2008 07/08
Asymmetric Instabilities in stellar core collapse
Institute Henri Poincare, Paris

Magnetorotational collapses and jet formations

arXiv:0712.1949 and beyond

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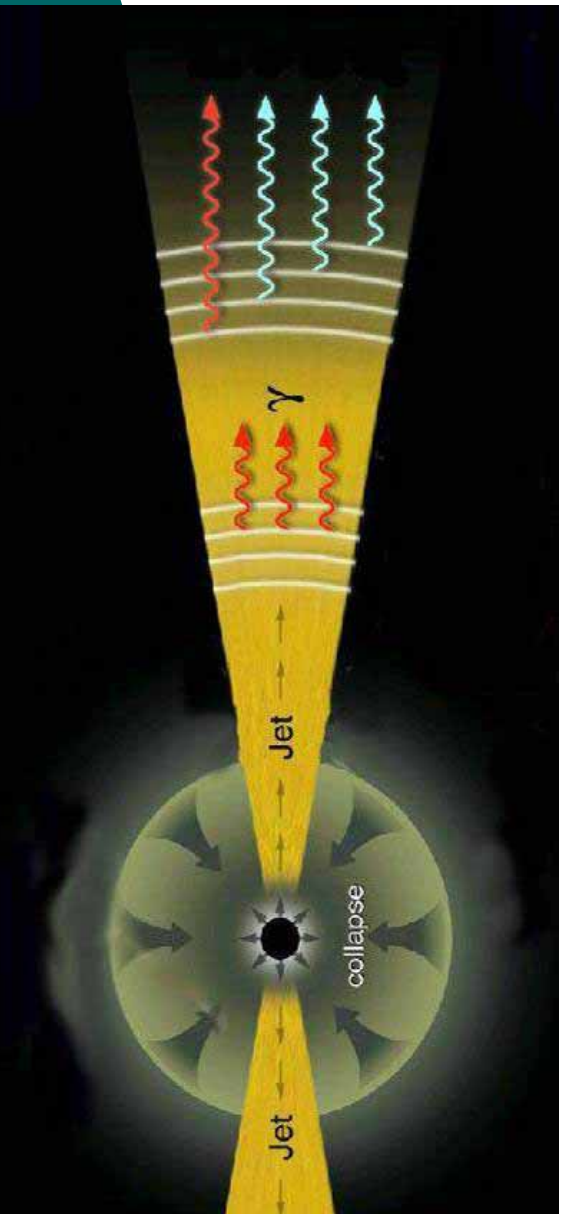
GRB Jet and Magnetorotational Collapse



•GRBs are sudden burst event of ~200keV gamma-rays emitted from the relativistic jets.
•GRBs-SNe association
-> Central engine: stellar collapse
•Collapsar Model:
Rotation Energy is converted to kinetic energy of the jet.

Those mechanism are studied in the context of magnetorotational collapses (Symbalisty'84 etc…).

If the rotation and magnetic field is sufficiently strong, jet-like outflow can be launched.

Motivation of the Study

Wide variety of phenomena are related to the core-collapses.

- Supernovae
- Gamma Ray Bursts (GRB)
- Black Hole formations
- Neutron star formations
- Magnetar formations

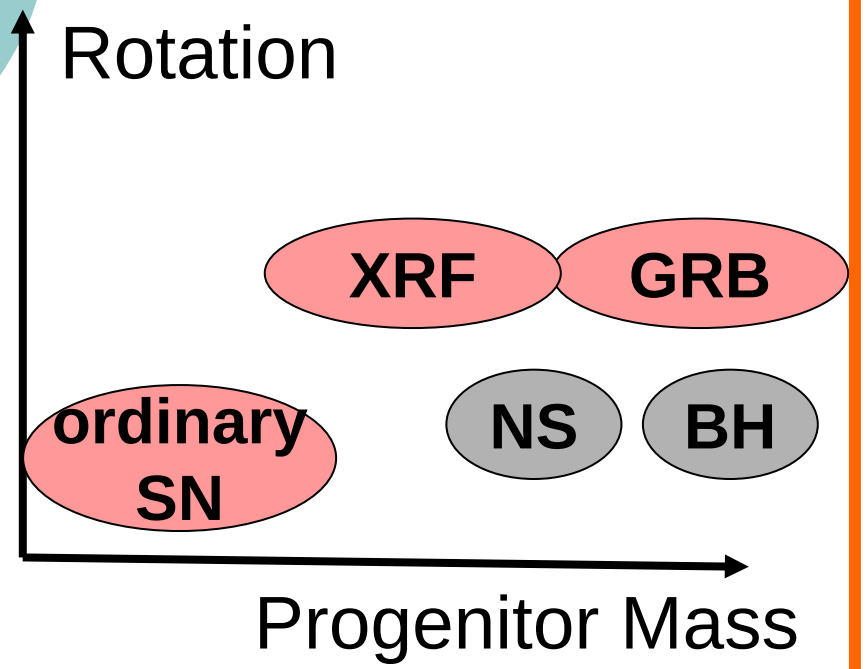
The difference might be originate from the properties of progenitors.

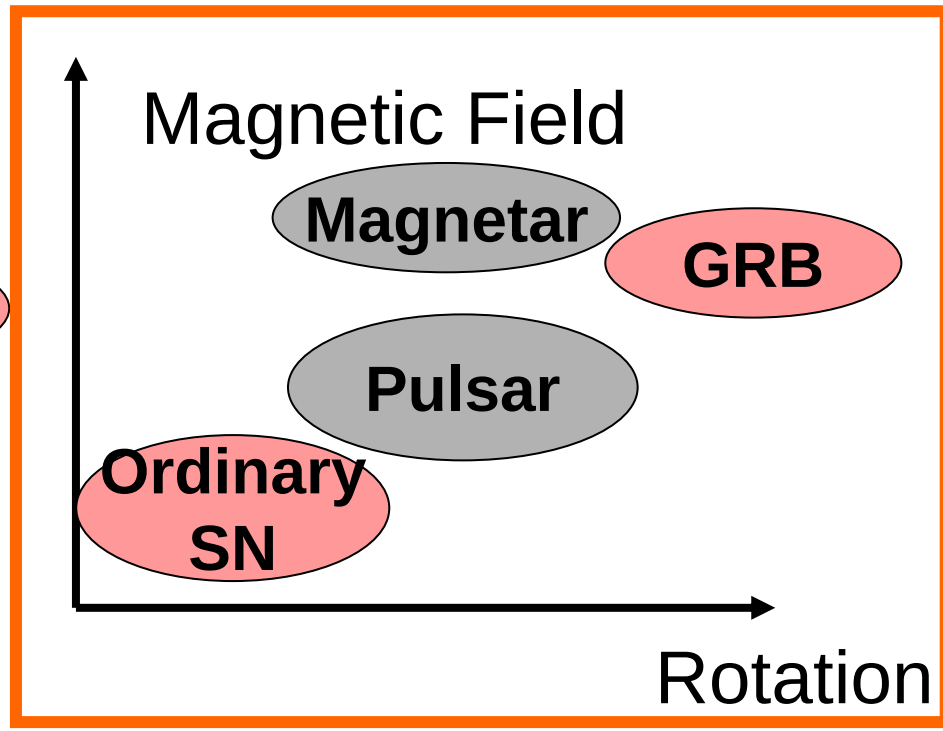
- Mass
- Rotation
- B-field
- Chemical abundance …

We investigate dependence of the property of the progenitor on the magnetic explosion and jet formations.

Some Expectations

Expectations for the relation between the property of progenitor star and phenomena is shown.

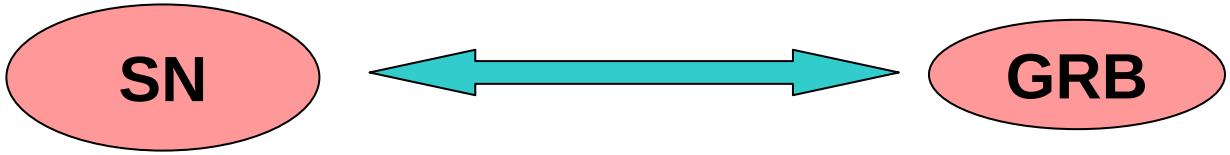




In this talk we mainly focus on magnetic field and rotation.

Intermediate State between GRB and ordinary SNe

GRBs and supernovae might be continuously aligned in the diagram.



-GRB030329, 980425 <-> SN2003dh, SN1998bw
-XRF060218 <-> SN2006aj
-X-ray transient 080109 <-> SN 2008d
-no counter part <-> Ordinary Supernovae

XRF and X-ray transient are low energy and low lorentz factor analogue of GRBs.

Intensive parametric studies are necessary.

Numerical Setups

Basic Equations

$$\frac{\partial D}{\partial t} + \frac{1}{\sqrt{\gamma}}\partial_i \sqrt{\gamma} D v^i = 0$$

$$\frac{\partial B^i}{\partial t} + \partial_j W v^j b^i - W v^i b^j = 0$$

$$\frac{\partial E}{\partial t} + \frac{1}{\sqrt{\gamma}}\partial_i \sqrt{\gamma} E v^i = -p\frac{\partial W}{\partial t} - \frac{p}{\sqrt{\gamma}}\partial_i \sqrt{\gamma} W v^i - L_\nu$$

$$\partial^k \partial_k \Phi = DhW - \left(p + \frac{|b|^2}{2}\right) - b^{02}$$

$$\frac{\partial S_i - b^t b_i}{\partial t} + \frac{1}{\sqrt{\gamma}}\partial_j \sqrt{\gamma}\left(S_i v^j - b_i b^j\right) = -\frac{1}{2}\left(\rho h (W v_k)^2 - (b_k)^2\right)\partial_i \gamma^{kk}$$

$$- \partial_i p + \frac{\|b\|^2}{2} - (DhW - b^{02})\partial_i \Phi$$

$$D = \rho W, E = eW, h = (1 + e/\rho + p/\rho) + |b|^2/\rho \qquad \mathbf{S} = \rho h W^2 \mathbf{v}$$

enthalpy

ρ : density

$\mathbf{v}$: velocity

e :internal energy

p : pressure (Shen et al. 1998)

Φ : gravitational potential

L_ν :cooling rate by neutrinos

W : Lorentz factor

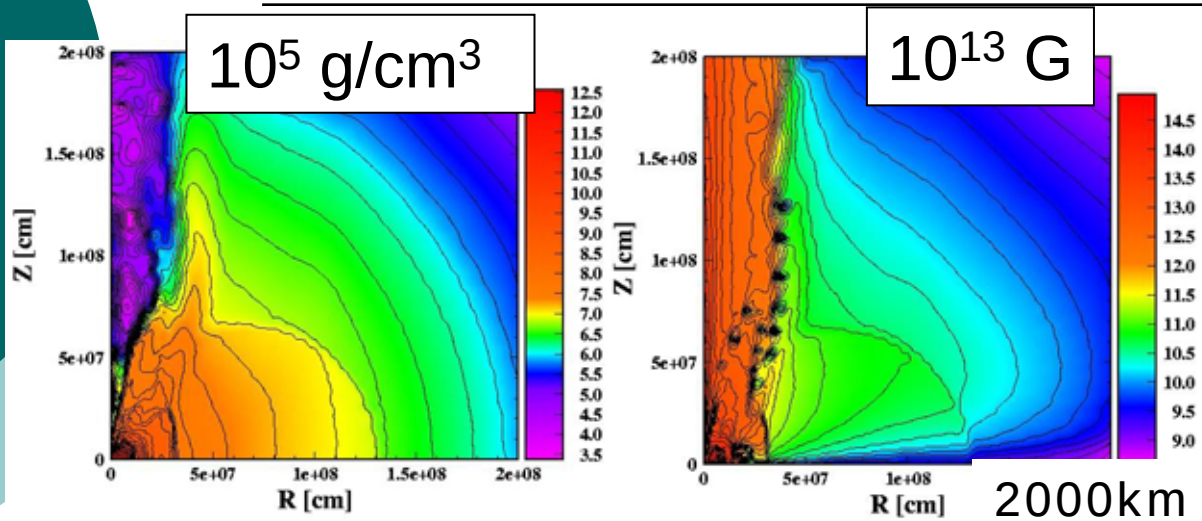
$\mathbf{B}$ Magnetic Field

γ_{ij} : spatial metric (spherical coordinate)

$\mathbf{b}$:magnetic field at fluid system

e.g. De Villiers & Hawley 2003

Requirement for the simulation



Propagation of Magneto-driven jet

(Takiwaki et al. 2004)

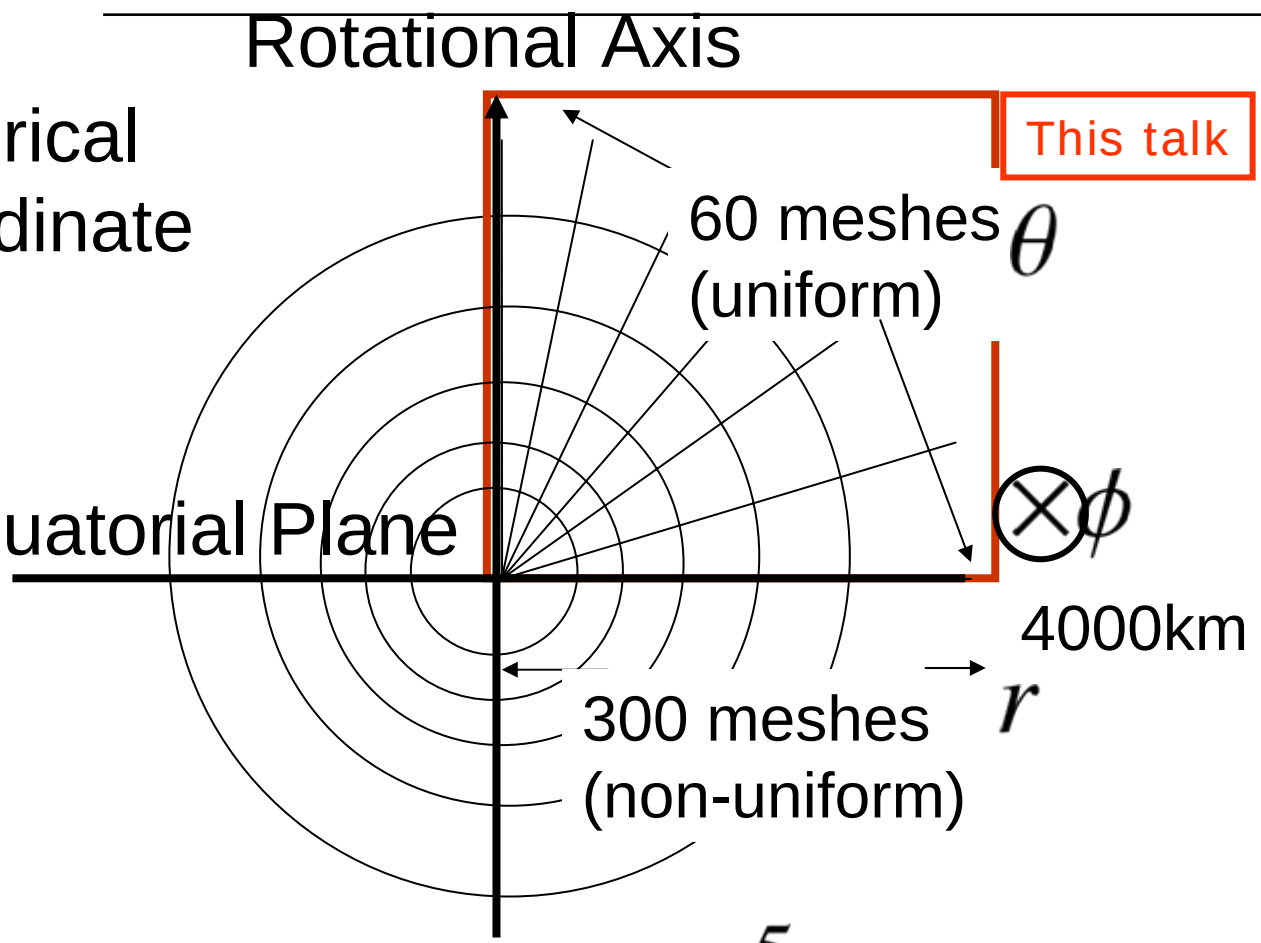
The speed of Alfven wave exceed the speed of light.

$$v_{\mathrm{Alfven}} \equiv \frac{B}{\sqrt{\rho}} \approx 3\times 10^{10}\mathrm{cm/s}\frac{B/10^{13}\mathrm{G}}{\sqrt{\rho/\left(10^5\mathrm{g/cm^3}\right)}}$$

$$\frac{B}{\sqrt{\rho}} \rightarrow \frac{B}{\sqrt{\rho + B^2/c^2}}$$

Under special relativity, the speed becomes below the speed of light.

Coordinate & Grid



Finest mesh is $\Delta r = 10^5 \mathrm{cm}$ (1km)

This size is not enough to resolve MRI.
(e.g. M. Obergaulinger's talk)

Hydrodynamical Setups

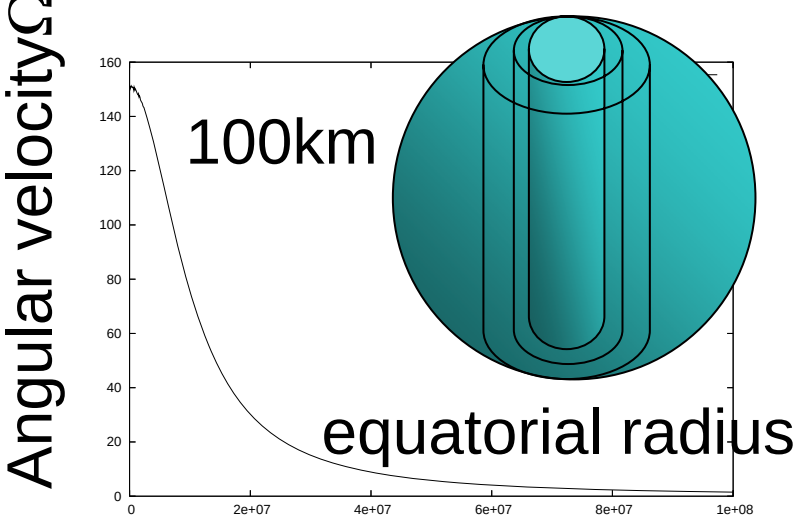
Density, internal energy and electron fraction

25M– @ZAMS (Heger et al. 2000)

Differential, Cylindrical rotation

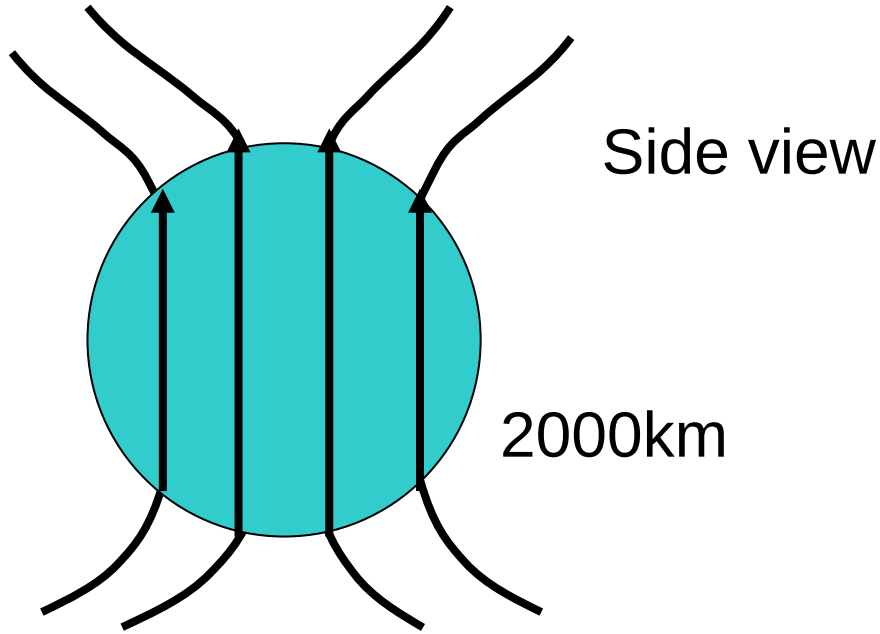
$$\Omega = \Omega_0 \frac{X_0^2}{X^2 + X_0^2} \frac{Z_0^4}{Z^4 + Z_0^4}$$

$$X_0 = 100\text{km}, Z_0 = 1000\text{km}$$



Uniform magnetic field

$$B_z = B_p, B_\phi = 0$$



Angular Velocity is 38-151rad/s B field10^{10}-10^{12}G

Features of Initial Setups

Rotation

Our setups

T/|W|= 0.25e-2 - 1e-2

Ω = 38 - 151 rad/s

-> Rapidly Rotaing

(but ordinary magnetorotational mechanism)

Heger's original model

T/|W|= 1.5e-5

Ω = 0.2 rad/s

Magnetic Field

E_m/|W|=2.5e-8 – 2.5e-4

B=10e10-10e12 G

→mildly strong

Previous works

E_m/|W|= 1e-4-1e-2

Observation

High Field White Dwalf 10e9 G

-> 10e11G Fe core

Rotation Energy is dominant.

And wider parametric range for magnetic fields are adopted.

Results:

two MHD explosions -prompt and delayed

Duration for MHD explosion

We found MHD Explosions for all the models below.
The times when the shocks escape iron core are in table.

	38 rad/s 0.25%	76 rad/s 1%	151 rad/s 4%
10^{10}G 2.5e-6%	122 ms	96 ms	104 ms
10^{11}G 2.5e-4%	72 ms	27 ms	32 ms
10^{12}G 2.5e-2%	32 ms	20 ms	25 ms

There are two types of MHD explosion.
One is prompt and another is delayed.

Dependence on the Magnetic field

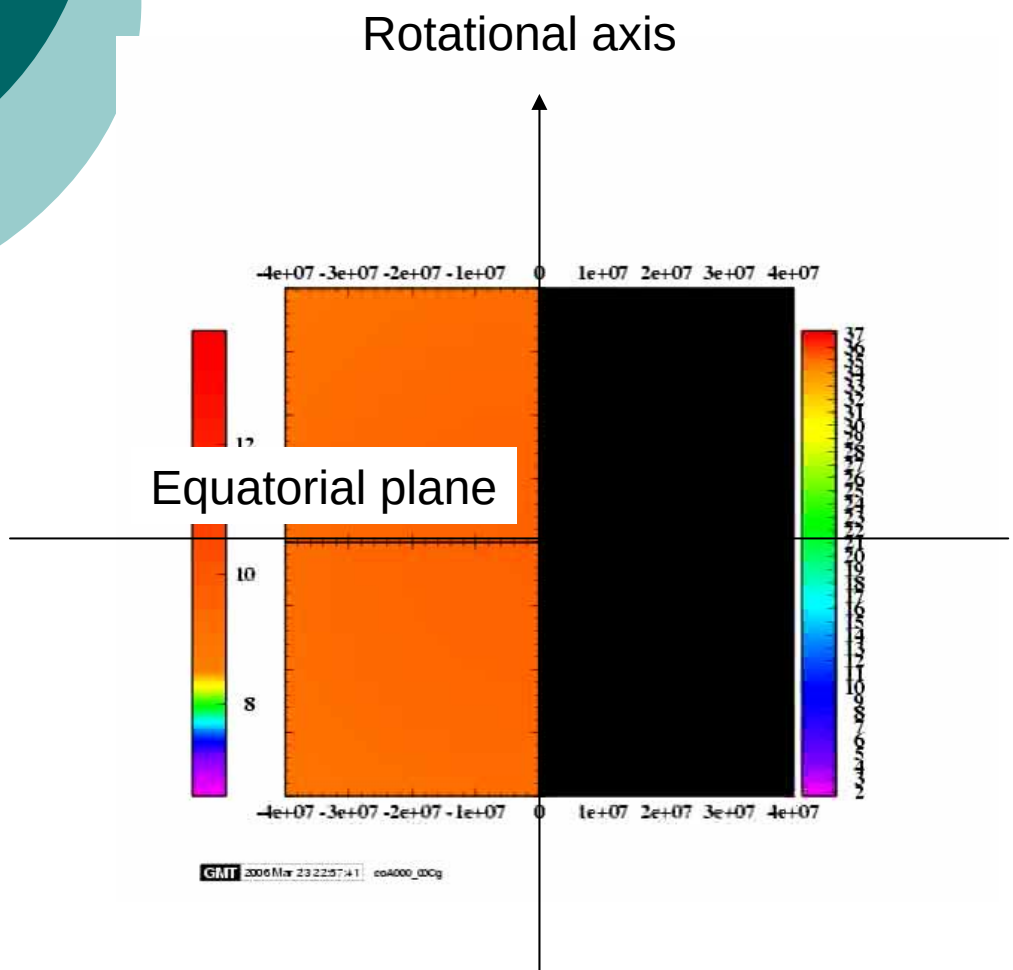
To see dependence on the magnetic field we focus on this line.

	38 rad/s 0.25%	76 rad/s 1%	151 rad/s 4%
10^{10}G 2.5e-6%	122 ms	96 ms	104 ms
10^{11}G 2.5e-4%	72 ms	27 ms	32 ms
10^{12}G 2.5e-2%	32 ms	20 ms	25 ms

Focus on this line

Prompt MHD Explosion

Left: log density (g/cc)
Right: entropy (kB)



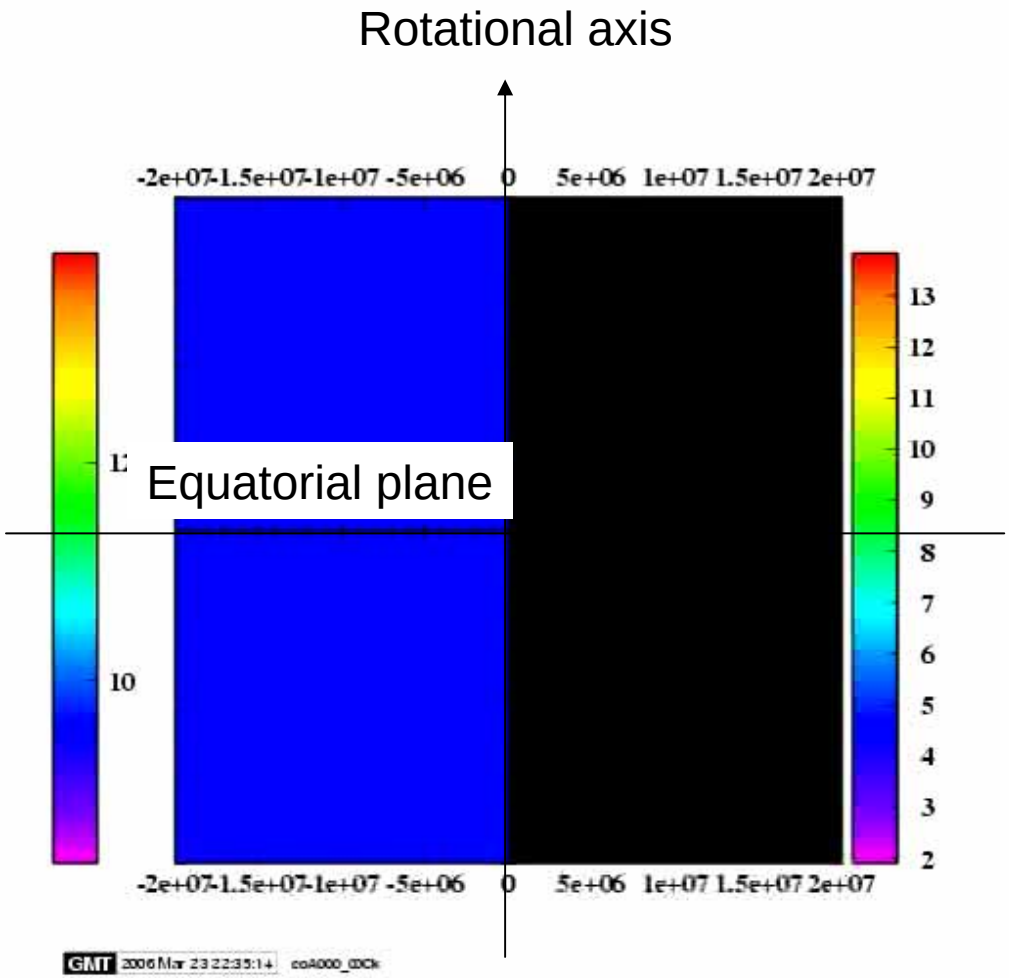
Initial strong magnetic field: B field:10^{11}G

Rotational Energy / Gravitational Energy

T/|W|=1.0%

Collapse
-> bounce
-> shock stall
-> collimated jet

Delayed MHD Explosion



Left: log density (g/cc)
Right: entropy (kB)

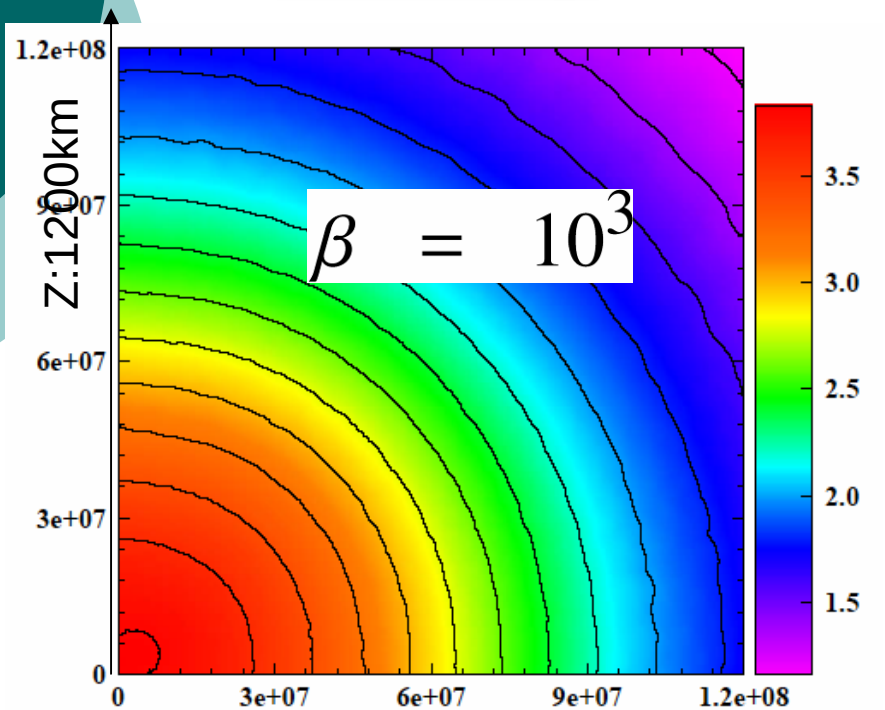
Initial weak magnetic field: B field:10^{10}G
Rotational Energy / Gravitational Energy
T/|W|=1.0%

Collapse
->bounce
->shock stall
->oscilation
->collimated jet

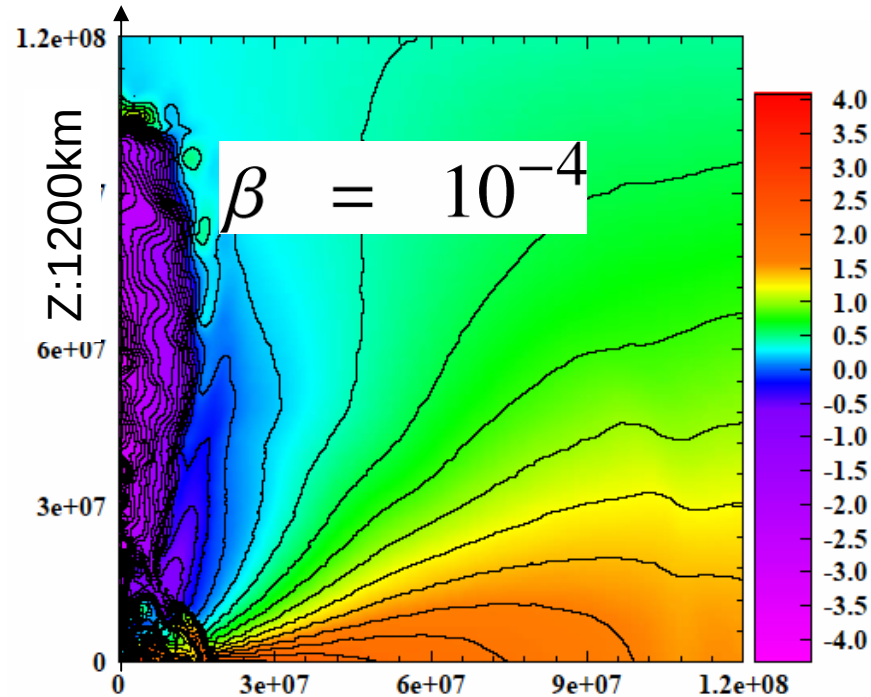
Mechanics of the MHD Explosions

Magneto-Driven Jet

Plasma $\beta = \frac{P}{P_B}$ In initial and final phases.



Initial State (B_ini=10^{12}G)

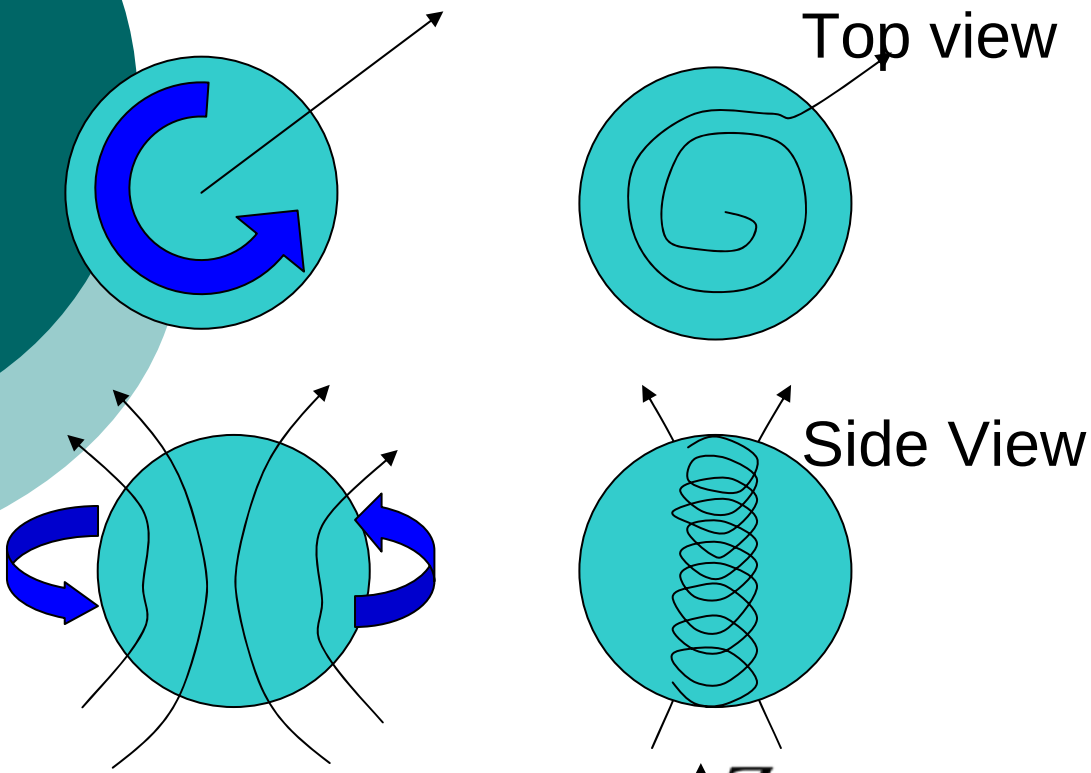


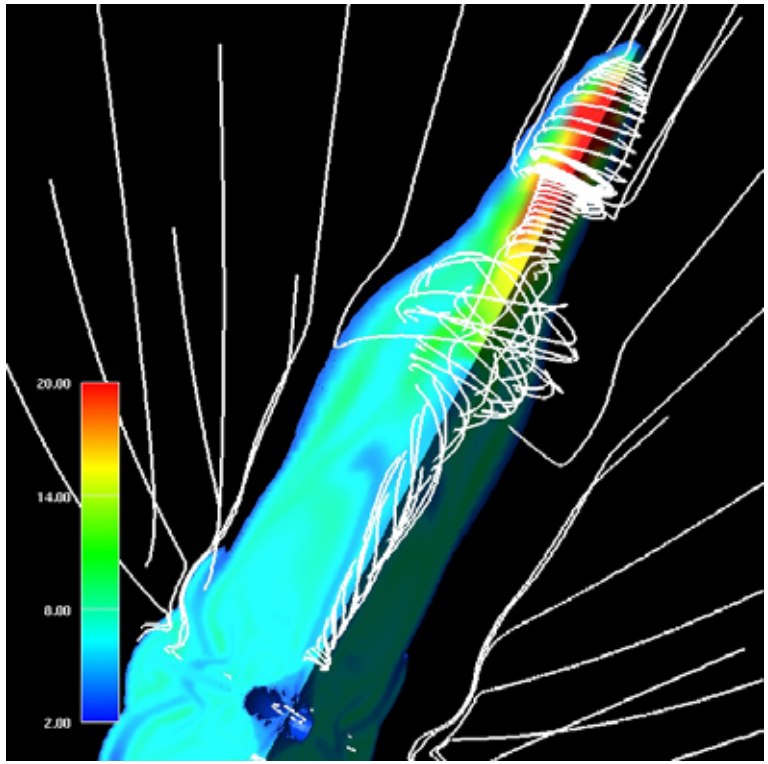
Low beta Jet

X:1200km

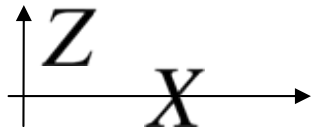
Even if beginning with high beta, in the end of simulations, strong magnetic fields are generated.

Winding Magnetic Field



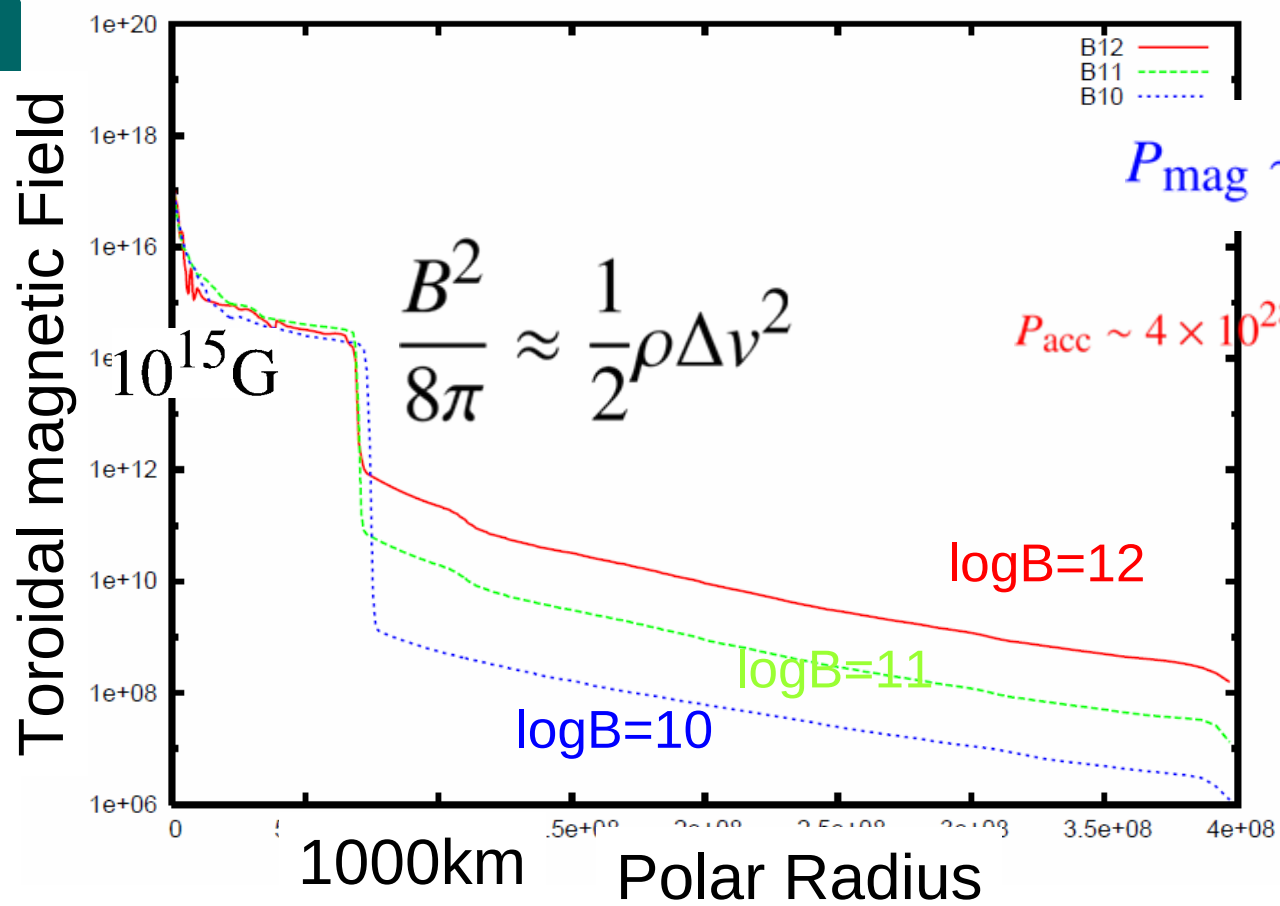


$$\frac{\mathrm{d}B_\phi}{\mathrm{d}t} = B_X X \frac{\mathrm{d}\Omega}{\mathrm{d}X} + B_Z X \frac{\mathrm{d}\Omega}{\mathrm{d}Z}$$



If the center of massive star spins rapidly compared to outer region, magnetic fields are wounded. And spring like strong magnetic field is generated along the axis.

Coherence of Toroidal Magnetic Field

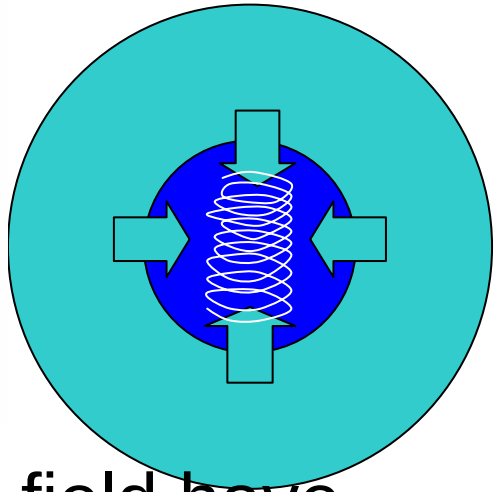


$$\frac{B^2}{8\pi} \approx \frac{1}{2}\rho\Delta v^2$$

T/|W|=1%

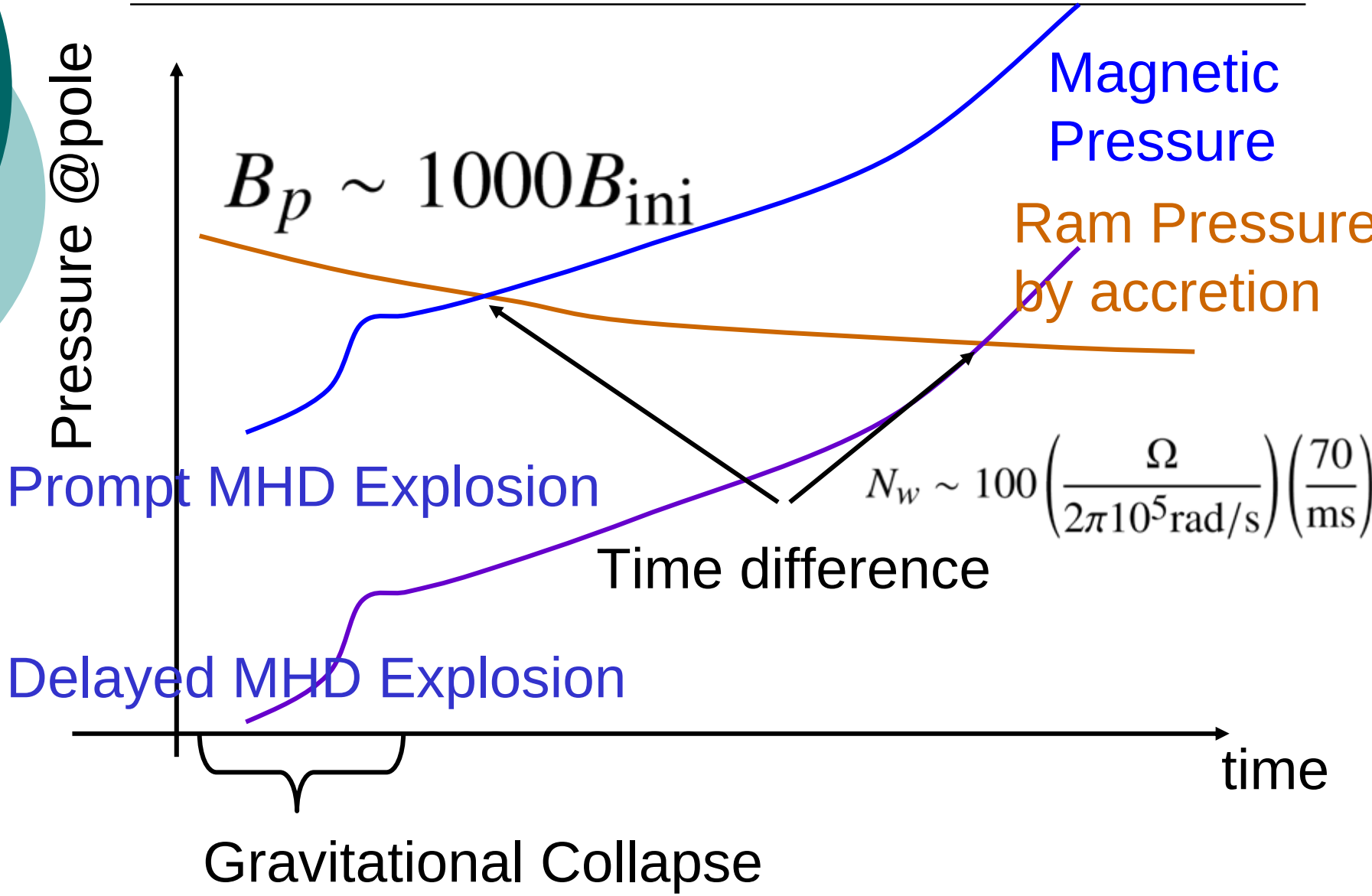
$$P_{\rm mag} \sim 4\times 10^{28}\left(\frac{B}{10^{15}{\rm G}}\right)$$

$$P_{\rm acc} \sim 4\times 10^{28}\left(\frac{\rho}{10^{10}{\rm g/cm^3}}\right)\left(\frac{\Delta v}{2\times 10^{9}{\rm cm/s}}\right)^2$$



Surprisingly, strengths of the toroidal B-field have same value! That is determined by the ram pressure!

MHD Explosion & Accretion



Rotational Dependence

Comparison on Explosion Energy

We show the explosion energy of each model here.

$$E_{\mathrm{exp}_{1000\mathrm{km}}} = \int_{\mathrm{D}} \mathrm{dV}\ \mathrm{e_{local}} = \int_{\mathrm{D}} \mathrm{dV}\,(\mathrm{e_{kin}} + \mathrm{e_{int}} + \mathrm{e_{mag}} + \mathrm{e_{grav}})$$

Strength of Magnetic Fields [G]

B \ T/|W|	0.25%	1%	4%
10^{10}	0.02	0.094	0.006
10^{11}	0.05	0.23	0.1
10^{12}	1.3	1.4	1.0

$t_{\mathrm{expl}} \approx 100\mathrm{ms}$

$t_{\mathrm{expl}} \approx 30\mathrm{ms}$

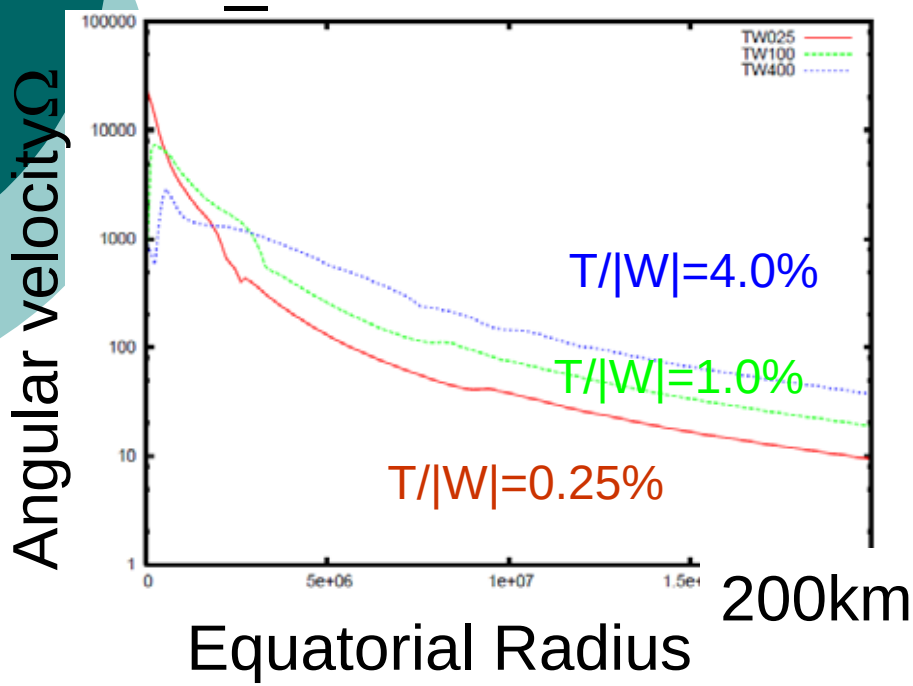
Explosion Energy [10^{50}ergs] when shock fronts reach10^{8}cm

Basically strong magnetic field and strong rotation is favorable for MHD explosions

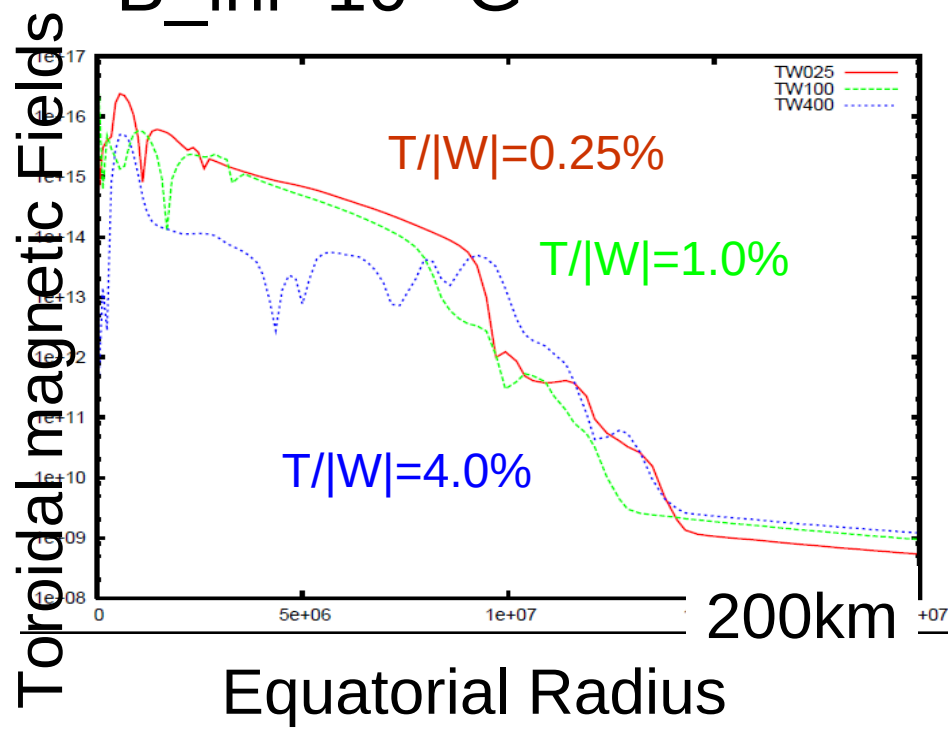
Most rapid case

Angular Velocity and Toroidal Field

B_ini=10^{10}G



B_ini=10^{10}G



In case of T/|W|=4.0%,
Central rotation is not strong!
Consequently toroidal magnetic field cannot grow!

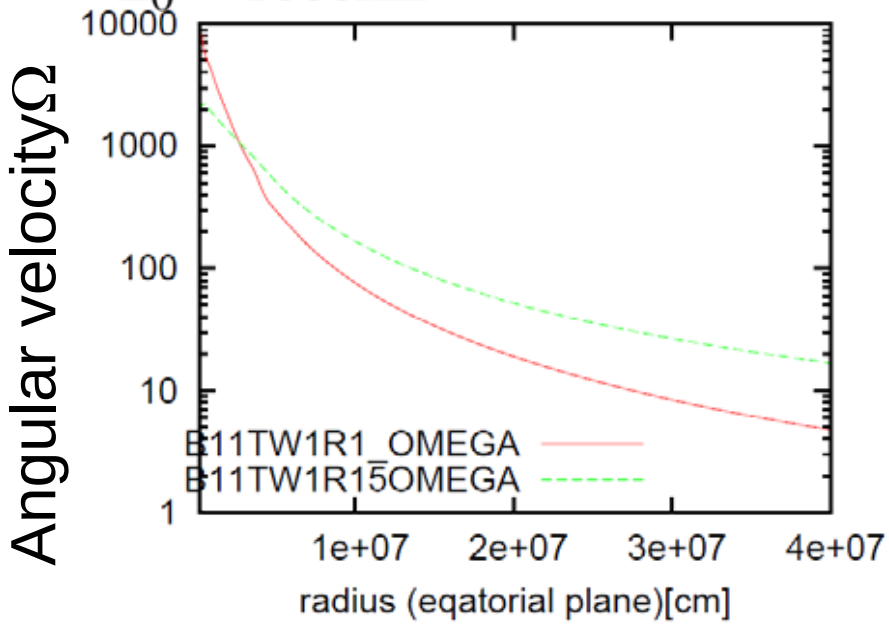
Some Generalization of our Result

In case of Rigid Rotation

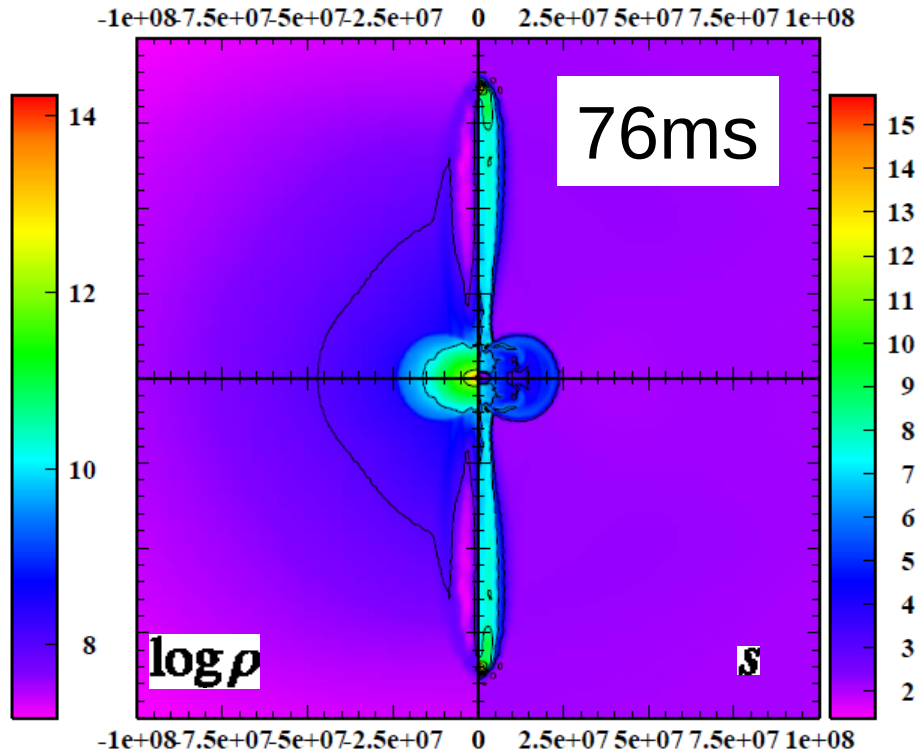
$$\Omega = \Omega_0 \frac{X_0^2}{X^2 + X_0^2} \frac{Z_0^4}{Z^4 + Z_0^4}$$

$X_0 = 100, 500, 1000, 1500\mathrm{km}$

$Z_0 = 1000\mathrm{km}$



$X_0 = 1500\mathrm{km}, B_{\mathrm{ini}} = 10^{11}\mathrm{G}, T/|W|_{\mathrm{ini}} = 1.0\%$



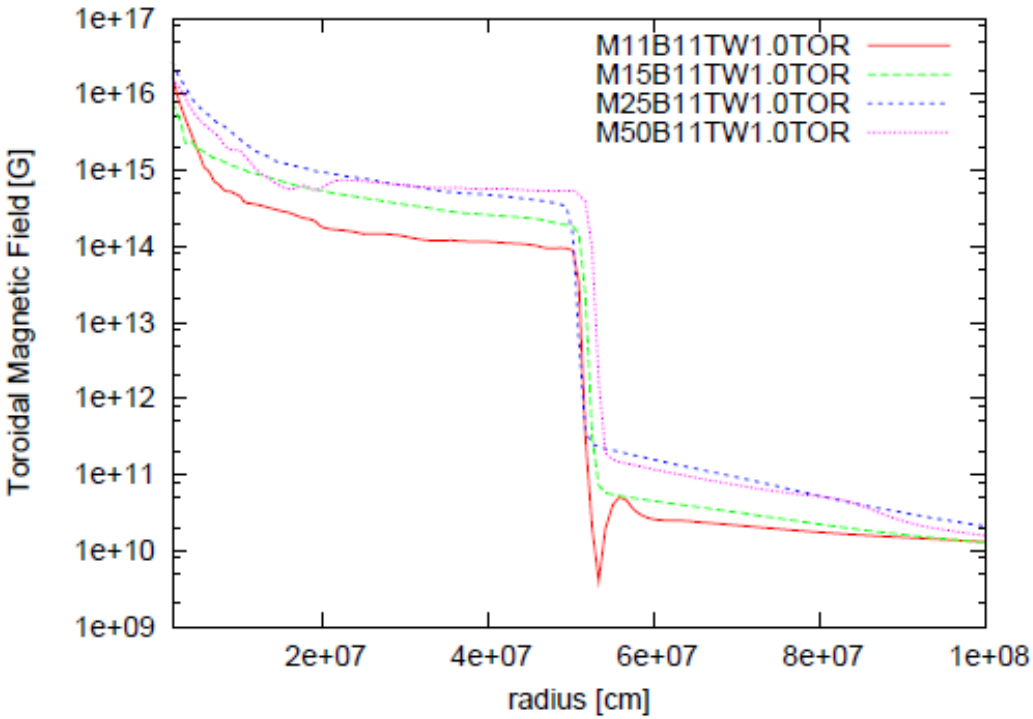
30ms at $X_0 = 100\mathrm{km}$

76ms at $X_0 = 1500\mathrm{km}$

Delayed Explosion are also found in case of rigid rotation.

Dependence on Accretion Rate

We change the mass of progenitor.



	Core mass	Accretion Rate
M11:	1.5 M_s,	13.2 M_s/s
M15:	1.9 M_s,	14.3 M_s/s
M25:	2.1 M_s	15.1 M_s/s
M50:	2.4 M_s	23.6 M_s/s

(Woosley 95)

Critical magnetic fields become weaker when accretion rate becomes low.

Summary

We develop new SRMHD code and perform 2-dimentional simulations for magneto-rotational collapse of iron core.

1. MHD explosions found even if the initial magnetic field is weak. Because the rapid rotation makes strong magnetic field at last.
2. The required magnetic field is determined by the ram pressure of accreting matter.
3. Varying rotation and magnetic field, different explosion energies and time scales are found.

See arXiv:0712.1949 for details

Appendix

GRB Supernovae

Table 2. GRB/SN Properties

	GRB 980425 SN 1998bw	GRB 031203 SN 2003lw	GRB 060218 SN 2006aj
redshift	0.0085	0.1055	0.033
fluence (10^{-6} erg cm^{-1})	2.8 ± 0.5^{a}	2.0 ± 0.4^{b}	6.8 ± 0.4^{c}
total duration (s)	~ 40	~ 40	> 2000
$E_{p,i}$ (keV)	55 ± 15^{d}	158 ± 51^{d}	$< 10^{e}$
E_{iso} (1×10^{50} erg)^{f}	0.010 ± 0.002^{d}	1.0 ± 0.4^{d}	0.65 ± 0.15^{e}
I-band T_{peak} (days)^{g}	17.7 ± 0.3^{h}	$18 - 28^{i}$	$12.7^{+2.0}_{-1.8}$
peak M_I (mag)	-19.27 ± 0.05^{h}	-19.0 to -19.7^{i}	-19.02 ± 0.09
I-J SN color, $\sim T_{peak,\ I}$	0.5^{l}	$\sim 0.4^{i}$	$\sim 0.0^{m}$

GRB03029
SN2003dh

0.1687
>25s
-
100
-

SN 2006aj and the nature of low-luminosity gamma-ray bursts
B. E. Cobb, C. D. Bailyn, P. G. van Dokkum, P. Natarajan ast-ph/0603832

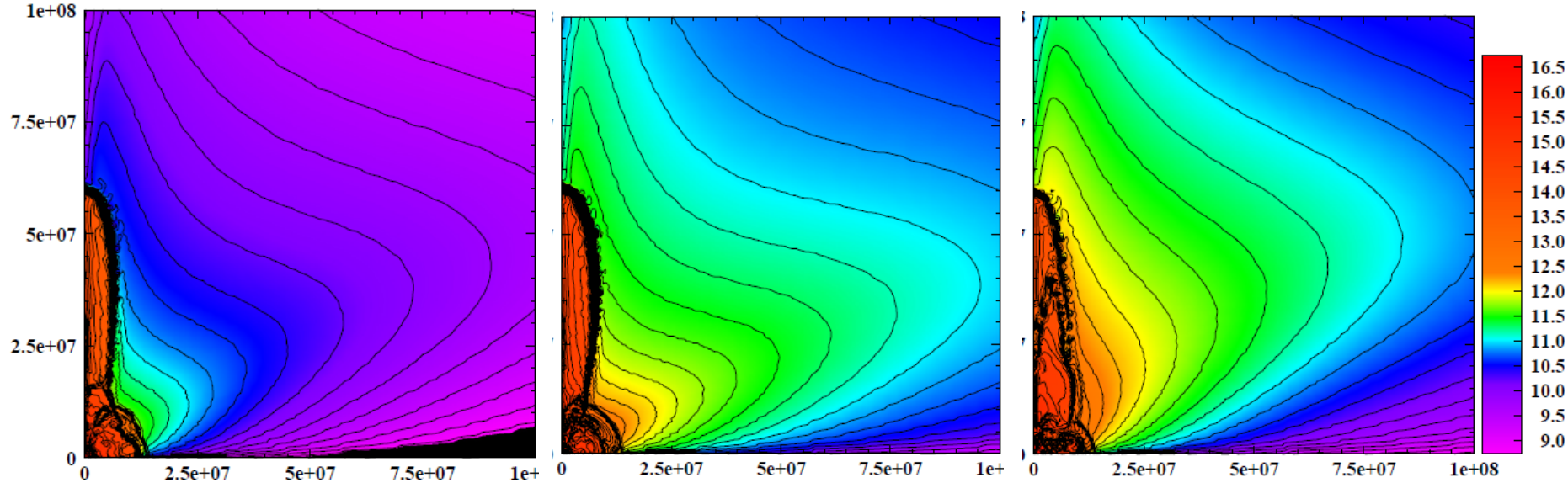
Comparison of EOSs (2)

	LS-EOS	Shen-EOS
K [MeV]	180, 220, 375	281
A_{sym} [MeV]	29.3	36.9
Max. NS mass [M_{sol}]	1.8, 2.0, 2.7	2.2

- Symmetry energy effect is large cf. non-rel.
  - Neutron Star: Y_p=0.1~0.2 for rel-EOS Sumiyoshi et al. NPA595 (1995
- Difference of composition, chemical potential
  - e-capture, ν-scattering rates → supernova dynamics

Spatial distribution of toroidal magnetic field

T/|W|=0.25%



logB=10 0.02×10^{50}[ergs]

logB=11 0.05×10^{50}[ergs]

logB=12 1.3×10^{50}[ergs]

In weak magnetic model strong toroidal magnetic field is confined to the rotational axis.
That reflects the difference of the explosion energy.

Neutrino Reaction

All kinds of Neutrinos are considered $\nu_e, \bar{\nu}_e, \nu_X$

Reaction for cooling：

$$e^- + (Z, A) \rightarrow \nu_e + (Z-1, A)$$

•Electron Capture (heavy nuclei, proton)
(Kotake et al. 2003,Epstein & Pethick 1981)
•Positron Capture (neutron) (Fuller et al. 1985)
•Pair reaction、plasmon decay (Itoh et al. 1989)

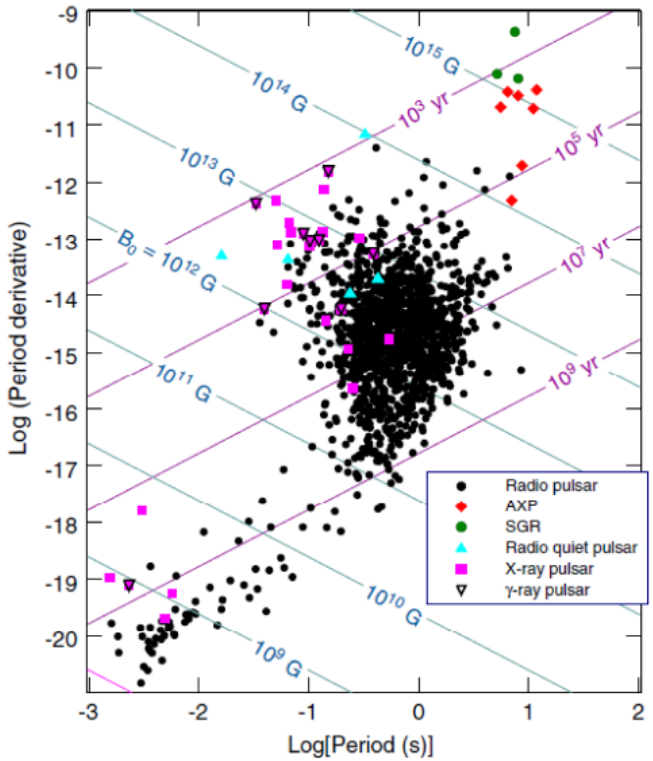
Reaction for opacity：

$$\nu + (Z, A) \rightarrow \nu + (Z, A)$$

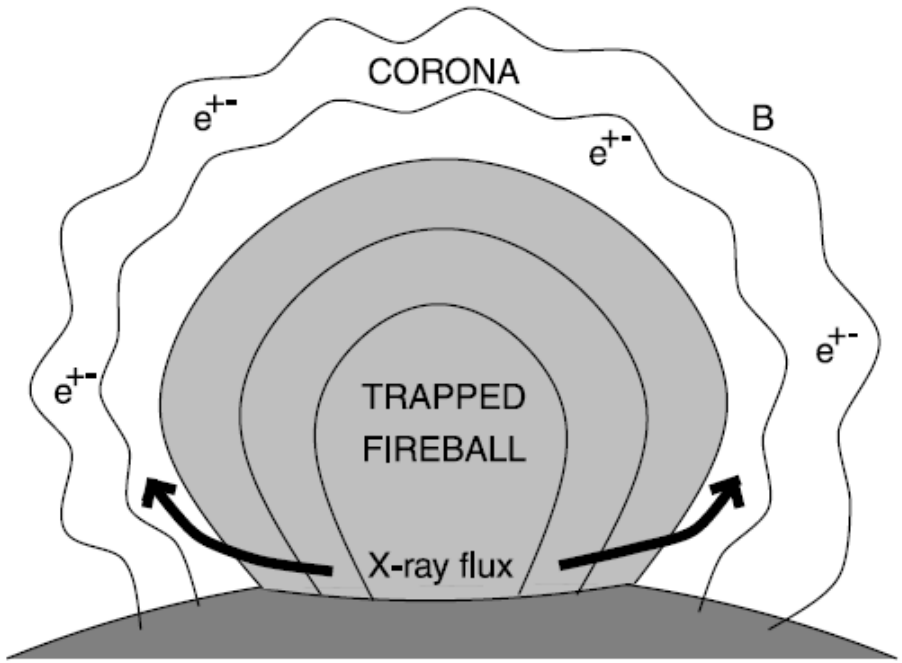
•Coherent scattering (heavy nuclei)
•Scattering by proton and neutron
•Absorption by Proton and neutron

(Kotake et al. 2003, Rosswog & Liebendoerfer 2003,Burrows et al. 2003, Horowitz 1996)

High field radio pulsar and magnetars



Harding and Lai 2006



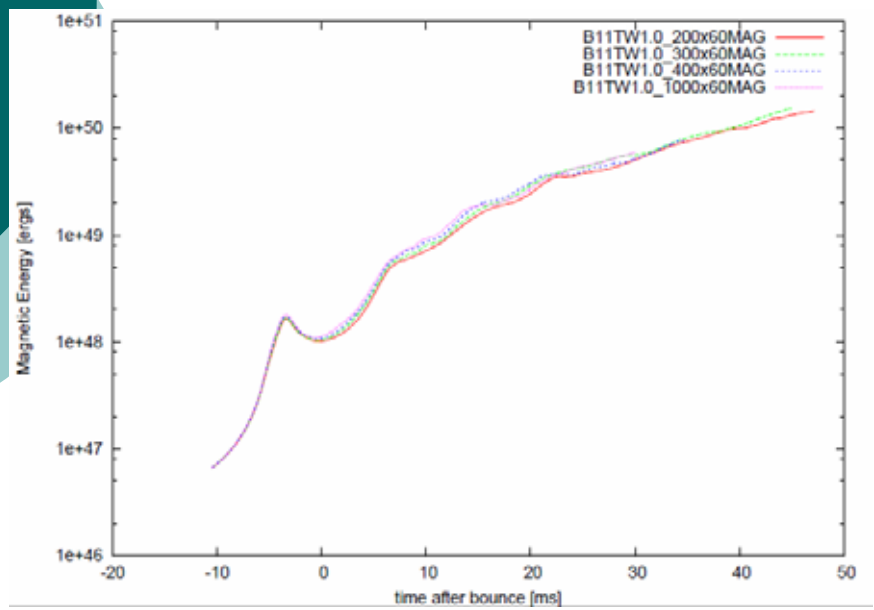
Thompson & Duncan 2001

Magnetars are not only characterized by its strong magnetic field but also by the flares.

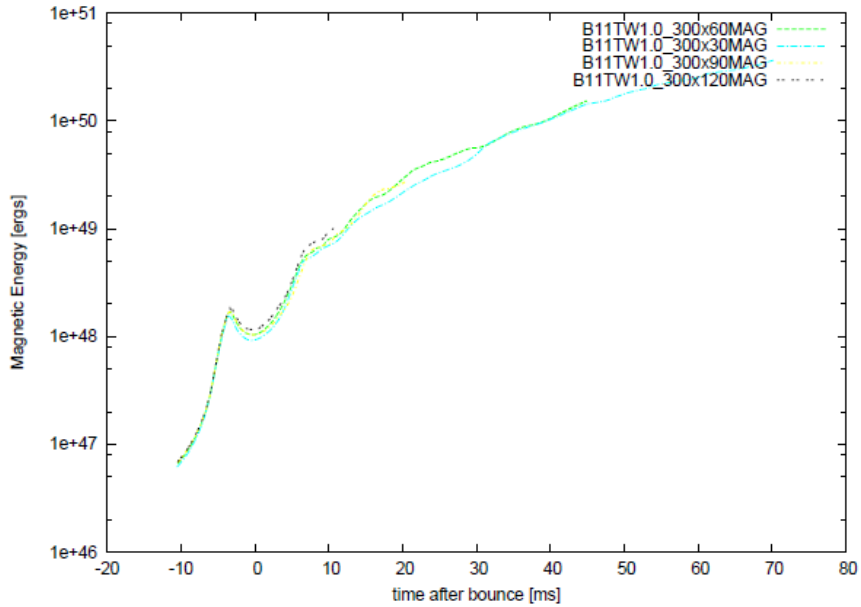
The structure of the magnetic field of neutron star should be important.

Resolution study

Evolutions of the magnetic energy.



r: 200,300,400,1000



θ : 30, 60, 90, 120

The difference from fiducial resolution (300x60) is 3%.

The effect of resolution on the magnetic field is minor.