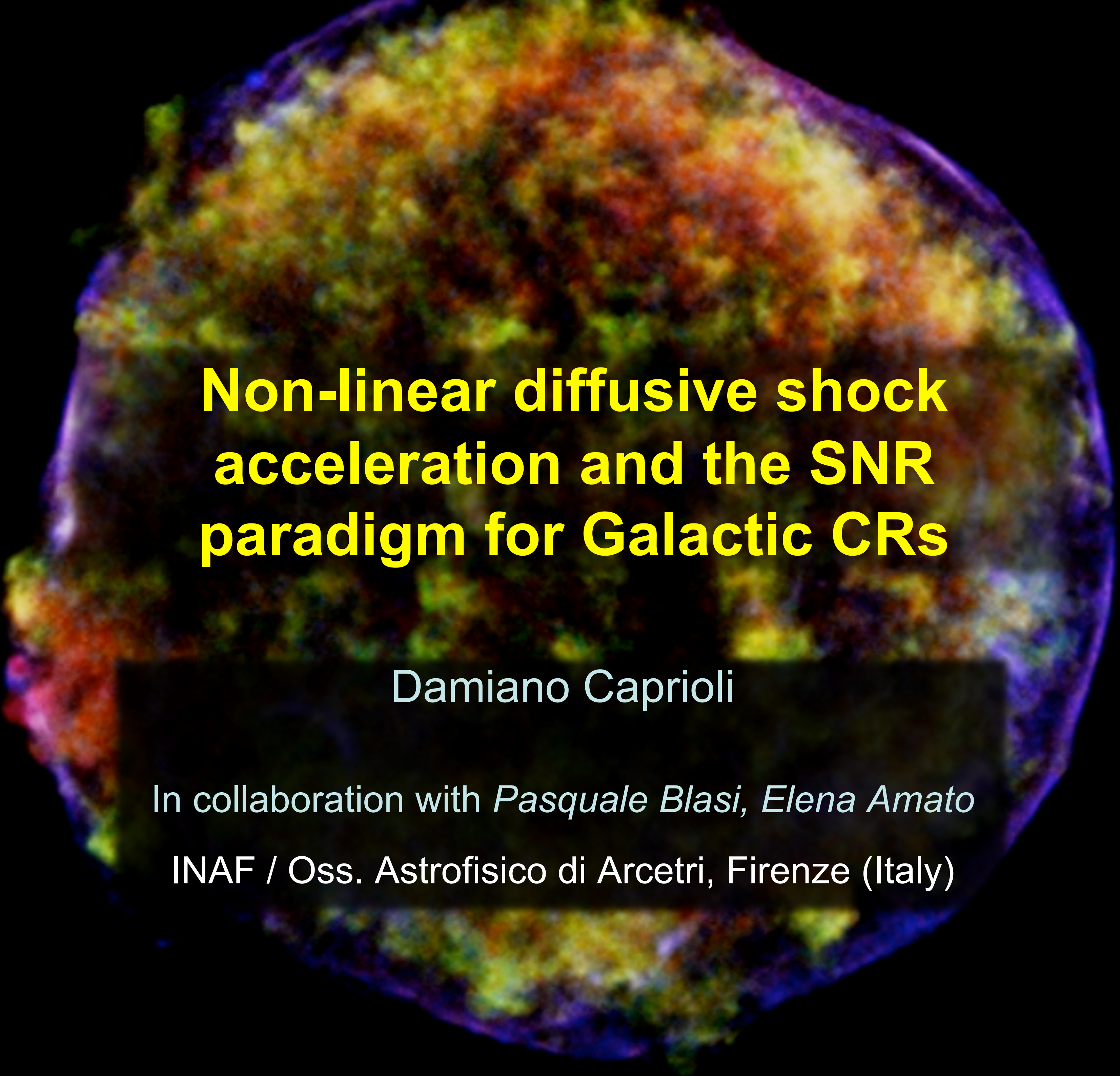


Non-linear diffusive shock acceleration and the SNR paradigm for Galactic CRs

Damiano Caprioli

In collaboration with *Pasquale Blasi, Elena Amato*

INAF / Oss. Astrofisico di Arcetri, Firenze (Italy)

The SNR paradigm for galactic CRs

- SNe may account for Galactic CR energetics
- Diffusive Shock Acceleration provides power law spectra (E^{-2}) with the *correct* index

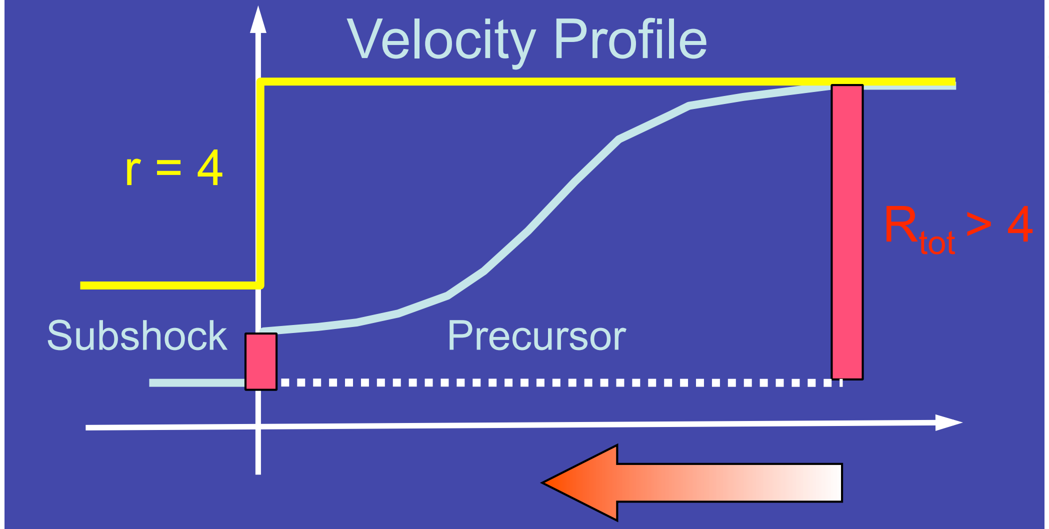
BUT

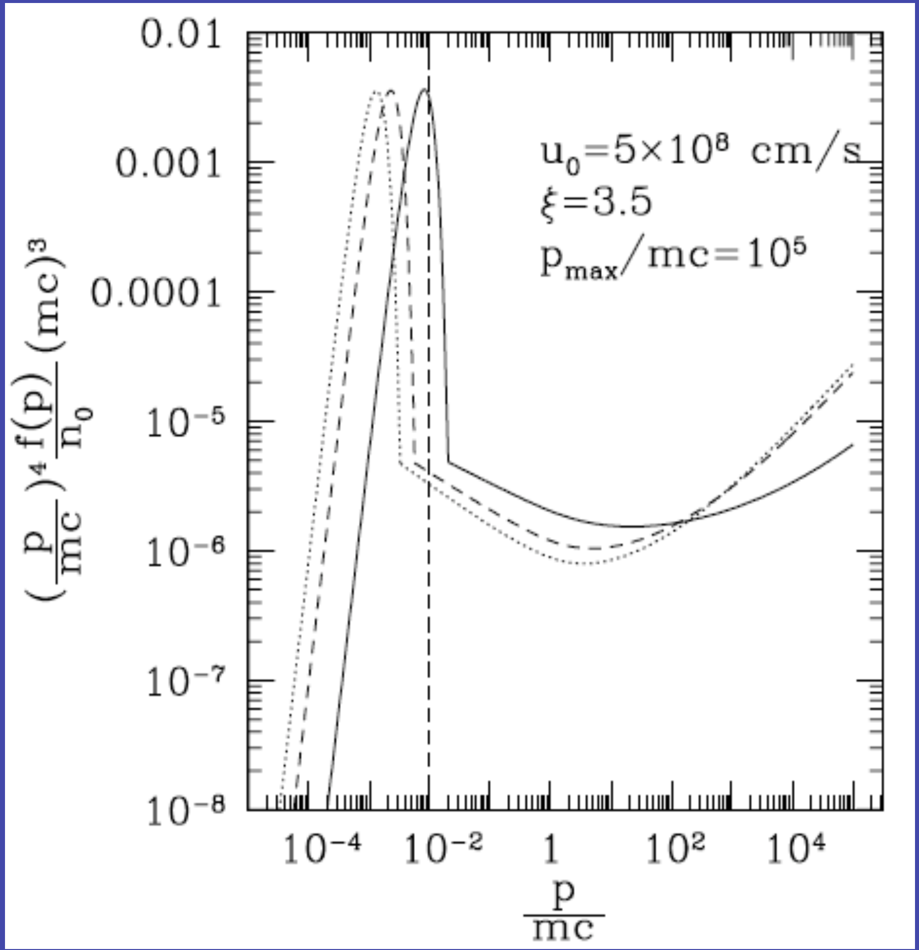
- Are CRs passive spectators of the shock dynamics?
- What is the maximum energy achievable in SNRs?
- How are particles released in the Galaxy?

Need for a Non-Linear theory of DSA

CR-modified shocks

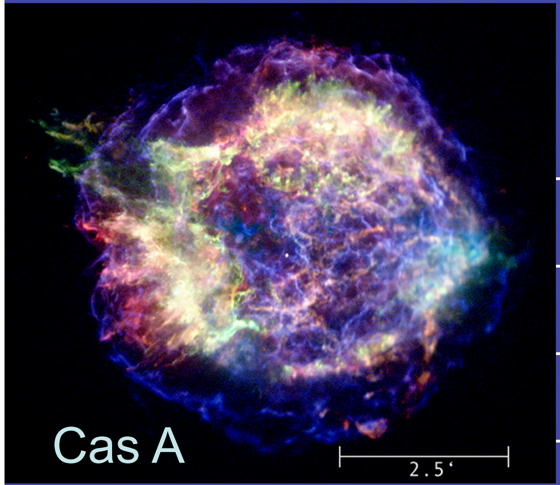
- CR pressure around the shock: the upstream fluid is slowed down and becomes more compressible

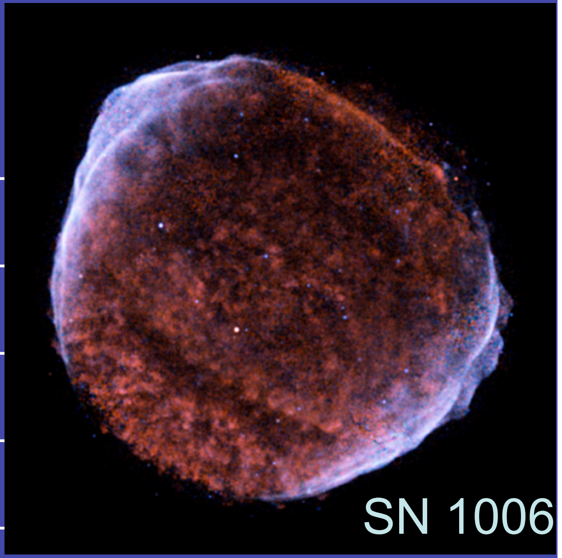




- *"Standard"* calculations leads to very efficient acceleration (R_{tot}~10-100)
- The spectra of the accelerated particles is concave (and even as flat as $E^{-1.2}$)
  - At odds with multi-wavelength observations!

Magnetic Field Amplification





The width of the rims requires

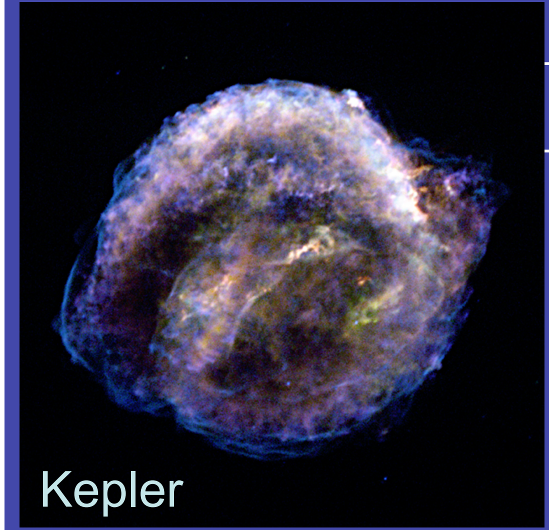
- $B_{ds} \approx$ 70-500 μG $\gg B_0$

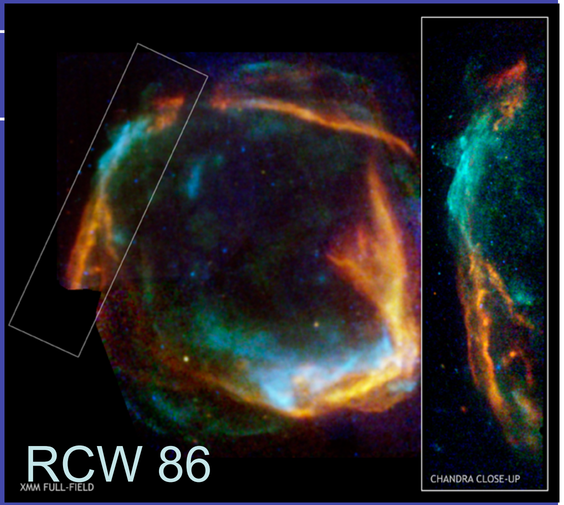
SNR	B_{ds} (μG)	$P_{B,ds}$(%)
Cas A	250-390	3.2-3.6
Kepler	210-340	2.3-2.5
Tycho	240-530	1.8-3.1
SN1006	90-110	4.0-4.2
RCW 86	75-145	1.5-3.8

Völk, Berezhko & Ksenofontov 2005

Parizot et al. 2006

The downstream magnetic pressure is at most 2 - 4% of the bulk pressure





But upstream P_B very likely dominates over P_{gas}, since:

$$\frac{B^2}{8\pi} > nkT \Rightarrow B > 6\mu G \, \mathrm{n}^{1/2} \left(\frac{\mathrm{T}}{10^4 K} \right)$$

The dynamical feedback of MFA

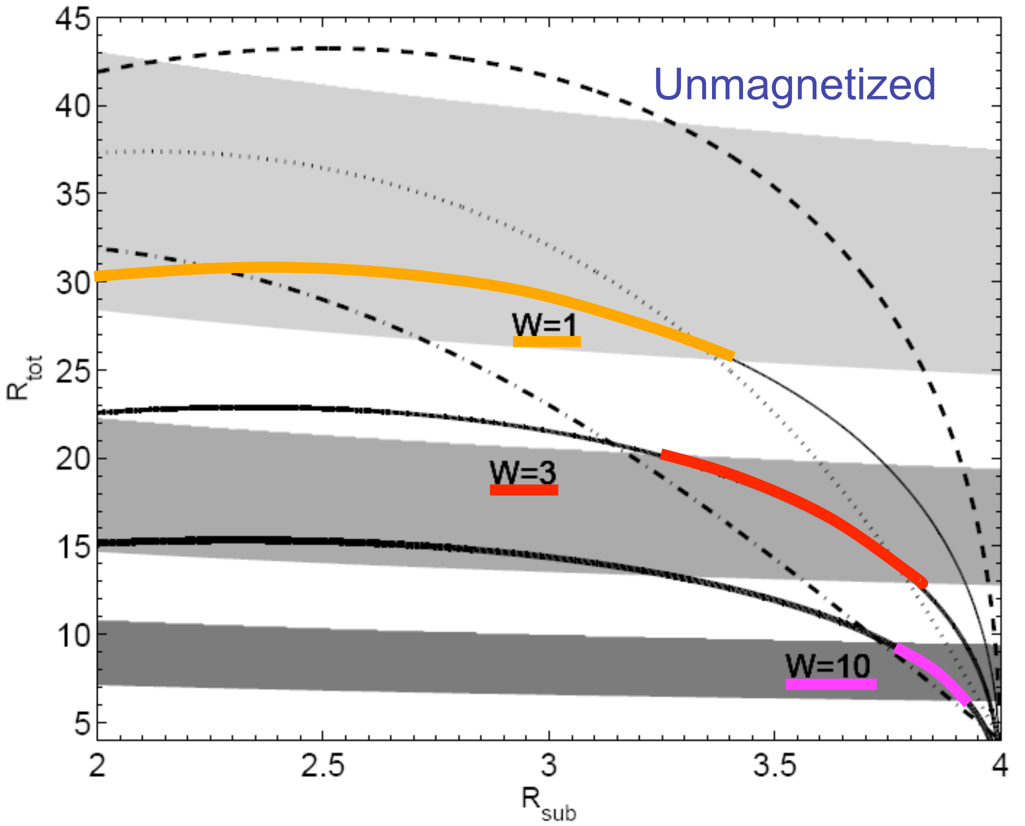
- Three-fluid model with Alfvén waves excited by streaming instability

$$R_{tot}^{\gamma+1} = \frac{M_0^2 R_{sub}^{\gamma}}{2} \left[\frac{\gamma + 1 - R_{sub}(\gamma - 1)}{1 + \Lambda_B} \right]$$

$$\Lambda_B = W \left[1 + R_{sub} \left(2/\gamma - 1 \right) \right]$$

$$W = \frac{P_{B,ups}}{P_{gas,ups}}$$

Ratio between magnetic and plasma pressure upstream

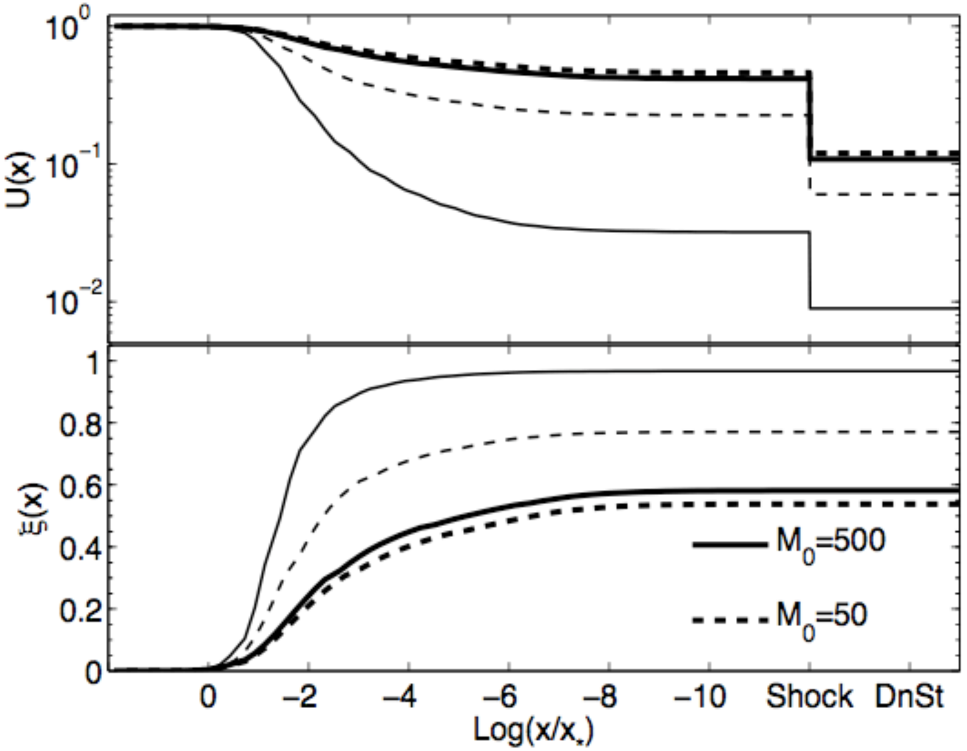


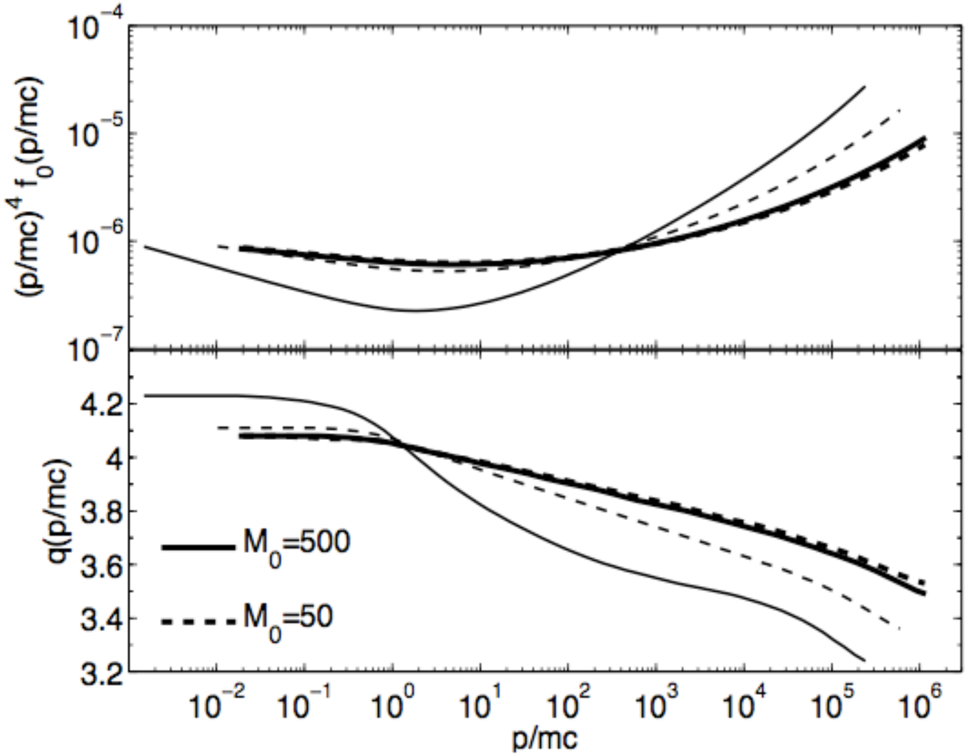
DC, Blasi, Amato,Vietri 2008

- In young SNRs: W ≈ 1-100

The magnetic turbulence feedback cannot be neglected and provides a smoothening of the precursor

Magnetic feedback on the spectra





U_0=5900 km/s; SNR age=1000yr

DC, Blasi, Amato, Vietri 2009

$T_0(\mathrm{K})$	Λ_B	ξ_1	$p_{max}(10^6 GeV)$	R_{sub}	R_{tot}	$T_2(10^6\,\mathrm{K})$
10^4	No	0.97	0.24	3.58	112.1	0.88
10^4	Yes	0.58	1.17	3.84	9.22	126.5
10^6	No	0.77	0.59	3.76	16.6	42.3
10^6	Yes	0.54	1.14	3.84	8.44	154.8

Turbulent (Alfvèn) Heating

- Often explained as due to non-linear Landau damping of the magnetic turbulence and invoked in order to reduce the precursor, but it:
  - Is expected to be relevant only if $V_{sh} < 4000\,(T/10^5\,K)^{1/2}$ km/s

    Völk & McKenzie 1981; Ptuskin & Zirakasvhili 2005
  - Cannot be too efficient, otherwise no MFA!!

$$\zeta = \Gamma_{damp} / \Gamma_{growth} < 1$$

$$R_{tot}^{\gamma+1} = \frac{M_0^2 R_{sub}^{\gamma}}{2} \left[\frac{\gamma + 1 - R_{sub}(\gamma - 1)}{(1+\Lambda_B)(1+\Lambda_{TH})} \right]$$

$$\Lambda_{TH} = \zeta(\gamma - 1)\frac{M_0^2}{M_A}\left[1 - \left(\frac{R_{sub}}{R_{tot}}\right)^{\gamma}\right]$$

ζ	ξ_1	$p_{max}(10^6 GeV)$	R_{sub}	R_{tot}	B_1/B_0	W	$B_2(\mu G)$	$T_2(10^6\ \mathrm{K})$
0	0.60	1.17	3.76	9.52	25.3	1.941	475.6	114.6
0.5	0.66	0.84	3.65	10.96	20.8	0.390	379.6	132.6
0.8	0.65	0.53	3.68	10.76	12.8	0.115	232.5	128.3
0.99	0.55	0.12	3.85	8.69	2.26	0.005	43.5	162.2

B_0=10 μG; Age=1000 yr; $T_0=10^5$ K

DC, Blasi, Amato, Vietri 2009

May lead to a too large downstream temperature

and too large thermal emissivity, see RX J1713.7-3946)

Kinetic approaches to NLDSA

- ***MONTE CARLO***: account for CR anisotropy
  - Jones, Ellison 1991; Ellison et al. 1990;1995; Vladimirov, Ellison, Bykov 2006

- ***FULLY NUMERICAL***: time-dependent
  - Kang, Jones 1997;2005;2008; Berezhko, Völk 1997;2004;2007; Zirakashvili, Aharonian 2009; Ptuskin, Zirakashvili, Seo 2010

- ***SEMI-ANALYTICAL***: versatile, computationally extremely fast
  - Malkov 1997; Blasi 2002; 2004; Amato, Blasi 2005; 2006, DC et al. 2009; 2010b

- *All methods* require an *a priori* description of:
  - Particle transport (diffusion and convection)
  - Magnetic field amplification
  - Injection into the acceleration process
  - Particle escape from the source

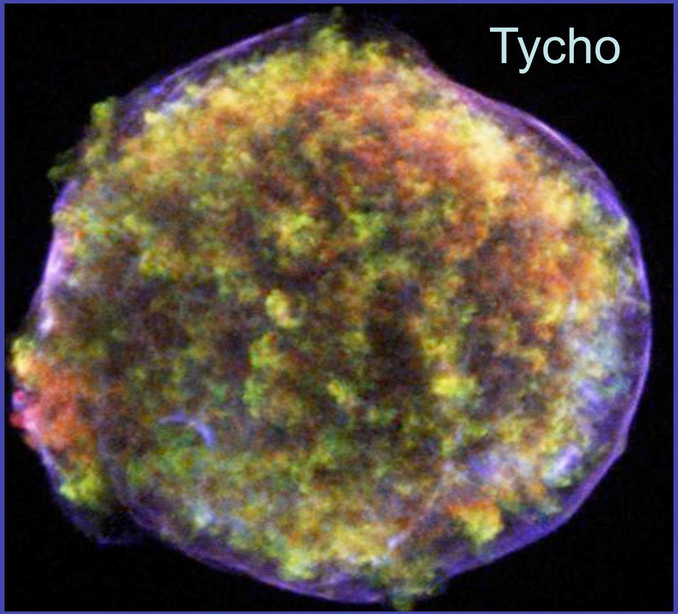
- For reviews on NLDSA see e.g. :

Drury 1983; Blandford,Heicler 1987; Jones, Ellison 1991; Malkov, Drury 2002

Why semi-analytical?

- The developed formalism is a very powerful tool since it:
  - is very fast (a run takes 10”-1’ on a laptop)
  - has virtually no dynamical range limitation on P_{max}, M_0, …
  - allows to scan a wide range of environmental parameters
  - allows the inclusion of nuclei

- Applications to SNR shocks:
  - Hydro + Multi-wavelength analysis of single SNRs
  - Test the SNR paradigm for the origin of galactic CRs



A semi-analytic approach

- Solution of the stationary diffusion-convection equation
  - With momentum boundary p_{max} (Amato & Blasi 2005; 2006; Blasi, Amato & DC 2007)
  - With escape boundary x_0 (DC, Amato & Blasi 2010b)

$$\tilde{u}(x)\frac{\partial f_i(x,p)}{\partial x} = \frac{\partial}{\partial x}\left[D_i(x,p)\frac{\partial f_i(x,p)}{\partial x}\right] + \frac{p}{3}\frac{\mathrm{d}\tilde{u}(x)}{\mathrm{d}x}\frac{\partial f_i(x,p)}{\partial p} + \frac{\eta_i n_0 u_0}{4\pi p_{inj,i}^2}\delta(p-p_{inj,i})\delta(x)$$

CR transport equation

Injection

$$f_i(x,p) = f_{sh,i}(p)\exp\left[-\int_x^0 \mathrm{d}x'\frac{\tilde{u}(x')}{D_i(x',p)}\right]\left[1-\frac{W_i(x,p)}{W_{i,0}(p)}\right];$$

$$\phi_{esc,i}(p) = -\frac{\tilde{u}_0 f_{sh,i}(p)}{W_{i,0}(p)},$$

$$W_i(x,p) = \tilde{u}_0\int_x^0 \frac{\mathrm{d}x'}{D_i(x',p)}\exp\left[\int_{x'}^0 \mathrm{d}x''\frac{\tilde{u}(x'')}{D_i(x'',p)}\right]$$

$$U_{p,i}(p) = \tilde{U}_1 - \int_{x_0}^0 \mathrm{d}x\frac{\mathrm{d}\tilde{U}(x)}{\mathrm{d}x}\frac{f_i(x,p)}{f_{sh,i}(p)}$$

$$f_{sh,i}(p) = \frac{\eta_i n_0}{4\pi p_{inj,i}^3}\frac{3R_{tot}}{\tilde{R}_{tot}U_{p,i}(p)-1}\exp\left\{-\int_{p_{inj,i}}^{p}\frac{\mathrm{d}p'}{p'}\frac{3\tilde{R}_{tot}}{W_{0,i}(p')}\frac{W_{0,i}(p')U_{p,i}(p')-1}{\tilde{R}_{tot}U_{p,i}(p')-1}\right\}$$

$$\xi_c(x) = 1+\frac{1}{\gamma_g M_0^2} - U(x) - \frac{1}{\gamma_g M_0^2}U(x)^{-\gamma_g}$$

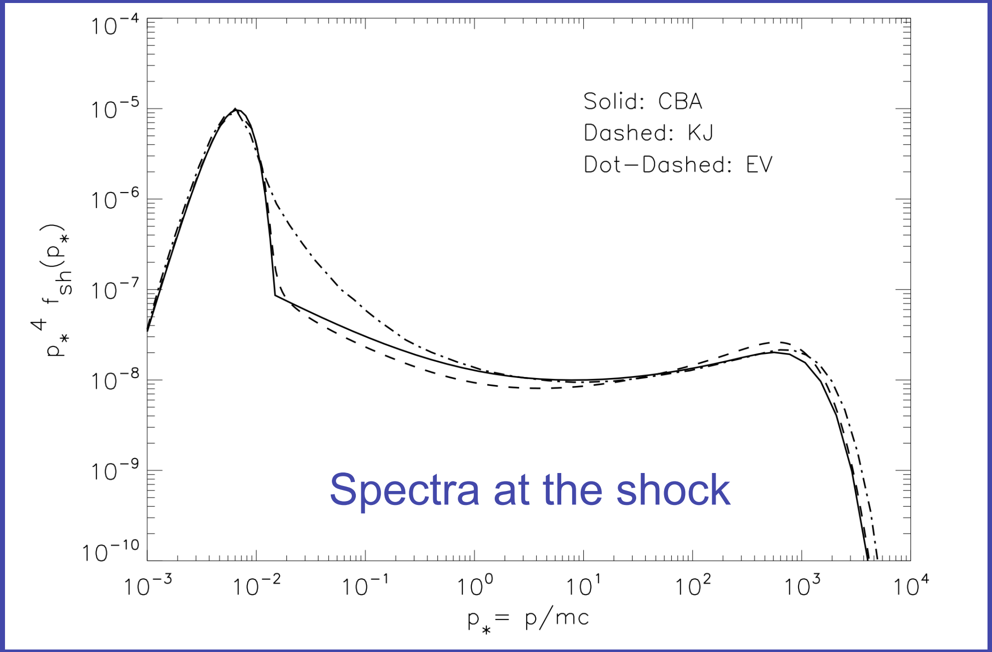
$+$

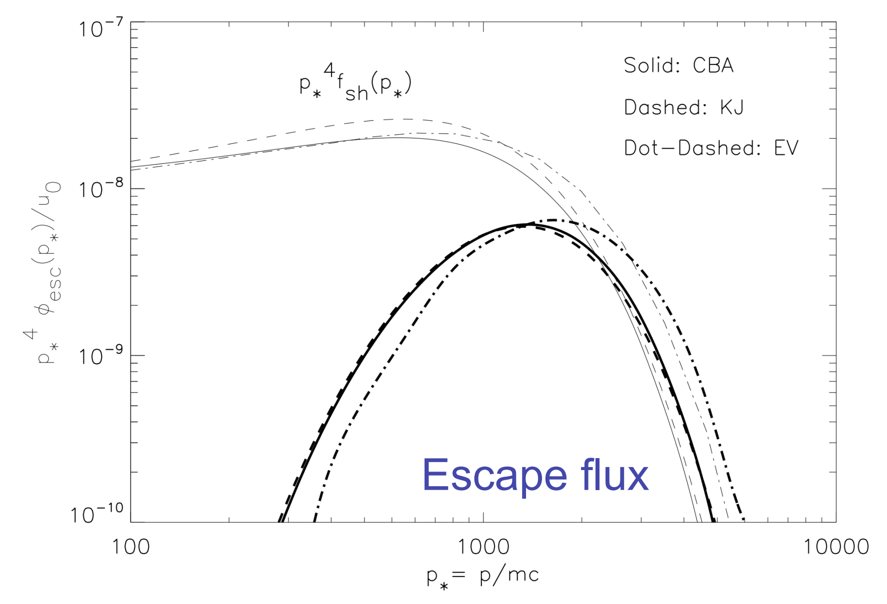
$$\xi(x) = \frac{1}{3\rho_0 u_0^2}\int_{p_{inj}}^{p_{max}}\mathrm{d}p\,4\pi\,p^3 v(p) f(x,p)$$

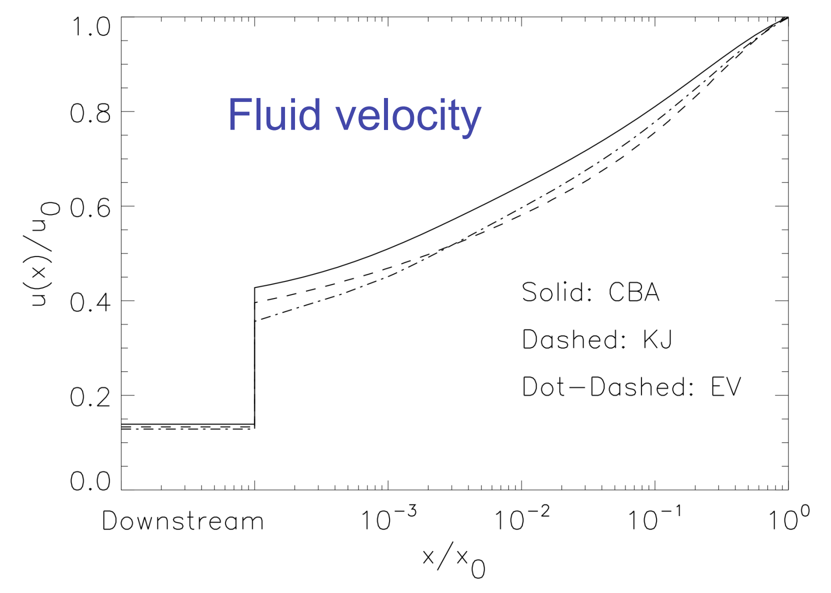
Momentum conservation

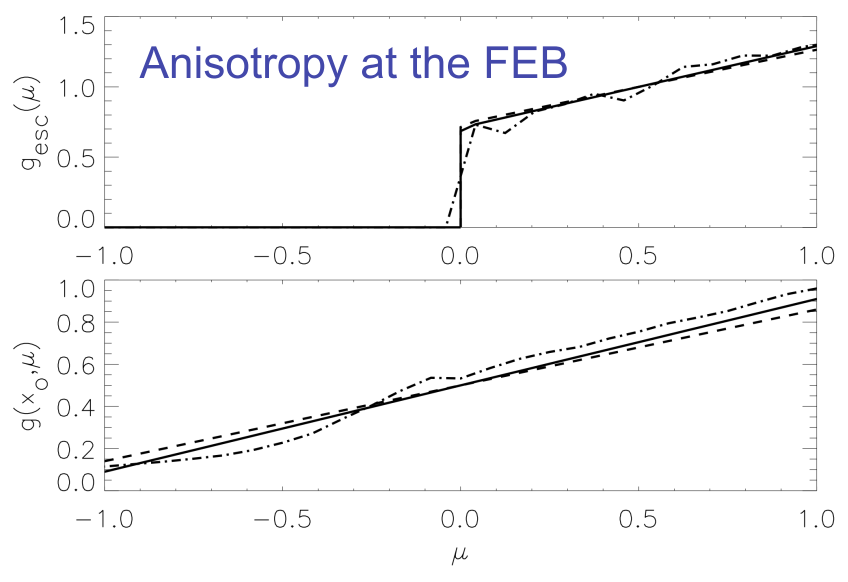
CR pressure

Vs Numerical and Monte Carlo approaches


Spectra at the shock


Escape flux


Fluid velocity


Anisotropy at the FEB

DC, Kang, Vladimirov, Jones 2010

Scattering centre velocity

- The velocity of the scattering centres naturally enters the transport equation

$$\tilde{u}(x)\frac{\partial f(x,p)}{\partial x} = \frac{\partial}{\partial x}\left[D(x,p)\frac{\partial}{\partial x}f(x,p)\right] + \frac{d\tilde{u}(x)}{dx}\frac{p}{3}\frac{\partial f(x,p)}{\partial p} + Q(x,p)$$

$$\tilde{u}(x) = u(x) + v_W$$

How does v_W depend on the nature of the turbulence?

- It strongly affects the CR spectrum:

$$R_{sub} = \frac{u_1 + v_{w1}}{u_2 + v_{w2}} \qquad R_{tot} = \frac{u_0 + v_{w0}}{u_2 + v_{w2}}$$

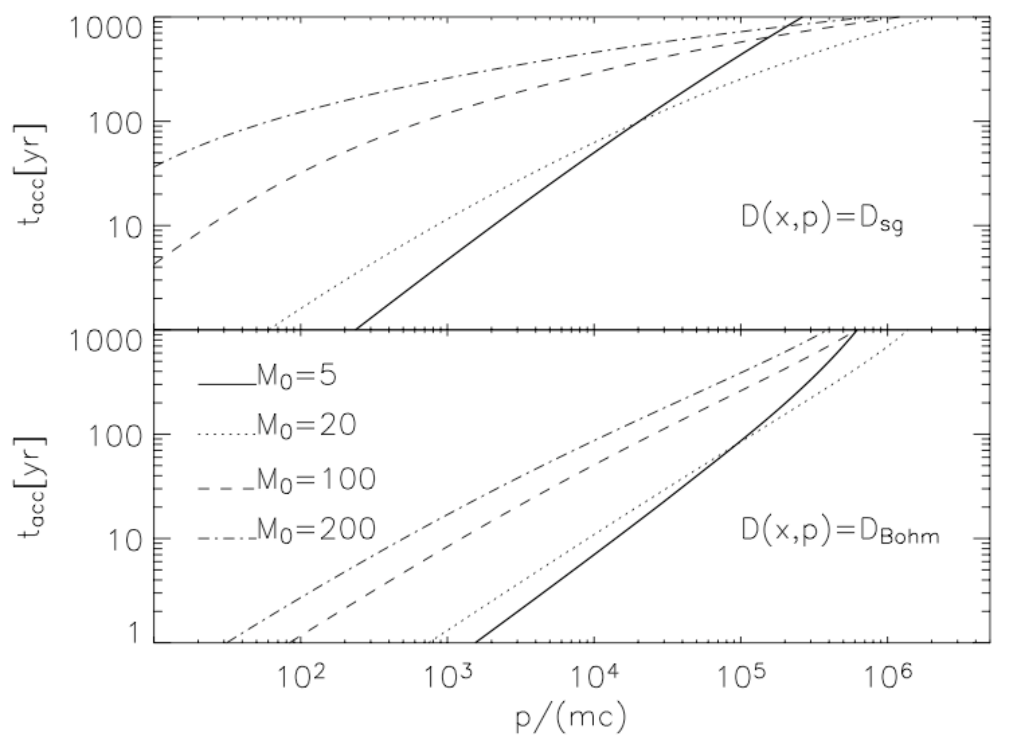
- Resonant streaming instability (Skilling 1975)
  - *UPSTREAM*: countergoing Alfvèn waves excited by CRs
  - *DOWNSTREAM*: isotropy? Reflection + transmission? Other instabilities?
  - In the background field or in the amplified field?

- Evidences of magnetic field amplification suggest:

$$v_w = -v_A = -\frac{\delta B}{\sqrt{4\pi\rho}} \approx -(0.01 \div 0.1)u$$

From accelerated particles to CRs

- Ejecta dominated stage
  - The magnetic turbulence and P_{max} increase with time
- Sedov-Taylor stage
  - V_{sh}, P_{max} and δB decrease, and so does the SNR confining power
  - Particles with momentum close to $P_{max}(t)$ escape the system



Blasi, Amato, DC 2007

- For constant $F_{esc}(t)$ and $R_{sh} \alpha\ t^{2/5}$ i.e. the adiabatic self-similar solution:

$$d\mathcal{E}(t) = \mathcal{F}_{esc}(t)\frac{1}{2}\rho V_{sh}^3(t) 4\pi R_{sh}(t)^2 dt$$

$$d\mathcal{E}(p) = 4\pi p^2 N_{esc}(p) pc\, dp$$

$$N_{esc}(p) \propto p^{-4} t^{5\nu-2} \mathcal{F}_{esc}(t)$$

$$R_{sh}(t) \propto t^{\nu}$$

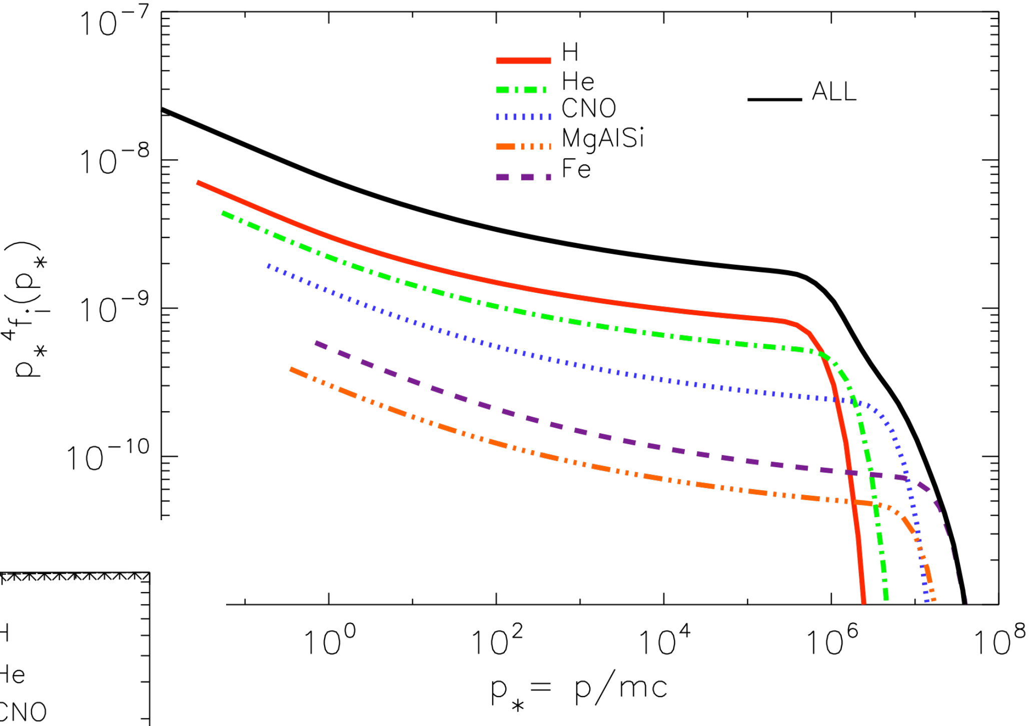
$$\boxed{N_{esc}(p) \propto p^{-4}}$$

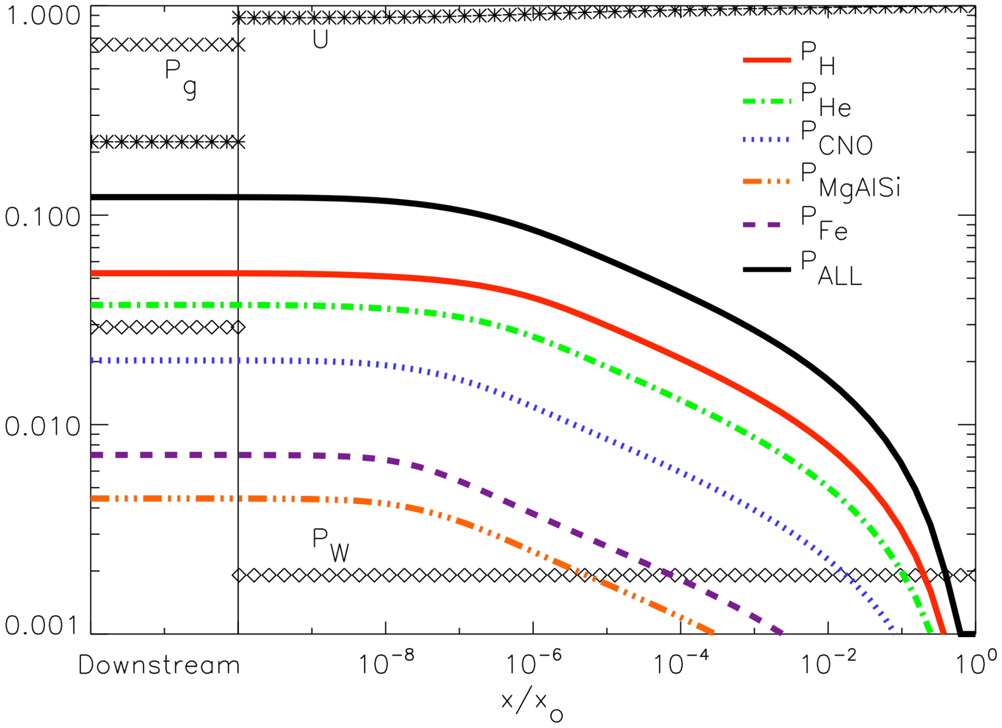
The released spectrum is the convolution over time of 3 contributions:
Escape from upstream+ Leakage from downstream + Relic advected CRs

DC, Amato, Blasi, 2010a

A snapshot from a benchmark SNR

- CSM density = 0.01 part/cm^{-3}
- CSM temperature = 10^{6} K
- Diffusion in the amplified magnetic field
- Chemical abundances tuned to fit the observed ones (Hörandel 2003; Blümer et al. 2009)





DC, Blasi, Amato astroph:/1007.1925

Nuclei are as relevant as protons for the shock dynamics!

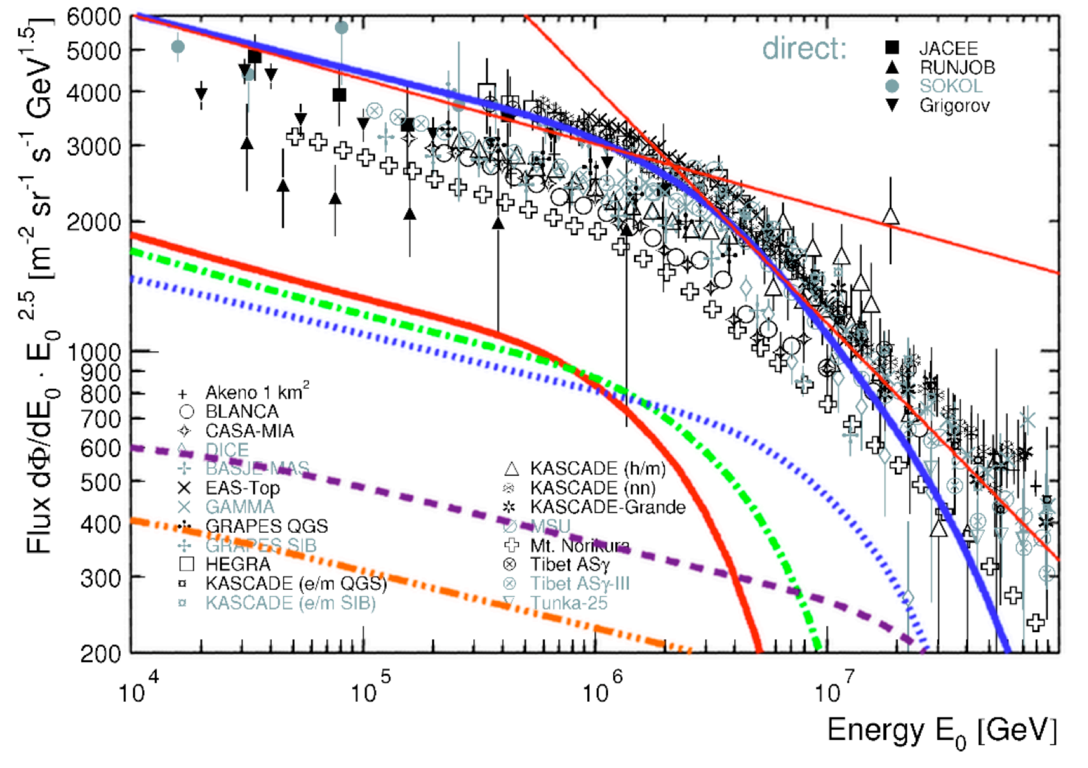
CRs at Earth

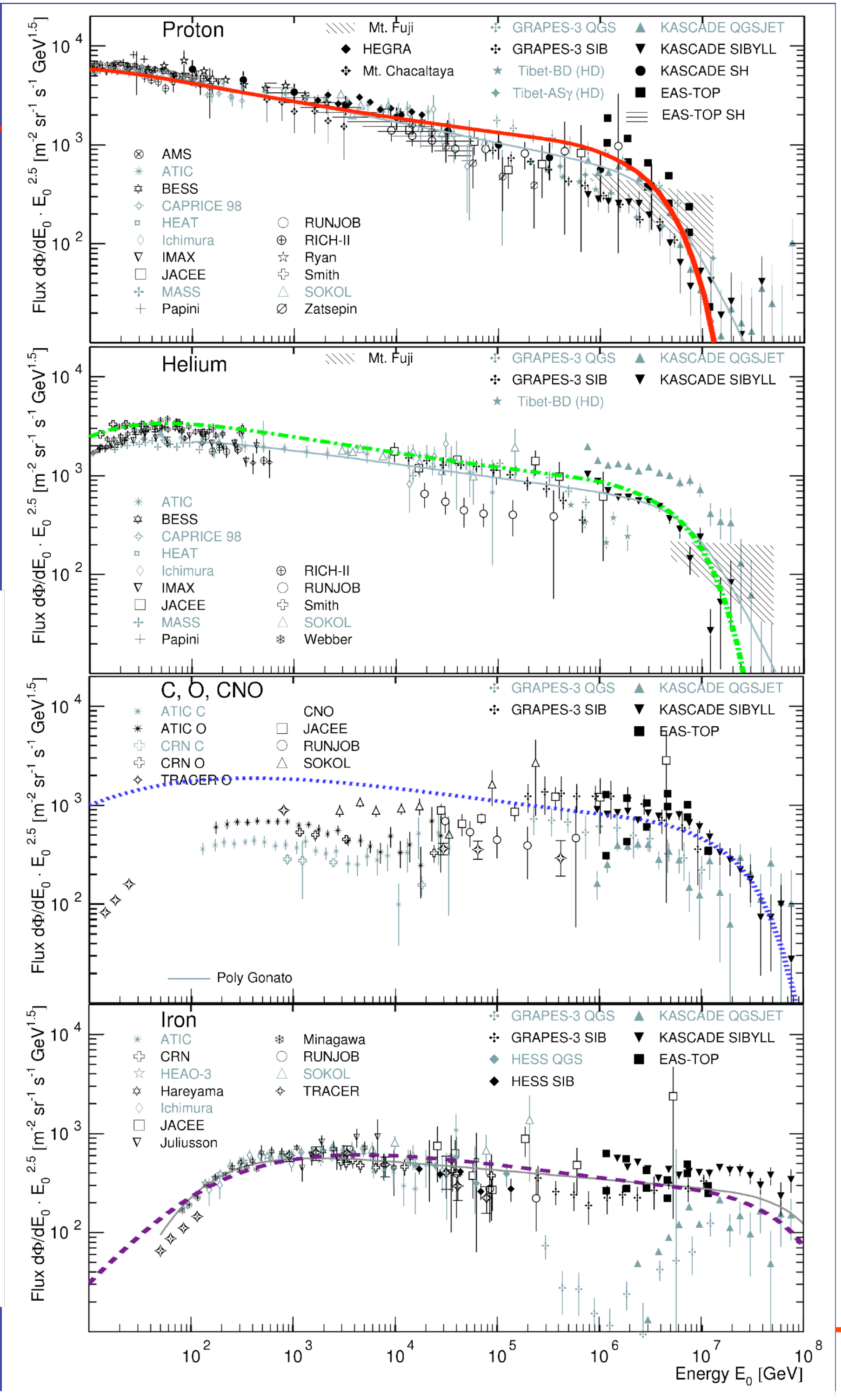
- Account for propagation in the Galaxy

$$\phi_{Earth}(E) = \frac{\eta_{SN} N_{SNR}(E)\tau_{esc}(E)}{4\pi R_{Gal}^2}$$

$$\tau_{esc}(E) = 15\,Myr\left(\frac{1}{Z}\frac{E}{10GeV}\right)^{-0.55} \quad ; \quad \eta_{SN} = \frac{3}{100\,yr}$$

- + spallation (at lower energies)





DC, Blasi, Amato astroph:/1007.1925

Open issues about SNR paradigm

- What is the contribution by Type I/II SNe?
  - Role of pre-SN stages (winds, hot bubbles, chemical composition…)
- What is the nature of magnetic turbulence in modified shocks?
  - Are they resonant and/or non-resonant modes? (Bell 2004)
  - Velocity of the scattering centres ➔ CR spectrum
  - Details of the magnetic feedback ➔ SNR hydrodynamics
- How does injection of heavy nuclei work?
  - C,O in molecular form, Fe in grain form…
- How is the diffusion around a SNR (Bohm-like or Galactic like)?
  - Relevant for predicting the spectrum illuminating Molecular Clouds