Towards a MHD version of SASI

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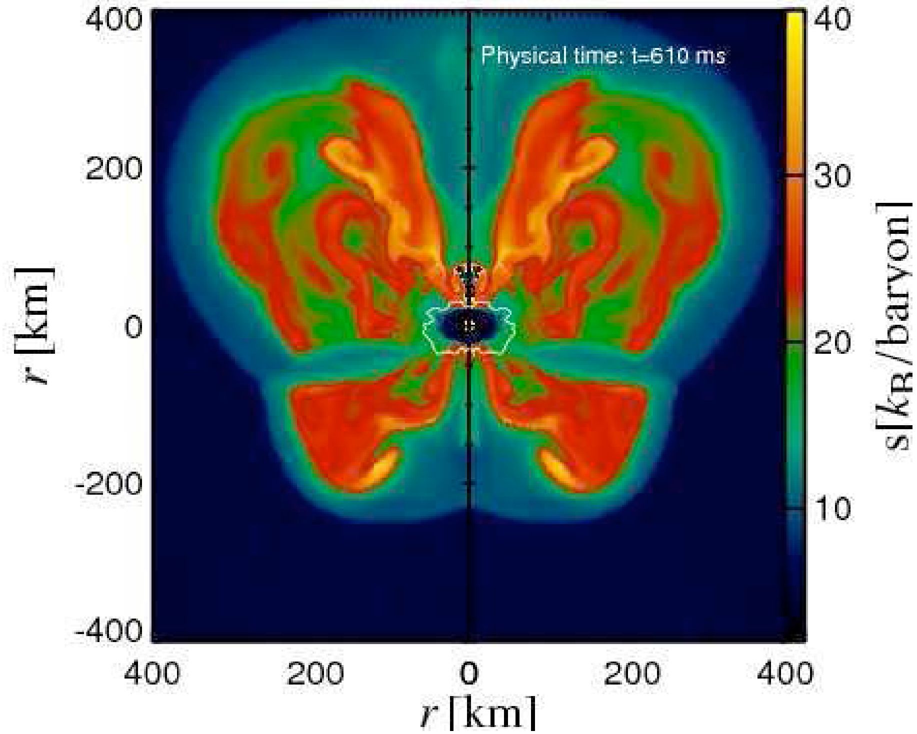
Outline

- Core collapse with a moderate magnetic field
- Studying MHD SASI : The set up and formalism
- Effect of the magnetic field in different geometries :
  - B perpendicular to the shock
  - B parallel to the shock
- Conclusion and discussion

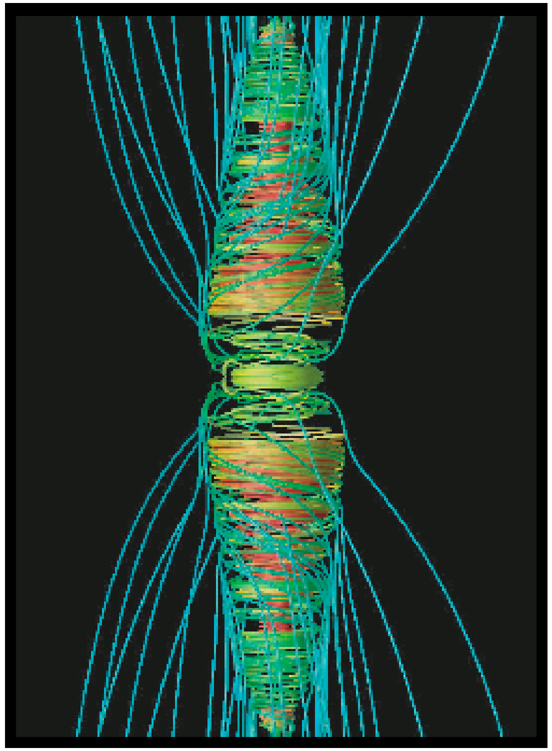
Non magnetic SN



Marek & Janka 2008

SASI plays a crucial role

Magnetic explosion



Burrows *et al.* 2007

Jet explosion with no time for SASI to develop

A few questions

- What happens with a moderate rotation & magnetic field? Mixed features?
- Could a moderate magnetic field help the neutrino explosion?
- What is the effect of the magnetic field on the SASI?
- How strong a magnetic field should be to have a significant effect on SASI?

A few numbers

- Strength of B needed to have $v_{Alfven} = v_{flow}$?
  - r_{sh} = 150km : B ~ 3.10^{13} G
  - r ~ 50 km : B ~ 10^{14} G

- Magnetic field from flux conservation :
  - Heger *et al.* 2005 + flux conservation :
    - At r=150km : B_ϕ ~ 3.10^{11} G too small
    - at r=50km : B_ϕ ~ 2.10^{12} G

- Scaling of the saturation of MRI (from Burrows *et al.* 2007) :
  - Supposing a period of the core P_{core} ~ 20s
    - R = 150km : time scale too long (>500ms)
    - R = 50km : t_{growth} ~ 50ms, B_{sat} ~ 3.10^{14} G

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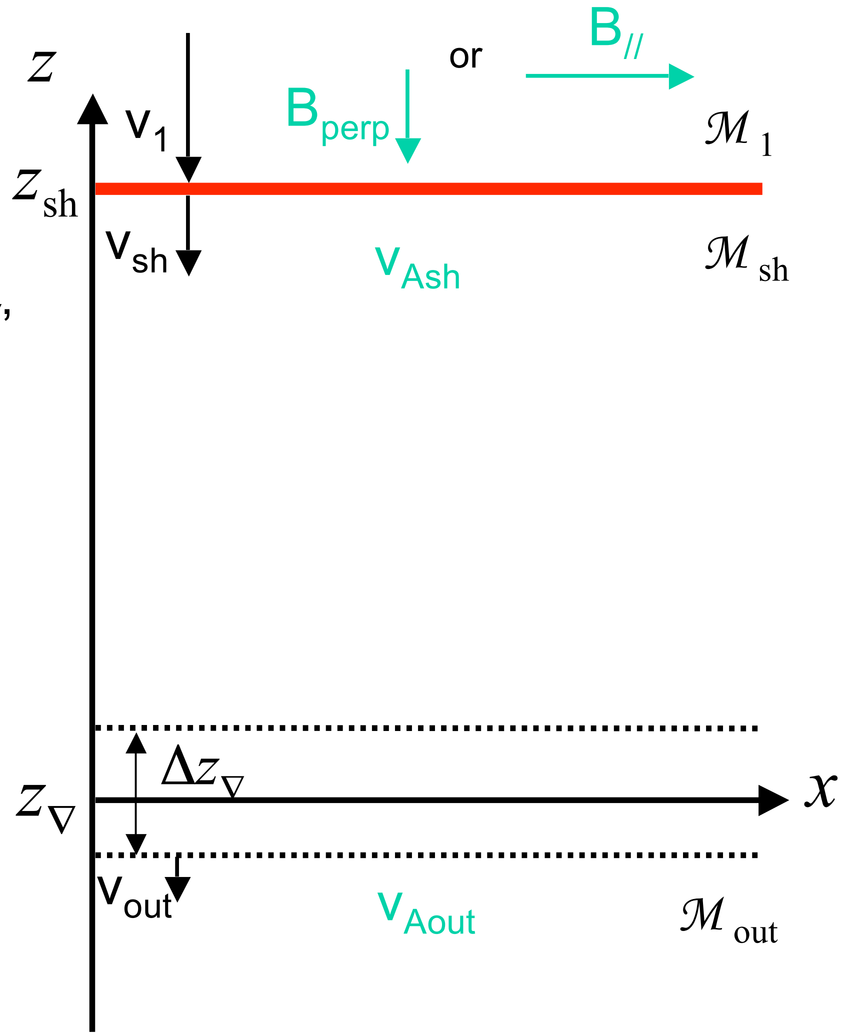
The physical mechanism behind SASI

- Two proposed mechanisms :
  - Advective-acoustic cycle : Foglizzo 2000, 2002, 2007, Blondin *et al.* 2003
  - Purely acoustic mechanism : Blondin & Mezzacappa 2006, Laming 2007
- We will focus on the advective-acoustic interpretation
- The mechanism relies on the decomposition of the mode into waves
- Cycles between the shock and a region of gradient which couples advected waves to acoustic waves

A (very) simplified set up

- Based on the hydro toy model of Foglizzo 2006 (Talk in KITP)
- Parallel adiabatic flow ($\gamma=4/3$)
- Localized coupling with a potential jump (Δz_∇, c_{in}/c_{out})
- Lower boundary condition : no wave propagating up from below
- Periodic box of aspect ratio 4
- 2 geometries of magnetic field are studied : parallel and perpendicular to the shock
- 3D perturbations
- Perturbative approach :

  → only the linear phase…



What does B change?

Decomposition of the perturbations : 3 waves -----> 7 waves!

2 acoustic waves : → 2 fast magnetosonic waves

1 advected wave :

- entropy → 1 advected entropy wave
- Vorticity → 2 slow magnetosonic waves
- Vorticity → 2 Alfven waves

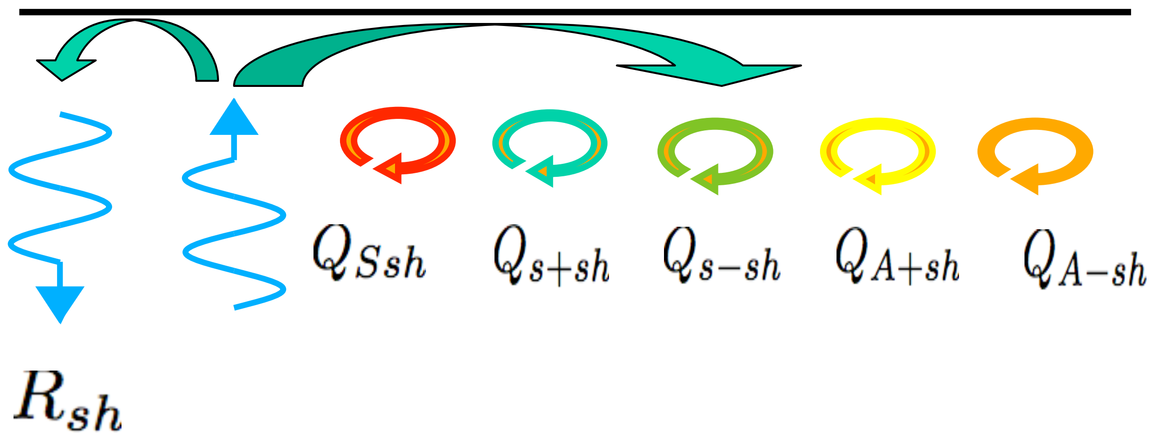
Vorticity and entropy decouple : vorticity can propagate!

But : slow or alfven waves cannot propagate up to the shock…

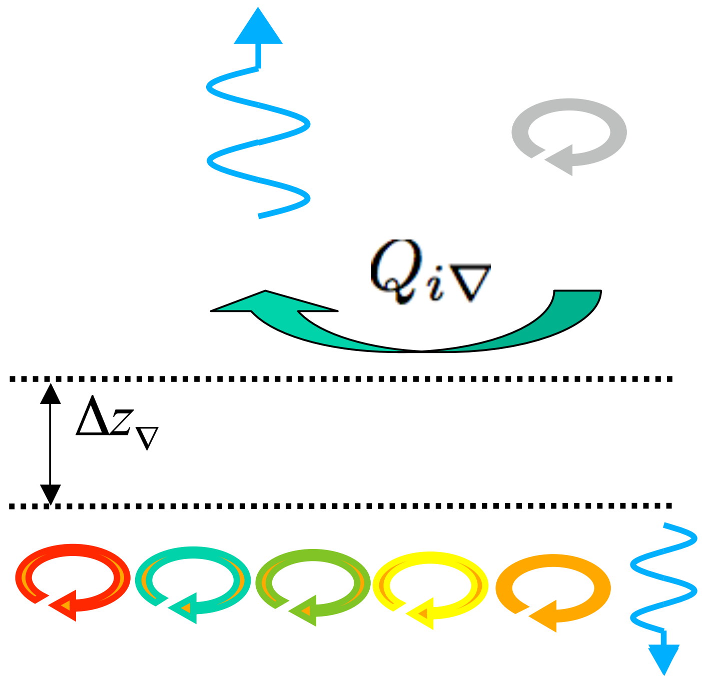
(Condition of evolutionarity of the fast MHD shock wave : Akhiezer *et al.* 1959)

The 6 cycles

Coupling at the shock



Coupling at the potential jump



Cycle constants :

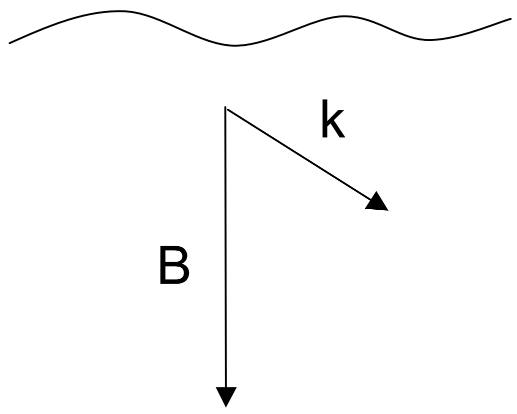
$$Q_i = Q_{ish} \times Q_{i\nabla}$$

$$R = R_{sh} \times R_{\nabla}$$

The non magnetic limit

- Which one of slow or Alfven waves contains the vorticity of the hydro cycle?

  It depends on the geometry… can be understood by geometric arguments.
  - If B perp shock : slow waves
  - If B // shock :
    - If k_{tr} // B : slow waves
    - If k_{tr} perp B : Alfven waves
    - If k_{tr} oblique : both
- What is the relative contribution of the vortical and entropic cycles ?
  - Depends on the details of the coupling region : $c_{in}/c_{out}, \Delta z_{\nabla}$
  - Usually in this set up : $Q_S/Q_{vort} \sim 2\text{-}5$



Formalism for studying the 6 cycles

$$Qe^{i\omega\tau_Q} + Re^{i\omega\tau_R} = 1$$

↓

$$Q_S e^{i\omega\tau_S} + Q_{s+} e^{i\omega\tau_{s+}} + Q_{s-} e^{i\omega\tau_{s-}} + Q_{A+} e^{i\omega\tau_{A+}} + Q_{A-} e^{i\omega\tau_{A-}} + Re^{i\omega\tau_R} = 1$$

entropy | Slow waves | Alfven waves | fast waves

Usefull quantities :

$$Q_{tot} \equiv \sum_i Q_i e^{i\omega_r\tau_i} + Re^{i\omega_r\tau_R} \qquad \omega_i \sim \frac{\log Q_{tot}}{\tau}$$

$$Q_{vort} \equiv Q_{s+} e^{i\omega_r\tau_{s+}} + Q_{s-} e^{i\omega_r\tau_{s-}} + Q_{A+} e^{i\omega_r\tau_{A+}} + Q_{A-} e^{i\omega_r\tau_{A-}}$$

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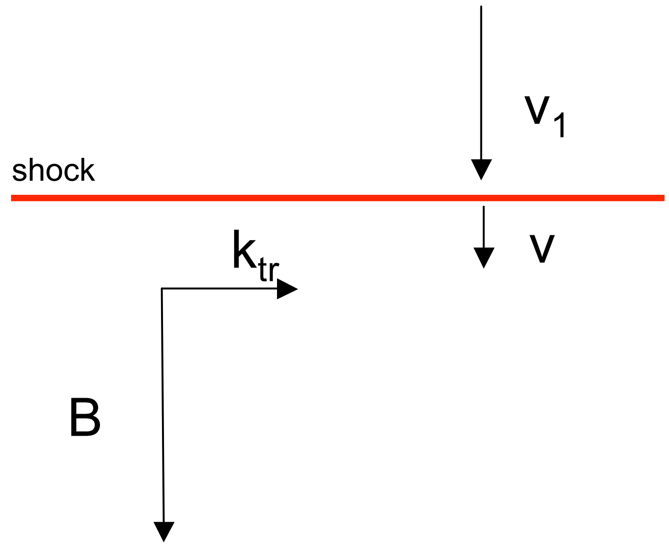
Magnetic field perpendicular to the shock

- Corresponds to a radial field in core collapse (split monopole)
- If $v=v_A$: a singular point appears which is unstable (Williams 1975)

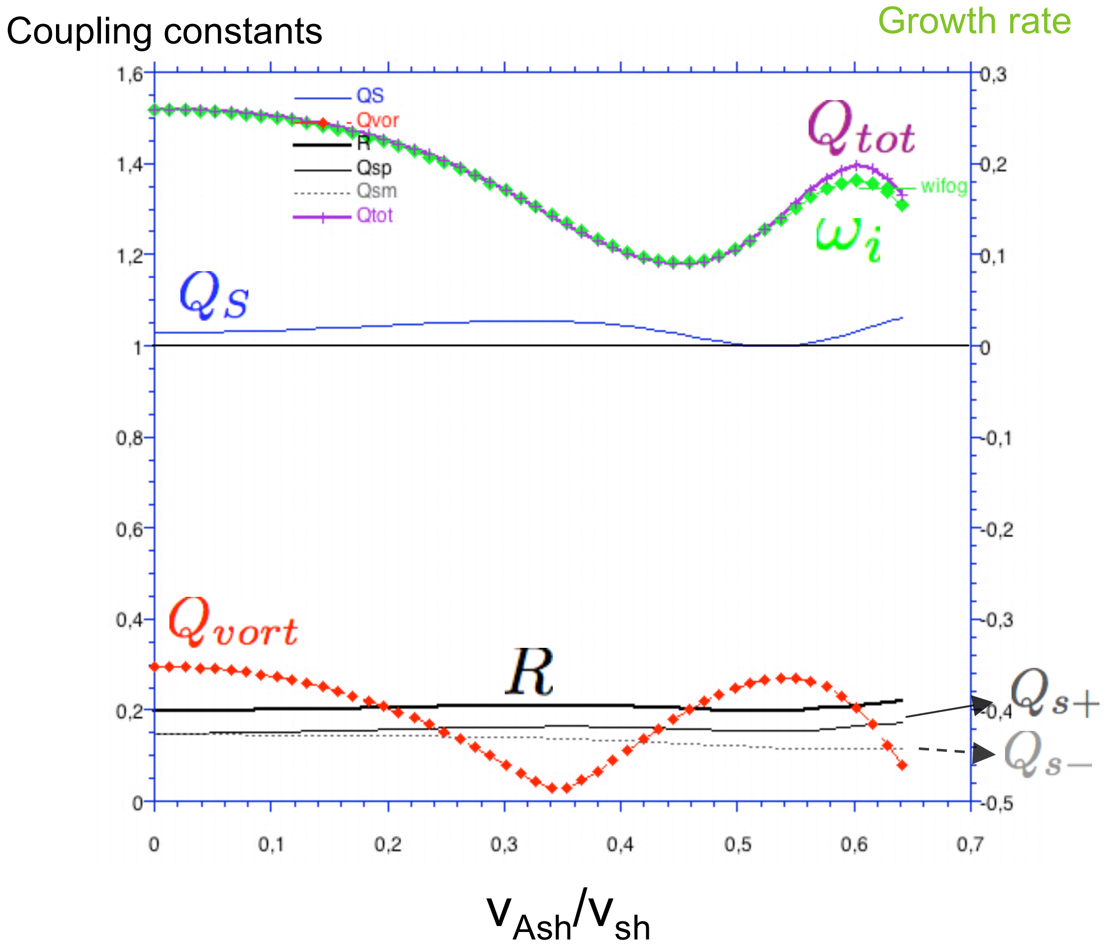
  ➔We restrict our study to $v_A<v$ everywhere
- 2D perturbations are enough to describe the flow : Alfven waves are not produced at the shock
- Vorticity propagates through slow waves along the magnetic field direction



n=1

- Effect on the coupling : not much…
  - Q_{s+} and Q_{s-} change very little
- Dominant effect : interferences between the 3 cycles : Q_S, Q_{s+}, Q_{s-}

→ Stabilization…



n=6, h=6

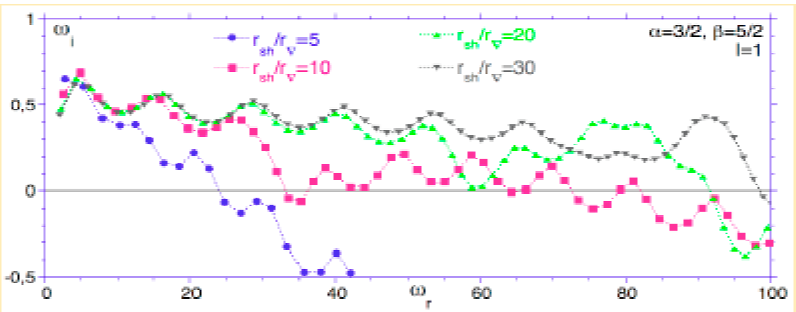
–Slow « + » : v-v_A $$k_z \sim \frac{\omega}{v - v_A}$$

–Slow oscillations

–Slow « - » : v+v_A $$k_z \sim \frac{\omega}{v + v_A}$$

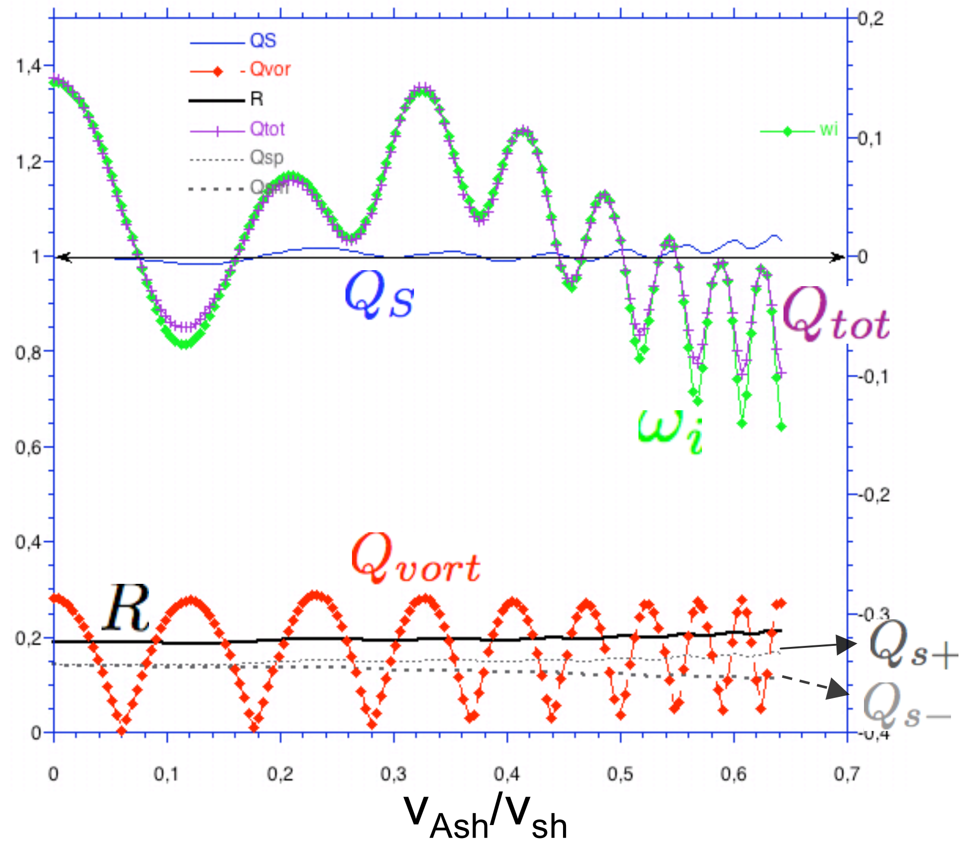
–Fast oscillations

Similar to the oscillations in the eigenspectrum : due to the acoustic interferences (Foglizzo et al 2007)



Coupling constants

Growth rate



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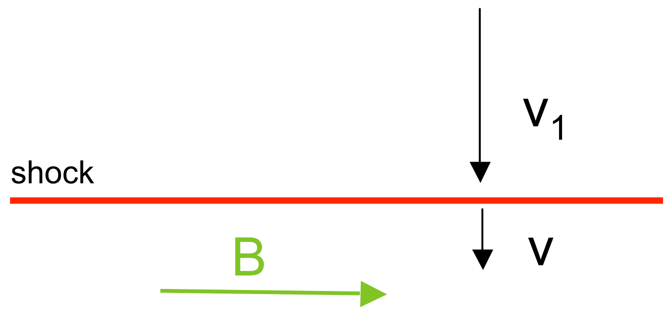
Magnetic field parallel to the shock

- Transverse magnetic field is amplified by compression through the shock and in a decelerating region
- Winding by rotation creates this geometry (cf magnetic explosions)

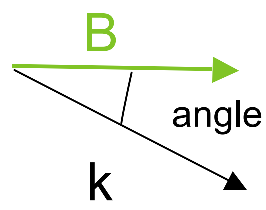
  → More realistic than a perpendicular field

- Alfven waves propagate only in a transverse direction
- Slow waves propagate mainly along the field line : transverse direction as well

$$k_z \sim \frac{\omega}{v} \pm k_{//B} \frac{v_A}{v}$$

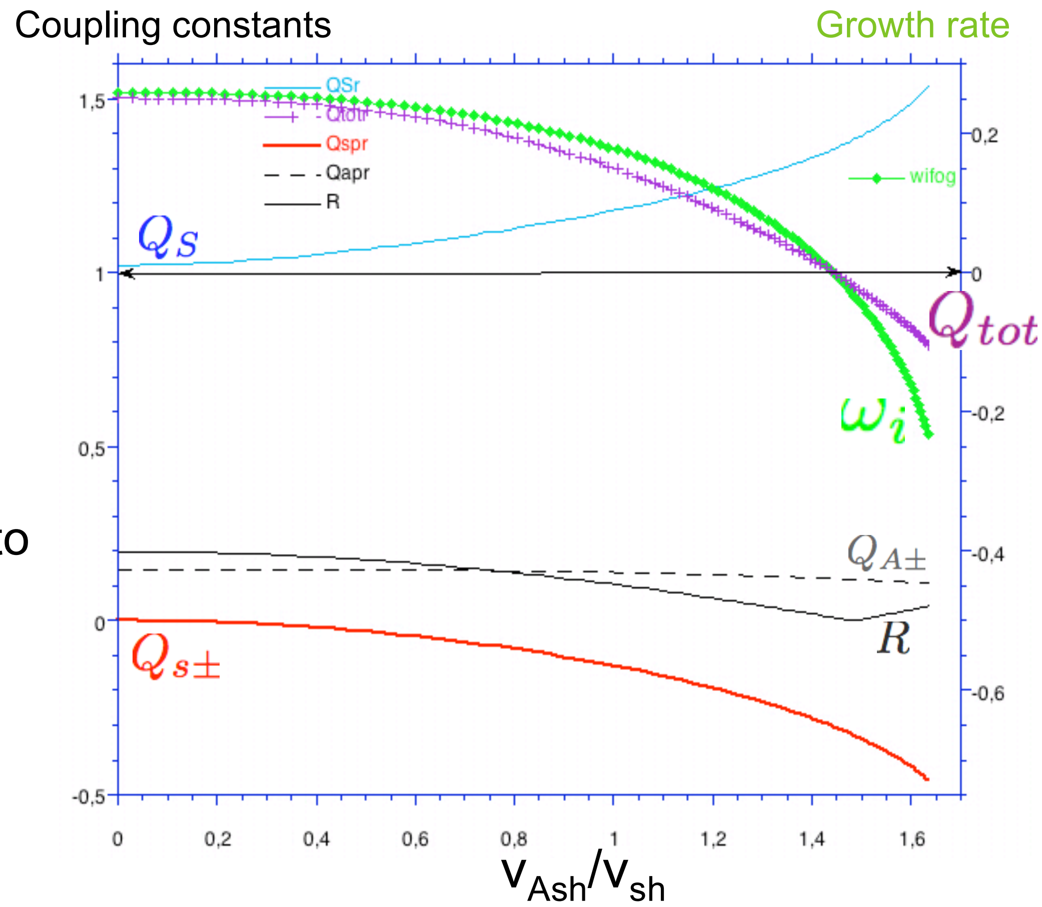


Viewed from above :



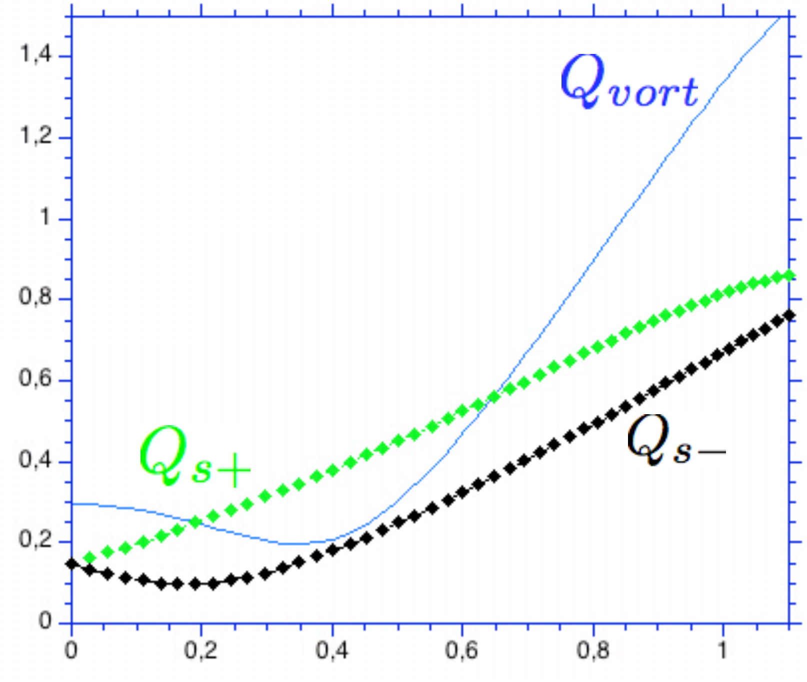
K perpendicular to B

- Slow and Alfven waves simply advected
- If vA=0 : only Alfven waves
- With a magnetic field : slow waves excited but transverse velocity associated to it = 0
- Stabilizing effect due to the slow waves which have an opposite effect to entropy…



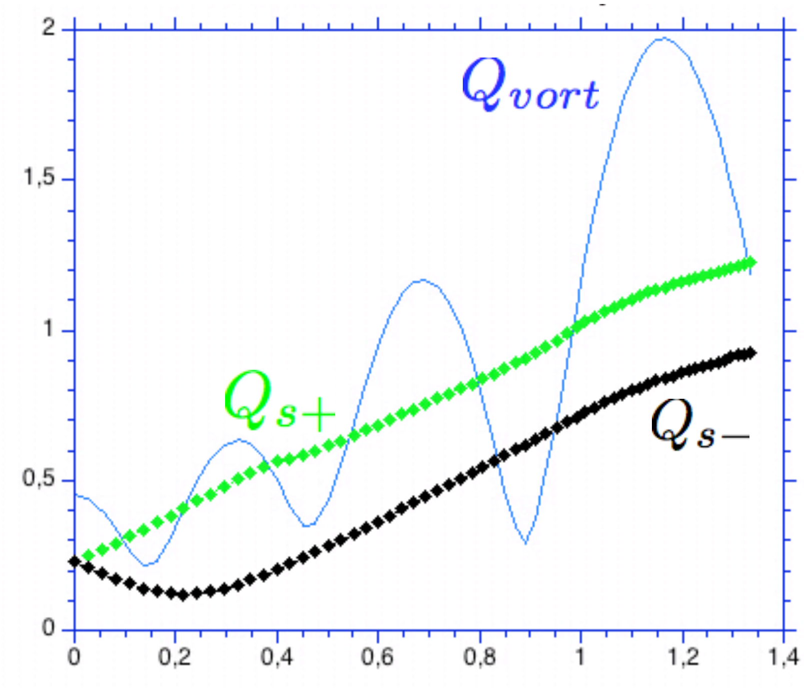
K parallel to B : the 'vortical' cycles

Q_s n=1



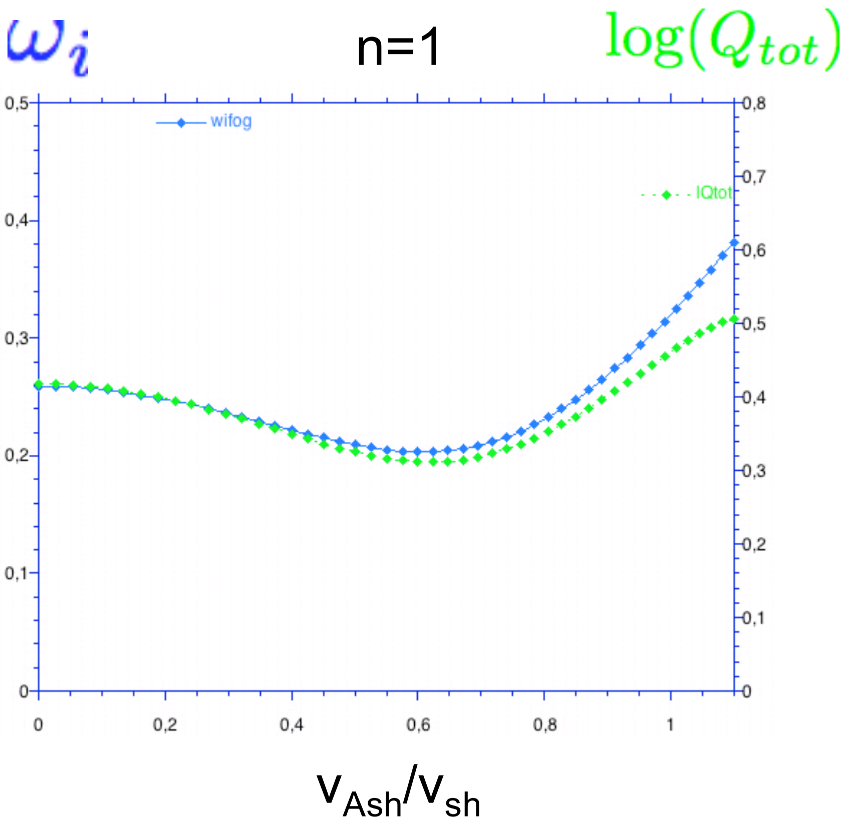
v_{Ash}/v_{sh}

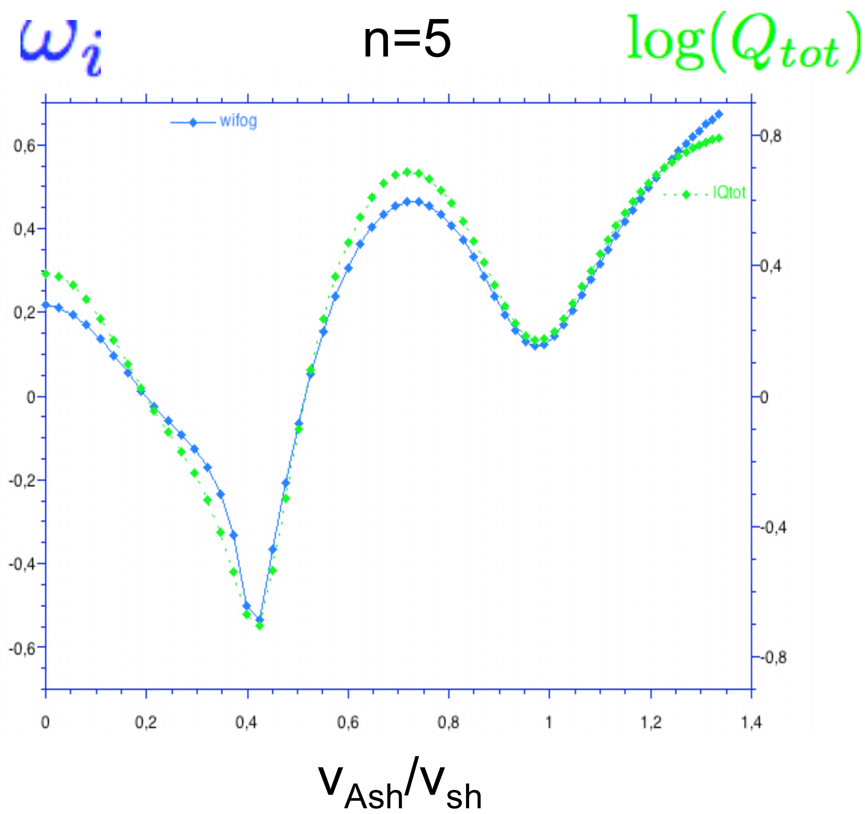
Q_s n=5



v_{Ash}/v_{sh}

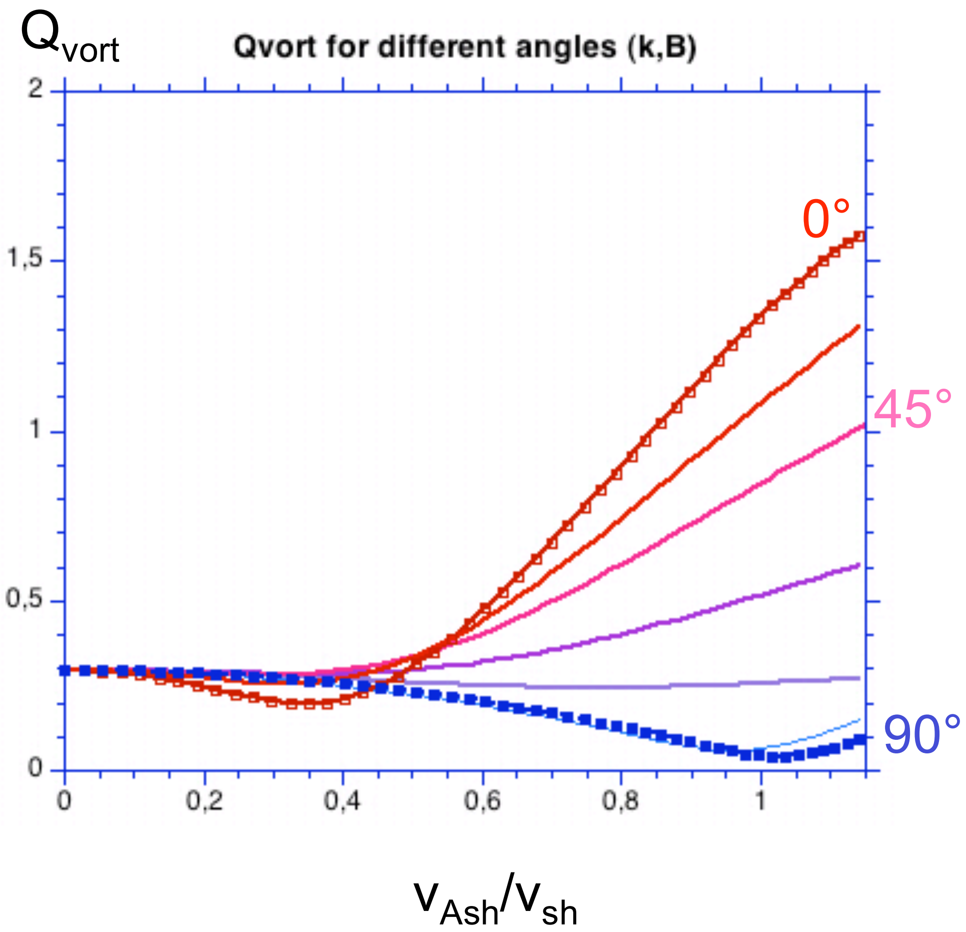
Growth rate : stabilization or destabilization

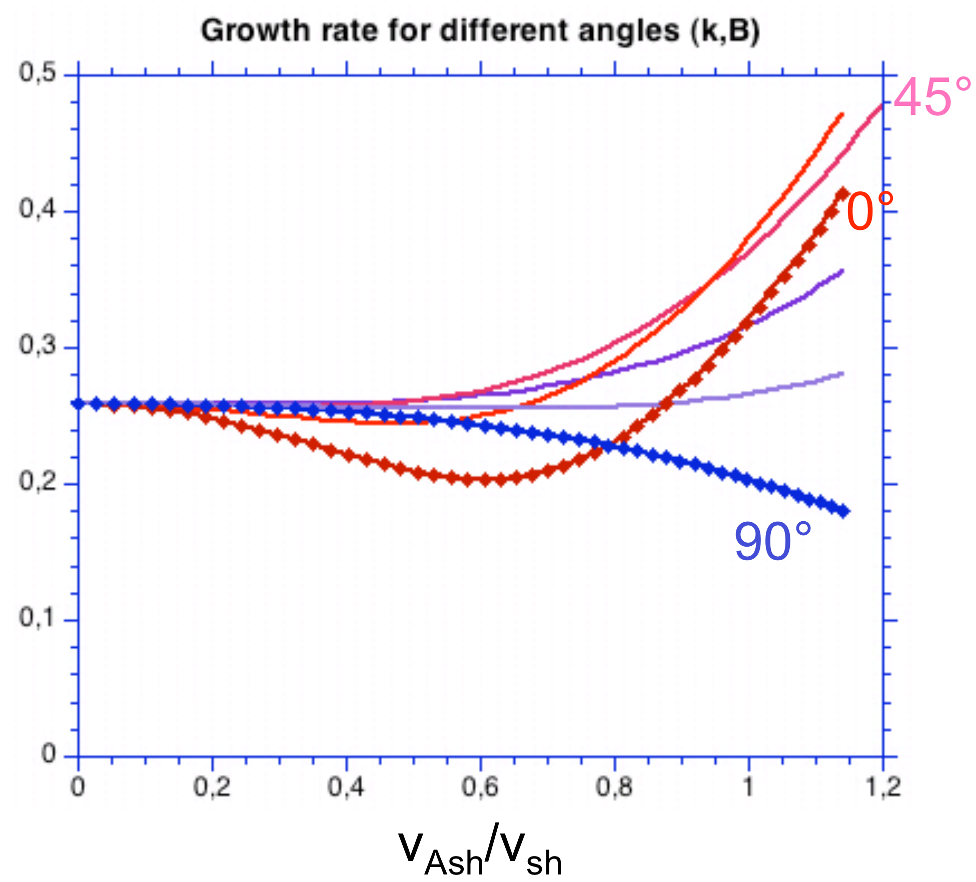




Stabilization at small B, destabilization for strong B

Oblique wave vector





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Summary

- Effect always of the second order (~ v_A^2)
- No significant effects on the acoustic cycle : remains stable.
- Vorticity can propagate through up to 4 different waves : not in phase with entropy or with each other…

  → Interference between the different cycles : stabilizing effect…

- If B perpendicular to the shock :

  no strong effect on the coupling, only the interference effect

- If B parallel to the shock :
  - K perp B : magnetic perturbation associated to entropy

    → stabilization…

  - K // B : strong amplification of vortical cycles

    → Possible destabilization

Towards more realism

- Missing ingredients : cooling and convergence (and gravity).
- Effect of cooling : Favor the vortical cycle compared to the entropic cycle (?)
  - Effects due to the vortical cycle are accentuated :
    - If B perpendicular : more stabilized
    - If B // :
      - If k // B : the destabilization could be much more efficient
      - If k.B : effect independant of vorticity (associated to entropy) : less stabilized?
  - New effect?
- Effect of convergence :
  - Strength needed to have a strong effect? v_A/v at the shock or deep inside are very different…
- Future works :
  - Include cooling, convergence in cylindrical geometry
  - Magnetic field + rotation…