Formation of Large Scale Structure: the role of dark matter

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IAP

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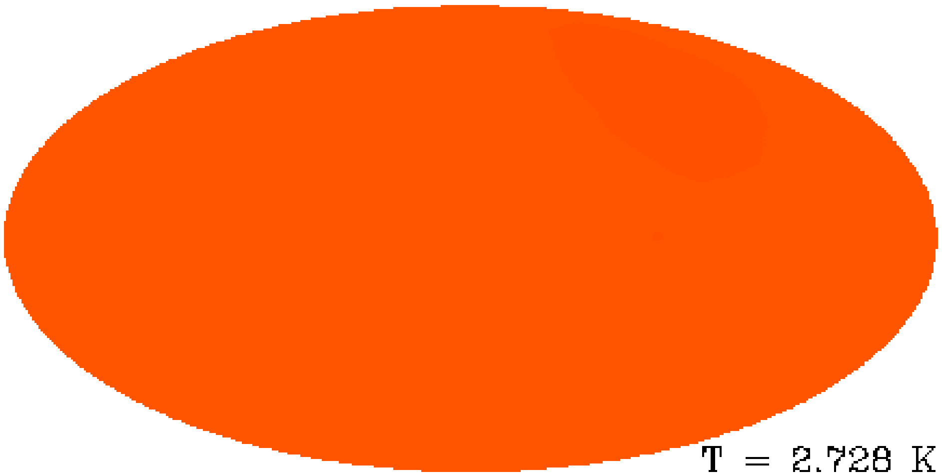
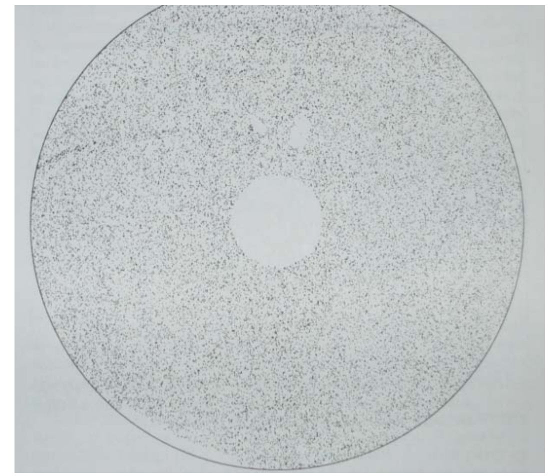
- The amount and properties of matter in the Universe changes the cosmic history of structure formation
- → observations of fluctuations (mass density, temperature) in the Universe may reveal
  - propreties of dark matter,
  - its nature ?

The Homogeneous and Isotropic Universe

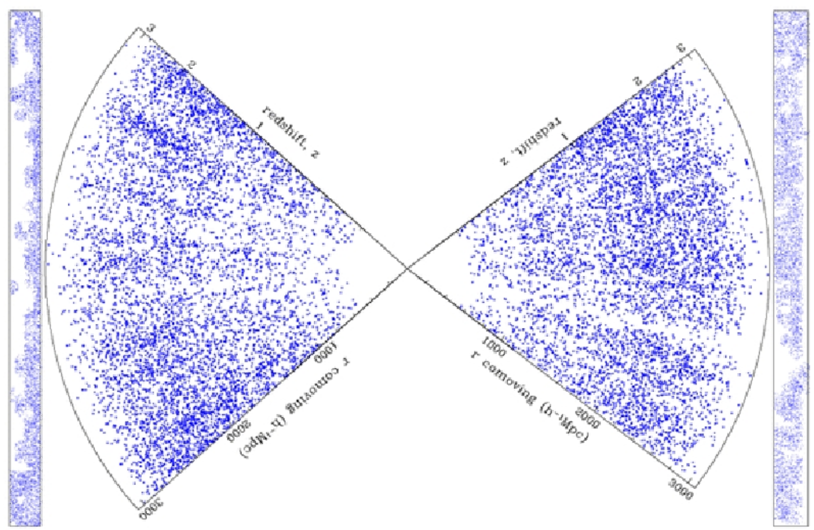
CMB



Radio galaxies



Quasars



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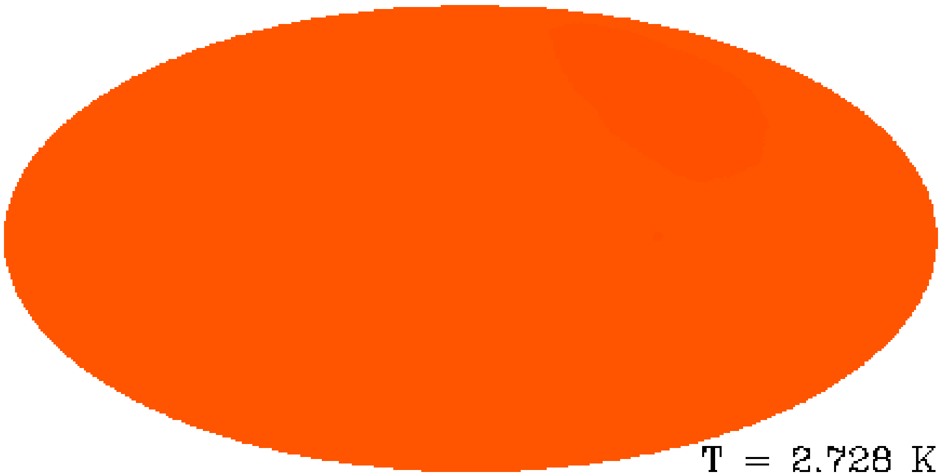
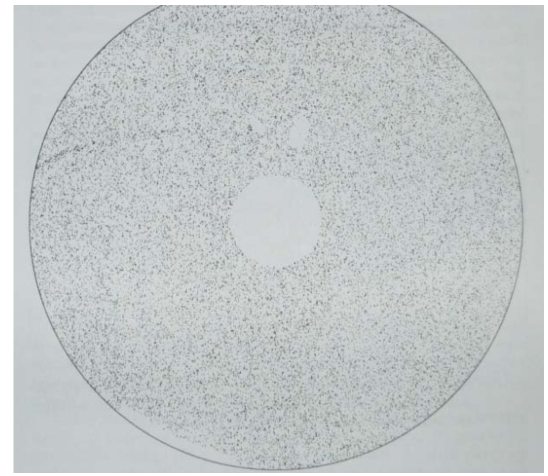


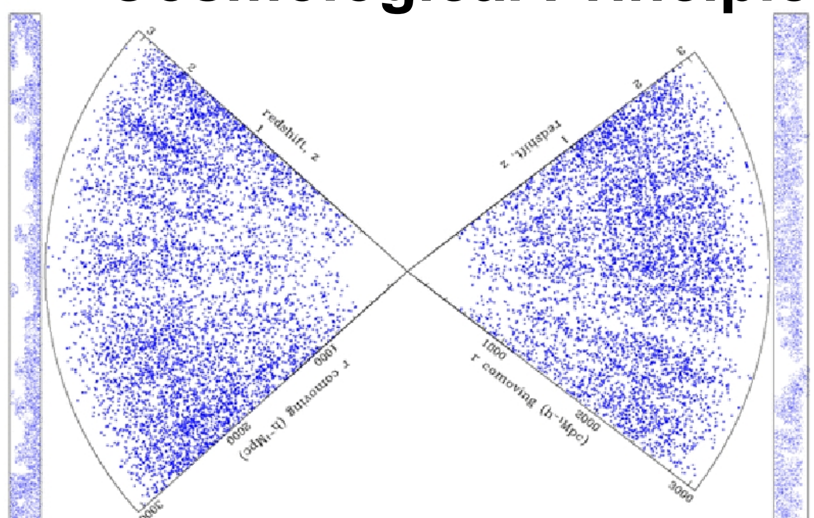
$T = 2.728$ K

Radio galaxies



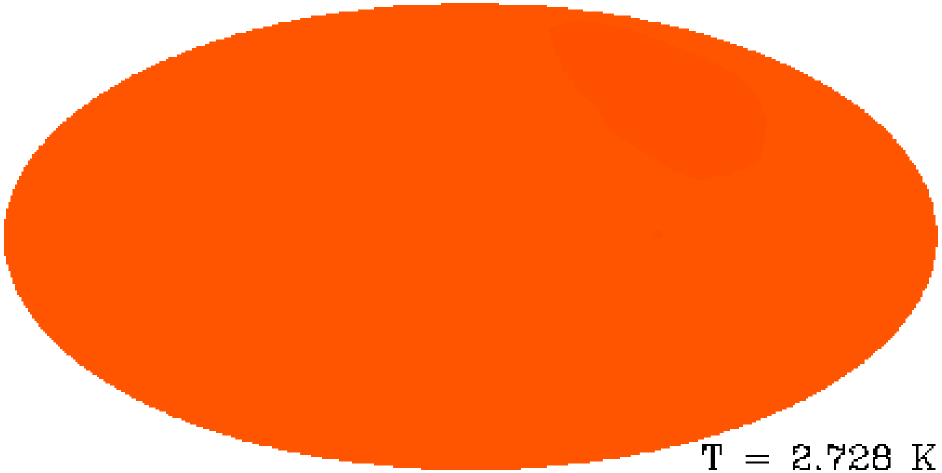
→ Cosmological Principle

Quasars



→ assume the cosmological principle valid

CMB



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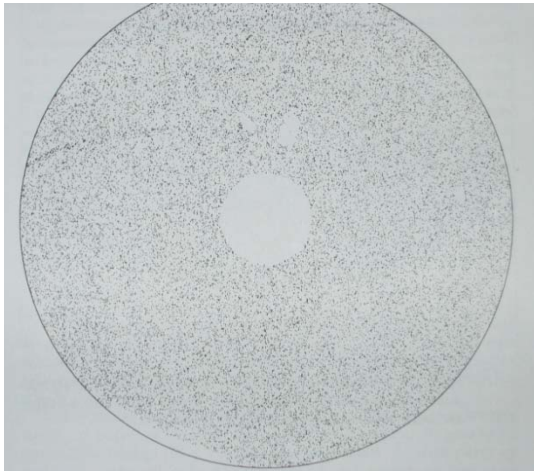

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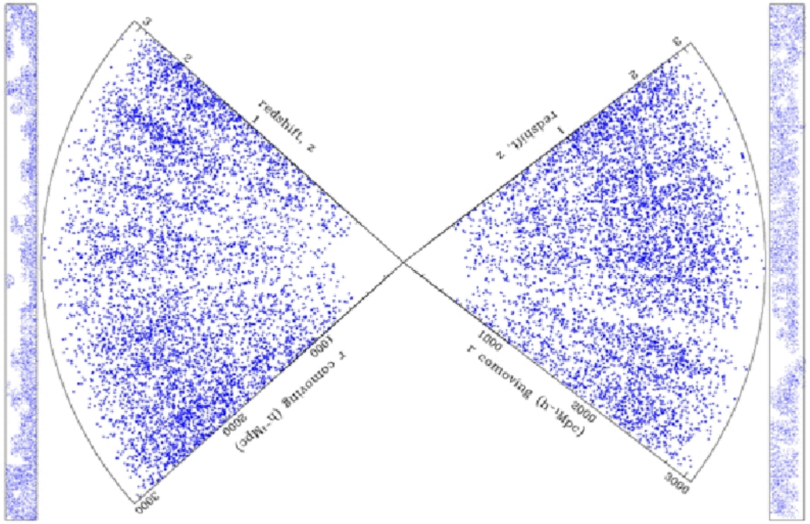

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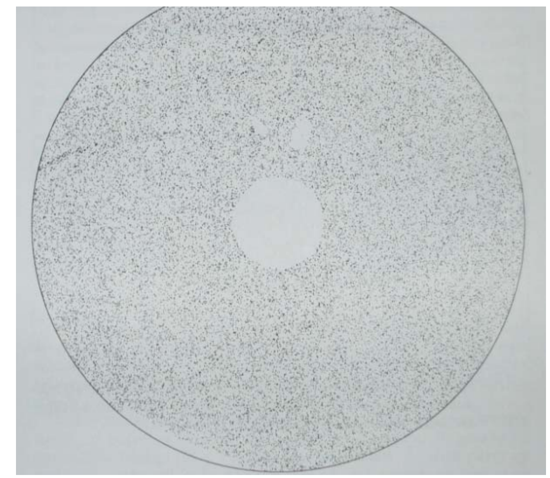
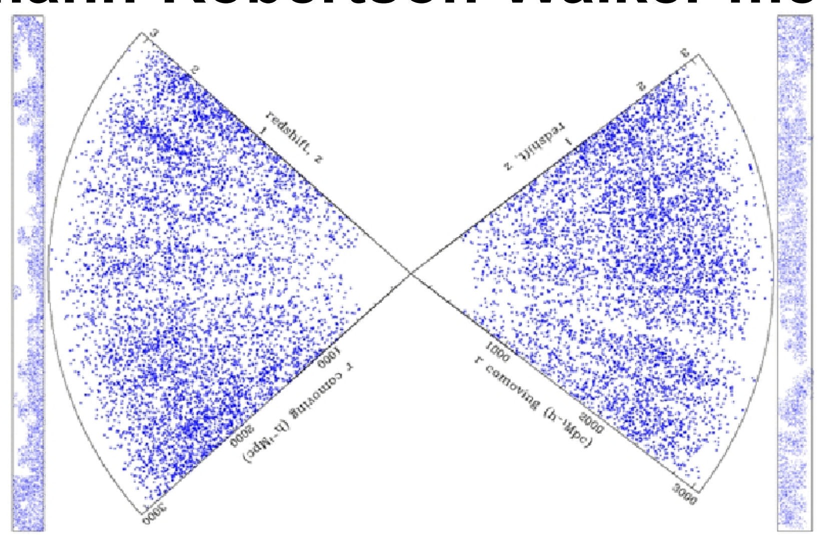


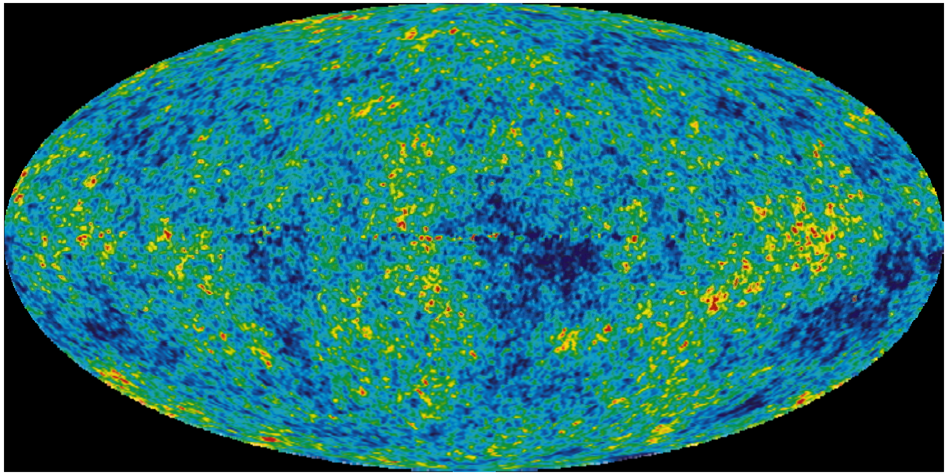
→ Friedmann-Robertson-Walker metrics

Quasars

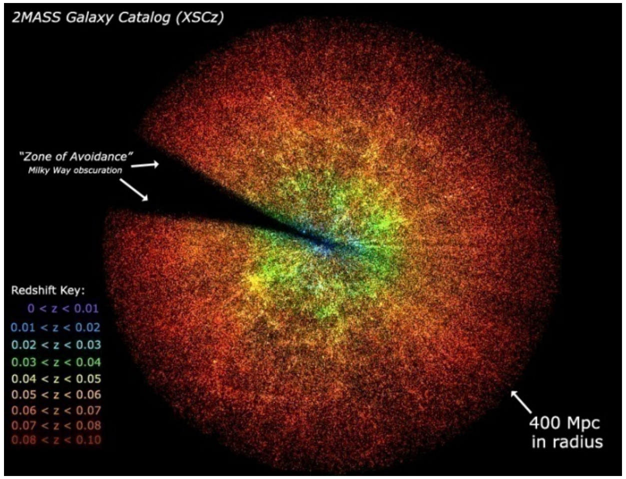


The Inhomogeneous Universe

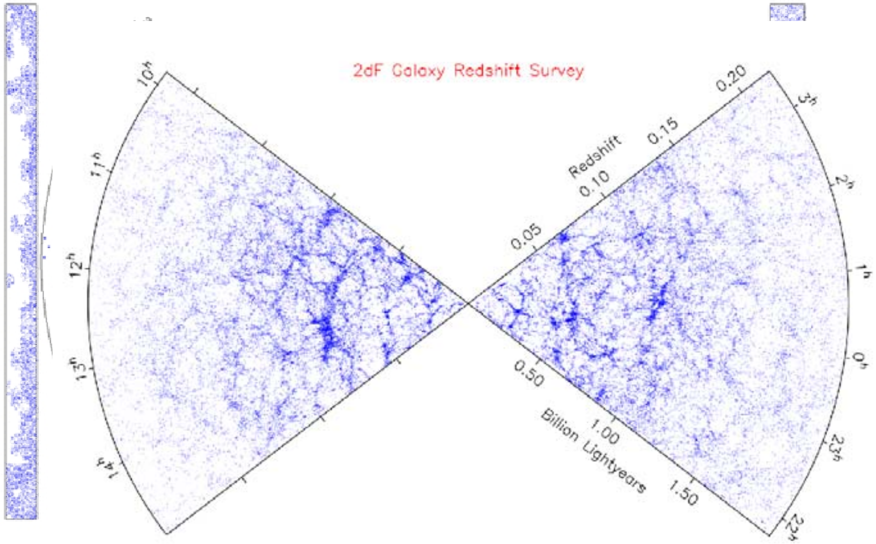
CMB WMAP-5



2MASS galaxies

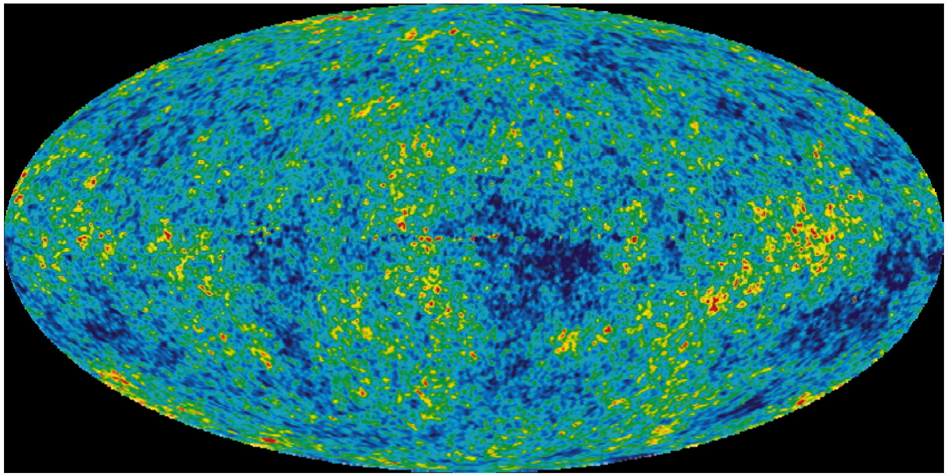


2dF Galaxies



The Inhomogeneous Universe

CMB WMAP-5 z=1000 $\Delta T/T = 10^{-5}$



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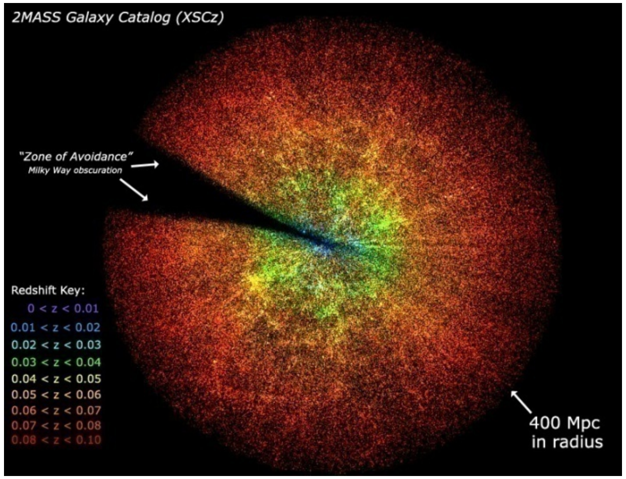
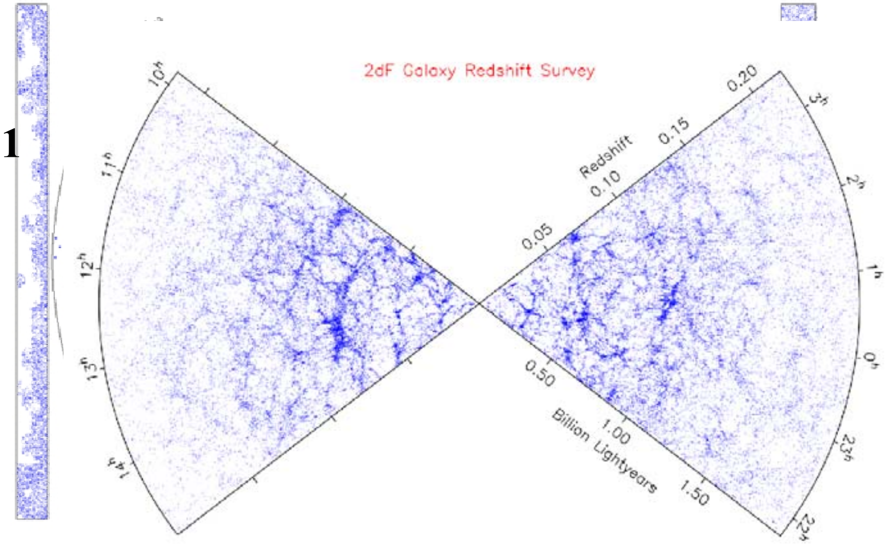


2dF Galaxies
z < 1 $\Delta\rho/\rho \gg 1$



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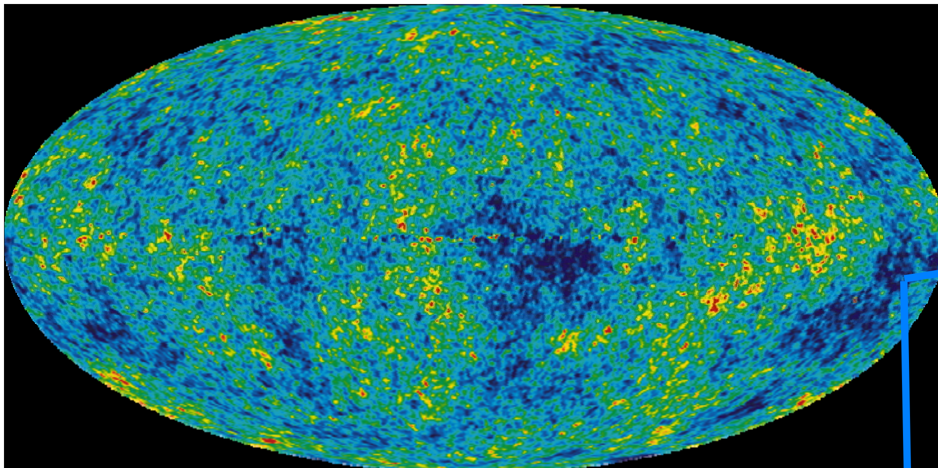

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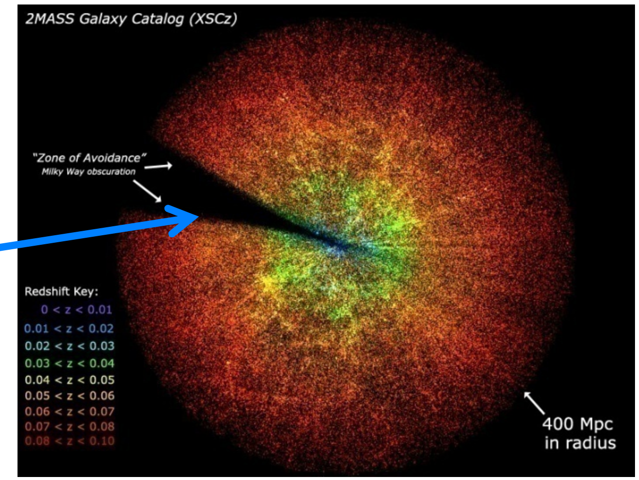
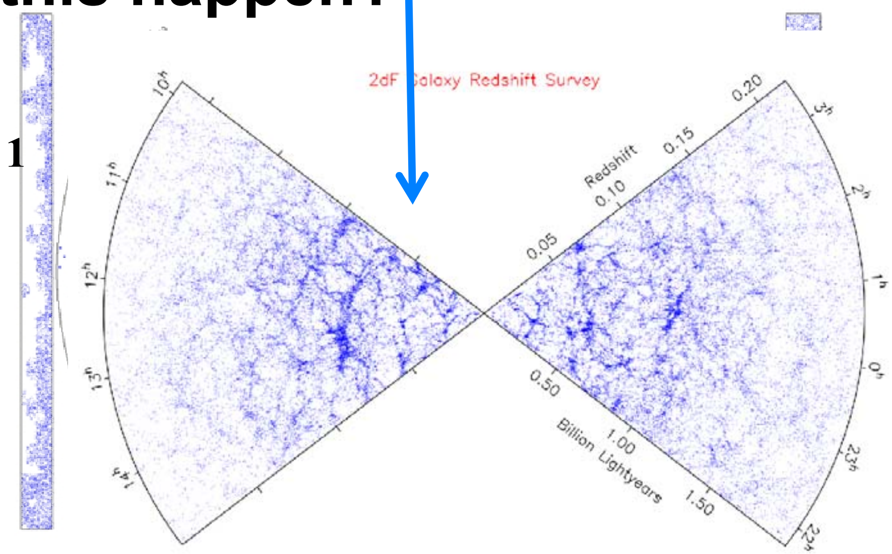


Why and how did this happen?

2dF Galaxies
z < 1 $\Delta\rho/\rho \gg 1$



The origin and formation of structures

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  - perturbation theory, linear growth of structures,
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- Theoretical description:
  - perturbation theory, linear growth of structures,
  - non linear evolution of structures,

    in an expanding, adiabatically cooling, universe
  - Thermal history of the universe is important:
    - 2 eras: radiation and matter dominated periods (= at z ~3500),
    - decoupling (the rate of Compton scattering is slower than the expansion of the Universe: baryons and photons are no longer coupled fuilds): decoupling at z ~ 1000

The origin and formation of structures

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- Theoretical description:
  - perturbation theory, linear growth of structures,
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    in an expanding, adiabatically cooling, universe
  - Thermal history of the universe is important:
- Primary components:
  - Collisional baryonic matter,
  - Photons,
  - Collisionless dark matter

The origin and formation of structures: complications I …

- Evolution of perturbation depends on
  - The expansion rate
  - The component (Dark matter, photon, baryon)
  - The era:
    - Before/After photon/matter decoupling
  - Key periods and transitions: t_{dec}

Theoretical framework

- General Relativity
- Cosmological Principle: Friedmann Robertson Walker metrics

$$ds^2 = c^2dt^2 - R(t)^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

- Friedmann equation :

  - Equation of state *P*= $\omega\ \rho$

  - $\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right)R \quad (a)$

  - $\dot{R}^2 + kc^2 = \frac{8\pi G}{3}\rho R^2 \quad (b)$

Cosmological background

The FRW metrics with $a(t)=R(t)/R_0$

$$c^2 d\tau^2 = c^2 dt^2 - a^2(t)\left[d\omega^2 + f_K(\omega)\, d\Omega^2\right]$$

$a(t)$ is the **scale factor** , such that $a(t = t_0) = 1$;
$d\Omega^2$ is the solid angle:

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$$

$$f_K(\omega) = \begin{cases} K^{-1/2}\sin\left(K^{1/2}\omega\right) & K > 0 \\ \omega & K = 0 \\ (-K)^{-1/2}\sinh\left((-K)^{1/2}\omega\right) & K < 0 \end{cases}$$

Cosmological background

Equations of the scale factor

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}$$

p is the pressure, ρ the total density.

$$\rho = \rho_b + \rho_{dm} + \rho_r + \rho_\nu + \rho_v$$

Equation of state parameter w:

$$p = w\ \rho c^2$$

Cosmological background

Models with zero curvature

$$\frac{\ddot{a}}{a} = \frac{-H_0}{2}\left(\Omega_{0m}(1+z)^3 + 3\sum_i w_i \Omega_{0i}(1+z)^{3(1+w_i)}\right)$$

$$H_0 = \left(\frac{\dot{a}}{a}\right)_0 = 72 \pm 7\text{km/sec/Mpc}$$

$$\Omega_i = \frac{\rho_i}{\rho_c} \;;$$

$$\rho_c = \frac{3H^2}{8\pi G} = 1.88\,10^{-29}\,h^{-2}\,\text{g.cm}^{-3}$$

$$\left(\frac{H(z)}{H_0}\right)^2 = \sum_i \Omega_{0i}(1+z)^{3(1+w_i)}$$

Theory of Structure Formation

Overview

- Very small perturbations are assumed to exist at high redshift (whatever ther origin)

- Perturbations then grow from gravity (gravitational instability)

- The growth of perturbation will by modified by other physical effects : free streaming, damping, pressure

- Because of pressure, damping and free streaming,
    . baryonic and non-baryonic matter grow differently
    . hot, warm and cold dark matter grow differently

- All components have their own equations, but they are coupled

Gravitational instability

Equation of gas dynamics for a fluid in a gravitational field

Contitiuty equation *(Conservation of mass)*

$$\frac{\partial\rho\left(\vec{r};t\right)}{\partial t}+\nabla_r.\left[\rho\vec{u}\left(\vec{r};t\right)\right]=0$$

Euler equation *(Equation of motion for an element of the fluid)*

$$\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}.\nabla_r\right)\vec{u}=-\nabla_r\varphi$$

Poisson equation *(Gravitational potential in the presence of a density distribution ρ)*

$$\nabla_r^2\varphi=4\pi G\rho-\Lambda$$

Gravitational instability

Homogeneous solution

$$\rho(\vec{r},t) = \bar{\rho}(t)$$

$$\vec{u}(\vec{r},t) = \frac{\dot{a}}{a}\vec{r}$$

Poisson equation is satisfied: $$\varphi(\vec{r},t) = \frac{1}{6}(4\pi G\bar{\rho} - \Lambda)\,|\vec{r}|^2$$

Continuity equation $\longrightarrow$ $\dot{\bar{\rho}} + \frac{3\dot{a}}{a}\bar{\rho} = 0 \longrightarrow \bar{\rho} = \rho_0 a^{-3}$ = Friedmann equation!

Euler equation $\longrightarrow$ $\frac{\ddot{a}}{a} = -\frac{4\pi G\bar{\rho}}{3} + \frac{\Lambda}{3}$ = Friedmann equation!

Gravitational instability

Transformation to comoving coordinates

$$\vec{x} = \frac{\vec{r}}{a(t)}$$

$$\rho(\vec{r}, t) = \rho' \left(\frac{\vec{r}}{a(t)}, t \right) = \rho'(\vec{x}, t)$$

$$\vec{u}(\vec{r}, t) = \dot{a}\vec{x} + \vec{v}(\vec{x}, t) = \frac{\dot{a}}{a}\vec{r} + \vec{v}\left(\frac{\vec{r}}{a(t)}, t \right)$$

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Expansion

peculiar velocity

Gravitational instability

Continuity equation in comoving coordinates:

$$\frac{\partial \rho(\vec{r},t)}{\partial t} + \vec{\nabla}_r.\left(\rho \vec{u}(\vec{r},t)\right) = 0$$

$$\frac{\partial \rho'}{\partial t} - \frac{\dot{a}}{a^2}\vec{r}.\nabla_x \rho' + \frac{1}{a}\left(\left(\dot{a}\vec{x} + \vec{v}\right).\nabla_x \rho' + \rho'\left(3\dot{a} + \nabla_x.\vec{v}\right)\right) = 0$$

$$\frac{\partial \rho'}{\partial t} + \frac{3\dot{a}}{a}\rho' + \frac{1}{a}\nabla_x.\left(\rho'\vec{v}\right) = 0$$

Euler equation in comoving coordinates:

$$\frac{\partial u_i}{\partial t} + \left(u_j \frac{\partial}{\partial r j}\right) u_i = 0$$

$$\frac{\partial v_i}{\partial t} + \ddot{a}x_i + \frac{\dot{a}}{a}v_i + \frac{1}{a}\left(\vec{v}.\nabla_x\right) v_i = -\frac{\partial \varphi}{\partial r_i}$$

Gravitational instability

Comoving gravitational potential:

$$\Phi(\vec{x},t) = \varphi(a\vec{x},t) + \frac{\ddot{a}a}{2}|\vec{x}|^2$$

$\longrightarrow$ Poisson equation:

$$\nabla_x^2 \Phi(\vec{x},t) = a^2 \left(\nabla_r^2 \varphi + \frac{3\ddot{a}}{a} \right) = 4\pi G \left(\rho'(\vec{x},t) - \bar{\rho}(t) \right)$$

Gravitational instability

Euler equation becomes

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}\left(\vec{v}.\nabla_x\right)\vec{v} = -\frac{1}{a}\nabla_x\Phi$$

Density contrast :

$$\delta(\vec{x};t) = \frac{\rho(\vec{x};t)-\bar{\rho}(t)}{\bar{\rho}(t)}$$

Continuity →

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x.\left[\left(1+\delta\right)\vec{v}\right] = 0$$

And the Poisson equation writes:

$$\nabla_x^2\Phi = \frac{3H_0^2}{2a}\Omega_m\delta$$

Gravitational instability

Linear perturbation equations

If we only consider terms linear in v and δ

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla_x . \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{1}{a} \nabla_x \Phi$$

Together with the Poisson equation , these equations are linear and homogeneous in spatial variables.

Gravitational instability: linear perturbation

Fourier decomposition

$$\delta(\vec{x};t) = \int \frac{d^3k}{(2\pi)^2} \tilde{\delta}\left(\vec{k};t\right) e^{i\vec{k}.\vec{x}}$$

$$\longrightarrow \qquad \frac{\partial^2 \delta}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial \delta}{\partial t} - \frac{3H_0^2}{2a^3}\Omega_m \delta = 0$$

Linear perturbation equations

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Hubble drag (or Hubble friction) = 2.*H*(z)

- Friction term opposing to growth
- Slow down the growth as compared to gravitational instability in a non-expanding sphere
- $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda}{3}$: depends on amount of matter, curvature, DE

Linear perturbation equations

Fourier decomposition

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$$\frac{\partial^2\delta}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial\delta}{\partial t} - \frac{3H_0^2}{2a^3}\Omega_m\delta = 0$$

Usually written as

$$\ddot{D} + \frac{2\dot{a}}{a}\dot{D} = \frac{3H_0^2}{2a^3}\Omega_m D$$

The growing modes, D_+, imply that δ grow as

$$\delta(\vec{x};t) = D_+(t)\delta_0(\vec{x})$$

and it can be shown that

$$\boxed{D_+(t) = D_{in}\ H(a) \int_0^a \frac{\mathrm{d}a'}{[a'H(a')]^3}}$$

where D_{in} is such that $D_+(t_0) = D_+(a=1) = 1.$

D_+ is called the growth factor

Linear perturbation equations

Example: EdS universe ($\Omega_m = 1$):

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

The differential equation:

$$\ddot{D} + 2\frac{\dot{a}}{a}\dot{D} - \frac{3H_0^2\Omega_m}{2a^3}D = 0$$

writes

$$\ddot{D} + \frac{4}{t}\dot{D} - \frac{2}{3t^2}D = 0$$

This equation has solution of type $D \propto t^n$.

Inserting this solution we find that the growing modes are:

$$D_+ = a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Including pressure perturbation

Let us include a pressure perturbation:

$$\frac{\partial \vec{U}}{\partial t} + 2\frac{\dot{a}}{a}\vec{U} = -\frac{1}{a^2}\nabla_x \Phi - \frac{\nabla \delta p}{\bar{\rho}}$$

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Using the same equations as before and Fourier-transform them lead to

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = \left(4\pi G\bar{\rho} - c_s^2 \frac{k^2}{a^2}\right)\delta$$

where

$$c_s = \left(\frac{\partial p}{\partial \rho}\right)^{1/2}$$

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Both the Hubble drag and the pressure oppose to the growth of instabilities

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•The growth of structure will depends on the Gravity/Pressure balance:

- $\left(4\pi G\bar{\rho} - c_s^2\frac{k^2}{a^2}\right)\delta$ > 0 perturbation can grow
- $\left(4\pi G\bar{\rho} - c_s^2\frac{k^2}{a^2}\right)\delta$ < 0 oscillatory solution = large k, small scales

Structure Formation

The Jeans Length

Equation of State for an ideal Gas $P = \omega \rho c^2 = \frac{kT}{\mu} \rho = \frac{\bar{v}_s}{3} \rho \quad (\bar{v}_s << c)$ $\bar{v}_s$ = the sound speed

In absence of pressure, an overdense region collapses on order of the free fall time : $\tau_{ff} = (4\pi G \overline{\rho})^{-1/2}$

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Pressure Gradient : resists collapse if a pressure gradient can be created over a timescale given by $\tau_J < \tau_{ff}$

$$\tau_J \approx \frac{r}{\bar{v}_s}$$

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Define a critical length over which density perturbation will be stable against collapse under self gravity

$$r_{critical} = \lambda_J \sim \bar{v}_s \tau_{ff} \sim \sqrt{\frac{\bar{v}_s^2}{G\bar{\rho}}}$$

JEANS LENGTH $\lambda_J = \left(\frac{\pi \bar{v}_s^2}{G\bar{\rho}}\right)^{1/2} = 2\pi \bar{v}_s \tau_{ff}$

JEANS MASS $M_J = \frac{4\pi}{3}\bar{\rho}\lambda_J^3$

Structure Formation

Jeans Mass at the decoupling epoch

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Free Fall Time $\tau_{ff} = (4\pi G\bar{\rho})^{-1/2}$ and $H^2 = \frac{8\pi G\bar{\rho}}{3}$ $\rightarrow$ $H^{-1} \approx \tau_{ff}$

Jeans Length $\lambda_J = 2\pi \bar{v}_s \tau_{ff} \approx 2\pi(2/3)^{1/2} \frac{\bar{v}_s}{H}$ Photon sound speed :ω=1/3 $\bar{v}_s = \frac{c}{\sqrt{3}} \approx 0.6c$

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Jeans Length $\lambda_J = 2\pi \bar{v}_s \tau_{ff} \approx 2\pi (2/3)^{1/2} \frac{\bar{v}_s}{H}$ Photon sound speed :ω=1/3 $\bar{v}_s = \frac{c}{\sqrt{3}} \approx 0.6c$

Before epoch of decoupling, photons and baryons bound together as a single fluid: pressure is driven by photon contribution: strong opposition to gravity

At decoupling (z=1089) $\lambda_{J,\gamma} \approx 3\frac{c}{H} \Rightarrow \lambda_{J,\gamma}(dec) \approx 0.6 Mpc$

Super-horizon scales (sound speed~c)
→ Sub horizon scales cannot grow

$$M_{J,baryon} \approx 36\pi\bar{\rho}\left(\frac{c}{H}\right)^3 \Rightarrow M_{J,baryon}(dec) \approx 10^{18} M_o$$ (Supercluster scale)

Structure Formation

Jeans Mass after decoupling

After epoch of decoupling, photons and baryons behave as separate fluids: pressure of baryons much smaller than photons. Baryon Jeans mass drop by a huge factor

Photon sound speed $\bar{v}_{s,\gamma} = \frac{c}{\sqrt{3}} \approx 0.6c$

Baryon sound speed $\bar{v}_{s,baryon} = \left(\frac{kT}{mc^2}\right)^{1/2} c \approx 0.00001c$

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Jean's Length $\lambda_J = \left(\frac{\pi \bar{v}_s^2}{G\bar{\rho}}\right)^{1/2}$ After decoupling $\lambda_J = \frac{\bar{v}_{s,baryon}}{\bar{v}_{s,\gamma}} \lambda_J(dec) \sim 2\text{x}10^{-5} \lambda_J(dec)$

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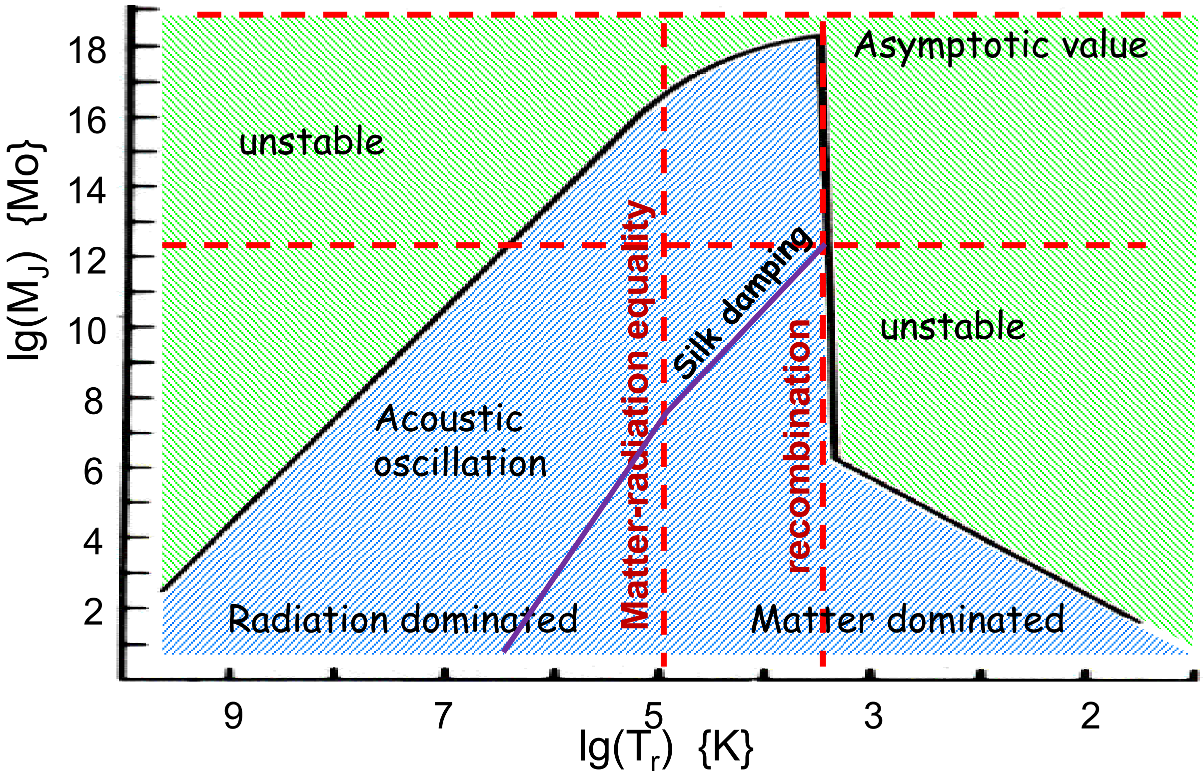
Jean's Mass after decoupling $$M_{J,baryon} = \left(\frac{\lambda_J(dec)}{\lambda_J}\right)^3 M_{J,baryon}(dec) \sim 10^5 M_o$$

This mass is approximately the same mass as Globular Cluster today

Until decoupling, structures over scales of globular clusters up to superclusters could not grow

Structure Formation

Jeans Mass, Silk damping, Silk Mass and the decoupling epoch



$$\lambda_{J,\gamma}(dec) \approx 0.6 Mpc$$

$$M_{J,baryon}(dec) \approx 10^{18} M_o$$

$$\lambda_{J,\gamma} \approx 12 pc$$

$$M_{J,baryon} \approx 10^{5} M_o$$

Silk dampiing: Close to decoupling / recombinationn:

- Baryon/photon fluid coupling becomes inefficient
- Photon mean free path increases → diffuse / leak out from over dense regions
- Photons / baryons coupled → smooth out baryon fluctuations

Damp fluctuations below mass scale corresponding to distance traveled in one expansion time scale

Structure Formation in baryon dominated universe

Density fluctuations in a flat, matter dominated Universe grow as $\delta \propto A\, t^{2/3} \propto R(t) \propto \frac{1}{(1+z)}, \qquad \delta << 1$

- δ <<1 (linear regime)
- Baryonic Matter fluctuations can only have grown after recombination (z ~1000) $\rightarrow$ by a factor $(1+z_{dec})$ ~ 1000 by today

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- δ~0.001 : $\delta T/T$ ~0.001 at recombination
- But CMB : $\delta T/T$ ~10^{-5} !!!

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MATTER PERTURBATIONS DO NOT HAVE TIME TO GROW IN A BARYON DOMINATED UNIVERSE

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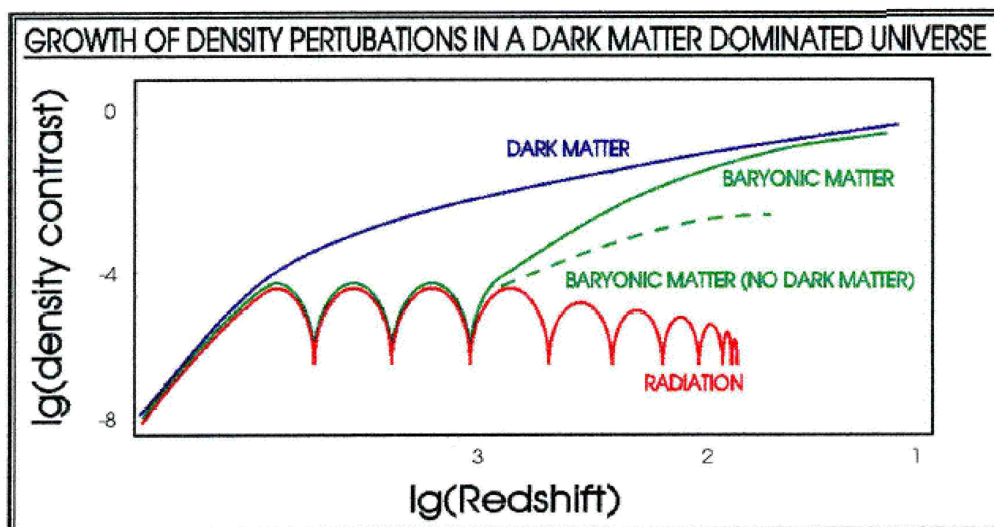
MATTER PERTURBATIONS DON'T HAVE TIME TO GROW IN A BARYON DOMINATED UNIVERSE

- Dark matter needed:
  - Condensed at earlier time (no pressure)
  - Matter then fall into DM gravitational wells



The origin and formation of structures: complications II …

- Evolution of perturbation depends on
  - The expansion rate
  - The component (Dark matter, photon, baryon)
  - The era:
    - Radiation vs matter dominated era of the universe
    - Before/After photon/matter decoupling
  - The physical size of perturbation with respect to the horizon size
  - Key periods and transitions: t_{eq} , t_{dec} , $t_{enter\_horizon}$

Structure Formation: the horizon scale

- Comoving horizon size:

$$d_H = \frac{c}{H_0}\,\Omega_m^{-1/2}\,a^{1/2}\,\left(1+\frac{a_{eq}}{a}\right)^{-1/2}$$

• For scale larger than the horizon size, Newtonian perturbation theory is no longer valid

• Perturbation theory must be carried out in a full General Relativity framework

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• One find
  • In the radiation dominated era, δ grows like a^2 for superhorizon scales
  • In the matter era, δ grows like a for superhorizon scales

Structure Formation: horizon vs. radiation

- Comoving horizon size:

$$d_H = \frac{c}{H_0} \Omega_m^{-1/2} a^{1/2} \left(1 + \frac{a_{eq}}{a}\right)^{-1/2}$$

If perturbation enters the horizon during the radiation dominated (R) period then:

the Hubble drag term is large and dominates:

expansion time scale ~ $(G\rho_R)^{-1/2}$ < collapse time scale ~ $(G\rho_{DM})^{-1/2}$

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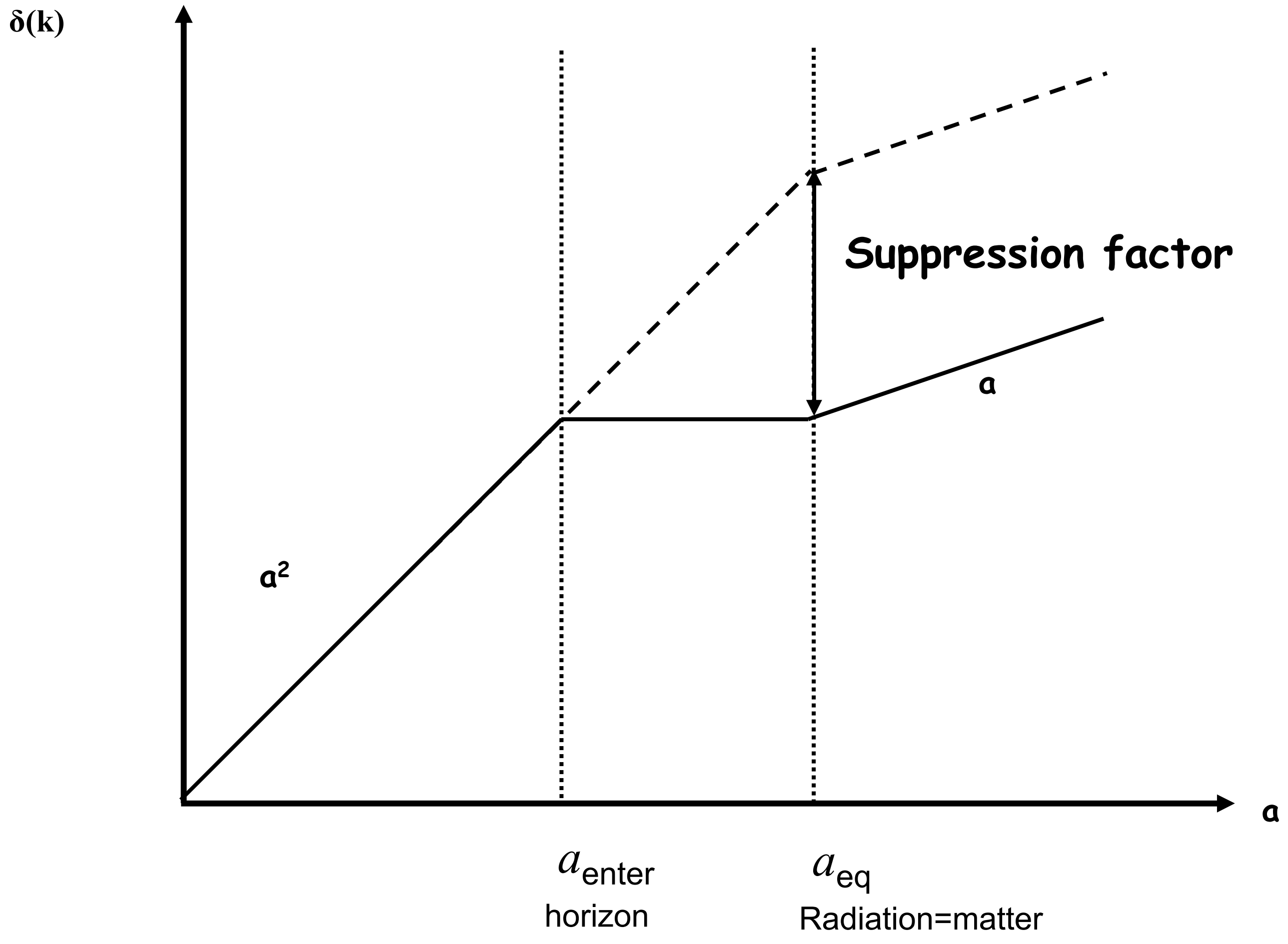
→ Radiation prevents growth of perturbation

Structure Formation: horizon vs. radiation



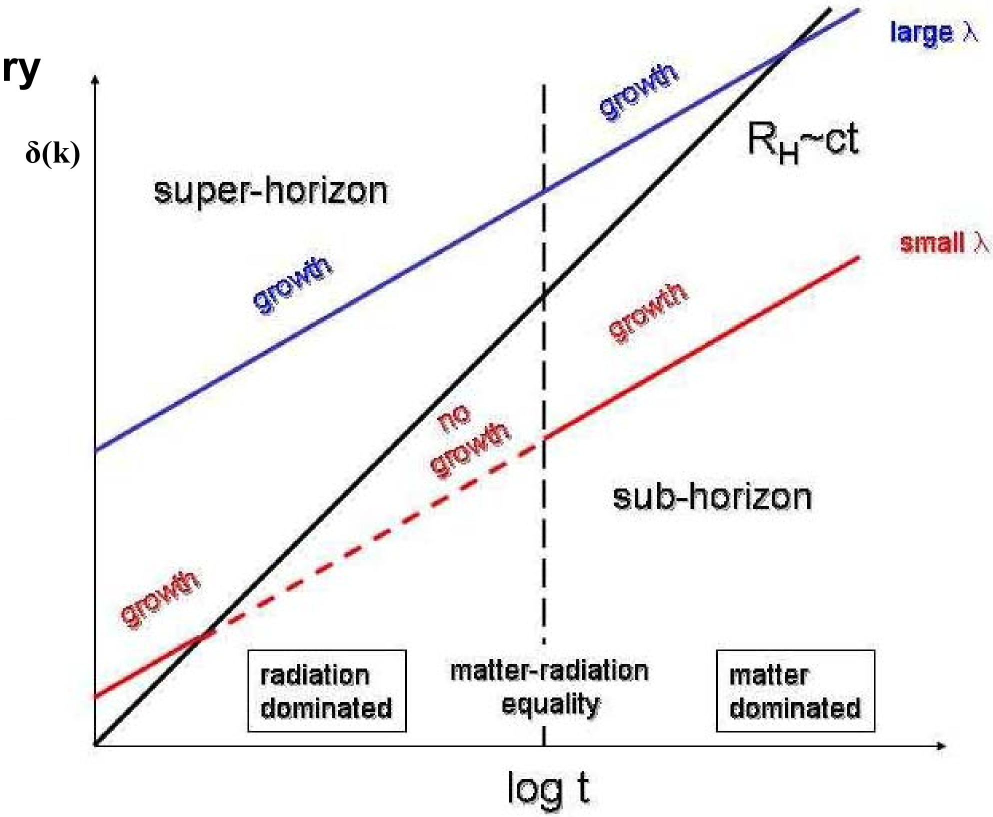
Growth of structure… summary

Super-horizon fluctuations

- General relativistic perturbation theory

Radiation dom: $\delta_+(t)$ - t

Matter dom: $\delta_+(t) – t^{2/3}$



Sub-horizon fluctuations

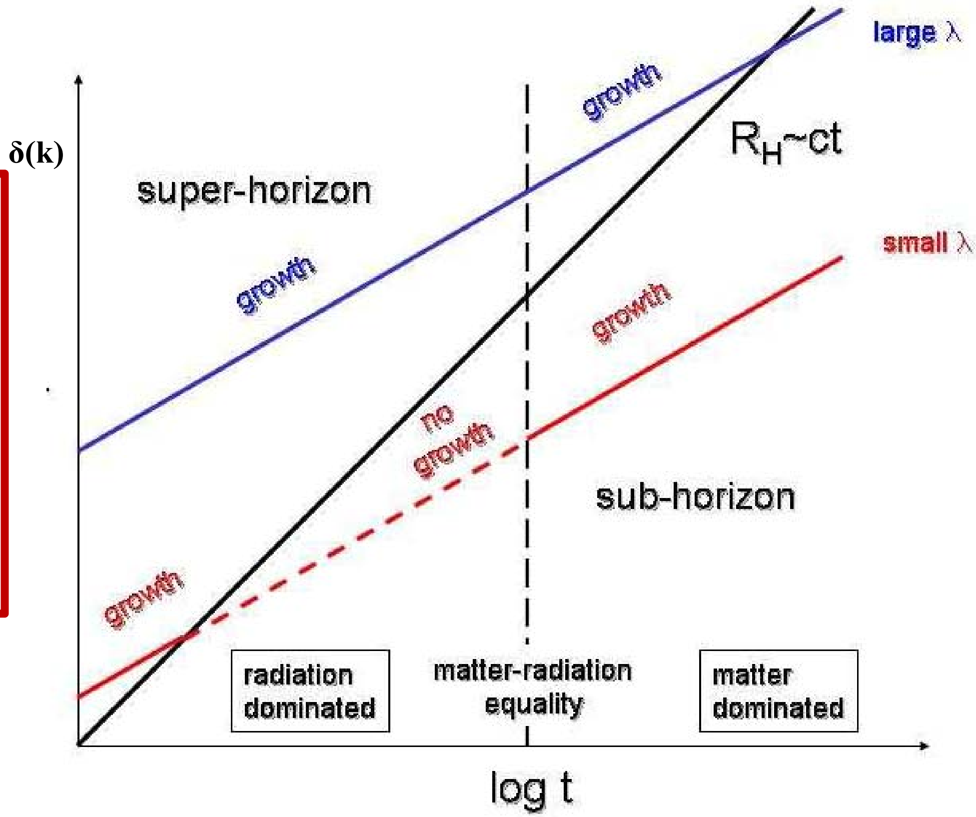
- Newtonian Jeans analysis

Radiation dom: $\delta_+(t)$ - const

Matter dom: $\delta_+(t) – t^{2/3}$

Growth of structure… summary

Epoch	δ_{DM}	δ_B
$t < t_{enter} < t_{eq}$	$\sim a^2$	$\sim a^2$
$t_{enter} < t < t_{eq}$	~const	oscillate
$t_{eq} < t < t_{dec}$	$\sim a$	oscillate
$t_{dec} < t$	$\sim a$	$\sim a$



Structure Formation in a dark matter dominated universe

Depend on the nature/properties of Dark Matter

- COLD DARK MATTER: non relativistic at decoupliing:
WIMPS (Heavy neutrinos, SUSY particles), Axions

- HOT DARK MATTER: relativistic at decoupling:
Light neutrinos

- COSMIC DEFECTS: symmetry defects
Monopoles, Cosmic Strings, Domain Walls, Cosmic Textures

Structure Formation in a dark matter dominated universe

- Weakly interacting → no photon damping
- Structure formation proceeds before epoch of decoupling
- Provides Gravitational ‘sinks’ for baryons
- Baryons fall into sinks after epoch of decoupling
- Model of formation depends on whether Dark Matter is Hot/Cold
- Hot /Cold DM Decouple at different times → Different effects on Structure Formation

Structure Formation in a dark matter dominated universe

Hot Dark Matter

- Any massive particle that is relativistic when it decouples will be HOT
- → Characteristic scale length / scale mass at decoupling given by Hubble Distance $c/H(t)$

Structure Formation in a dark matter dominated universe

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- → Characteristic scale length / scale mass at decoupling given by Hubble Distance *c/H(t)*

Radiation Dominates

$\rho \propto R^4 \quad (R = R(t))$

Friedmann eqn. $\quad H(t)^2 = \Omega_{r,o}\left(\frac{R_o}{R}\right)^4 H_o^2$

Radiation dominated $\quad H(t) = \frac{1}{2t}$

For radiation (photons)

1+z ~ 3500

Matter/Radiation Equality

⟷

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Substituting for (R_o/R), The Hubble Distance at t_{eq} is $\frac{c}{H(t_{eq})} = \frac{c}{H_o}\frac{\Omega_{r,o}^{3/2}}{\Omega_{m,o}^2} \equiv 2ct_{eq} \approx 30kpc$

Mass inside Hubble volume $M_{eq} = \frac{4\pi}{3}\left(\frac{c}{H(t_{eq})}\right)^3 \rho(t_{eq}) = \frac{4\pi}{3}\left(\frac{c}{H_o}\right)^3\left(\frac{\Omega_{r,o}^{3/2}}{\Omega_{m,o}^2}\right)^3 \Omega_{m,o}\rho_{c,o}\left(\frac{R_o}{R_{eq}}\right)^3 = \frac{\pi}{3}\left(\frac{c}{H_o}\right)^3 \frac{\Omega_{r,o}^{3/2}}{\Omega_{m,o}^2}\rho_{c,o} \sim 10^{17} M_o$

>> $M_{Supercluster}$

Structure Formation in a dark matter dominated universe

Hot Dark Matter

Other relativistic particles

Epoch of equality defined when $k_B T \sim mc^2$ $1+z \sim >3500$, $M_H < 10^{17} M_o$

At a time given by $$T = \left(\frac{32\pi G a}{3c^2} \right)^{-1/4} t^{-1/2} \approx 1.5 \text{x} 10^{10} t^{-1/2}$$

Result obtained by solving Friedmann equation in a radiation era:

$a^2 = (32 \ \pi \ G \ \varepsilon_0 \ / \ 3 \ c^2)^{1/4} \ t^{1/2}$ and $\varepsilon = \varepsilon_0 \ a^{-4} = k_B \ T$; with ε = energy

Structure Formation in a dark matter dominated universe

Case of a hot neutrino, mass m_ν *(eV/c²)* :

$$T_{eq} \approx \frac{m_\nu}{k} \approx 11600 m_\nu \; \{K\} \quad \Rightarrow \quad t_{eq} = 1.7\text{x}10^{12} (m_\nu)^{-2} \; \{s\}$$

- Before t_{eq}, neutrinos are relativistic and move freely in random directions
- Absorbing energy in high density regions and depositing it in low density regions
- Effect → smooth out any fluctuations on scales less than ~ ct_{eq}

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This Effect is the FREE STREAMING

Fluctuations suppressed on mass scales of $$M = \frac{4\pi}{3}\lambda_o^3 \Omega_{m,o}\rho_{c,o} \sim \frac{10^{16}}{m_\nu^3} M_o$$

Large Superstructures form first in a HDM Universe → TOP-DOWN SCENARIO

Structure Formation in a dark matter dominated universe

Cold Dark Matter

Case of a cold dark matter mass *m_{CDM} ~ 1 GeV* :

$$T_{eq} \approx \frac{m_{CDM}}{k} \approx 10^9 \ \{K\} \quad \Rightarrow \quad t_{eq} = 5\,s$$

$$\frac{c}{H} = 2ct = 3.10^9\,m \Rightarrow \lambda_o = \left(\frac{R_o}{R_{eq}}\right)\lambda_{eq} \sim \left(\frac{T_{eq}}{2.73}\right)\lambda_{eq} \approx 0.04kpc$$

$$M = \frac{4\pi}{3}\lambda_o^3 \Omega_{m,o} \rho_{c,o} << M_o$$

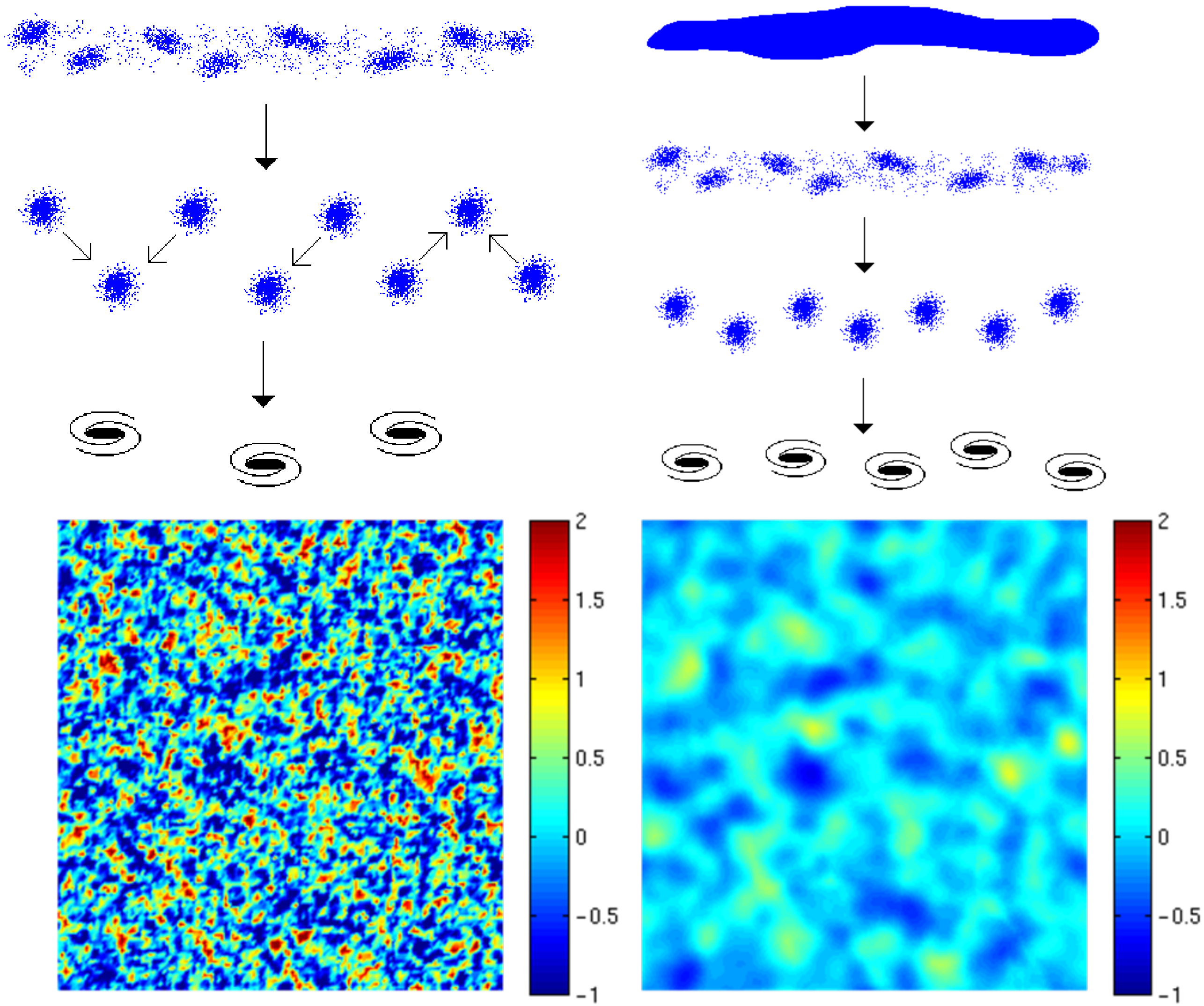
Much smaller mass limit than neutrinos

Structure forms hierarchically in a CDM Universe → BOTTOM-UP SCENARIO

Structure Formation in a dark matter dominated universe

CDM - Bottom-Up
Hierarchical Scenario

HDM - Top-Down
Pancake Scenario



Structure formation and observations

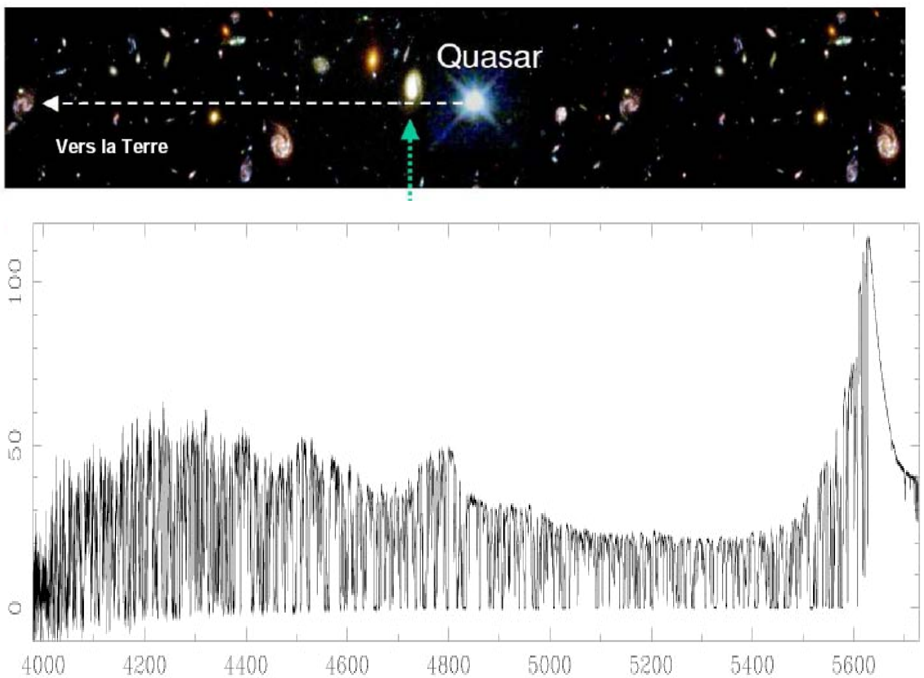
Cosmic history will be visible on

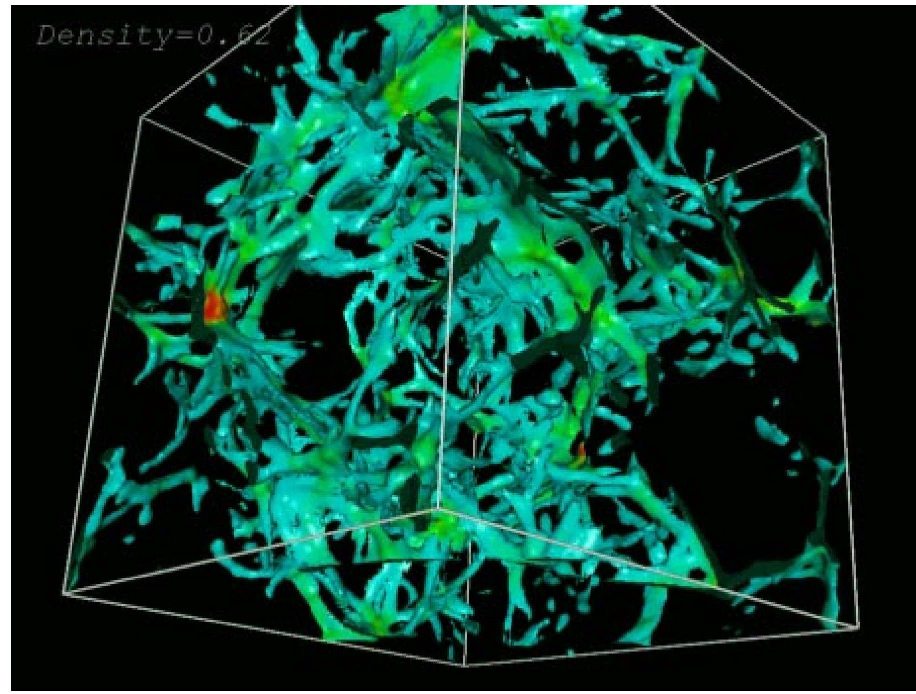
- CMB temperature fluctuations
- Dark matter distribution
- Galaxy distribution
- Absorption line distribution in quasar spectra (Lyman alpha forest)

$\rightarrow$ From these observations :

derive the power spectrum $P(k)$

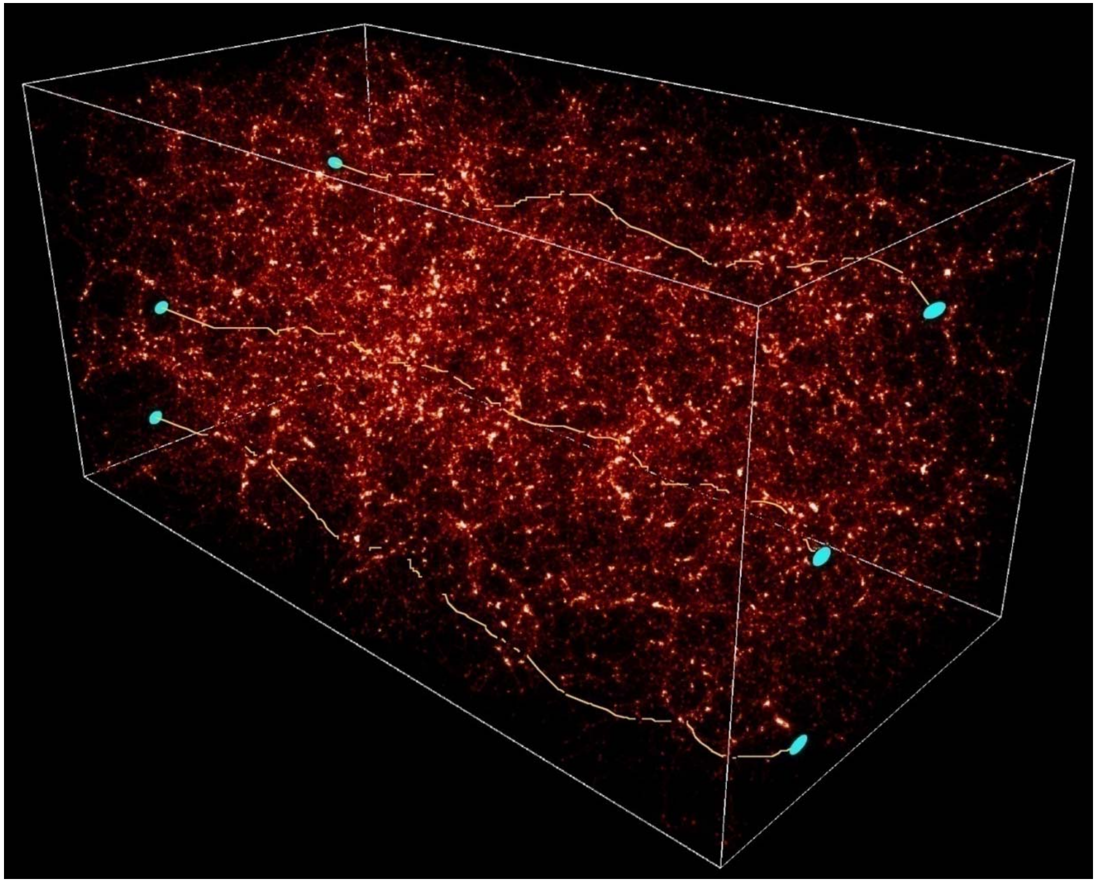
Baryon distribution from the Lyman-alpha forest





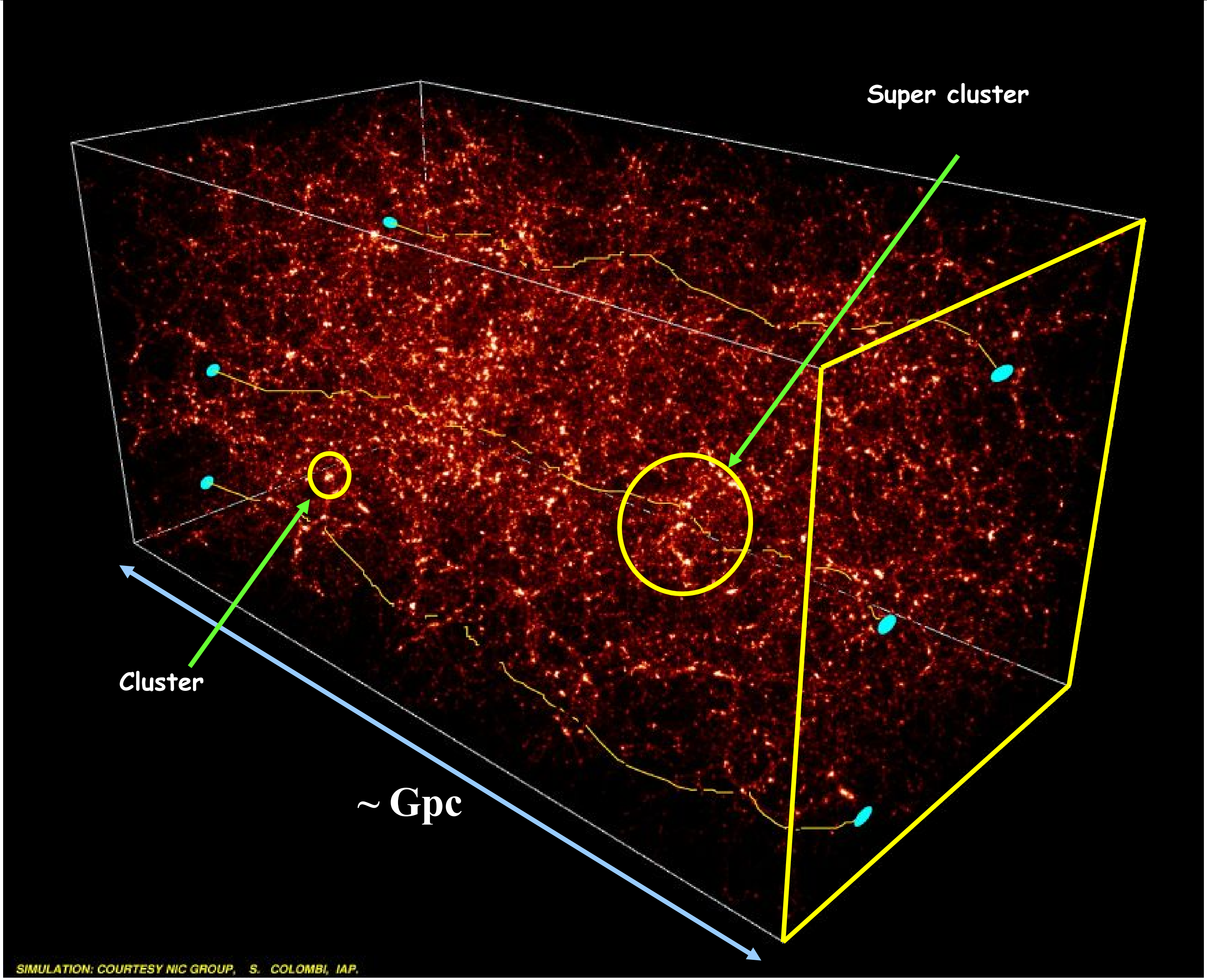
Lyman alpha clouds have very low mass density contrast: linear physics applies

Matter distribution from weak gravitational lensing

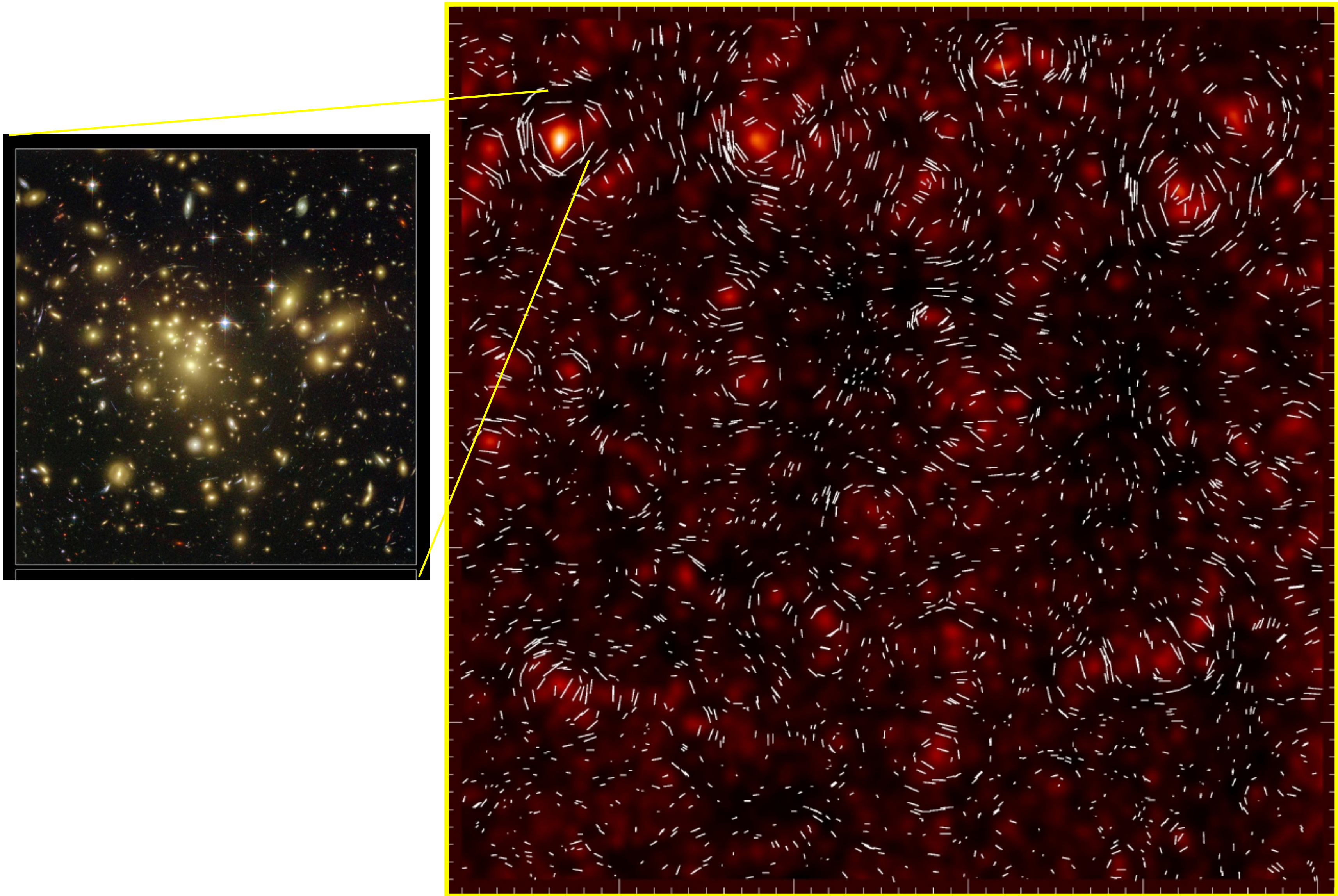


Weak lensing by large scale structures of (dark) matter : linear and non- linear

Cosmic shear : **propagation of light through the cosmic web**

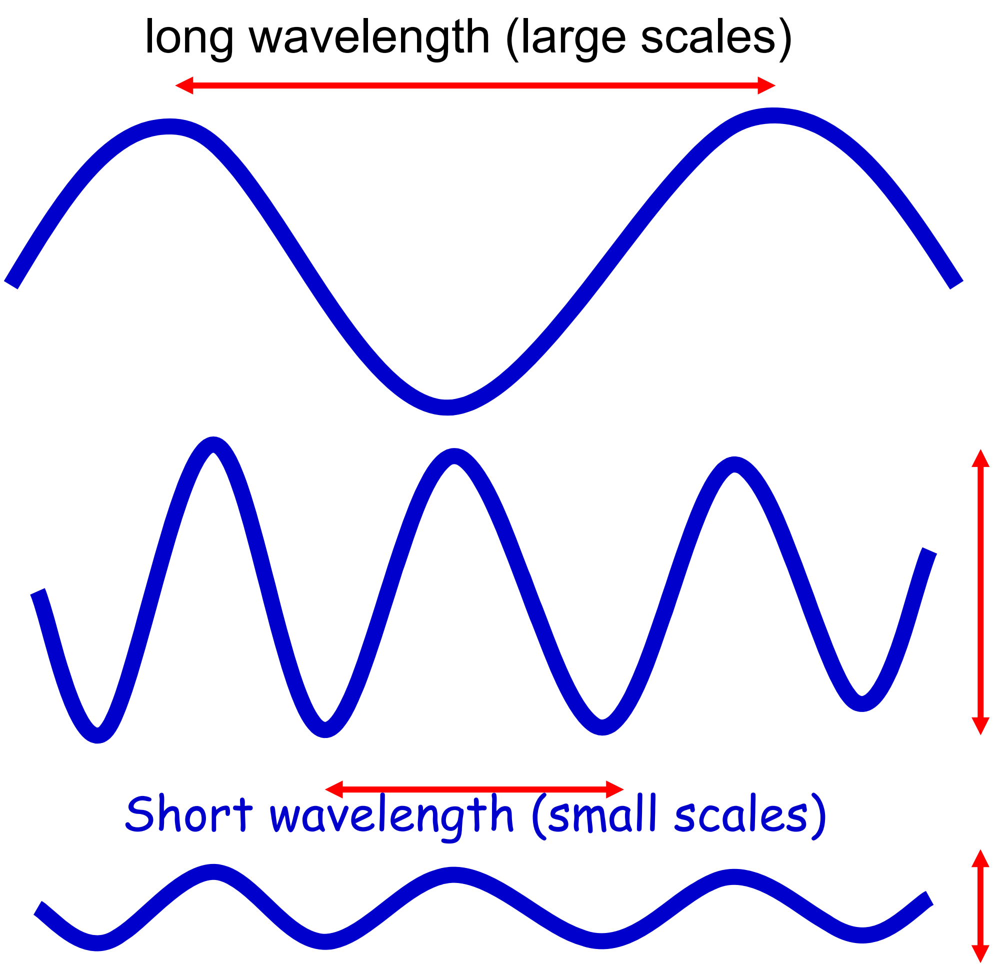


Gravitational shear power spectrum= projected matter density power spctrum



The power spectrum

Need to quantify the power in the density fluctuations on different scales



Density fluctuation field $\delta(\vec{r}) = \dfrac{\rho - \bar{\rho}}{\bar{\rho}} = \dfrac{\Delta\rho}{\bar{\rho}}$

Fourier Transform of Density fluctuation field $\delta_k = \sum \delta(\vec{r}) e^{-i\mathbf{k}.\mathbf{r}}$

Power of the density fluctuations $P(k) = \left\langle \left| \delta_k \right|^2 \right\rangle$

High Power (large amplitude)

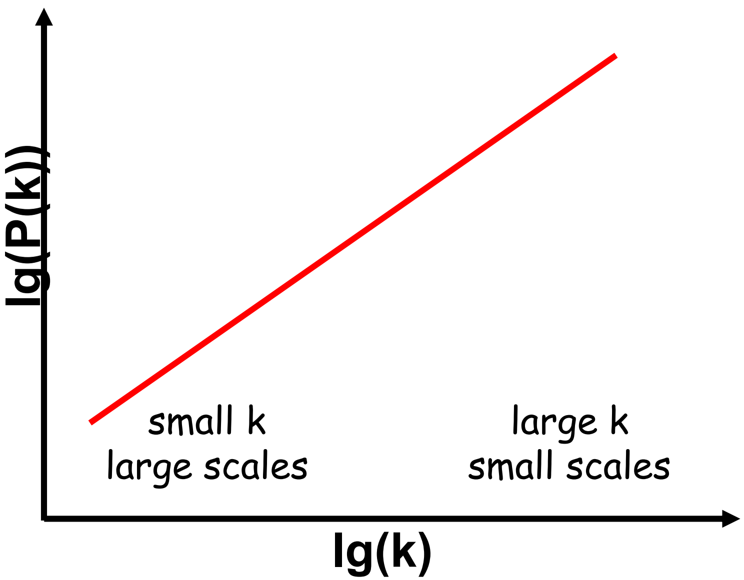
Low Power (small amplitude)

The power spectrum

- Inflation → Scale Free Harrison - Zeldovich spectrum model: $P(k) = \left\langle \left|\delta_k\right|^2 \right\rangle \propto k^n, \qquad n = 1$
- **Fluctuations have the same amplitude when they enter the horizon** ~ $\delta \sim 10^{-4}$
- Inflation field is isotropic, homogeneous, Gaussian field (Fourier modes uncorrelated)
- For a Gaussian field **All information contained within the Power Spectrum** *P(k)*
- Value of $\delta(r)$ at any randomly selected point drawn from GPD

$$\wp(\delta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\delta}{2\sigma^2}\right)}$$

$$\sigma = \frac{V}{(2\pi)^3}\int P(k)d^3k = \frac{V}{2\pi^2}\int P(k)k^2dk$$



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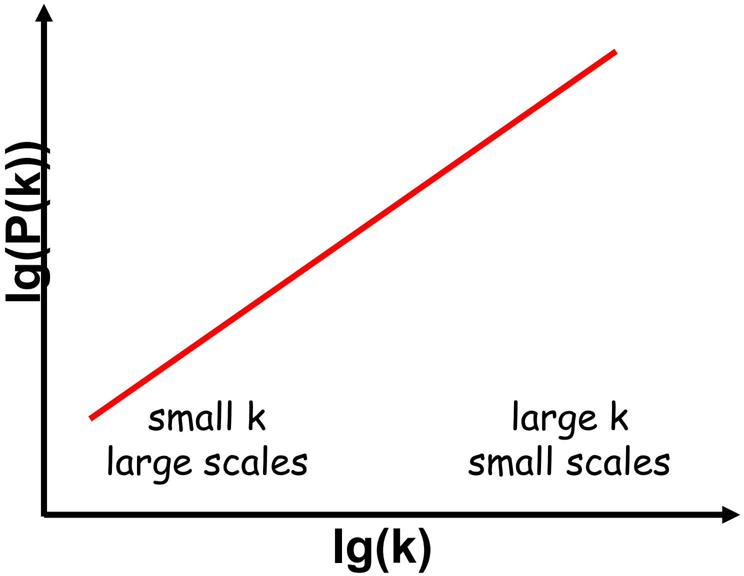
$$\sigma = \frac{V}{(2\pi)^3} \int P(k) d^3k = \frac{V}{2\pi^2} \int P(k)\, k^2 dk$$



- Average mass contained with a sphere of radius λ $(=2\pi/k)$

$$\langle M \rangle = \frac{4\pi}{3}\left(\frac{2\pi}{k}\right)^3 \rho$$

- Mean squared mass density within sphere

$$\left\langle \left(\frac{M - \langle M \rangle}{\langle M \rangle} \right)^2 \right\rangle \propto k^3 P(k) \equiv \left(\frac{\delta M}{M}\right)^2$$

Instead of $P(k)$ → plot $[k^3 P(k)]^{1/2}$ the *rms* mass fluctuations

The power spectrum

The Transfer Function

• Matter-Radiation Equality: Universe matter dominated but photon pressure → baryonic acoustic oscillations

• Recombination → Baryonic Perturbations can grow !

• Dark Matter "free streaming" & Photon "Silk Damping" → erase structure (power) on smaller scales (high k)

• After Recombination → Baryons fall into Dark Matter gravitational potential wells

The transformation from the density fluctuations from the primordial spectrum

The power spectrum

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The transformation from the density fluctuations from the primordial spectrum

$$P(k,t) = T(k)^2 P(k, t_{primordial})$$

- through the radiation domination epoch
- through the epoch of recombination
- to the post recombination power spectrum,

given by :

TRANSFER FUNCTION $T(k)$, contains *physics* of evolution of density perturbations

The power spectrum

The Transfer Function

$$P(k,t) = T(k)^2 P(k, t_{primordial})$$

The transformation from the density fluctuations from the primordial spectrum

TRANSFER FUNCTION $T(k)$ *depends on the nature of dark matter*

HDM

$$T(k) = 10^{-\left(\frac{k}{k_\nu}\right)^{1.5}}$$

$k_\nu \approx 0.4 \Omega_o h^2 Mpc^{-1}$ (for a 30eV neutrino)

$\Rightarrow$ supress all fluctuation modes $\lambda < \frac{2\pi}{k_\nu} \approx \frac{120}{m_\nu(eV)} Mpc$

CDM

$$T(k) = f(\Gamma) = \left[\left(1 + ((ak) + (bk)^{3/2} + (ck)^2)\right)^\nu\right]^{-1/\nu}$$

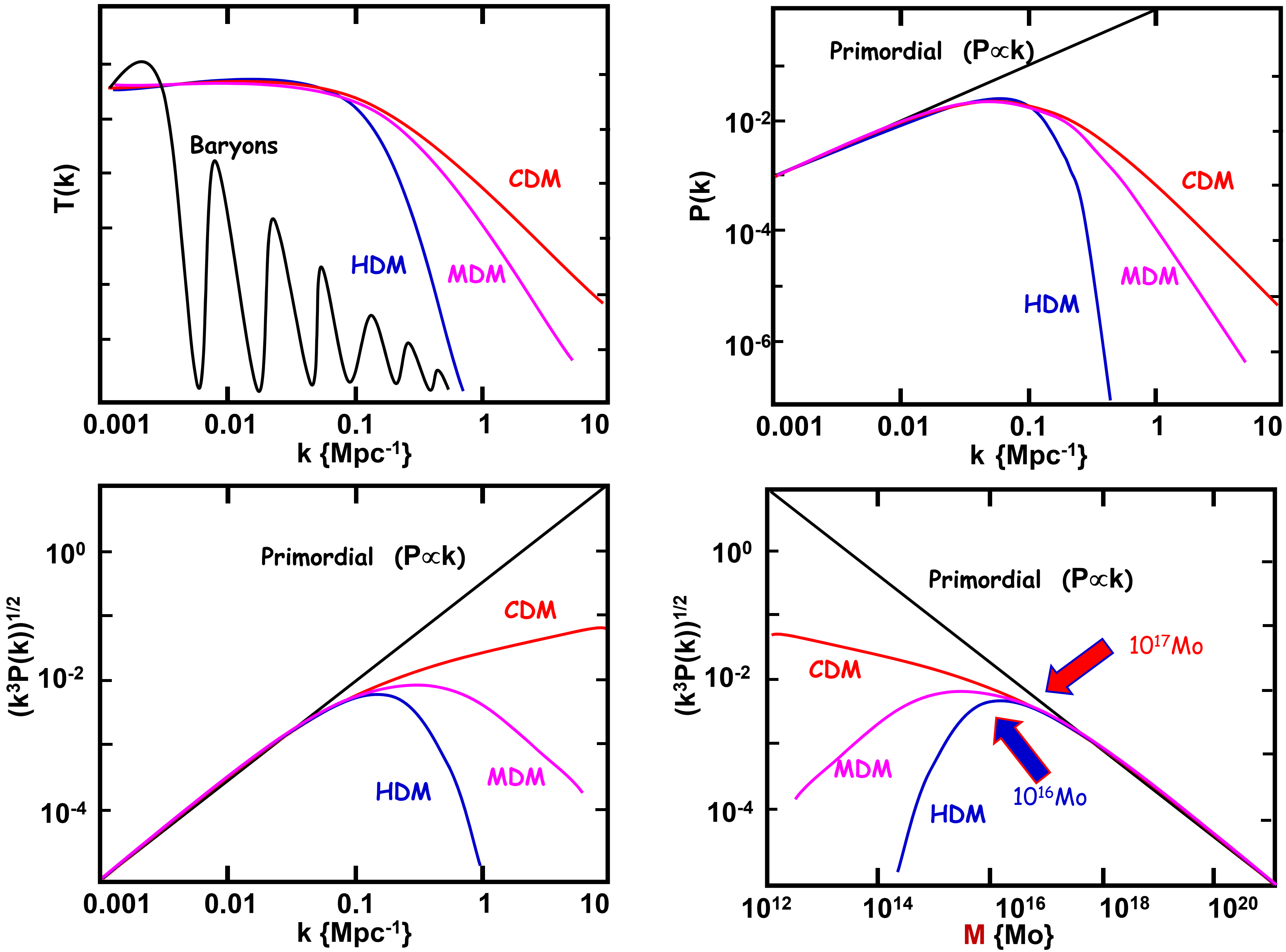
$a = 6.4(\Omega_o h^2)^{-1}$ $\quad b = 3.0(\Omega_o h^2)^{-1}$

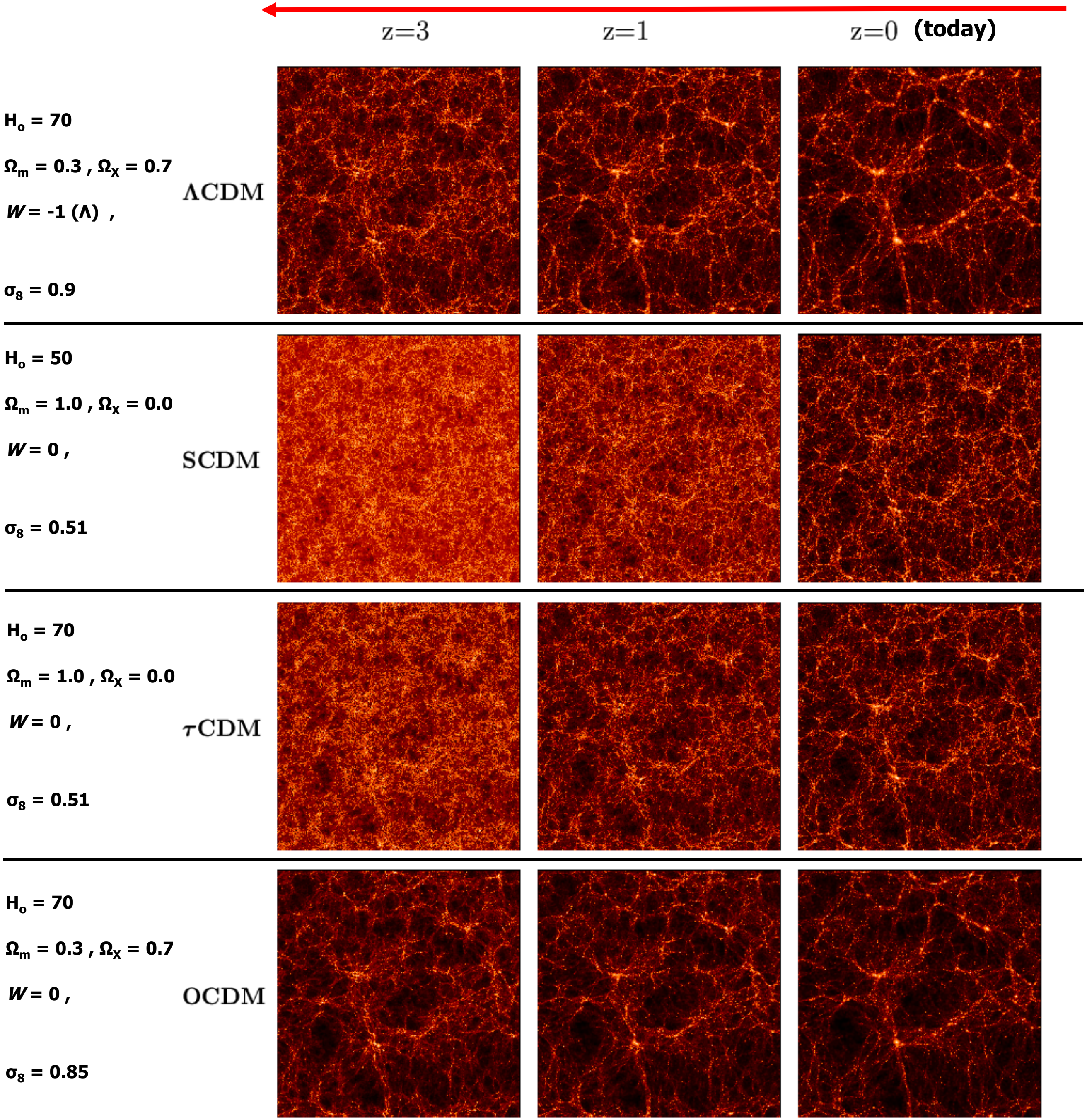
$c = 1.7(\Omega_o h^2)^{-1}$ $\quad \nu = 1.13$

$k \to 0, \quad T(k)^2 \to 1 \quad \Rightarrow P(k) \propto k \Rightarrow$ unchanged! $\quad \Gamma =$ Shape Parameter

$k \to \infty \quad T(k) \propto k^{-2} \quad \Rightarrow P(k) \propto k^{-3} \Rightarrow$ Small scale power!

Power spectrum and Transfer function





The transfer function at work

numerical simulations

$$P(k) \sim \sigma^2_8 \, k^{\,n}$$

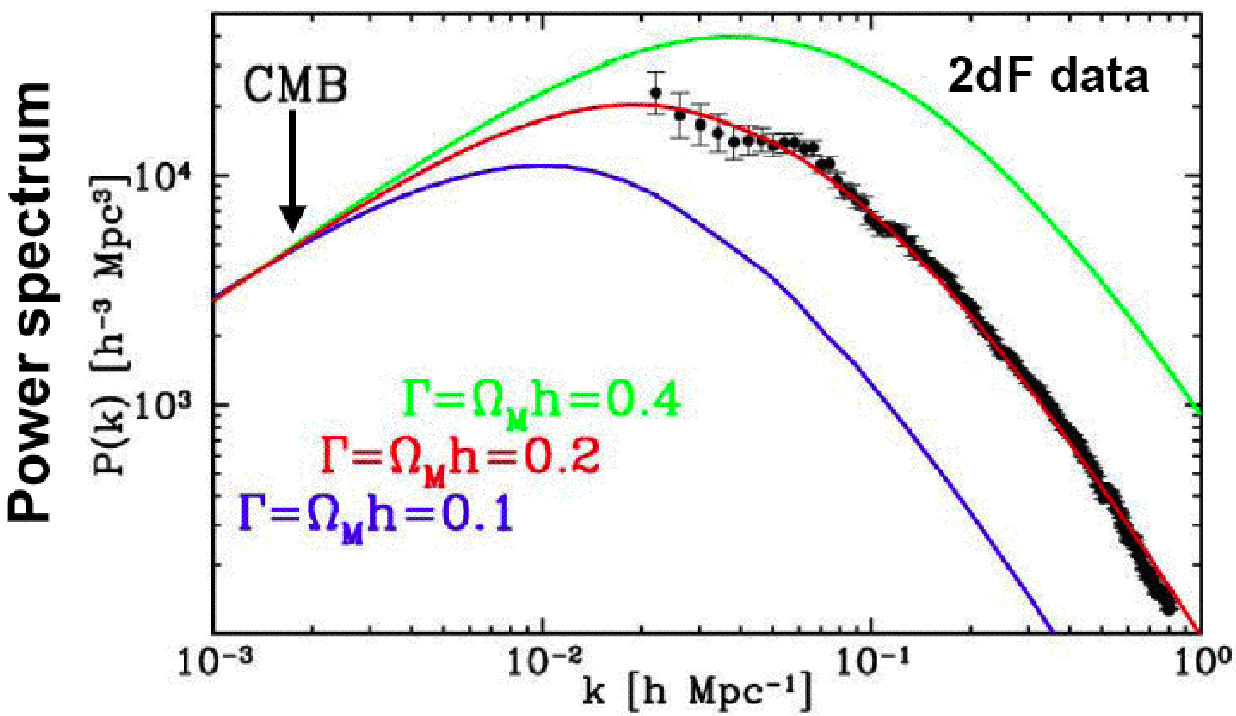
The power spectrum:

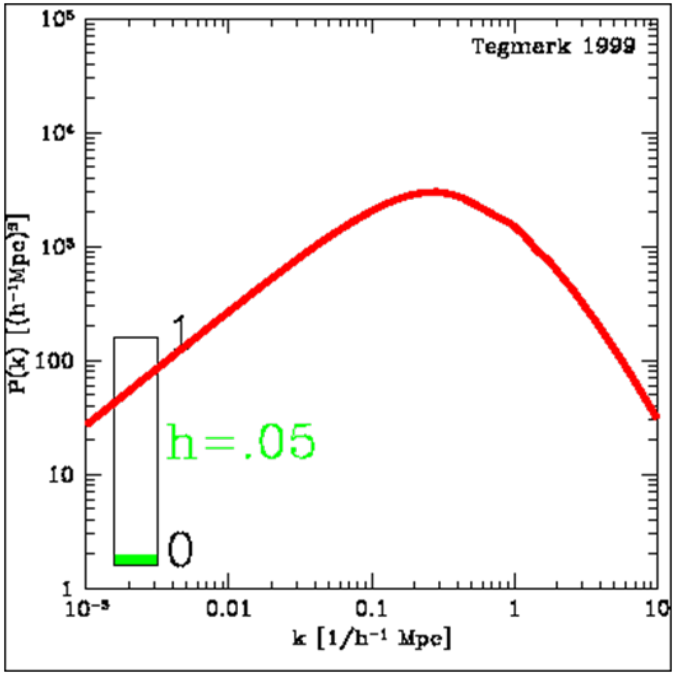
sensitivity to shape parameter Γ and h

Shape factor

$$\Gamma \sim \Omega_M h \sim 0.25 \pm 0.05$$

$\Omega_M h$	Ω_M	h
0.25	1	0.25
0.25	0.35	0.70





Tegmark 2003

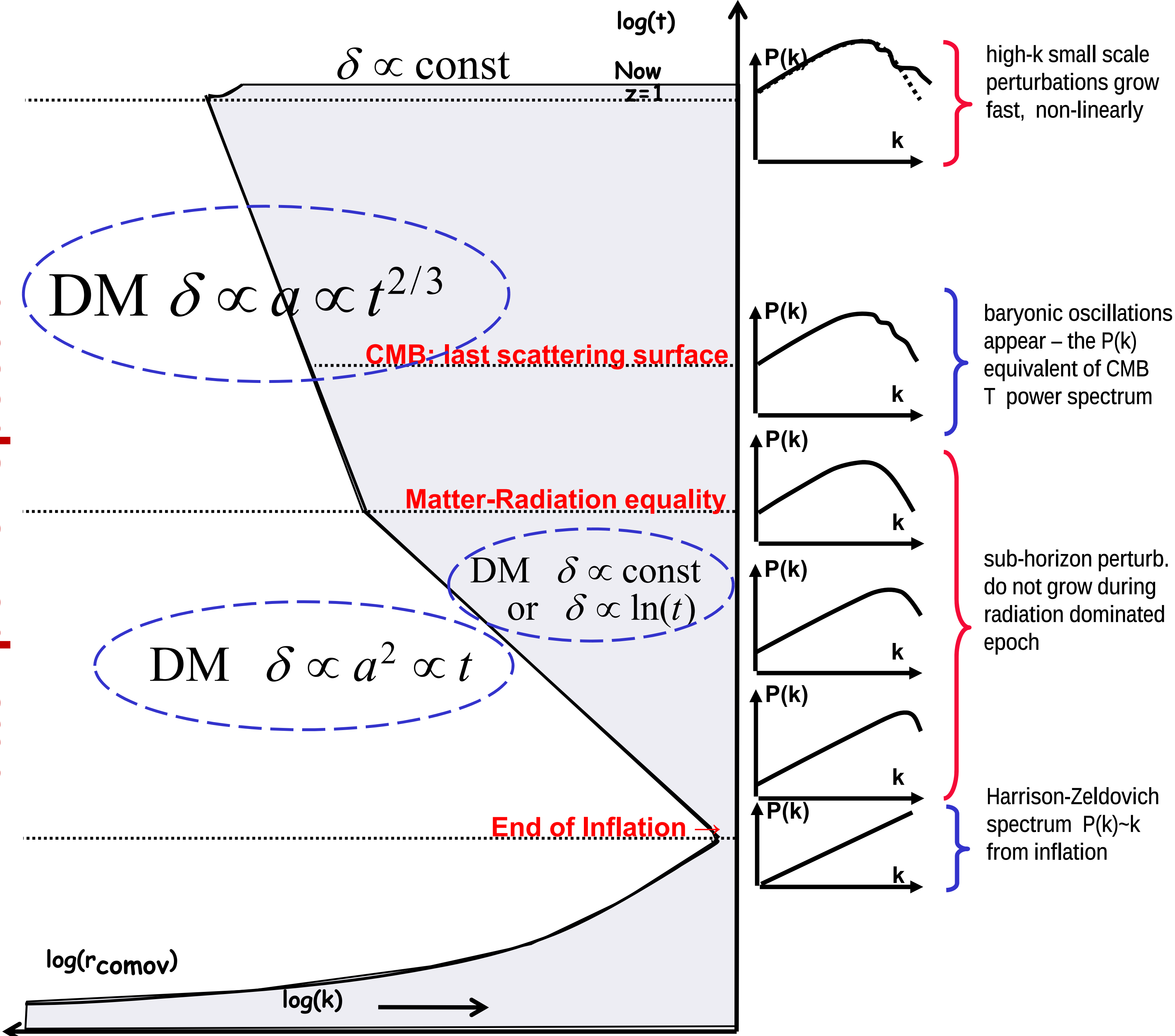
The power spectrum

For a simple power law, $P_i(k) \propto k^n$, with $n = 1$ the asymptotic behavior of the power spectrum is

$$P_\delta(k) \propto k \text{ for small } k$$
$$P_\delta(k) \propto k^{-3} \text{ for large } k$$

- The steep decrease at large k is due to the suppression os small scale perturbations with entered the horizon before matter radiation equality.
- The turn over depends on $d_h(a_{eq})$. It is the most characteristic scale in the dark matter power spectrum

Summary: evolution of the (dark) matter power spectrum



The power spectrum

In summary the power spectrum

- is determined with the spectral index, n, and
- the shape parameter, Γ.
- But the normalisation is missing...
- The non-linear evolution is not described (numerical simulation, relation between the linearly evolved power spectrum to the fully non linear power spectrum, use halo models..)

Normalisation of the power spectrum

3 ways of getting the normalisation:

- Normalisation from CMB anisotropy on large scale
(= temperature fluctuations on linear scales)

- Density fluctuation inside a sphere:
(= mass (weak lensing), or galaxies (number density))

- Number density of clusters of galaxies

Normalisation of the power spectrum

The observed variance of galaxy counts in a sphere of 8 Mpc
Dispersion of the smoothed density field:

$$\sigma^2(R) = \langle \delta_R^2(\vec{x}) \rangle = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} |\hat{W}_R(k)|^2 P(k)$$

The normalisation of the power spectrum can in principle be measured from observations.

From galaxies. Inside a sphere of radius 8 h^{-1} Mpc, galaxy catalogues show that

$$\frac{\Delta N}{N} = \frac{\left\langle (N - \langle N \rangle)^2 \right\rangle}{\langle N \rangle^2} = 1$$

Dispersion of a smooth density field

$$\sigma^2(R) = \left\langle \delta_R^2(\vec{x}) \right\rangle = \int W_R(|\vec{x} - \vec{y}|) \delta(\vec{y})\, \mathrm{d}^3 y$$

If the galaxies follow the dark matter then

$$\sigma_8 = \sigma^2(8h^{-1}\mathrm{Mpc}) = \frac{\Delta N}{N} = 1$$

Normalisation of the power spectrum

BUT: galaxies may not trace the dark matter very well. However, one can assume that, to first order, there is a simple linear relation between the fluctuation of the number of galaxies and the mass density contrast:

$$\frac{\Delta n}{n} = b\,\frac{\Delta \rho}{\rho}$$

and b is the *bias factor*. In that case:

$$\sigma_8 = \frac{1}{b}$$

Normalisation of the power spectrum

A structure with density contrast that reaches $\sigma_8 = 1$ enters into the non-linear regime. Today, it correspond to mass such that:

$$M = \frac{4\pi}{3}\bar{\rho}\left[8\ h^{-1} Mpc\right]^3 \quad (3)$$

$$M = \frac{4\pi}{3}\Omega_m \frac{3H_0^2}{8\pi G}\left[8\ h^{-1} Mpc\right]^3 \quad (4)$$

that is

$$M = 5 \times 10^{14}\ \Omega_m\ h^{-1}\ M_\odot \quad (5)$$

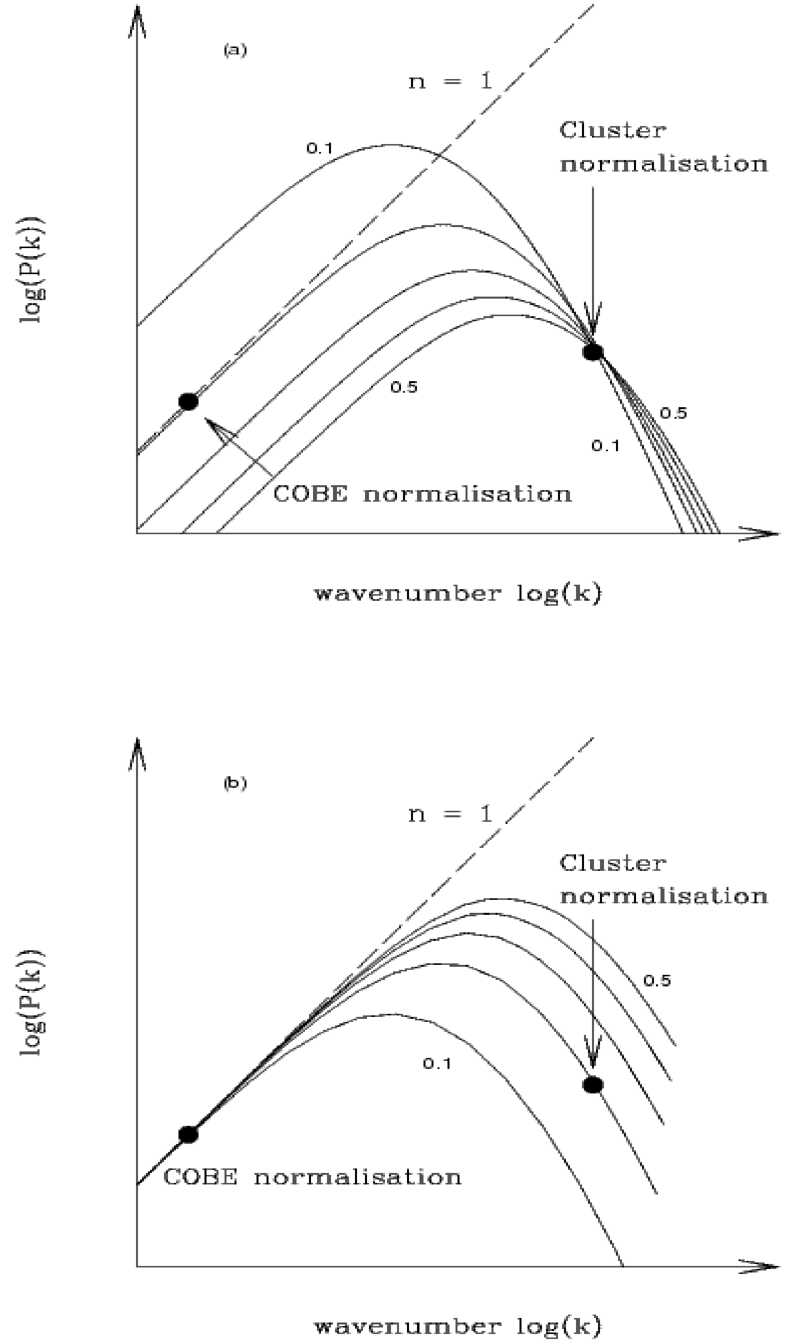
which corresponds to clusters of galaxies.

Normalisation of the power spectrum

to check the validity of CDM power specturm; it is important to check the normalisation at large and small scales:

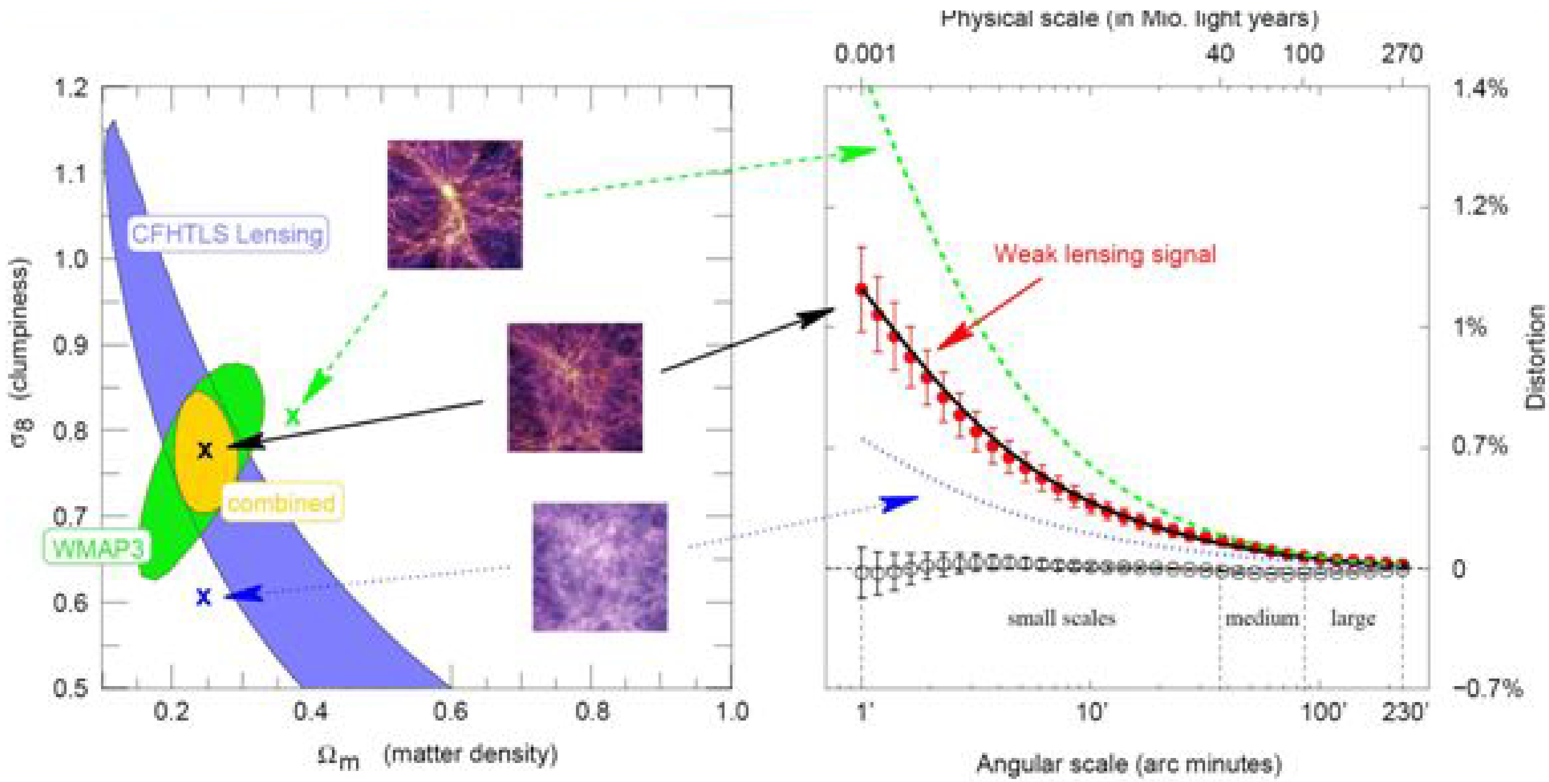
- Linear scales: CMB (COBE)

- Non linear scales: clusters of galaxies, galaxies, Lyman-alpha forest, weak lensing (σ_8)



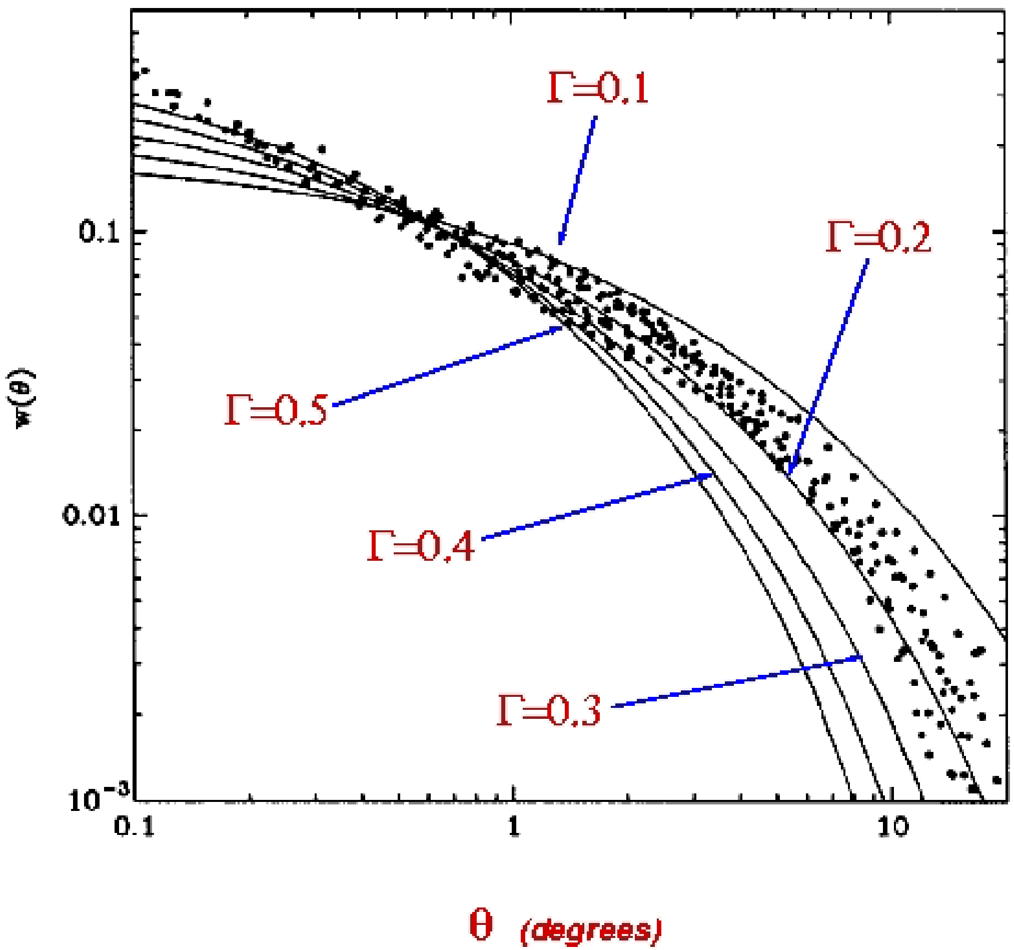
Constrains on Ω_m-σ_8 from CMB + Weak Lensing

Clustering of dark matter and power spectrum normalisation



Ω_m=0.248+/- 0.019 σ_8= 0.771 +/-0.029

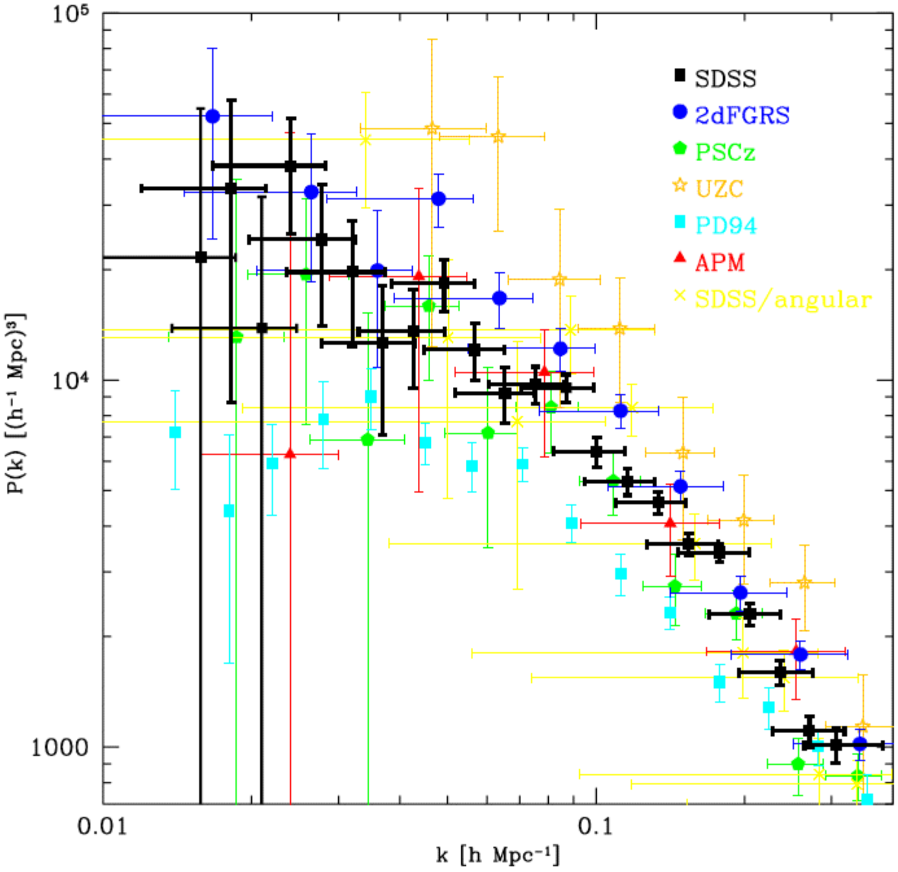
The observed shape parameter of the power spectrum

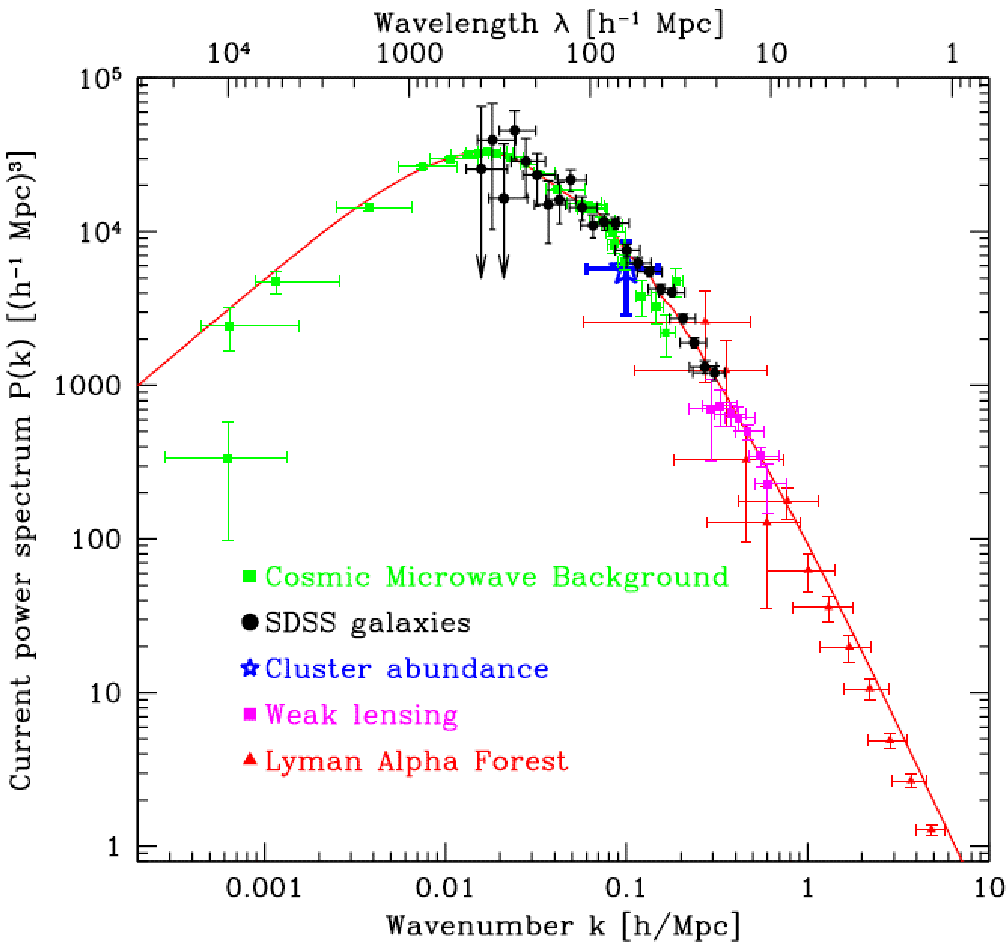


Fitting the shape parameter:

the APM galaxy power spectrum

The observed power spectrum

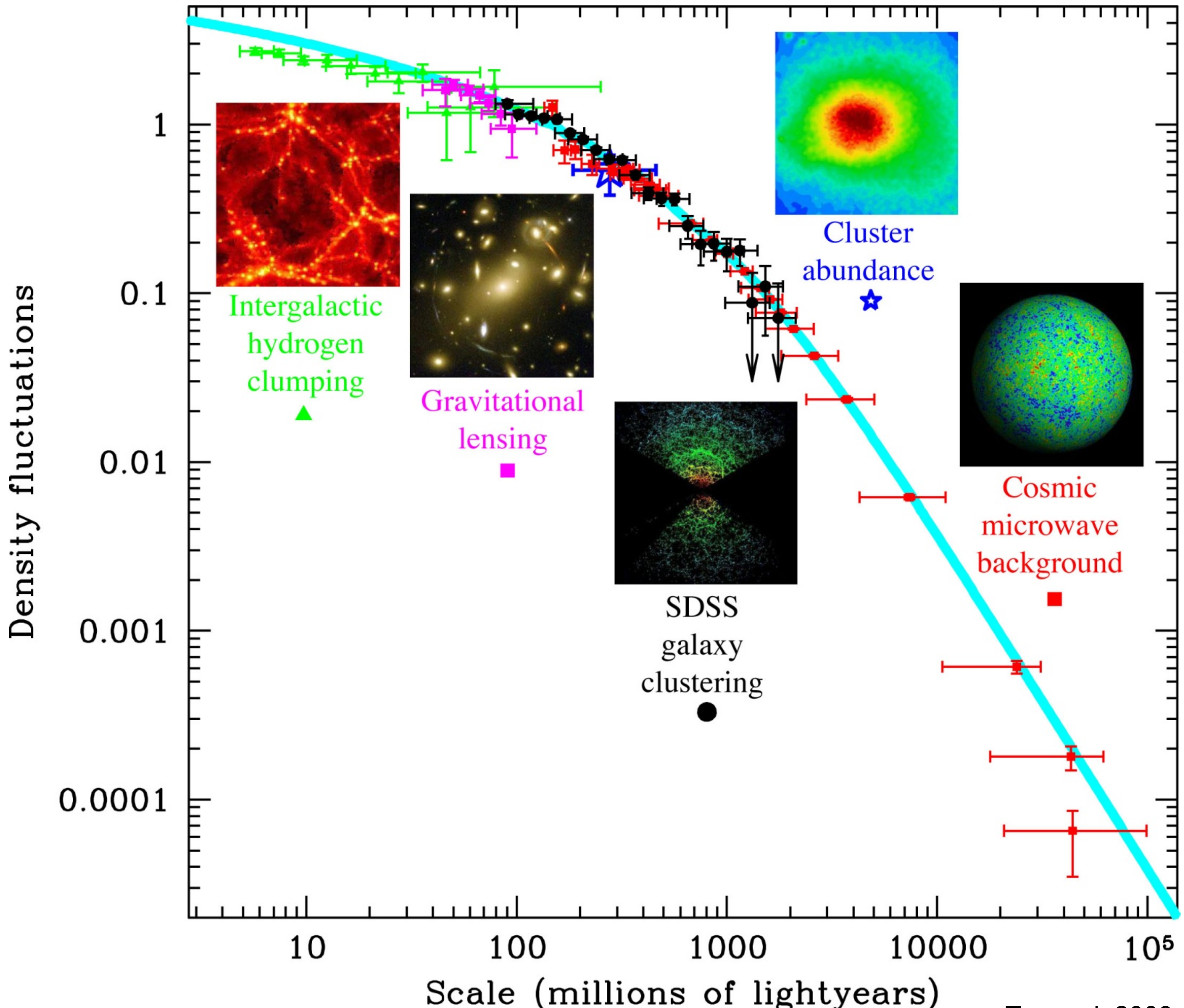




Best fit : Ω_Λ=0.72, Ω_m=0.28, Ω_b=0.04, H_0=72, τ=0.17, b_{SDSS}=0.92

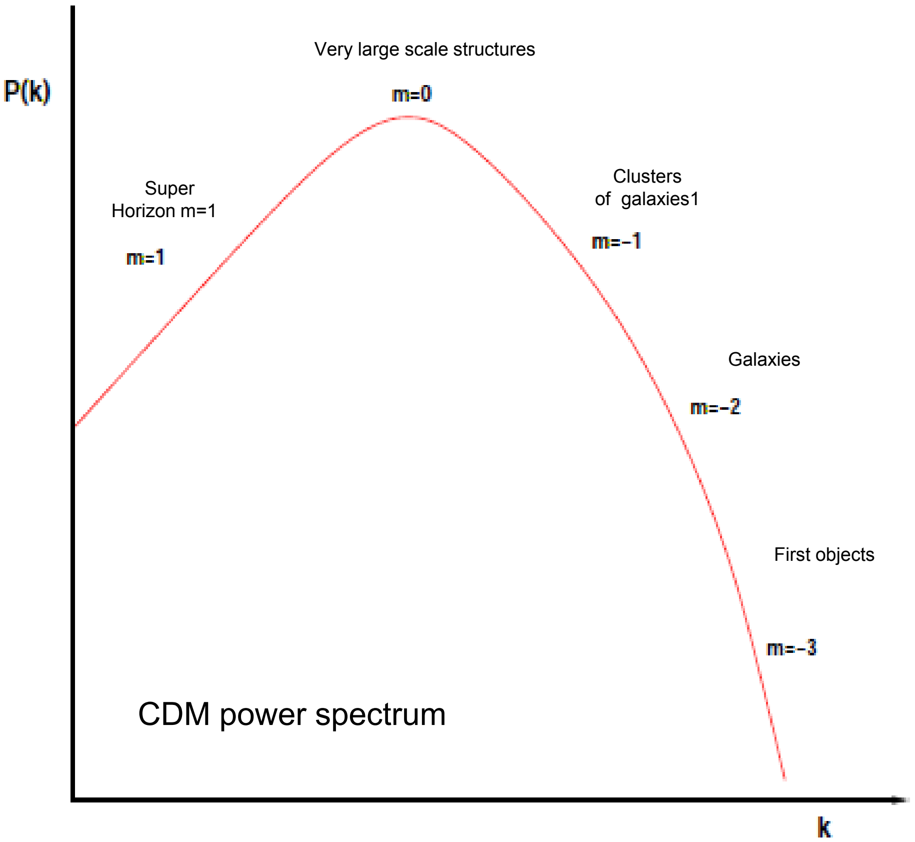
Tegmark 2003

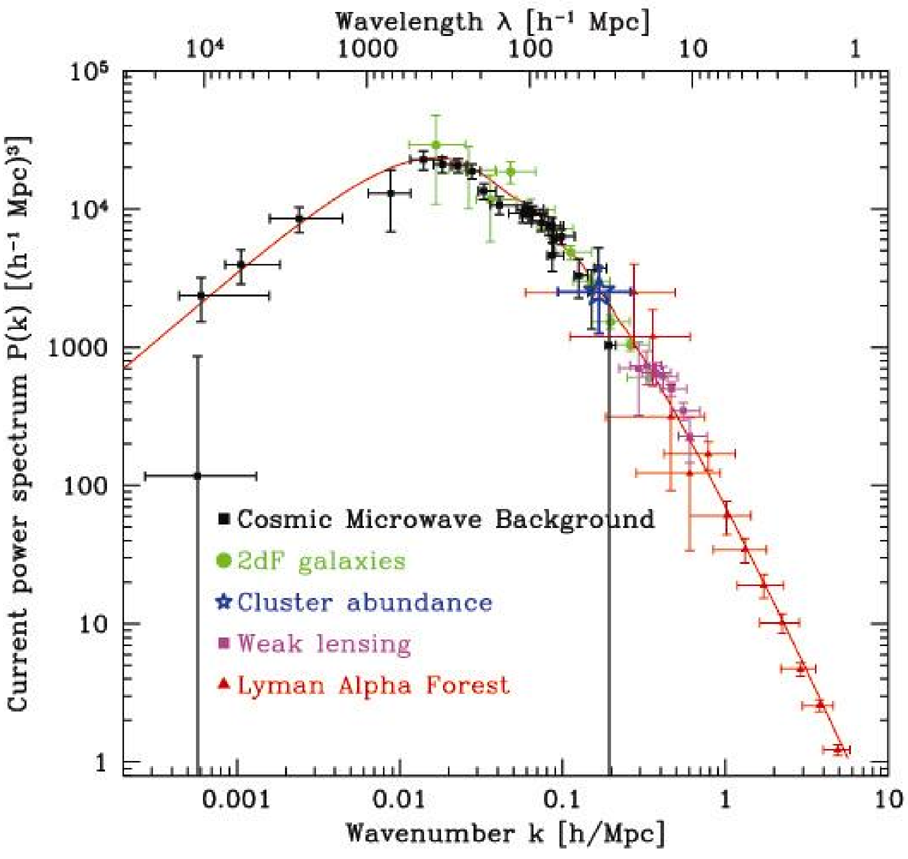
The observed power spectrum



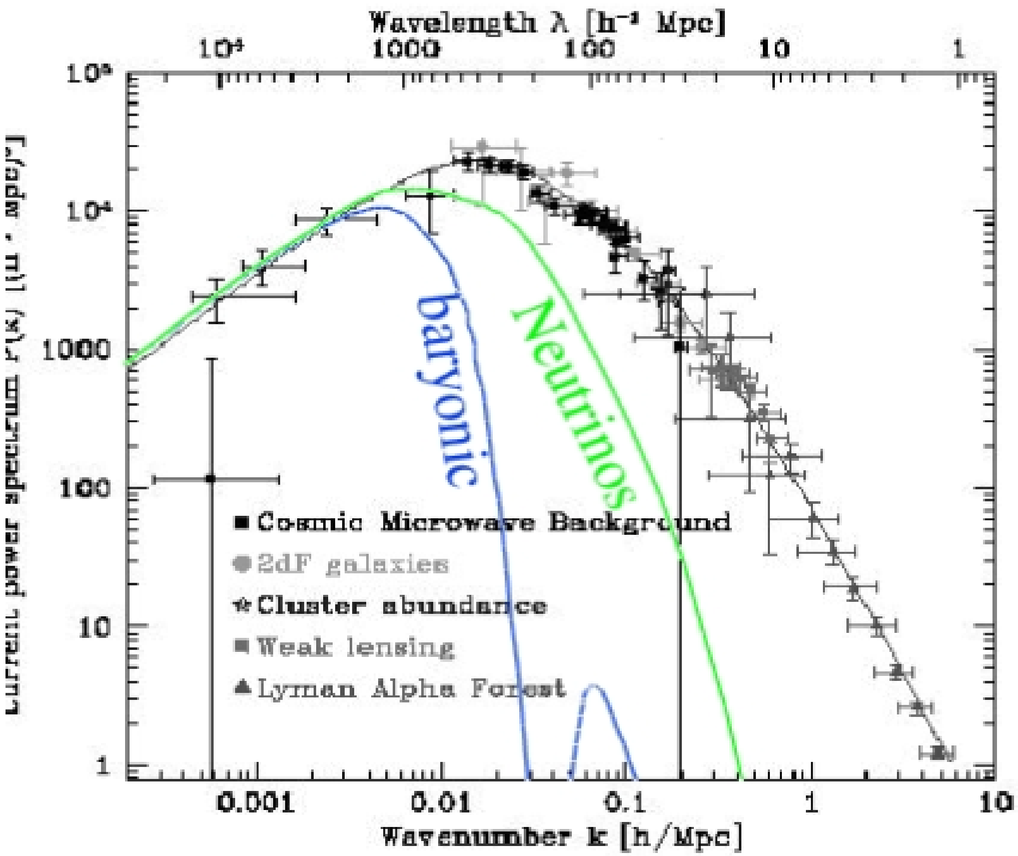
Tegmark 2003

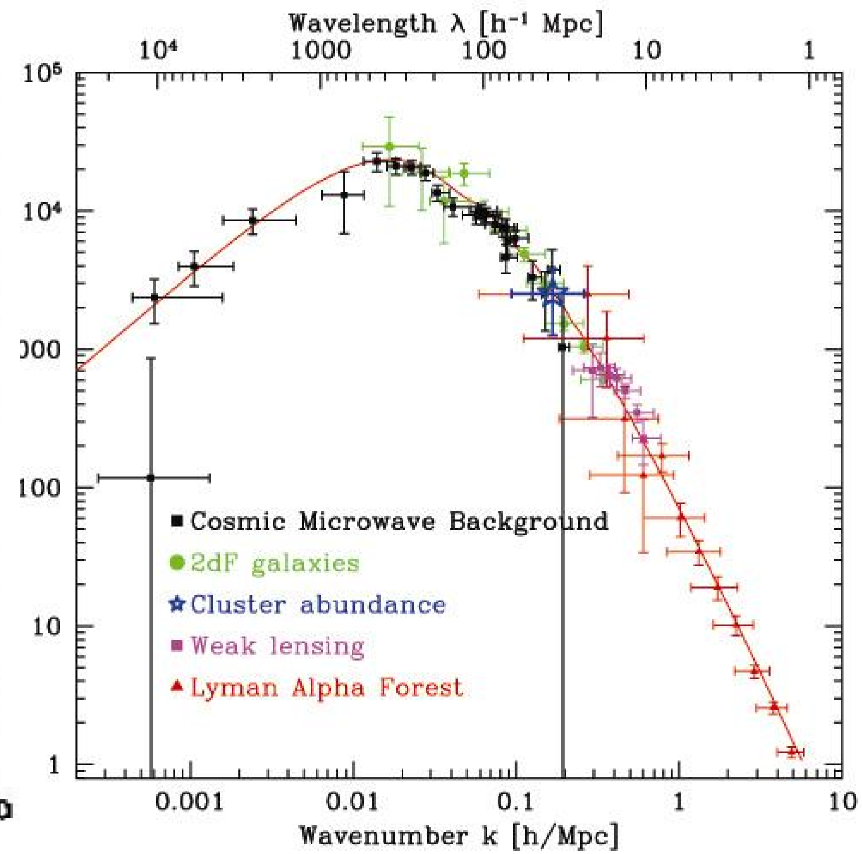
Comparing observations with the predictions of cold dark matter dominated universes





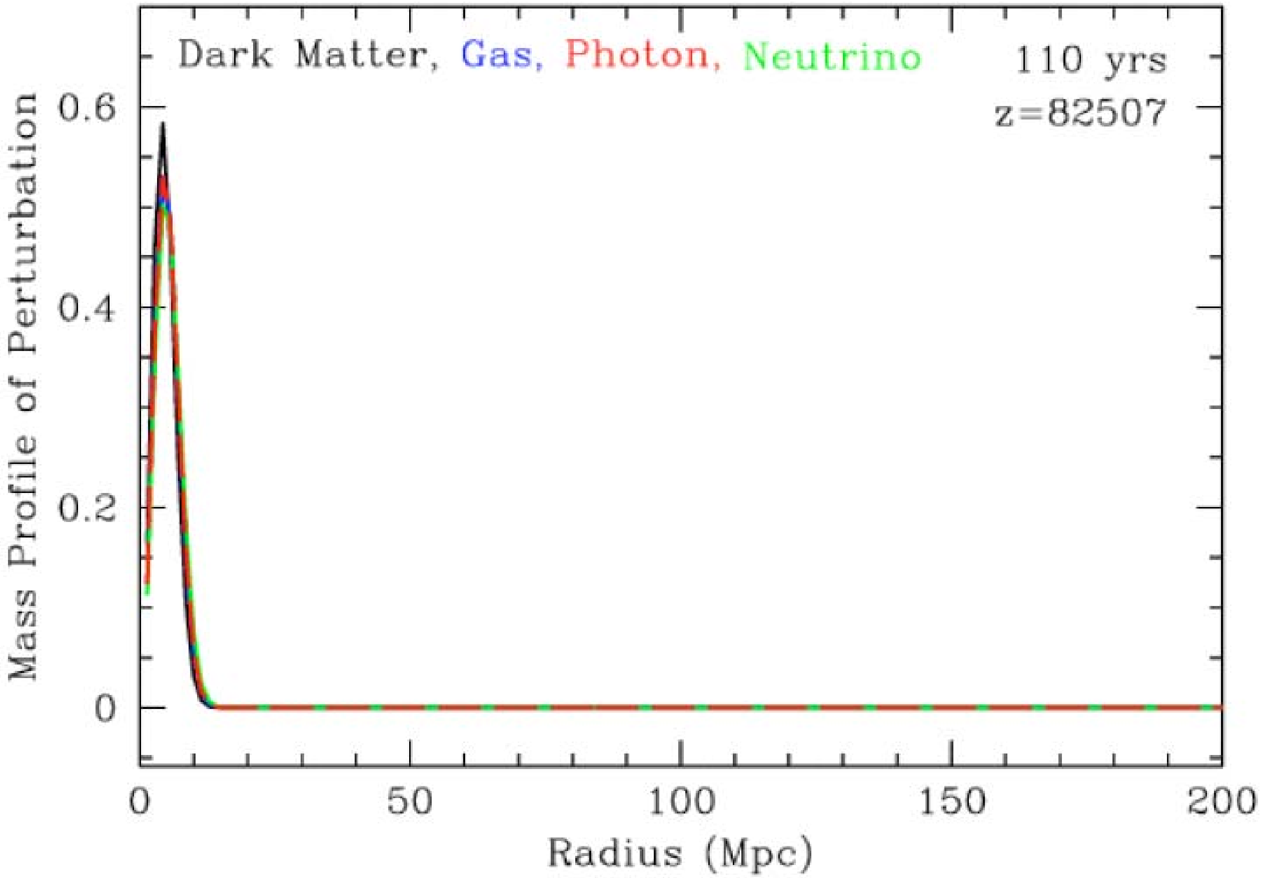
Comparing observations with the predictions of cold-dark matter dominated universe



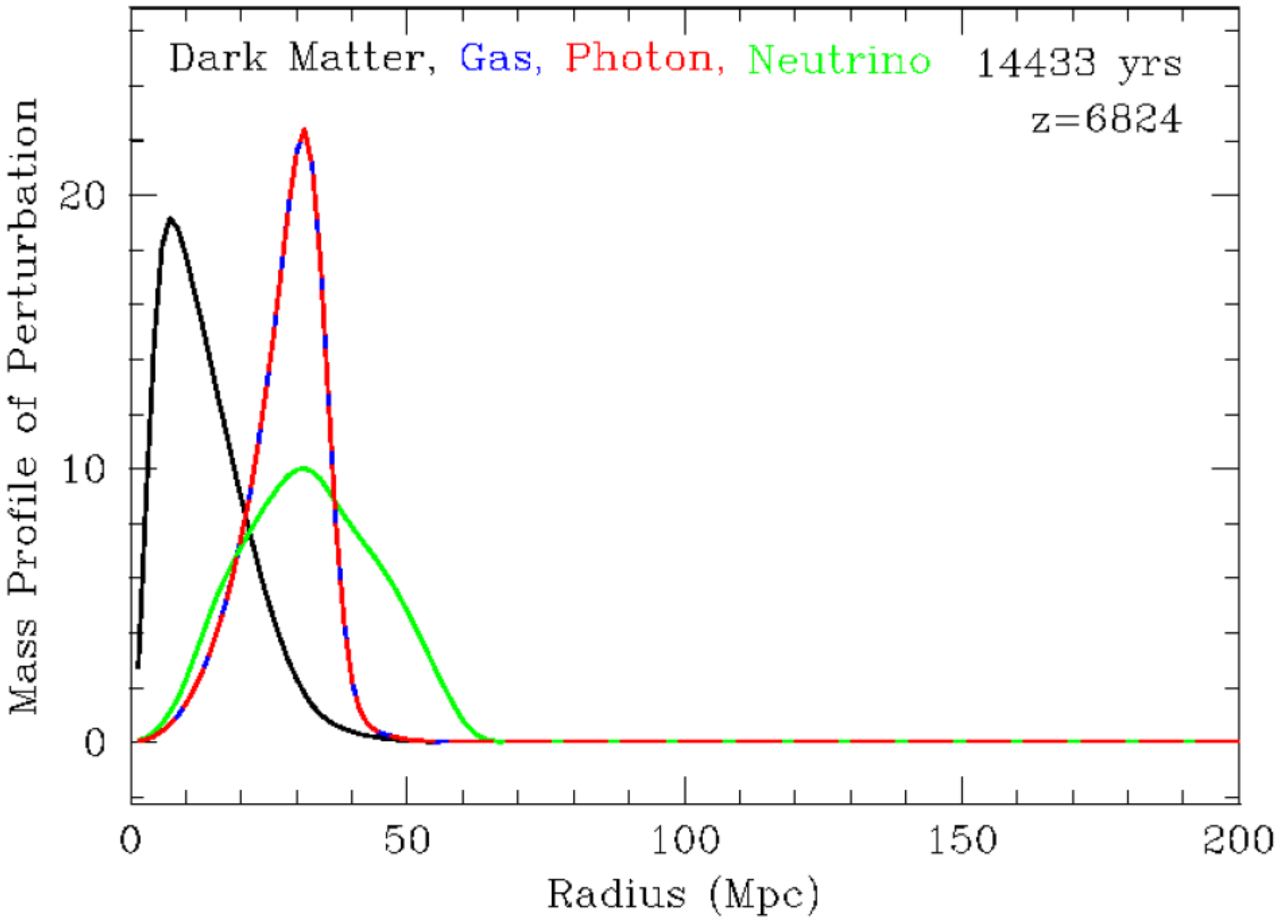


Consequence of baryon oscillations

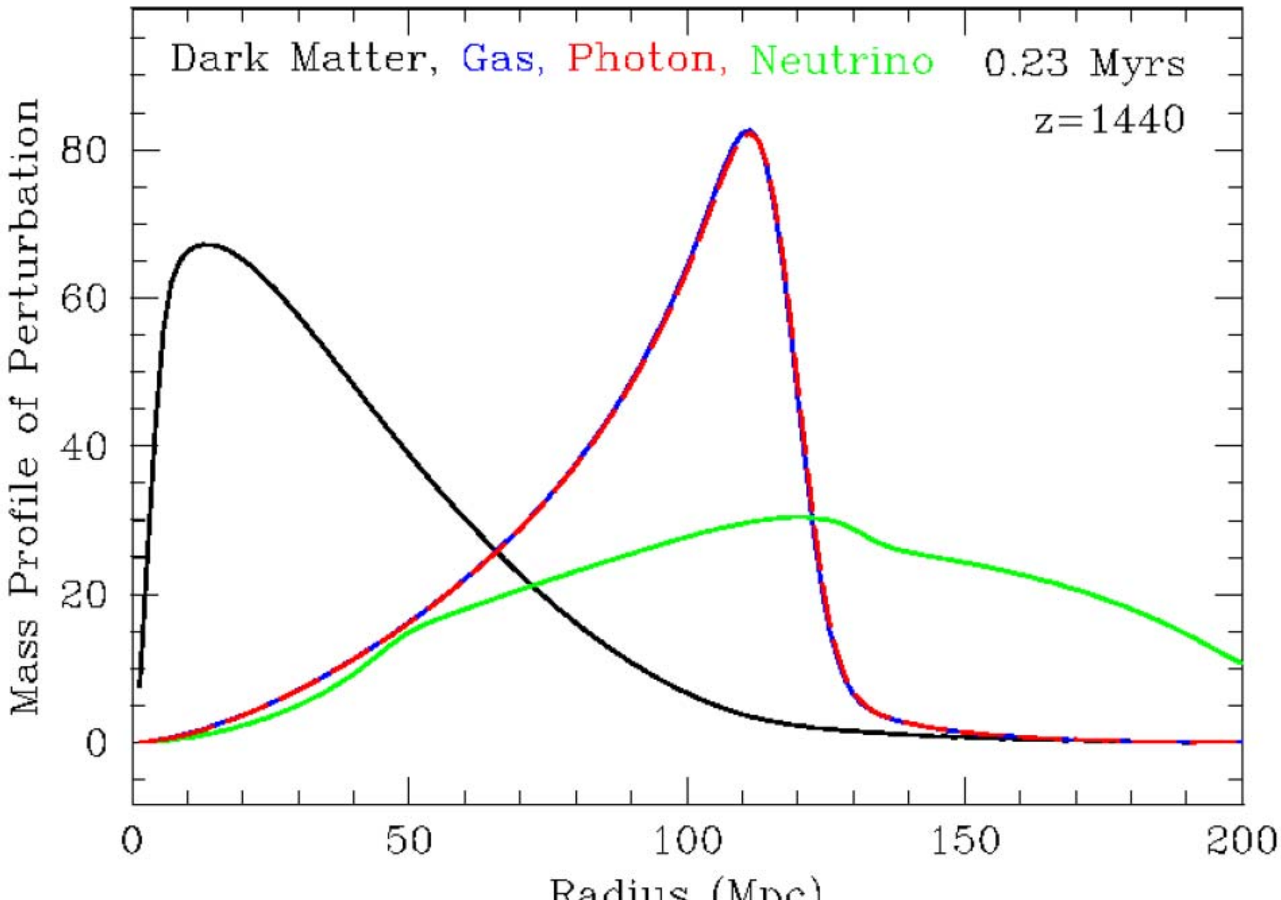
Initial perturbation



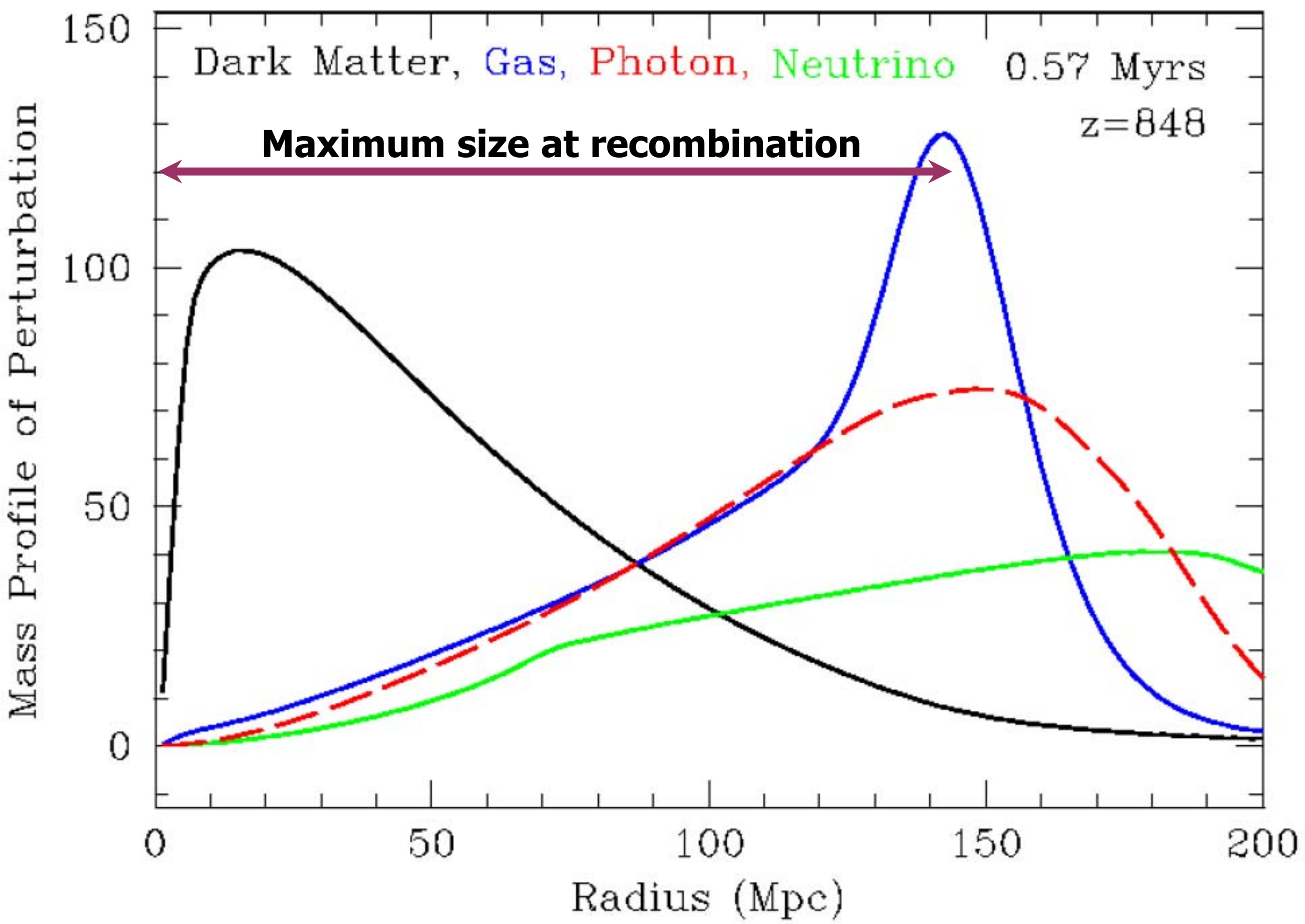
Consequence of baryon oscillations



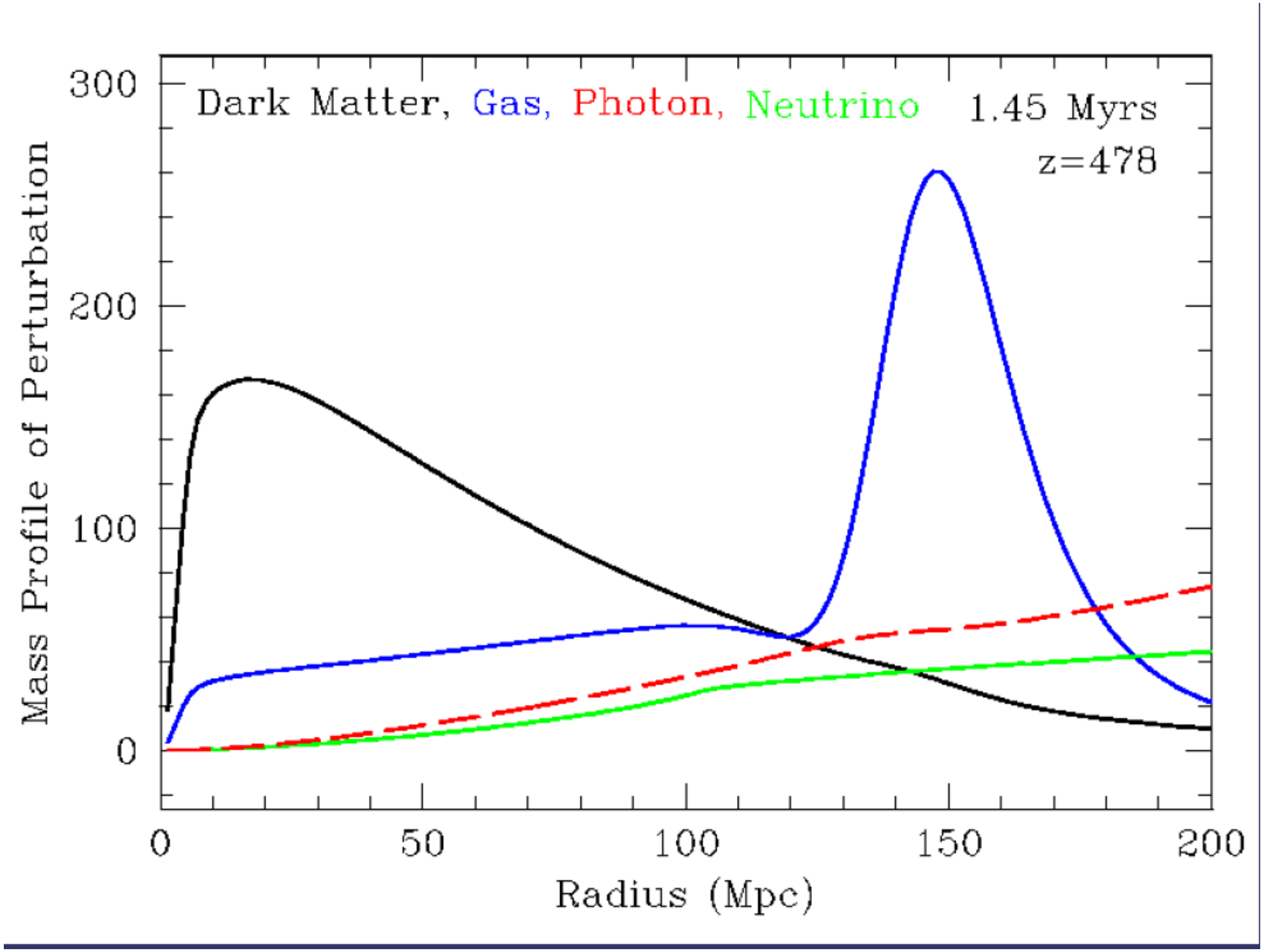
Consequence of baryon oscillations



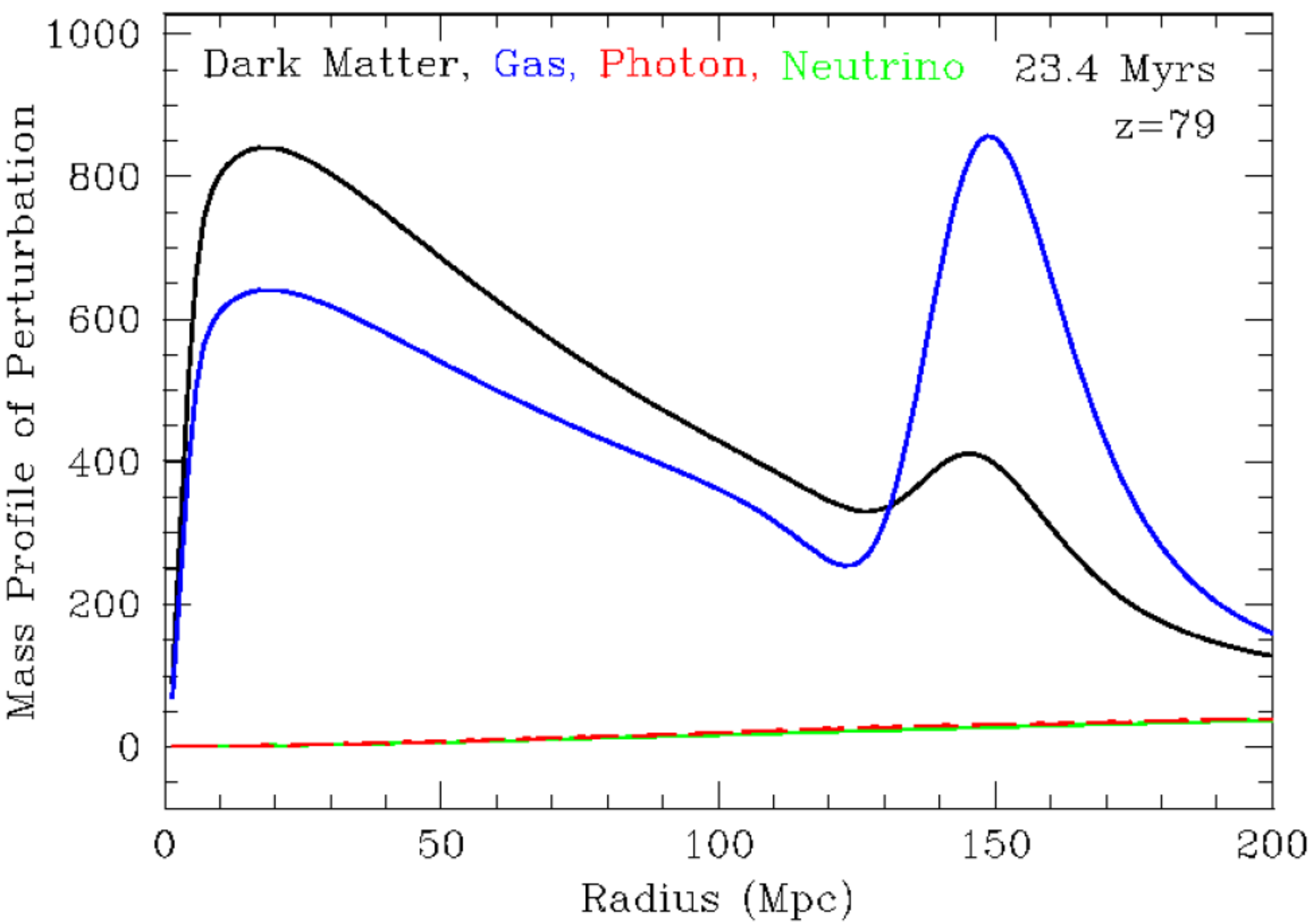
Consequence of baryon oscillations



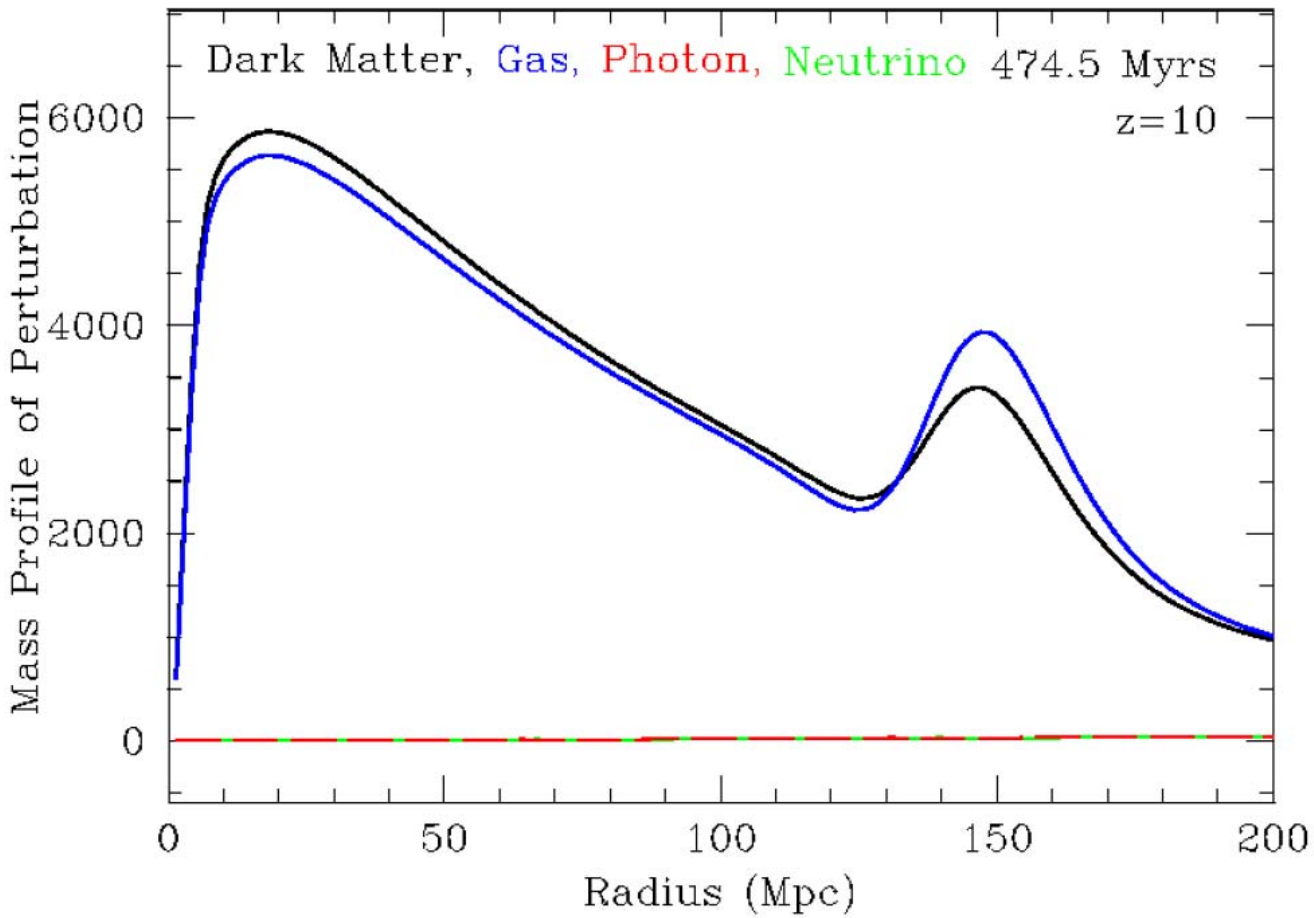
Consequence of baryon oscillations



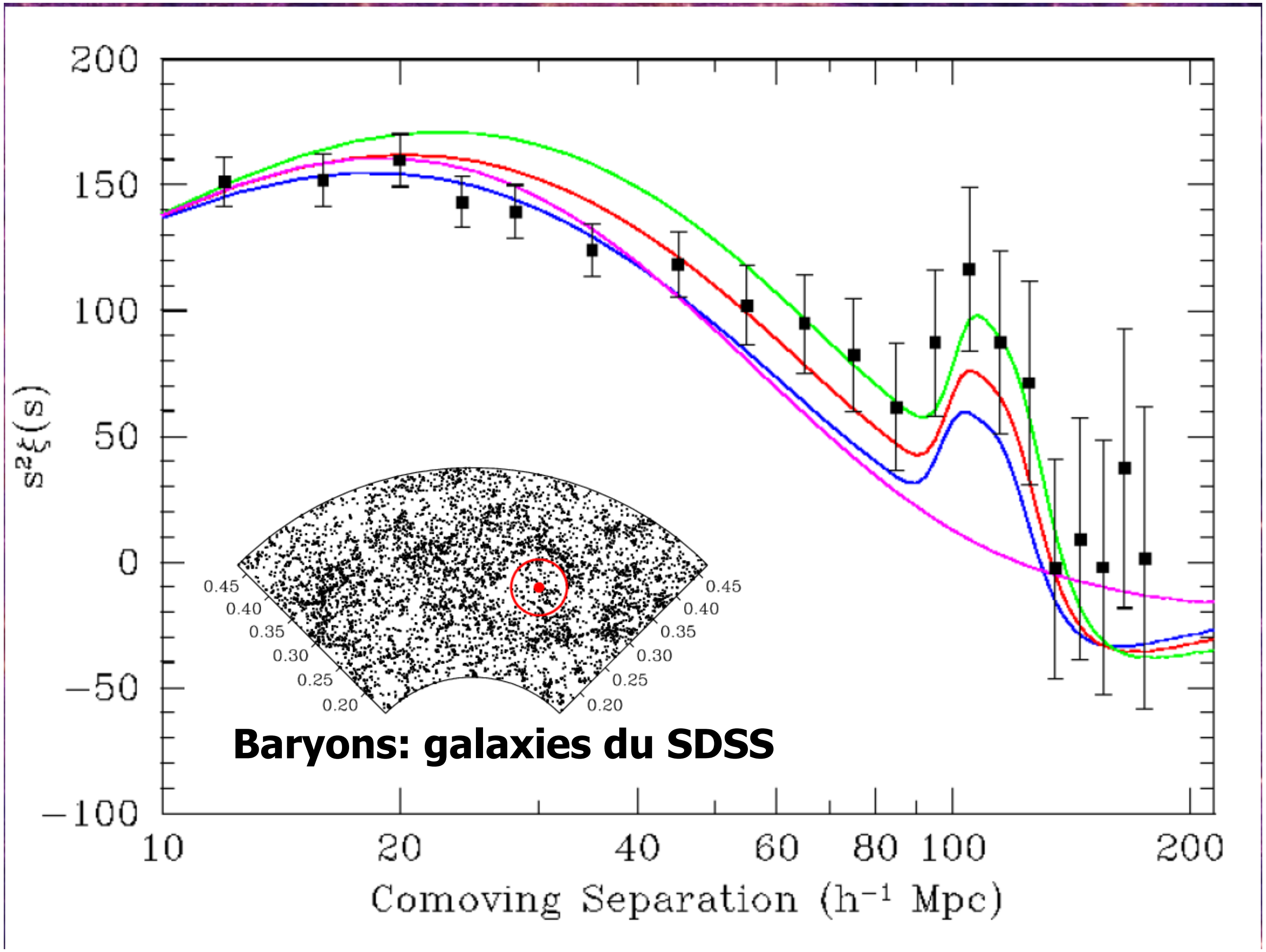
Consequence of baryon oscillations



Consequence of baryon oscillations

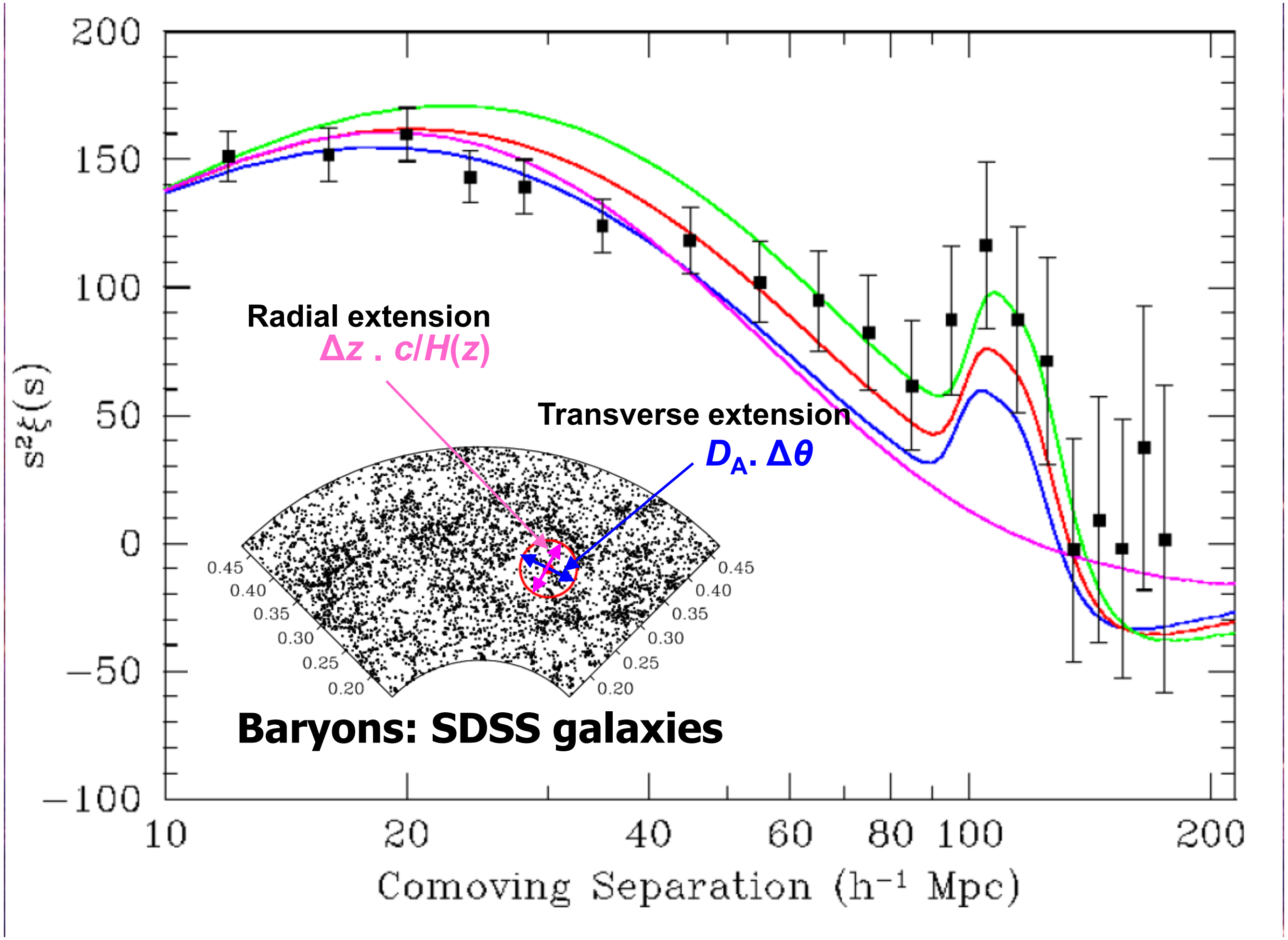


Consequence of baryon oscillations



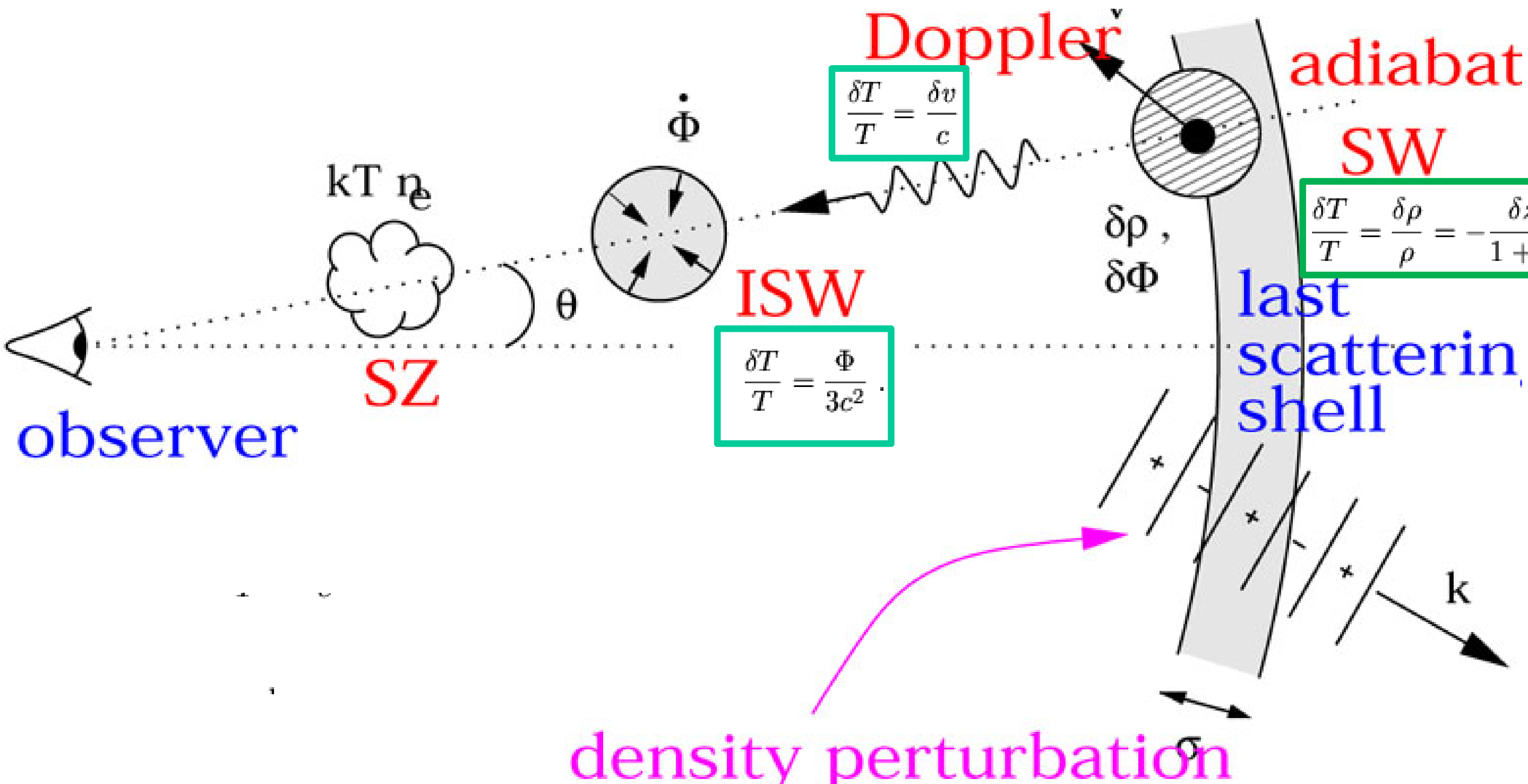
Consequence of baryon oscillations

A remarkable predictions of gravitational instability scenario and the behavior of baryons

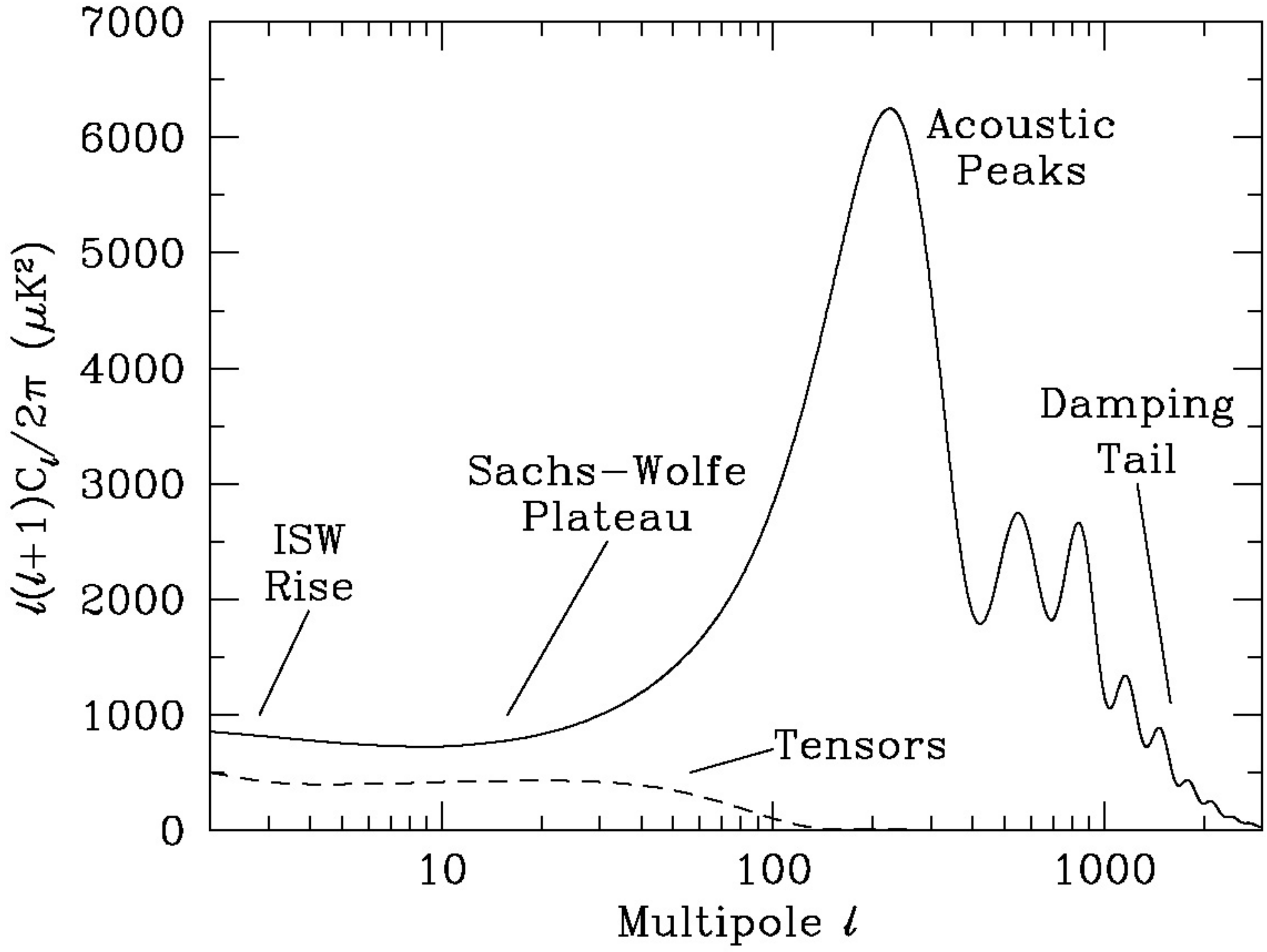


Primary CMB anisotropies:

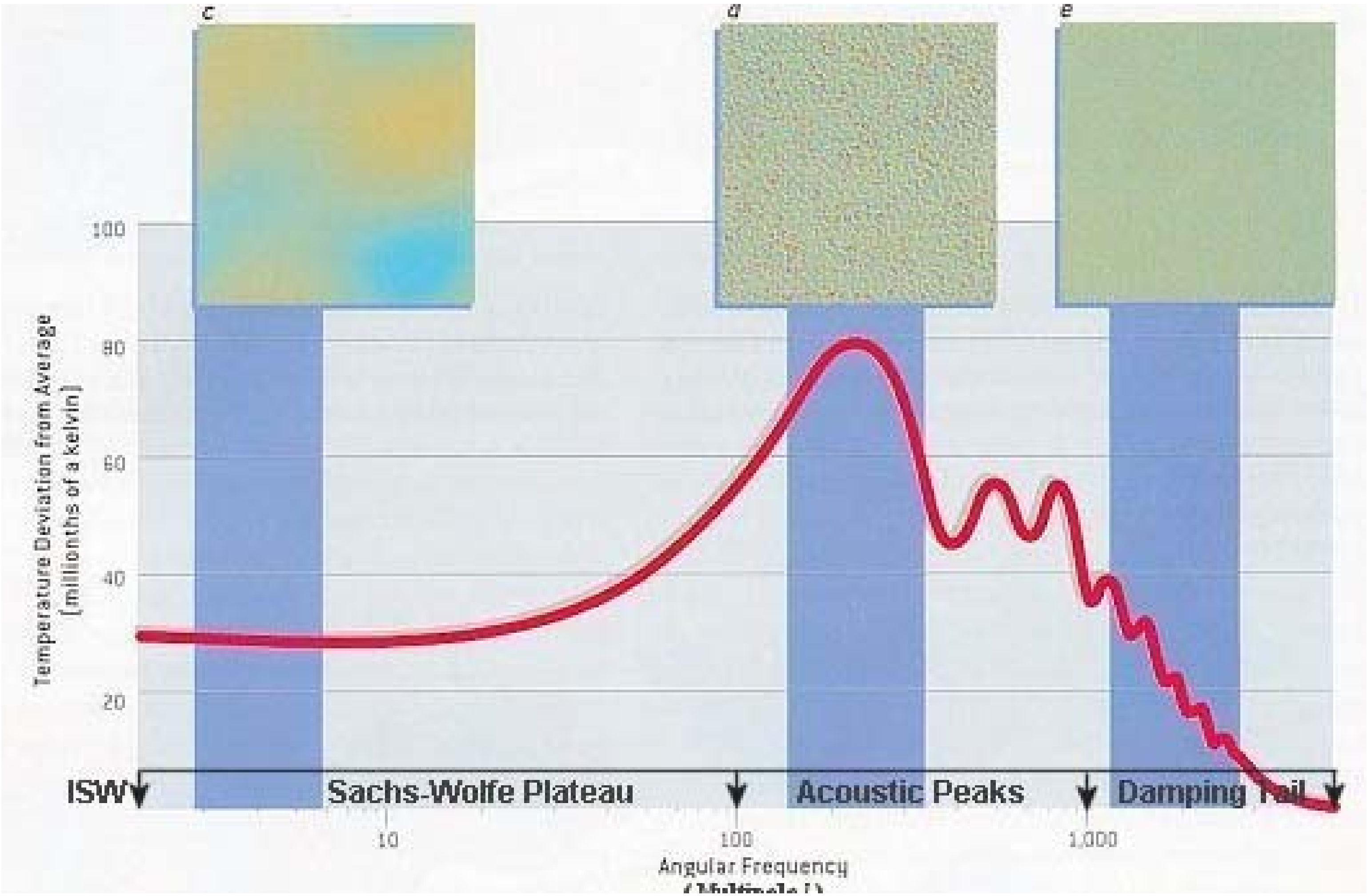
results from density, temperature velocity perturbations



Cosmological contribution to C_l



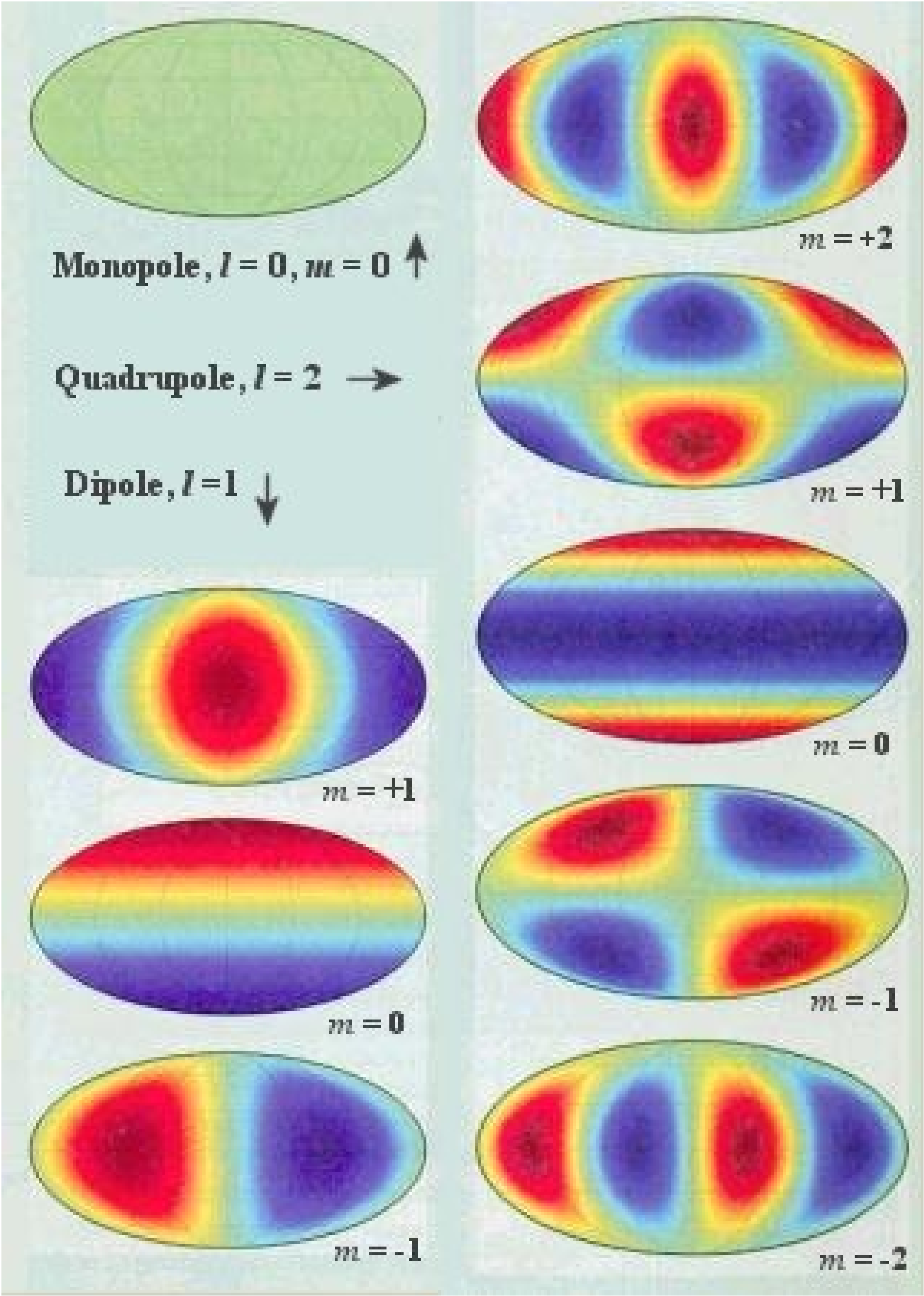
Cosmological contribution to C_l



CMB anisotropies decomposition

$$\frac{\Delta T}{T} = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$$

Multipole expansion

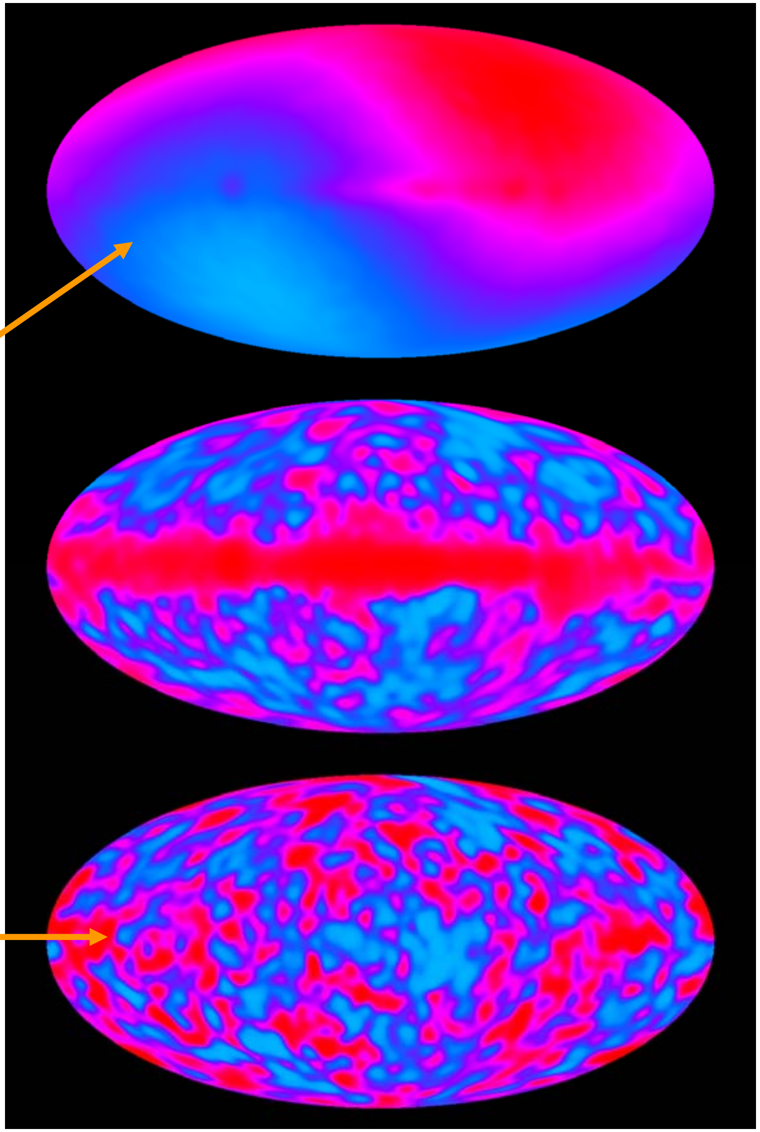


CMB anisotropies: contamination

$$\frac{\Delta T}{T} = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$$

$$T(\theta) = T_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \left(1 + \frac{v}{c}\cos\theta\right)$$

$$C_l = \frac{1}{2\pi} \left(\frac{H_0}{c}\right)^4 \int_0^\infty \frac{P(k)}{k^2} j_l^2 \left(2ck/H_0\right) \, \mathrm{d}k \, ,$$



Sensitivity of primary peaks

1st peak:

- Primarily depends on curvature: low curvature more the first peak toward larger l (smaller scales)

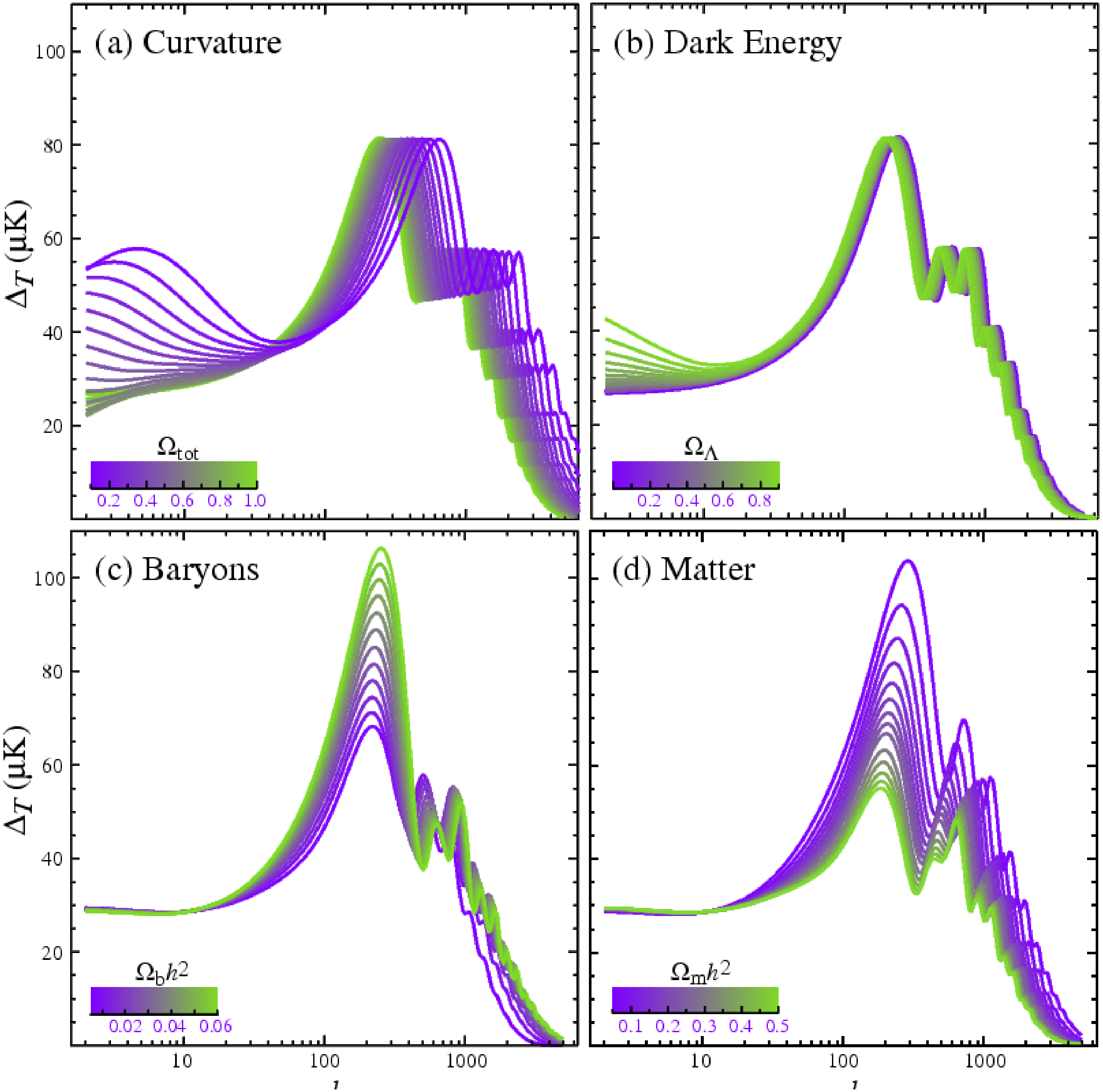
2^{sd} peak:

- Mainly sensitive to the amount of baryons and the baryon/photon ratio

Other peaks:

- Primarily sensitive to Ω_m

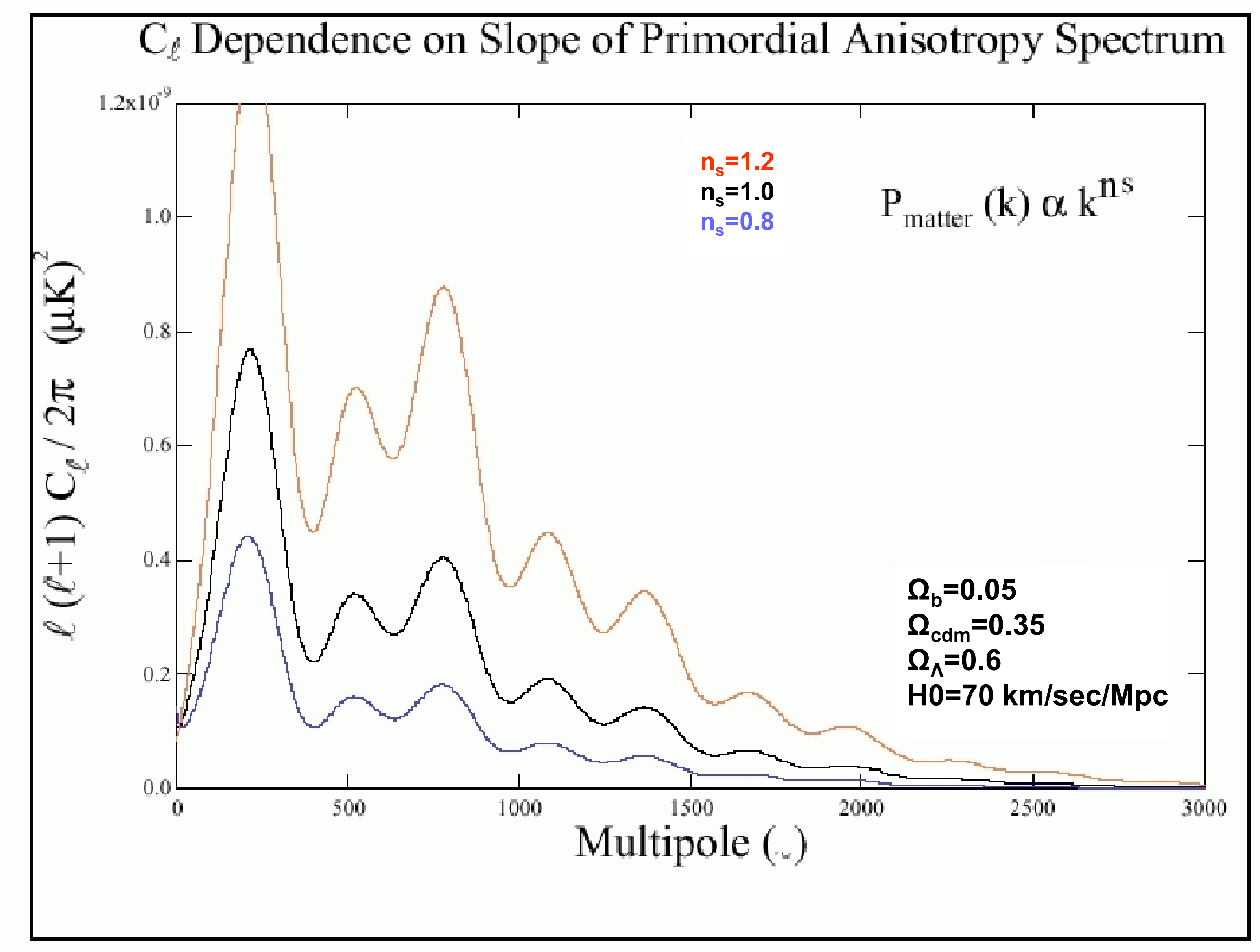
CMB peaks and Ω_i



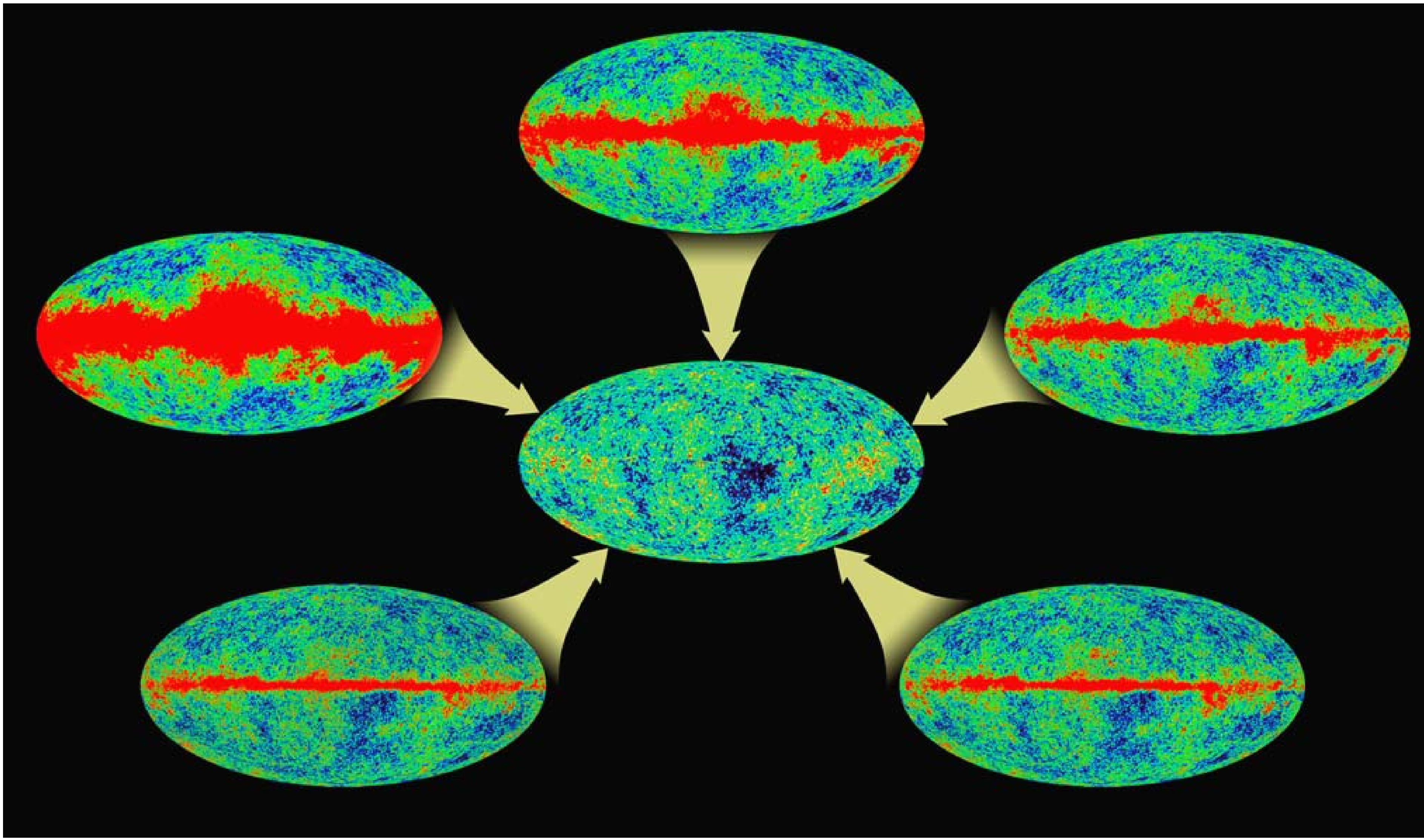
$(\Delta T/T)_\theta \sim [l(l+1)C_l/2\pi]^{1/2}$

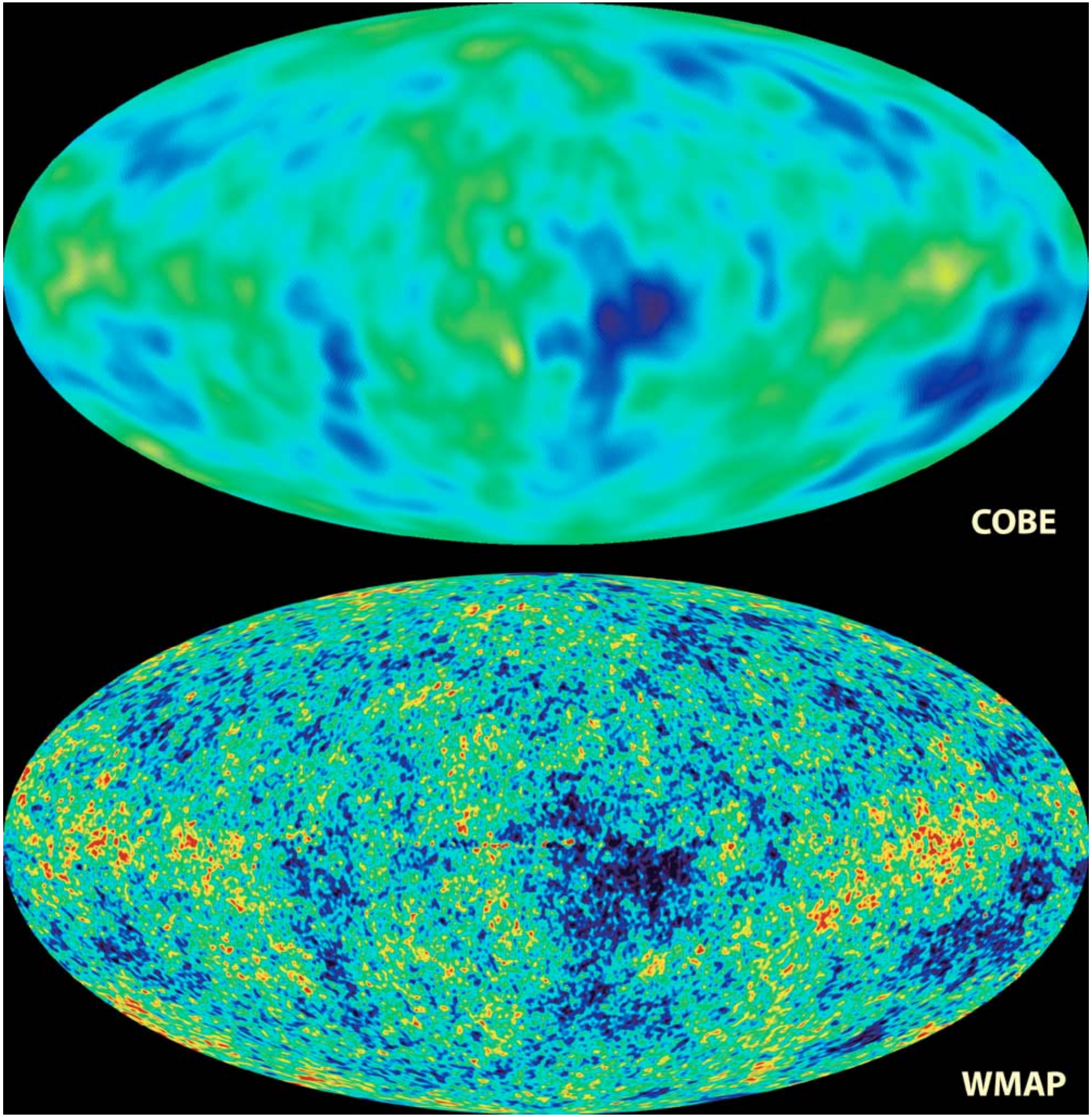
$l \sim 100^\circ/\theta \qquad \theta = 1' \sim 2$ Mpc

CMB peak and the spectral index



CMB : full signal

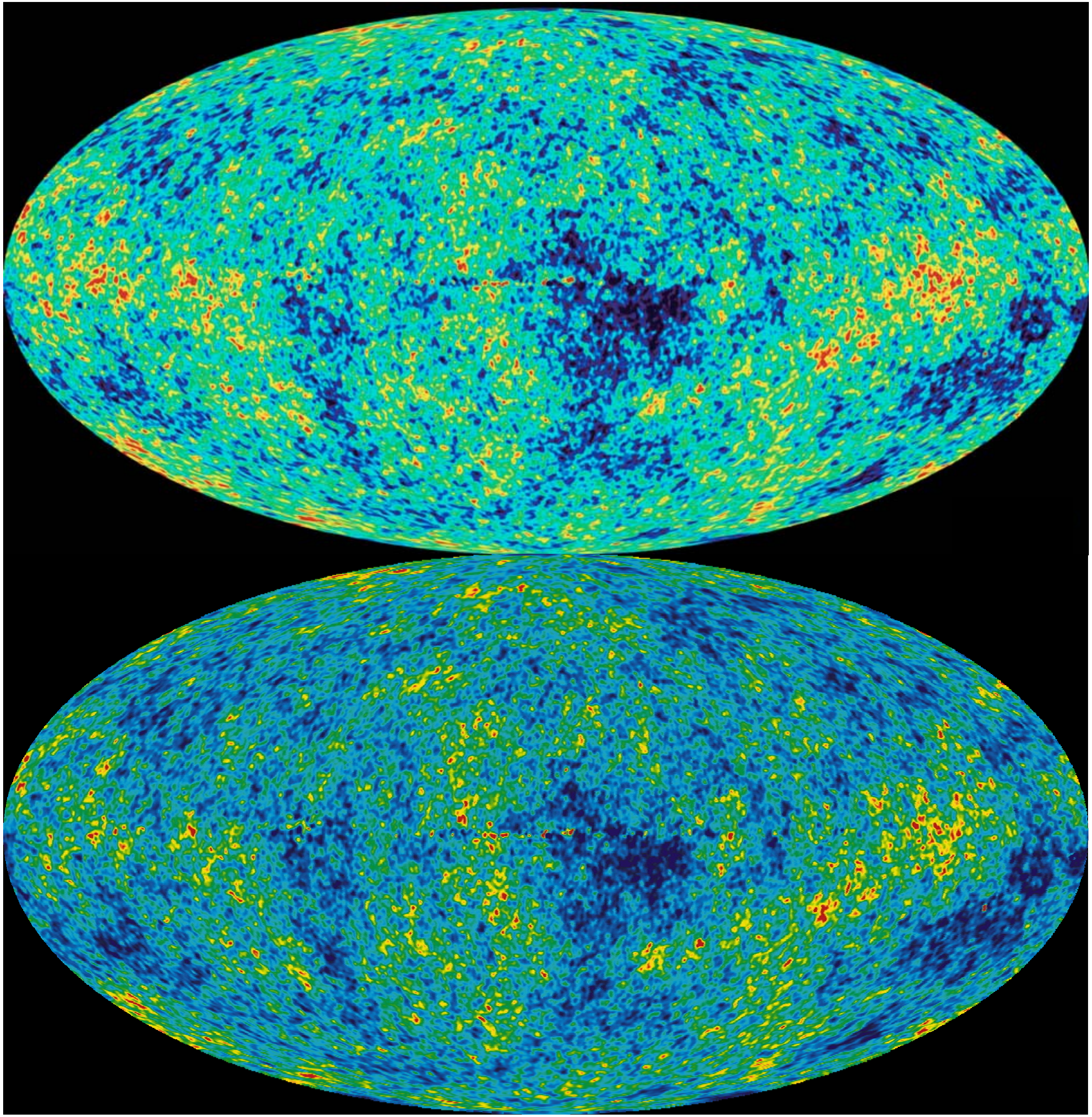




COBE

and

WMAP

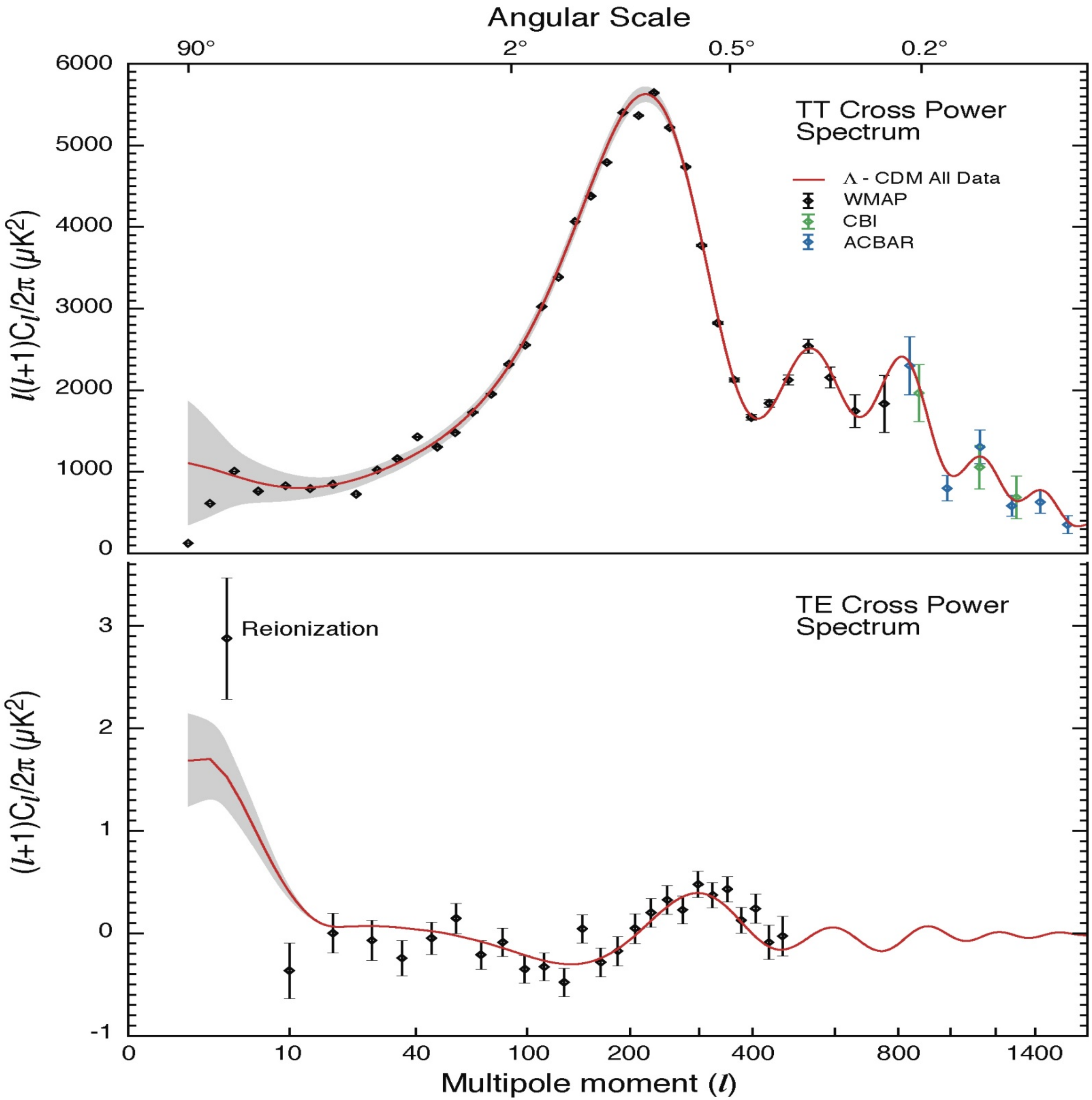


WMAP-1

and

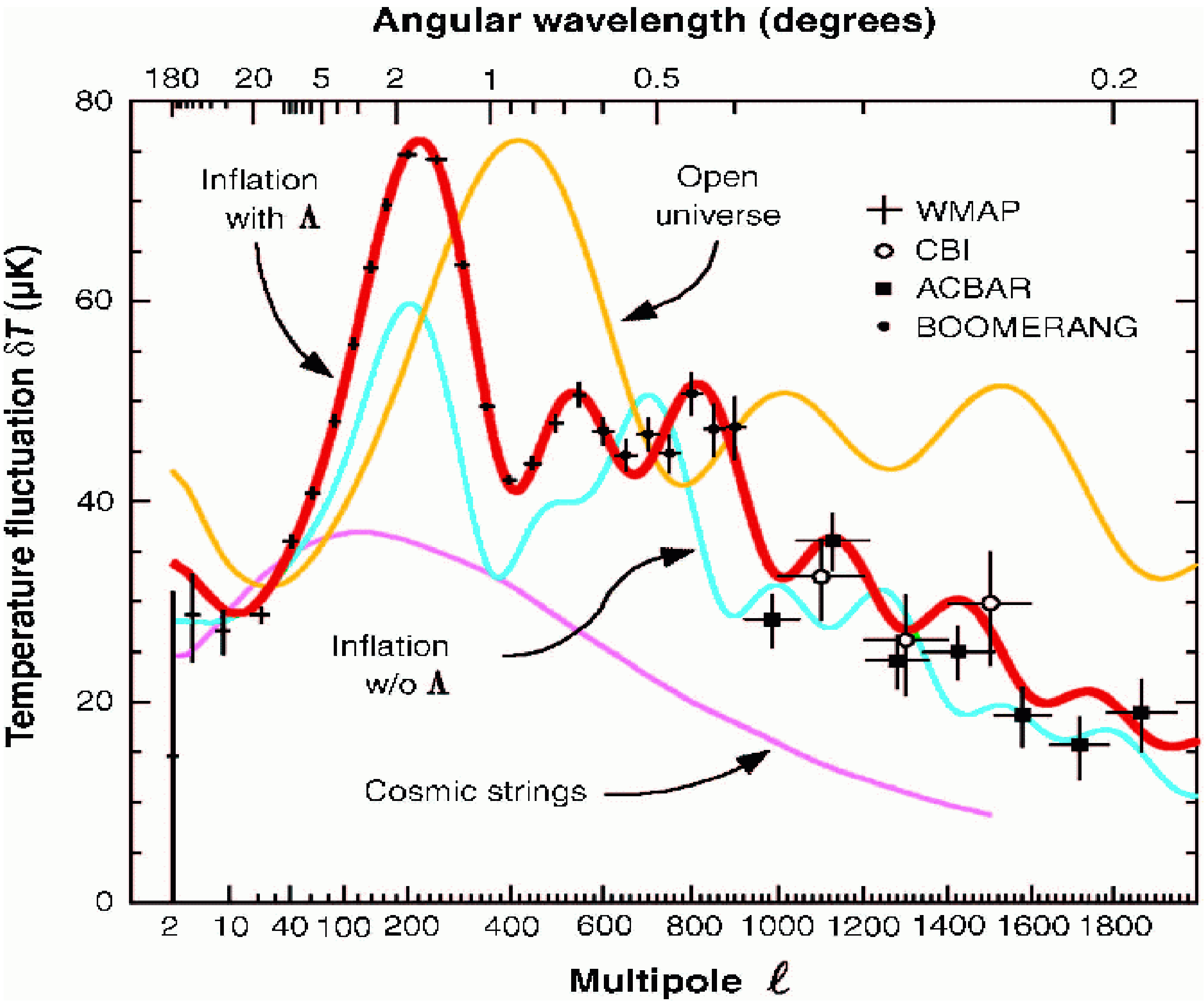
WMAP-5

WMAP



Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
l(l+1)C_l/2π (μK²)
3
2
1
0
-1
Reionization
TE Cross Power Spectrum
(l+1)C_l/2π (μK²)
0
10
40
100
200
400
800
1400
Multipole moment (l)

CMB anisotropies can constrain properties of dark matter (and much more)

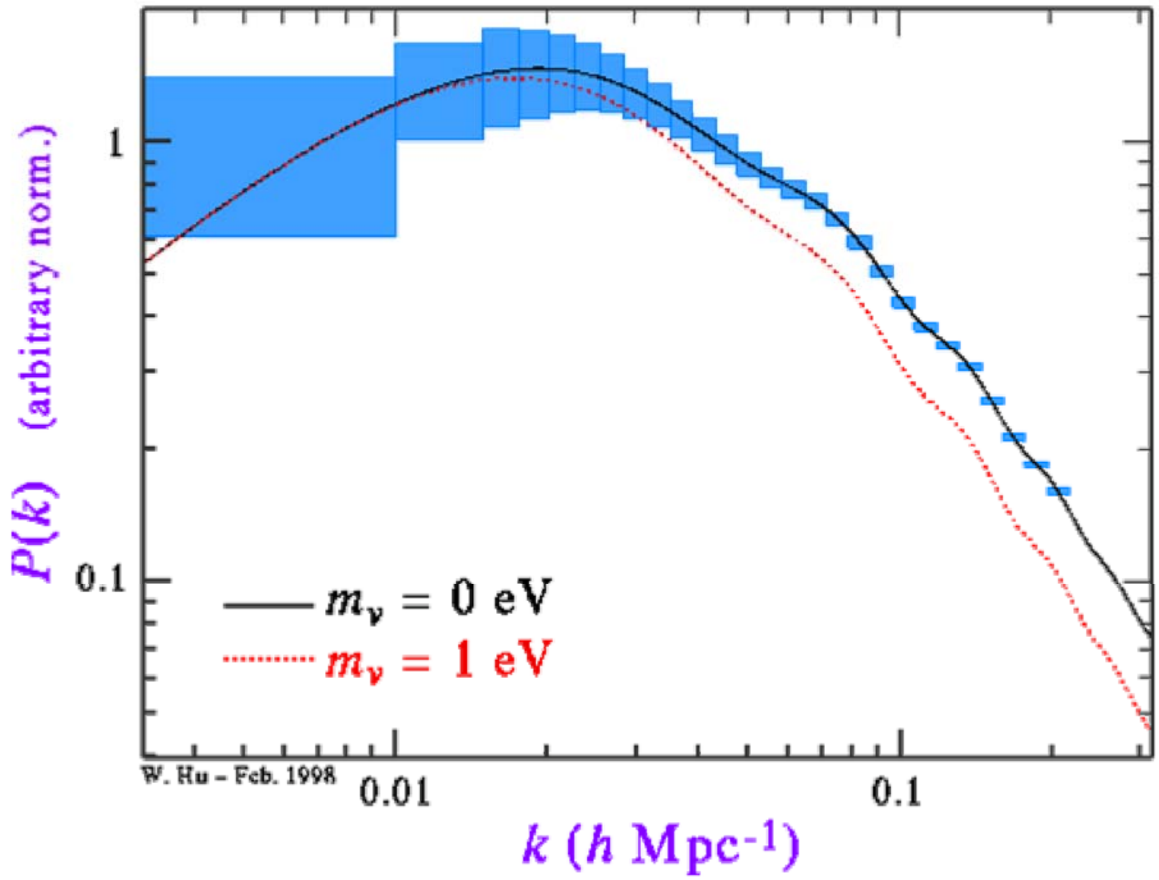


Parameter	Values derived from WMPA-5 only
h	$0.719^{+0.026}_{-0.027}$
Ω_{tot}	$1.099^{+0.100}_{-0.085}$
w	$-1.06^{+0.41}_{-0.42}$
$\Omega_b h^2$	0.02273 ± 0.00062
Ω_b	0.0441 ± 0.0030
$\Omega_{dm} h^2$	0.1099 ± 0.0062
Ω_{dm}	0.214 ± 0.027
Ω_Λ	0.742 ± 0.030
σ_8	0.796 ± 0.036
n_s	$0.963^{+0.014}_{-0.015}$
t_0	$13.69 \pm 0.13 \times 10^9$ ans
z_{eq}	3176^{+151}_{-150}
$d_A(z_{eq})$	14279^{+188}_{-191}Mpc
z_{dec}	1090.51 ± 0.95
$d_A(z_{dec})$	14115^{+188}_{-191}Mpc
t_{dec}	380081^{+5843}_{-5841} ans
$t_{reionis}$	$427^{+88}_{-65} \times 10^6$ ans
$r = T/S$ (à $k_0 = 0.002$ Mpc^{-1})	< 0.43 (95% C.L.)
$dn_s/d\ln k$ (à $k_0 = 0.002$ Mpc^{-1})	-0.037 ± 0.028
$\Omega_\nu h^2$	< 0.014 (95% CL)
$\sum m_\nu$	1.3 eV (95% CL)
$N_{\nu eff}$	> 2.3
$\tau(reionis)$	0.087 ± 0.017
$z(reionis)$	11.0 ± 1.4

WMAP-5

Limits on neutrino mass from the shape of the matter power spectrum at large scale

galaxy redshift surveys



Limits on neutrino mass from all cosmological probes of the matter power spectrum

Probes/data	Authors	Neutrino mass limits
2dF	Elgaroy et al (2002)	$\sum m_\nu < 1.8$ eV
WMAP-3+Lyα+SDSS	Seljak et al (2004)	$\sum m_\nu < 0.17$ eV
WMAP-3 +BAO+SNIa	Komatsu et al (2005)	$\sum m_\nu < 0.67$ eV
WMAP-3 seul	Fukugita et al (2006)	$\sum m_\nu < 2.0$ eV
CMB + 2dF	Sanchez et al (2005)	$\sum m_\nu < 1.2$ eV
CMB+BAO+LSS+SNIa	Goobar et al (2006)	$\sum m_\nu < 0.62$ eV
WL[CFHTLS-T01+autre]+WMAP-5+SNIa	Li et al (2008)	$\sum m_\nu < 0.47$ eV
WL[CFHTLS-T03]+WMAP5+SNIa	Tereno et al (2008)	$0.03 < \sum m_\nu < 0.54$ eV
WL[CFHTLS-T01+autre]+SNIa+BAO+ RAG	Gong et al (2008)	$\sum m_\nu < 0.80$ eV
WMAP-5 +BAO+SNIa	Komatsu et al (2008)	$\sum m_\nu < 0.61$ eV
WL[CFHTLS-T03]+WMAP5+SNIa+BAO	Ichiki et al (2009)	$\sum m_\nu < 0.54$ eV

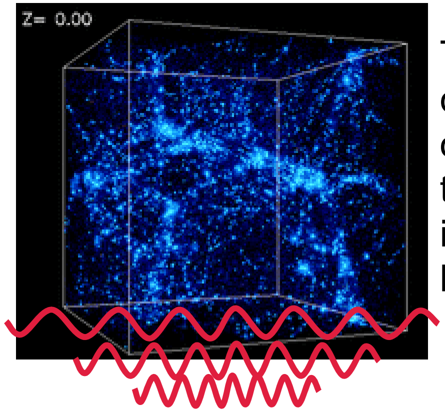
Summary - I

- The predictions of the standard cosmological model is remarkably succesful on large scale
- All data compatible with adiabatic primordial density fluctuation field following a scale invariant primordial power spectrum
- All data comptatible with cold matter particles
    - Non relativistic at decoupling
    - Collisionless : intercats mainly through gravity
    - Dissipasionless : cannot cool by radiating photons
    - Long lived particles

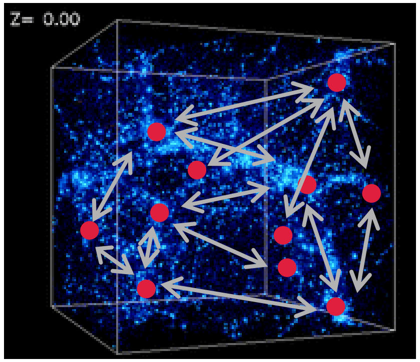
The non-linear Regime

- Primordial Fluctuations → the seeds of structure formation
- Fluctuations enter horizon → grow linearly until epoch of recombination
- Post recombination → growth of structure depends on nature of Dark Matter
- Fluctuations become non-linear i.e. $\delta > 1$
- How can we model the non-linear regime?

Quantifying structures on linear and non-linear regimes



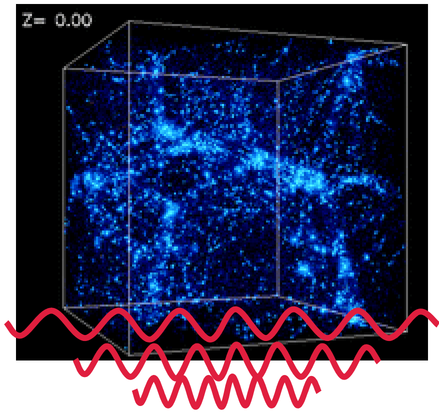
The power spectrum quantifies clustering on spatial scales larger than the sizes of individual collapsed halos



The 2pt correlation fcn is another way to quantify clustering of a continuous fluctuating density field, or a distribution of discrete objects, like collapsed DM halos.

LINEAR REGIME

Quantifying structures on linear and non-linear regimes



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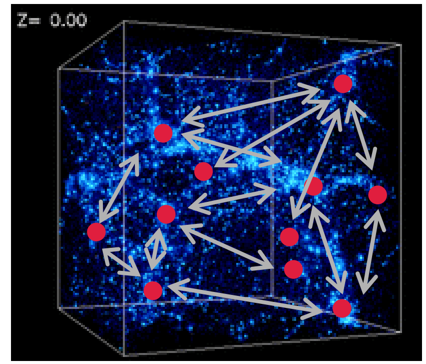

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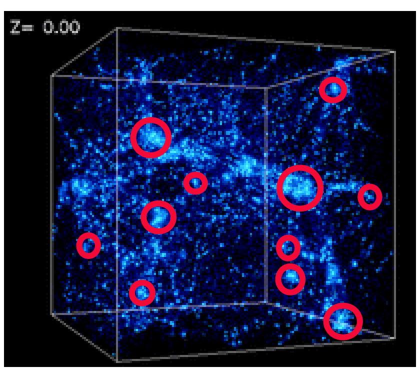
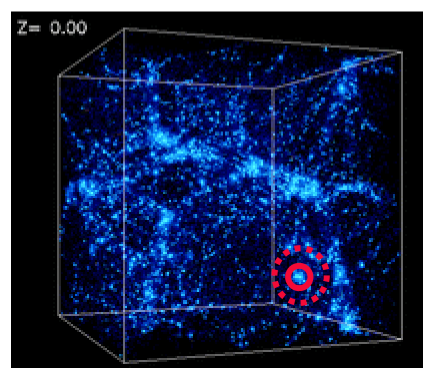
LINEAR REGIME

NON LINEAR REGIME



The mass function of discrete objects is the number density of collapsed dark matter halos as a function of mass - n(M)dM.
This was evaluated analytically by
Press & Schechter (1974)



Internal structure of individual collapsed halos: one can use an analytical description for mildly non-linear regimes, but numerical N-body simulations are needed to deal with fully non-linear regimes.

The non-linear Regime

(1) N-Body Simulations

- PP Simulations:
  - Direct integration of force acting on each particle
- PM Simulations: Particle Mesh
  - Solve Poisson eqn. By assigning a mass to a discrete grid
- P3M: Particle-particle-particle-Mesh
  - Long range forces calculated via a mesh, short range forces via particles
- ART: Adaptive Refinment Tree Codes
  - Refine the grid on smaller and smaller scales

PP	Direct summation	$O(N^2)$	Practical for $N<10^4$
PM, P^3M	Particle mesh	O(N logN)	Use FFTs to invert Poisson equation.
	ART codes	O(N logN)	Multipole expansion.

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PM, P^3M	Particle mesh	$O(N \log N)$	Use FFTs to invert Poisson equation.
	ART codes	$O(N \log N)$	Multipole expansion.

- Strengths
  - Self consistent treatment of LSS and galaxy evolution
- Weaknesses
  - Limited resolution
  - Computational overheads

The non-linear Regime

(2) SAM - Semi Analytic Modelling

- Merger Trees; the skeleton of hierarchical formation
- Cooling, Star Formation & Feedback
- Mergers & Galaxy Morphology
- Chemical Evolution, Stellar Population Synthesis & Dust

The non-linear Regime

(2) SAM - Semi Analytic Modelling

- Merger Trees; the skeleton of hierarchical formation
- Cooling, Star Formation & Feedback
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- Chemical Evolution, Stellar Population Synthesis & Dust

- Hierarchical formation of DM haloes (Press Schecter)
- Baryons get shock heated to halo virial temperature
- Hot gas cools and settles in a disk in the center of the potential well.
- Cold gas in disk is transformed into stars (star formation)
- Energy output from stars (feedback) reheats some of cold gas
- After haloes merge, galaxies sink to center by dynamical friction
- Galaxies merge, resulting in morphological transformations.

The non-linear Regime

(2) SAM - Semi Analytic Modelling

- Merger Trees; the skeleton of hierarchical formation
- Cooling, Star Formation & Feedback
- Mergers & Galaxy Morphology
- Chemical Evolution, Stellar Population Synthesis & Dust

- Hierarchical formation of DM haloes (Press Schecter)
- Baryons get shock heated to halo virial temperature
- Hot gas cools and settles in a disk in the center of the potential well.
- Cold gas in disk is transformed into stars (star formation)
- Energy output from stars (feedback) reheats some of cold gas
- After haloes merge, galaxies sink to center by dynamical friction
- Galaxies merge, resulting in morphological transformations.

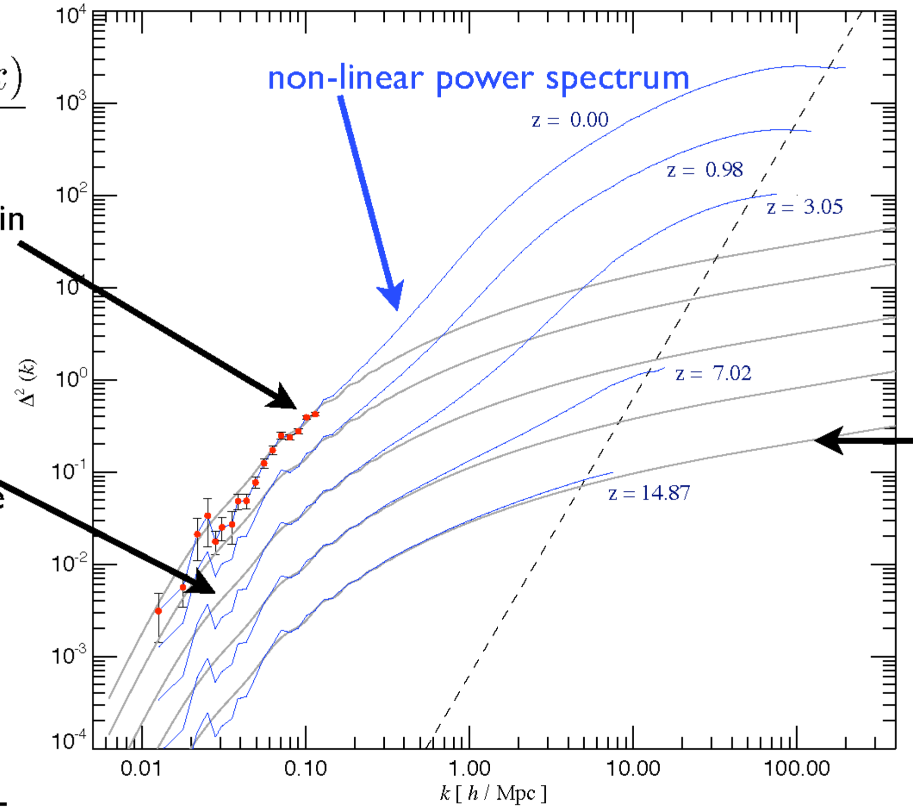
- Strengths
  - ❑ No limit to resolution
  - ❑ Matched to local galaxy properties
- Weaknesses
  - ❑ Clustering/galaxies not consistently modelled
  - ❑ Arbitrary functions and parameters tweaked to fit local properties

Non-linear evolution of power spectrum

$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$$

largest scales are still in the linear regime

finite volume box; large modes have noise



Springel et al 2005

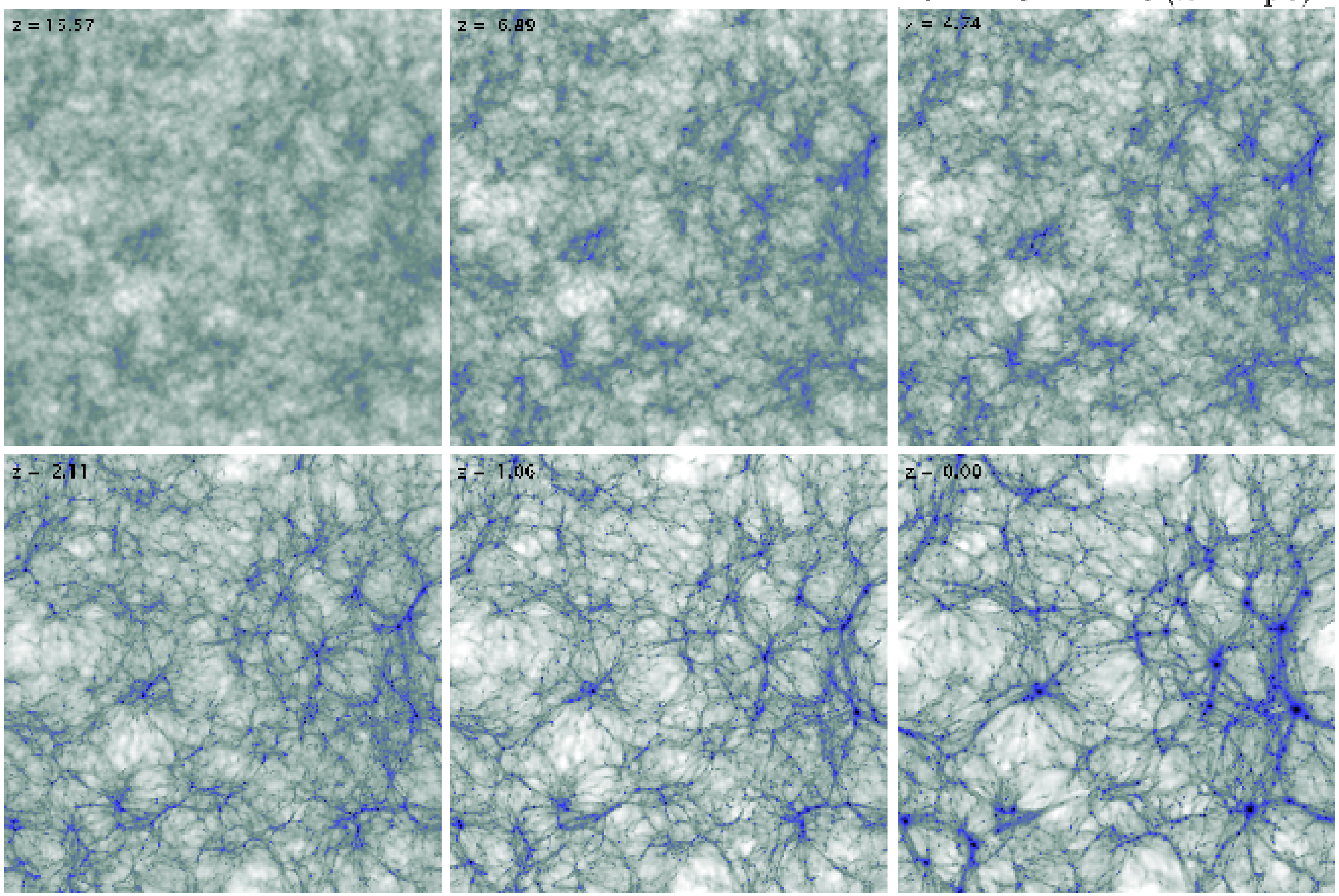
Numerical simulation: CDM models

Large-scale structure arises from Gaussian initial conditions seeded by inflation

ΛCDM, $N = 2 \times 224^3$

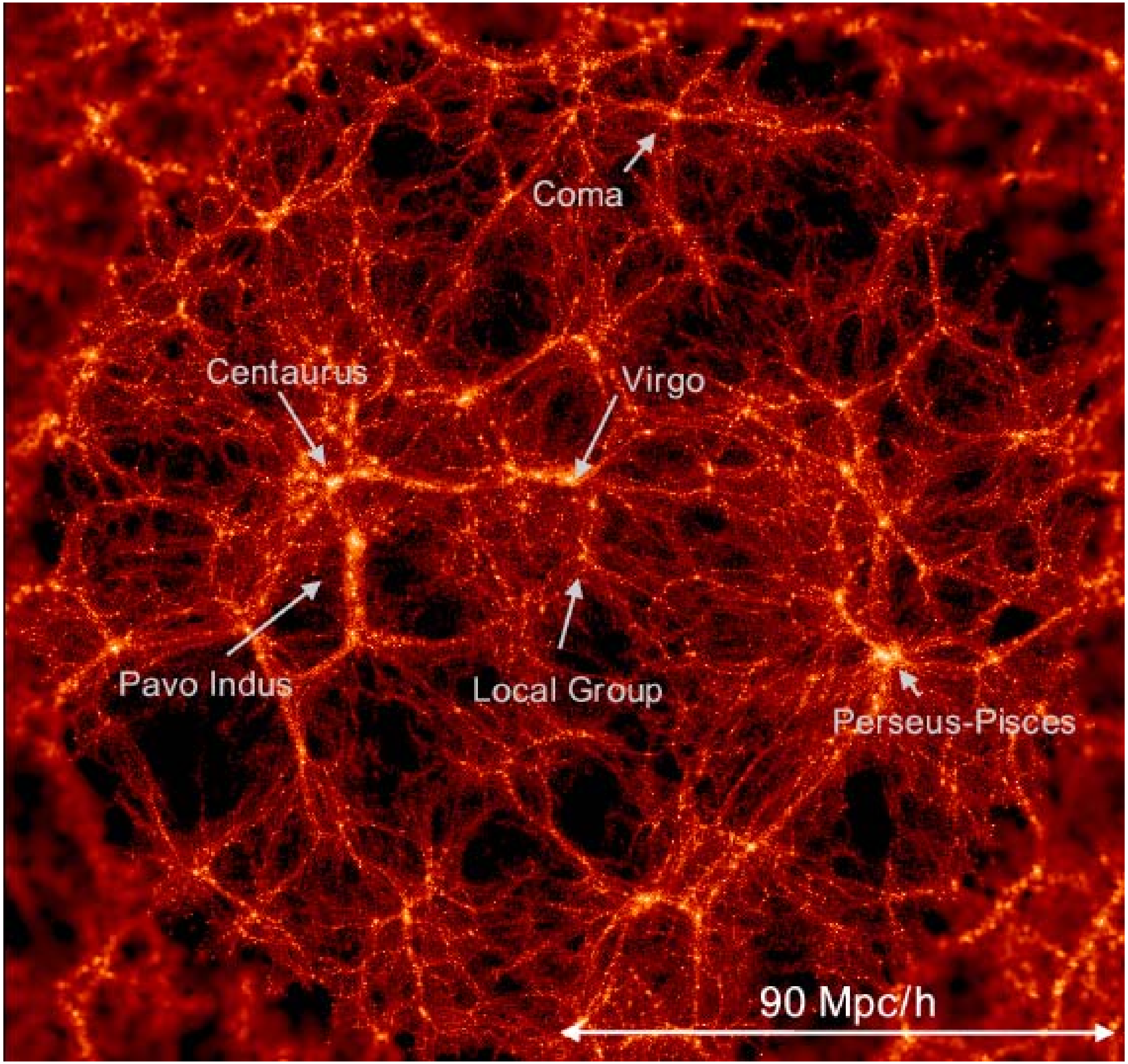
$134 \times 134 \times 22.3\,(h^{-1}\mathrm{Mpc})^3$

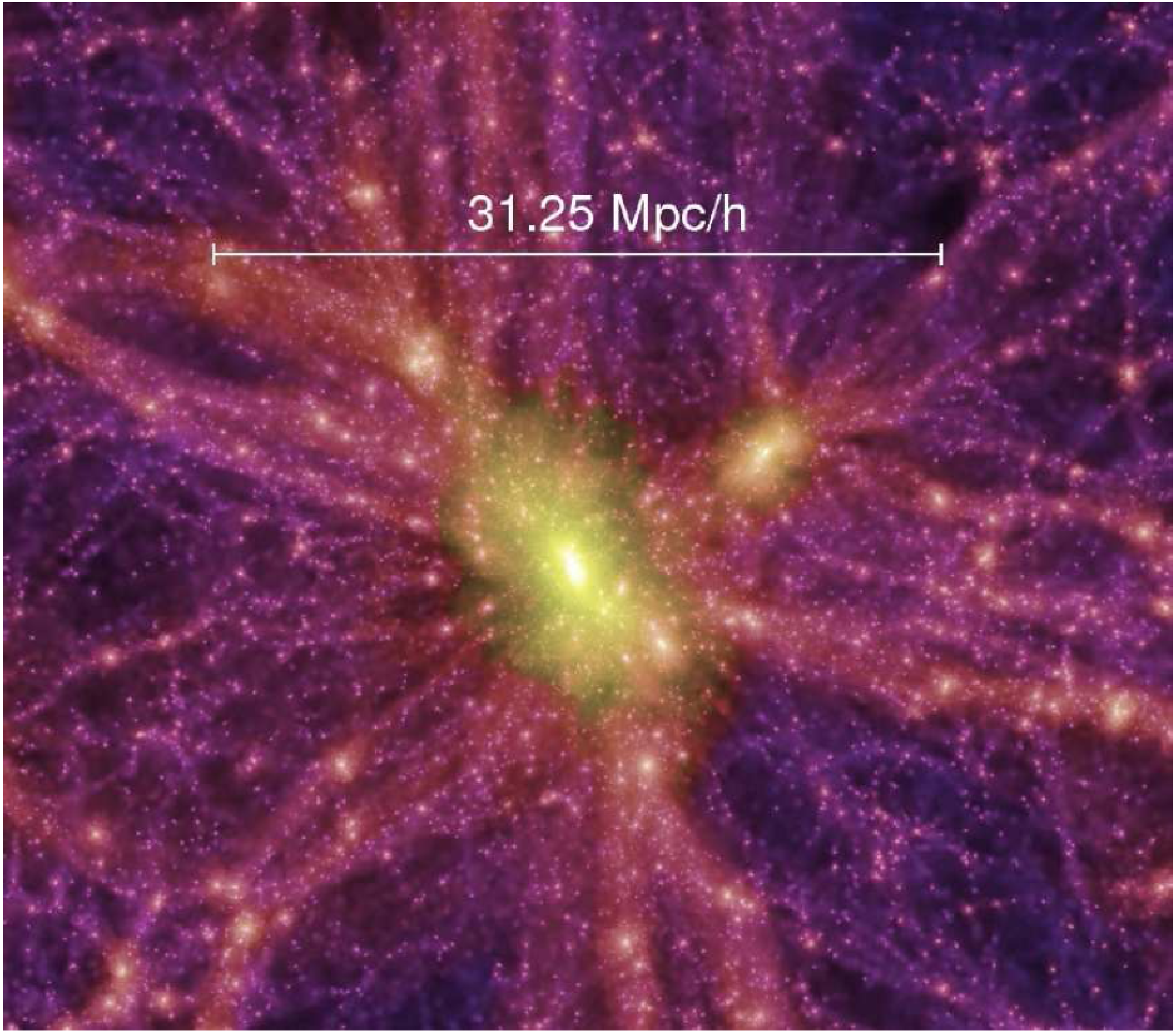
EVOLUTION OF STRUCTURE



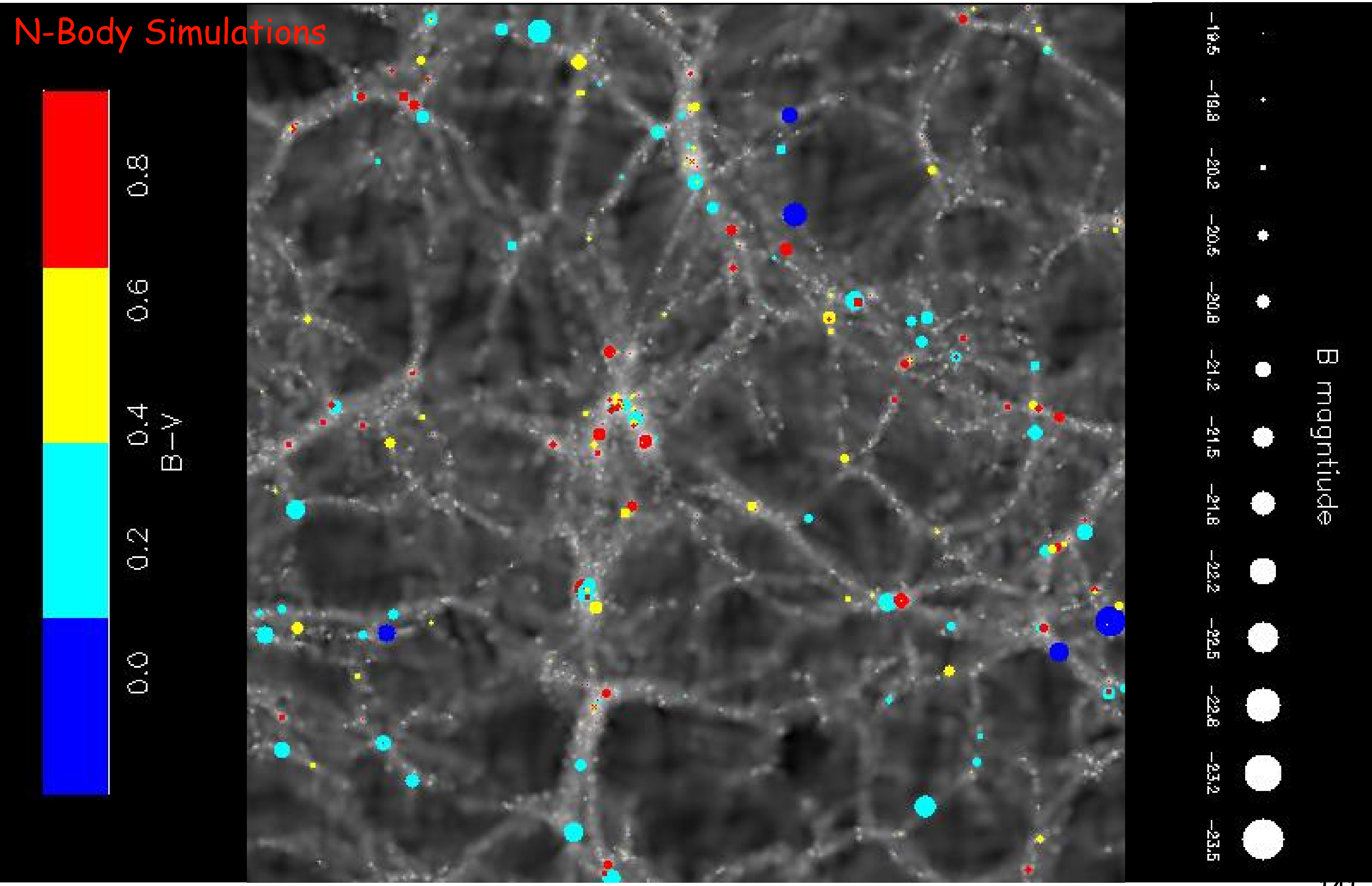
Springel, Hernquist & White (2000)

Numerical simuations: CDM models



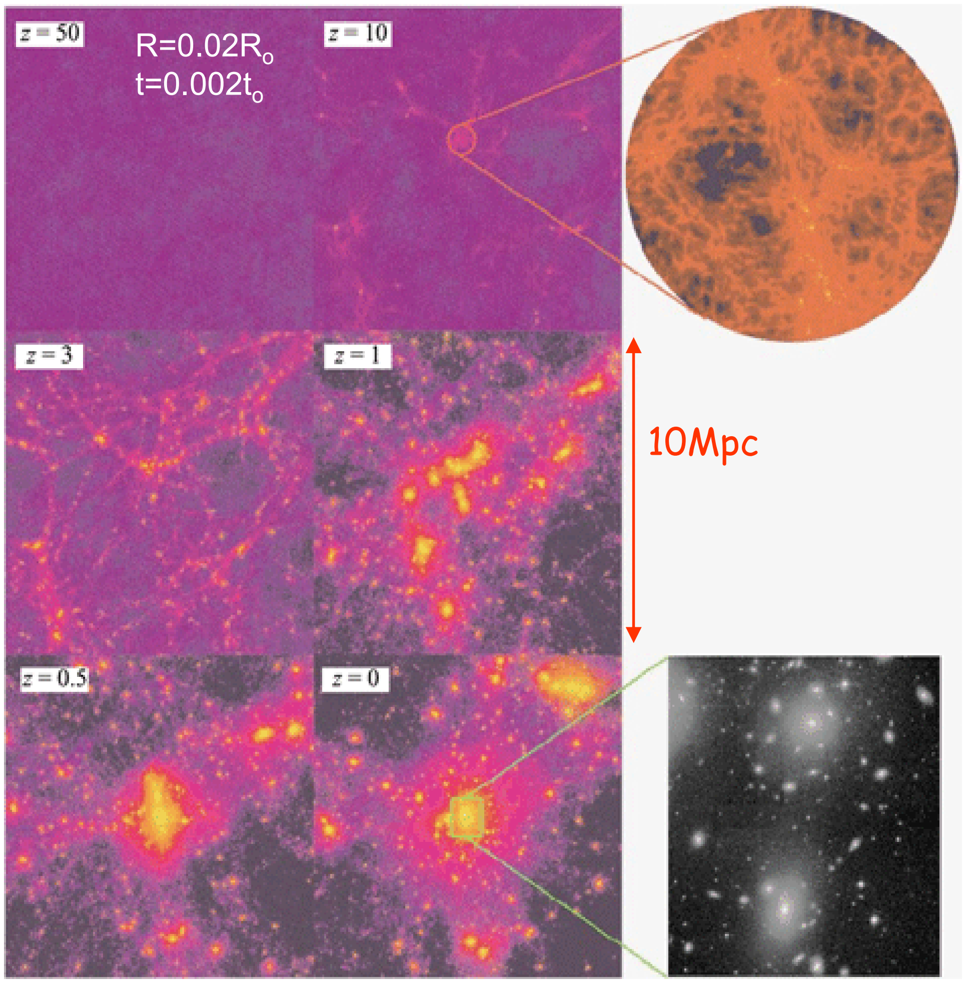

31.25 Mpc/h

Simulations with SAM : dark matter haloes + "galaxies"



Bevis & Oliver 2002

Cold Dark Matter Halo properties are predicted (using simulations)



- The hierarchical evolution of a galaxy cluster in a universe dominated by cold dark matter.

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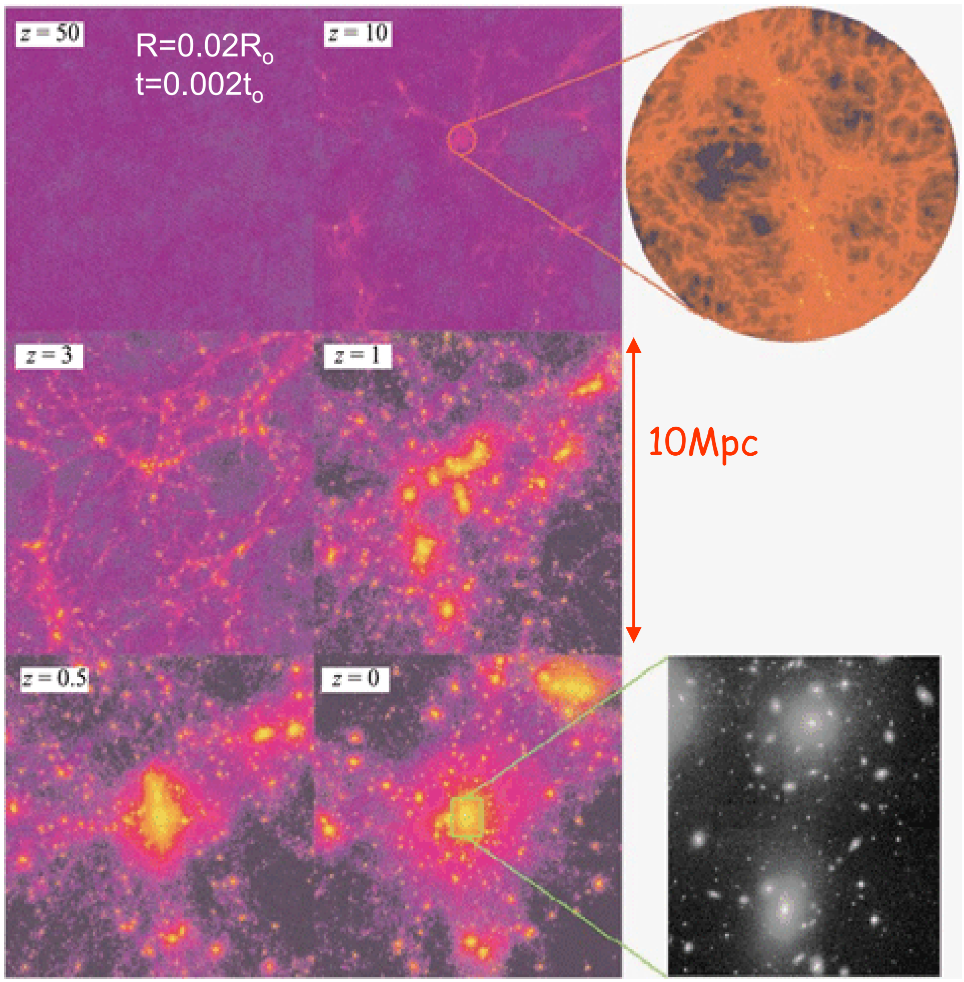

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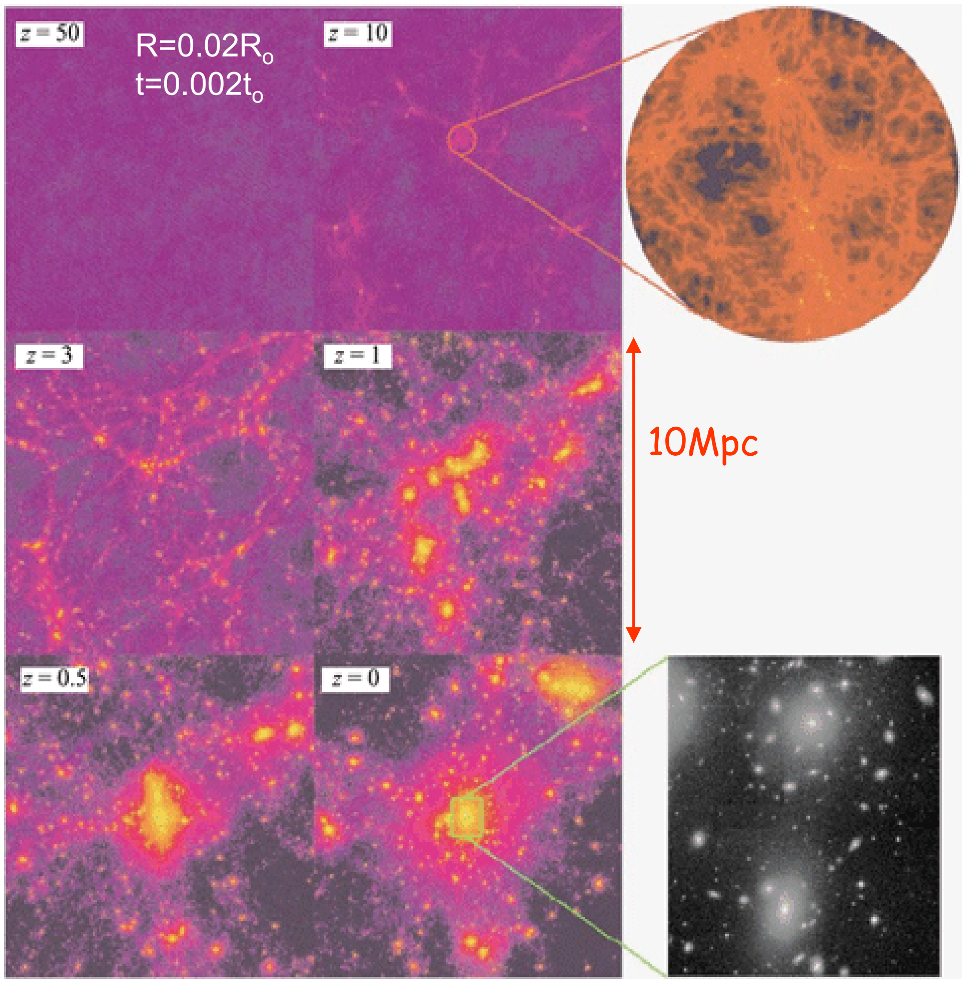

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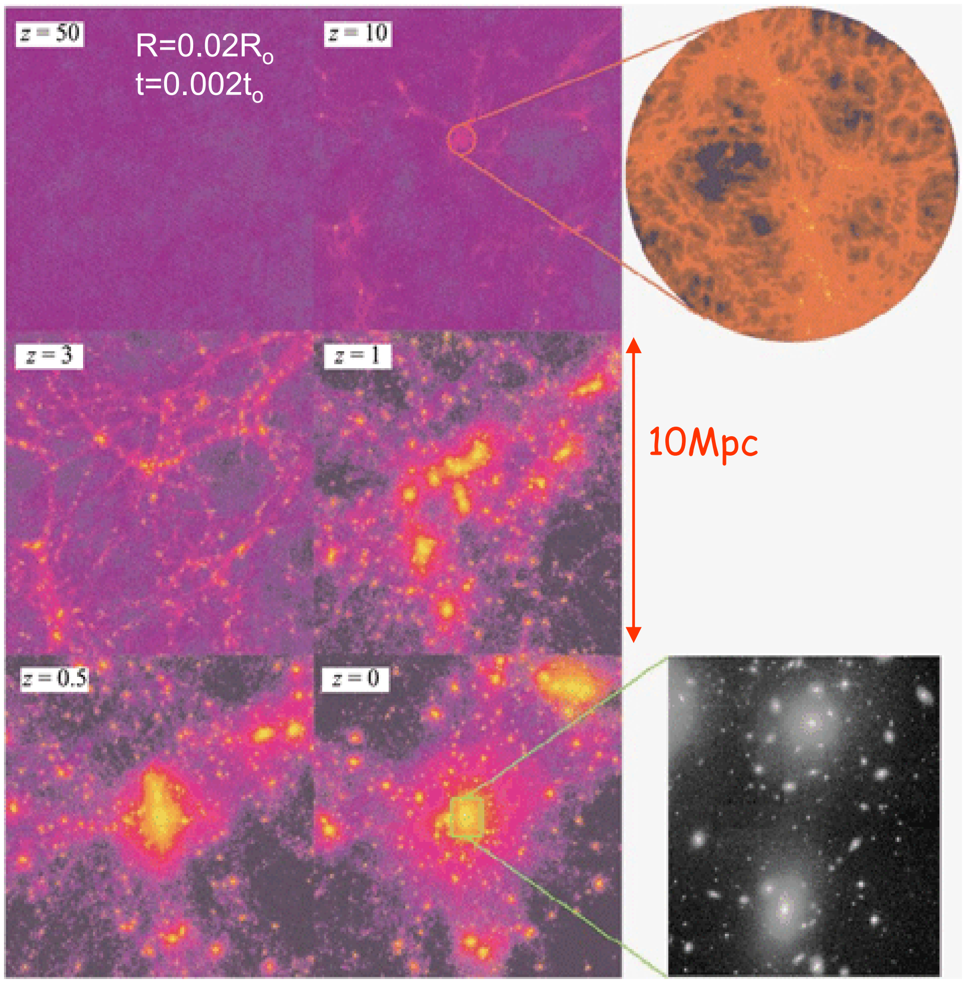

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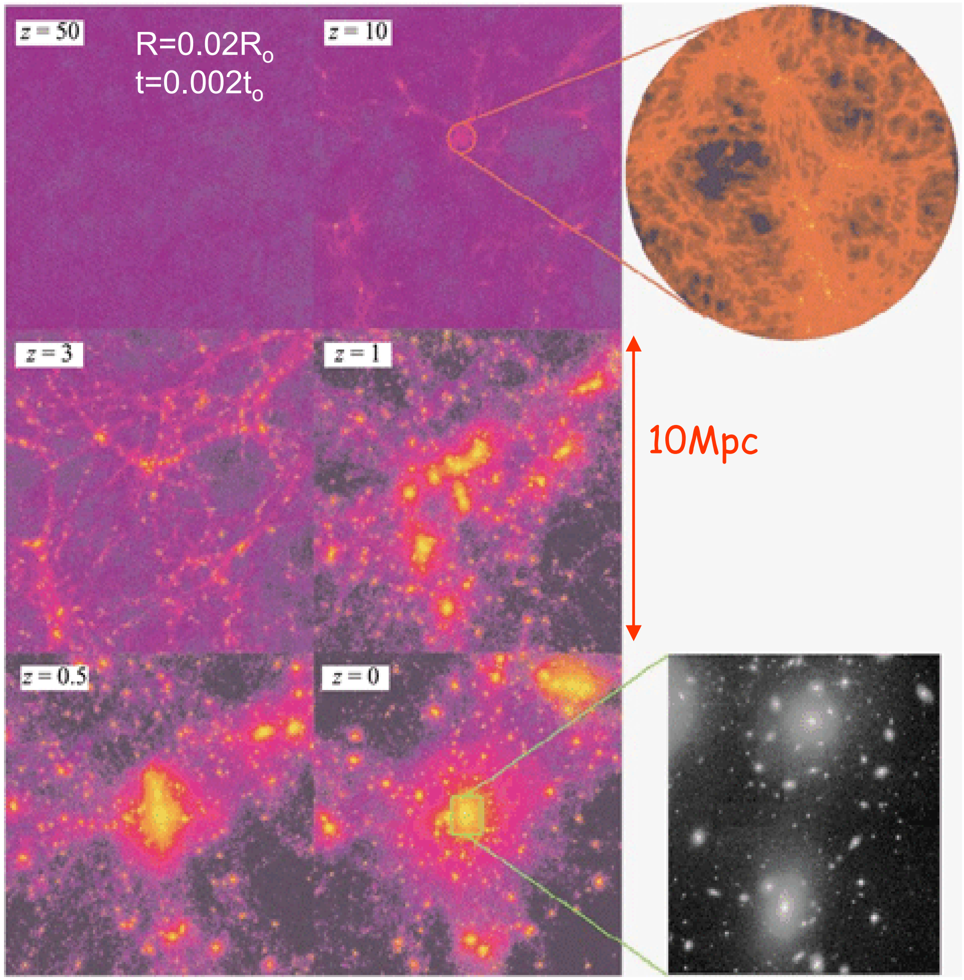
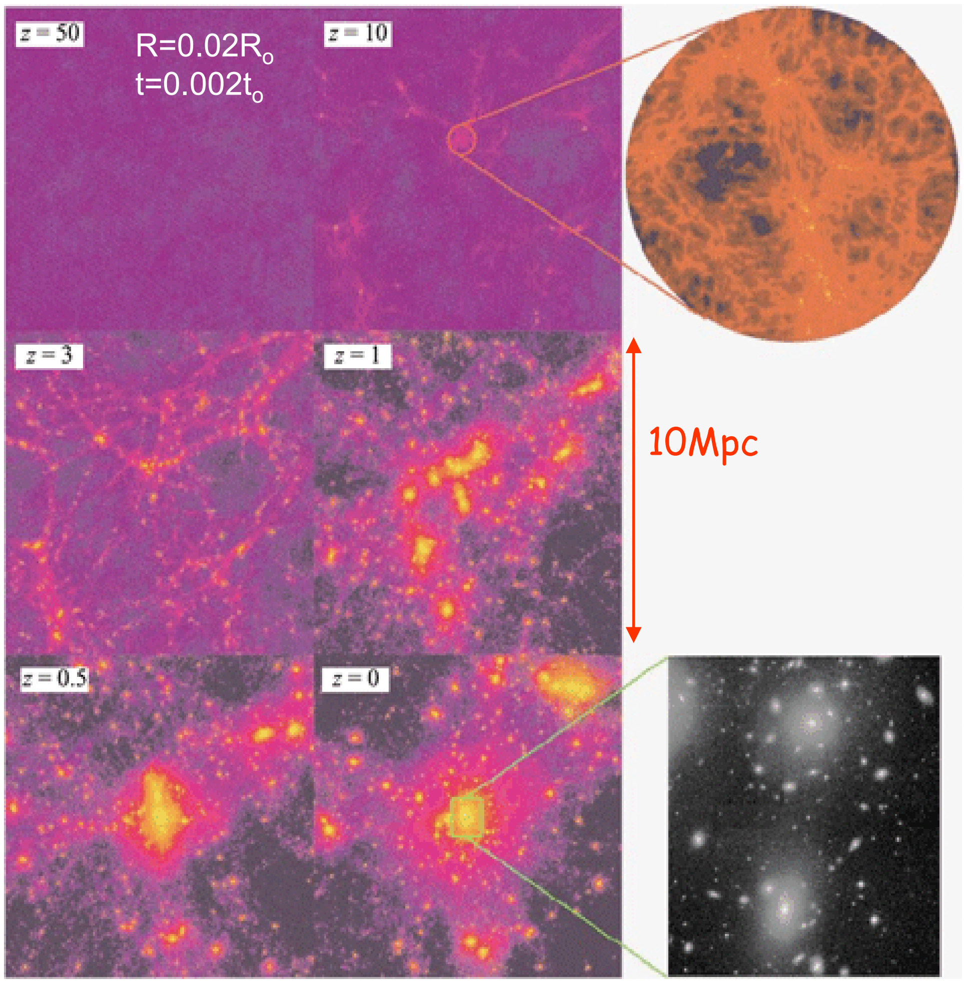
- Haloes merge and most massive are surrounded by sub-haloes

Cold Dark Matter Halo properties are predicted (using simulations)



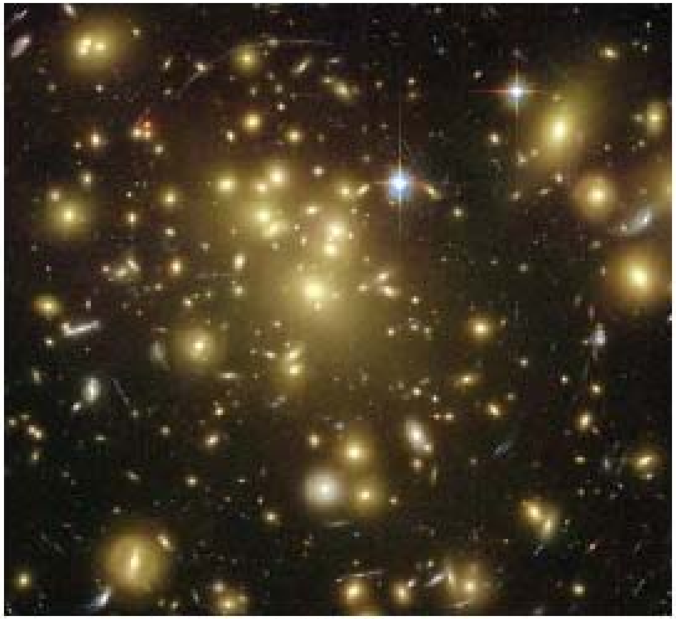
- Prediction of a universal mass density profile for dark matter haloes
- Predictions of the fraction of substructures inside haloes: consequence of a hierarchical process of structure formation
- Prediction of the luminosity function of galaxy populations
- Prediction of the number density of elliptical/red galaxies at high redshift

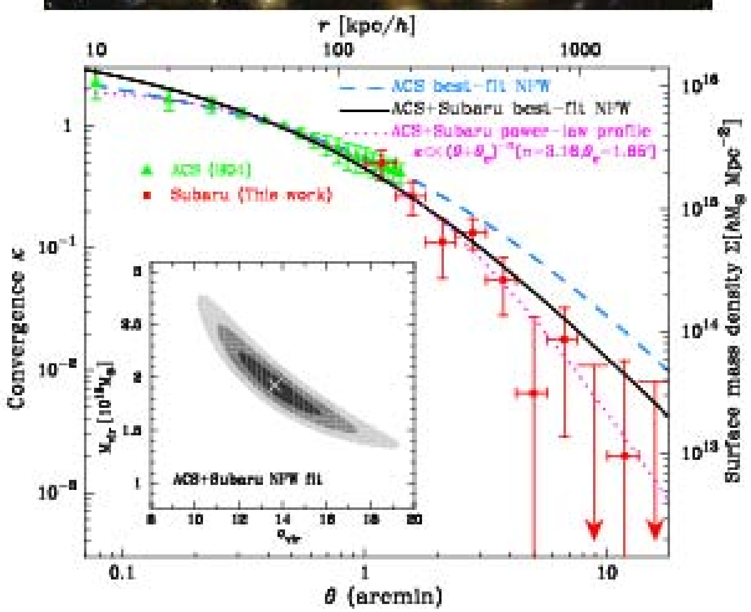
Cold Dark Matter Halo properties are predicted (using simulations)



- CDM predicts an universal NFW profiles: observations still debated
- CDM haloes are tri-axial: observations still debated
- Cold dark matter haloes are cuspy: not confirmed in LSB galaxies
- CDM galaxy halos have many substrucrures: not seen in galaxies
- CDM halos are assembled through a sequence of merger events: seems not compatible with the angular momentum and thinness of stellar disk
- The number of high redshift massive elliptical galaxies seem to contradict CDM predictions

Cold Dark Matter Haloes : NFW profile





- Slope as function of radial distance

$$\rho(r) = \frac{200}{3} \frac{c_{200}^3}{\ln(1+c_{200}) - c_{200}/(1+c_{200})} \frac{\rho_c(z)}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2},$$

- NWF and isothermal do equally well ?

- Predicted concentration c:
    - c= 6- 8 for galaxies
    - c = 3-4 for clusters of galxies
- Observations for 3 samples of clusters: not yet unanimous view

$$c_{vir} = 14.8 \pm 6.1 (1+z)^{-1} \left(\frac{M_{vir}}{1.3 \times 10^{13}\ M_\odot}\right)^{-0.14\pm0.12},$$

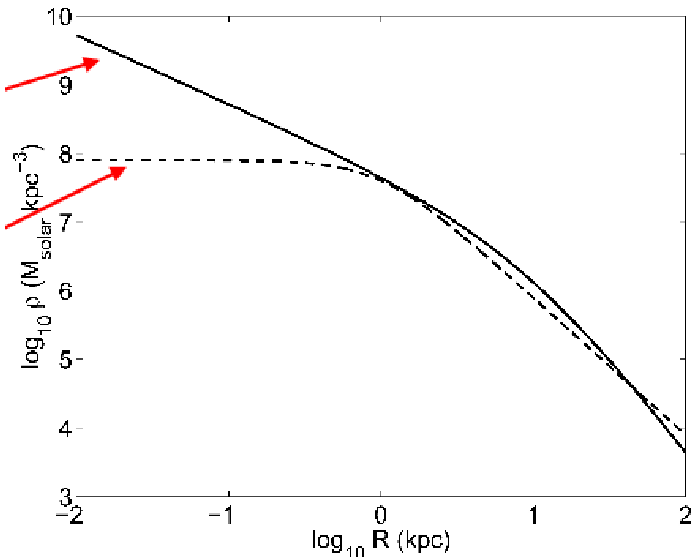
$$c_{200} = 4.1 \pm 1.5 \left(\frac{M_{200}}{1.0 \times 10^{14}\ M_\odot}\right)^{-0.12\pm0.04}$$

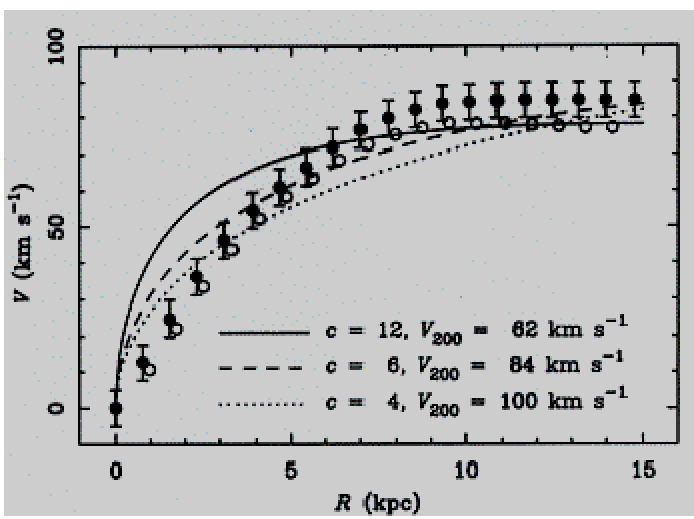
$$c_{200} = 4.6 \pm 0.7 \left(\frac{M_{200}}{1.0 \times 10^{14}\ M_\odot}\right)^{-0.13\pm0.07}$$

Cold Dark Matter Haloes : cuspy profile

- Predicted by CDM: density cusp

- Favored by observations: density core





- Cold dark matter haloes are cuspy?

- Not confirmed in LSB galaxies

- Contradictory results. Still debated but some galaxies show discrepancies with CDM predictions.

- Pb with observations?

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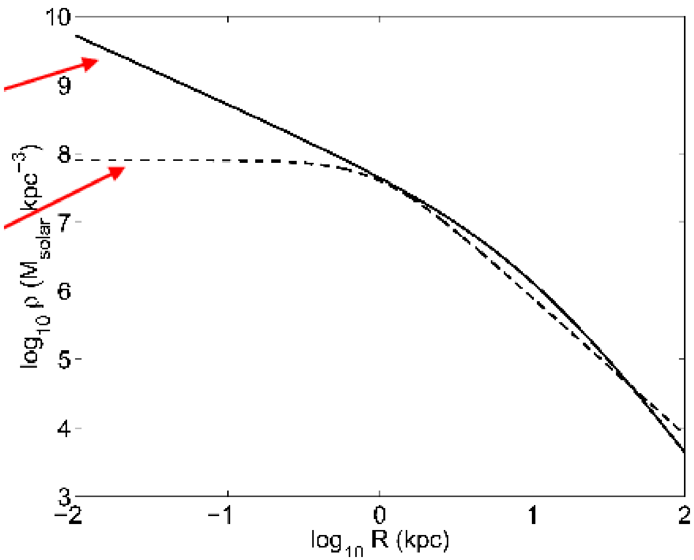

- Cold dark matter haloes are cuspy?
- Not confirmed in LSB galaxies
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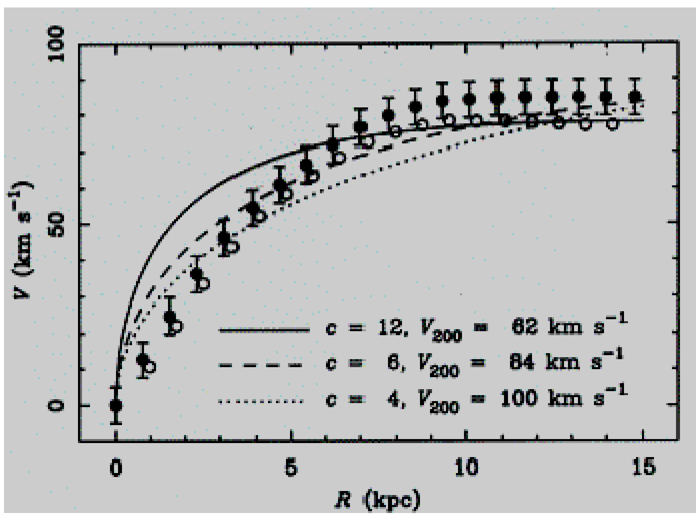
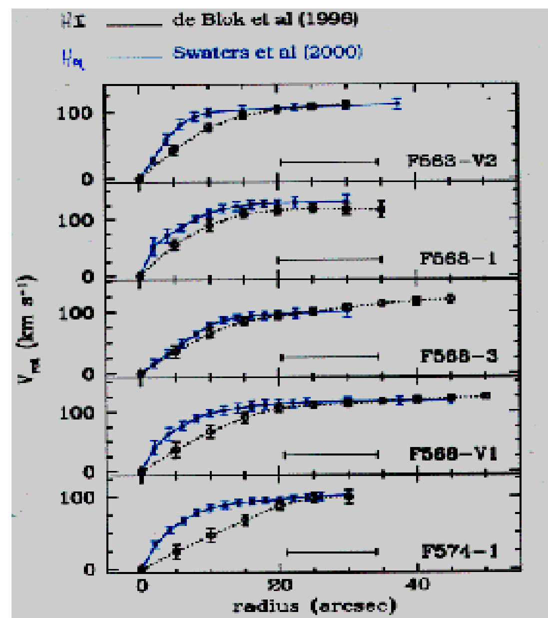
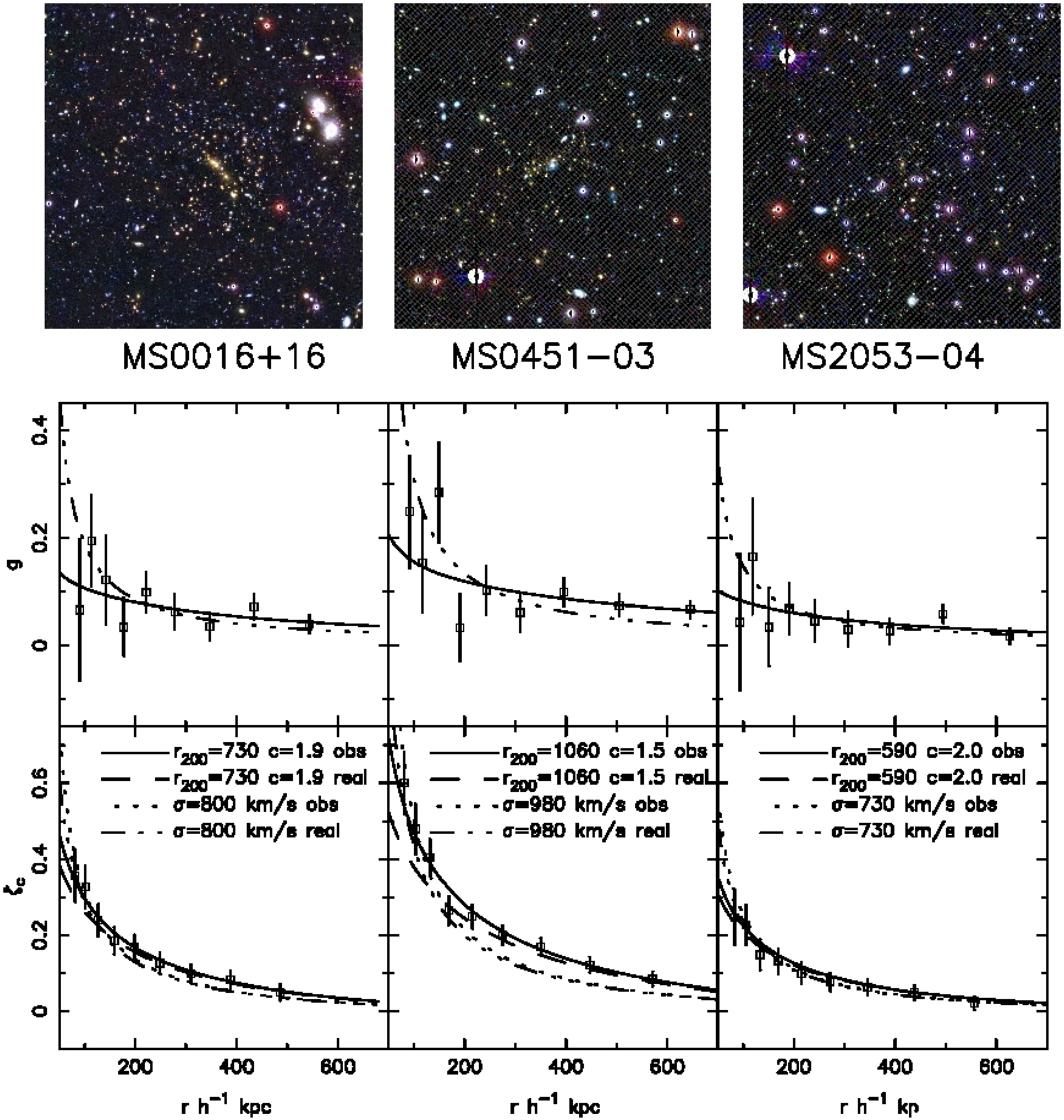
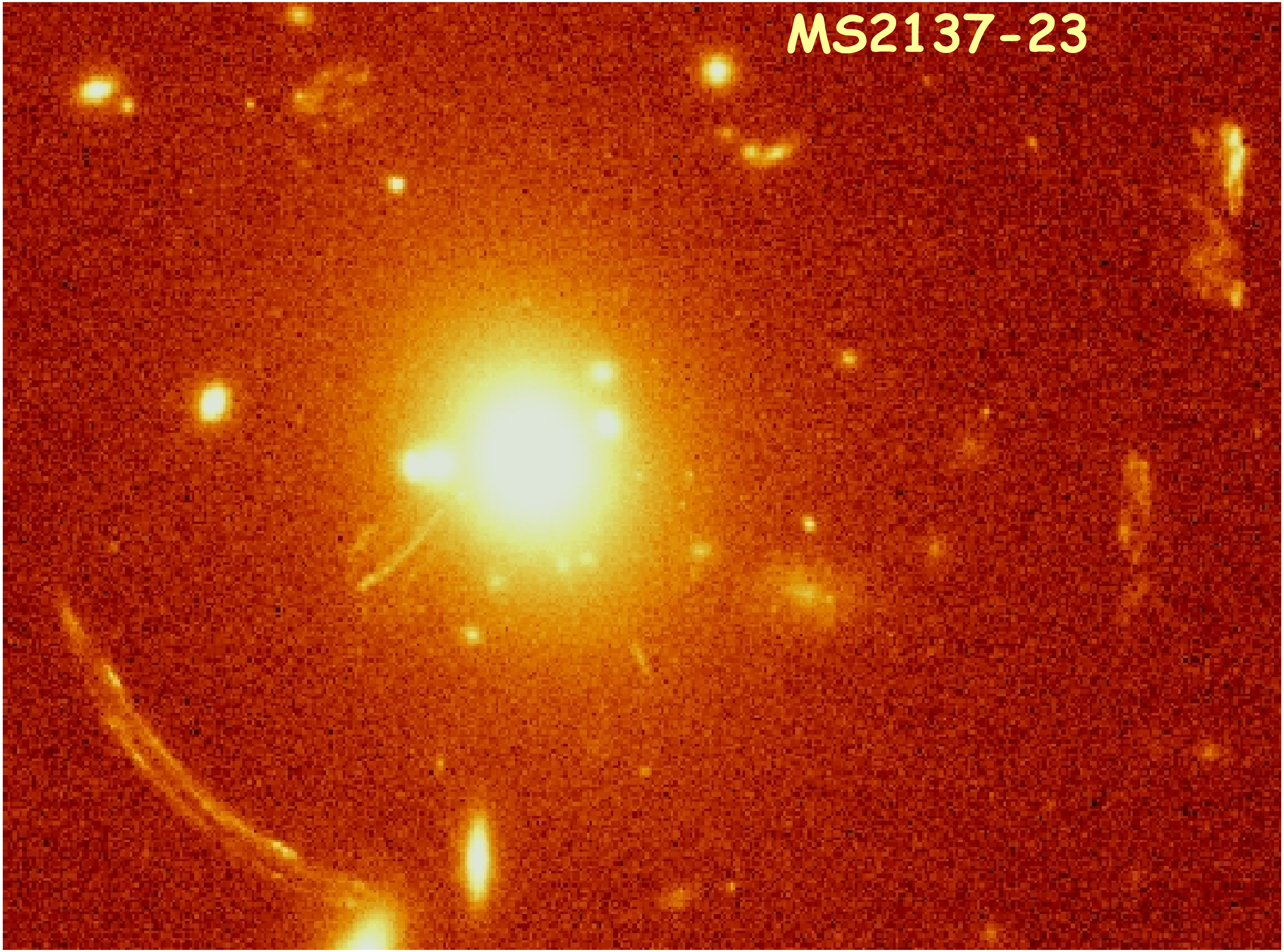
- Pb with observations: beam smearing effect?



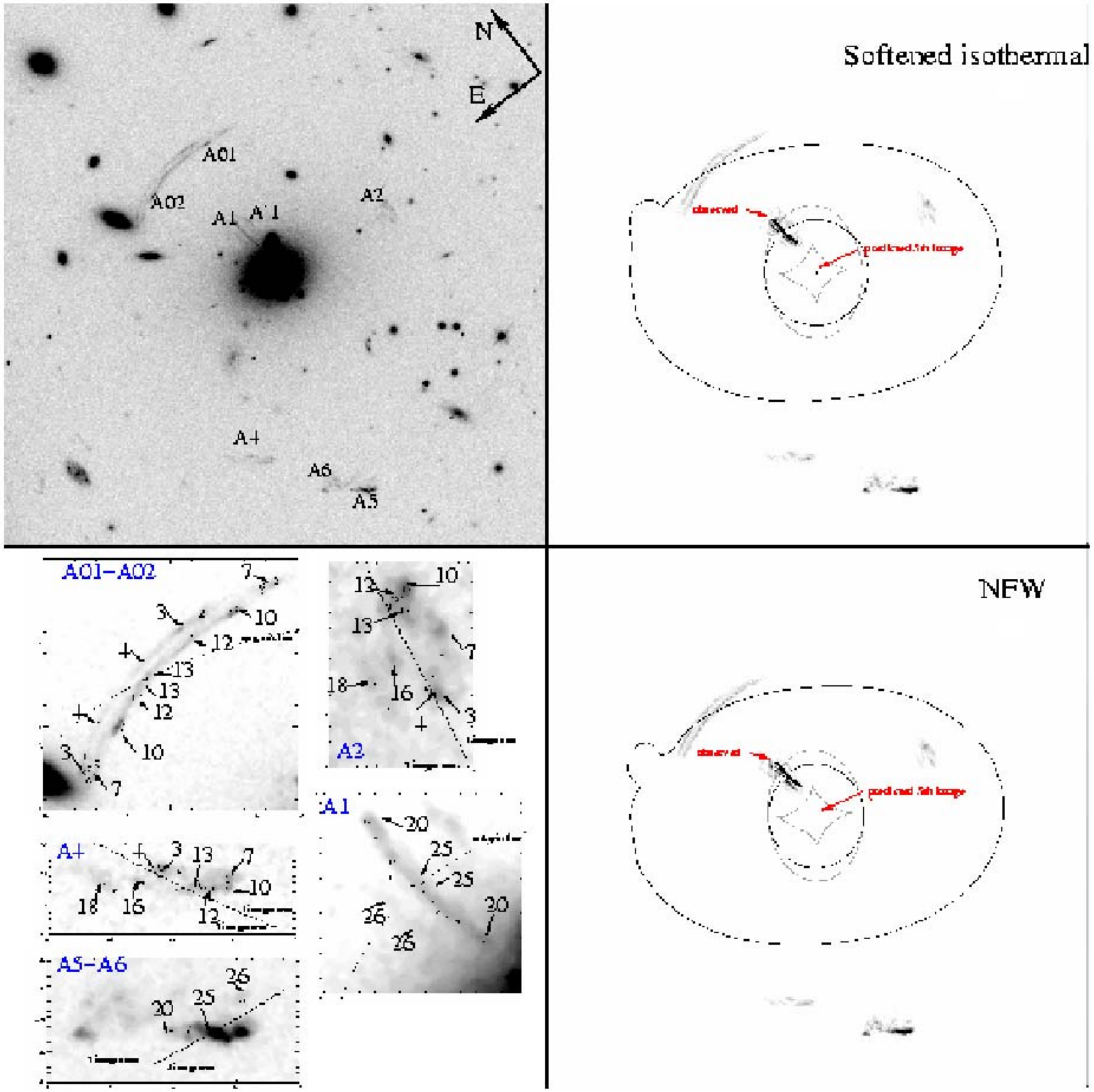




Mass profile: NFW vs. Isothermal sphere?

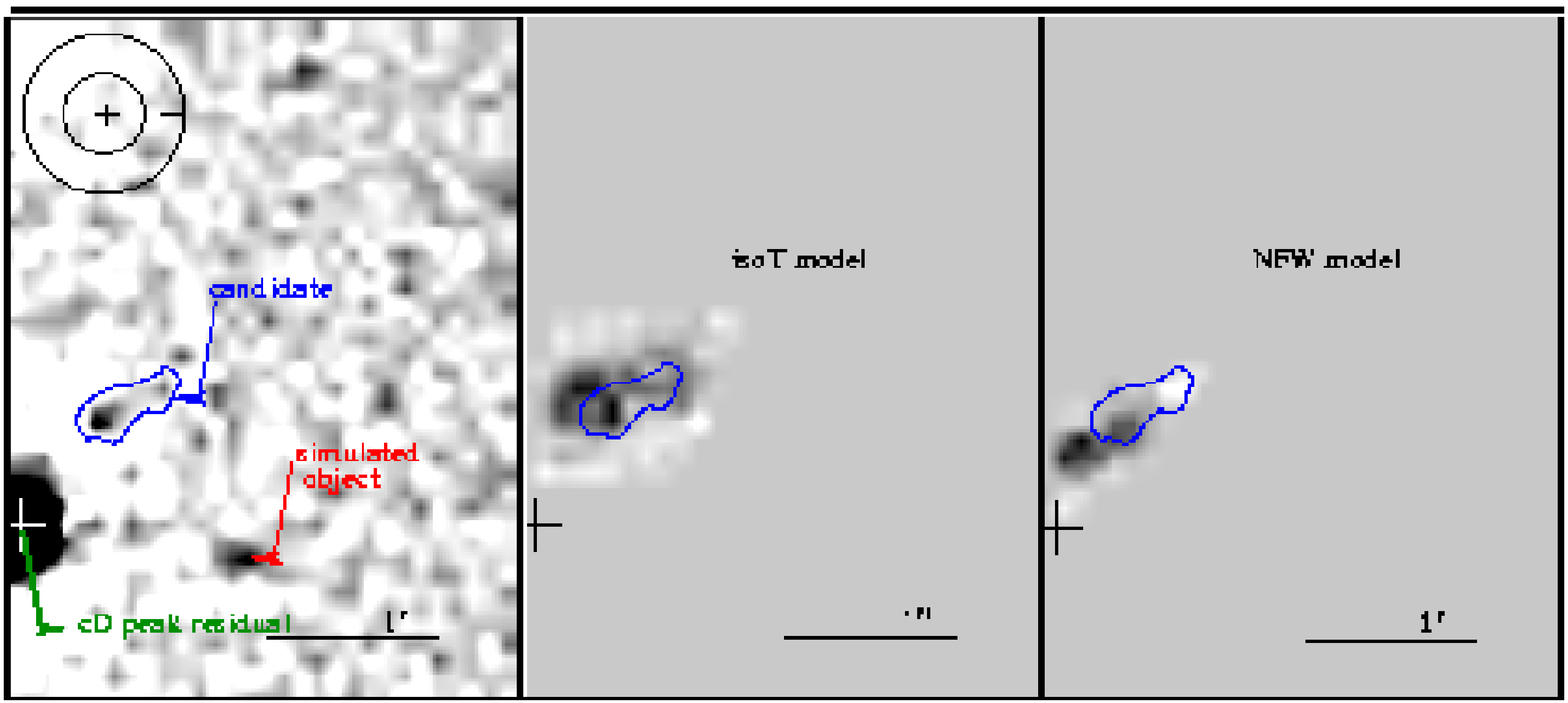

MS2137-23

Can gravitational lensing solve the cusp issue?



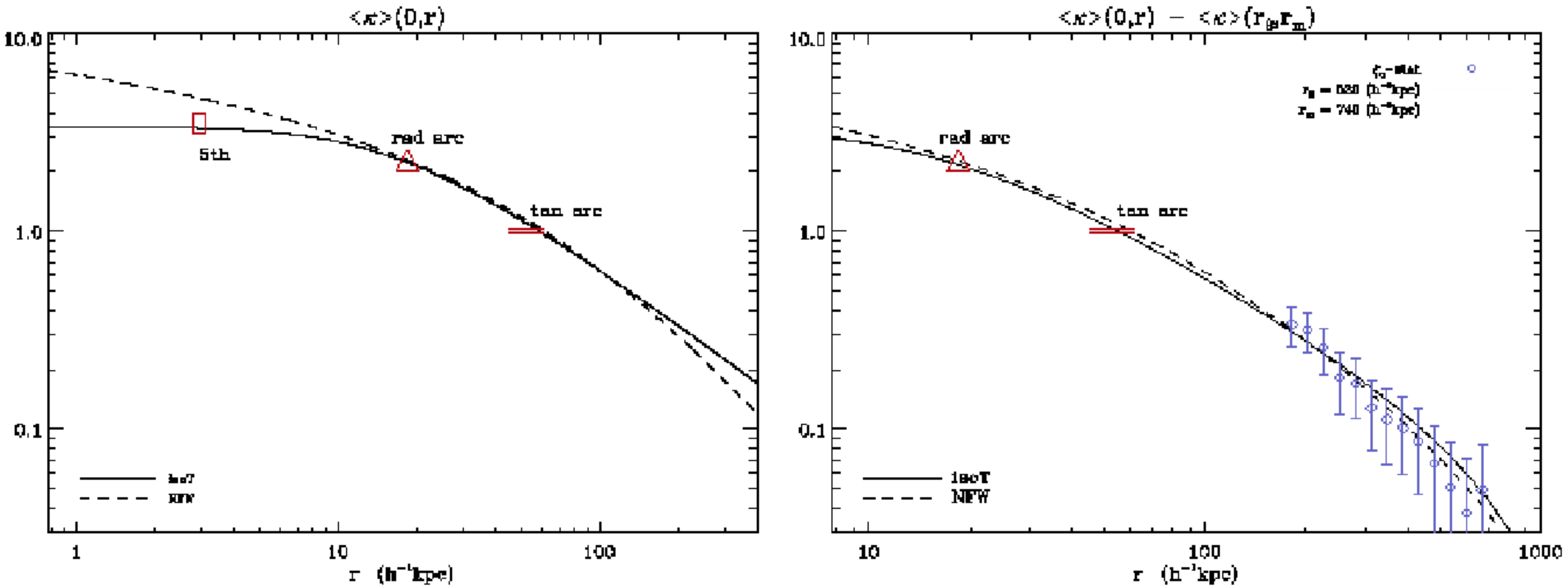
Can gravitational lensing solve the cusp issue?

- Find the 5th image

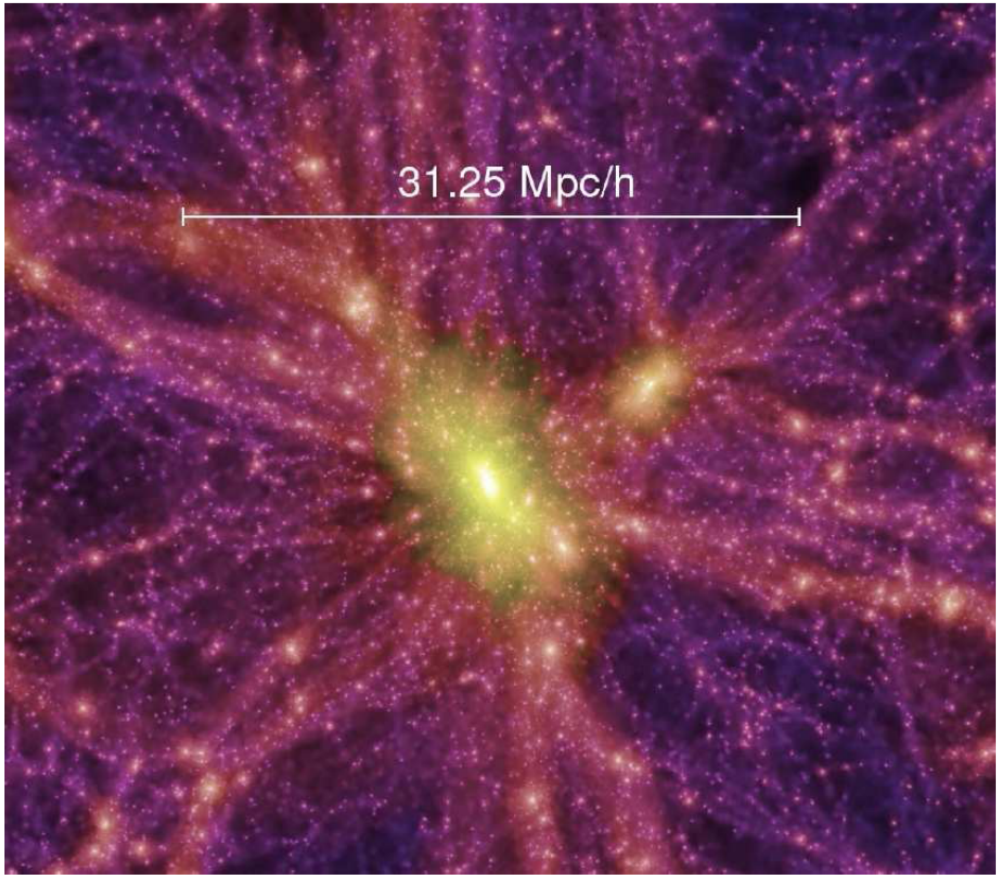


Can gravitational lensing solve the cusp issue?

Mass profile from strong + weak lensing with and without the 5th image



Cold Dark Matter Haloes : abundance of substructure



- Due to continuous merging processes in hierachical growth of structures, dark haloes are not perfectly smooth

- $M_{subhaloes} < 0.1\ M_{haloes}$

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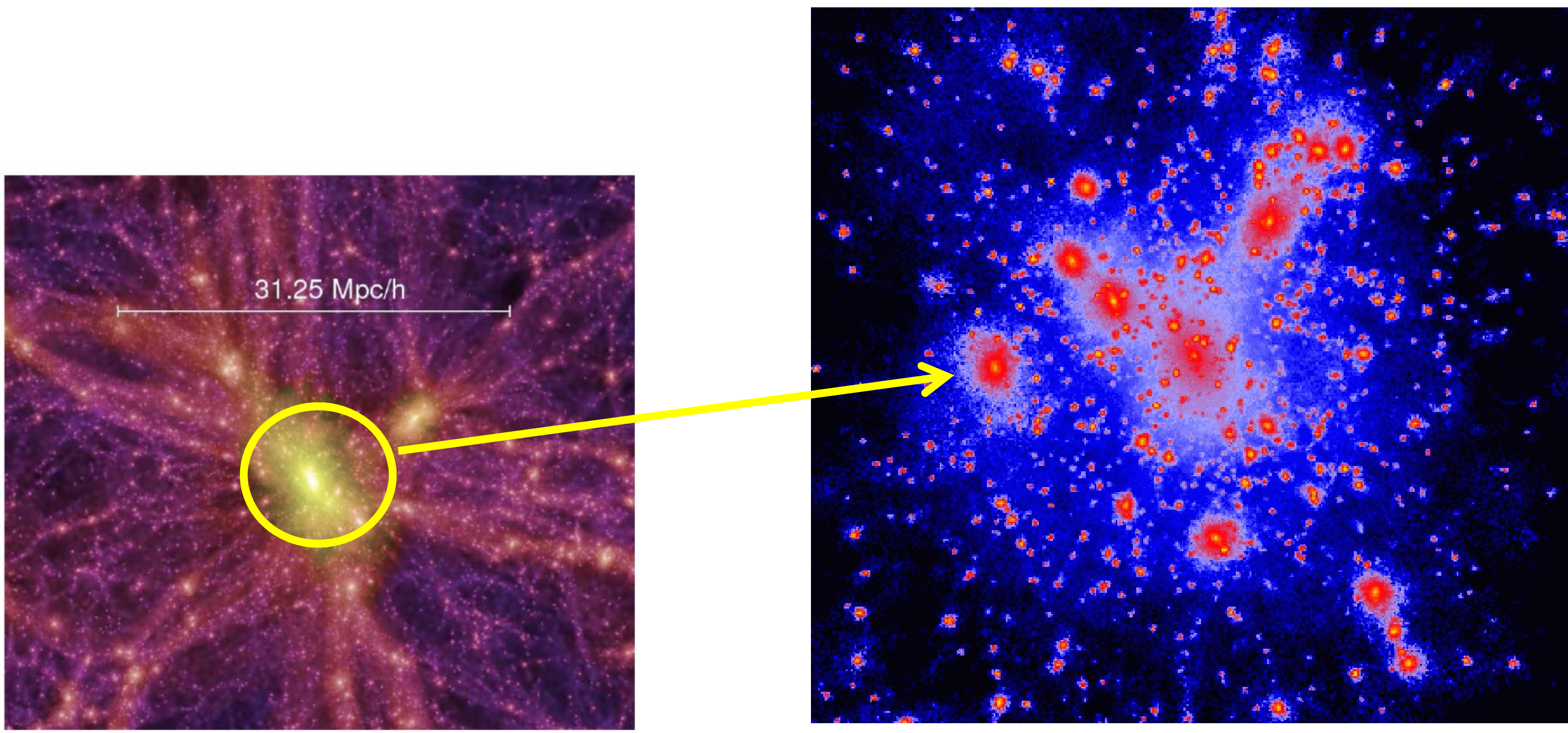

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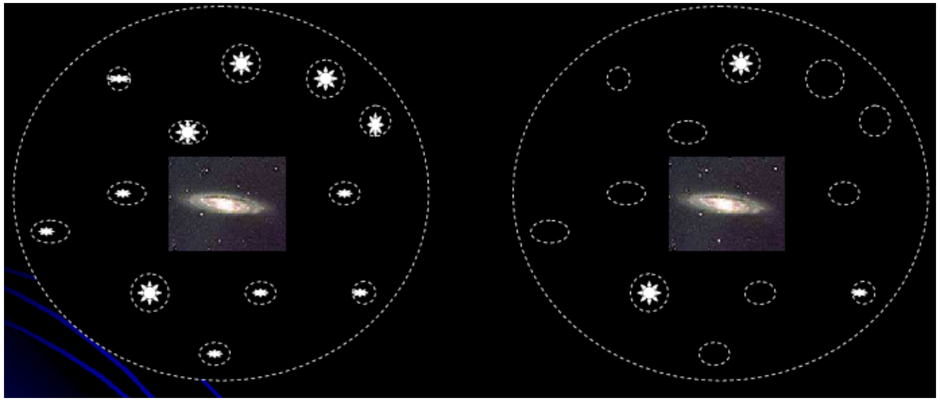
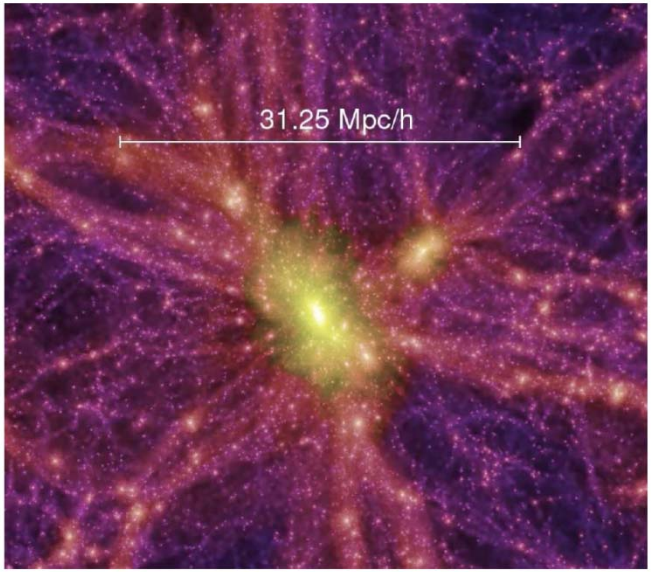


A factor 10-100 too few satelitte galaxies around the Milky Way ?



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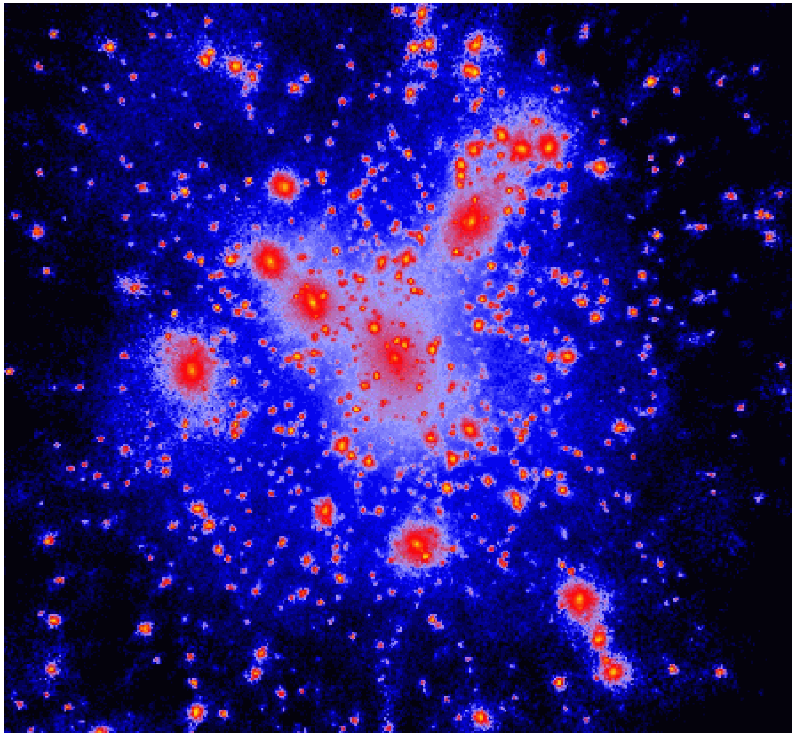
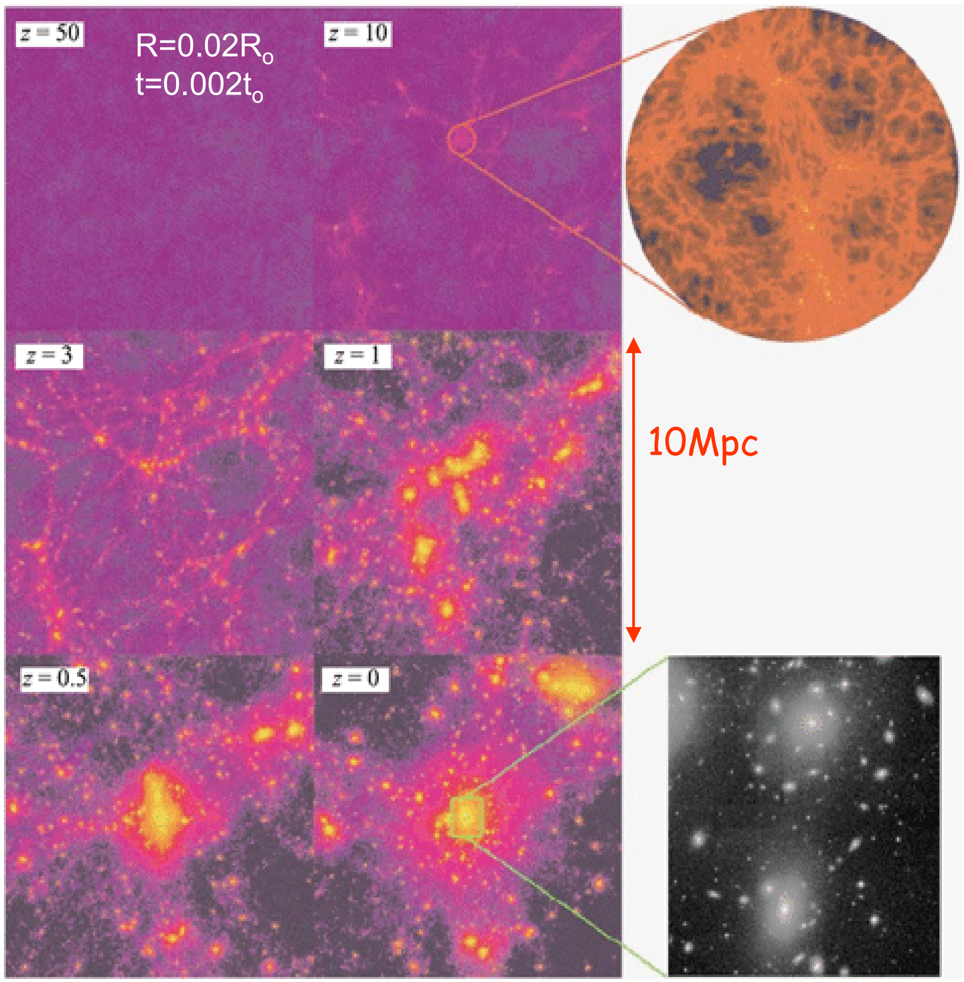


Cold Dark Matter in trouble



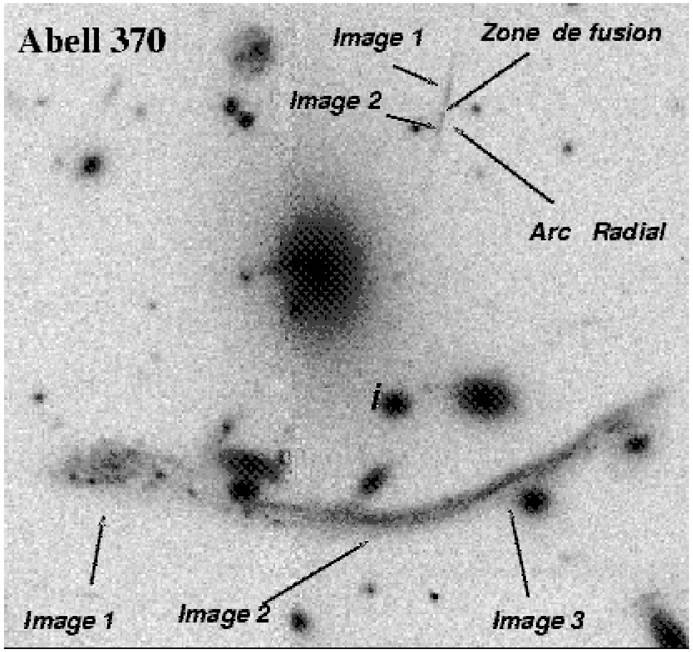
- Observations not good enough?
- e.g. Subtructures: count light haloes, not mass haloes.
- Could be fixed with gravitational lensing + perturbation theory (applied to lenses)

Masse strong lensing: isothermal

Magnification, shear and convergence

$$\begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} = \begin{pmatrix} 1-\partial_{xx}\varphi & -\partial_{xy}\varphi \\ -\partial_{xy}\varphi & 1-\partial_{yy} \end{pmatrix}$$

$$\varphi = 4\pi \frac{\sigma^2}{c^2}\frac{D_{LS}}{D_{OS}} r \qquad \boldsymbol{\theta}_S = \boldsymbol{\theta}_I - 4\pi\frac{\sigma^2}{c^2}\frac{D_{LS}}{D_{OS}}\frac{\boldsymbol{\theta}_I}{|\boldsymbol{\theta}_I|}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 - 4\pi\frac{\sigma^2}{c^2}\frac{D_{LS}}{D_{OS}}\frac{1}{|\boldsymbol{\theta}_I|} \end{pmatrix}$$

$$\theta_{SIS} = 4\pi\frac{\sigma^2}{c^2}\frac{D_{LS}}{D_{OS}} \approx 16'' \left(\frac{\sigma}{1000\mathrm{km.sec}^{-1}}\right)^2$$

$$M(\theta) = 0.57 \times 10^{14}\ h^{-1}\ M_\odot \left(\frac{\theta}{30''}\right)\left(\frac{\sigma}{1000\mathrm{km.sec}^{-1}}\right)^2$$

Masse strong lensing:
isothermal haloes+ isothermal sub-haloes

Magnification, shear and convergence

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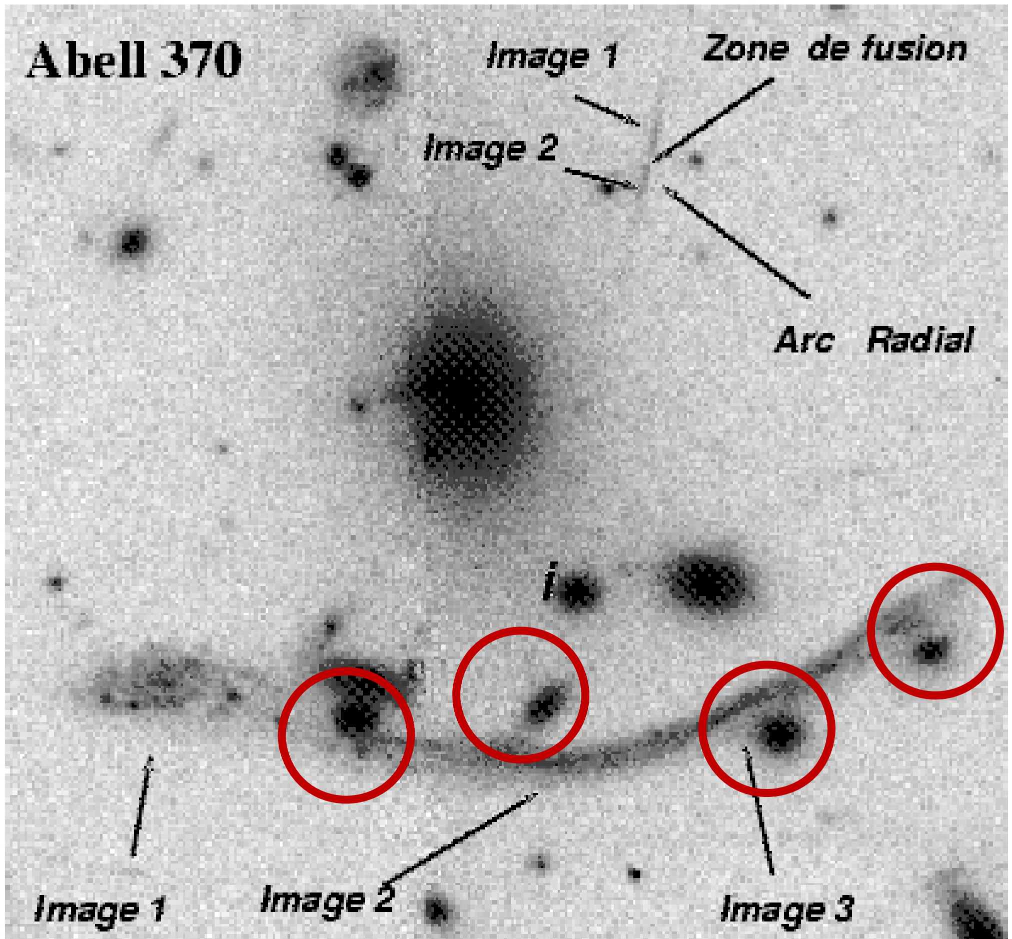
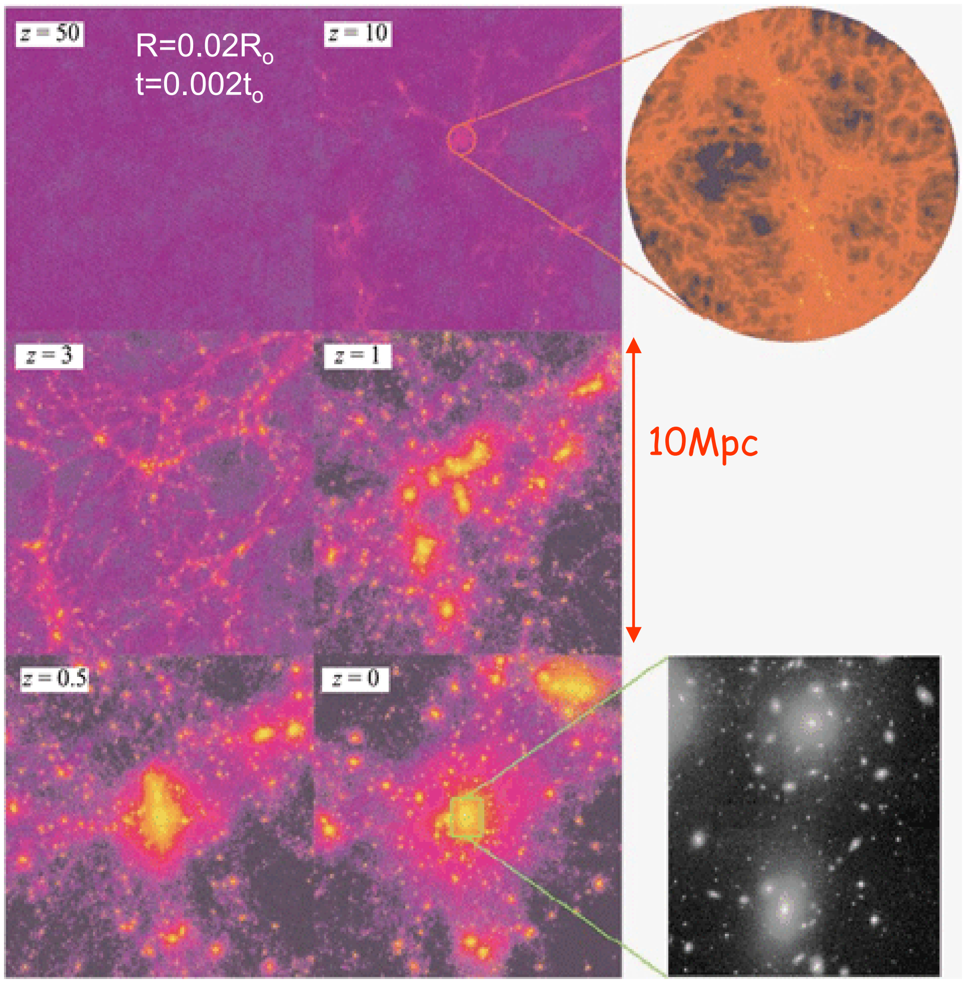


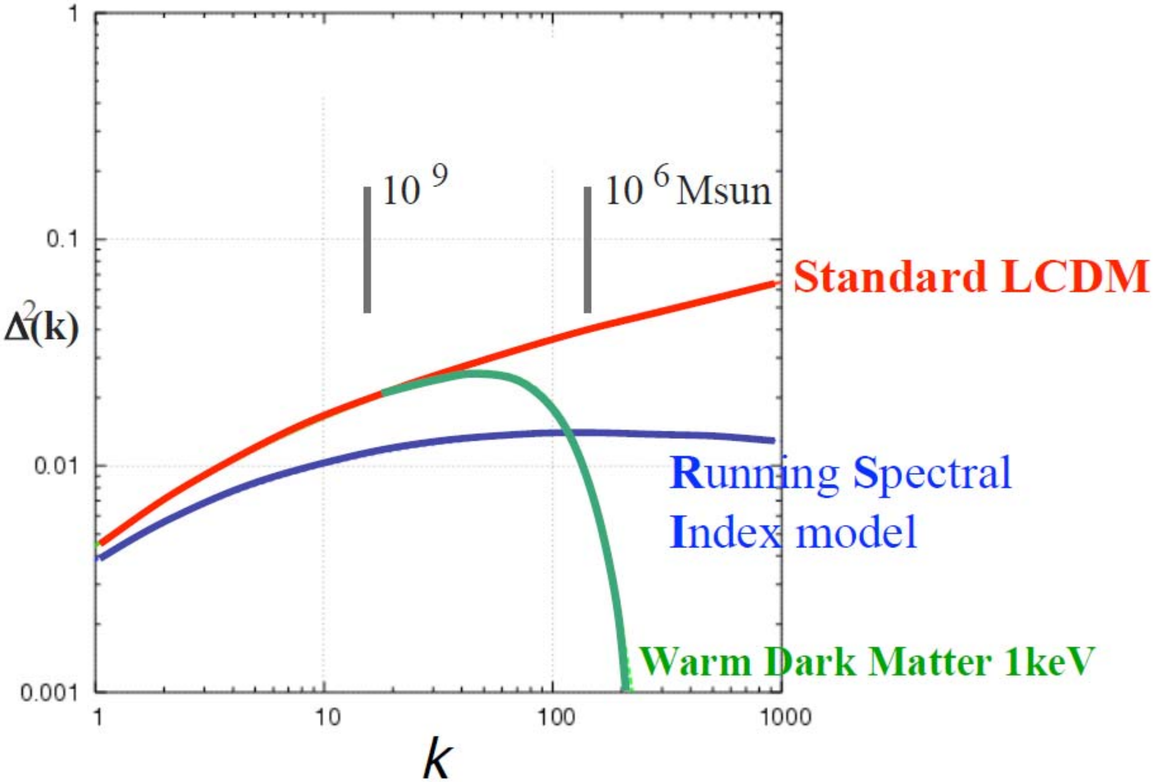
Sub-haloes break giant arcs: M_{haloes} around the giant arc in A370 < 3 10^{10} M_{sol}

Cold Dark Matter in trouble



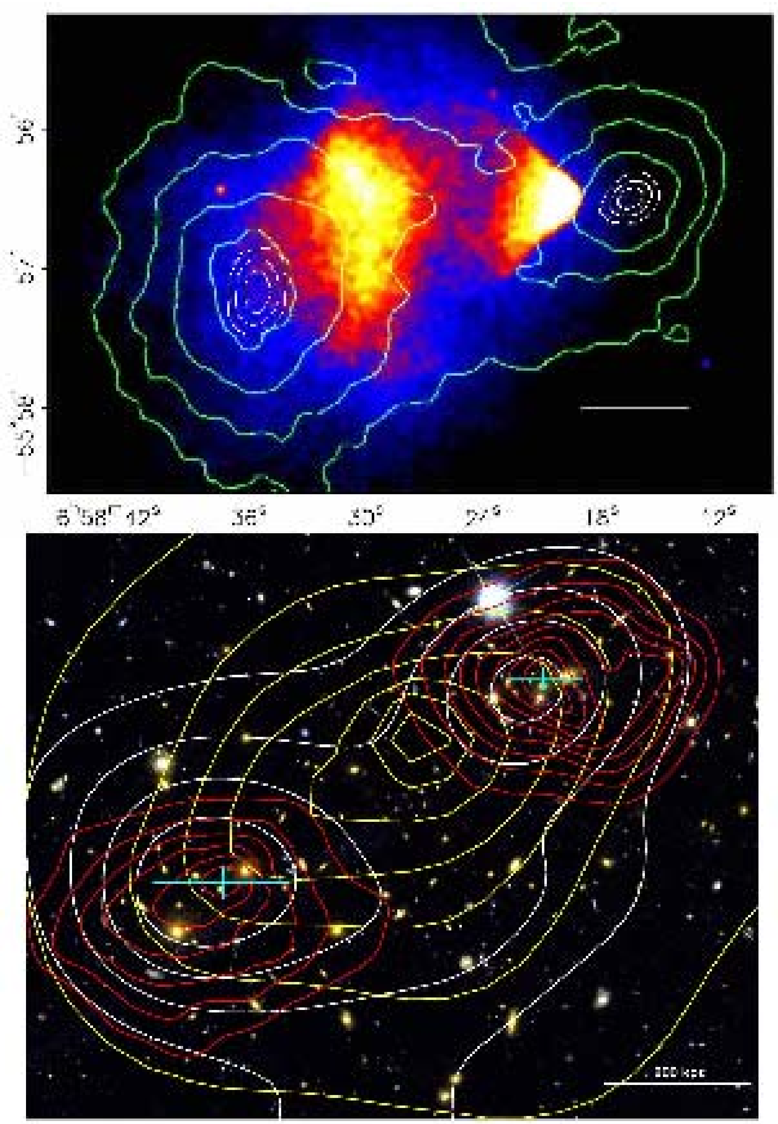
- Observations not good enough?
- Change dark matter
  - Self interacting dark matter?
  - Warm dark matter?
- tilted or running initial power spectrum
- Non-linear physical processes not well taken into account ?
- Not enough resolution in simulations ?
- No dark matter: change gravity?
- Change h ?
- Change the cosmological model?

Cold Dark Matter in trouble



- How can we get less numerous small objects?
- Warm dark matter?
- tilted or running initial power spectrum

Cold Dark Matter in trouble



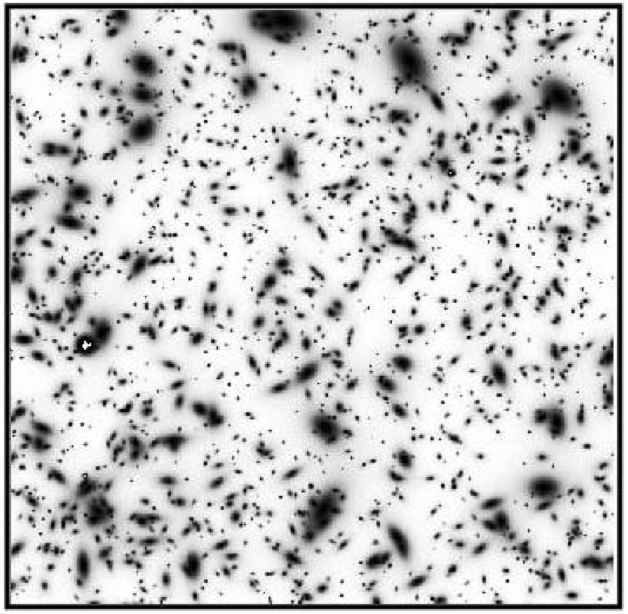
Clowe et al 2006 (top), Bradac et al 2008 (bottom)

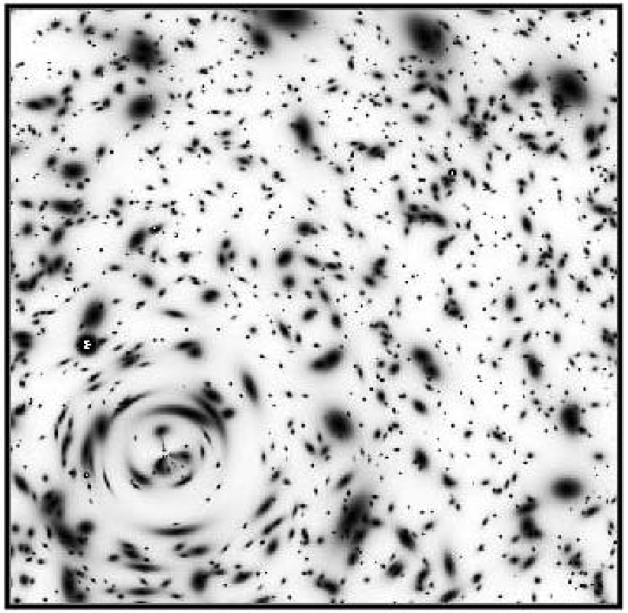
- No dark matter: gravitation theory wrong?

- Gravitational lensing seems to favor dark matter rather than modified gravity

- Hot plama (baryons) does not follow the dark matter (weak lensing + strong lensing (arcs)) , nor the galaxy distributions

Cold Dark Matter in trouble





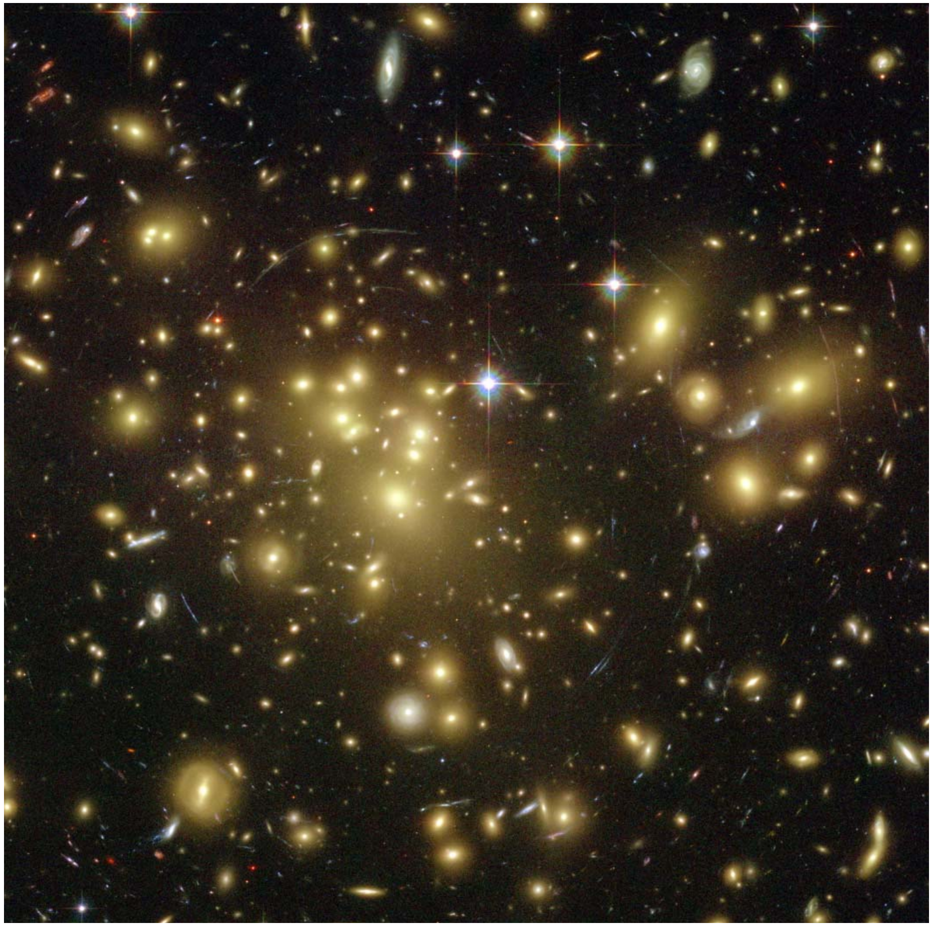
• No dark matter: gravitation theory wrong?

• Gravitational lensing show the matter distribution woithout regards its dynamical stage, and its nature

• The distortion of galaxies is the gravitational shear field.

• The gravitational shear field provide the convergence field= mass density field

Cold Dark Matter in trouble



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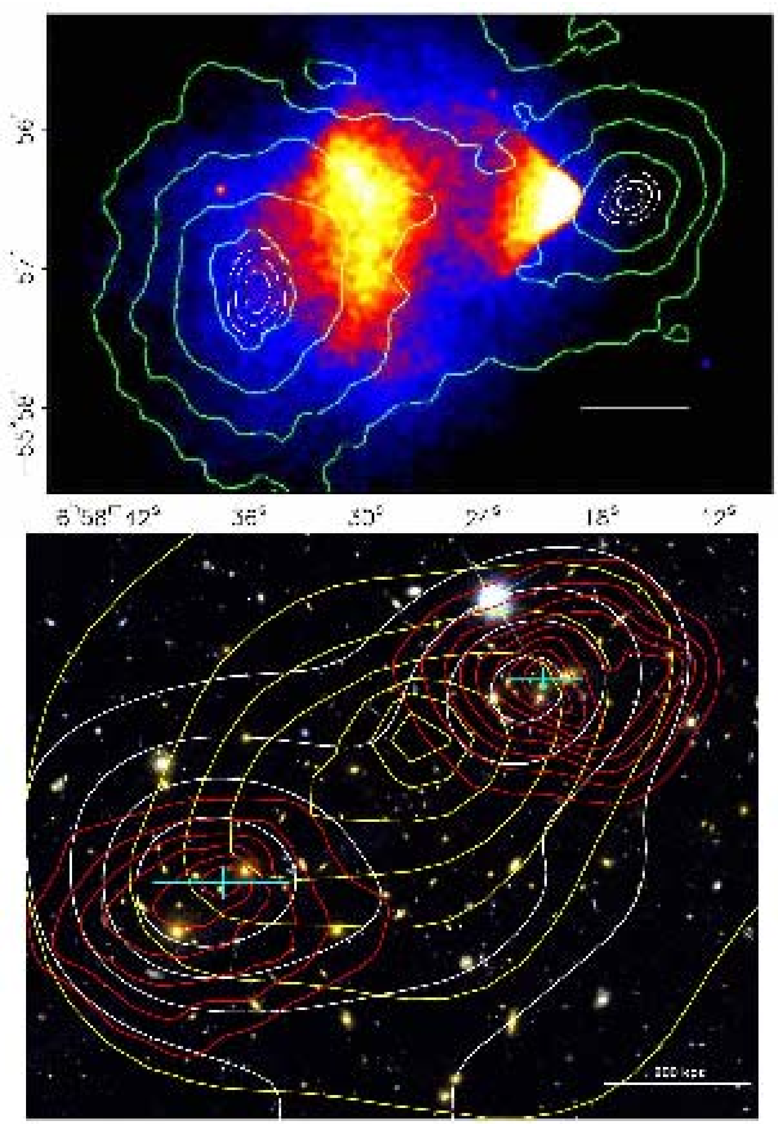
•This is observed !

Cold Dark Matter in trouble



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- In modified gravity:
  - this is not possible: matter is baryonic only : DM exist then !!
  - or large fraction of massive neutrinos (wait for Katrin experiment)
  - or shear map is not kappa map ?
  - or measurements are wrong ?

Summary - II

- The predictions of the standard cosmological model at small scale show several discrepancies with respect to the standard CDM model
- One most critical in the abundance of sub-haloes
- One more debated : the mass density profile:
    - cuspy,
    - triaxiality,
    - slope,
    - concentration
- So far, the assumption of a dominant contribution of cold dark matter particles is still the most successfull