# 2D Multi-Angle Multi-Group Radiation-Hydrodynamic Simulations of Postbounce Supernova Cores

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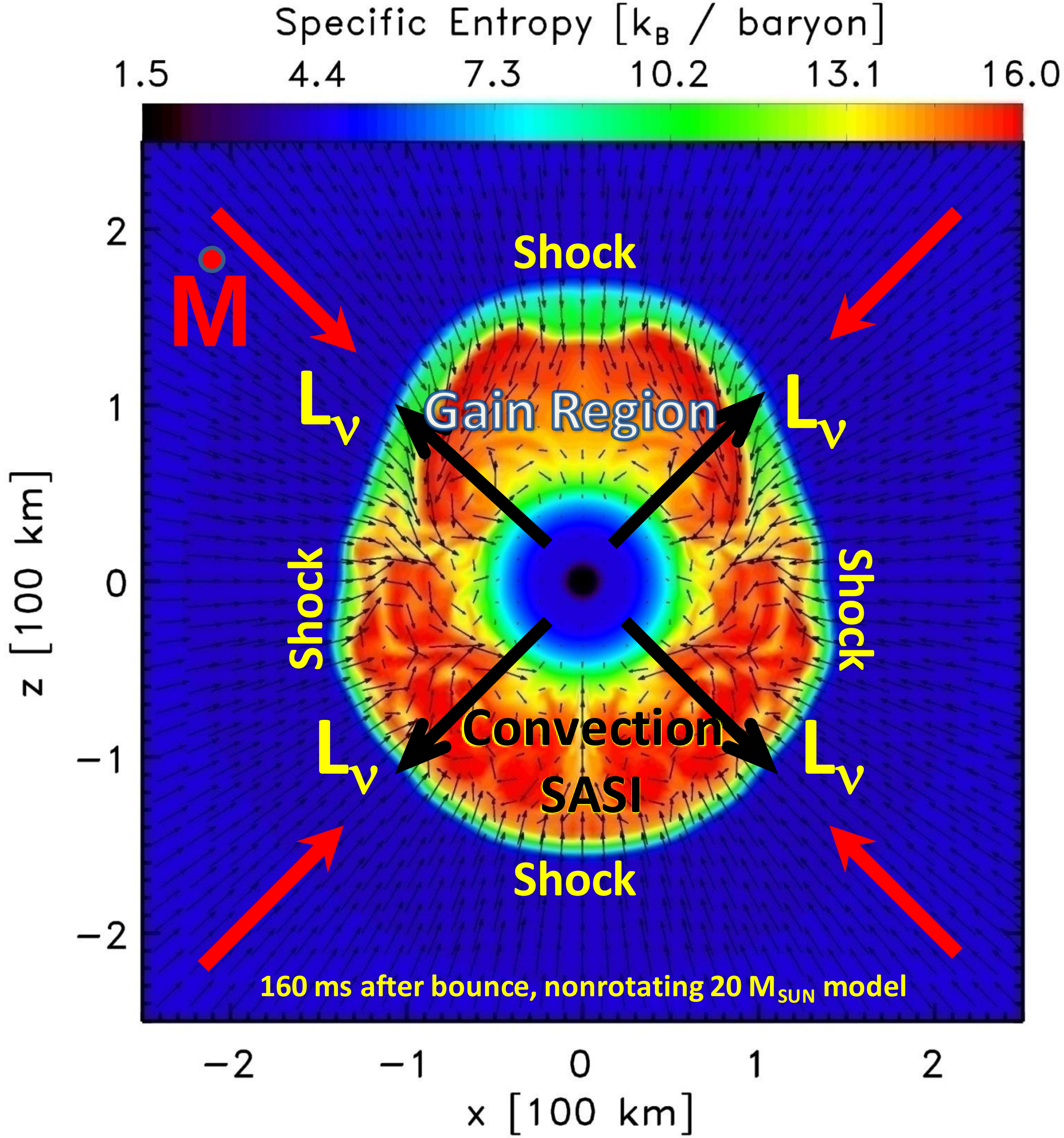
# Postbounce Situation




- Neutrino cooling of PNS.
- Neutrino heating in gain region.
- Convection and SASI.

Neutrino Mechanism fails in 1D.

Can we make it work in 2D (3D)?

No neutrino-driven explosions in MGFLD VULCAN/2D -- Sensitivity to transport method?

# Core-Collapse SNe and Neutrino Transport

Hiding ugly details:

$$\frac{1}{c}\frac{\partial I(\vec{r},\vec{n},\epsilon_\nu)}{\partial t} + \vec{n}\,\vec{\nabla} I(\vec{r},\vec{n},\epsilon_\nu) = \Xi[I(\vec{r},\vec{n},\epsilon_\nu),\rho,T,Y_e]$$

$$J = \frac{1}{4\pi}\oint I d\Omega \qquad \vec{H} = \frac{1}{4\pi}\oint \vec{n} I d\Omega \qquad \mathbf{K} = \frac{1}{4\pi}\oint \vec{n}\cdot\vec{n}\, I d\Omega$$

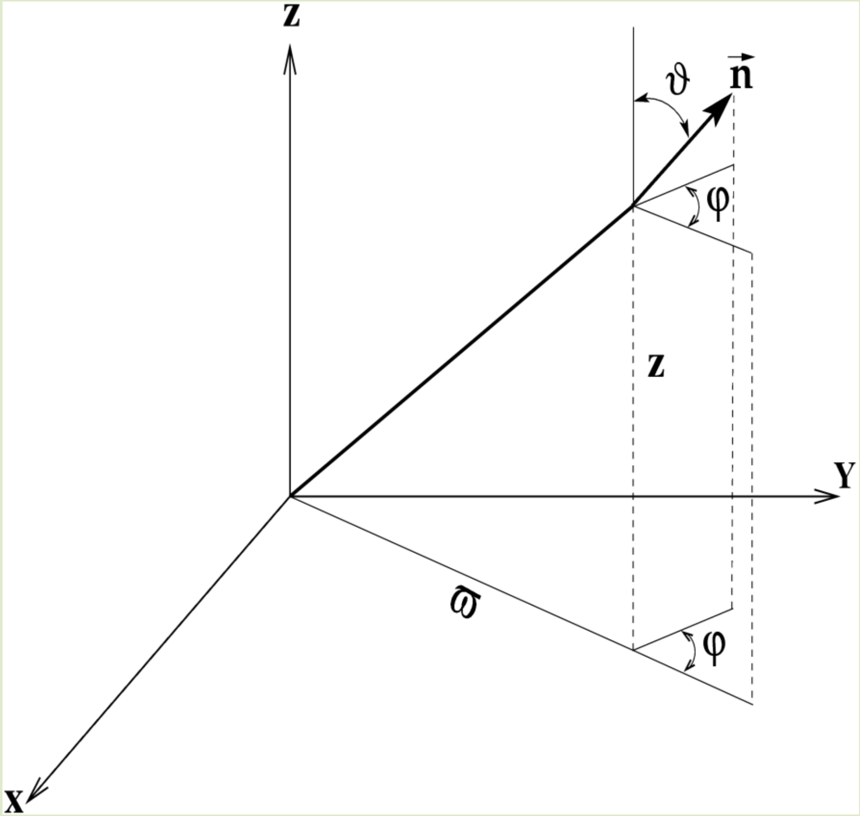
- Full Boltzmann problem: 7D -> 3D space, 3D momentum space, time.
- Frequent approximations in multi-D core-collapse SN simulations:
  - **Ray-by-Ray**: 1D space, 2D momentum space [e.g., Buras et al. 06, Bruenn et al. 06, Marek & Janka 07]
  - **2D multi-group flux-limit diffusion** (MGFLD): [e.g., Swesty & Myra 2006, Burrows 2007a] evolve mean-intensity J; 2D space, 1D momentum space (energy).
  - Additional various common simplifications to (1) collision/source/sink term and (2) fluid-velocity dependence.

- Here: **approach solution of 6D problem** (2D space, 3D momentum space, time)
  - Multi-Group multi-angle discrete-ordinate ($S_n$) solver in the radiation-MHD code **VULCAN/2D**. [Livne et al. 2004]
  - Comparison with MGFLD within VULCAN/2D; MGFLD “good enough”?

# Our Work: Setup and Implementation

[Ott et al. 2008, arxiv:0804.239, ApJ accepted]

- **VULCAN/2D**:
  - Unsplit 2D ALE (magneto-)hydrodynamics.
  - MGFLD & **discrete-ordinate ($S_n$) Boltzmann solver.** [Livne et al. 2004] But: No energy redistribution/ inelastic scattering, no velocity dependence.
  - MGFLD Flux limiter: 2D variant of Bruenn 1995.
  - $S_n$ calculations with 8, 12, 16 $\vartheta$-angles in momentum space. In 2D: also radiation-momentum $\varphi$-angles $[0,\pi]$ -> 40, 92, or 162 angular points at each spatial location, tiling the hemisphere uniformly in solid angle.
  - 16 energy groups, 3 neutrino "species".

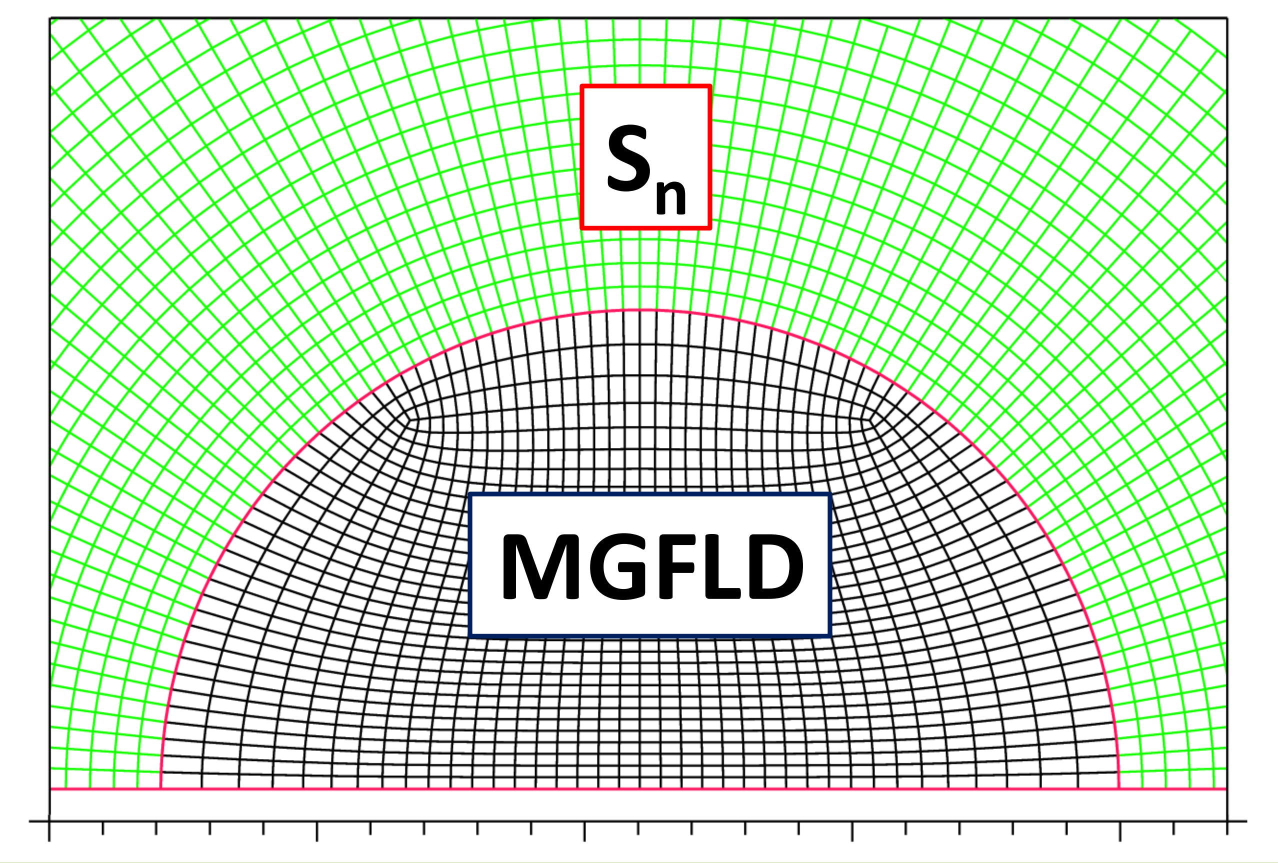


# Modifications to the Solver

[Ott et al. 2008, arxiv:0804.239, ApJ accepted]

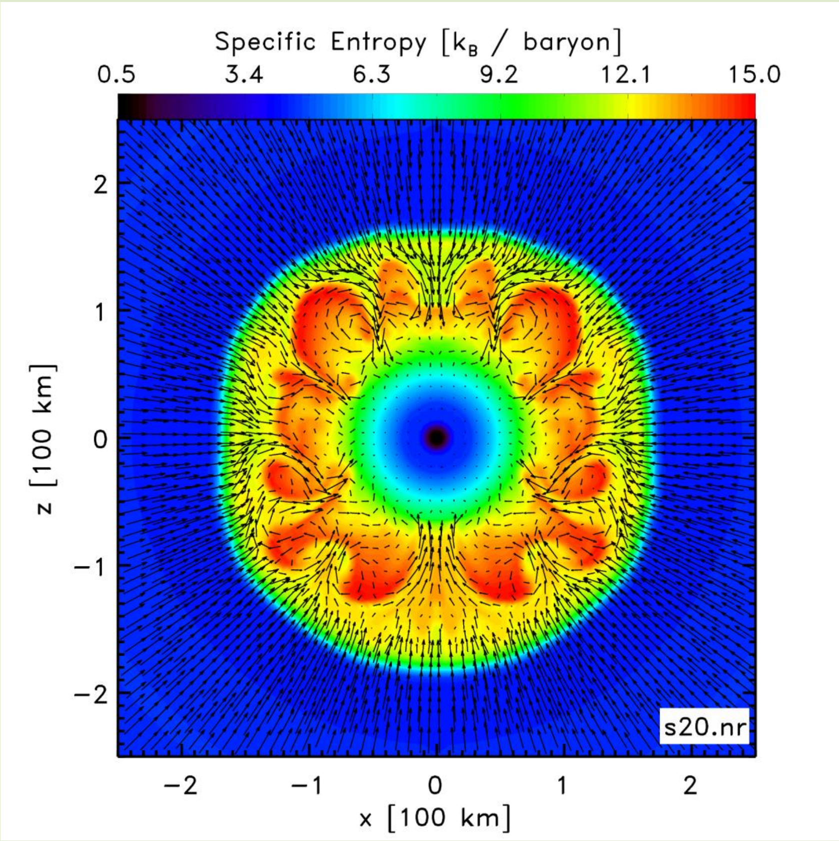
- $S_n$ – MGFLD hybrid scheme:
  - $S_n$ solver in VULCAN/2D converges slowly at high optical depth. -> time step limitation.
  - Idea: Use MGFLD at high optical depths and transition to Sn at intermediate optical depth.
  - Set up boundary data according to Eddington approximation:

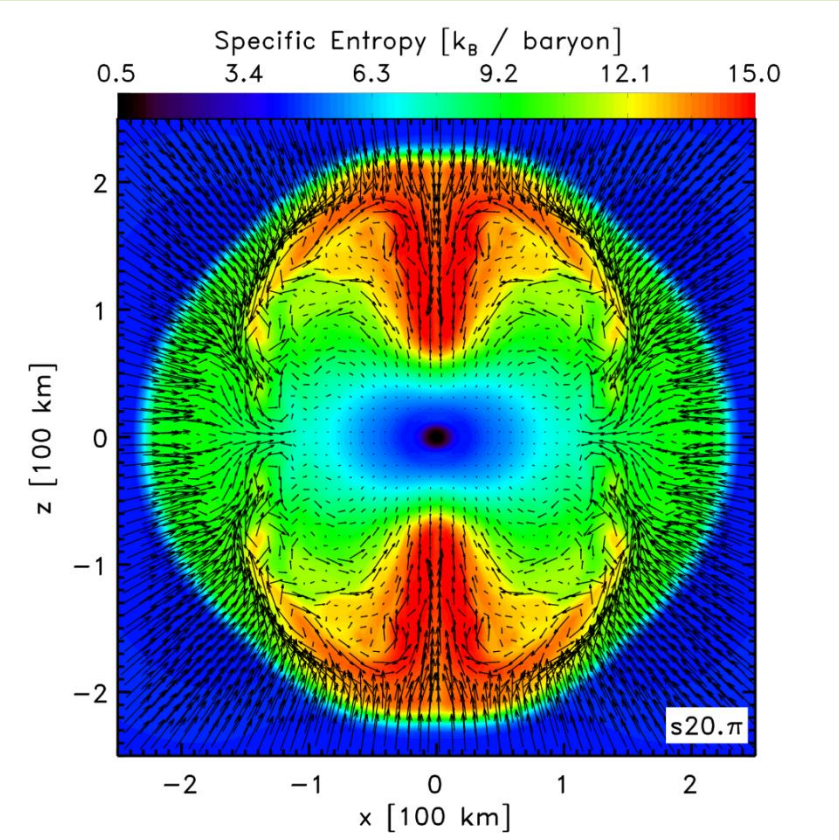
$$I(\vec{n}) = J_{\rm MGFLD} + 3(\vec{n} \cdot \vec{H}_{\rm MGFLD})$$




- Matching at $\tau \geq 2$, R = 20 km.
- Efficient at high optical depths and accurate in semi-trans-parent regions.

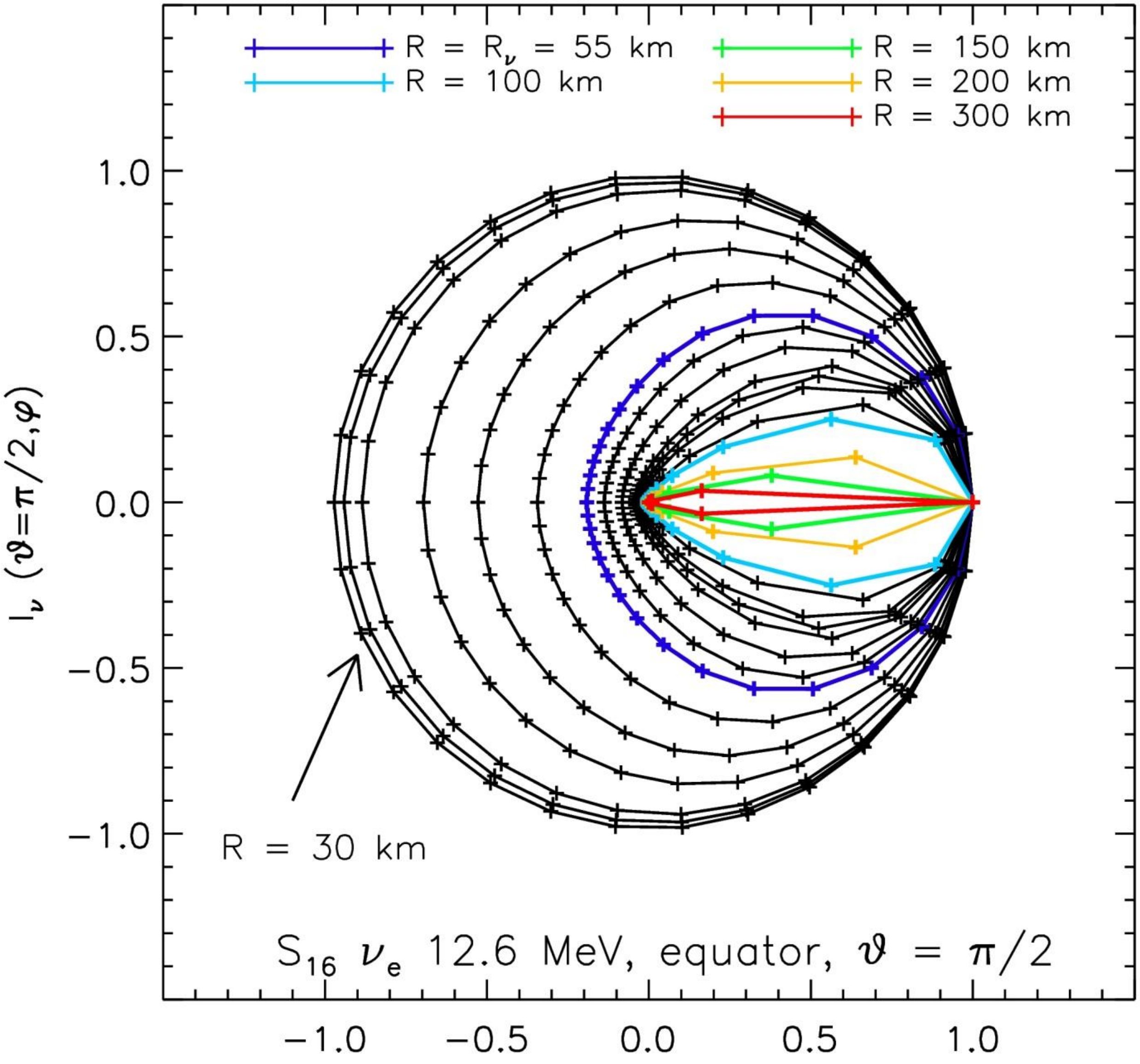
# Postbounce SN Models:

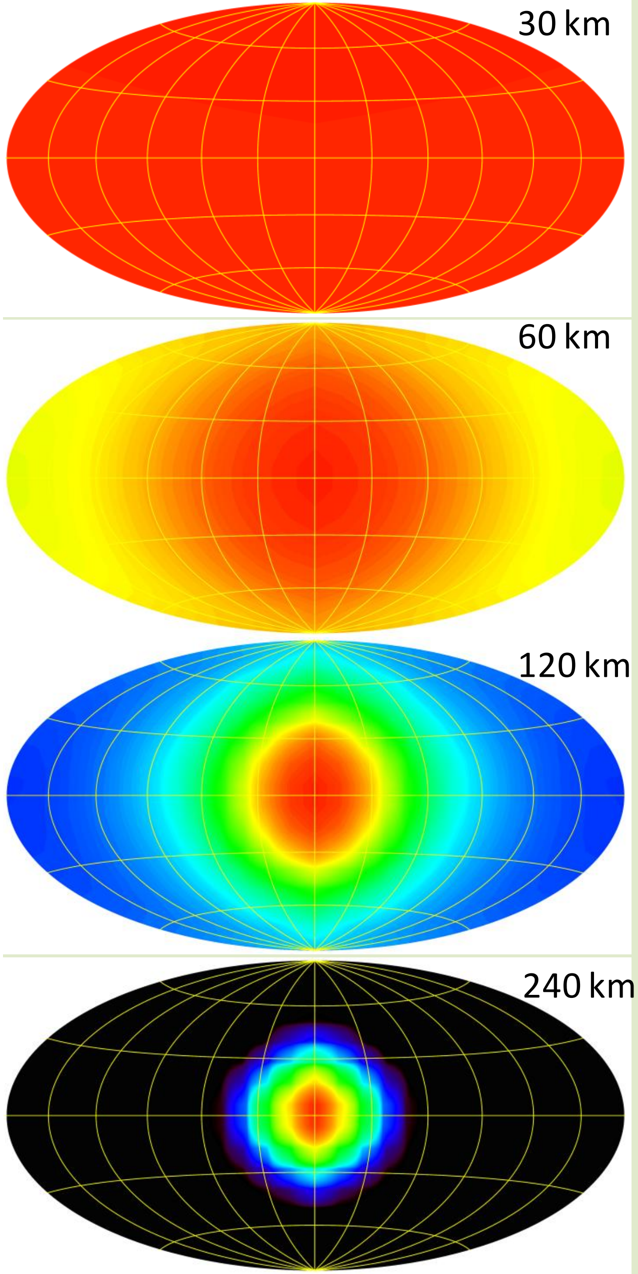



- 20-solar mass pre-SN model of Woosley, Heger & Weaver 2002.
- Nonrotating (s20.nr) and rotating model (s20.π, precollapse central $P_0$ = 2 s, $\Omega_0$ = π rad/s).
- Evolved to 160 ms postbounce with MGFLD, then stationary-state $S_n$ solution.
- Steady-State solutions with $S_8$,$S_{12}$,$S_{16}$ -> 40,92,162 total angular zones.
- Long-term (~400 ms) time-dependent calculations with $S_8$.

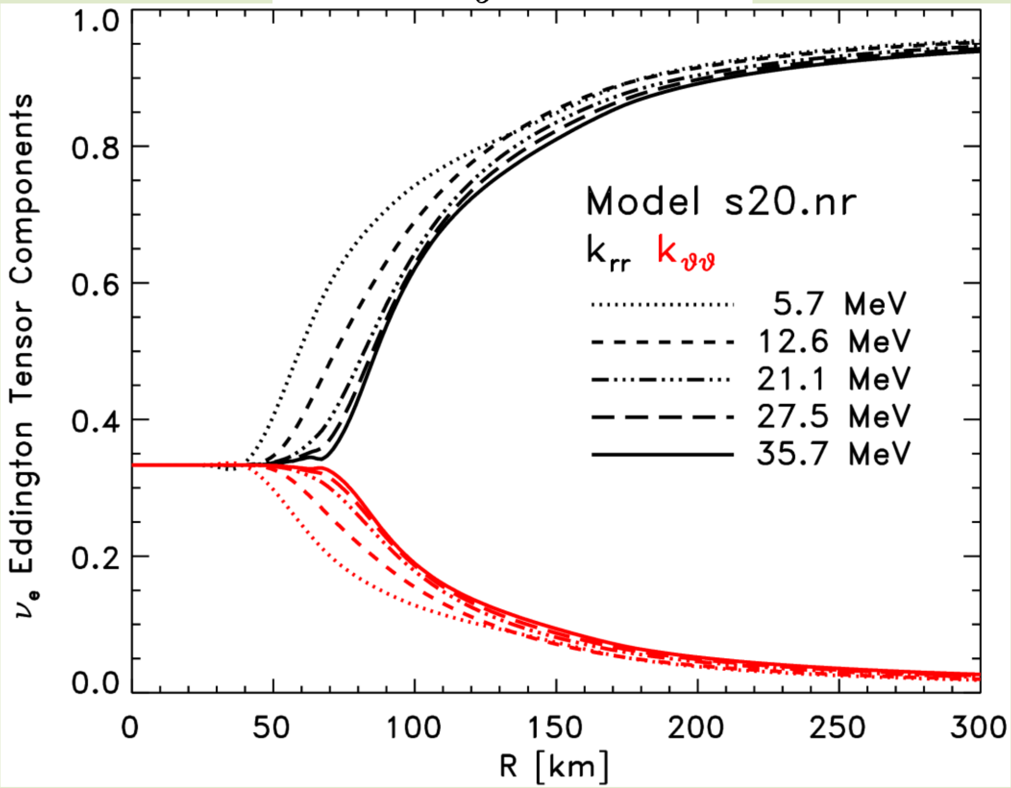
# The Radiation Field



# Eddington Tensor Components

$$\mathbf{K} = \frac{1}{4\pi}\oint \vec{n}\cdot\vec{n}\, I\, d\Omega$$




- In axisymmetry and without velocity dependence:
  4 independent components (3 diagonal, 1 off-diagonal).
  (note: 1D/Ray-by-Ray -> only one “Eddington factor”)
- Here: spherical coordinates; off-diagonal term $K_{r\vartheta}$ small (<1%).
- $S_n$ “striping” considerable outside R ~200 km.

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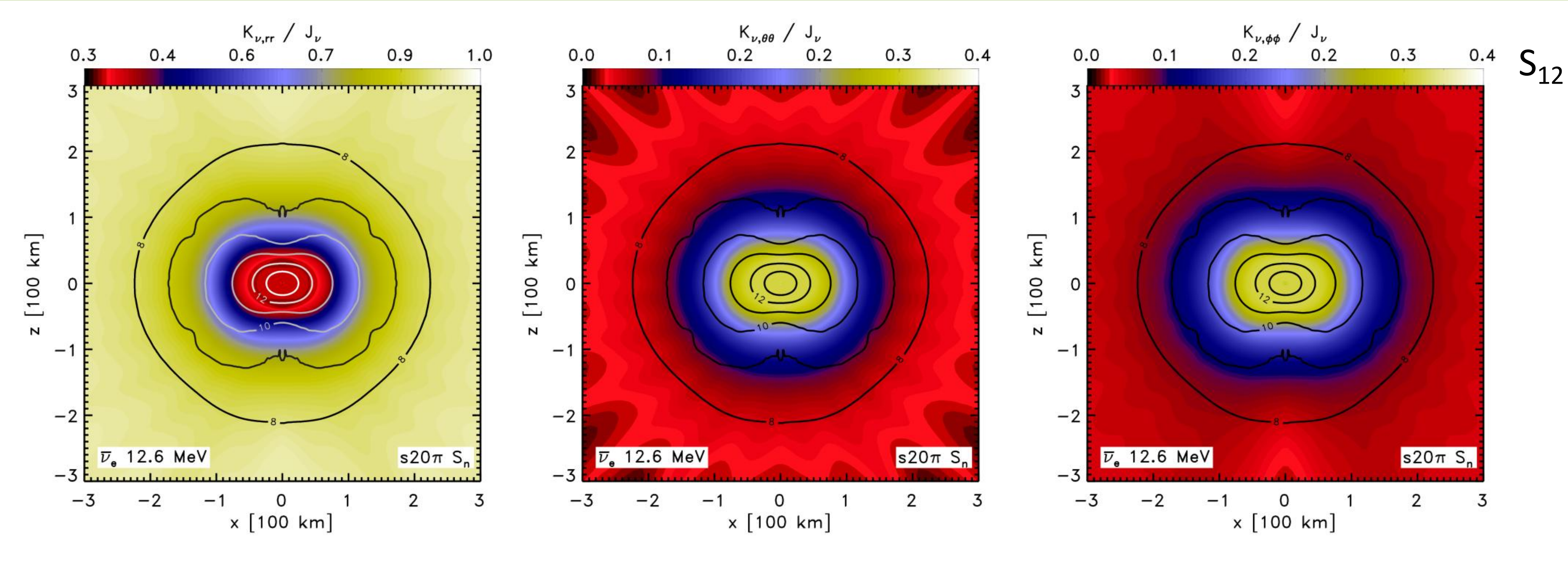


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# Comparing with MGFLD.

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- MGFLD reminder:
  - Operates on mean intensity J.
  - Good approximation in the diffusion limit.
  - Can handle streaming limit with flux limiter.
  - Must "interpolate" between diffusion / streaming using the flux limiter.
- Neutrino heating:

$$\dot{\epsilon} = \frac{X_n}{\lambda_0^a} \frac{L_{\nu_e}}{4\pi r^2} \langle E_{\nu_e}^2 \rangle \left\langle \frac{1}{\mathsf{F}} \right\rangle + \frac{X_p}{\bar{\lambda}_0^a} \frac{L_{\bar{\nu}_e}}{4\pi r^2} \langle E_{\bar{\nu}_e}^2 \rangle \left\langle \frac{1}{\bar{\mathsf{F}}} \right\rangle$$

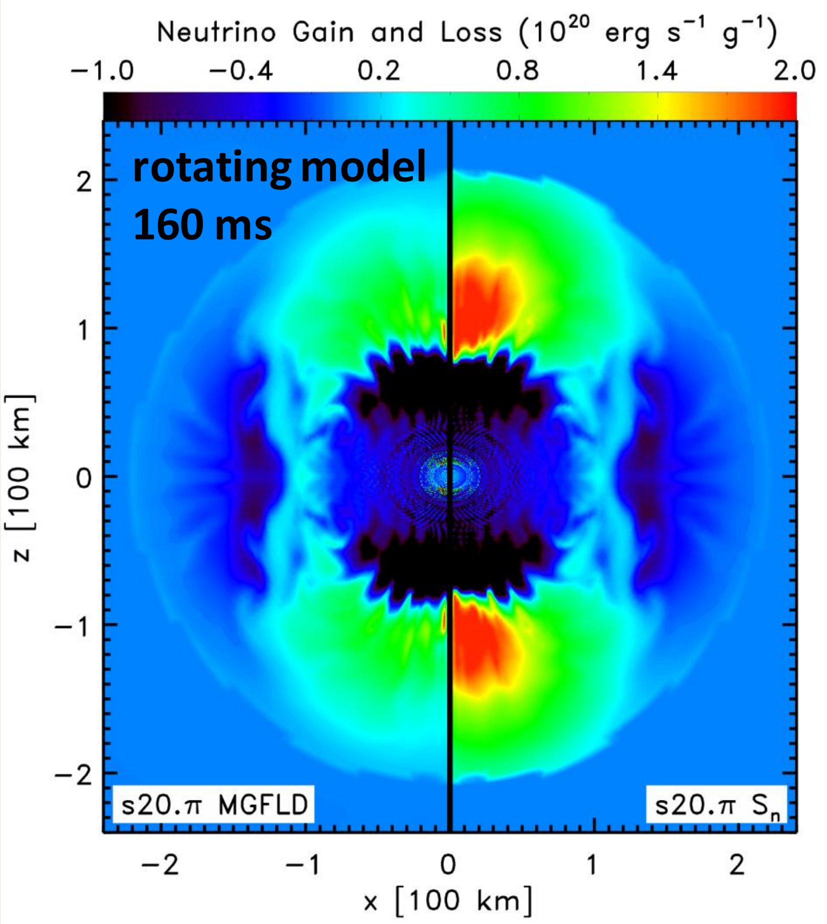
[Messer et al. 1998]

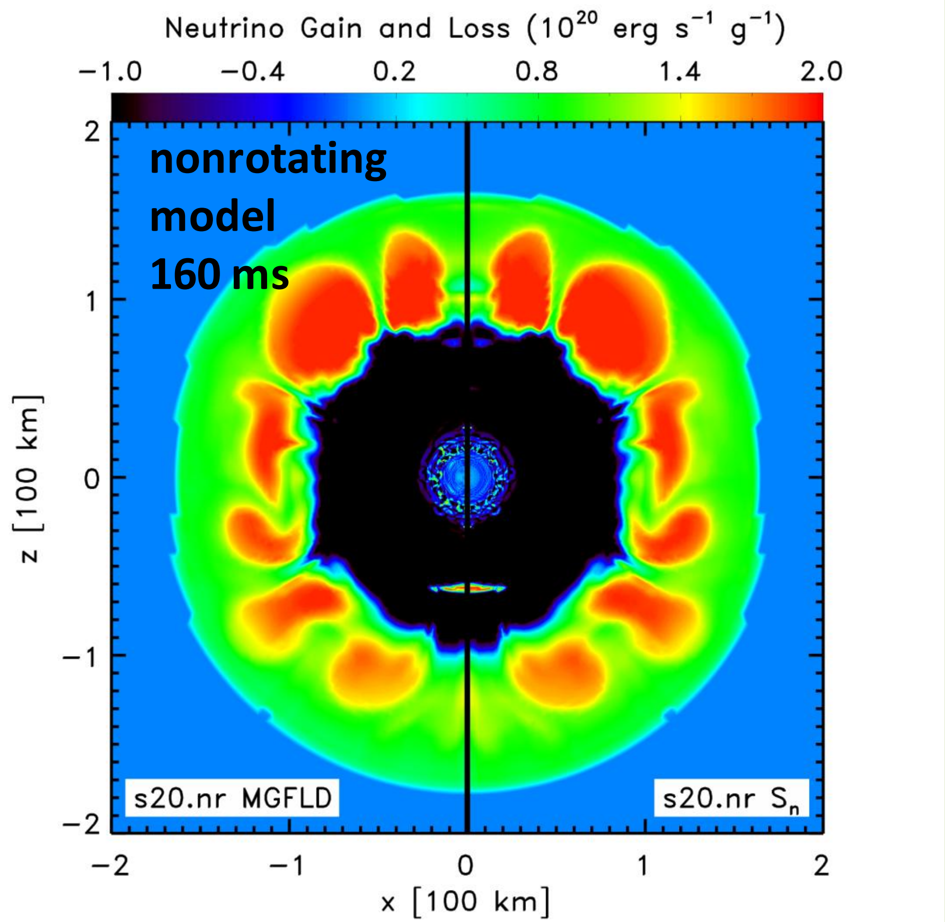
- Relevant quantities:
  Luminosity, mean inverse flux factor, mean rms neutrino energies.
- Mean inverse flux factor:

$$\left\langle \frac{1}{\bar{\mathsf{F}}_{\nu_i}} \right\rangle \frac{\int dE_\nu \, J(E_\nu, \nu_i)}{\int dE_\nu \, H_r(E_\nu, \nu_i)}$$

- Previous work all 1D: Janka et al. 1992, Yamada et al. 1999, Messer et al. 1998, Burrows et al. 2008 (all steady-state); Liebendörfer et al. 2004 (1D evolution).

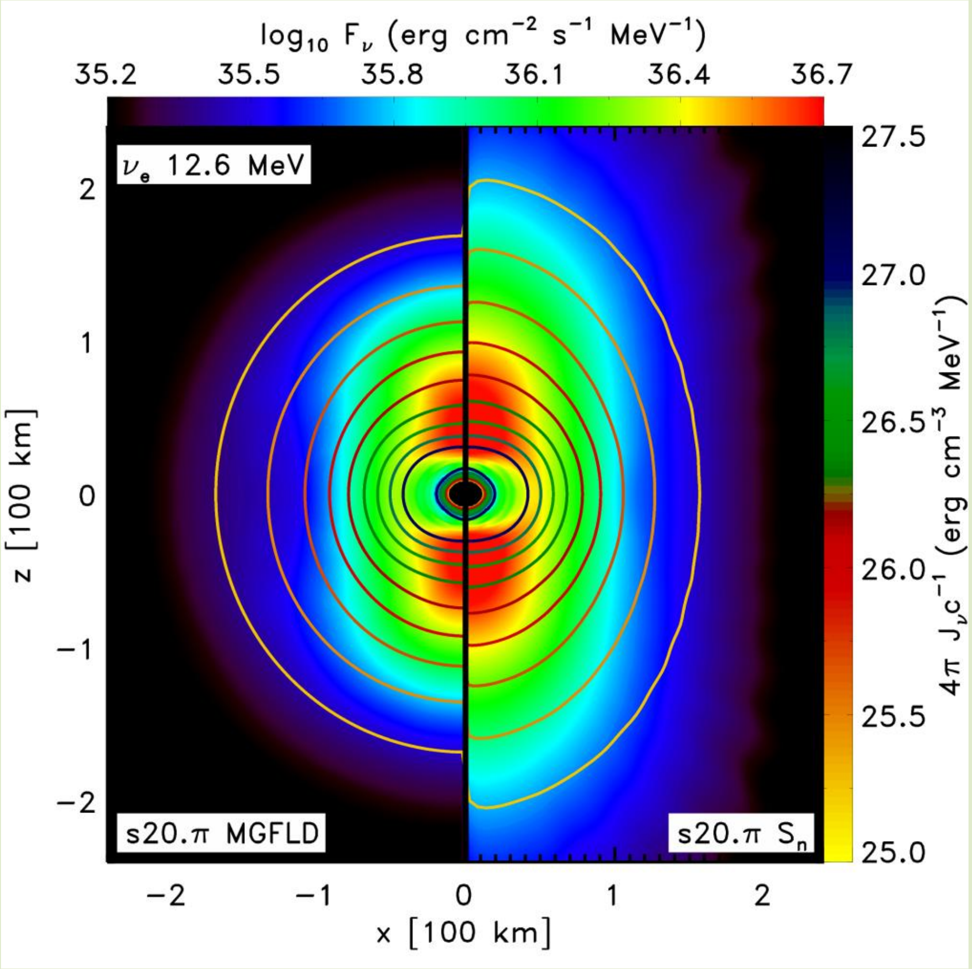
# Neutrino Energy Deposition



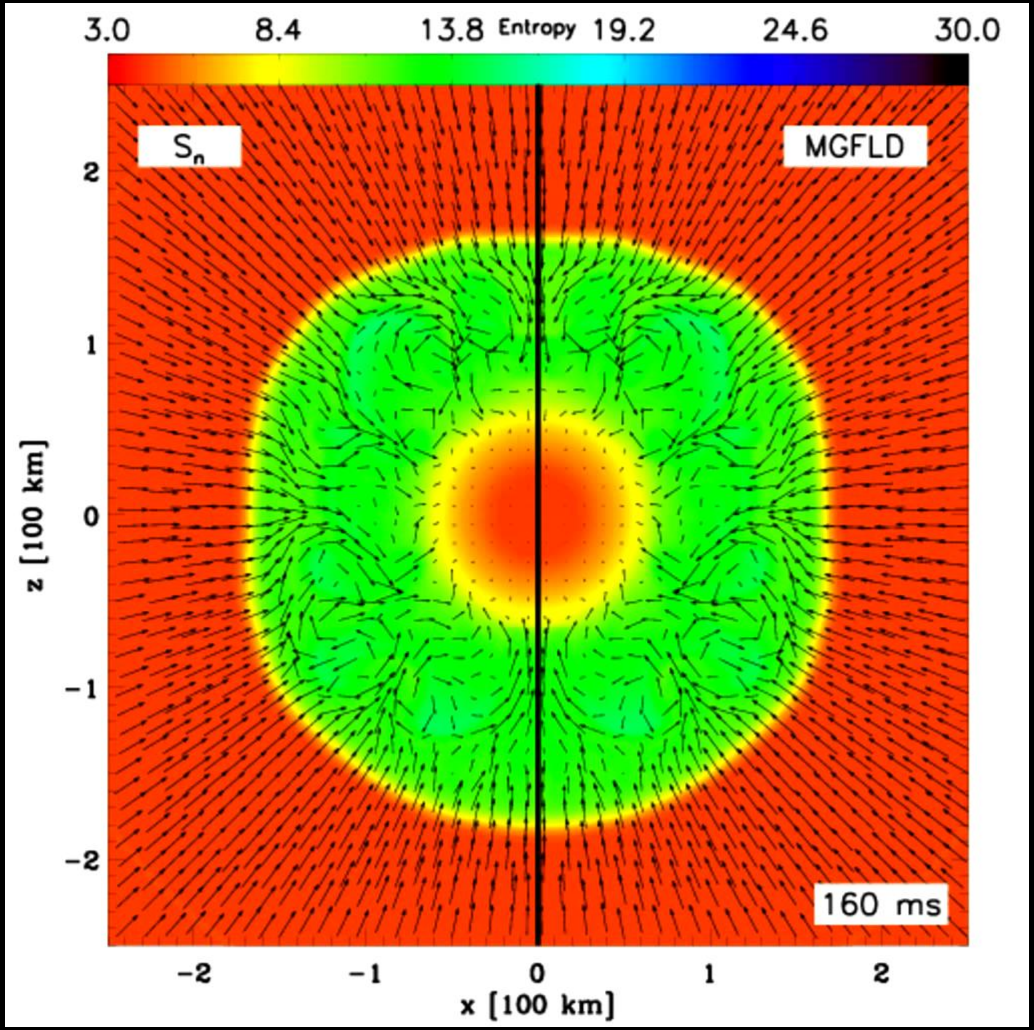

- s20.nr: Little difference between MGFLD and $S_n$ at 160 ms after bounce.
- s20.π: Large (factor ~3) polar differences in specific heating rates.

# The Rotating Model s20.π: Flux Asymmetry

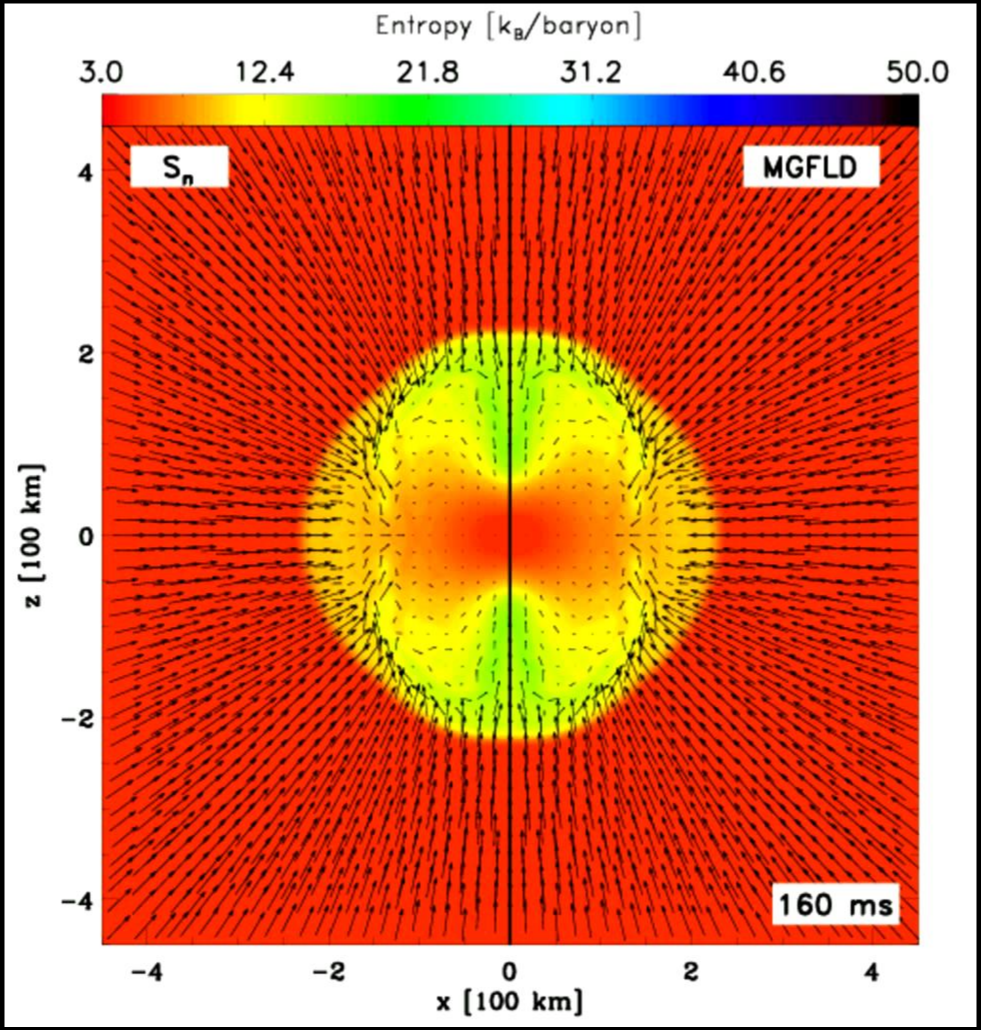



- Radiation field oblate in PNS, prolate outside. Strong flux-enhancement along poles.
- Snapshot at 160 ms: Pole/Equator flux asymmetry much better captured by $S_n$. MGFLD smoothes-out asymmetries at large radii/low optical depths.

# Evolution Calculations: Nonrotating s20.nr



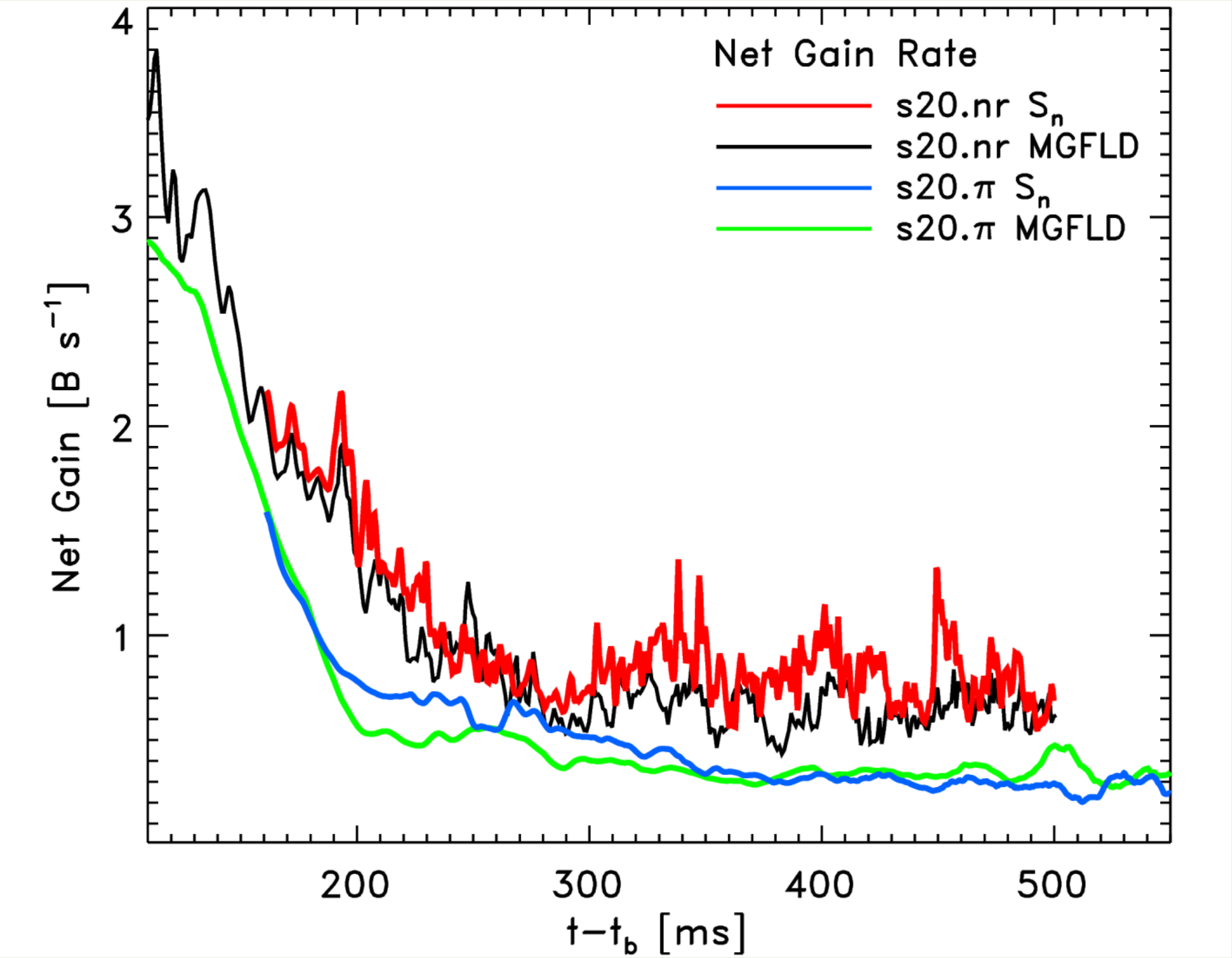
# Evolution Calculations: Rotating s20.π



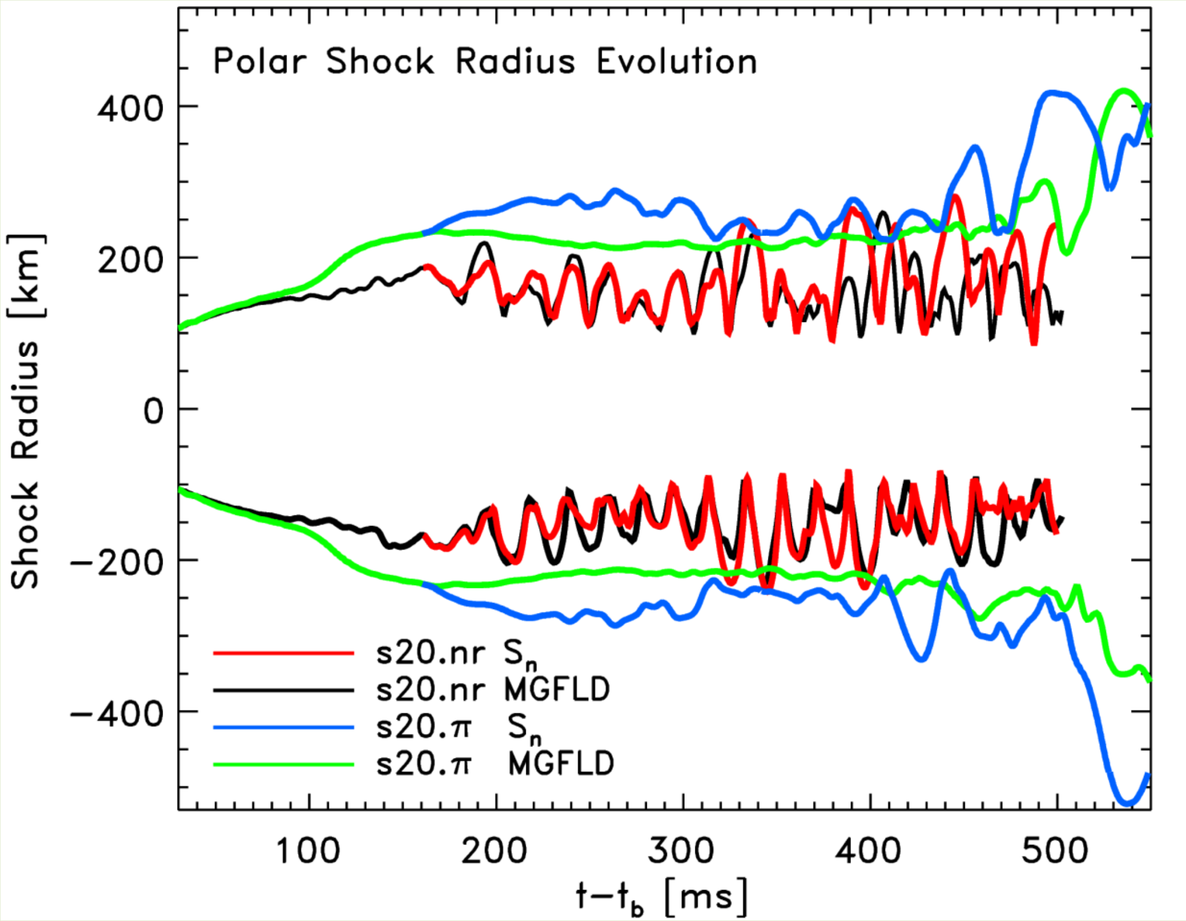
# Evolution Calculations: Energy Deposition

- s20.nr:
  160 -> 500 ms
  Up to 30% larger heating rates at late times predicted by multi-angle transport.

- s20.π:
  160 -> 550 ms
  Despite large polar enhancement no clear and consistent enhancement of total heating.



# Shock Radii




- $S_n$ leads to *somewhat* larger shock radii / greater excursions.
- Pronounced initial polar shock expansion in s20.π.
  Model appears to "settle" at new quasi-equilibrium.
- No sign of explosion.
- s20.π develops SASI at late times, faster/stronger in $S_n$ variant.

# Summary: 2D MGFLD vs. 2D $S_n$

- $S_n$ superior in capturing global and local radiation-field asymmetries associated with aspherical hydrodynamics.
- Increased (local) neutrino heating, in particular along the poles in the rotating model (-> earlier/stronger SASI); larger SASI shock excursions in nonrotating model.
- Strong feedback in the SN problem;
  **$S_n$ and MGFLD both do not produce neutrino-driven explosions in our VULCAN/2D simulations**.
- What else is needed?
  3D? GR? Microphysics/EOS? O(v/c) transport?