Radiation Hydrodynamics using moment method

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Radiative Transfer

We get (almost) **all our information from photons**

Radiative transfer is a diagnostic and interpretation tool

→ Detailed transfer, many lines ….

Radiative Hydrodynamics : dynamical effect of transfer

→ Only the global energy budget matters

Both aspects can be coupled …

Radiative Transfer

The transfer equation :

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}\cdot\nabla\right)I(\mathbf{x},t;\mathbf{n},\nu) = \sigma_a^\nu\left(B(\mathbf{x},t,\nu) - I(\mathbf{x},t;\mathbf{n},\nu)\right) - \sigma_s^\nu I(\mathbf{x},t;\mathbf{n},\nu)$$
$$+\sigma_s^\nu \int_{4\pi} p^\nu(\mathbf{n}.\mathbf{n}')I(\mathbf{x},t;\mathbf{n}',\nu)d\mathbf{n}'$$

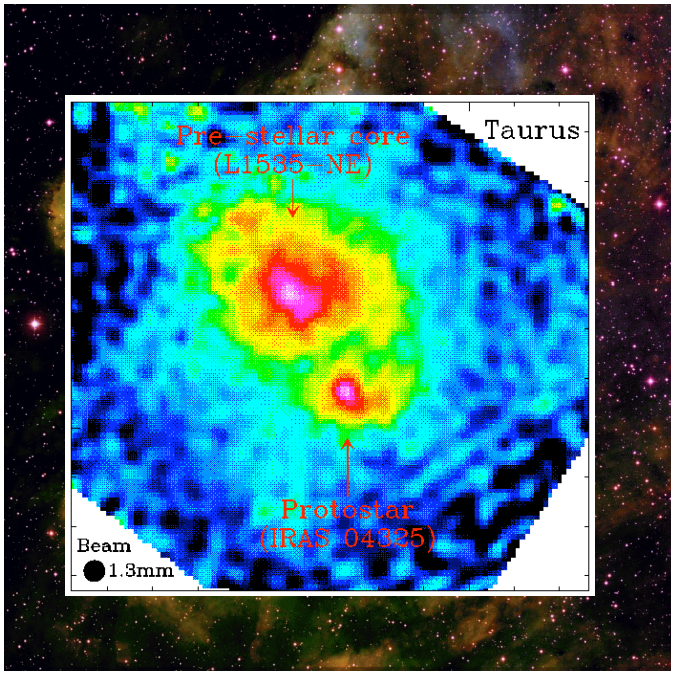
$$\eta_{\text{therm}}^\nu = \sigma_a^\nu B(\nu) \qquad \text{et} \qquad \eta_{\text{diff}}^\nu = \sigma_s^\nu \oint p^\nu(\Omega' \to \Omega)I(\Omega')d\Omega'$$

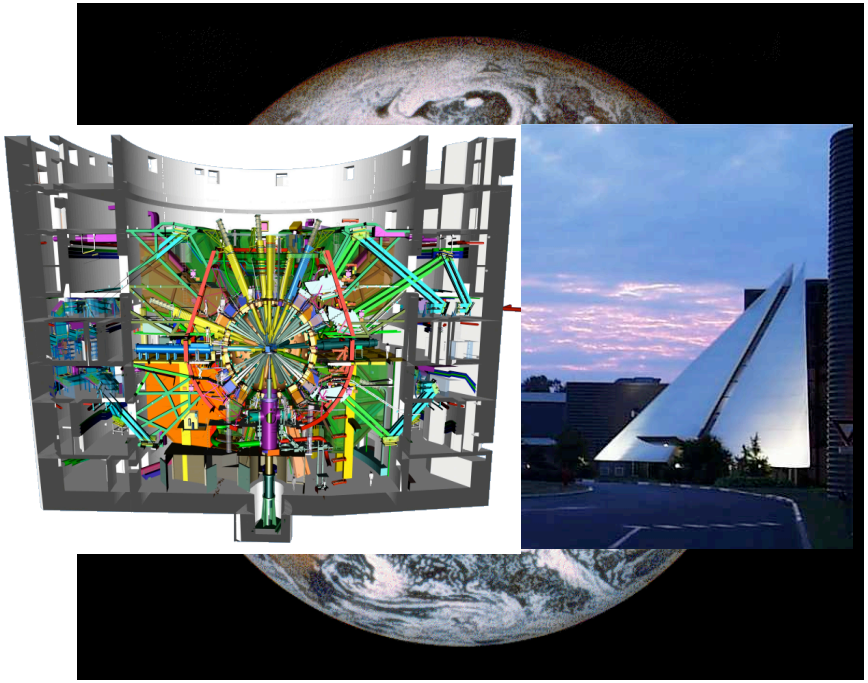
3 + (1) + 2 + 1 = 6

$N^6 >> 1 , \quad \forall N$

Quelques ordres de grandeur

$$\frac{E_r}{e} = \frac{a_r T^4}{3/2NkT} \simeq 36\frac{T^3}{N}$$





N ~ 10^4 - 10^8 cm^{-3}

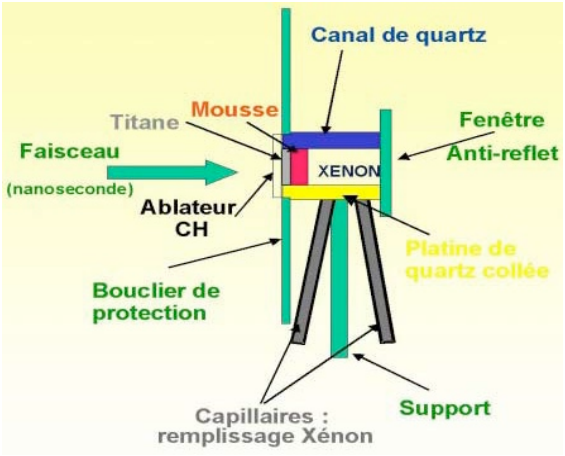
→ T_{eq} ~ 6 – 150 °K

N ~ 3. 10^{19}

→ T_{eq} ~ 10^6 °K ~ 100 eV

Quelques ordres de grandeur

$$\frac{F_r}{F} = \frac{1/4 a_r c T^4}{ve} \simeq 9 \frac{c}{v} \frac{T^3}{N} = \frac{1}{4} \frac{c}{v} \frac{E_r}{e}$$



T_{eq} ~ 10 eV

LULI, PALS, ...

Deux régimes asymptotiques

Régime de diffusion :

- $\lambda << L$
- Matière et rayonnement proche de l'équilibre
- Equation de diffusion (parabolique)

Régime de transport :

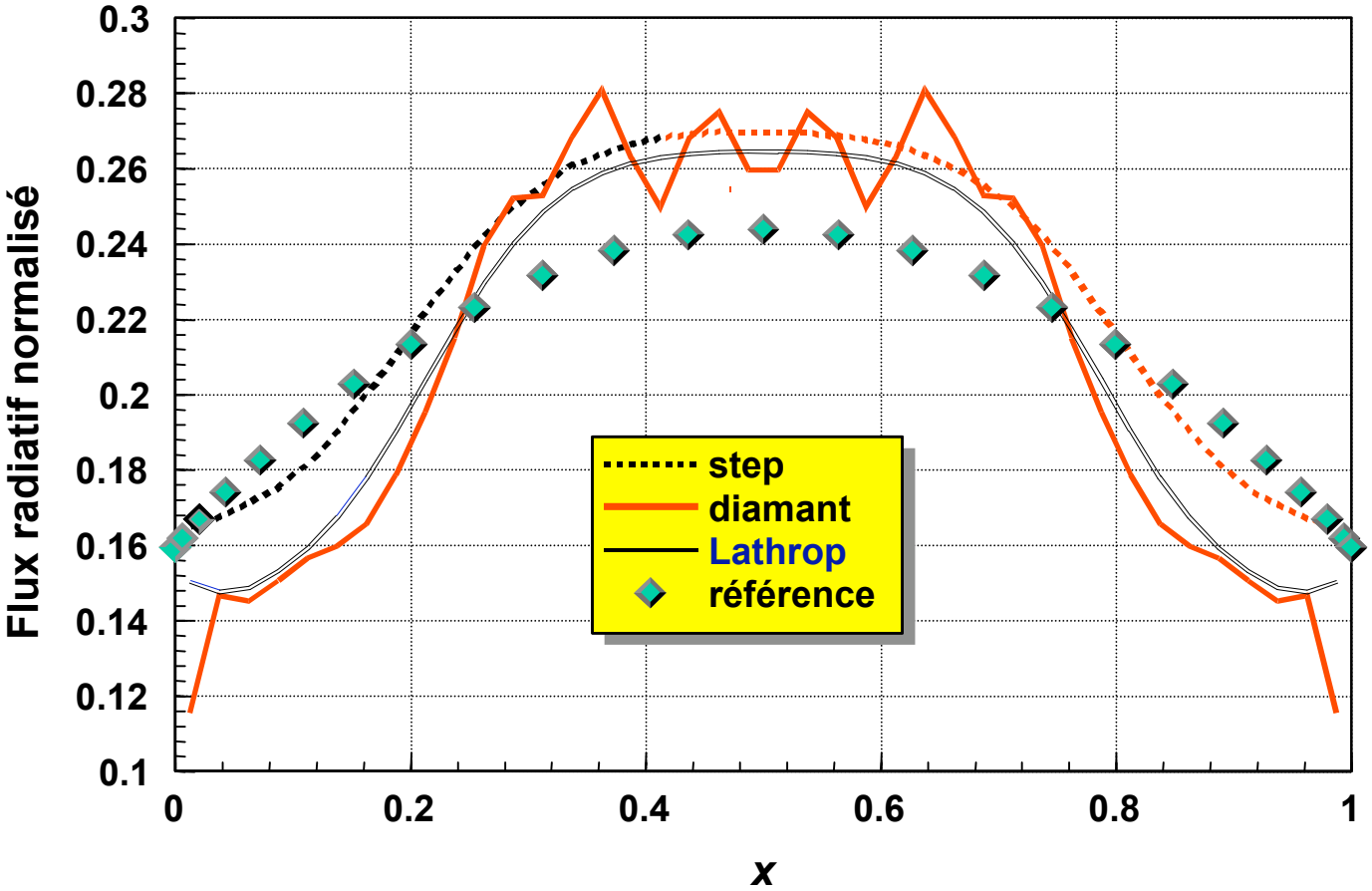
- **$\lambda >> L$**
- Matière et rayonnement peu couplé
- Régime de transport hyperbolique

How to solve the radiative transfer equation …?

- **Direct integration :**
  - ✓Very expensive
  - ✓ Geometrical constraints
  - ✓ Discretization problems
- **Monte-carlo Methods**
  - ✓Expensive in optically thick regions
  - ✓Photon noise
  - ✓Difficult to couple to hydro.
- **Moments Models**
  - ✓Approximate physical Model
  - ✓Geometrical constraints

Discretization of the space and the sphere

The simultaneous discretization of both the computation domain and all the directions at all points of this domain is a very delicate matter…..

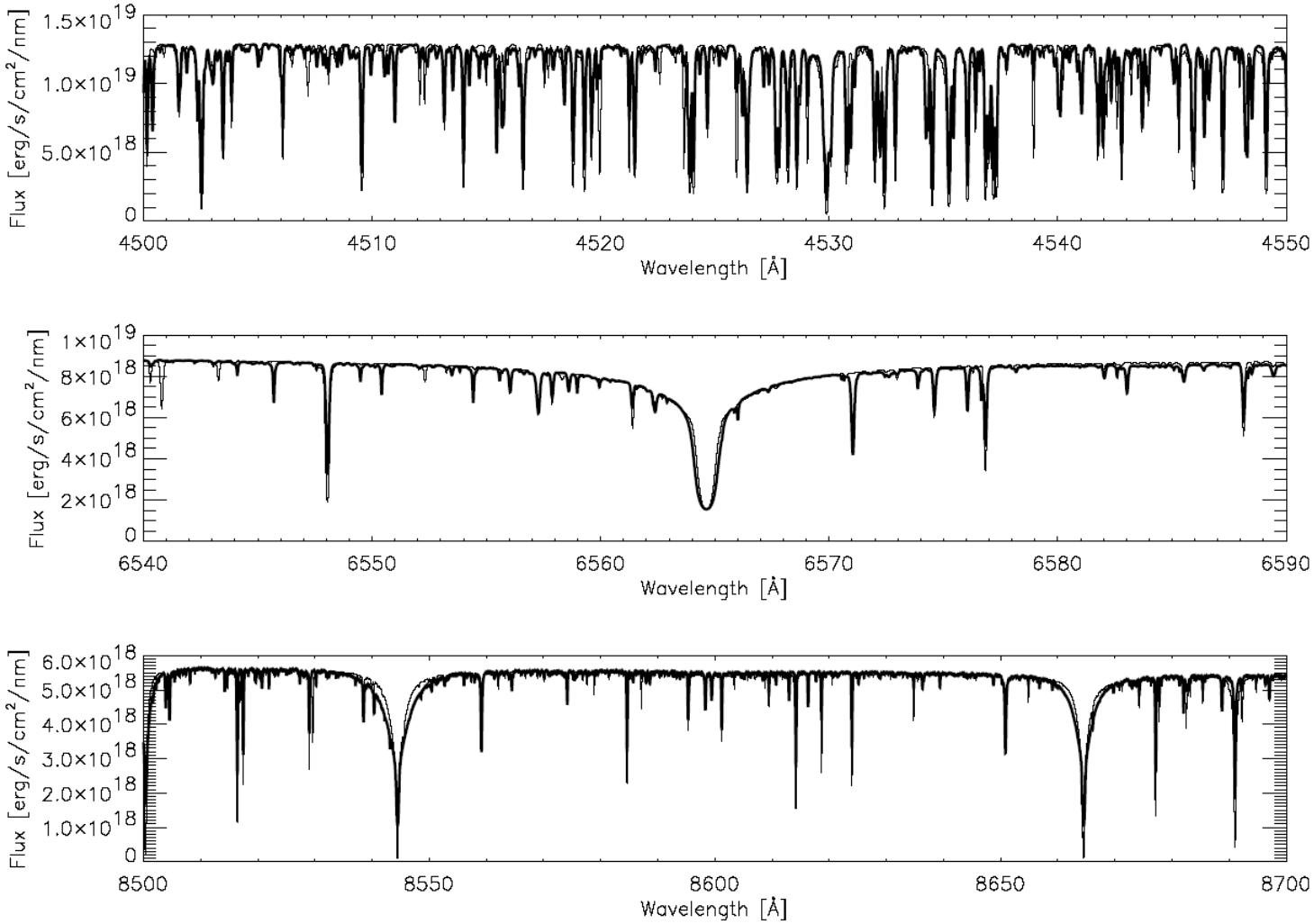


The Phoenix code

- «Exact» resolution of the transfer equation for a static atmosphere.
- Equation of state with 40 elements (up to 26 degres of ionisation) and 206 molecules
- Iteration to find the population of all levels
- 42 millions of atomic and ionic lines
- 700 millions of molecular lines
- Between 10^4 et 10^6 frequencies, 50 levels of stratifications

6 days of CPU ……

Spectra of the sun



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Moments Models

The transfer equation :

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}.\nabla\right) I(\mathbf{x}, t, \mathbf{n}, \nu) = \eta(\mathbf{x}, t, \mathbf{n}, \nu) - \chi(\mathbf{x}, t, \mathbf{n}, \nu) I(\mathbf{x}, t, \mathbf{n}, \nu)$$

- Too time and memory consuming
- Too precise !!

Moments models

$$E_r(\mathbf{x}, t, \nu) = \frac{1}{c}\int I(\mathbf{x}, t, \mathbf{n}, \nu) \quad d\omega$$ Radiative Energy

$$F_r(\mathbf{x}, t, \nu) = \frac{1}{c}\int I(\mathbf{x}, t, \mathbf{n}, \nu)\, \mathbf{n} \quad d\omega$$ Radiative Flux

$$P_r(\mathbf{x}, t, \nu) = \frac{1}{c}\int I(\mathbf{x}, t, \mathbf{n}, \nu)\, \mathbf{n} \otimes \mathbf{n}\, d\omega$$ Radiative pressure

Les Modèles aux Moments

$$\partial_t E_r + \nabla . F_r = \sigma c (a_r T^4 - E_r)$$

$$\partial_t F_r + c^2 \nabla . \mathbf{P}_r = -\sigma c F_r$$

Il faut ensuite une relation de fermeture pour clore le système :

$$\mathbf{P_r = f(E_r, F_r)}$$

Moments models

$$\partial_t E_r + \nabla . F_r = \sigma c (a_r T^4 - E_r)$$

$$\partial_t F_r + c^2 \nabla . \mathbf{P}_r = -\sigma c F_r$$

One now needs a closure relation to close the system :

$$\mathbf{P_r = f(E_r, F_r)}$$

Et les fréquences…

- Modèle spectrale
- Modèle multi-groupes
- Modèle gris

$$\sigma_P = \frac{\int_0^\infty \sigma_a^\nu B(\nu,T)d\nu}{\int_0^\infty B(\nu,T)d\nu}$$

$$\sigma_R = \frac{\int_0^\infty \frac{\partial B(\nu,T)}{\partial T} d\nu}{\int_0^\infty \frac{1}{\sigma_{tot}^\nu} \frac{\partial B(\nu,T)}{\partial T} d\nu}$$

Modèle de diffusion

$$\partial_t F_r \simeq 0 \quad \longrightarrow \quad F_r = -\frac{c}{\sigma}\nabla . P_r$$

On suppose un tenseur des pressions isotrope

$$\longrightarrow \quad P_r = \frac{1}{3} E_r$$

$$\partial_t E_r - \frac{c}{3}\nabla.\left(\frac{1}{\sigma}\nabla E_r\right) = \sigma c\left(a_r T^4 - E_r\right)$$

$$\sigma_R = \frac{\int_0^\infty \frac{\partial B(\nu,T)}{\partial T} d\nu}{\int_0^\infty \frac{1}{\sigma_{tot}^\nu} \frac{\partial B(\nu,T)}{\partial T} d\nu}$$

Le Modèle M1

Planckienne :

$$B(\nu, T) = \frac{h\nu^3}{c^2}\left(e^{\frac{h\nu}{kT}} - 1\right)^{-1}$$

Fonction de distribution de M1 :

$$B(\nu, \Omega, T) = \frac{h\nu^3}{c^2}\left(e^{\frac{h\nu}{kT}\left(1 - \frac{2-\sqrt{4-3f^2}}{f^2}\vec{f}.\vec{\Omega}\right)} - 1\right)^{-1}$$

- **Minimisation de l'entropie du rayonnement** (Dubroca & Feugeas)
- **Transformée de Lorentz d'une Planckienne** (Levermore)

The M1 model

The closure relation for the M1 model is :

$$\mathbf{P}_r = \mathbf{D} E_r$$

$$\mathbf{D} = \frac{1-\chi}{2}\mathbf{I} + \frac{3\chi-1}{2}\mathbf{n}\otimes\mathbf{n}$$

$$\chi = \frac{3+4f^2}{5+2\sqrt{4-3f^3}} \quad \text{avec} \quad f = \frac{F_r}{cE_r}$$

- Reasonably cheap
- Allows to take into account anisotropies in the radiation field
- Exact in free streaming and diffusive regime
- Allows to treat scattering (anisotropic)
- Allows «clean» averaging over opacities.

Lien entre diffusion et modèle à deux moments

$$\begin{array}{lcll} \partial_t E_r^\nu & + & c\nabla\cdot\mathbf{f}E_r^\nu & = & \sigma_a^\nu(4\pi B(\nu) - cE_r^\nu) \\ \partial_t \mathbf{f}E_r^\nu & + & c\nabla\cdot\mathbb{D}E_r^\nu & = & -\sigma_{tot}^\nu \mathbf{f}cE_r^\nu \end{array}$$

$$\cancel{[1/c\partial_t\mathbf{f} + \mathbf{f}\cdot\nabla\mathbf{f}]}E_r^\nu + \nabla\cdot[(\mathbb{D} - \mathbf{f}\otimes\mathbf{f})E_r^\nu] = -\omega\sigma_{tot}^\nu\mathbf{f}E_r^\nu$$

avec $\omega = \dfrac{\sigma_a^\nu 4\pi B(\nu) + (1-g^\nu)\sigma_s^\nu cE_r^\nu}{c\sigma_{tot}^\nu E_r^\nu}$

$$(\mathbb{D} - \mathbf{f}\otimes\mathbf{f})\mathbf{R} = \mathbf{f}$$ où $\mathbf{R} = -(1/\omega\sigma_{tot}^\nu)(\nabla E_r^\nu/E_r^\nu)$

$$\mathbf{f} = \lambda(R)\mathbf{R} \qquad R = \|\mathbf{R}\| = \frac{f}{\chi(f) - f^2}$$

Lien entre diffusion et modèle à deux moments

$$\mathbf{f} = \lambda(R)\mathbf{R} \qquad R = \|\mathbf{R}\| = \frac{f}{\chi(f) - f^2}$$

$$\lambda(R) = \chi(f) - f^2$$

$$\chi(f) = \lambda(R) + \lambda(R)^2 R^2$$

$$\mathbf{F}_r^\nu = -\frac{c}{\omega\sigma_{tot}^\nu}\lambda(R)\nabla E_r^\nu$$

Radiation hydrodynamics

$$
\begin{aligned}
\partial_t \rho + & \nabla.[\rho u] & = & \; 0 \\
\partial_t \rho u + & \nabla.[\rho u \otimes u + P\mathbf{I}] & = & \frac{\sigma}{c} F_r + F \\
\partial_t E + & \nabla.[u(E+P)] & = & -\sigma c(a_r T^4 - E_r) + (\frac{\sigma}{c} F_r + F).u \\
\partial_t E_r + \nabla.[E_r u] + & \nabla.F_r + \mathbf{P}_r : \nabla u & = & \sigma c(a_r T^4 - E_r) \\
\partial_t F_r + \nabla.[F_r u] + & c^2 \nabla.\mathbf{P}_r + (F_r.\nabla)u & = & -\sigma c F_r
\end{aligned}
$$

Approche Numérique

Le rayonnement doit etre implicite (dt < dx/c)

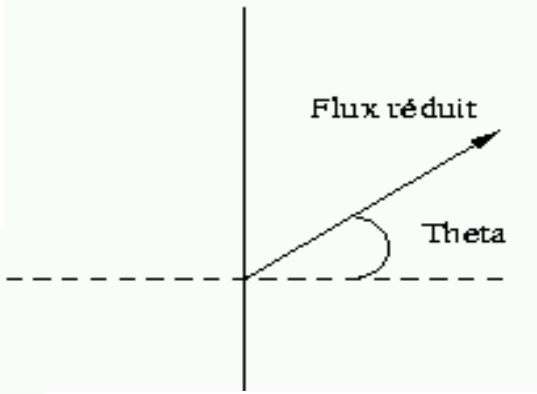
Splitting :

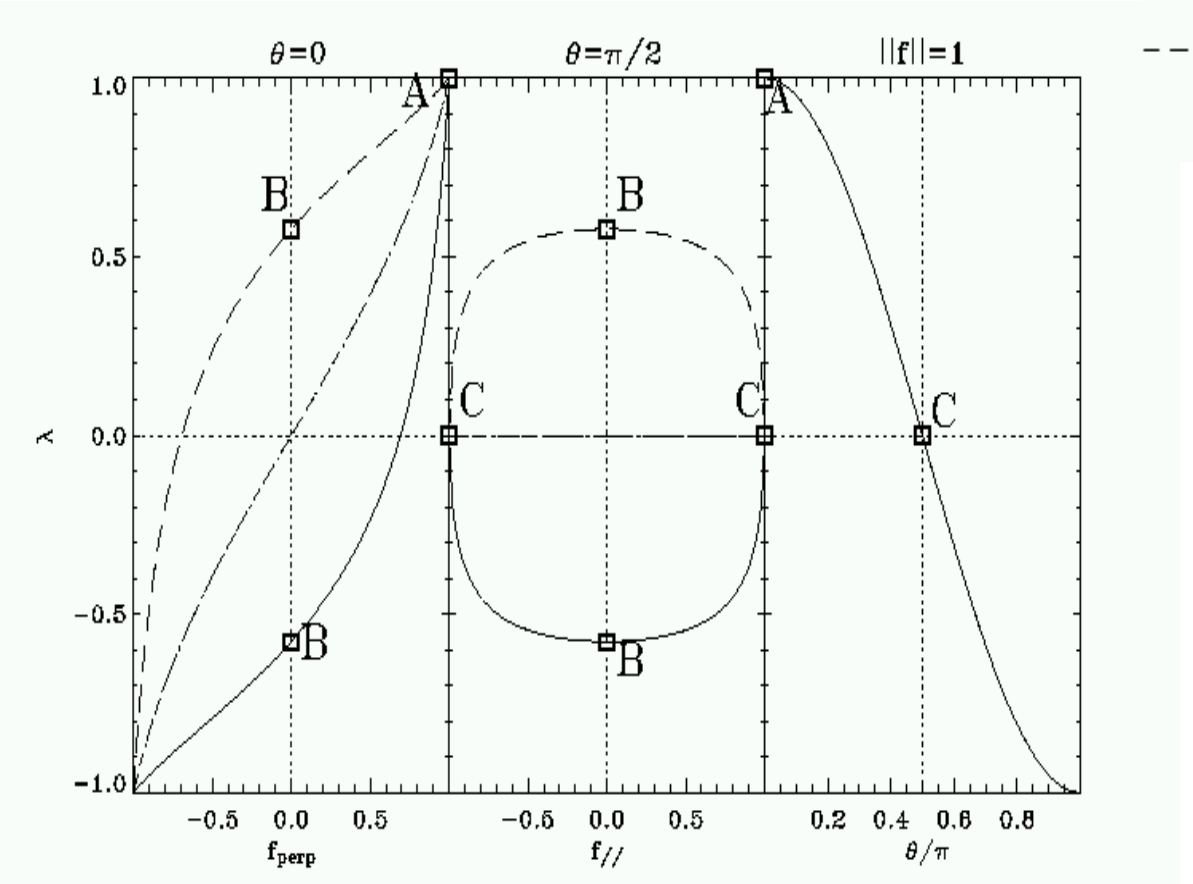
- hydrodynamique et advection **explicite**
  (schéma de MUSCL – Hancock d'ordre 2)
- rayonnement et termes de couplages **implicites**

(le terme "κρF/c" dans l'hydrodynamique est explicite)

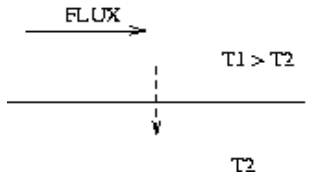
Le solveur Radiatif

Solveur HLLE à deux ondes





Transverse flux



La limite diffusive

$$\epsilon = \frac{1}{\sigma l} = \frac{\lambda}{l}$$

$$\partial_t E_r + \partial_x F_r = \frac{(aT_m^4 - E_r)}{\epsilon^2}$$

$$\partial_t F_r + \frac{1}{\epsilon^2}\partial_x \chi E_r = -\frac{1}{\epsilon^2} F_r$$

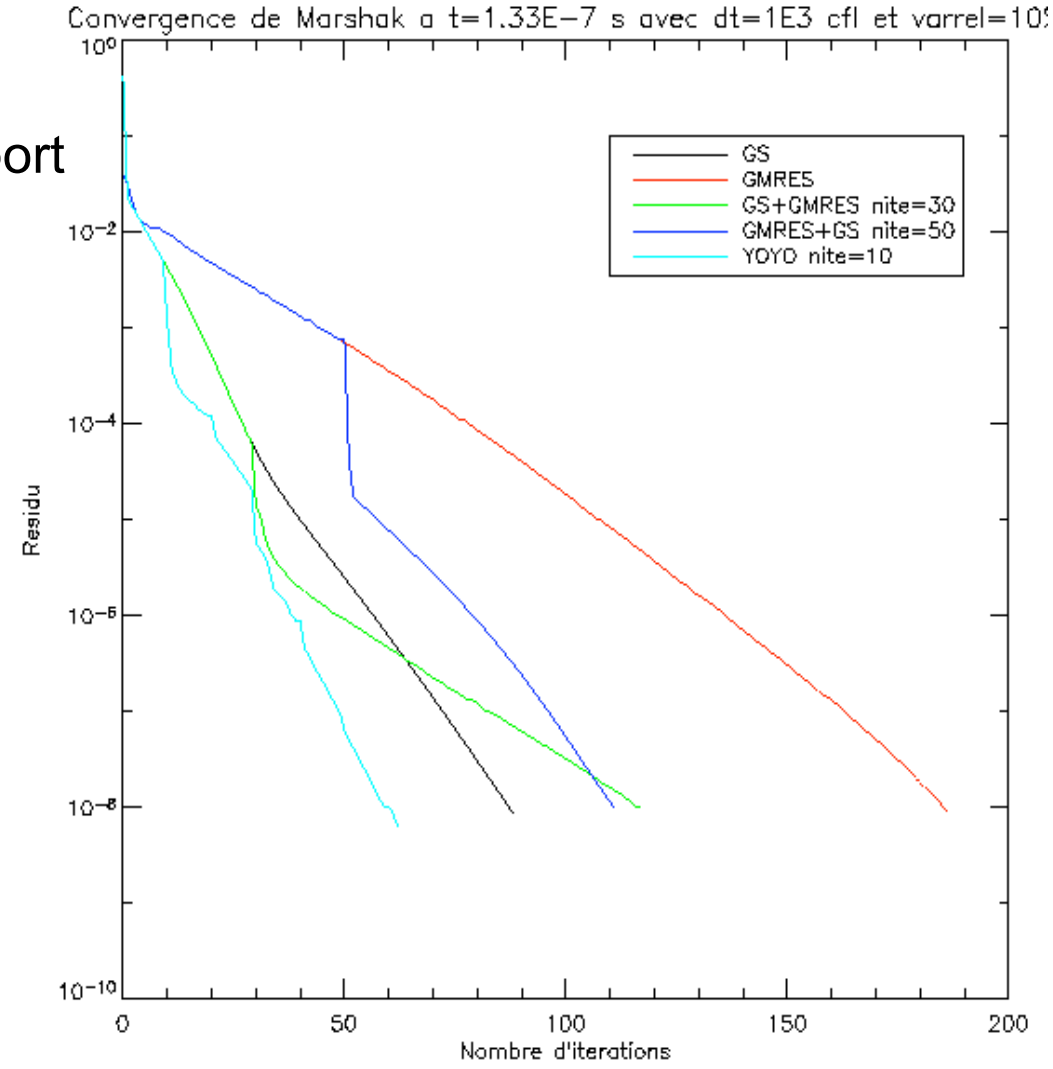
→ **La diffusion numérique devient plus grande que la diffusion physique !!**

$$\partial_t F_r + \partial_x \chi E_r = -\frac{1}{\epsilon^2}(F + (1-\epsilon^2)\partial_x \chi E_r)$$

Méthodes pour résoudre un système

Méthodes itératives

- Gauss-Seidel : limite transport
- GMRES : limite diffusive
- couplage

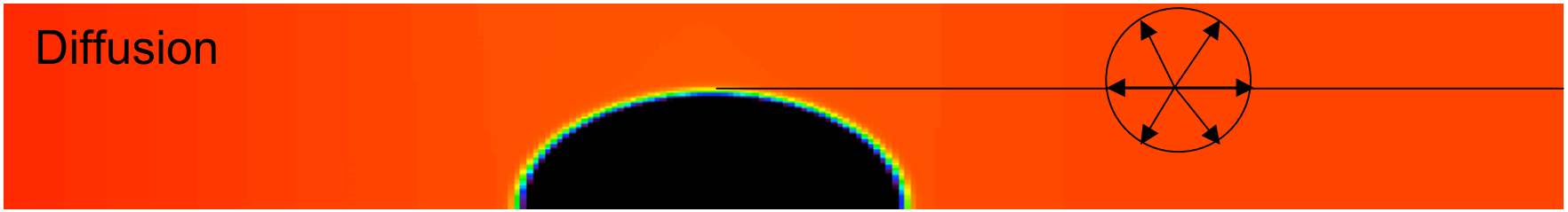


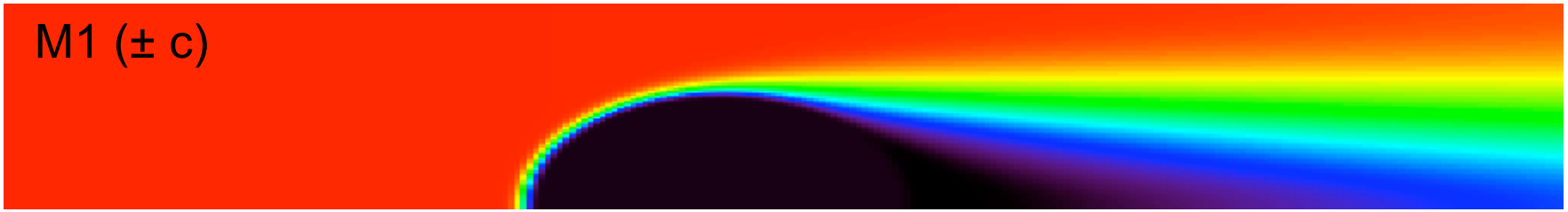
Parallélisation en MPI

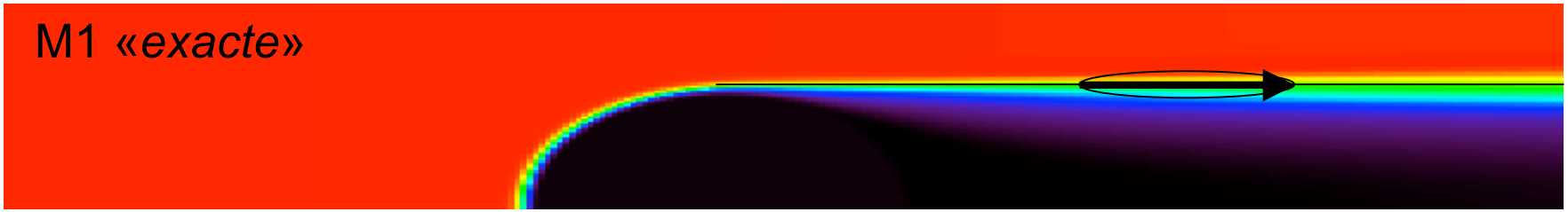
Utilisation du centre de calcul CCRT

Optimisation du code

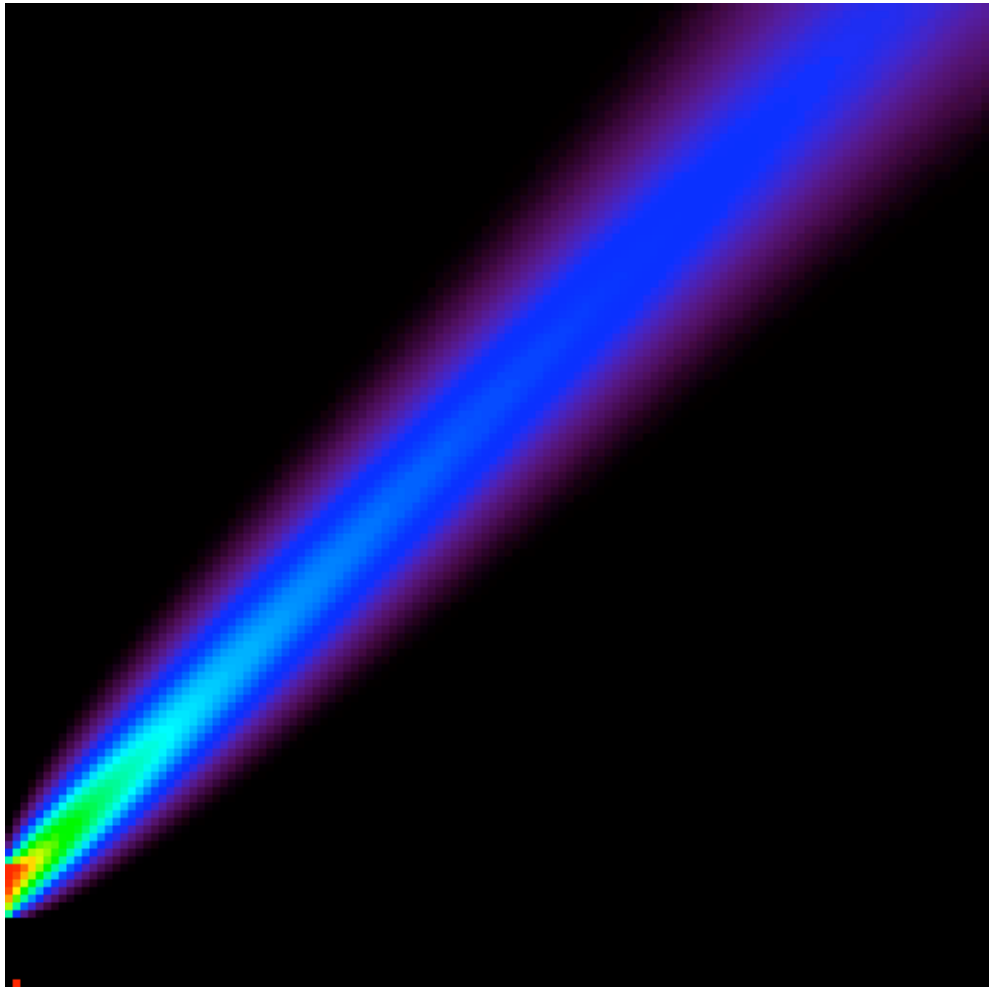
Ombre et lumière



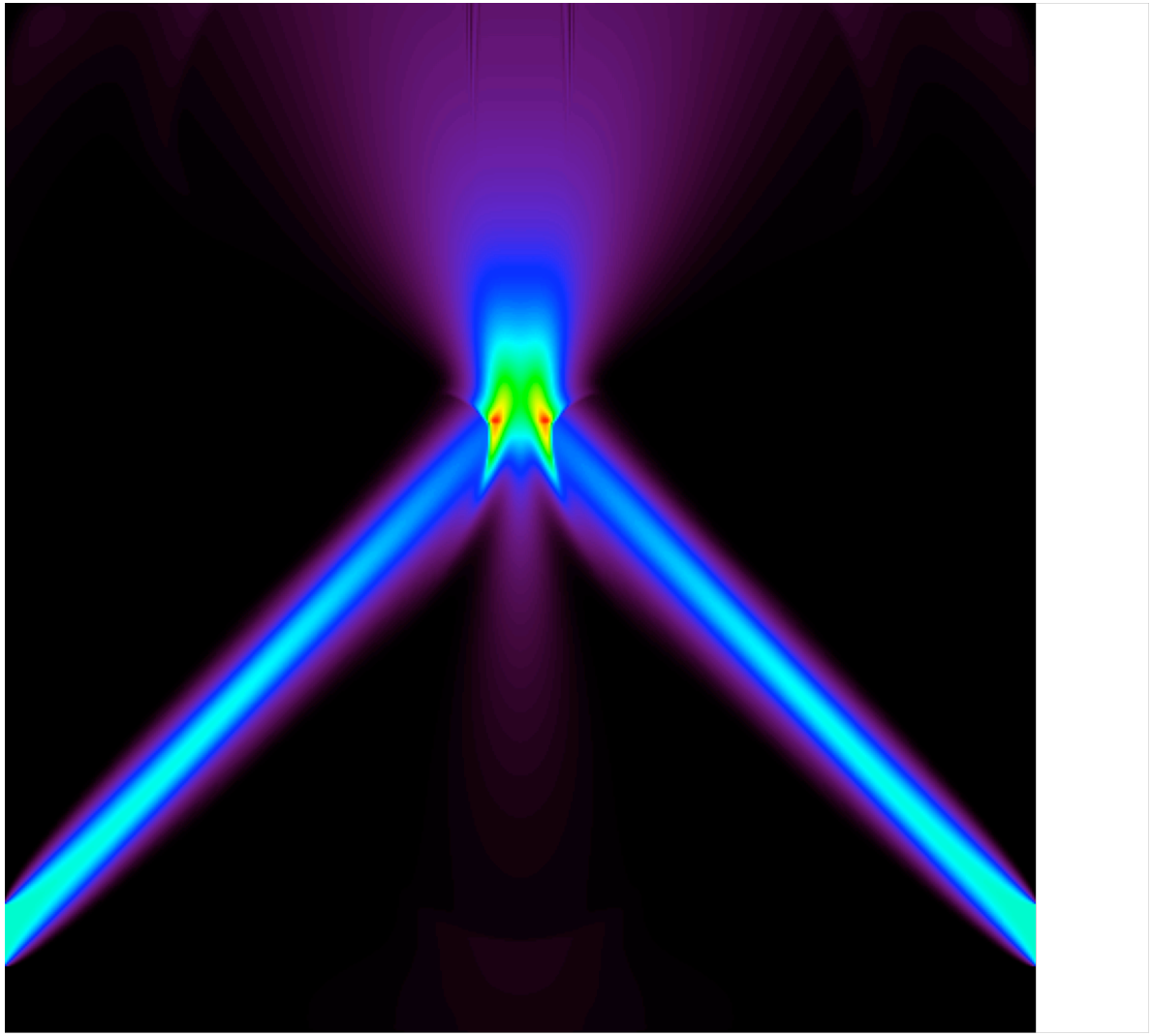




Un pinceau

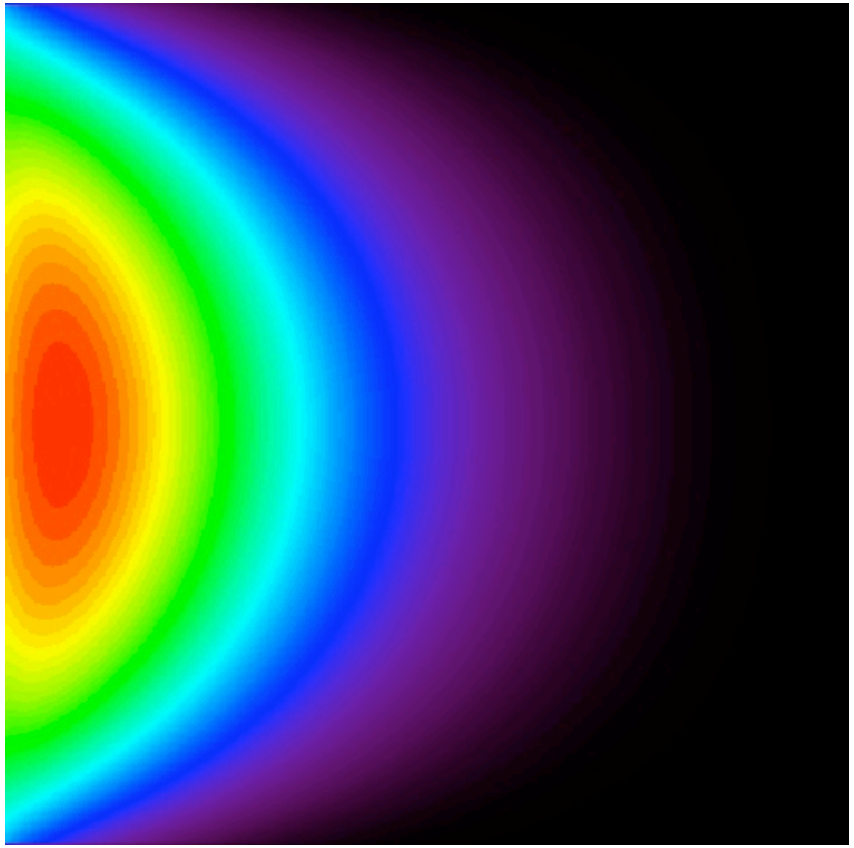


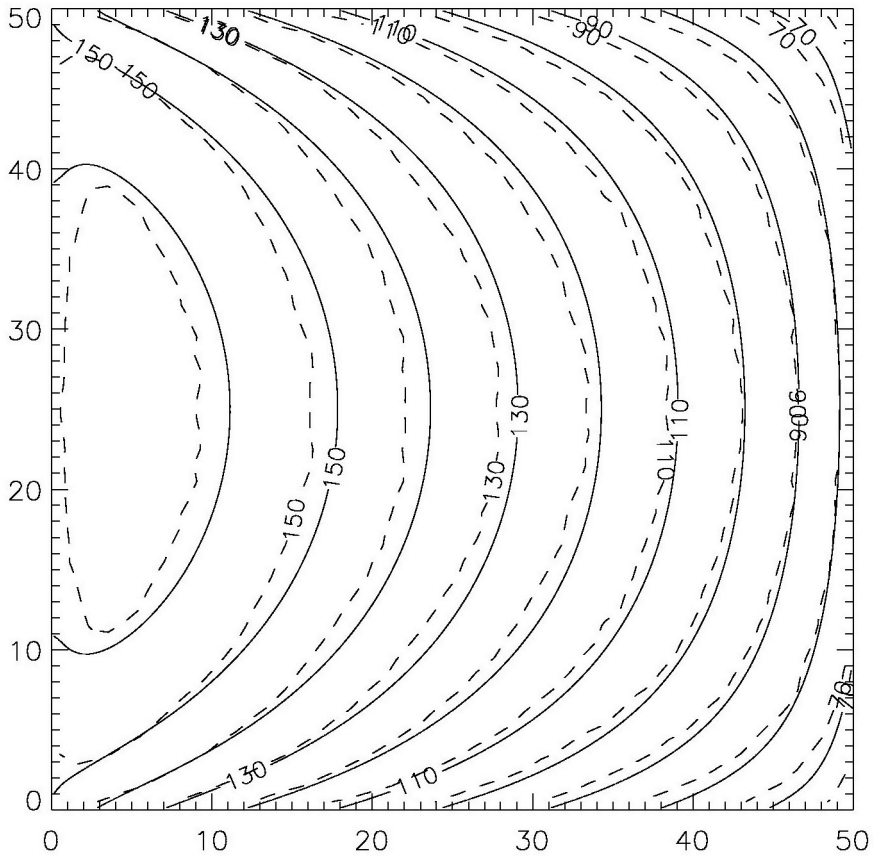
Deux pinceaux



M1 versus Monte-Carlo

Comparaison à un code Monte-Carlo (Dullemond & Natta 2003)





La diffusion (scattering)

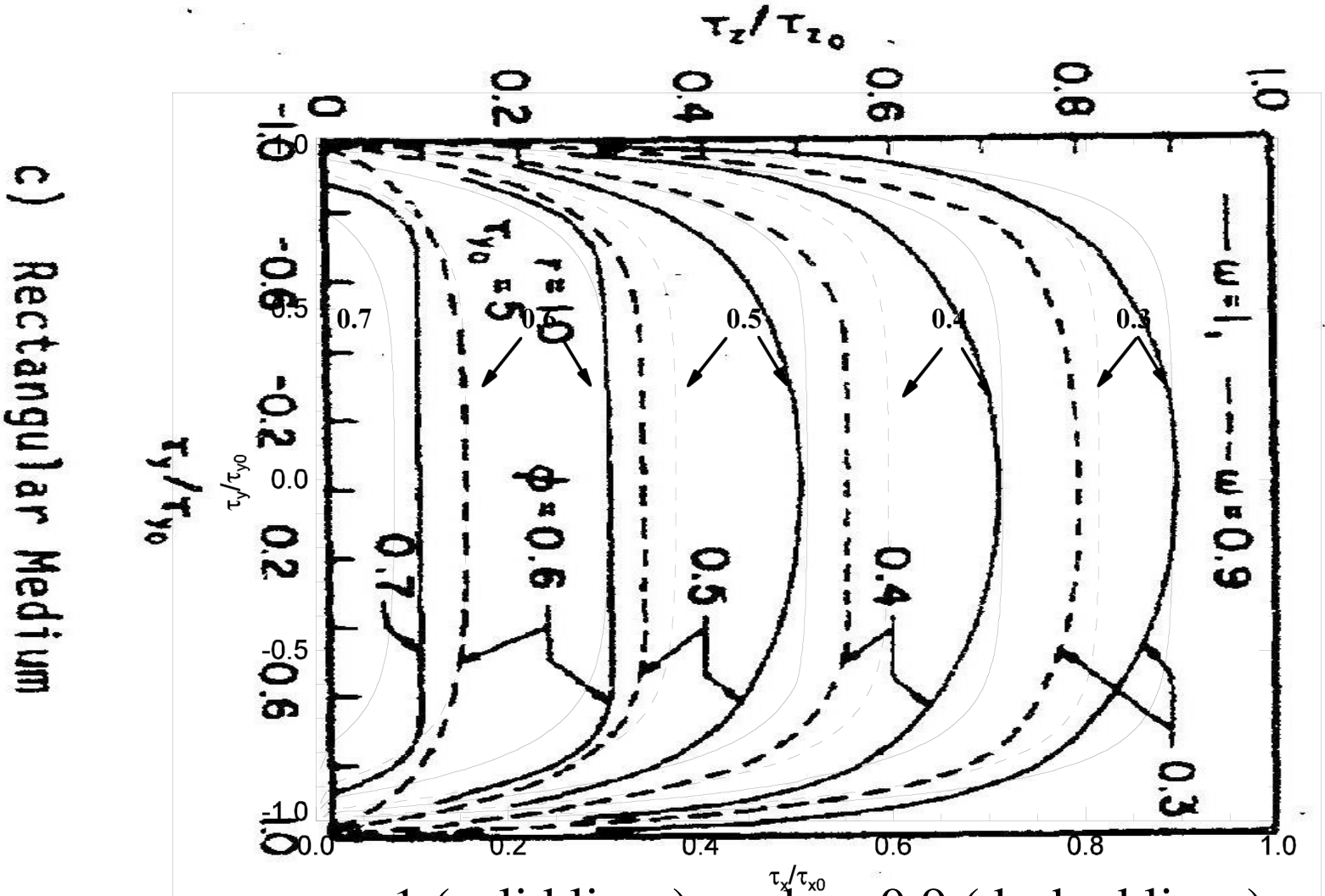
$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}\cdot\nabla\right)I(\mathbf{x},t;\mathbf{n},\nu) = \sigma_a^\nu\left(B(\mathbf{x},t,\nu) - I(\mathbf{x},t;\mathbf{n},\nu)\right) - \sigma_s^\nu I(\mathbf{x},t;\mathbf{n},\nu)$$

$$\boxed{-\sigma_s^\nu \int_{4\pi} p^\nu(\mathbf{n}.\mathbf{n}')I(\mathbf{x},t;\mathbf{n}',\nu)d\mathbf{n}'}$$

$$\begin{aligned}
\frac{\partial E_r^\nu}{\partial t} &+ \nabla\cdot\mathbf{F}_r^\nu &= \sigma_a^\nu(4\pi B(\nu) - cE_r^\nu) \\
\frac{1}{c}\frac{\partial \mathbf{F}_r^\nu}{\partial t} &+ c\nabla\cdot\mathbb{P}_r^\nu &= -(\sigma_a^\nu + \boxed{(1-g^\nu)\sigma_s^\nu})\mathbf{F}_r^\nu
\end{aligned}$$

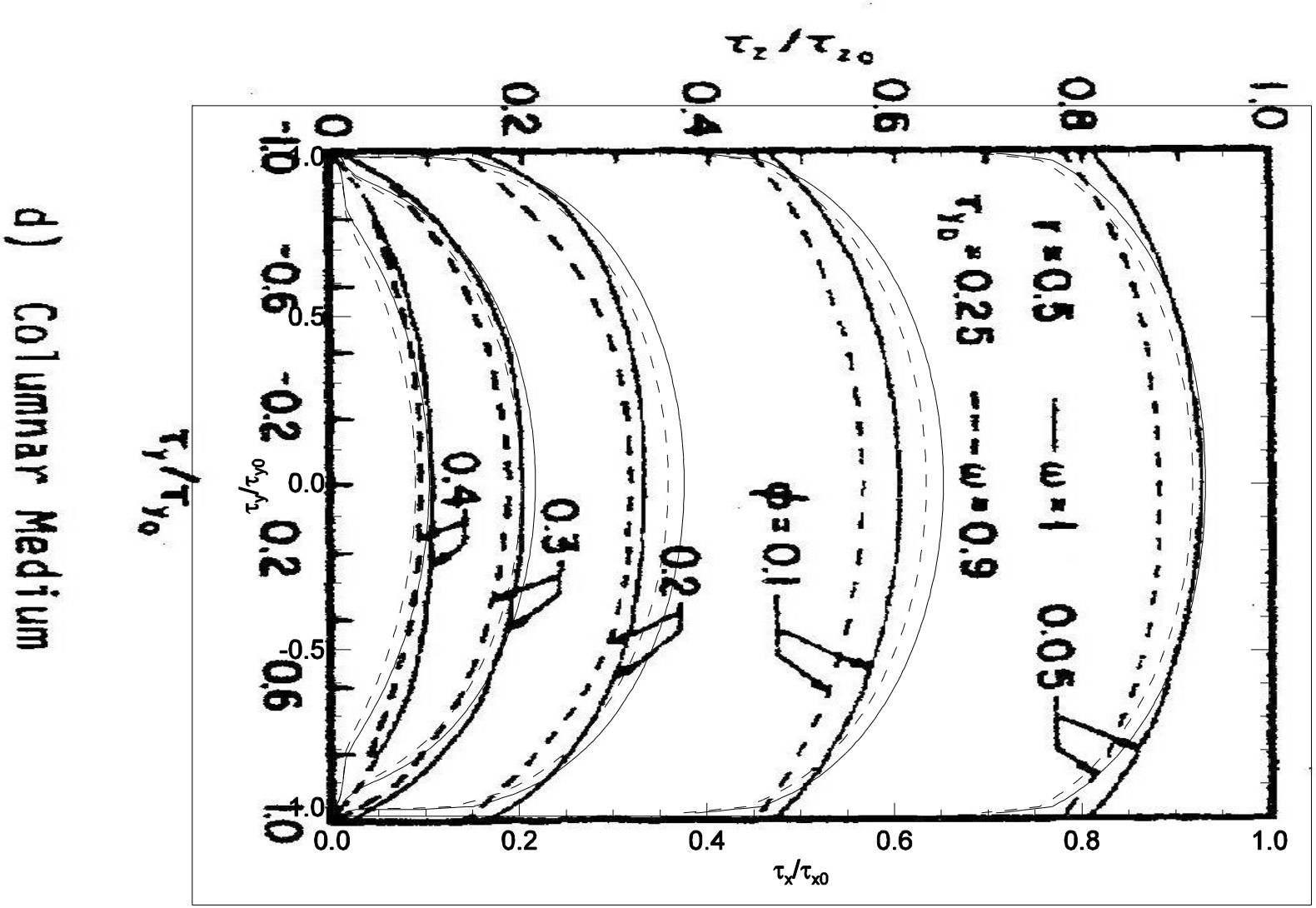
Albedo : $\omega = \dfrac{\sigma_s^\nu}{\sigma_a^\nu + \sigma_s^\nu}$

Albedo variable



ω=1 (solid lines) and ω=0.9 (dashed lines)

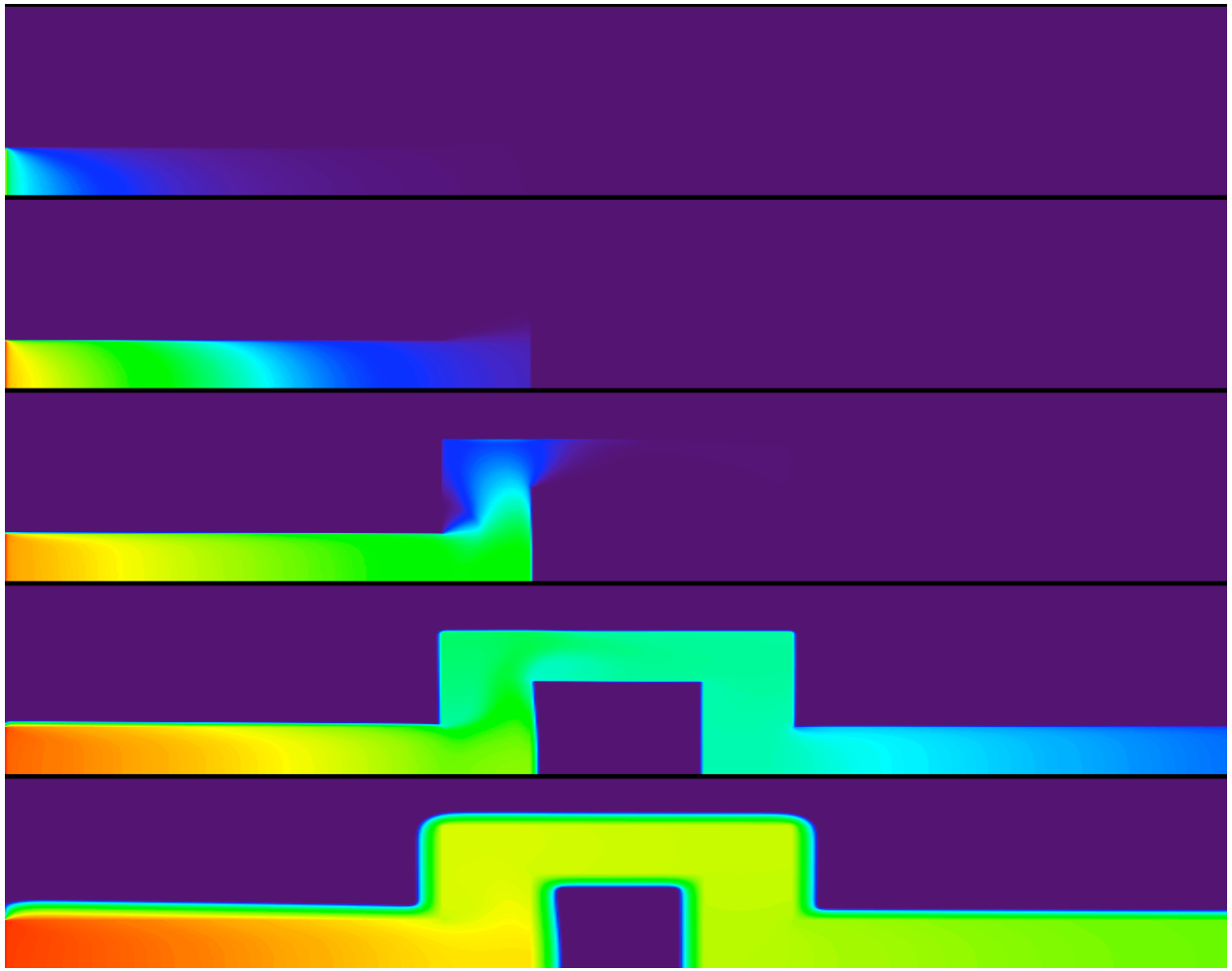
(Crosbie & Schrenker, JQSRT, 1984)



d) Columnar Medium

Test du « top hat »

(Gentile, JCP, 2001)



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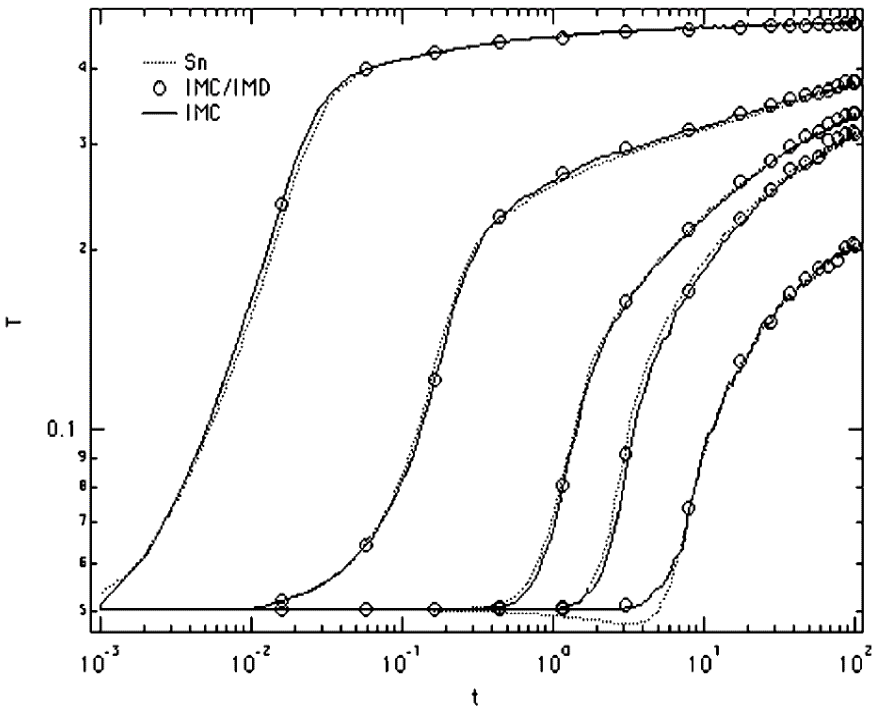
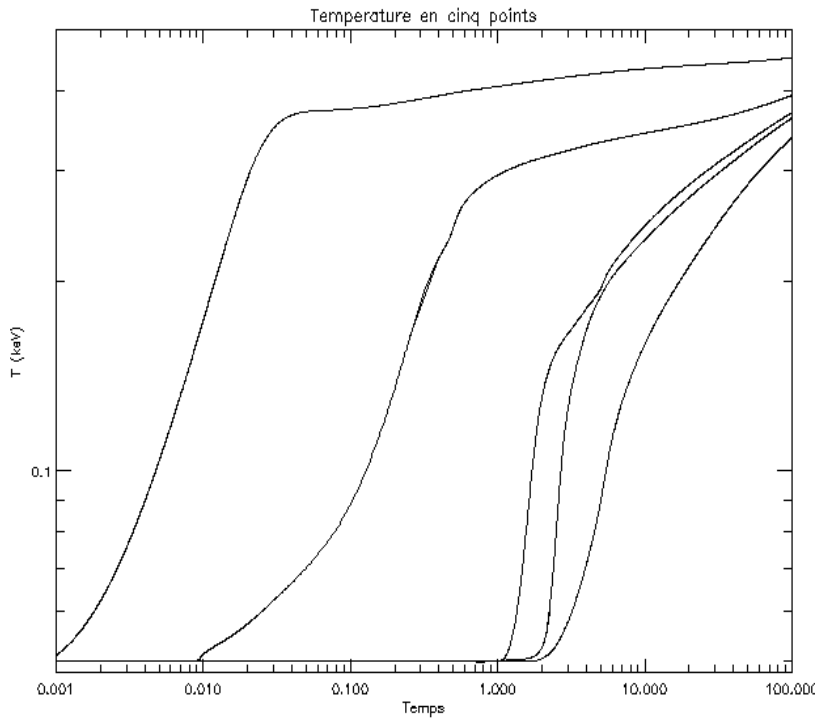


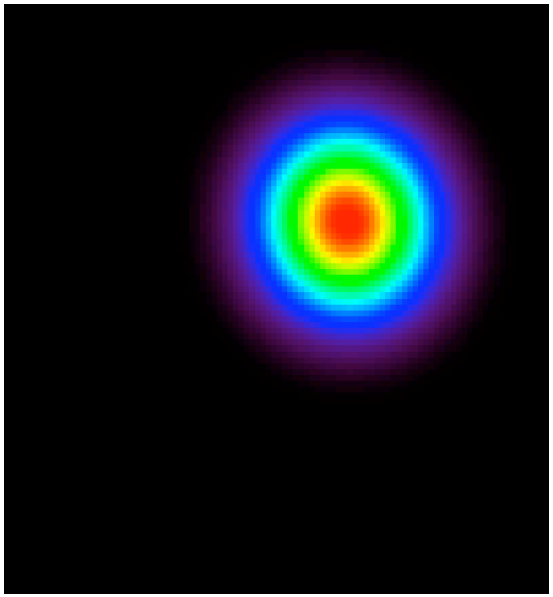
Monte-Carlo
(Gentile, JCP, 2001)



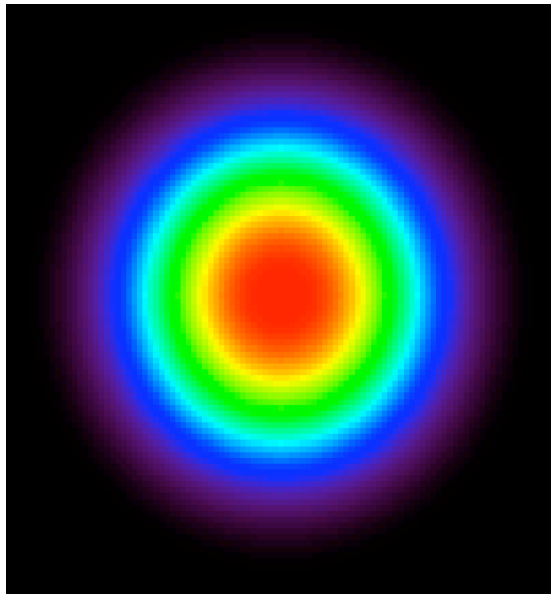
HERACLES

Température matière

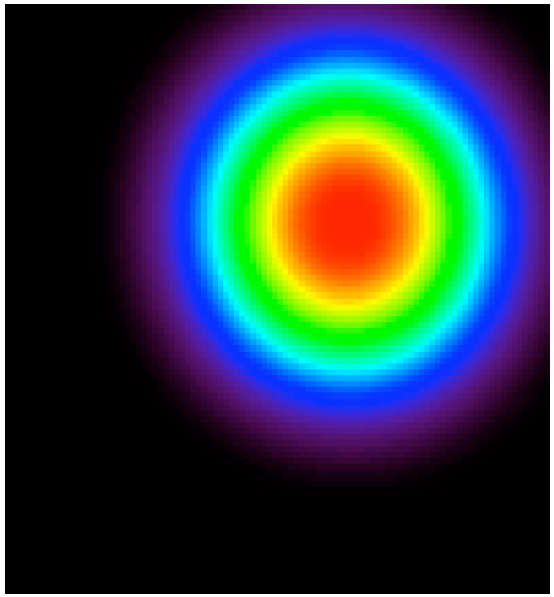
Test de diffusion dans un fluide



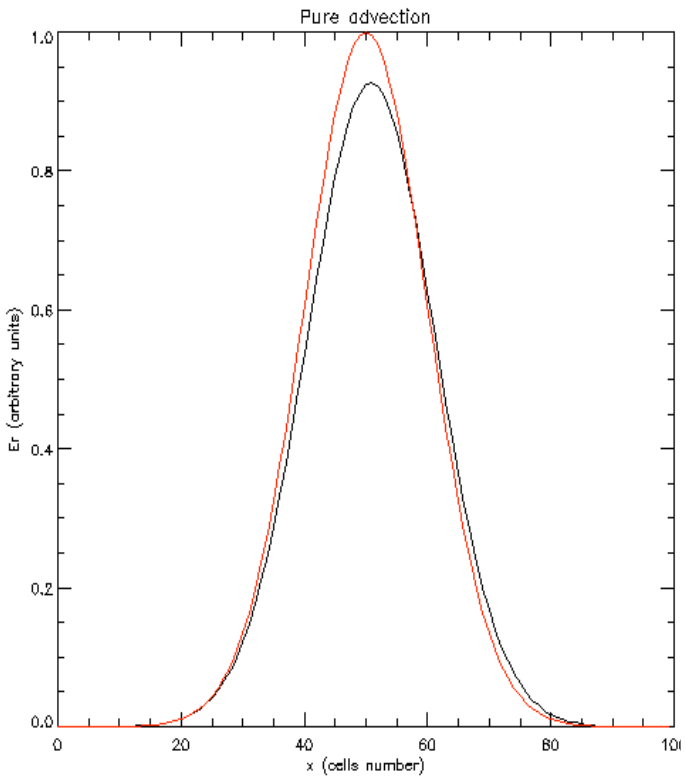
Advection pure

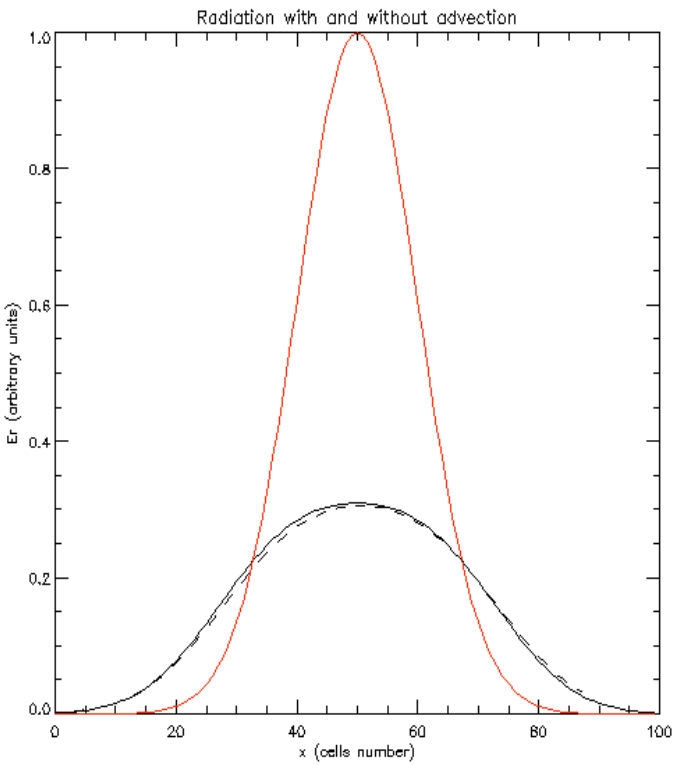


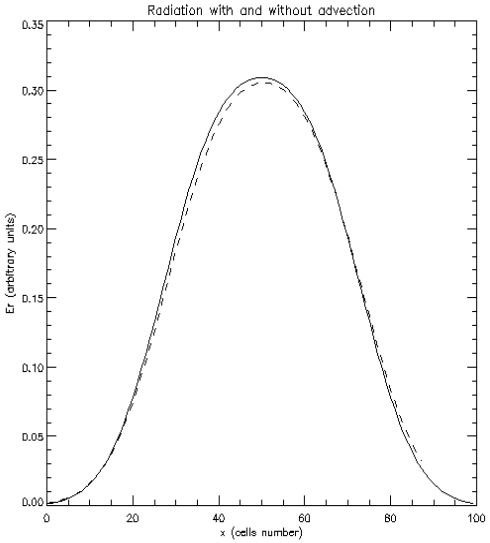
Fluide au repos



Fluide en mouvement

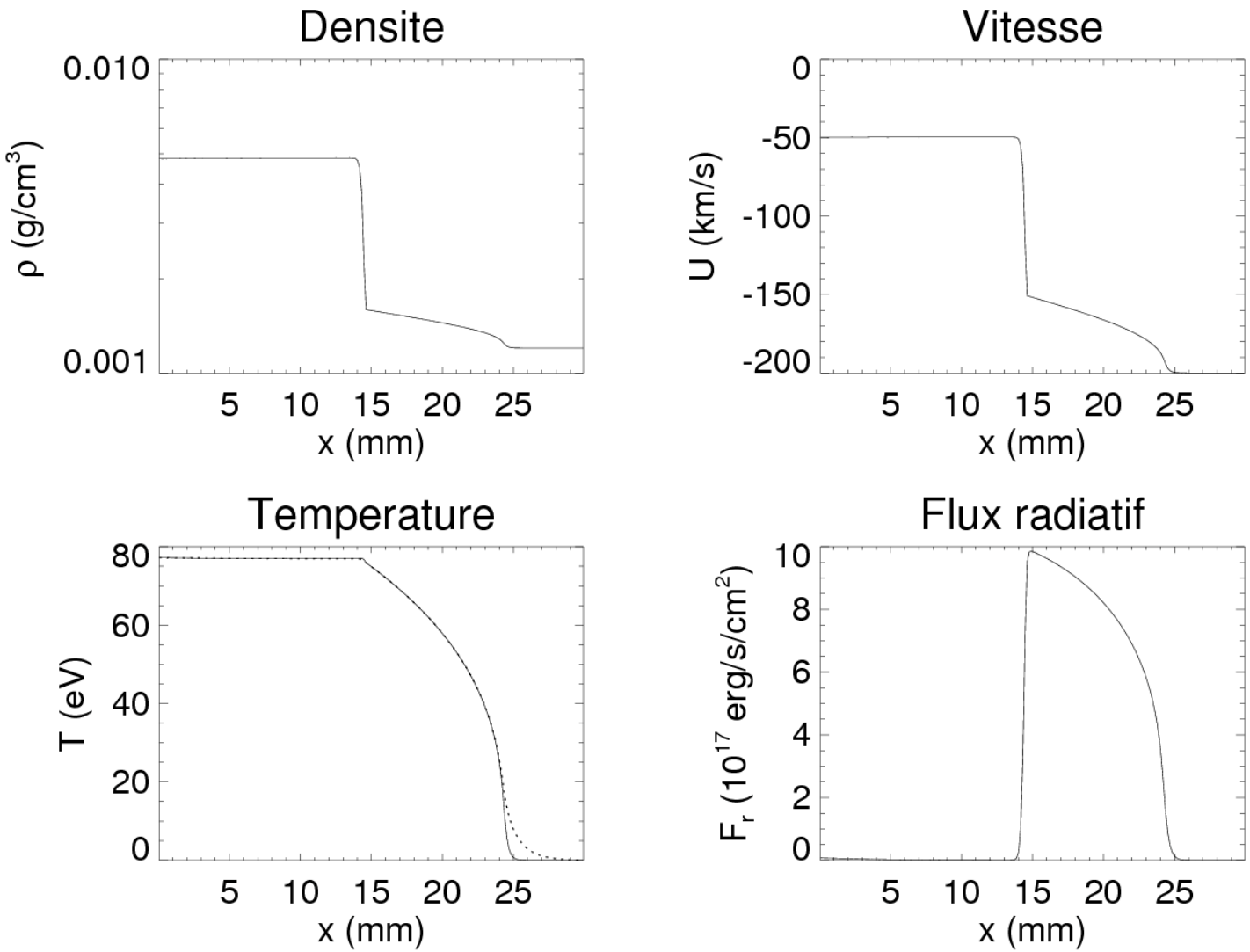

Pure advection
Er (arbitrary units)
x (cells number)


Radiation with and without advection
Er (arbitrary units)
x (cells number)

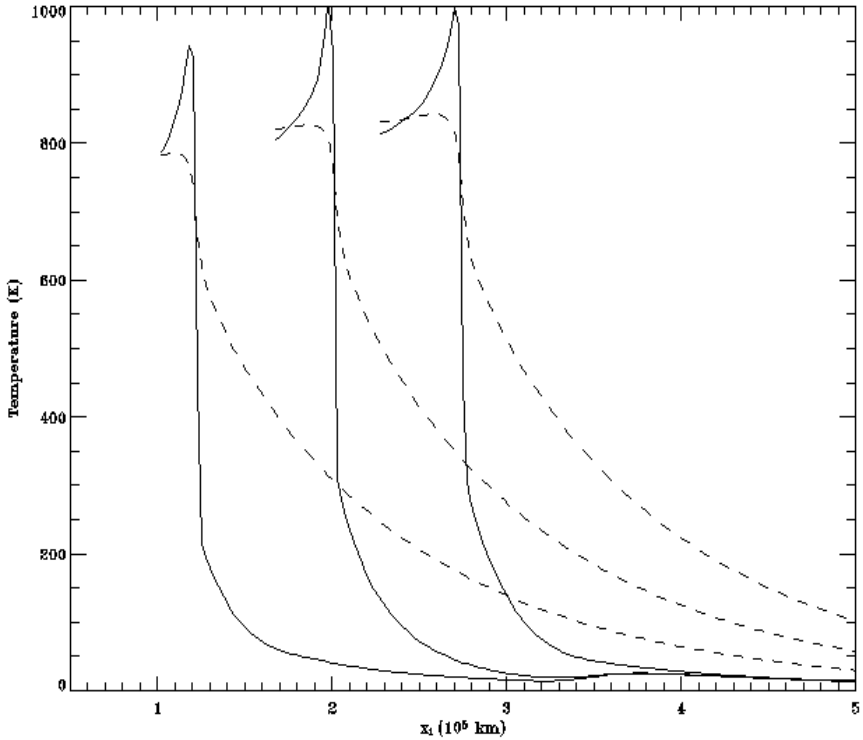

Radiation with and without advection
Er (arbitrary units)
x (cells number)

zoom

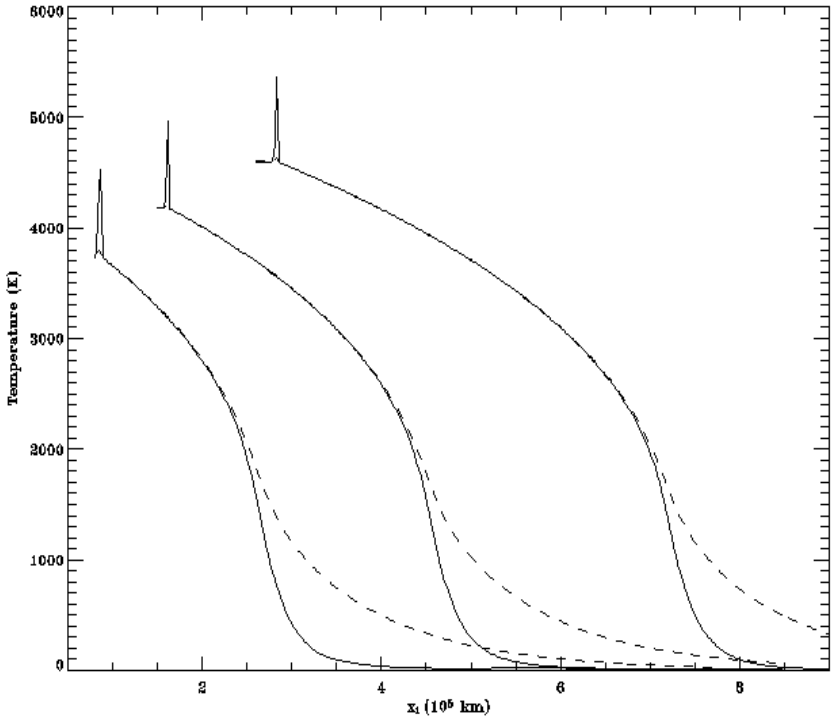
Choc Radiatif



Chocs Radiatifs



Subcritical

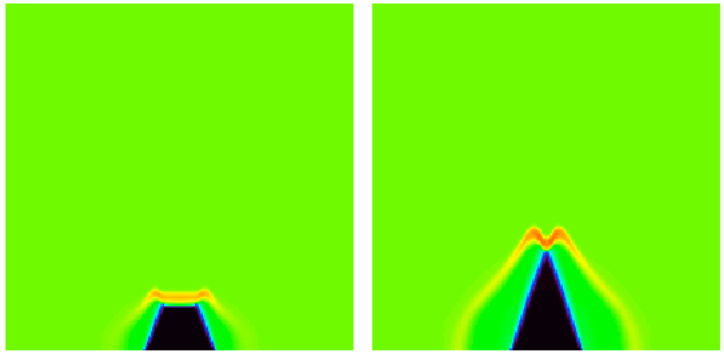


Supercritical

Bilan

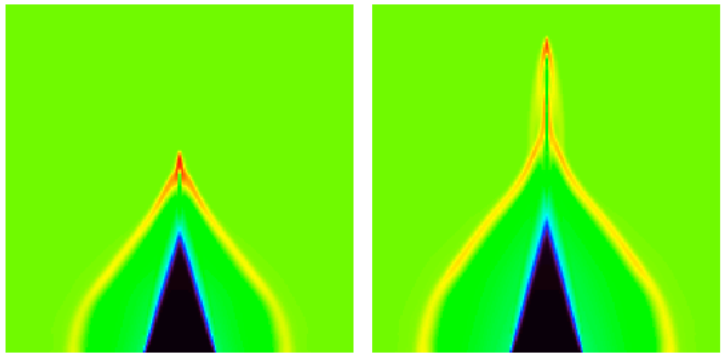
- **Modélisation des régimes de diffusion et de transport**
- **Coût maîtrisé (possibilité de faire du 3D)**
- **Couplage Naturel avec l'hydrodynamique**
- **Le modèle physique semble raisonnable**
- **Traitement de la diffusion**

- Le modèle physique
- Gris (ou quelques groupes)



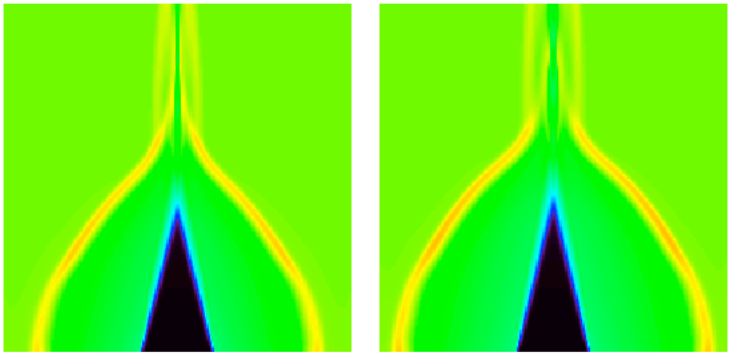
(a) t=480 ans

(b) t=960 ans



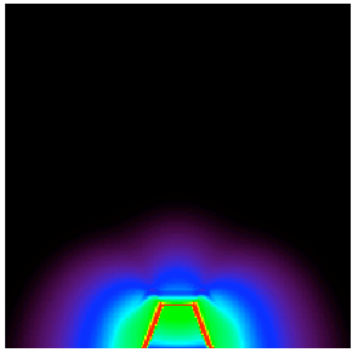
(c) t=1 440 ans

(d) t=1 920 ans

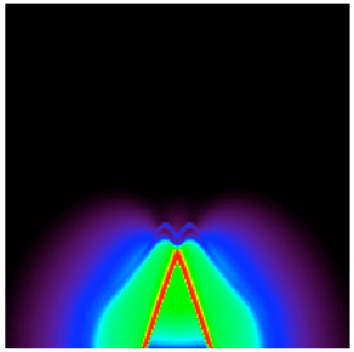


(e) t=2 400 ans

(f) t=2 880 ans



(a) t=480 ans



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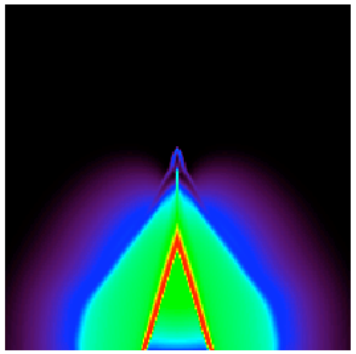
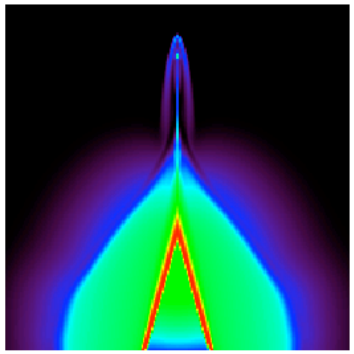
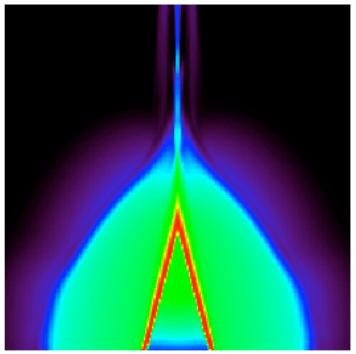


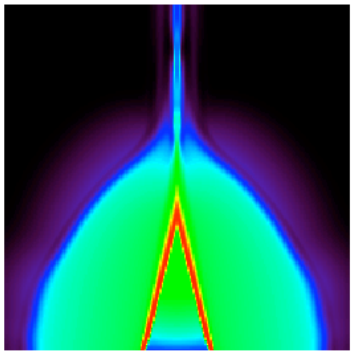
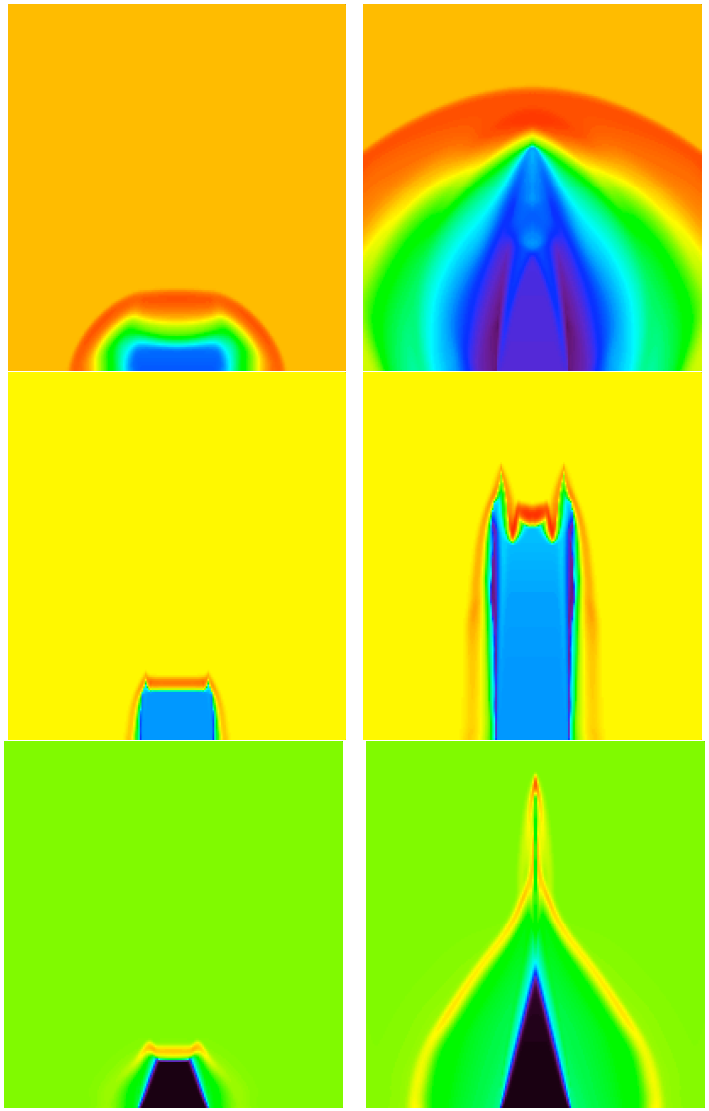
(c) t=1 440 ans



(d) t=1 920 ans



(e) t=2 400 ans



(f) t=2 880 ans