

New Results from 3-D supernova models with spectral neutrino diffusion

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Overview

- Introduction
- The Isotropic Diffusion Source Approximation
- Results
- Conclusions

Part I – Introduction

The Supernova Problem

- Simulations of core-collapse supernovae in general do not show robust explosions
 - The expanding shockwave stalls around 200 kilometres
 - Missing some input physics *or*
 - Restricted dimensionality
- Suggestions relate to:
 - Neutrino heating
 - Convection
 - Oscillating neutron star

Convection

- Matter near PNS is heated until buoyancy carries it to low density regions at large radii
- Essentially a Carnot cycle is established where convection drives thermal transport
- High efficiency due to large temperature contrast
- (e.g. Herant et al 1994)

NS oscillations

- Accreting PNS oscillates
- Excites core g-mode oscillations
- This acoustic power may be sufficient to cause an explosion
- (e.g. Burrows et al, 2006)

Neutrino Heating

- Neutrinos are produced in the proto-neutron star
- These neutrinos may be absorbed behind the stalled shock
- This may heat matter behind the shock and restarts the explosion
- (e.g. Colgate & White 1966; Bethe & Wilson 1985)

Simulation Requirements

- Desirable to have a computer model which includes all relevant physics:
 - Magnetohydrodynamics
 - Gravity
 - Neutrino Transport
 - Including interaction with the fluid
 - Spectral
 - Weak interaction rates
 - Nuclear equation of state
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Simulation Requirements II

- SN need to be modelled in three dimensions
 - Convection, for example, can only properly be modelled in 3D
 - Accretion
 - Accretion flow in 2D is an “accretion torus”
 - Magnetic fields
 - e.g. dynamo, Lorentz force

Simulation Requirements III

- Radiation hydrodynamics is computationally demanding
 - Direct solution of the Boltzmann transport equation is only feasible in one dimension
- Approximations are hence required
 - Grey neutrino transport (e.g. Fryer & Warren 2004, Scheck et al. 2004)
 - Parameterisations (e.g. Ott et al 2007, Scheidegger et al 2007)

Spherically symmetric Boltzmann Equation

$$\begin{aligned} & \frac{df}{cdt} + \mu \frac{\partial f}{\partial r} + \left[\mu \left(\frac{d \ln \rho}{cdt} + \frac{3v}{cr} \right) + \frac{1}{r} \right] (1 - \mu^2) \frac{\partial f}{\partial \mu} \\ & + \left[\mu^2 \left(\frac{d \ln \rho}{cdt} + \frac{3v}{cr} \right) - \frac{v}{cr} \right] E \frac{\partial f}{\partial E} \\ & = j (1 - f) - \chi f + \frac{E^2}{c (hc)^3} \\ & \times \left[(1 - f) \int R f' d\mu' - f \int R (1 - f') d\mu' \right]. \end{aligned}$$

Part II – The Isotropic Diffusion Source Approximation

Isotropic Diffusion Source Approximation

- Liebendörfer, Whitehouse and Fischer, 2007
- Goal is to implement the dominant features of radiative transfer *efficiently* and *consistently*
- To do this we decompose the problem into different subdomains and apply appropriate algorithms for each
 - Each subdomain can use a different method

Decomposition

- Decompose the distribution function f of transported particles

$$f = f^t + f^s$$

f^t = Trapped particles – opaque regime

f^s = Streaming particles – transparent regime

- The two components are evolved separately:

$$\begin{aligned} D(f^t) &= C^t - \Sigma \\ D(f^s) &= C^s + \Sigma \end{aligned}$$

Trapped Component

- Assumptions:
 - Distribution function f^t is isotropic
 - Source function Σ is isotropic
- Angular integration reduces Boltzmann eqn to

$$\frac{df^t}{cdt} + \frac{1}{3} \frac{d \ln \rho}{cdt} E \frac{\partial f^t}{\partial E} = j - (j + \chi) f^t - \Sigma.$$

Trapped Component II

- Evolution of the trapped particles must reproduce correct diffusion limit
 - Particles slowly drain or replenish in a fluid element
- Needs to be included in Σ term
- Comparison with diffusion gives us our diffusion source Σ

The Diffusion Source

$$\frac{1}{2} \int \Sigma d\mu = \frac{1}{r^2} \frac{\partial}{\partial r} \left(\frac{-r^2}{3(j+\chi+\phi)} \frac{\partial f^t}{\partial r} \right) + (j + \chi) \frac{1}{2} \int f^s d\mu$$

- LHS is the diffusion source
- First term on RHS is diffusive
- Last term on RHS is absorption of streaming particles
- Source is limited by emissivity to prevent unphysical particle fluxes in transparent regime

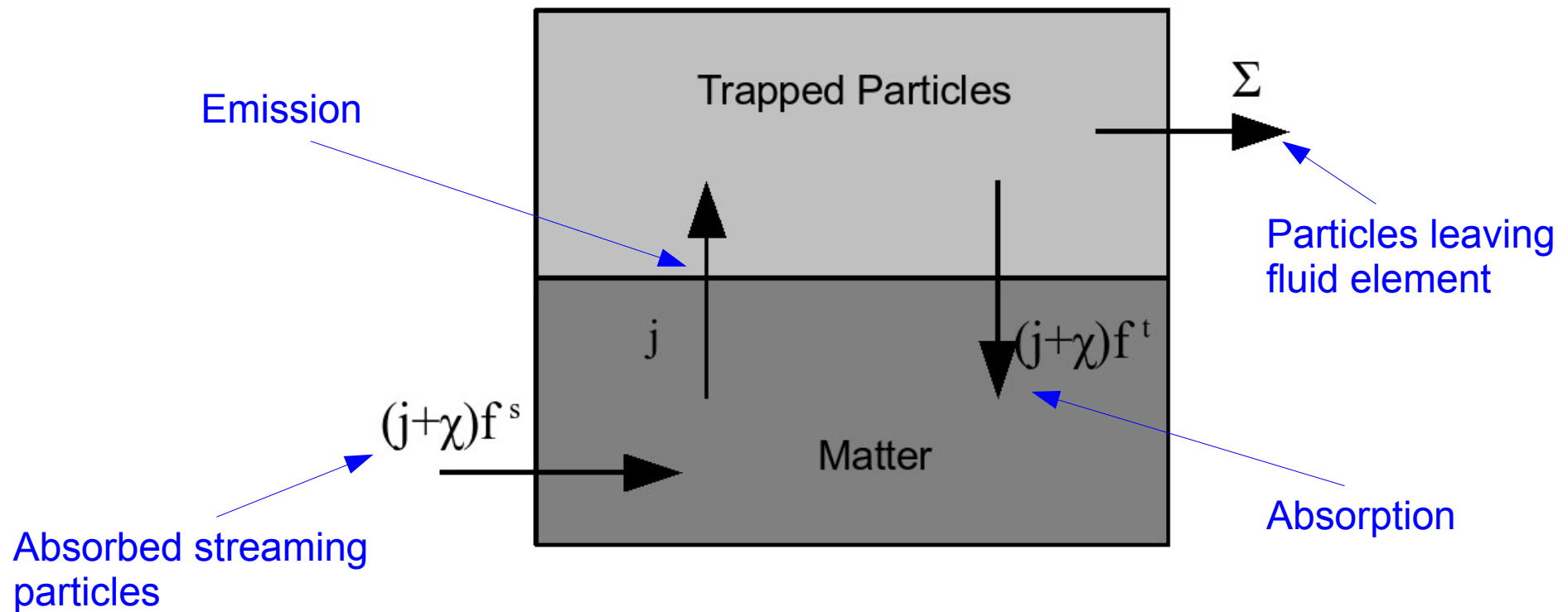
Streaming Component

- Weakly coupled to matter
 - Laboratory frame is more convenient
- Evolution equation becomes

$$\frac{\partial \hat{f}^s}{c \partial \hat{t}} + \hat{\mu} \frac{\partial \hat{f}^s}{\partial r} + \frac{1}{r} (1 - \hat{\mu}^2) \frac{\partial \hat{f}^s}{\partial \hat{\mu}} = - \left(\hat{j} + \hat{\chi} \right) \hat{f}^s + \hat{\Sigma}$$

- At present evolved using steady-state approximation
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A Fluid Element



Coupling to Hydrodynamics

$$\frac{\partial}{\partial t} U + \frac{\partial}{r^2 \partial r} (r^2 F) = 0$$

In 3D data storage is a problem

(600 x 600 x 600 zones) x (4 neutrino species) x
(12 energy bins) = easily 10x more memory
than a normal MHD simulation

- Not practical on today's computers
- Cannot evolve distribution functions with hydrodynamics

Coupling to Hydrodynamics II

- Use equilibrium conditions to describe f^t
- Use a thermal spectrum for f^t
- This assumption is **only made for trapped particles** within a fluid element
- Particles transported between fluid elements **retain their spectral information**

Neutrino Fraction & Energy

$$Y^t = \frac{m_b}{\rho} \frac{4\pi}{(hc)^3} \int f^t E^2 dE d\mu$$

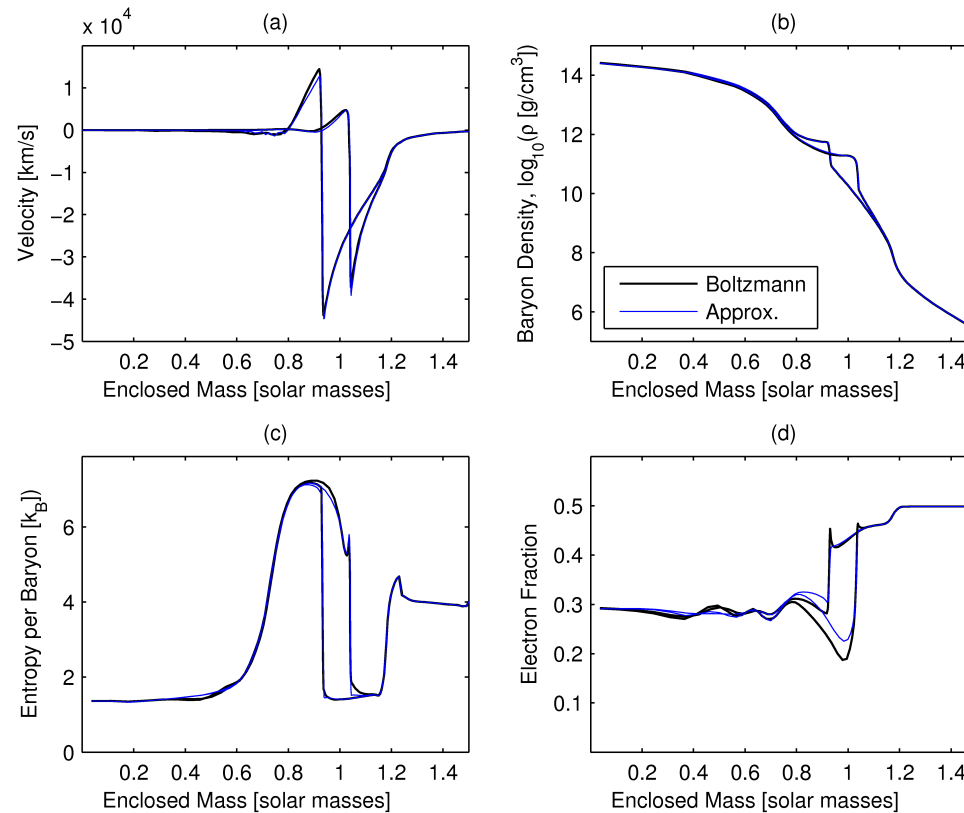
$$Z^t = \frac{m_b}{\rho} \frac{4\pi}{(hc)^3} \int f^t E^3 dE d\mu$$

Variables stored in 3D

$$U = \begin{pmatrix} \rho \\ \rho v \\ \rho \left(e + \frac{1}{2} v^2 \right) \\ \rho Y_e \\ \rho Y_i^t \\ (\rho Z_i^t)^{\frac{3}{4}} \end{pmatrix} \quad F = \begin{pmatrix} v \rho \\ v \rho v + p \\ v \rho \left(e + \frac{1}{2} v^2 + \frac{p}{\rho} \right) \\ v \rho Y_e \\ v \rho Y_i^t \\ v (\rho Z_i^t)^{\frac{3}{4}} \end{pmatrix}$$

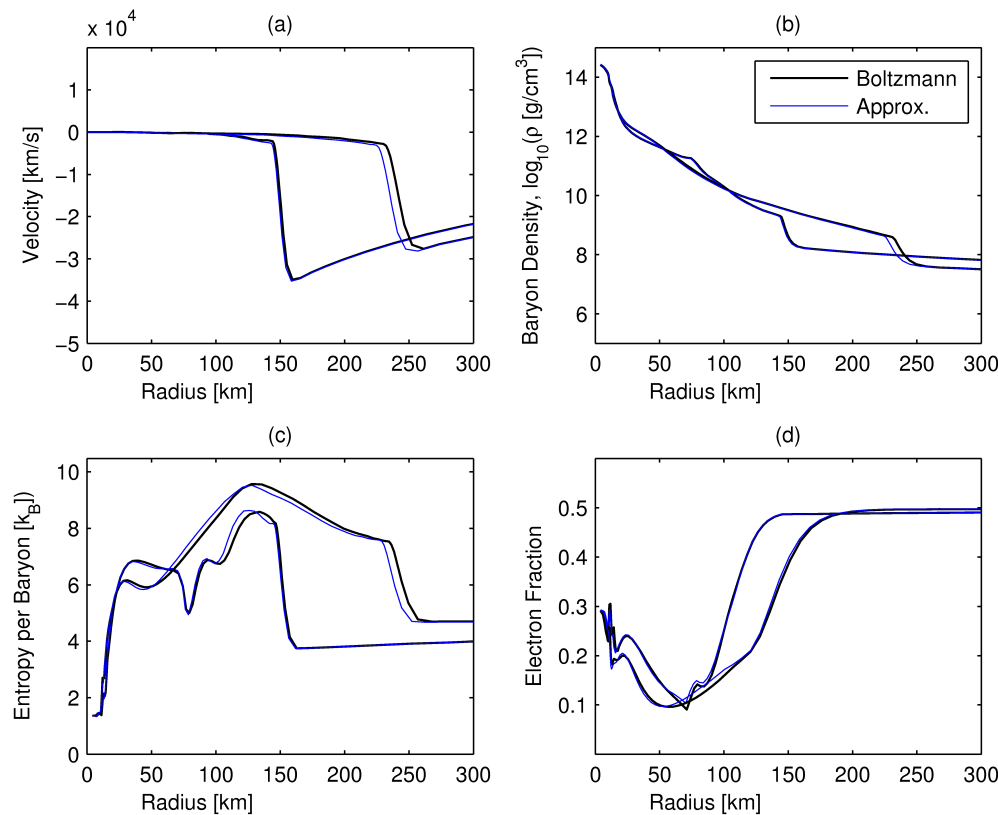
Verification in one dimension

- 1 ms and 3 ms



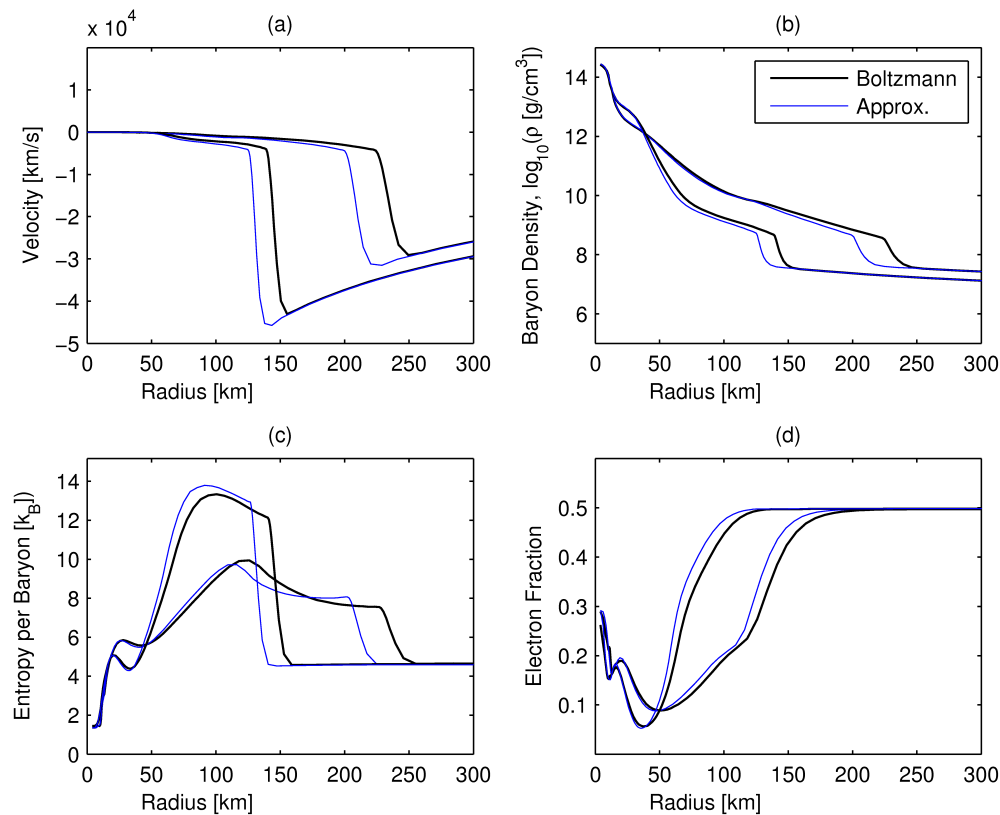
Verification in one dimension II

- 30 ms and 100 ms

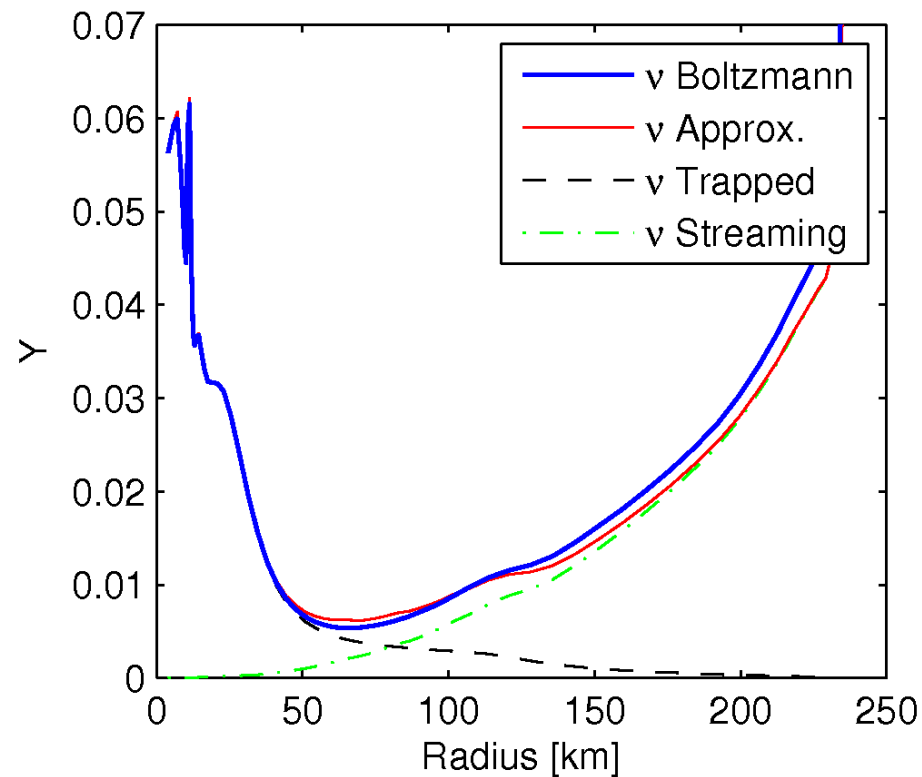


Verification in one dimension III

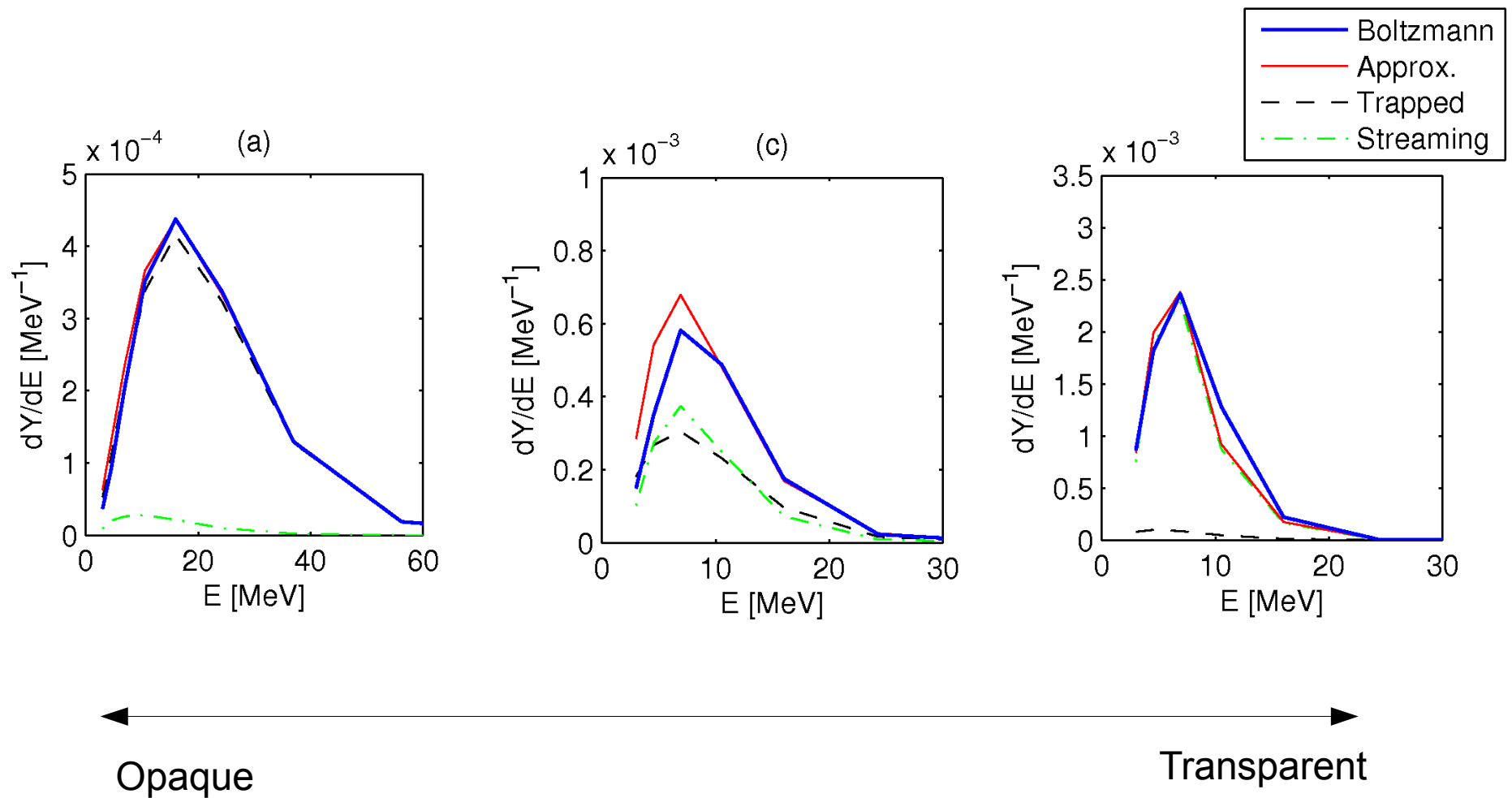
- 150 ms and 300 ms



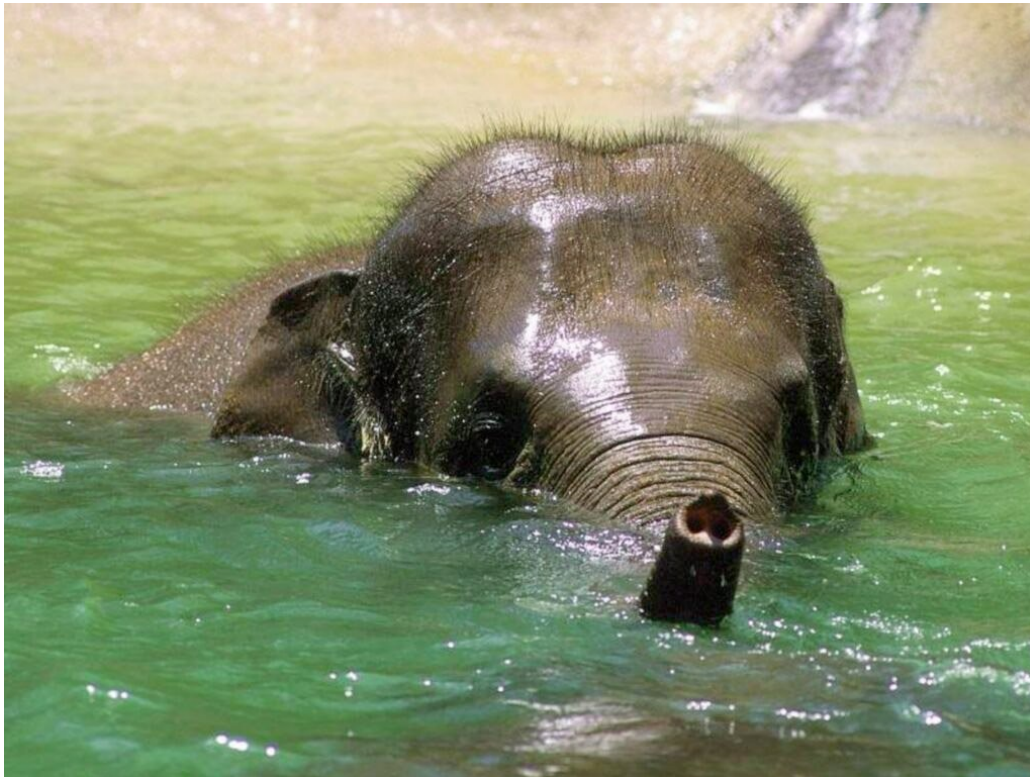
Verification in one dimension IV



1-D Energy Spectrum



ELEPHANT



- **E**legant
- **P**arallel
- **H**ydrodynamics
- With **A**pproximate
Neutrino **T**ransport

Elephant

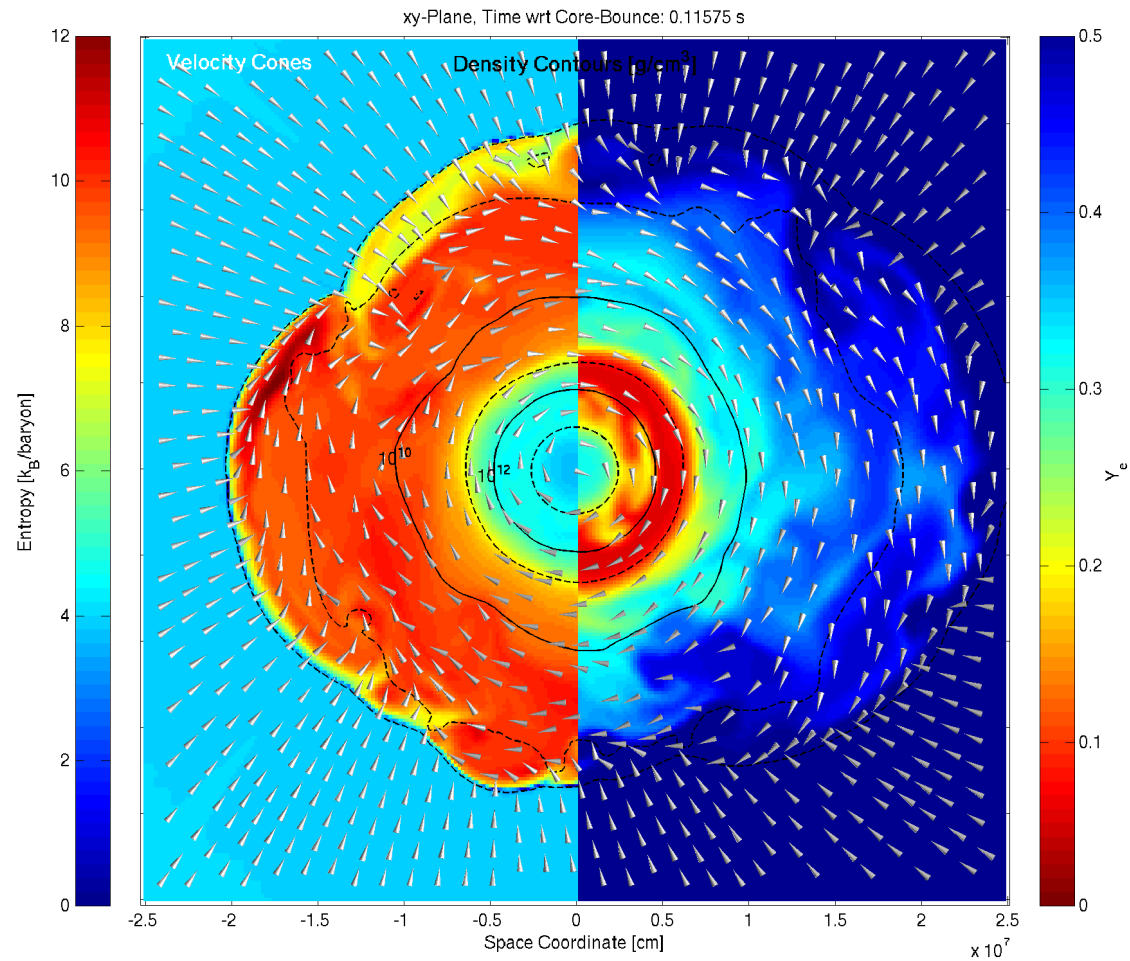
- 3-D MHD code
 - Parallelised using MPI
 - This simulation 300 x 300 x 300 zones at 2km resolution
 - Code does not include AMR
 - Grid with non-constant spacing is being developed
 - Up to 600x600x600 @ 1km tested
 - Lattimer & Swesty EoS
 - General Relativistic gravitational potential
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Elephant – IDSA Current Status

- Added neutrino diffusion
 - f^t only
 - Streaming component is currently under development
 - Effectively all streaming particles leave the computational domain with no further effect on the simulation
 - Makes simulation pessimistic
 - Electron neutrinos/anti-neutrinos only
-

Part III – Results

Neutrino Diffusion



Part IV – Conclusions

Neutrino heating

- Hydrodynamic conditions are spherically averaged
 - These conditions are then used to generate the fluxes in a radial manner
 - The fluxes are then used to generate a spherical distribution function f^s
 - The 3-D hydrodynamics then takes this f^s and uses it for its local calculations of neutrino transport
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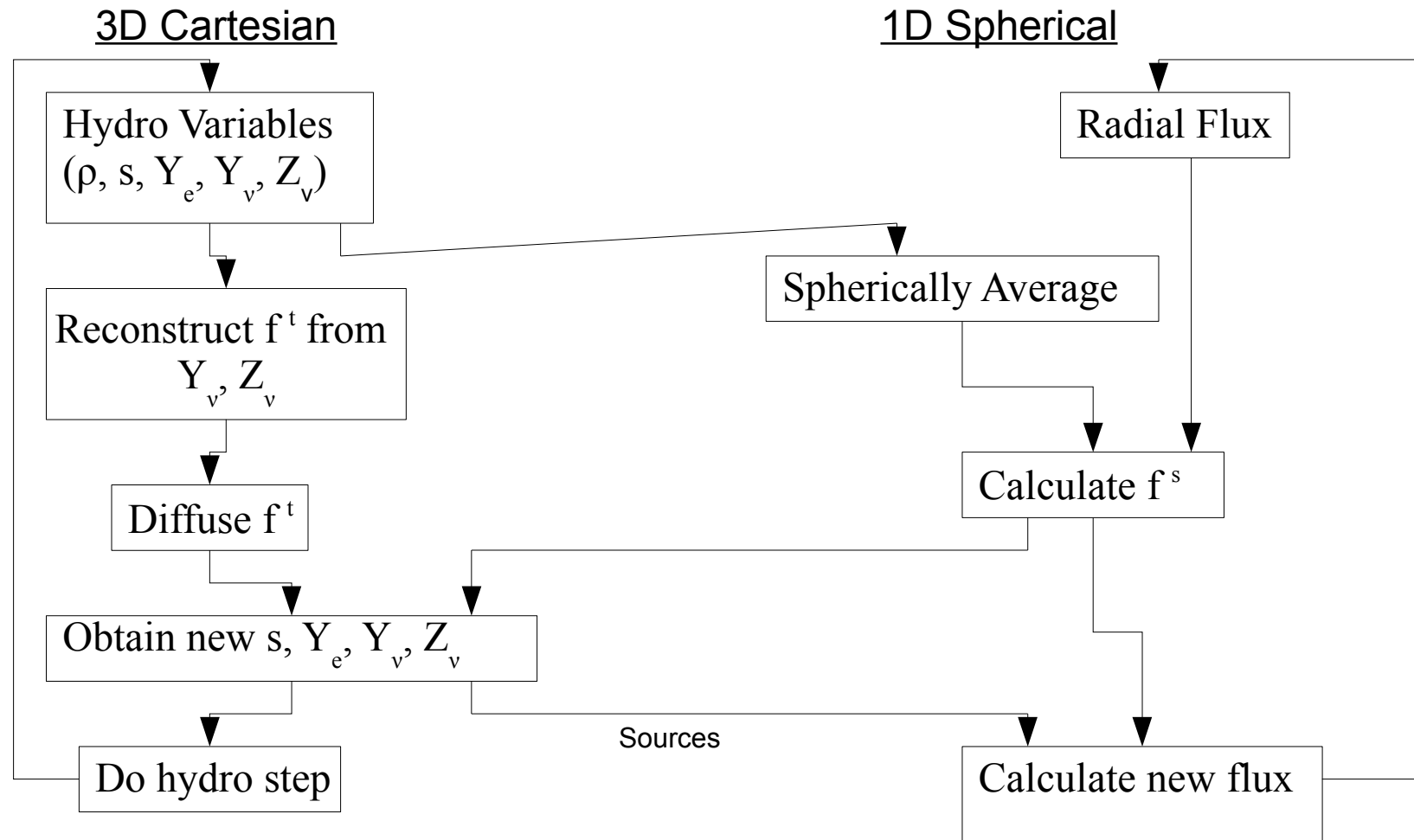
Neutrino heating II

- In the future each 3-D fluid element will calculate a local source
- This source will then be included in the relevant radial flux bin
- And then the distribution function will be propagated back to the 3D level

Neutrino heating III

- At present doesn't work
 - Probably due to badly resolved mean free paths in the spherically-averaged hydrodynamics
 - Work in progress

Flow chart of Code



Other Applications

- Advantage of supernovae is that $c_s \sim c$
 - So timesteps for radiation and hydro are of similar size
 - In most other areas of astrophysics in general $c_s \ll c$
 - Radiation transport typically requires some sort of implicit method for these cases
 - IDSA would require an implicit diffusion solver
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Future Work

- Get heating working
 - Add mu and tau neutrinos
 - Incorporate recent work done on unequally spaced grid
 - Improve streaming particles to use something more sophisticated than steady-state approximation
 - Poisson solver for gravity
 - General relativity?
-

Conclusions

- Isotropic Diffusion Source Approximation is a viable efficient way of including radiation transport in 3D simulations
 - Consistent with spherically symmetric Boltzmann transport including the same physics
 - Practical way of implementing neutrino transport for supernovae in 3D
 - Would have applications in many areas of astrophysics
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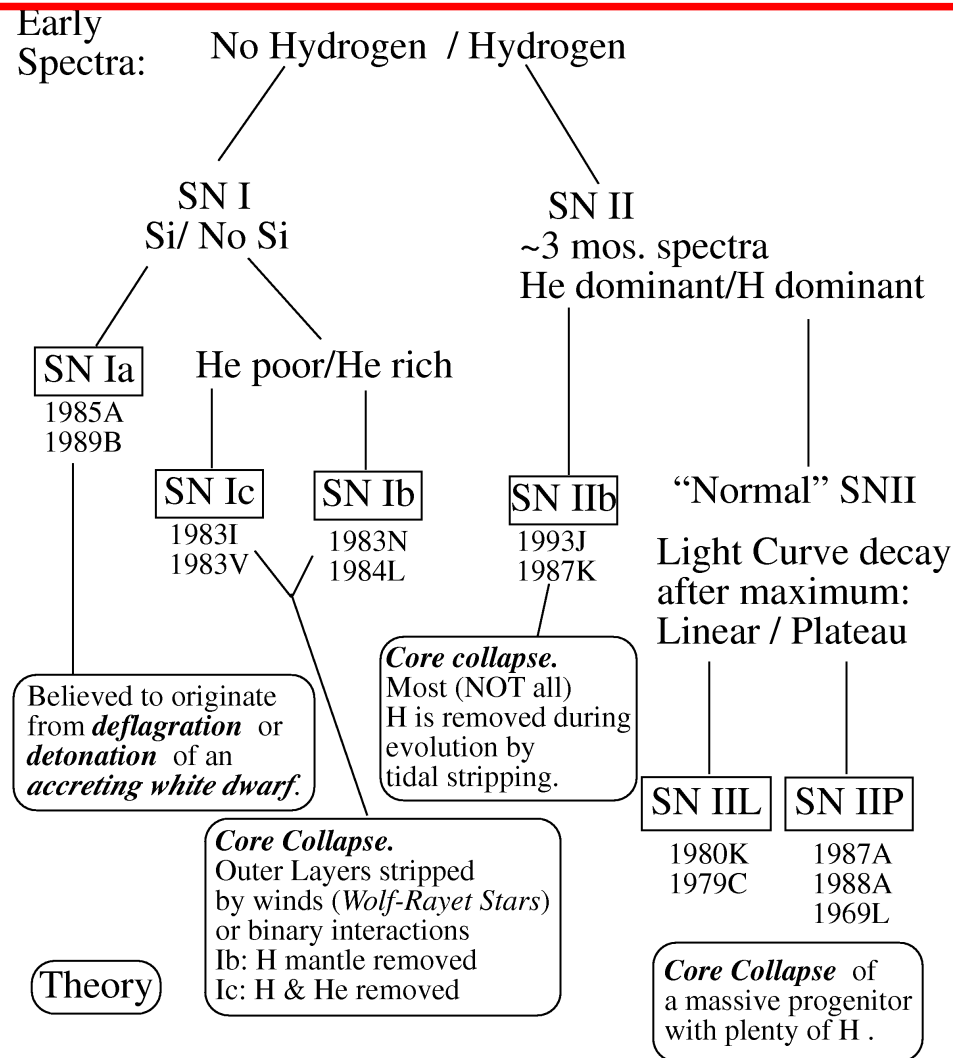
What is a Supernova?

- Huge stellar explosion
 - May outshine a galaxy for a brief period
- Explosion expels much of a stars material
- Leaves behind a *supernova remnant*
- Two different triggering mechanisms
 - One is sudden switching on of nuclear burning
 - The other is sudden switching off of nuclear burning

Motivation

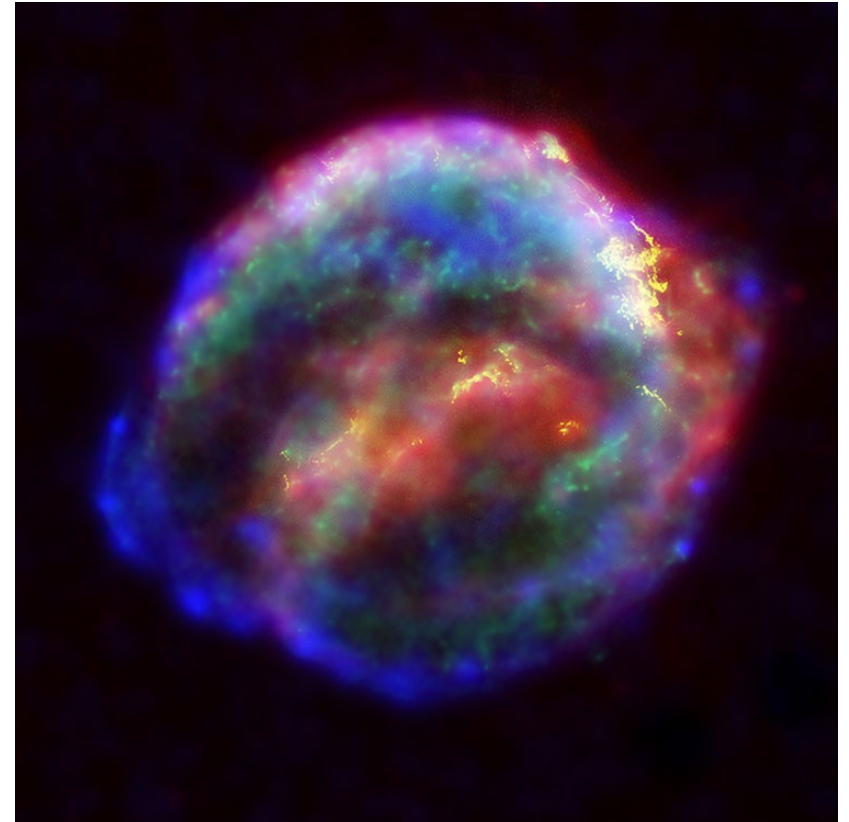
- Supernovae are laboratories for exotic physics
- Primary site of heavy element nucleosynthesis
- Type Ia supernovae are used as distance indicators of far-off galaxies
- Shockwaves from supernovae may trigger the next generation of star formation

A Supernovae Bestiary



Observational History

- First supernova detection possibly SN 185
 - Observed in China in 185 CE
- Most recent observed galactic supernova SN 1604
- Both thought to be Type Ia supernovae



NASA/ESA

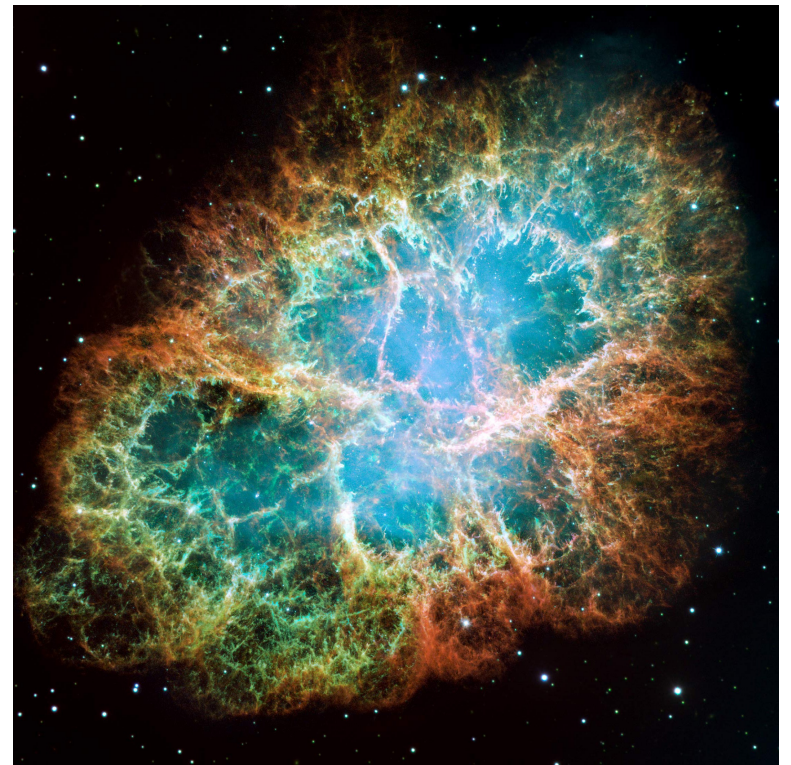
Type Ia Supernovae

- Accretion onto a C-O white dwarf
- As it approaches the Chandrasekhar mass central density & temperature core begins carbon fusion
- Within a few seconds most of the matter in the WD undergoes fusion, producing enough energy to unbind the star

Type II Supernovae

- Very massive ($>8 M_{\text{sun}}$) star burns core into Fe
- Cannot produce energy by fusing Fe
- Internal energy source is lost causing a loss of pressure support

Crab Nebula,
probably remnant of SN 1054
(NASA/ESA)



Type II Supernovae

- Gravitational collapse
 - Nuclear forces dominate in the core
- Outer layers bounce on proto-neutron star
 - Explodes as a supernova after a few 100ms

Type II Supernovae

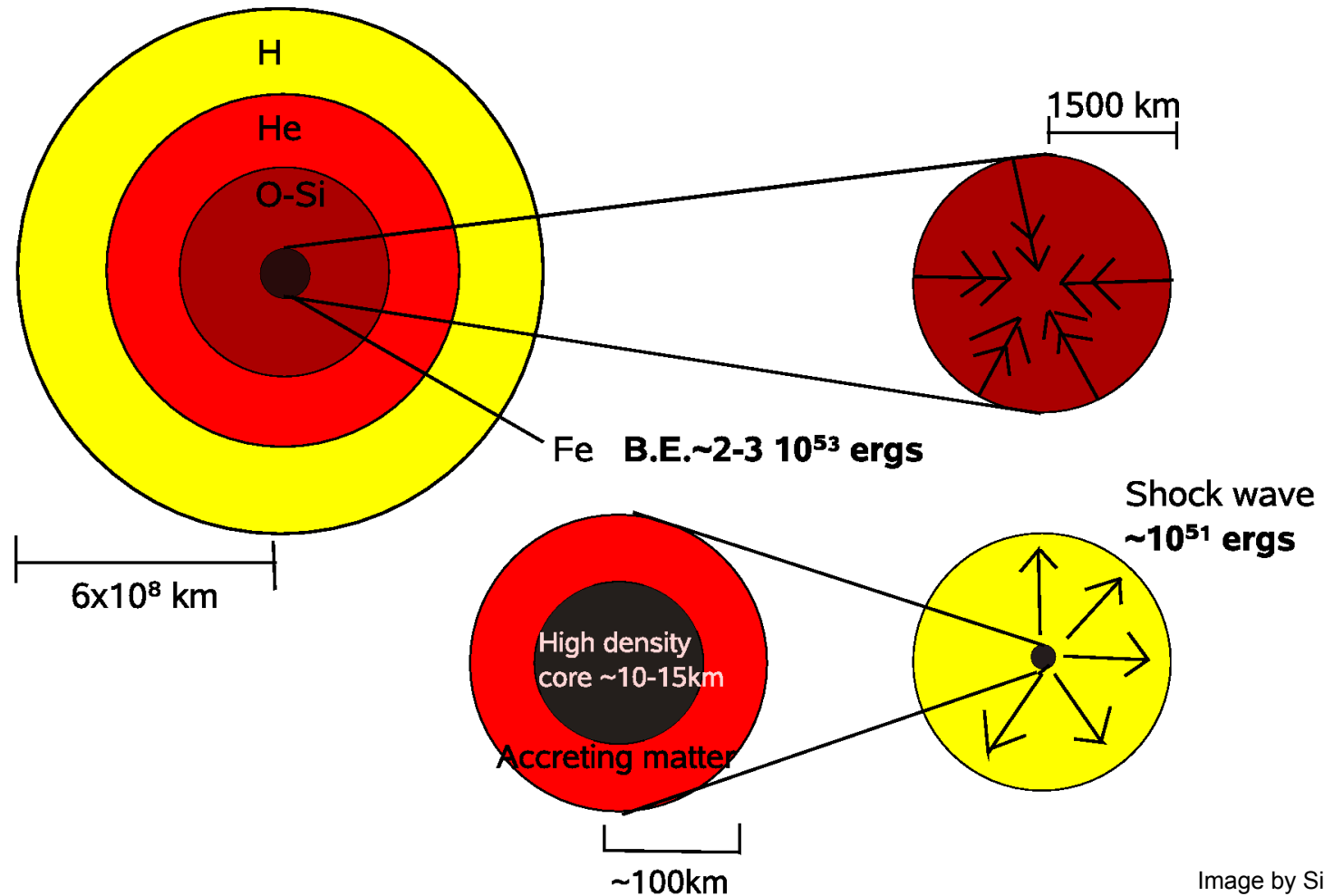


Image by Simon Scheidegger
based on one from Sanjay Reddy

SN 1987A

- First supernova visible to the naked eye since SN 1604
 - Progenitor star probably Sanduleak -69° 202a
 - B3 I blue supergiant
 - $L \sim 4 \times 10^{38}$ erg/s
 - Main sequence mass probably 16-22 M_{sun}
 - Located in the LMC
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IDS is not FLD

- IDS Differs from FLD
 - Agree in diffusion limit
 - FLD assumes flux is parallel to intensity gradient
 - Wrong in transparent regime
 - IDS takes flux as being directed away from source
 - IDS allows separate approximations for diffusive and streaming parts, e.g.
 - Co-moving frame vs laboratory frame
 - Thermal equilibrium vs stationary state
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