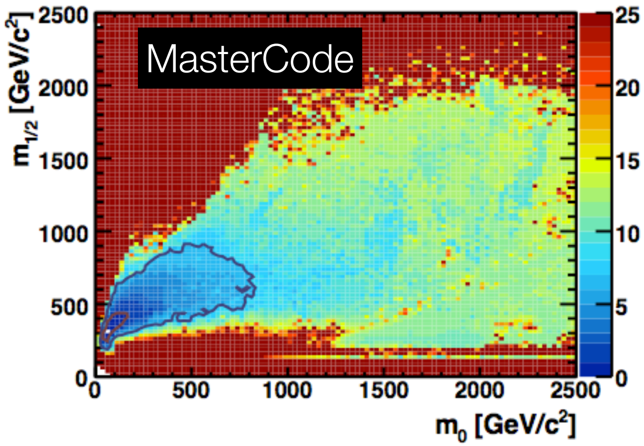

H. Flächer, M. Goebel, J. Haller, A. Höcker, K. Mönig, J. Stelzer



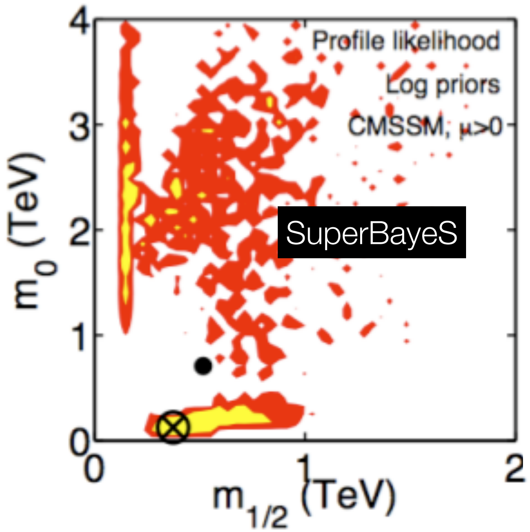

P. Bechtle, K. Desch, M. Uhlenbrock, P. Wienemann



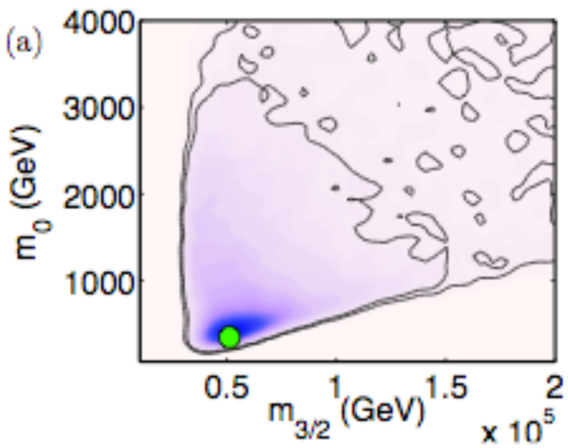

O. Buchmueller, R. Cavanaugh, A. De Roeck, J.R. Ellis, H.Flacher, S. Heinemeyer, G. Isidori, K.A. Olive, F.J. Ronga, G. Weiglein




F. Feroz, L. Roszkowski, R. Ruiz de Austri, R. Trotta




S.S. AbdusSalam, B.C. Allanach, M.J. Dolan, F. Feroz, M.P. Hobson



# Favoured regions: likelihood-based approach

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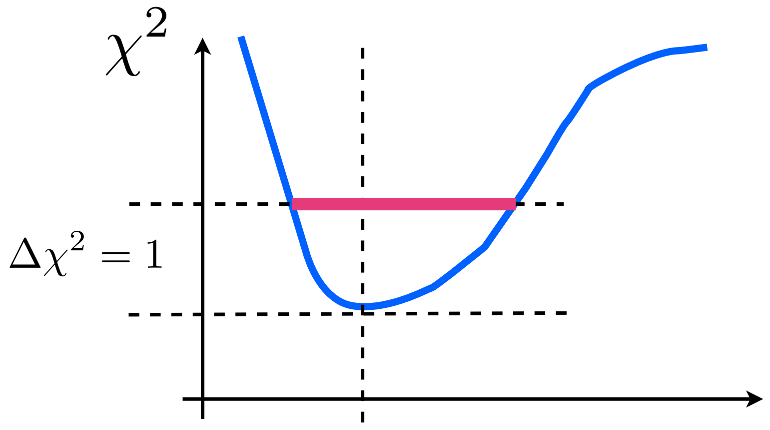
- Due to the weak nature of constraints, different scanning techniques and statistical methods will generally give different answers
- **Likelihood-based methods:** determine the best fit parameters by finding the minimum of -2Log(Likelihood) = chi-squared
    - Markov Chain Monte Carlo (MCMC)
    - MCMC and Minuit as "afterburner"
    - Simulated annealing
    - Genetic algorithm
- Determine approximate confidence intervals: Local Δ(chi-squared) method

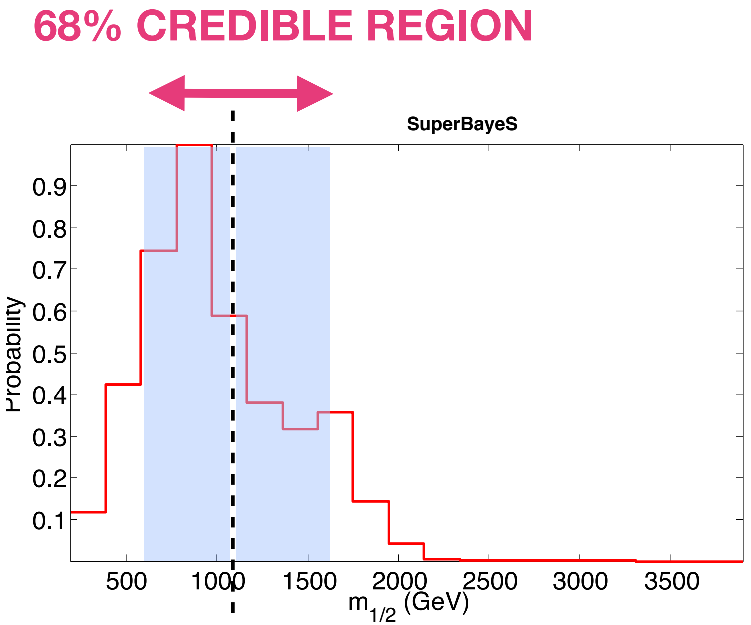



**≈ 68% CL**

# Favoured regions: Bayesian approach

Imperial College London

- Use the prior to define a metric on parameter space.
- **Bayesian methods:** the best-fit has no special status. Focus on region of large posterior probability mass instead.
    - Markov Chain Monte Carlo (MCMC)
    - Nested sampling
    - Hamiltonian MC
- Determine posterior credible regions: e.g. symmetric interval around the mean containing 68% of samples



# Marginalization vs profiling (maximising)

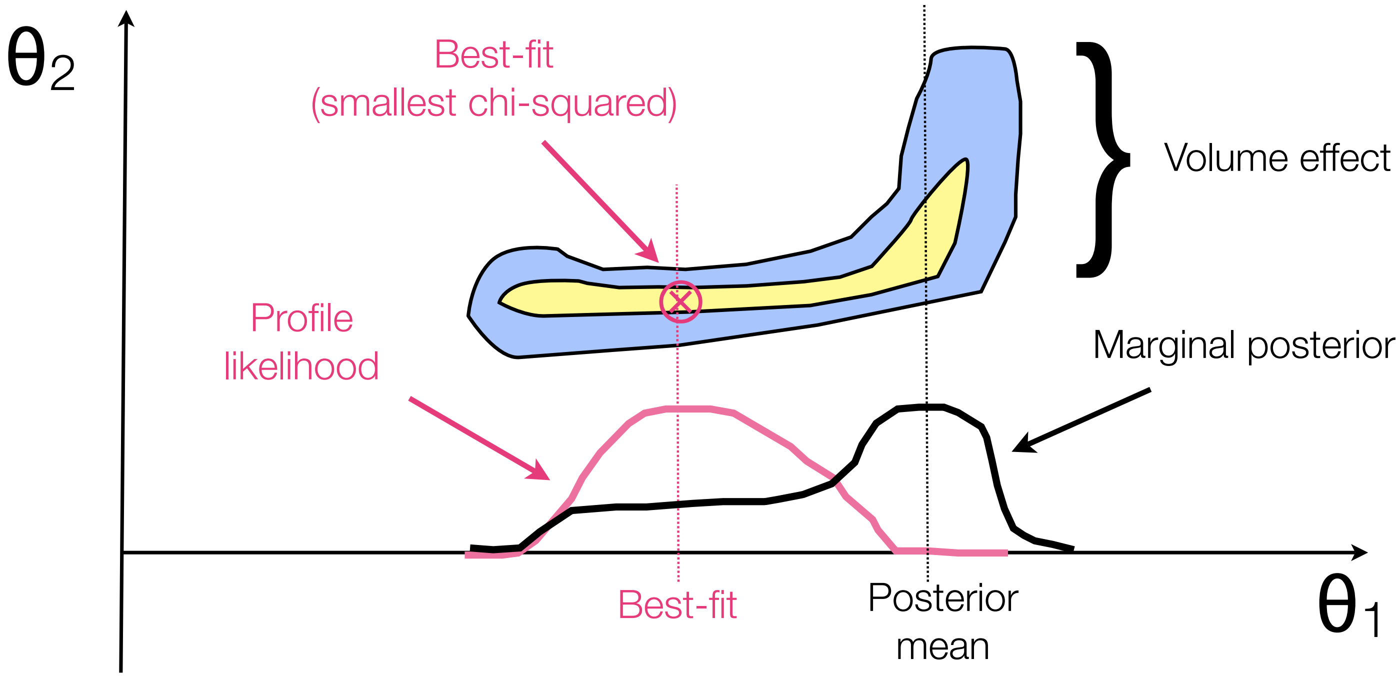
Imperial College London

## Marginal posterior:

$$P(\theta_1|D) = \int L(\theta_1, \theta_2)p(\theta_1, \theta_2)d\theta_2$$

## Profile likelihood:

$$L(\theta_1) = max_{\theta_2} L(\theta_1, \theta_2)$$




(2D plot depicts likelihood contours - prior assumed flat over wide range)

# Marginalization vs profiling (maximising)

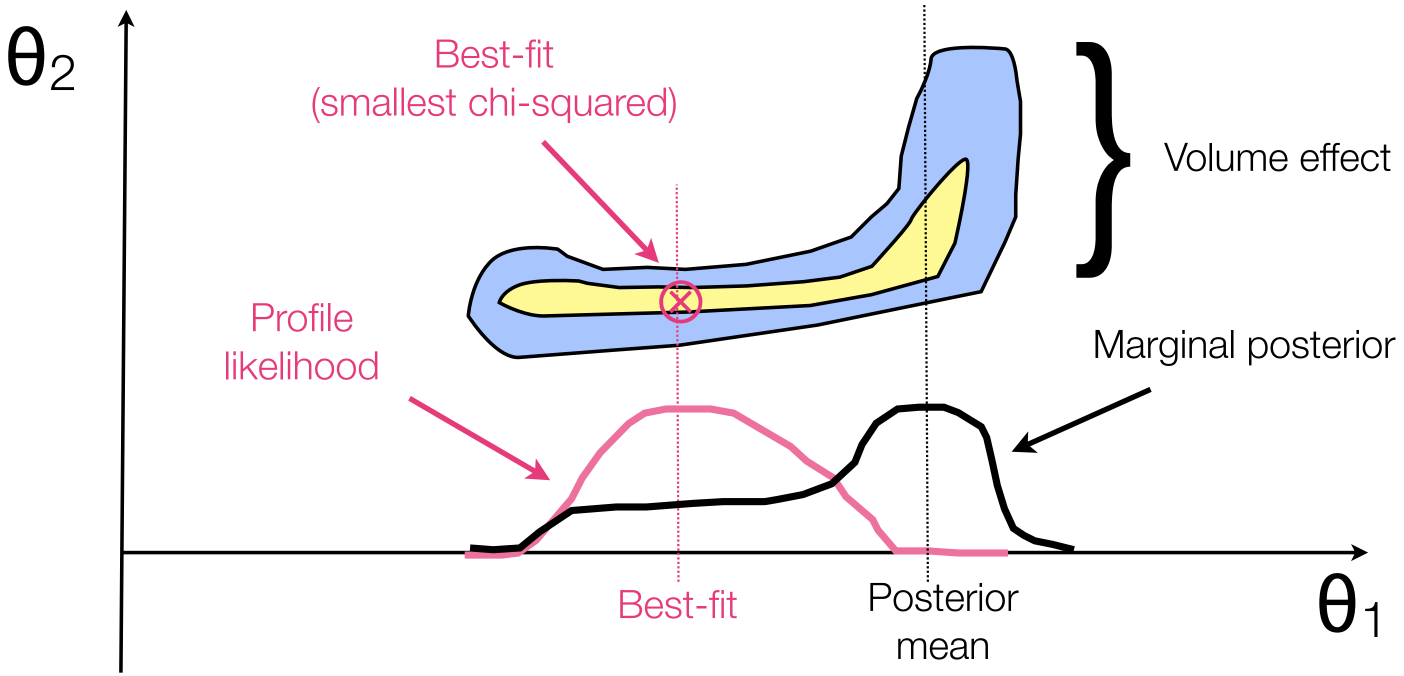


**Physical analogy:** (thanks to Tom Loredo)

Likelihood = hottest hypothesis
Posterior = hypothesis with most heat

**Heat:** $$Q = \int c_V(x) T(x) dV$$

**Posterior:** $$P \propto \int p(\theta) L(\theta) d\theta$$




(2D plot depicts likelihood contours - prior assumed flat over wide range)

# Constrained MSSM analysis pipeline

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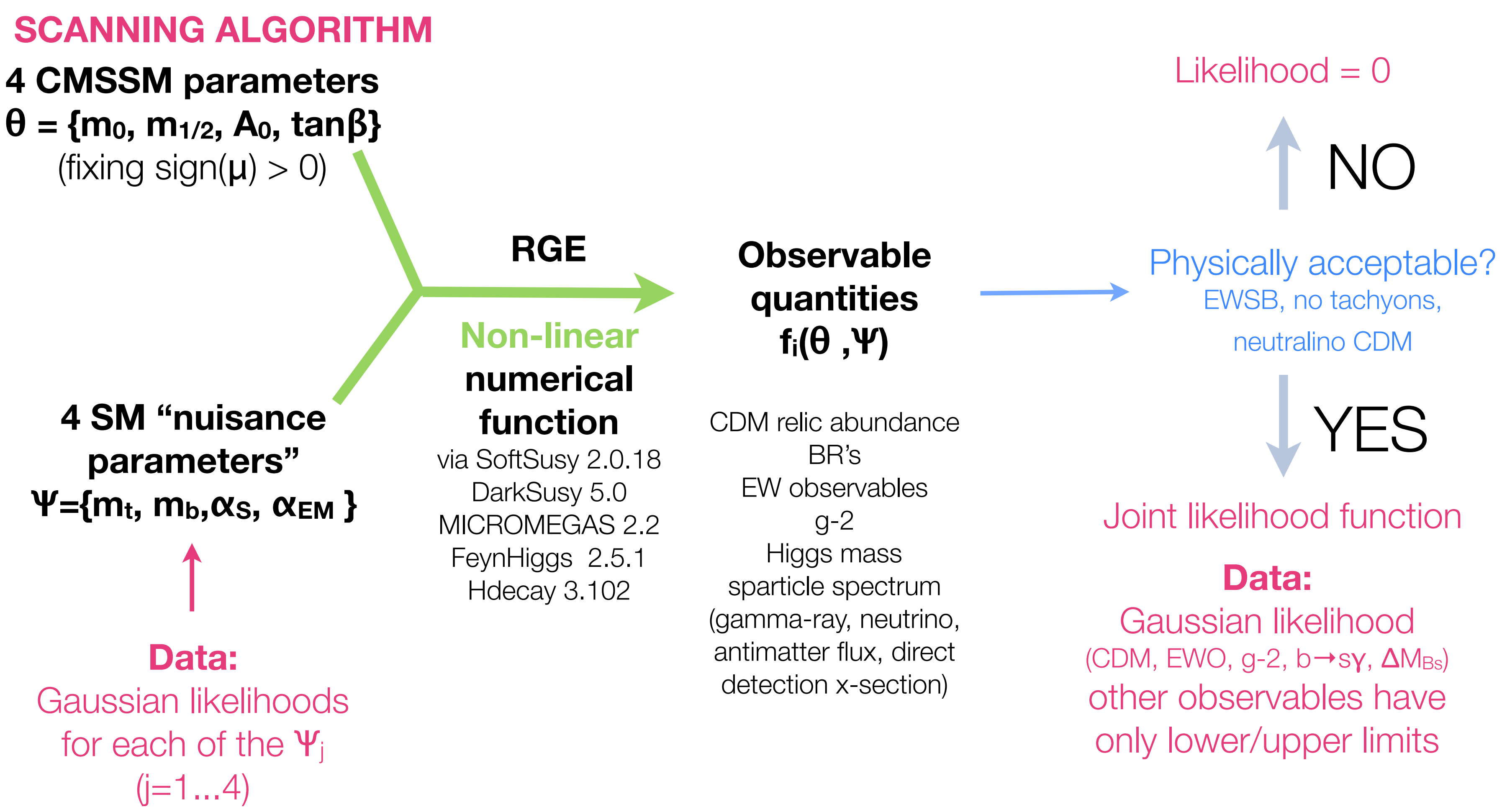

# Global CMSSM scans

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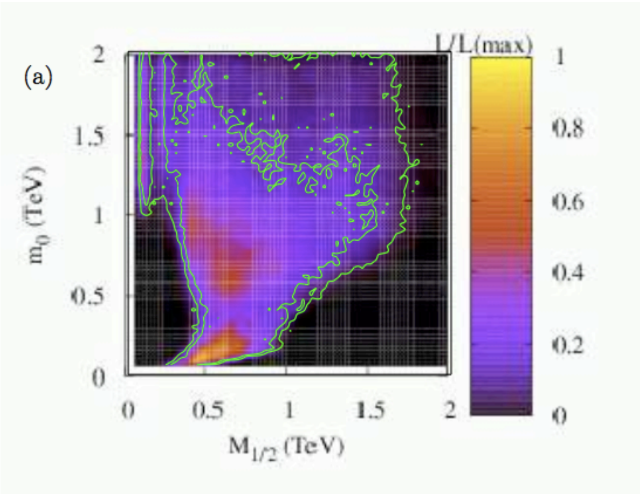
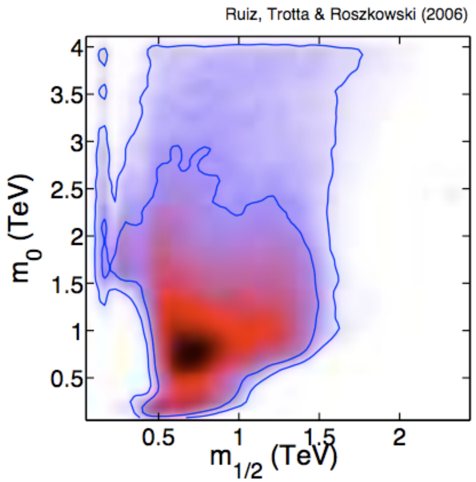
- Bayesian approach led by two groups (early work by Baltz & Gondolo, 2004):
- Ben Allanach (DAMPT) and collaborators (Allanach & Lester, 2006 onwards)
- Ruiz de Austri, Roszkowski & RT (Ruiz de Austri et al, 2006 onwards)
  + Feroz & Hobson (MultiNest), + Silk (indirect detection), + Strigari (direct detection), + Martinez et al (dwarfs), + de los Heros (IceCube), + Bertone et al (pMSSM)
  **SuperBayeS public code available from: superbayes.org**




Allanach & Lester (2006)




Ruiz de Austri, Roszkowski & RT (2006)

See also: Ellis et al (2004 onwards), Buchmuller et al (2008, 2009), Scott et al (2009), Akrami et al (2009)

# Key advantages of the Bayesian approach

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- **Efficiency:** computational effort scales ~ N rather than $k^N$ as in grid-scanning methods. Orders of magnitude improvement over grid-scanning.
- **Marginalisation:** integration over hidden dimensions comes for free.
- **Inclusion of nuisance parameters:** simply include them in the scan and marginalise over them.
- **Pdf's for derived quantities**: probabilities distributions can be derived for any function of the input variables (crucial for DD/ID/LHC predictions)

# www.superbayes.org

SuperBayeS
Supersymmetry Parameters Extraction Routines for Bayesian Statistics

- Implements the CMSSM, but can be easily extended to the general MSSM
- **New release (v 1.50) in June 2010:** linked to SoftSusy 2.0.18, DarkSusy 5.0, MICROMEGAS 2.2, FeynHiggs 2.5.1, Hdecay 3.102.
- Includes up-to-date constraints from all observables, plotting routines, statistical analysis tools, posterior and profile likelihood plots. Fully parallelized, MPI-ready, user-friendly interface
- MCMC engine (Metropolis-Hastings, bank sampler), grid scan mode, multi-modal nested sampling aka MultiNest (Feroz & Hobson 2008)
  **A full 8D scan now takes less than 2 days on 8 CPUs.**

# The future: “instantaneous” inference with neural networks

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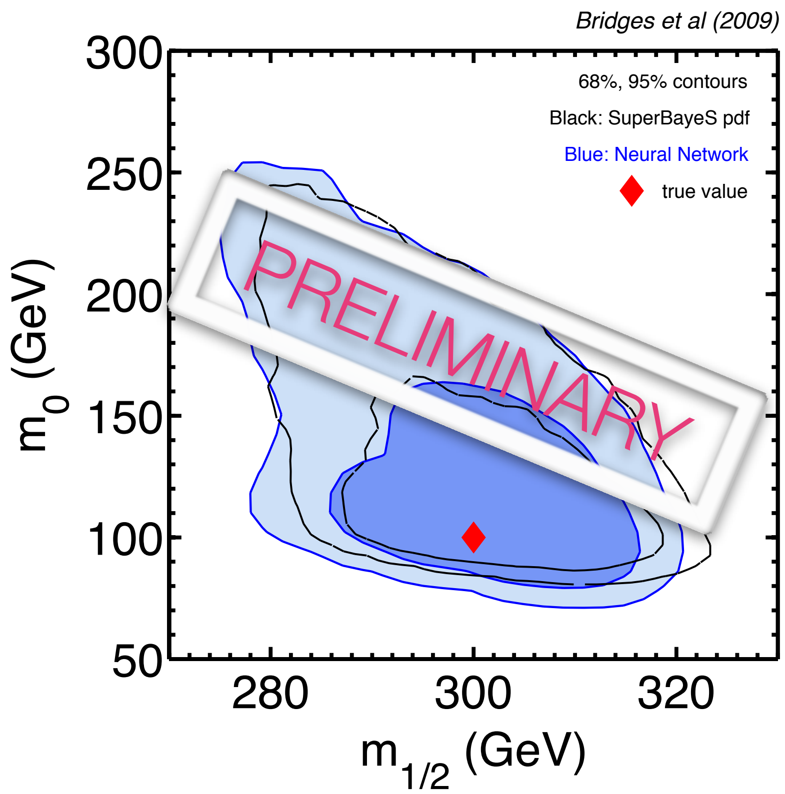
- **Standard MCMC**
  (SuperBayeS v1.23, 2006)
  **720 CPU days**

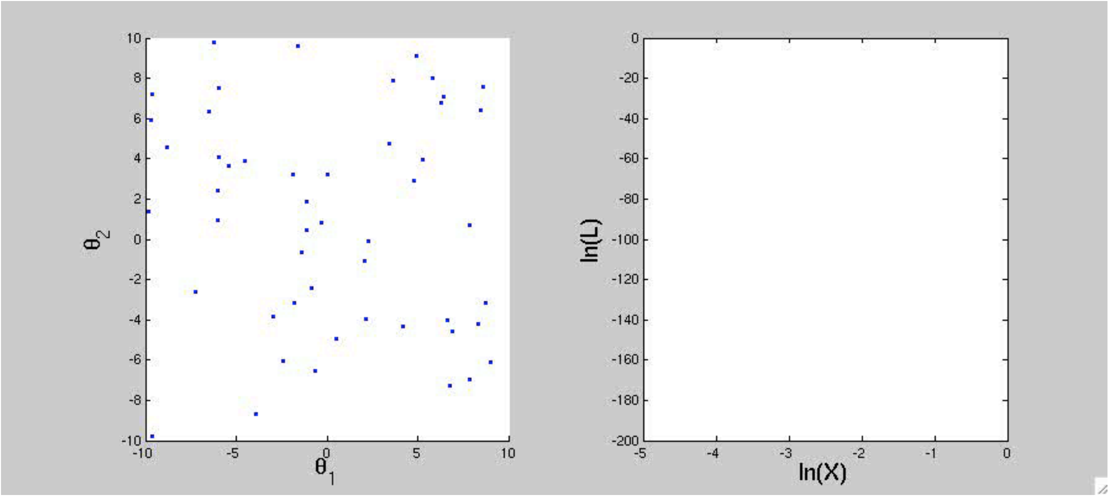
- **MultiNest**
  (SuperBayeS v1.5, 2010)
  **16 CPU days**
  **speed-up factor: ~ 50**

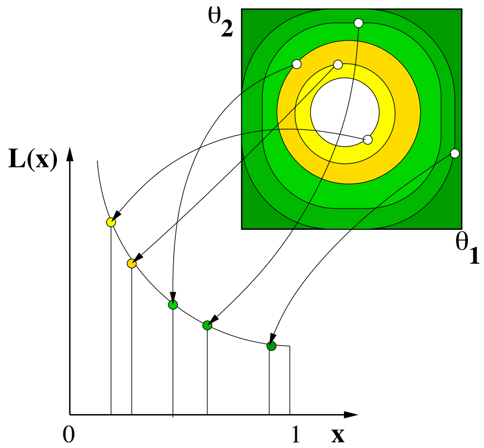
- **SuperBayeS+Neural Networks**
  (Bridges, RT et al, in prep)
  **15 CPU minutes**
  **speed-up factor: 70’000**

Simulated LHC data



# Nested sampling




(animation courtesy of David Parkinson)

An algorithm originally aimed primarily at the Bayesian evidence computation (Skilling, 2006):

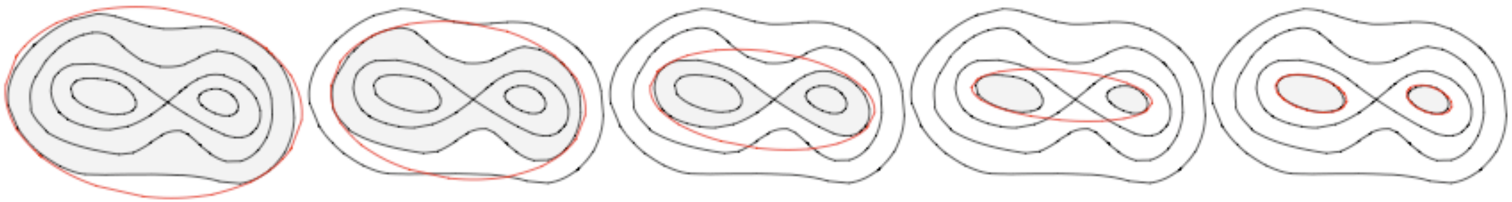
$$X(\lambda) = \int_{\mathcal{L}(\theta) > \lambda} P(\theta) d\theta$$

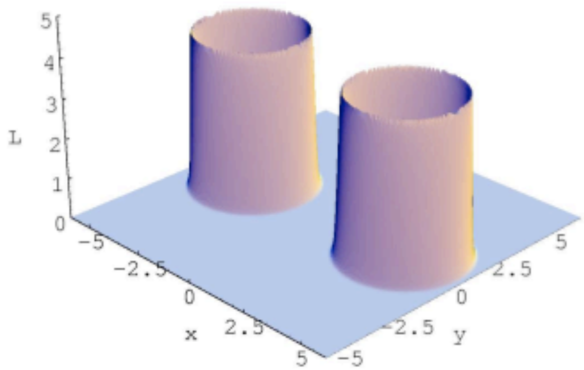
$$P(d) = \int d\theta \mathcal{L}(\theta) P(\theta) = \int_0^1 X(\lambda) d\lambda$$

Feroz et al (2008), *arxiv: 0807.4512*, Trotta et al (2008), *arxiv: 0809.3792*

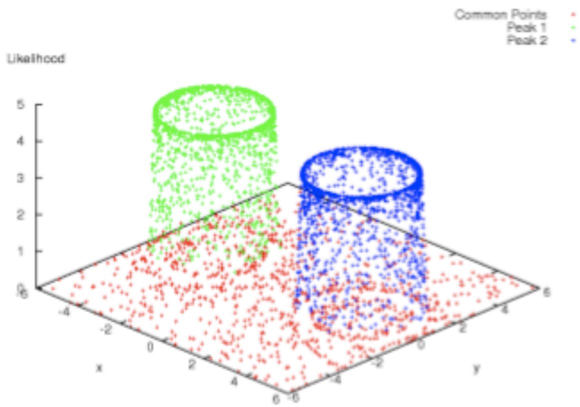
# The MultiNest algorithm

- MultiNest: Also an extremely efficient sampler for multi-modal likelihoods!
  Feroz & Hobson (2007), RT et al (2008), Feroz et al (2008)





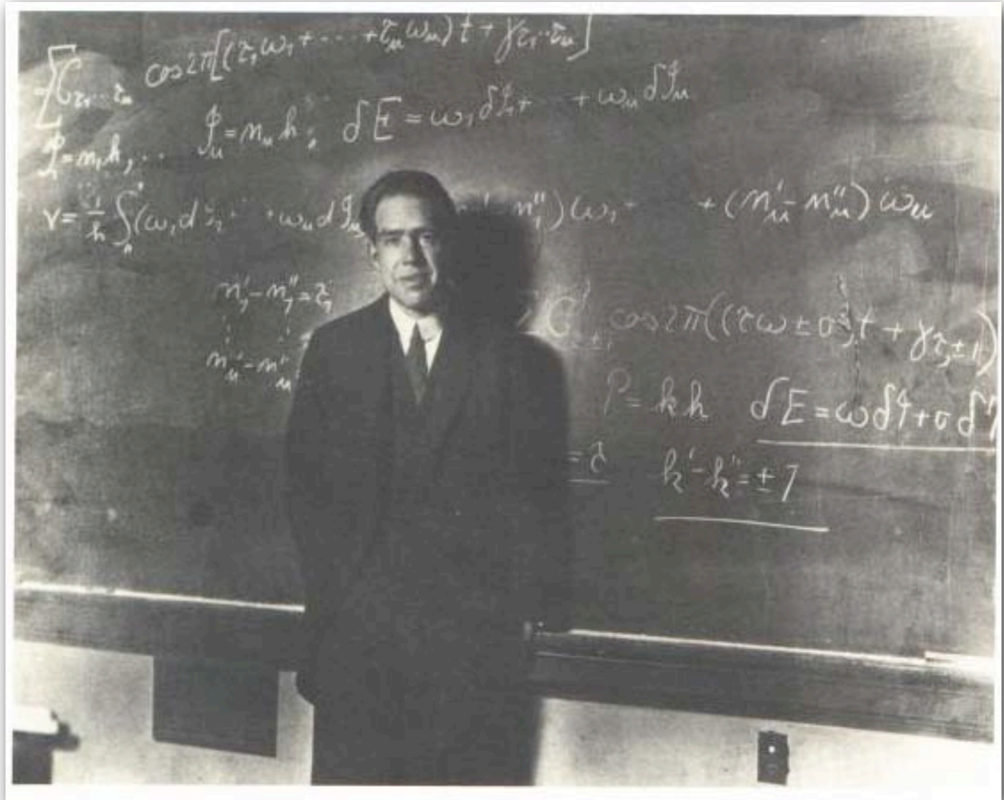

Target Likelihood




Sampled Likelihood

*Prediction is very difficult, especially about the future.*

Niels Bohr



# CMSSM today: likelihood-based results

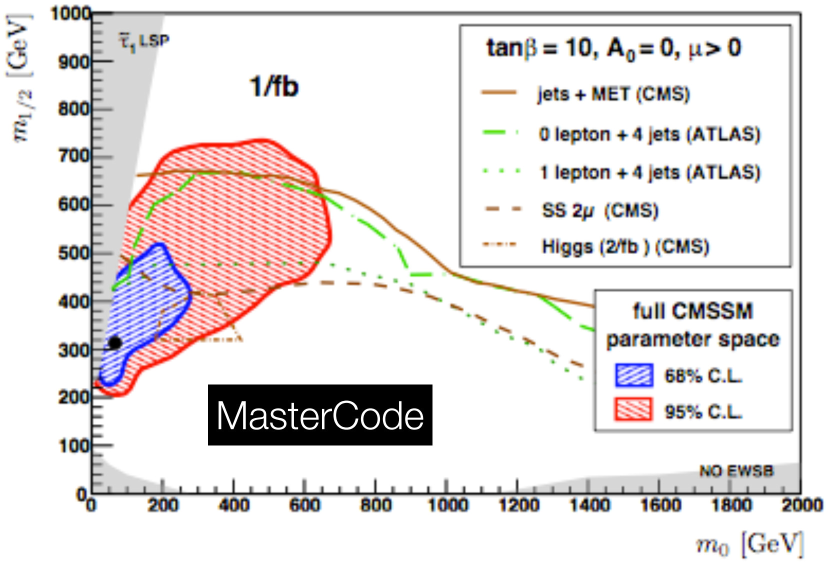
0907.4468 [hep-ph]
0808.4128 [hep-ph]

**Best fit points (μ>0)**

0907.2589 [hep-ph]

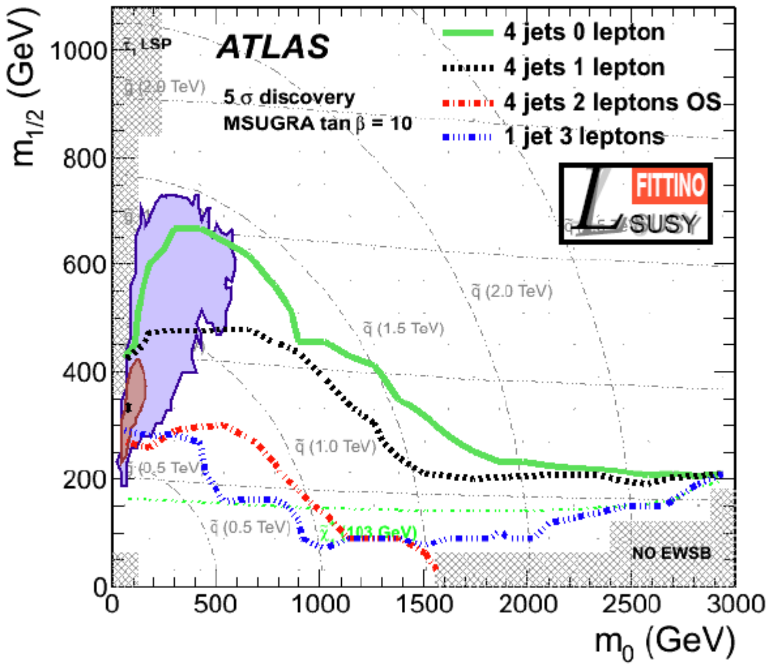
**MasterCode**

$M_0$=60 , $M_{1/2}$=310 $A_0$=130, tan**β**=11




**Fittino**

$M_0$=76 , $M_{1/2}$=332 $A_0$=383, tan**β**=13

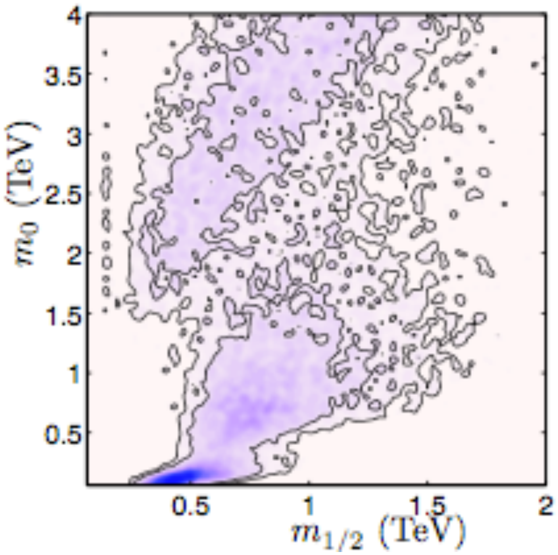


# CMSSM today: Bayesian results

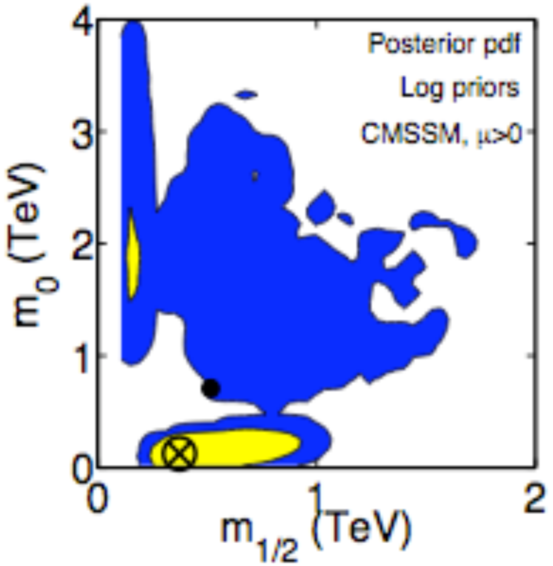
**"flat prior"**
Uniform in $M_0$,$M_{1/2}$,$A_0$,tan$\beta$

**"log prior"**
Uniform in log($M_0$), log($M_{1/2}$), $A_0$, tan$\beta$

**"naturalness prior"**
Penalizes regions of parameter space that are "fine tuned"




0807.4512 [hep-ph]

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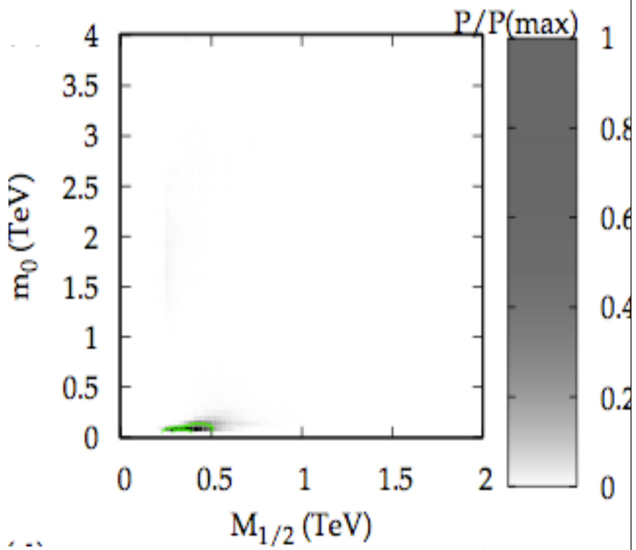
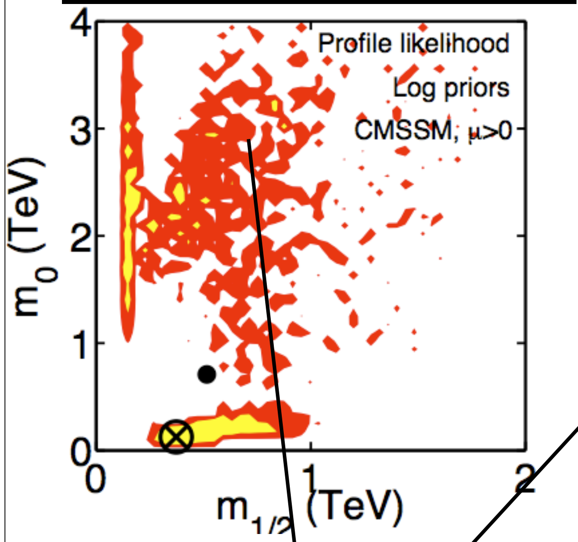



0705.0487 [hep-ph]

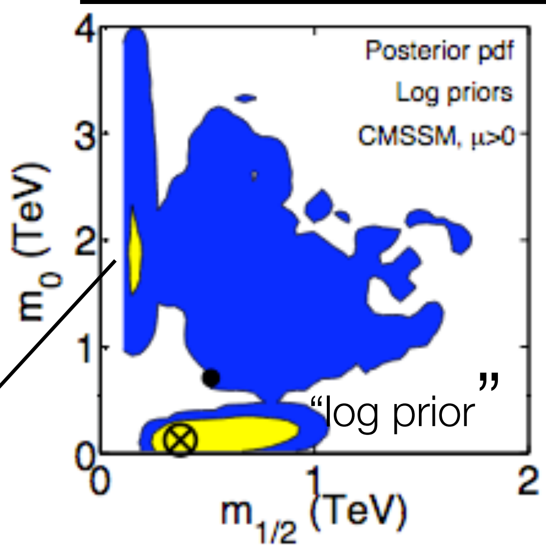
Posterior distributions

# CMSSM today: Frequentist vs Bayesian

SuperBayeS: profile likelihood

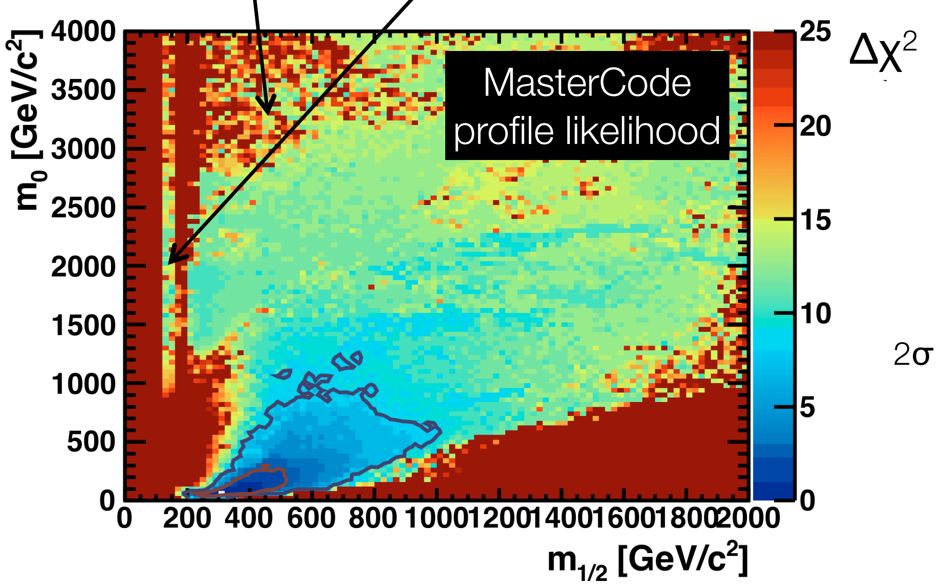


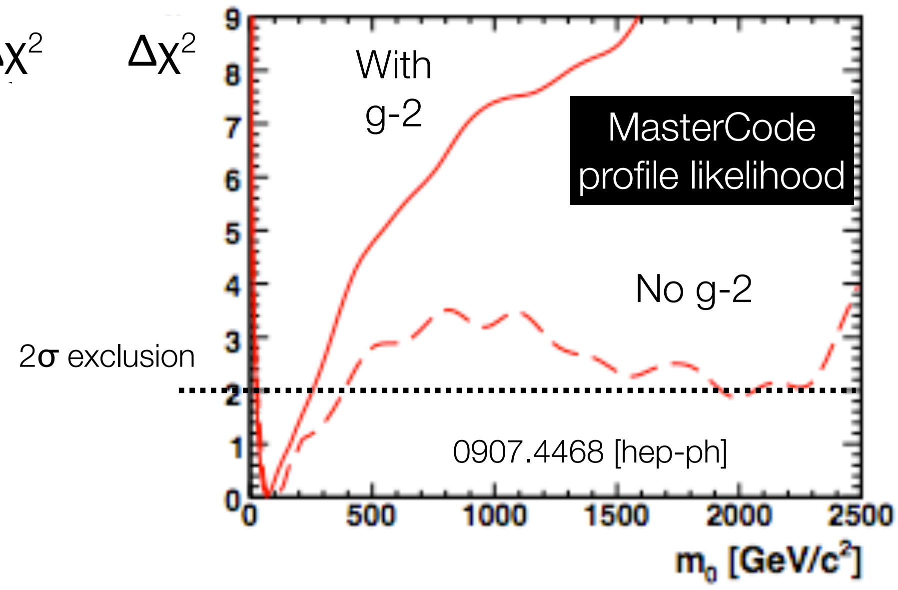

SuperBayeS: posterior




Both methods find a favoured low mass SUSY region: how constrained is it?

**The g-2 constraint is critical in robustly excluding TeV-scale masses in the frequentist approach**



# Profile likelihood results: comparison

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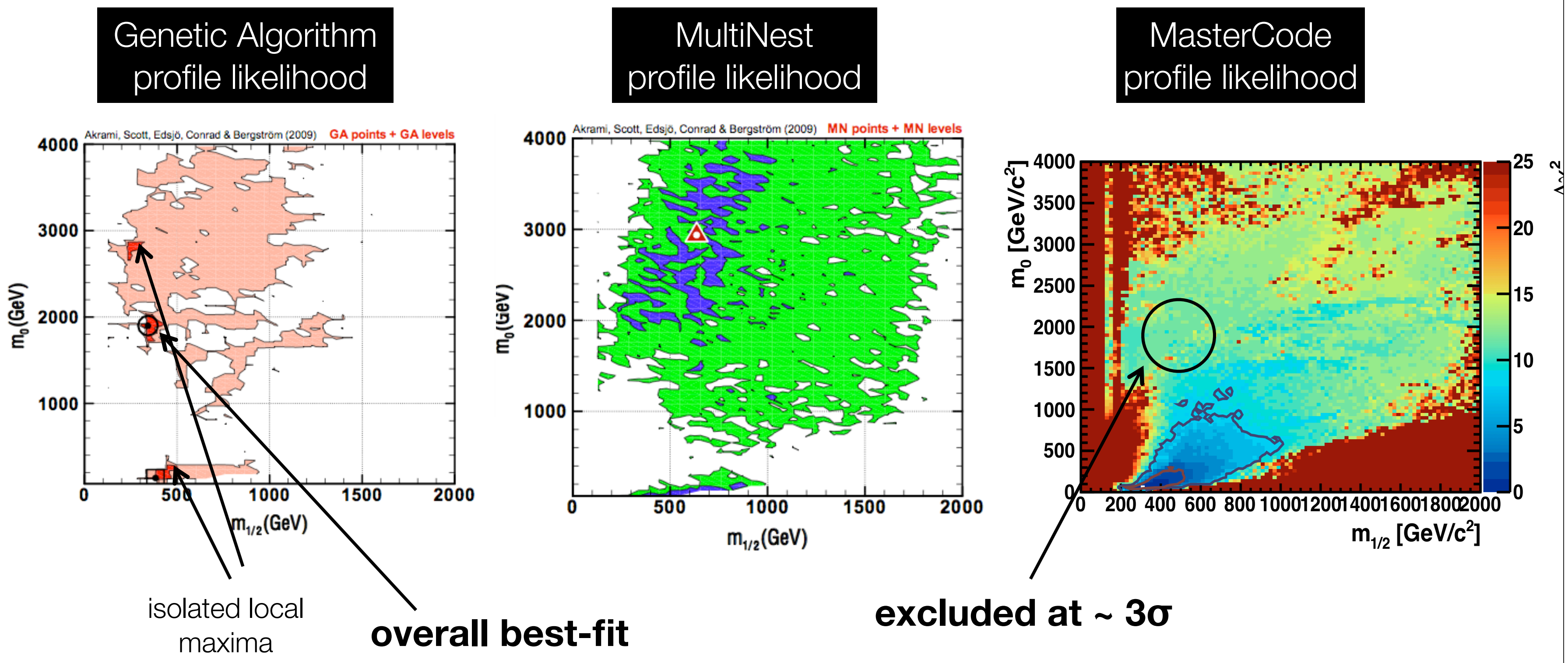
- Akram et al (0910.3950) adopted a genetic algorithm (GA) to map out the profile likelihood.
- This allows to find isolated spikes in the likelihood in the high-mass region: **is this something other frequentist fits might have missed?**



# Statistical conclusions

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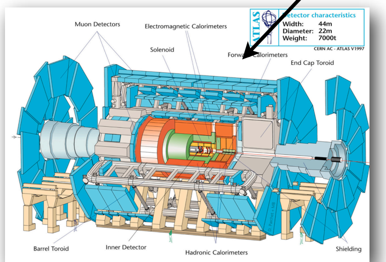
- Even one of the theoretically most constrained models (the CMSSM) shows signs of ambiguities in the statistical results
- This can be traced back to insufficiently constraining data (at present)
- Low-mass SUSY seems preferred but ~ TeV scale masses cannot be ruled out robustly
- **ALL ENSUING PREDICTIONS HAVE TO BE TAKEN WITH A LARGE GRAIN OF SALT**

grain of salt

ATLAS
(to scale)



# Direct detection prospects

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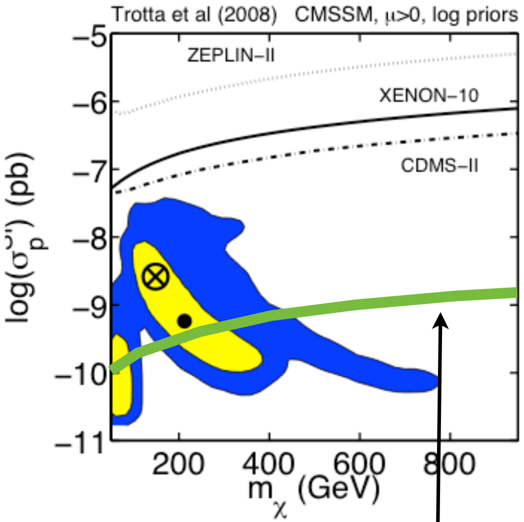
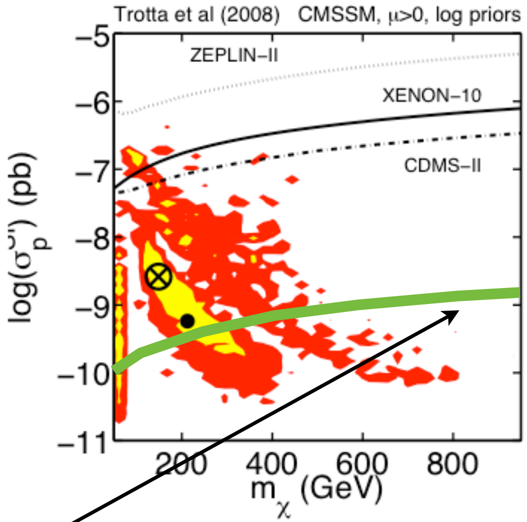
Generally favourable prospects for WIMP discovery in the CMSSM framework for upcoming detectors are **robust**, independently of the choice of statistics.

**Notice: canonical local density & velocity dispersion assumed**

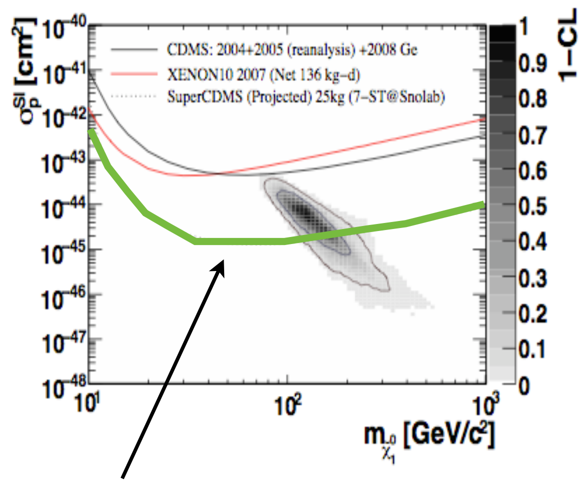
SuperBayeS posterior




SuperBayeS profile likelihood




MasterCode profile likelihood




reach of 1t of Xe detector

reach of 25 kg Ge

# More general models: NUHM

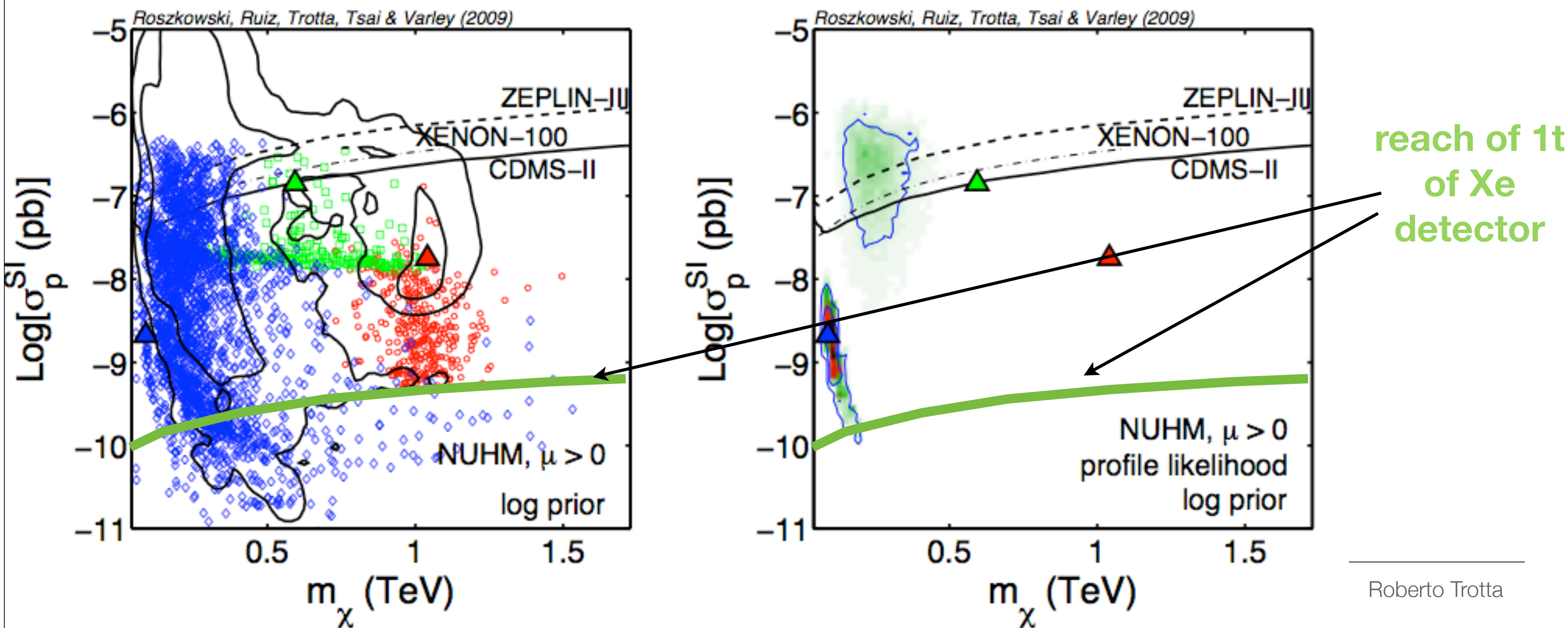


- Relaxing some of the universality assumptions at the GUT scale: Non-Universal Higgs Model with 2 non-universal Higgs masses. 6 SUSY + 4 SM free parameters:

$$\{m_0, m_{1/2}, \tan\beta, A_0, M_{H_u}, M_{H_d}\}$$

NUHM posterior

NUHM profile likelihood



AbdusSalam et al (0904.2548)

# More general models: pMSSM

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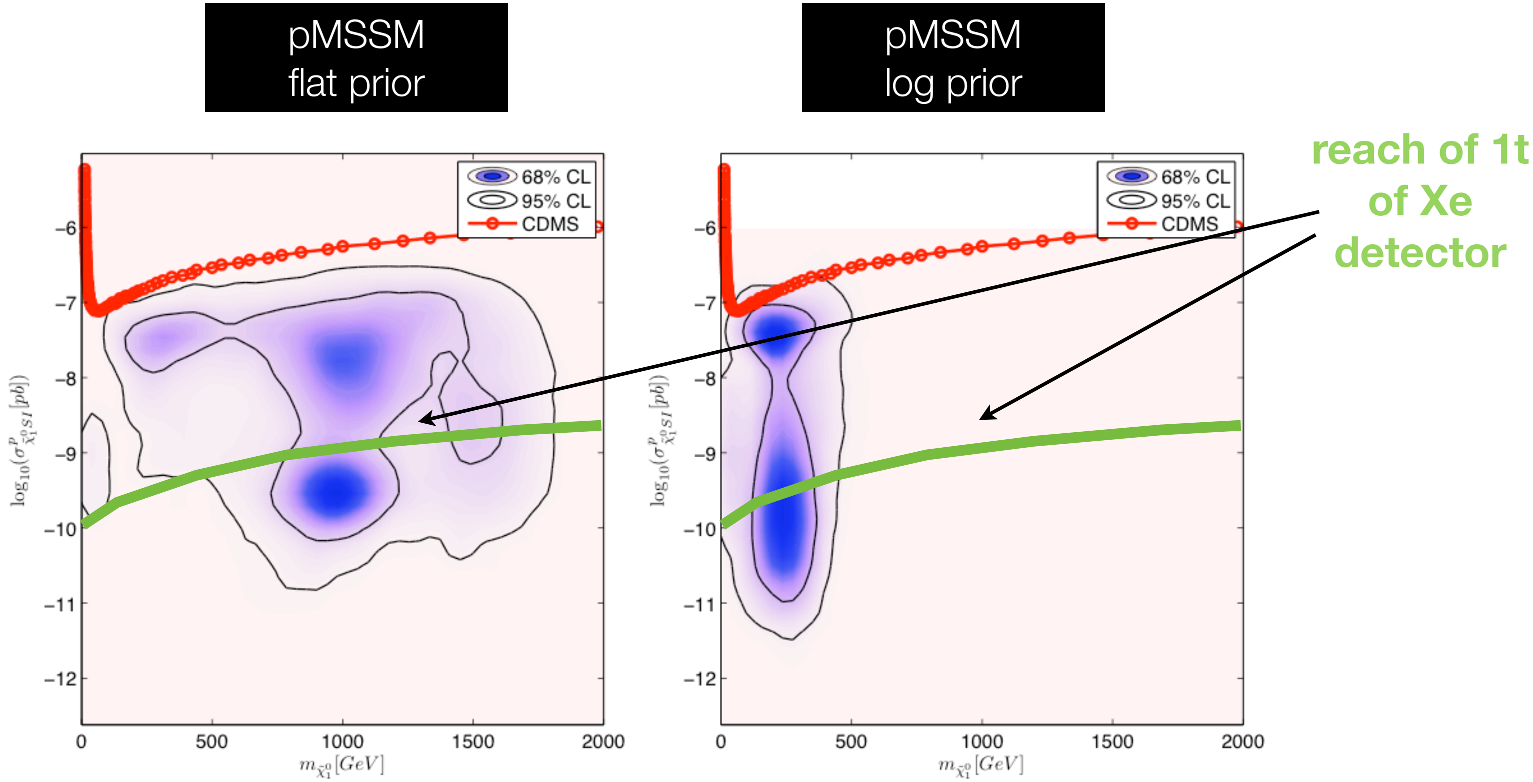
- Instead of using GUT-scale parameters, fit with EW-scale variables. 20 free parameters in the Phenomenological MSSM + 5 SM parameters.
  Prior dependence stronger than in the CMSSM due to much stronger volume effects.



# Astrophysical uncertainties



- The observable recoil rate in DD experiments depends both on particle physics properties and on astrophysical quantities which are poorly constrained:

$$\frac{dR}{dE} \sim \frac{\sigma}{\mu^2}\frac{\rho_\odot}{m_\chi} F^2(E) \int_{v>v_{\min}} d^3v \frac{f(v)}{v}$$

**local DM density**

**DM velocity distribution**

**(escape velocity, velocity dispersion, departures from Maxwell-Boltzman, anisotropic disperion, ...)**

Bertone & Serpico (1006.3268) tried to quantify such "systematic" uncertainties, estimating an envelope of **~ ± 30% uncertainty** for the recoil rate (excluding the impact of ρ)

# Determinations of the local density

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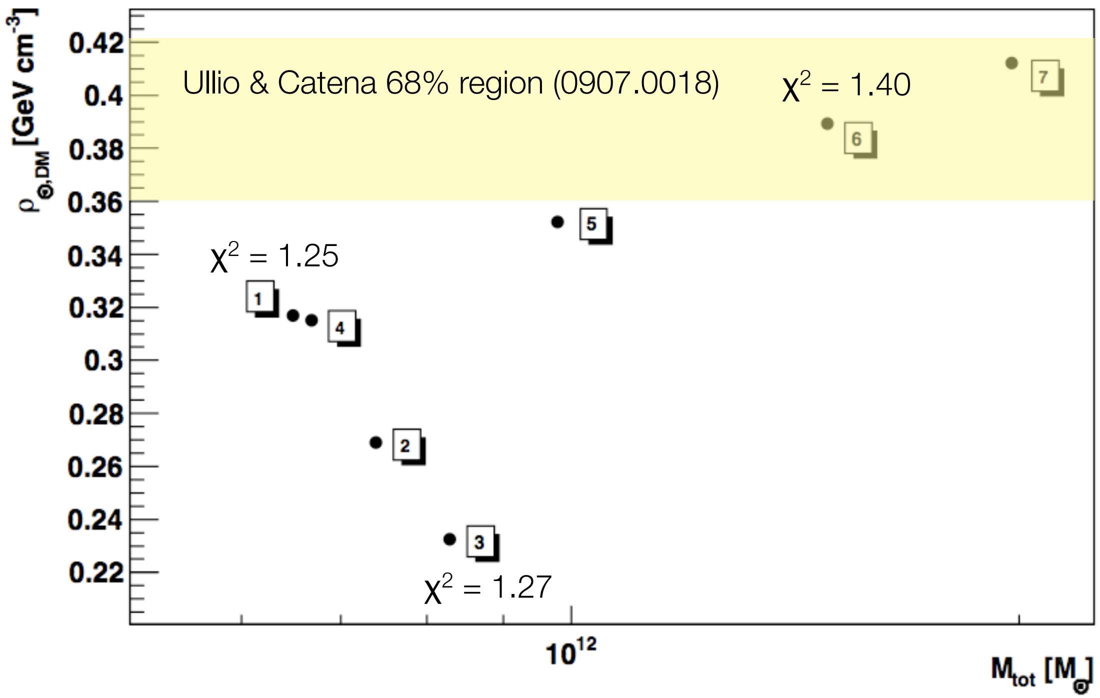
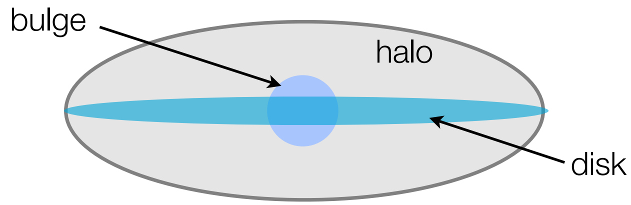
- Parameterized MW model constrained using a variety of kinematic data
- Ullio & Catena (2009): $\rho_{loc} = 0.39 \pm 0.03$ GeV/cm$^3$
- However, Weber & DeBoer (0910.4272) find a much larger spread with statistically indistinguishable fit
- Probably systematic errors from partial modeling: see Pato et al 1006.1322

Adapted from Weber & DeBoer (0910.4272)

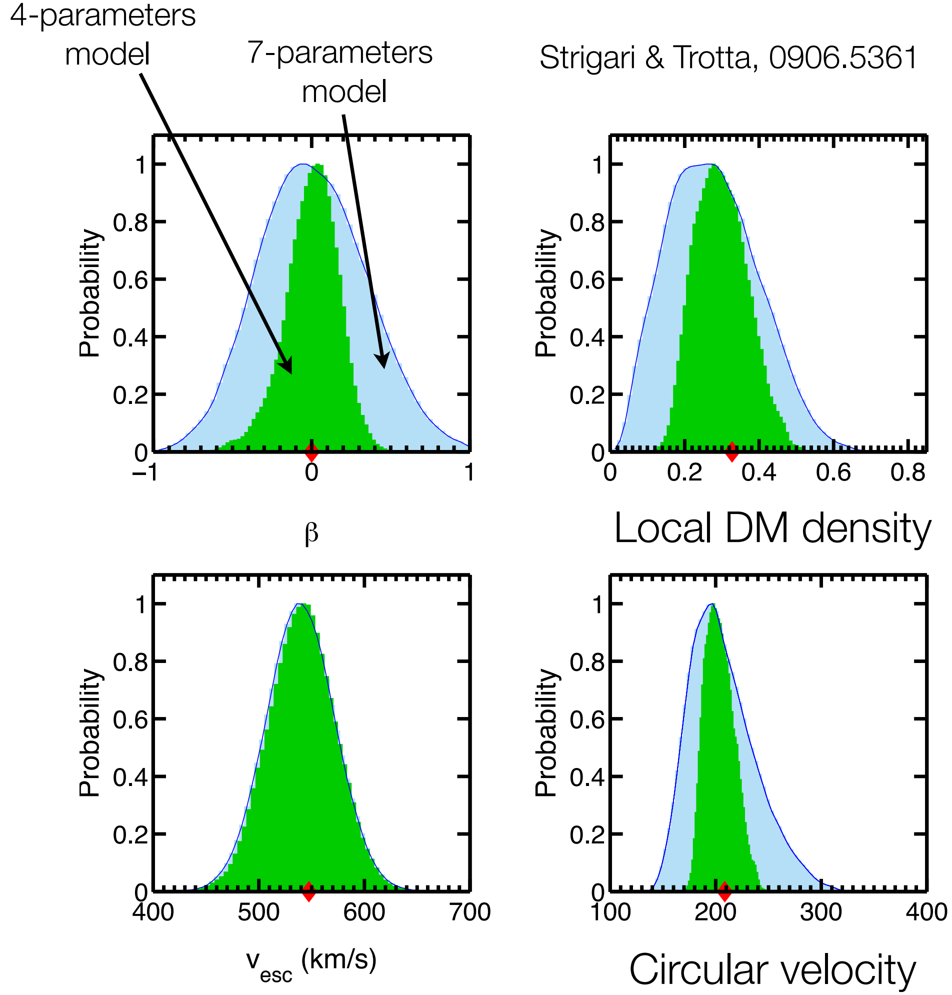


# Including astrophysical uncertainties

- Model for the Milky Way:
  bulge + disk + dark matter halo
- Line-of-sight velocity dispersion can be used to constrain the potential (using spherical Jeans equation). Assume Maxwellian velocity distribution.
- Use artificial Sloan-like l.o.s. data to estimate potential of the technique
- **Result:**
  projected error on $\rho_{loc}$ of order ~20%




*Simulated Sloan constraints*

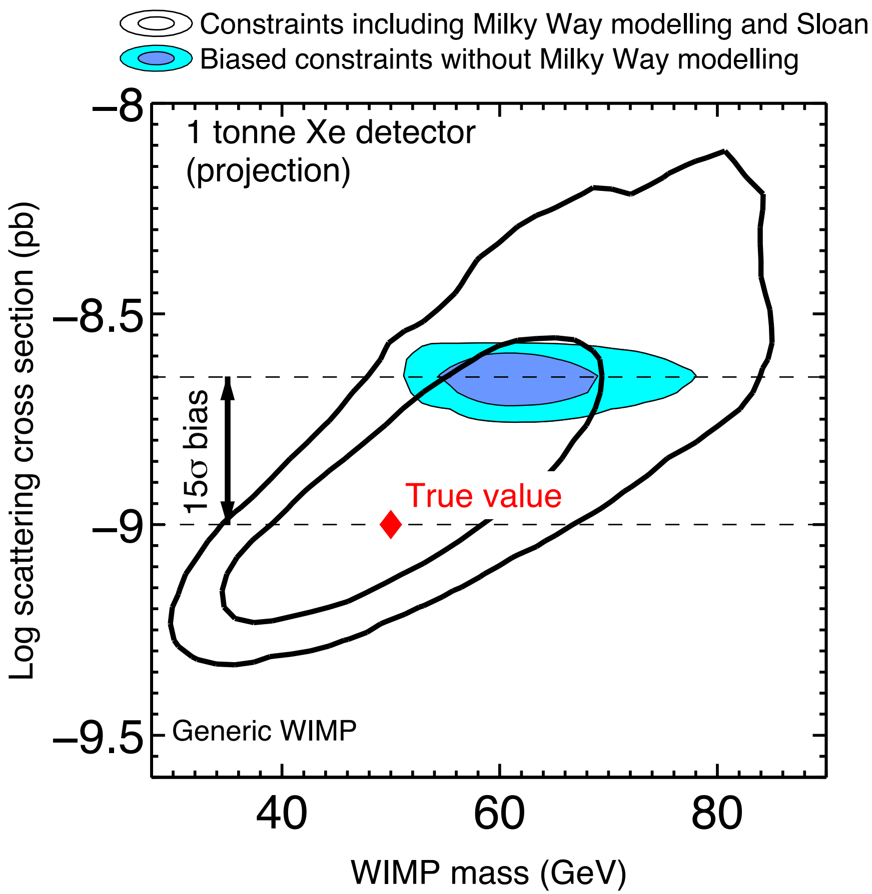


# The importance of modeling the MW

- Assuming an incorrect local density (by a factor of 2) can lead to **a 15 sigma bias** in the reconstructed cross section

- Accurate modeling of the MW may convert potentially catastrophic systematic errors into more manageable statistical errors

Strigari & Trotta (0906.5361)

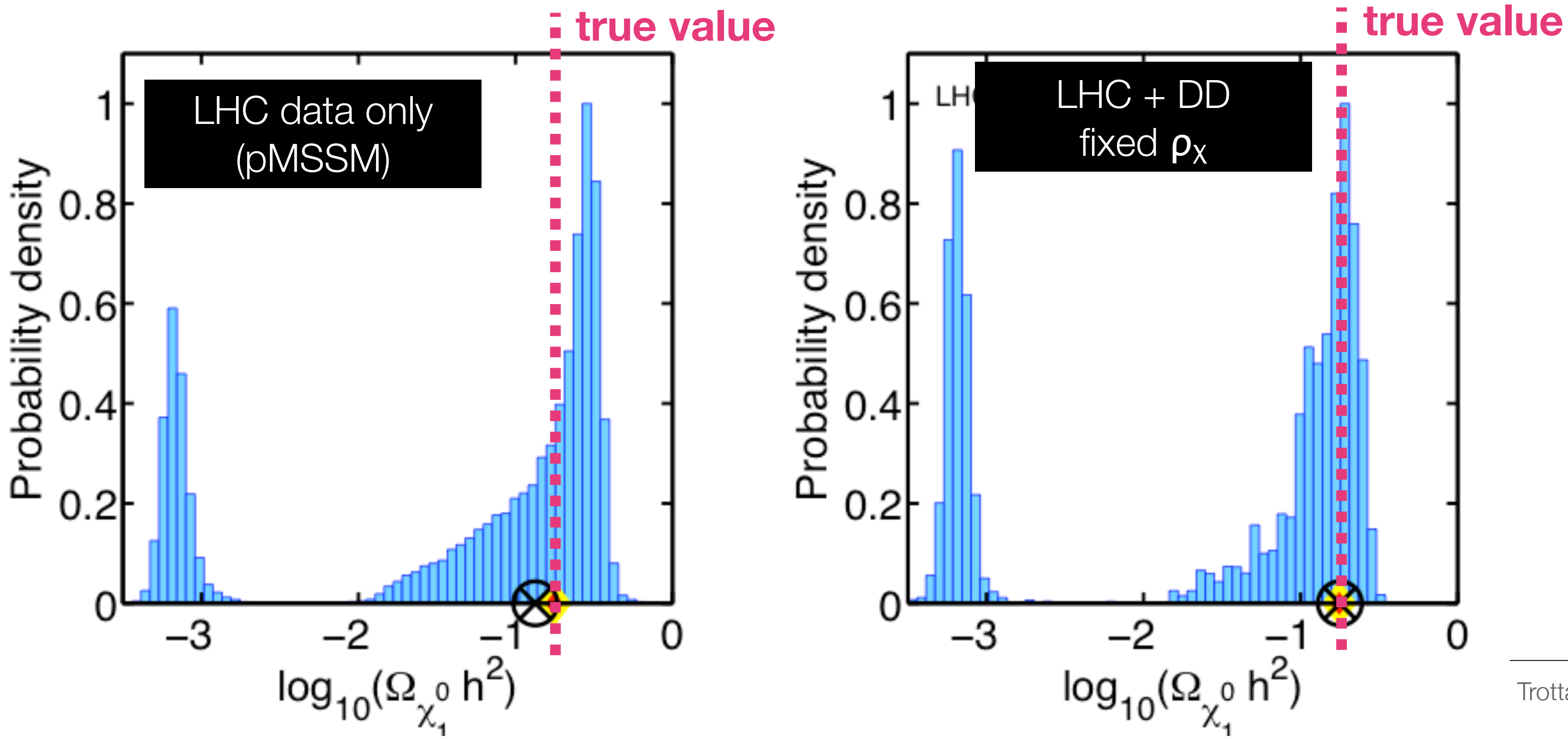


# Identification of the cosmological DM with direct detection + LHC

Bertone et al, 1005.4280

Imperial College London

- If a signal is seen both at the LHC and in direct detection detectors, how can we check that this WIMP makes up the bulk of the cosmological relic density?
- Fit low-energy SUSY parameters and try to predict $\Omega h^2$ from LHC data alone.
- Problem: LHC data alone are unable to constrain the relic abundance. Even DD data cannot break the degeneracy (if $\rho_X$ assumed fixed):




Trotta

# Identification of the cosmological DM with direct detection + LHC

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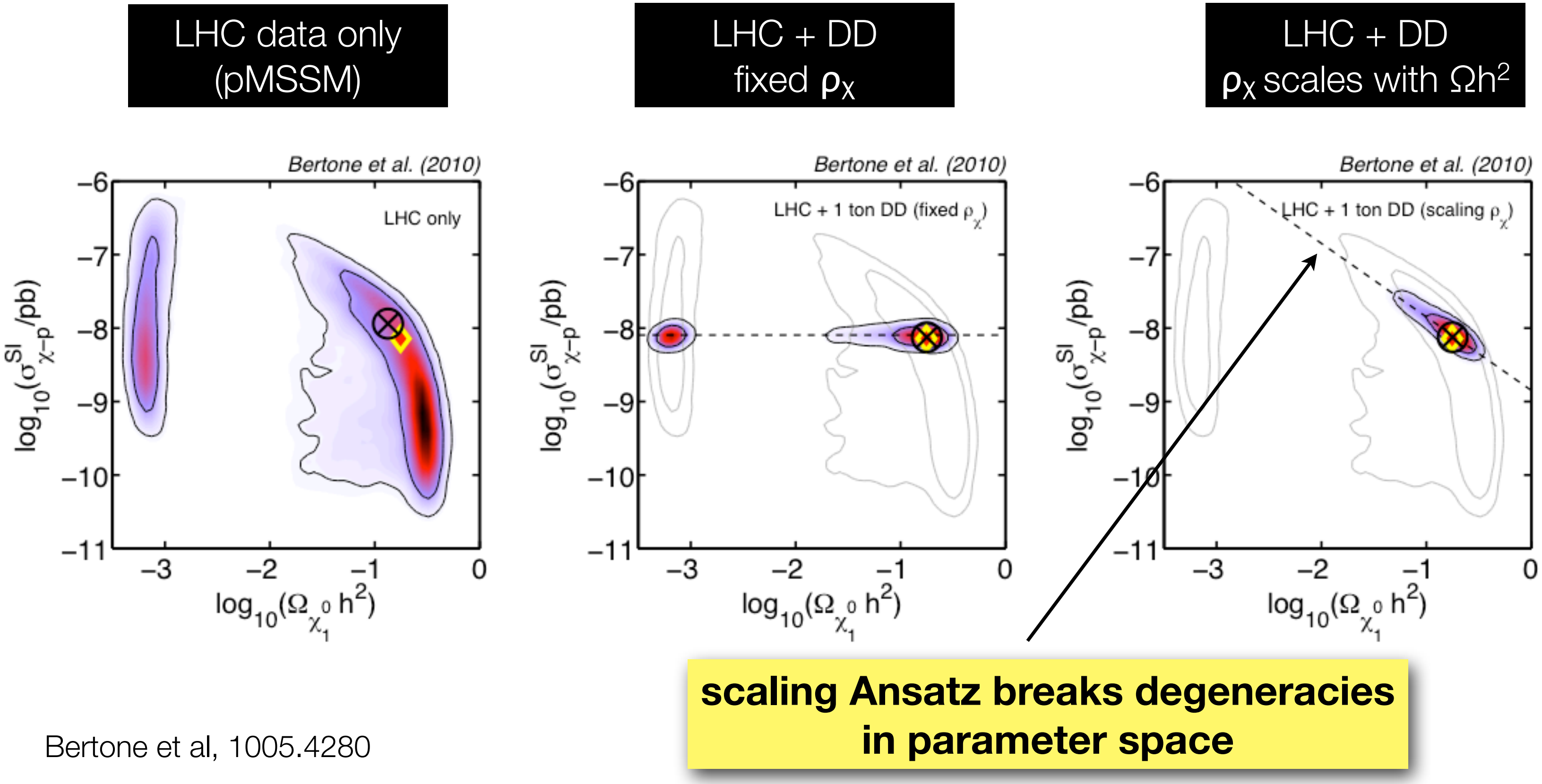
- Strategy: assume that the local density scales with the cosmological relic abundance (*scaling Ansatz*):

$$\rho_\chi \propto \Omega h^2_\chi$$




Bertone et al, 1005.4280

# Physics conclusions

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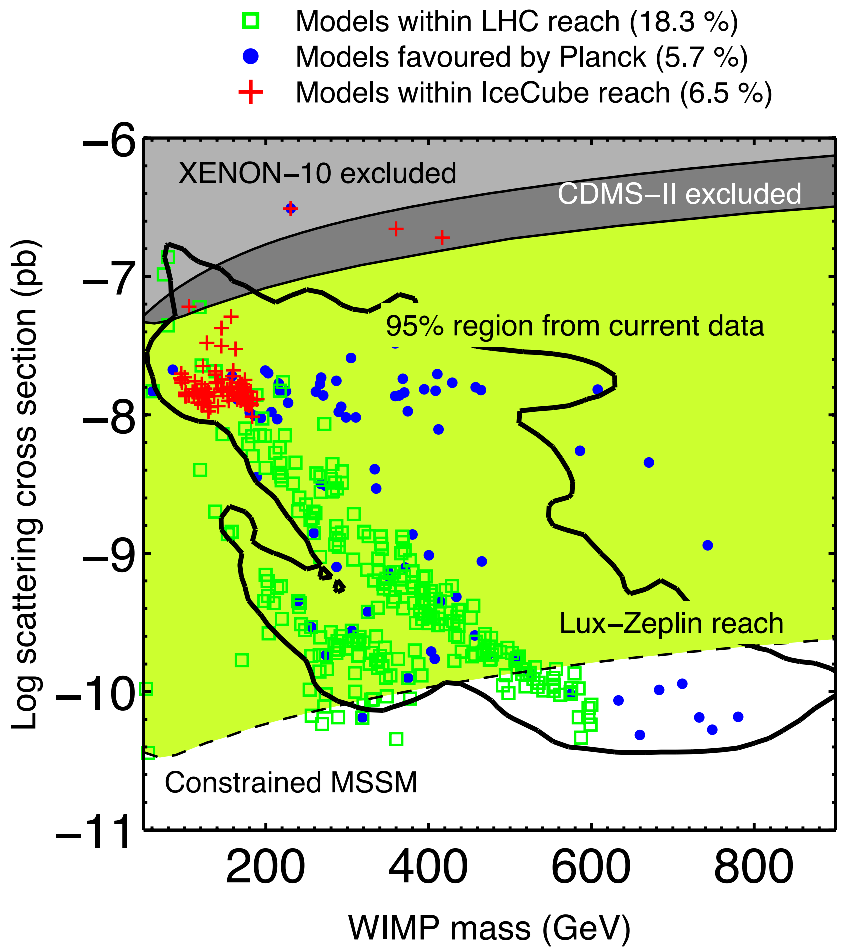
- No single probe can cover the whole favoured parameter space, not even the LHC.
- Astroparticle probes (direct and indirect detection) can increase the coverage of the favoured parameter space, and deliver increased statistical robustness.
- High complementarity of LHC reach with direct detection methods.




Cumberbatch, RT et al (in prep)

Thank you!