Stochastic Features of Gravitational-wave Signals in 3D SN Explosions with SASI and GWs from MHD Explosions

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Plan and Timetable of my talk

Section 1: General Introduction (5min)

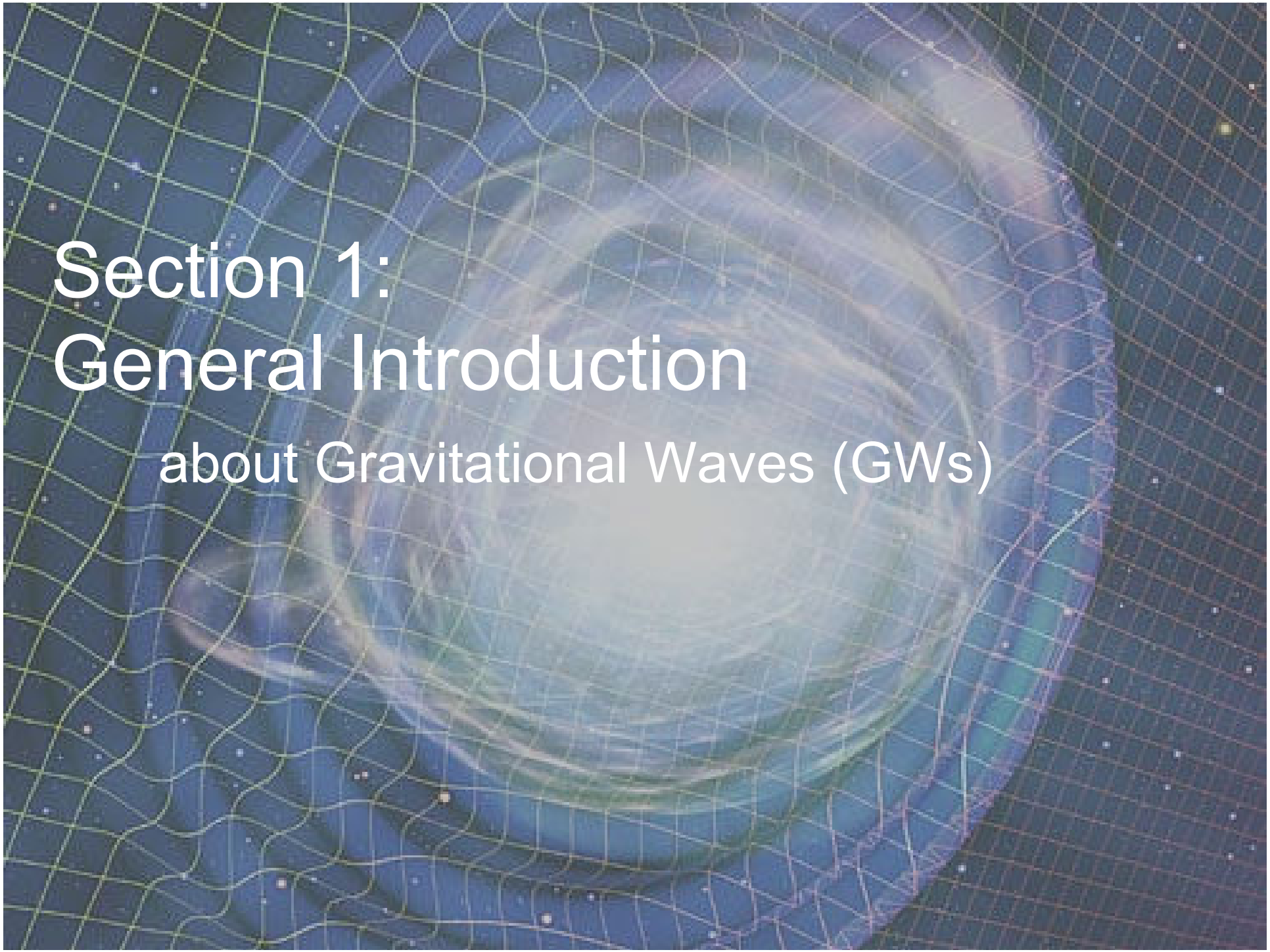
Gravitational Waves (GWs) from,

Section 2: MHD supernovae with rapidly rotating and magnetized cores (~20min)

Tomoya Takiwaki (RESCEU),
Kenta Kiuchi(Waseda), Nobutoshi Yasutake(NAOJ)

Section 3: 3D Supernova Explosions with SASI (～20min)

Wakana Iwakami, Naofumi Ohnishi (Tohoku Univ.)
Shoichi Yamada(Waseda Univ.)



Section 1:
General Introduction

about Gravitational Waves (GWs)

Fundamental about GWs

☆Gravitational Wave (GW) is
"a ripple" of space-time, predicted by
Einstein's theory of general relativity.

☆ GWs are emitted when matter
moves with acceleration.
(in analogous to the EM waves
from accelerating charged particles)

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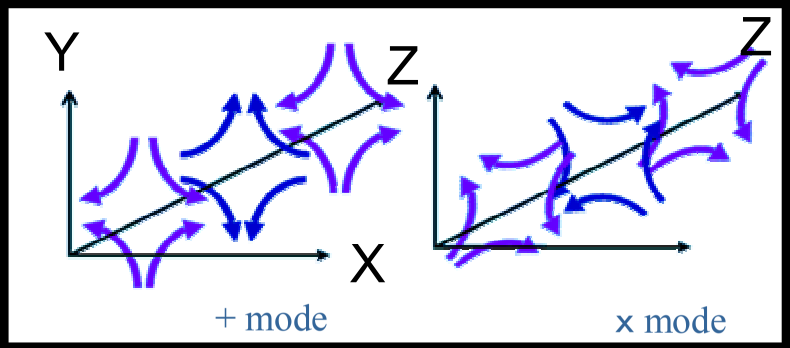
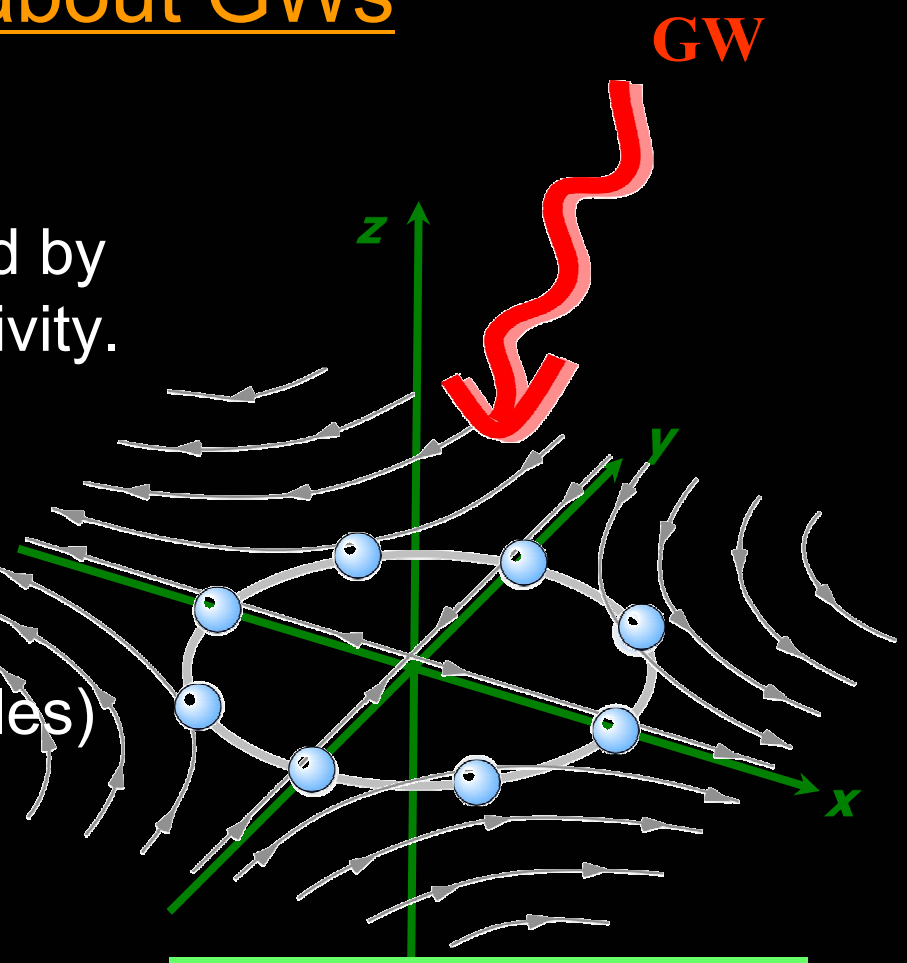


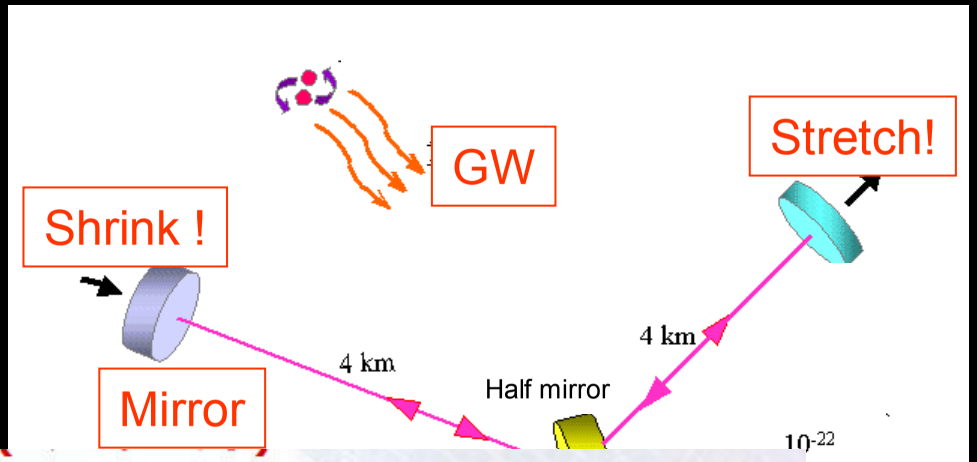


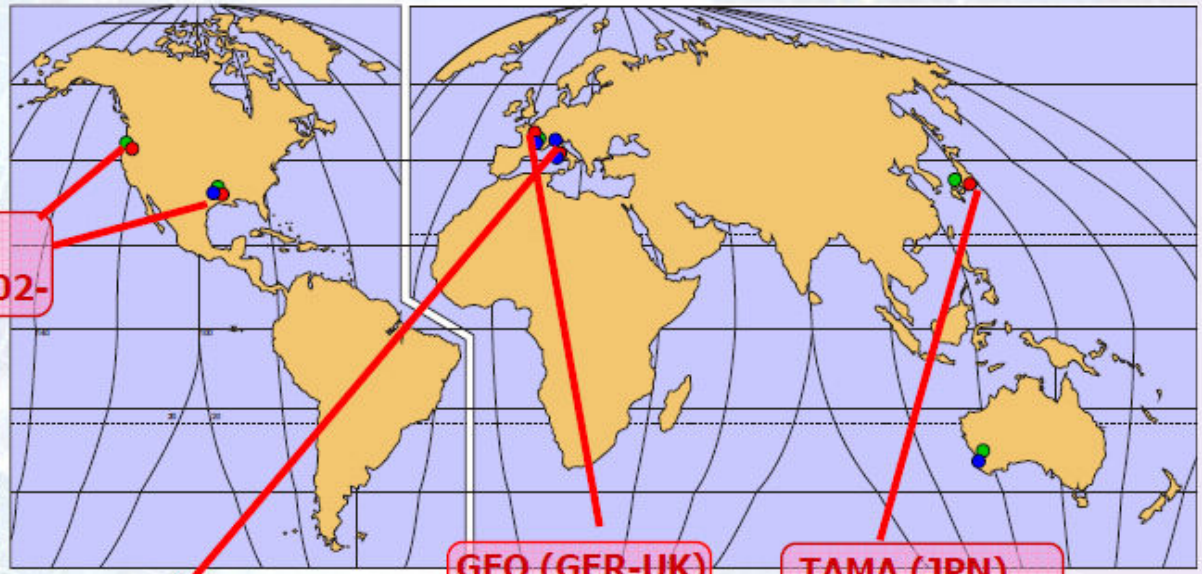
The GWs change the proper length (XY plane) as they pass by.

GWs have two polarizations

Towards the *direct* observations of GWs

4 projects are running





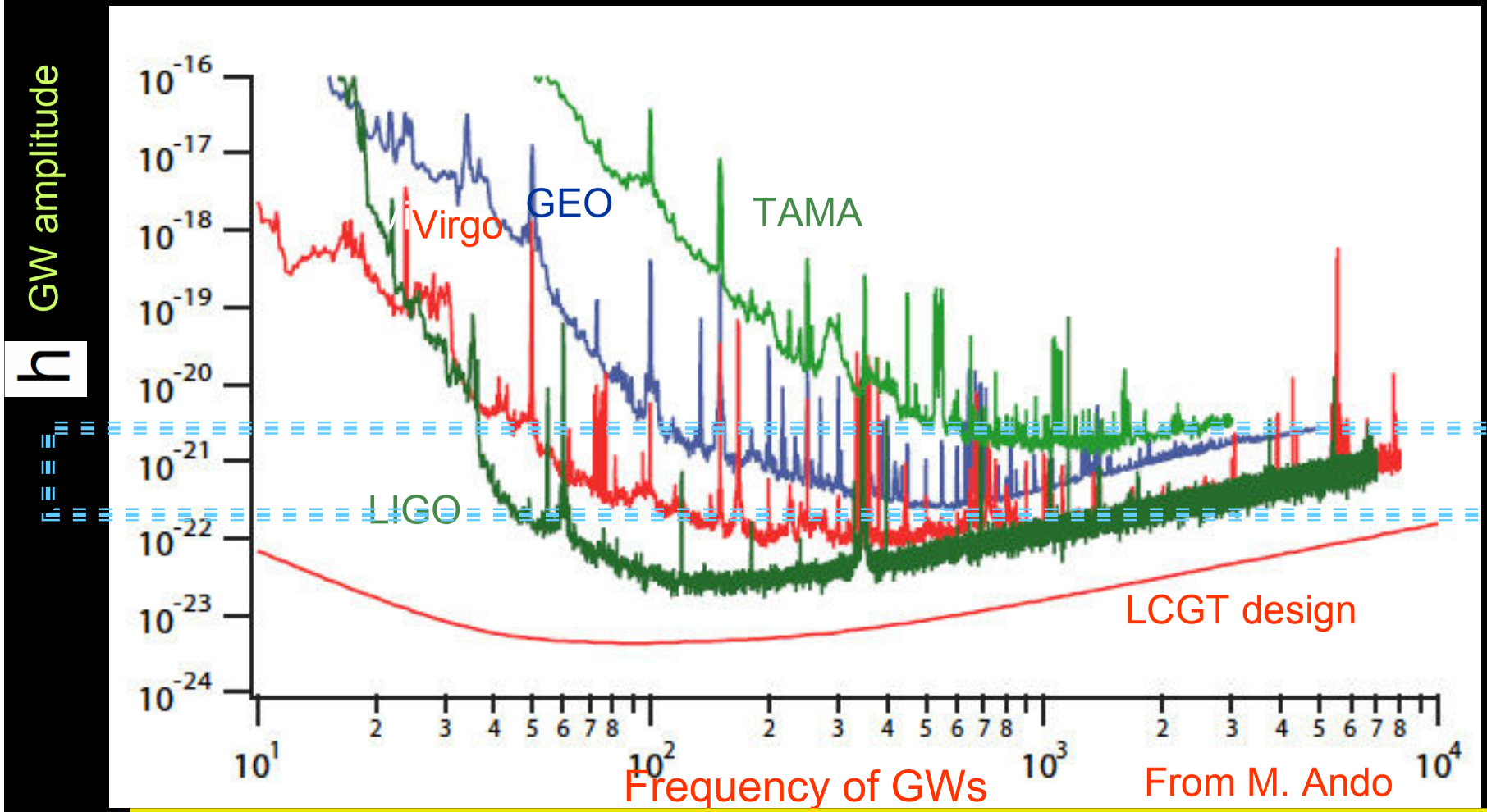
LIGO (USA)
4km x 2. 2002-

VIRGO (ITA-FRA)
3km. 2004-

GEO (GER-UK)
600m. 2002-

TAMA (JPN)
300m. 1999-

Sensitivity curves for the laser interferometers



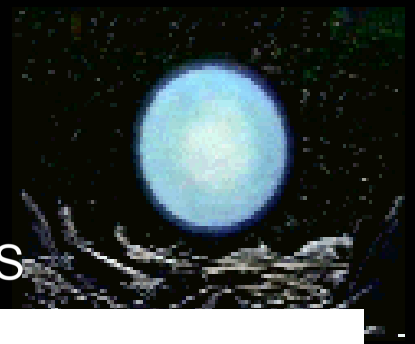
10**{-21~20} is the typical threshold for the currently running detectors.

Expected GW amplitudes from Stellar Collapse

GW amplitude from the quadrupole formula

$$h_{ij} = \frac{2G}{c^4 R}\frac{\partial^2}{\partial t^2}Q_{ij} \sim \frac{R_s}{R}\left(\frac{v}{c}\right)^2$$

Quadrupole moment



Typical values at the formation of NS

$$R_s = 3\ \mathrm{km}\left(\frac{M}{M_\odot}\right) \quad v/c = 0.1 \quad R = 10\ \mathrm{kpc}$$

$$h \sim 10^{-20}$$

- SN in our galaxy is the target of GWs.

More correctly,

$$h_{ij} = \epsilon\,\frac{R_s}{R}\left(\frac{v}{c}\right)^2$$

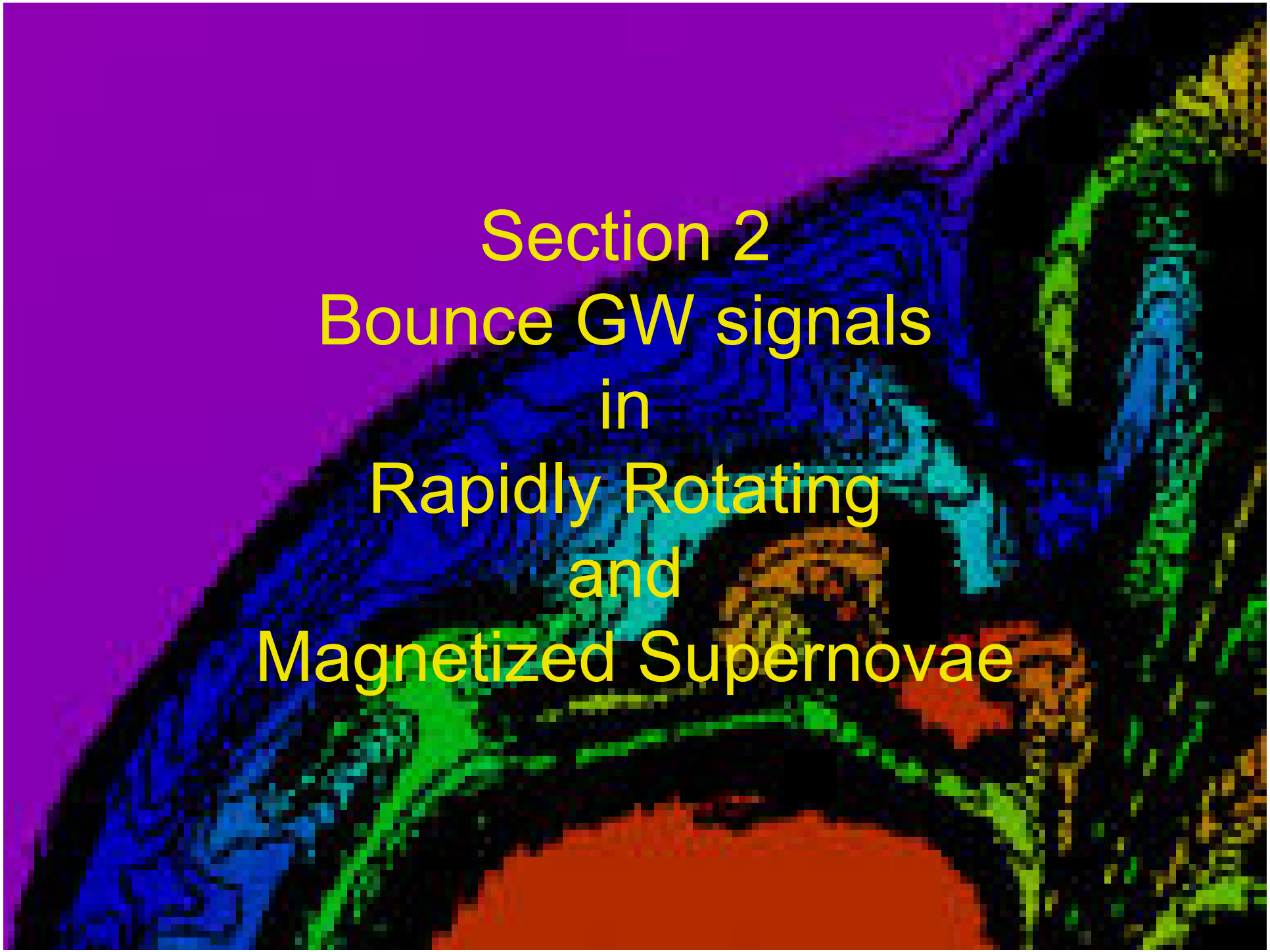
ϵ represents the degree of anisotropy.

If collapse proceeds spherically, $\epsilon = 0$
no GWs can be emitted.

What makes the collapse-dynamics deviates from spherical symmetry ?

When & How GWs are emitted from SNe ?

Origins	when	Why ? (Cause of asphericity)
Bounce origin	At bounce (duration、~100 msec)	Aspherical motions of inner core induced by rapid rotaion
Convection origin	After bounce (duration、~ 1 sec)	Aspherical motions of outer core after bounce
Neutrino origin	After bounce (duration、~ 1 sec)	Aspherical radiations of neutrinos

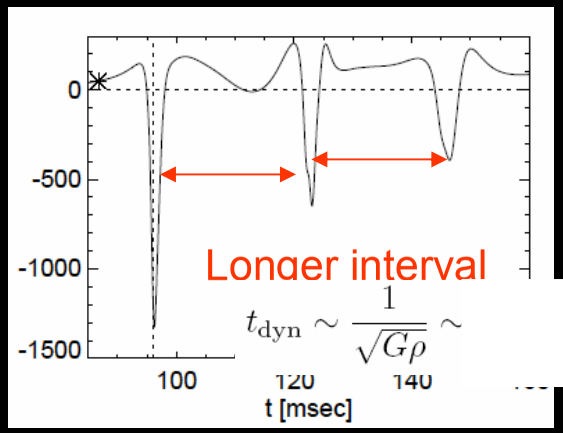


Section 2
Bounce GW signals in Rapidly Rotating and Magnetized Supernovae

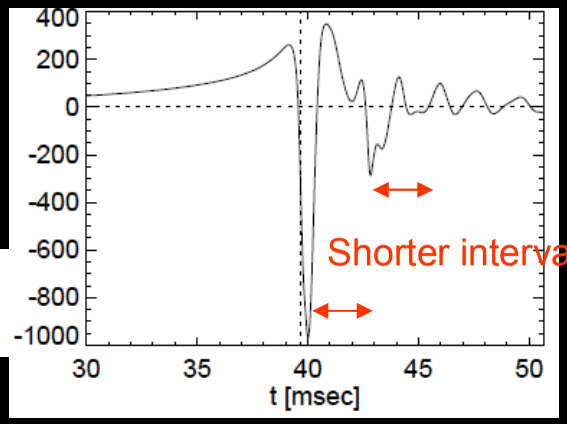
Three typical waveforms from Rotating SNe

See KK et al. (2006)for a review

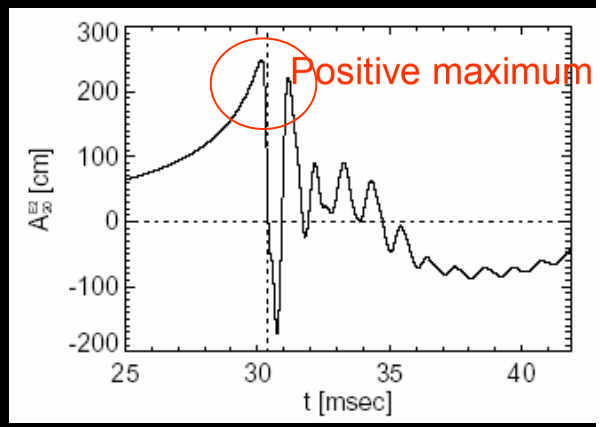
Type II



Type I



Type III



Zwerger & Mueller 1997

・Stiffness of EOS、

$P_{\rm cold} \propto \rho^{\Gamma_1}$

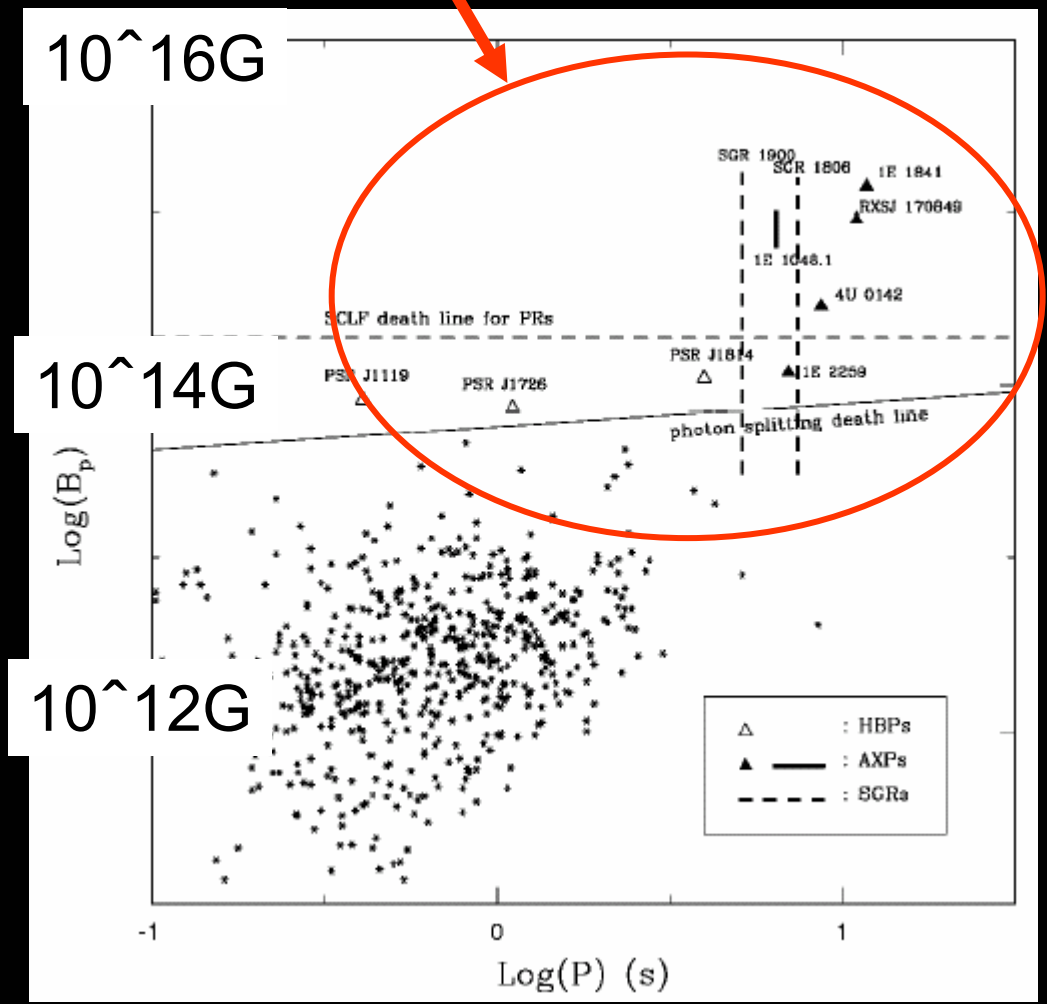
Γ_1 → SOFT

・Rotational profile

rapid → slow

Diff rotation large → Weak diff. rotation

Since Magnetars are minor…



Zhang et al. 00

Histrory of Magnetized Supernovae

Takiwaki et al.
Mikami et al
Scheidegger et al
Dessert et al.
Burrows et al.
Liebendoefer et al
Obergaulinger
Sawai et al.

Numer of papers

THE MAGNETIC FIELD STRENGTH IS PROPORTIONAL TO OUR IGNORANCE.

John Hawley

KK et al.
Yamada et al.

- Stellar evolution cal. is still uncertain.
- Possible link to the GRBs

LeBlanc & Wilson

1970 1976 1979 1983 1984 2004 2005 year

20year

Effects of Magnetic fields on the waveforms

KK et al. 2004 PRD

Derivations of quadrupole formula including B-field contributions

Dimensionless amplitude

$$h_{ij}^{\mathrm{TT}}(R) = \frac{2G}{c^4}\frac{1}{R}\frac{d^2}{dt^2}I_{ij}^{TT}\left(t - \frac{R}{c}\right)$$

Mass quadrupole

$$I_{ij} = \int \rho_*(x,t)\left(x_i x_j - \frac{1}{3}x^2\delta_{ij}\right)d^3x$$

$$\rho_* = \rho + \frac{B^2}{8\pi c^2}$$

The amplitude becomes,

$$h_{\theta\theta}^{\mathrm{TT}} = \frac{1}{8}\left(\frac{15}{\pi}\right)^{1/2}\sin^2\alpha\,\frac{A_{20}^{\mathrm{E2}}}{R}$$

$$A_{20}^{\mathrm{E2}} \equiv A_{20\,,\mathrm{quad}}^{\mathrm{E2}} + A_{20\,,\mathrm{Mag}}^{\mathrm{E2}}$$

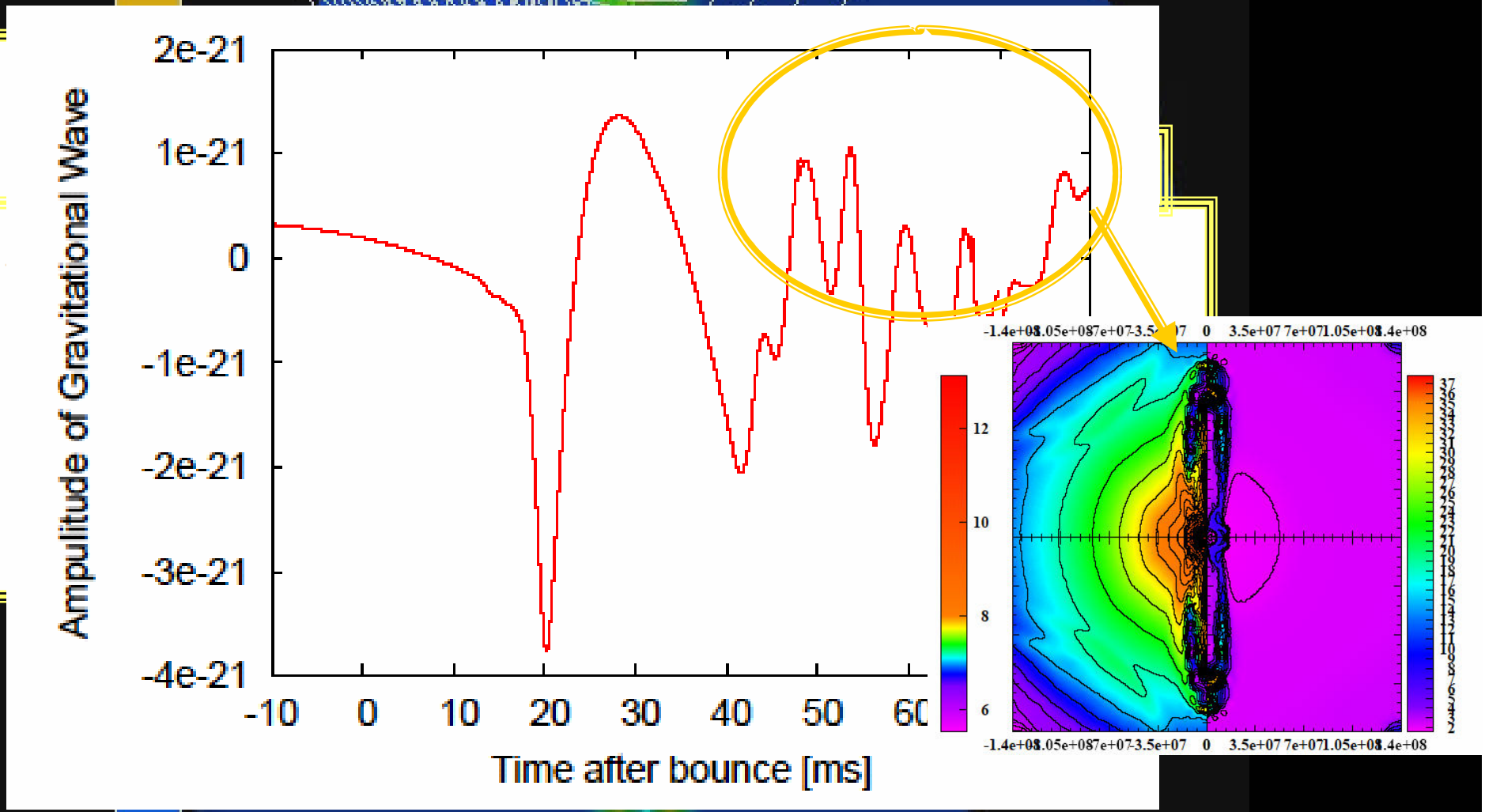
$A_{20\,\mathrm{Mag}}^{\mathrm{E2}} \equiv A_{20\,j\times B}^{\mathrm{E2}} + A_{20\,\rho_{\mathrm{m}}}^{\mathrm{E2}}$ is the contribution from the magnetic field:

$$A_{20\,j\times B}^{\mathrm{E2}} = \frac{G}{c^4}\frac{32\pi^{3/2}}{\sqrt{15}}\int_0^1 d\mu\int_0^\infty r^3\,dr\Big[(3\mu^2-1)\,\frac{1}{c}\,(\boldsymbol{j}\times\boldsymbol{B})_r - 3\mu\sqrt{1-\mu^2}\,\frac{1}{c}\,(\boldsymbol{j}\times\boldsymbol{B})_\theta\Big], \tag{18}$$

$$A_{20\,\rho_{\mathrm{m}}}^{\mathrm{E2}} = \frac{G}{c^4}\frac{32\pi^{3/2}}{\sqrt{15}}\int_0^1 d\mu\int_0^\infty dr\,\frac{1}{8\pi c}\frac{d}{dt}\Big[\frac{\partial}{\partial\theta}[B_r r^3(3\,\mu^2-1)]E_\phi - \frac{\partial}{\partial r}[B_\theta r^3(3\mu^2-1)]rE_\phi + \frac{\partial}{\partial r}[B_\phi r^3(3\mu^2-1)]rE_\theta - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}[B_\phi\sin\theta r^3(3\mu^2-1)]E_r\Big]. \tag{19}$$

Waveforms from Magneto-Driven Explosions

Takiwaki and KK in prep



GW memory from Jets

based on **Smarr PRD ('77)**

Gravitational field of a static point of mass M (@ rest frame)

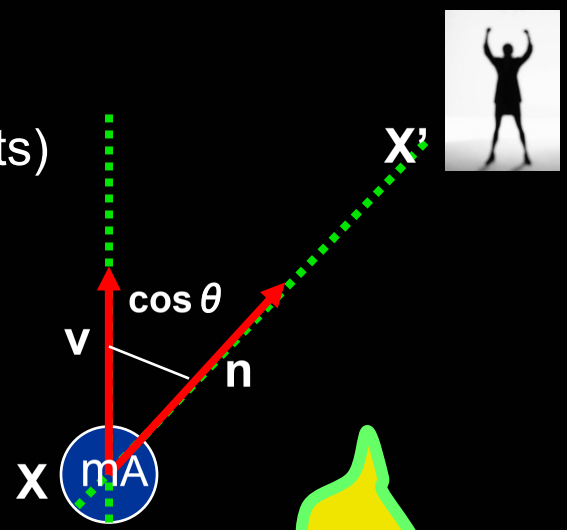
$$h_{00}^{TT} = \frac{4M}{r}, \quad h_{0j}^{TT} = h_{jk}^{TT} = 0$$

Lorentz boost ⇒ observer's rest frame

(p= γ u :4-momentum, k: null vector joining the 2 points)

$$k = r(1, \mathbf{n}), \quad p = M_A(\gamma, \gamma \mathbf{v})$$

$$h_{\mu\nu}^{TT} = \frac{4 p_\mu p_\nu}{-(k_\lambda p^\lambda)}$$

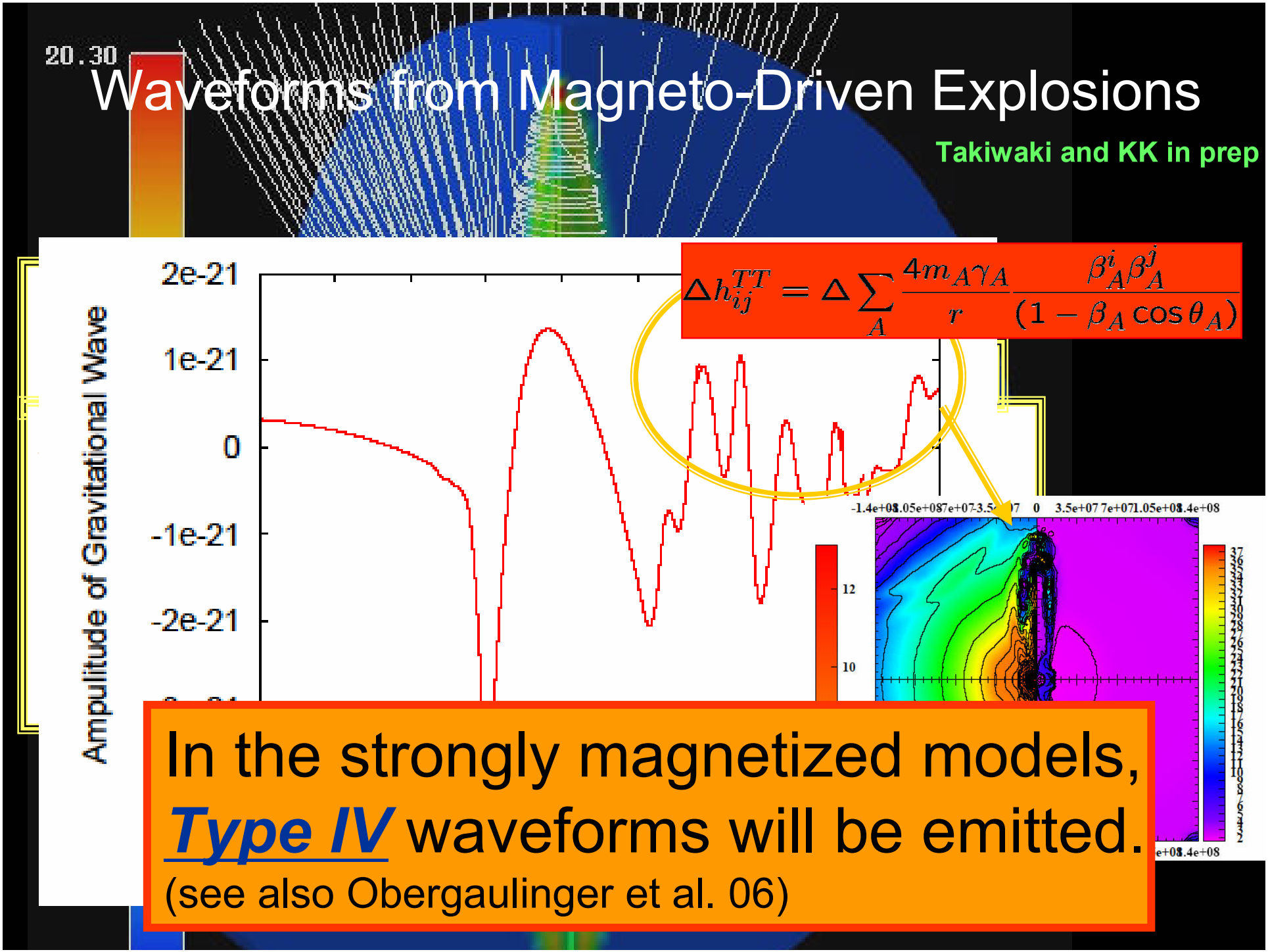


The GW is,

$$h_{ij}^{TT} = \frac{4M\gamma}{r} \frac{v_i v_j}{(1 - v\cos\theta)}$$

Waveforms from Magneto-Driven Explosions

Takiwaki and KK in prep



$$\Delta h_{ij}^{TT} = \Delta \sum_A \frac{4 m_A \gamma_A}{r} \frac{\beta_A^i \beta_A^j}{(1 - \beta_A \cos\theta_A)}$$

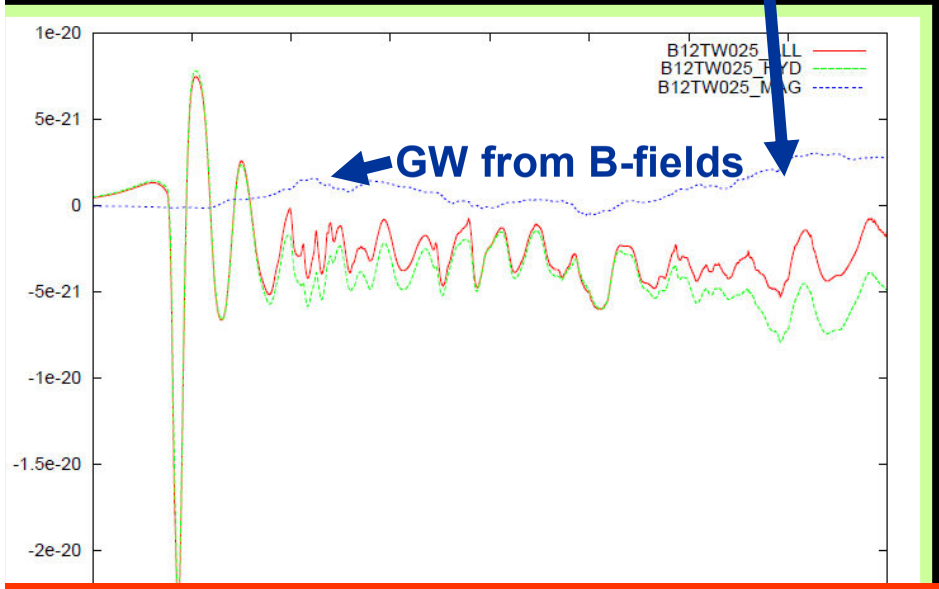
In the strongly magnetized models, ***Type IV*** waveforms will be emitted.

(see also Obergaulinger et al. 06)

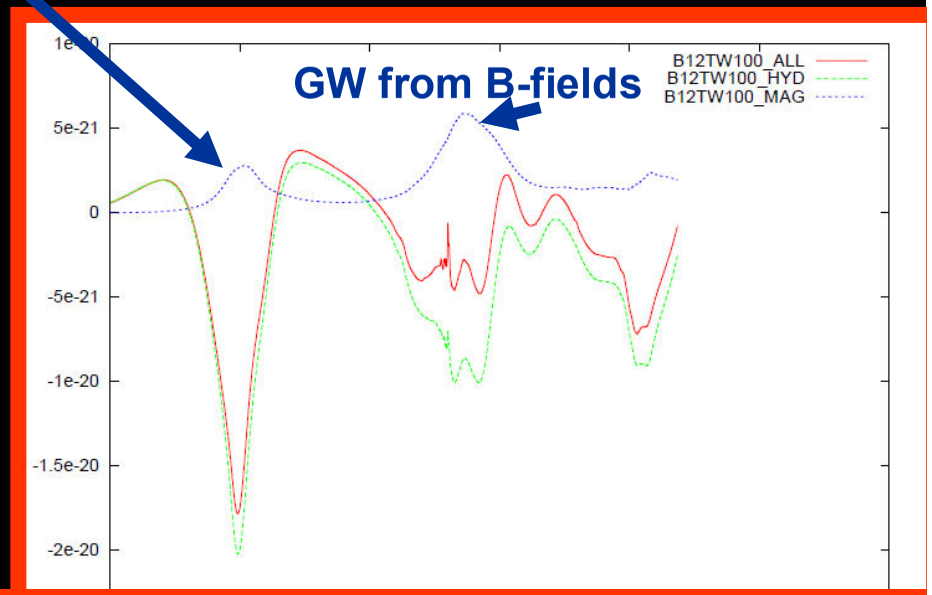
Magnetic Effects on GW signals

Takiwaki and KK in prep

Uniform Rotation



Differential Rotation



It is found that the type IV waveforms are generated due to the memory effect of jets and the magnetic contributions to the waveforms.

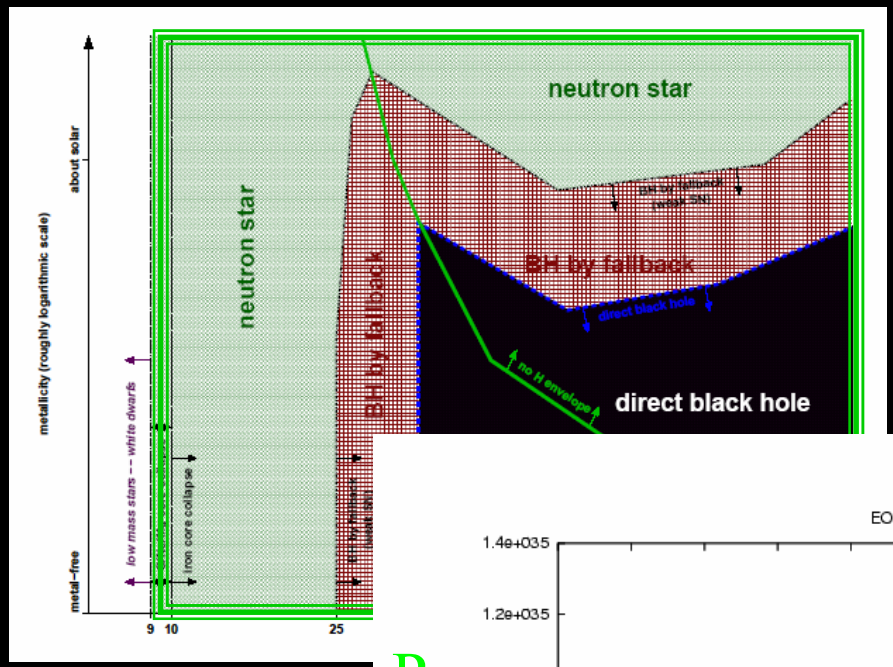
case.

$$\frac{B_c^2/8\pi}{\rho_c c^2} \sim 10\% \left(\frac{B_c}{\text{several} \times 10^{17}\,\text{G}}\right)^2 \left(\frac{\rho_c}{10^{13}\,\text{g cm}^{-3}}\right)^{-1}$$

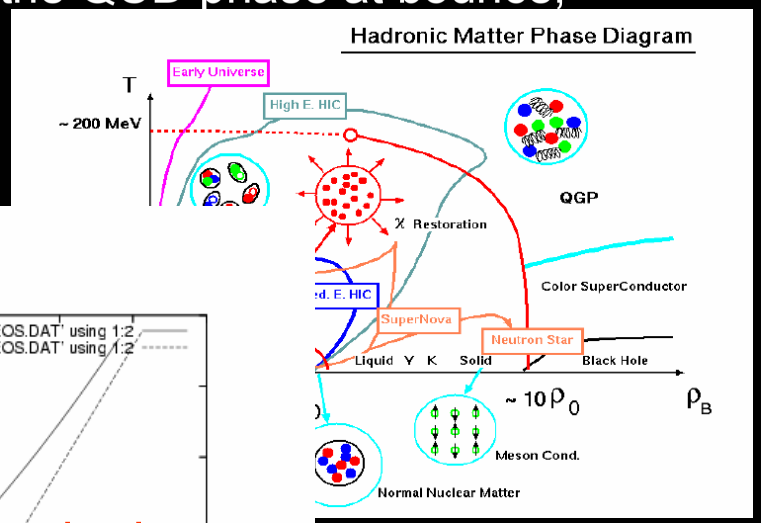
Effect of QCD phase transiton on GWs

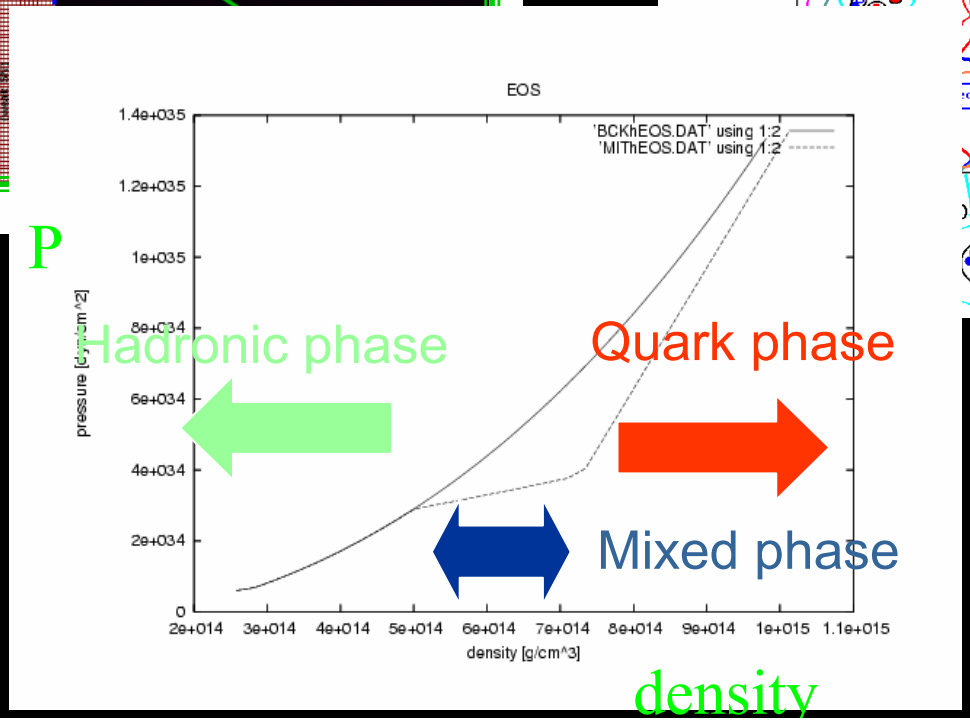
Yasutake, KK et al. PRD (2007)

During the collapse of massive stars,



The central core might experience the QCD phase at bounce,



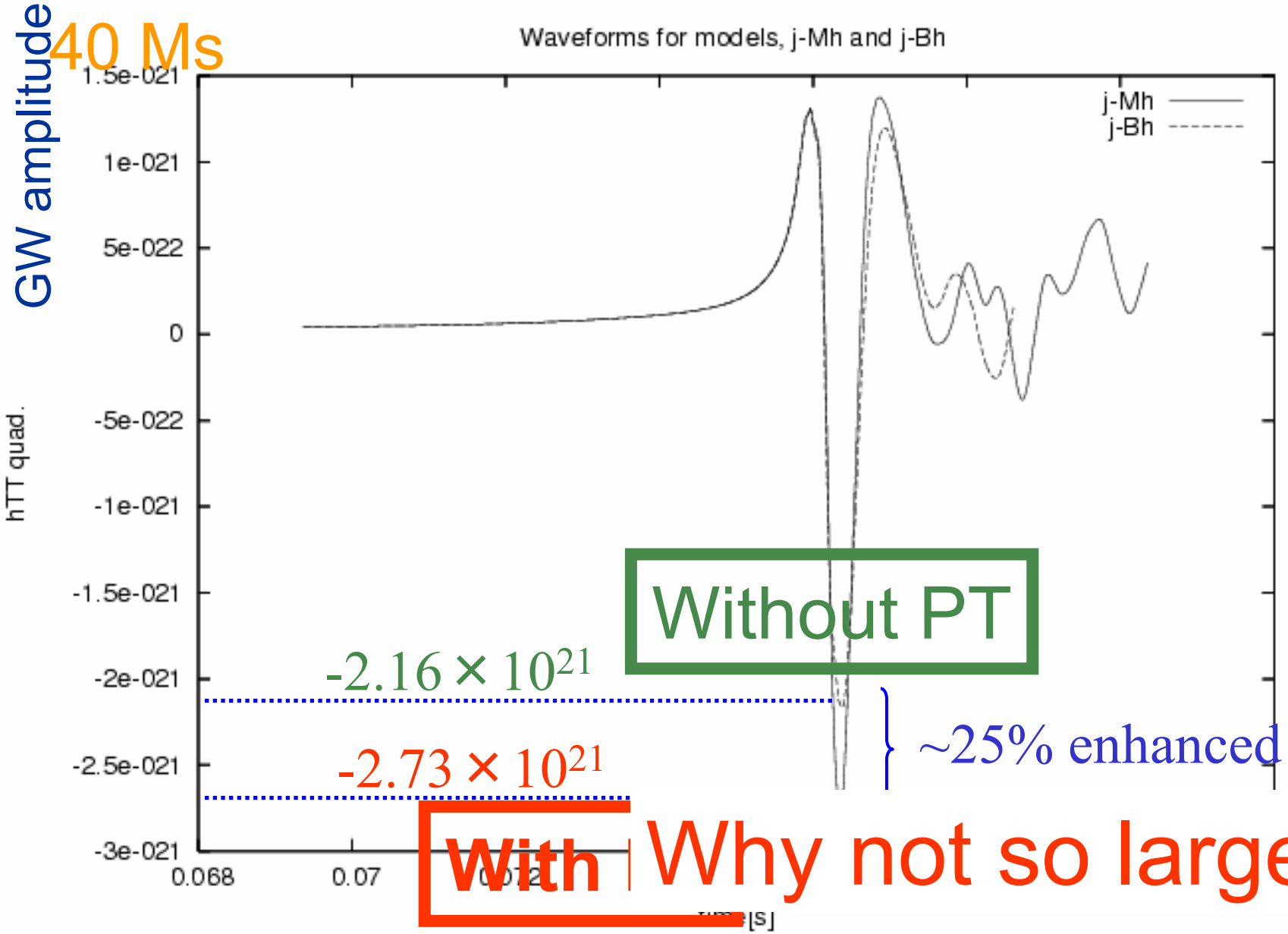


- Bag EOS

$$P = \frac{1}{3}(\rho_{\rm tol} - 4B),$$

$B^{1/4}$ (MeV)
164.8
163.8

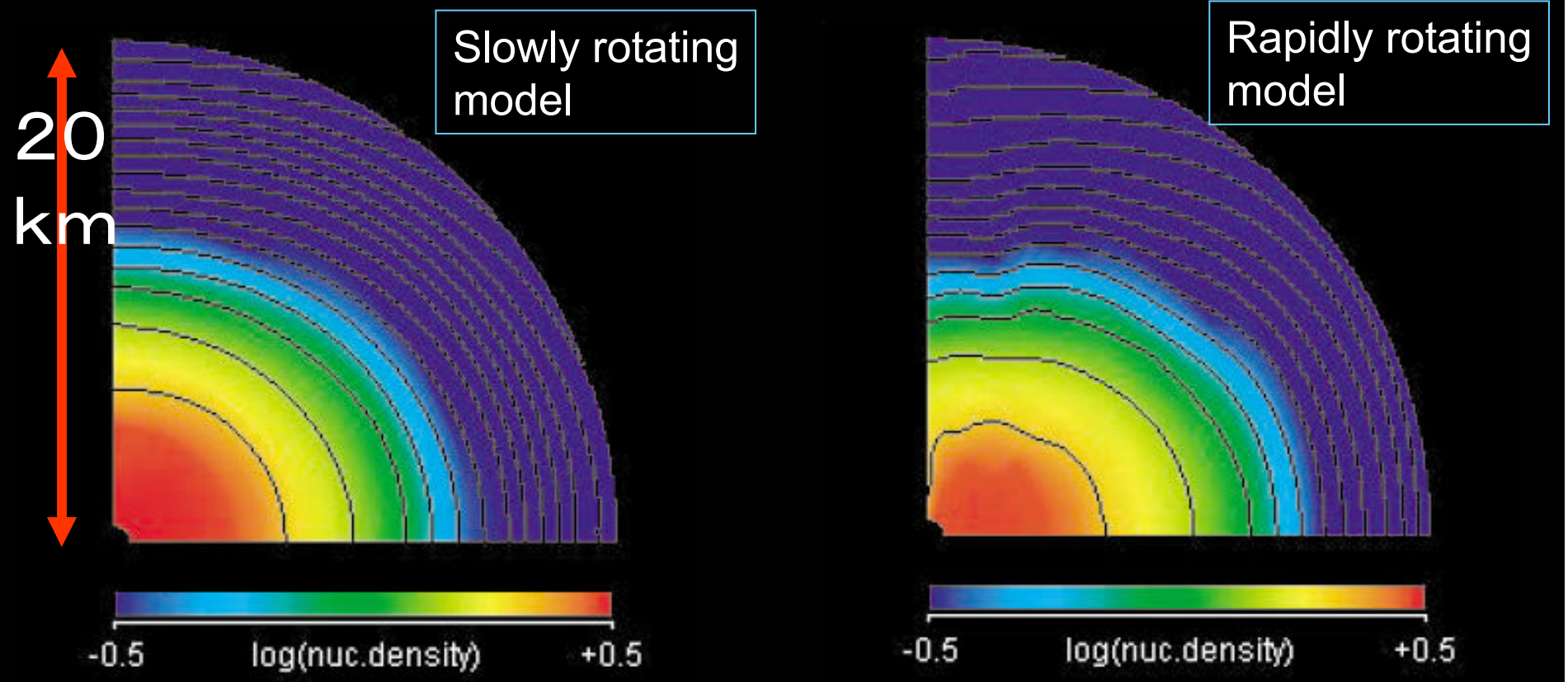
Yasutake, KK、Hashimoto, Yamada, PRD (2007)



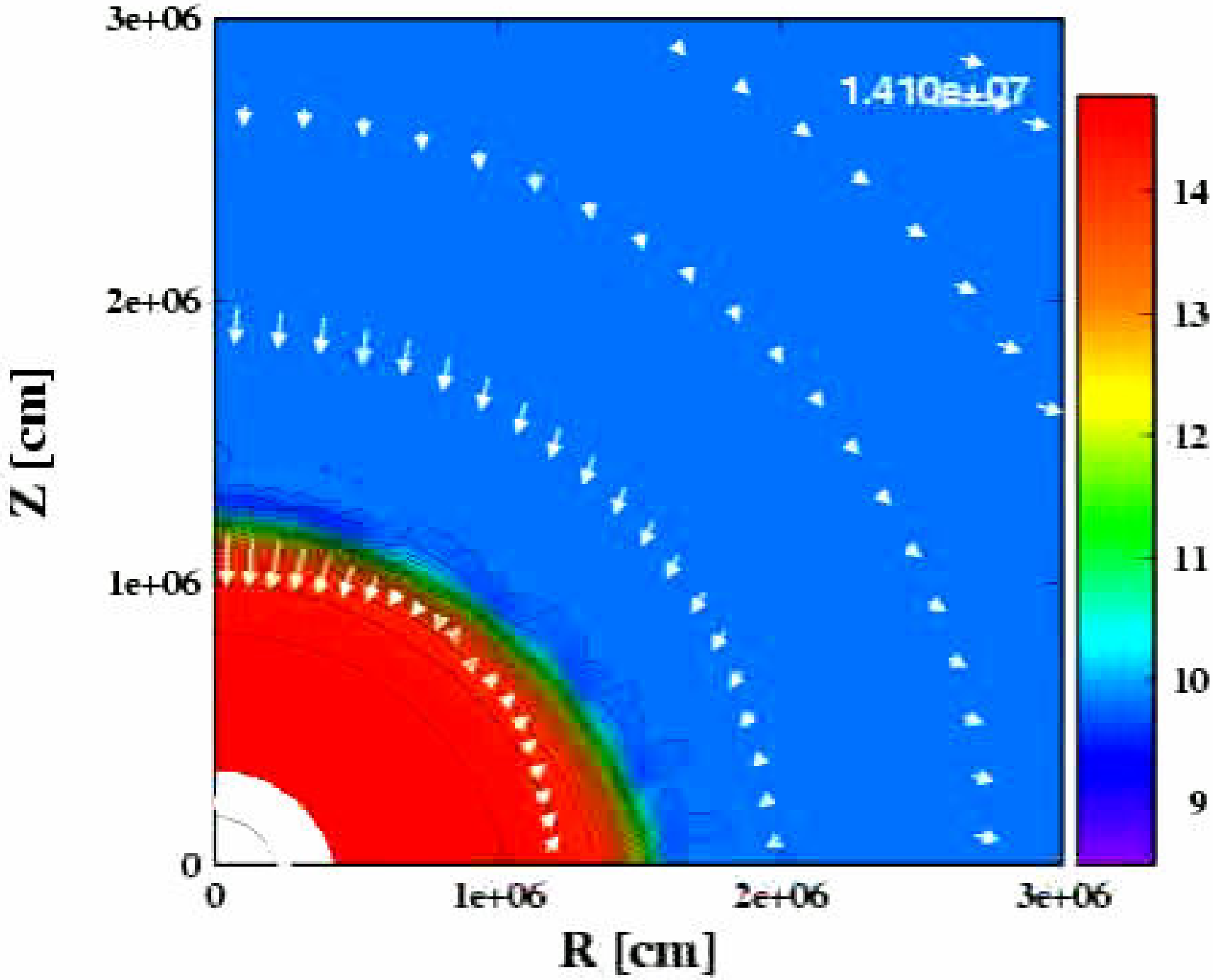
Why not so large ?

Density contour at the moment of Phase Transition

Yasutake, KK et al. PRD (2007)

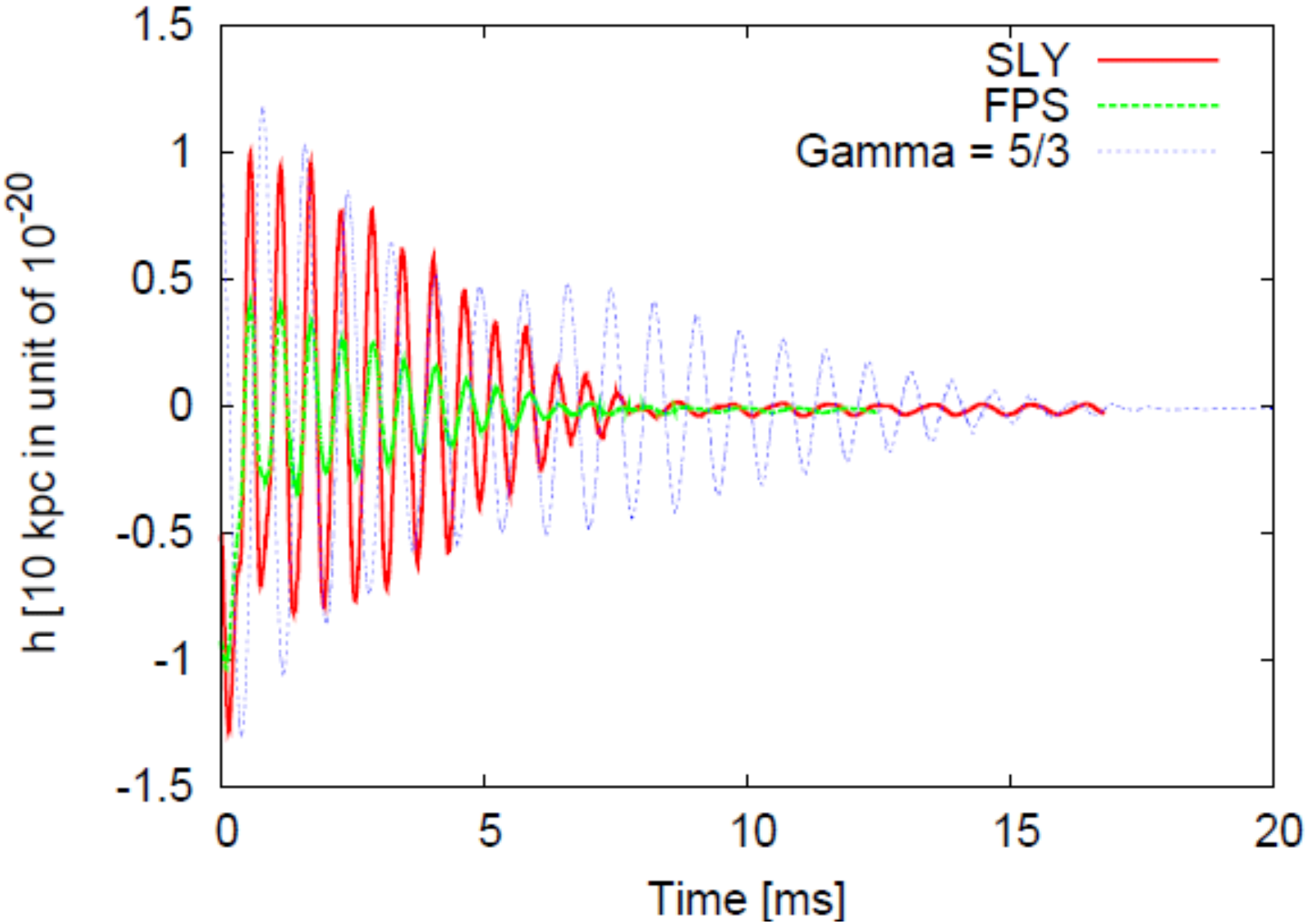


PT does not change the waveform because PT might occur in the relatively small regions (~1km) , leading to the smaller change in the quarupole mom..


3e+06
2e+06
1e+06
0
Z [cm]
0
1e+06
2e+06
3e+06
R [cm]
1.410e+07
14
13
12
11
10
9

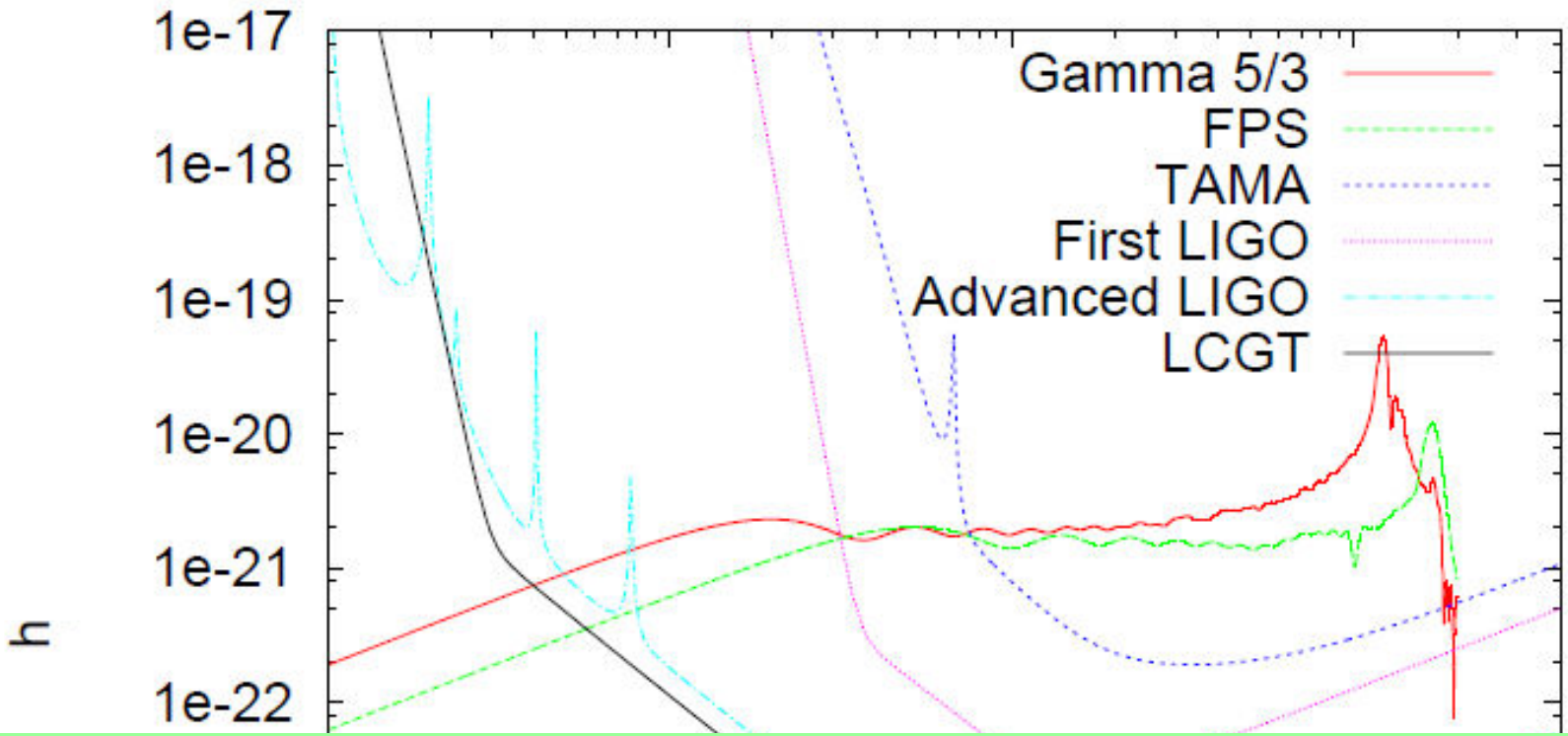
Effects of Different Equations of State on Gravitational-wave Signal

KK & Kiuchi in prep



Effects of Different EOSs on GW spectrum

KK & Kiuchi in prep



For the realistic EOS, typical freq. of GW becomes larger than Gamma =5/3 and within the detection limits of currently running detectors.

Summary of Bounce-originated GWs

Rotation:(Initial rotation rate, degree of differential rotation)
Mueller 1982,Moenchmeyer et al. '91,Yamada & Sato '95
Zwerger et al. '97, KK et al. (03), Ott et al. (04)……..

Magnetic fields:KK et al. (04), Obergaulinger et al. 05,Cerda et al.07,Takiwaki et al. in prep

EOS: (realistic or polytropic EOS, QCD phase transition)
KK et al. (03), Ott et al. (04), Yasutake,KK et al. (07) …

Dimension: (2D or 3D) Rampp et al. (98), Ott et al. (06),Scheidegger et al. (08)

GR Gravity: Dimmelmeier et al. 02,Shibata & Sekiguchi (05),Cerda et al.06,
Ott et al. 07…….

(Very roughly speaking,)

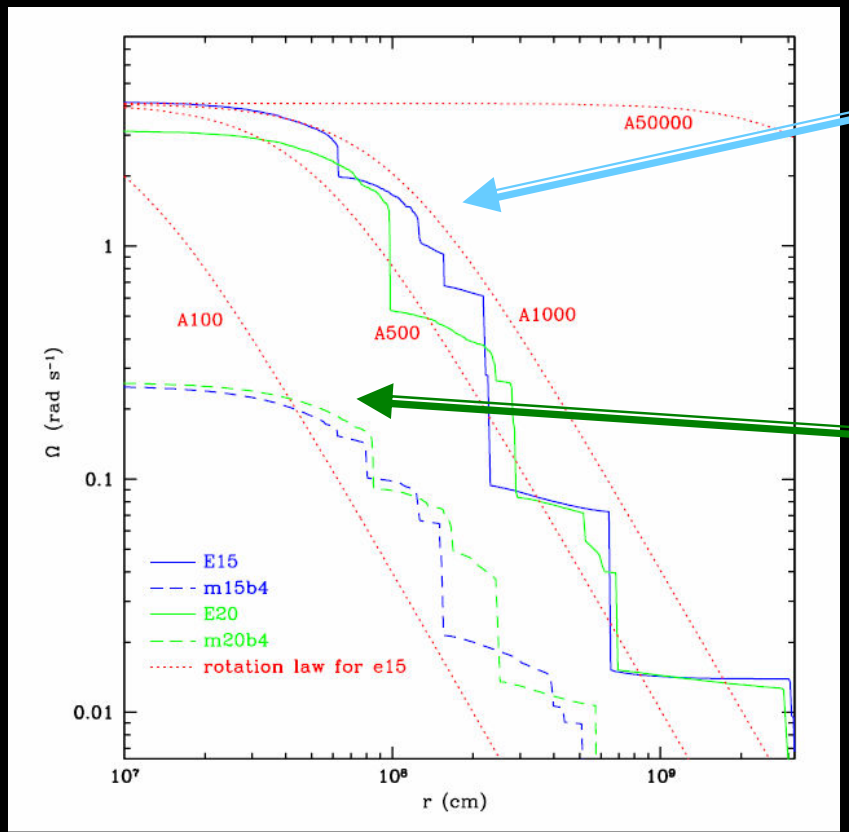
- Qualitatively, the waveforms are categorized to Types I, IV (II).
- Quantitatively, GWs from a supernova in our galaxy are within the detection limits for LCGT and 2nd LIGO (surely) and of TAMA and 1st LIGO (likely).

One concern of rotation-induced GWs is that,,

the amplitude of GW sharply depends on the initial rotation rate ε

$$|h| \sim 10^{-20}\left(\frac{M_{\rm PNS}}{1.4M_\odot}\right)\left(\frac{R_{\rm PNS}}{10\ {\rm km}}\right)^2\left(\frac{t_{\rm dyn}}{1\ {\rm msec}}\right)^{-2}\left(\frac{\epsilon}{0.1}\right)\left(\frac{R}{10\ {\rm kpc}}\right)^{-1}$$

Angular velocity in the iron core



Heger et al. 2000
Omega_0 = 4 rad/s

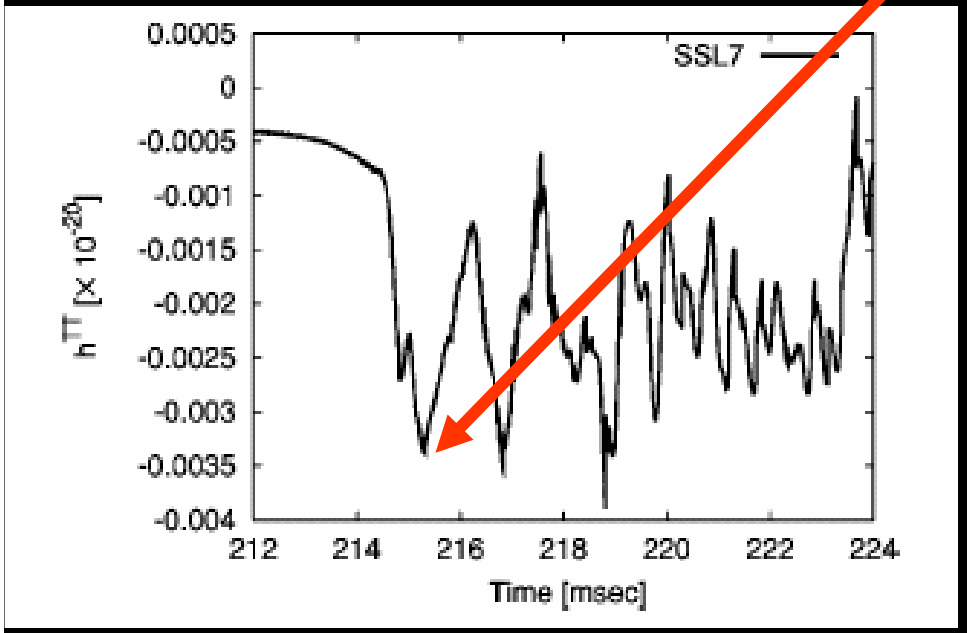
Heger et al. 2005
Omega_0 = 0.3~0.05 rad/s
due to the magnetic
breaking based on
Spruit 2002.

Ott et al. 2004

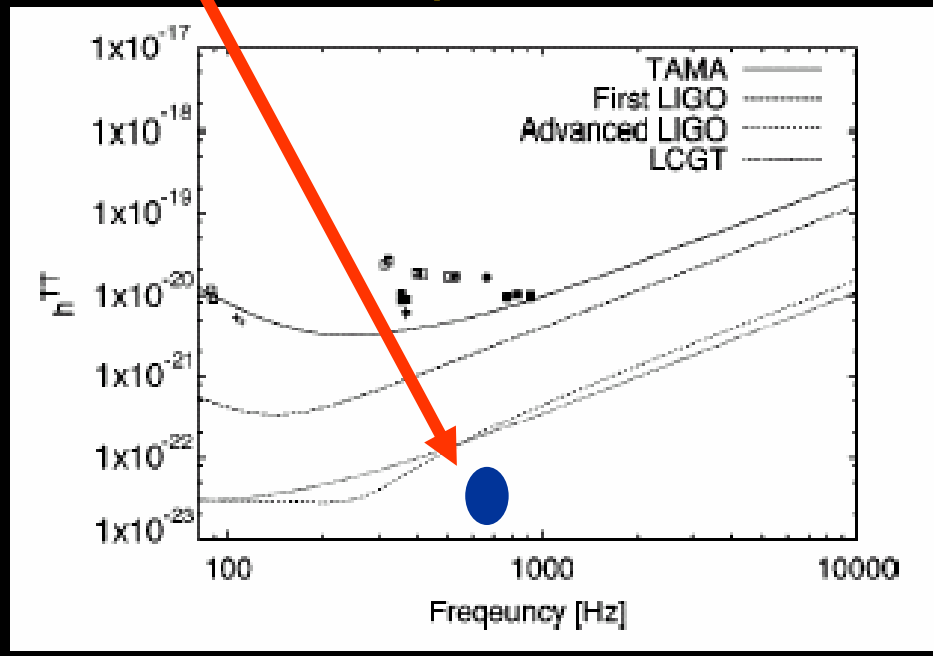
The GWs based on Heger 05 progenitor model

KK et al. 04

Waveform



amplitude



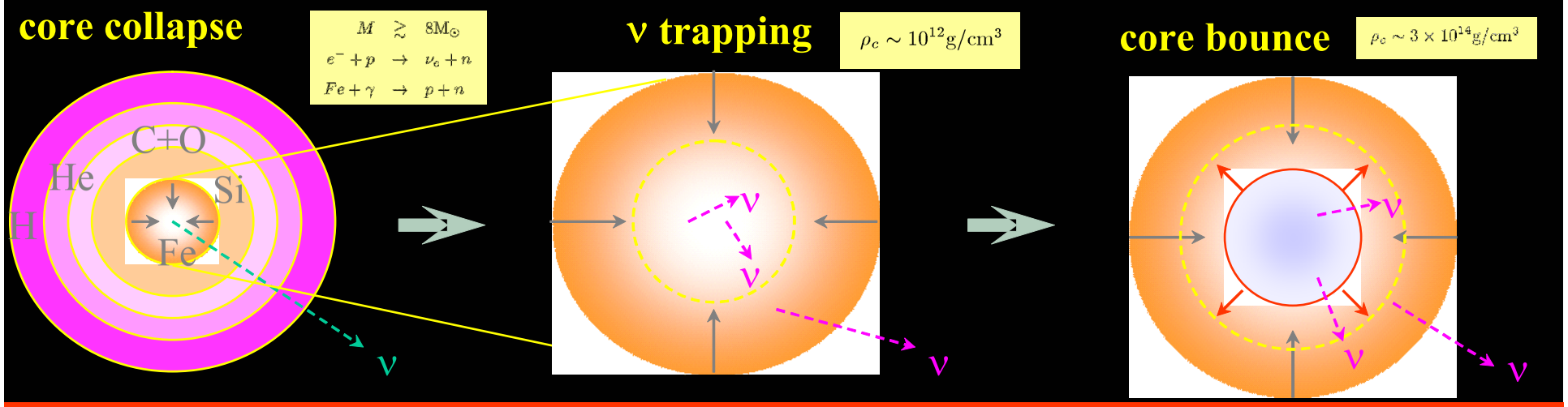
Waveform : little characteristics

amplitude : Too weak to be detected by the laser interferometers in the next generation

When & How GWs are emitted from SNe ?

Name	when	Why ? (Cause of asphericity)
Bounce origin	At bounce (duration、~100 msec)	Aspherical motions of inner core at bounce induced by rotaion
Convection origin	After bounce (duration、~ 1 sec)	Aspherical motions of outer core after bounce
Neutrino origin	After bounce (duration、~ 1 sec)	Aspherical radiations of neutrinos

Standard scenario of core-collapse SNe



After core-bounce, gravitational waveforms depend on the explosion scenarios.

Possible Explosion Mechanisms

- Neutrino-heating Mechanism
  may work for 1D in O-Ne-Mg massive stars, but
  fails to explode more massive stars (Kitaura et al.2006)

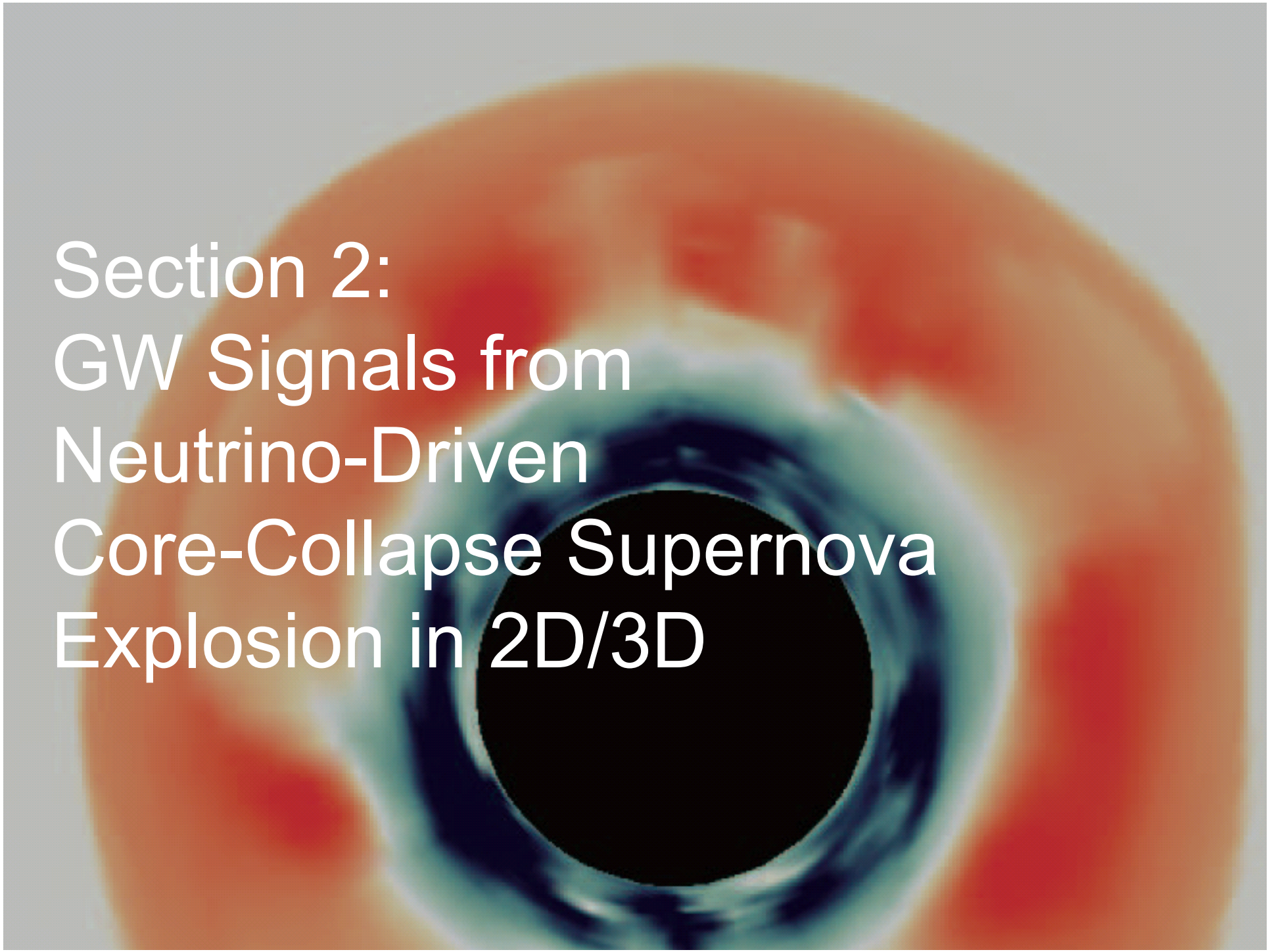
  .(Rampp&Janka 02,Liebendoerfer et al.02)

- Neutrino-heating Mechanism + Convection/SASI,
  : Explosion of 2D, low-stars (11.2 Ms), (Buras et al. 2006)
  : Nuclear-burning aided explosion (Bruenn et al. 07)
  : Onset of SASI-aided neutrino driven explosion of 15 Ms star (Marek &Janka 08)

- Acoustic mechanism (Burrows et al. 2005,6)

- Rapid rotation
  -- effect of rotation-induced anisotopic neutrino radiation (KK et al. 2003, Walder et al.)

- Magnetic fields:
  ・Jet like explosions may be naturally accompanied
  with the magnetar's formations.(KK et al.04,Takiwaki et al. 2005,6,
  Sawai et al. 2005,6,Burrows et al. 07…..)



Section 2: GW Signals from Neutrino-Driven Core-Collapse Supernova Explosion in 2D/3D

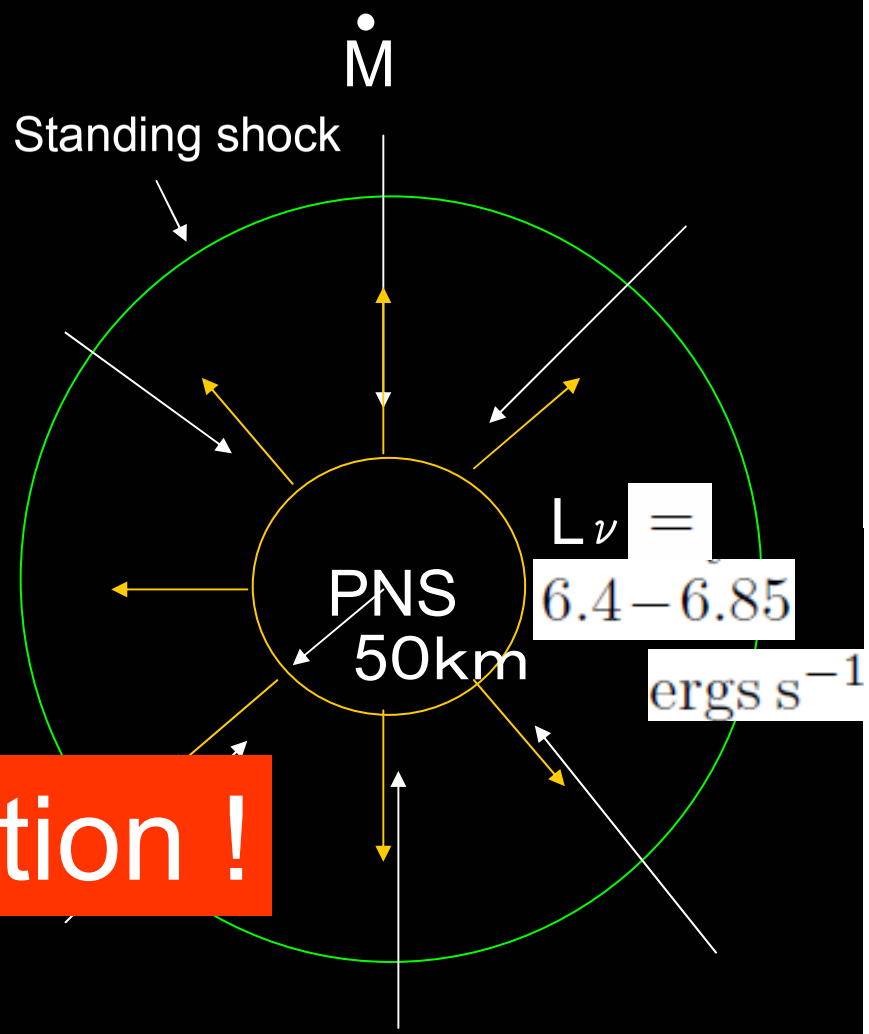
Numerical setting for our SASI simulations

Ohnishi, KK, Yamada,2006a,b,2007

- Contracting neutron star interior is replaced by the fixed boundary.
- To trigger explosion, we assume the neutrino luminosity at the boundary.
  (This simplification may be the only way, to date,
  to study the properties of GWs from
  the exploding SN in 3D.)

- Changing the neutrino luminosity,
  we can systematically investigate
  how the GW waveforms change
  with the explosion dynamics.

Numerical computations:
Hydro: ZEUS-2D/MP(Hayes et al. 06)
EOS: Shen et al (1998)
Neutrino transport: Light-bulb
approa
Mesh #: 300(r)*30(θ)*60(ϕ
2 weeks for 1 run (128CPUs)

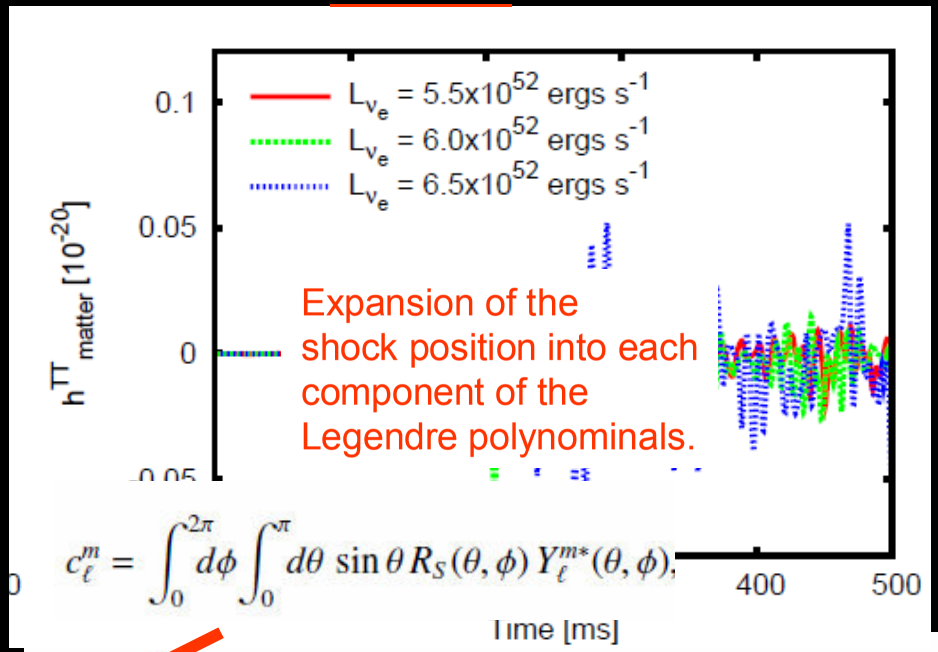


Animation !

Waveforms

KK et al. ApJ (2007)

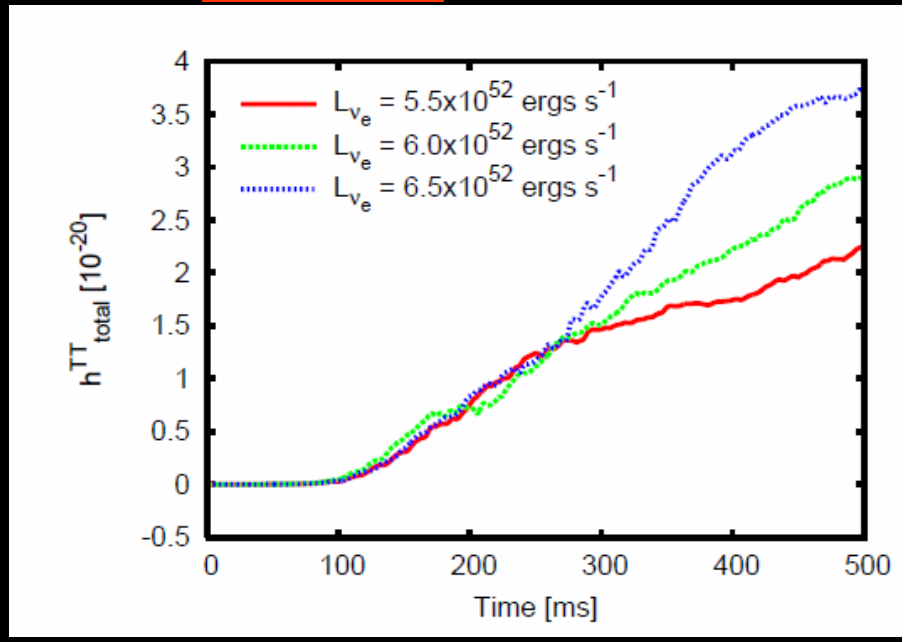
Matter

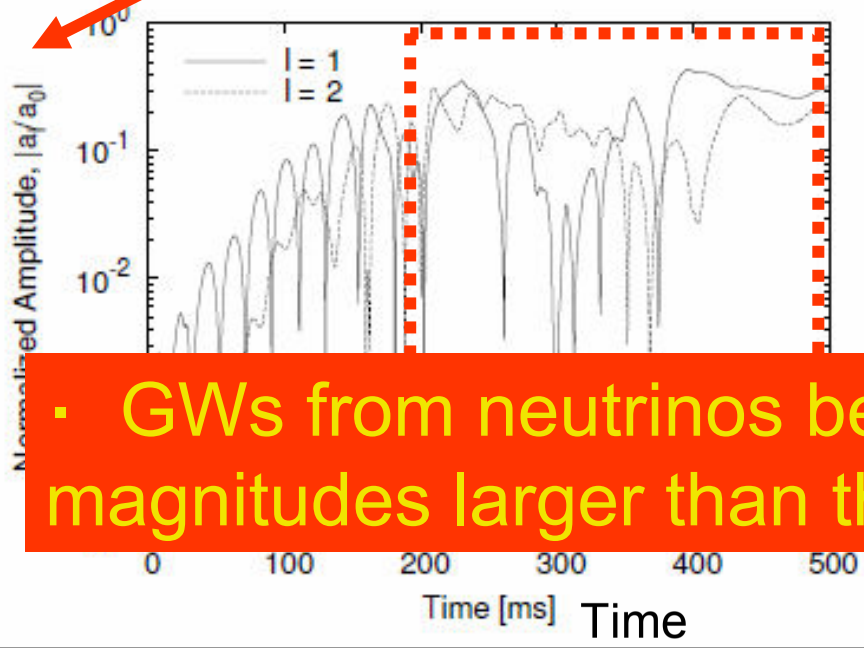


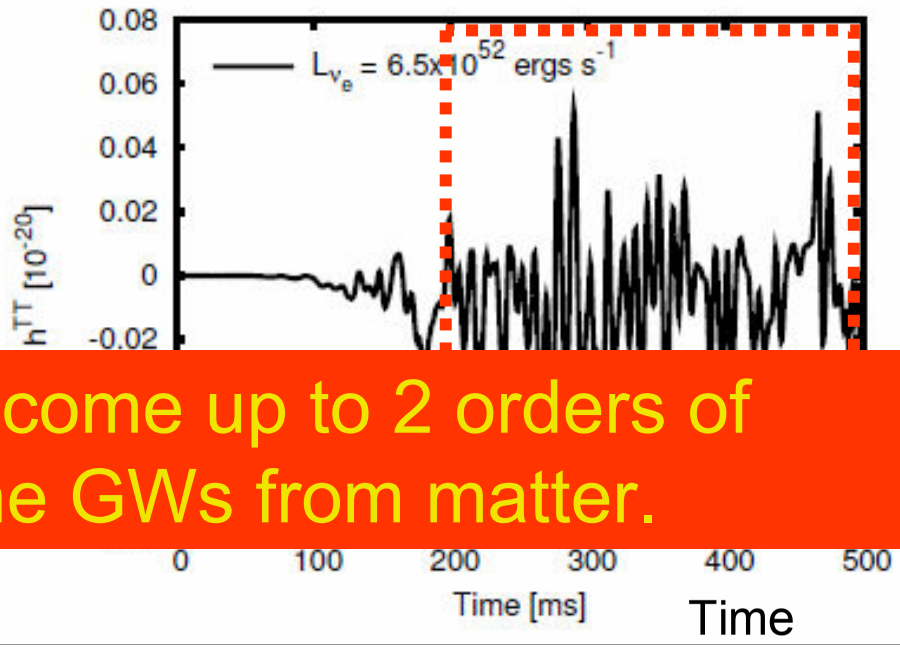
Expansion of the shock position into each component of the Legendre polynominals.

$$c_\ell^m = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \, \sin\theta \, R_S(\theta,\phi) \, Y_\ell^{m*}(\theta,\phi),$$

Neutrino







- GWs from neutrinos become up to 2 orders of magnitudes larger than the GWs from matter.

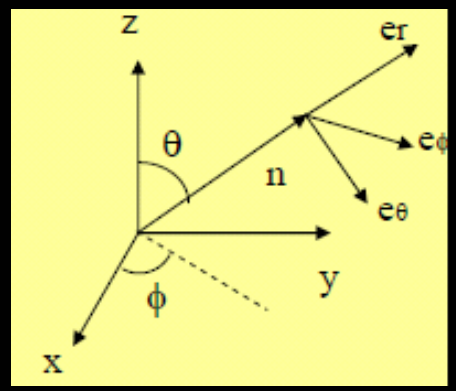
GWs from anisotropic neutrino emissions

$$h^{\mu\nu}(t,\boldsymbol{x}) = 4\int \frac{T^{\mu\nu}(t-|\boldsymbol{x}-\boldsymbol{x}'|,\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}d^3x'$$

$$T^{\mu\nu} = T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{neutrino}}$$

Anisotropic neutrino radiation field,
(exact form)

$$T^{\alpha\beta}_{\text{neutrino}} = \int \frac{d^3p}{2E_\nu} p^\alpha p^\beta f_\nu(x,p)$$



Epstein's formulae (1978) (very popular for estimating GWs from neutrinos)

$$T^{ij}(t,\boldsymbol{x}) = n^i n^j r^{-2} L_\nu(t-r) f(\Omega, t-r)$$

$$\boldsymbol{n} = \boldsymbol{x}/r$$

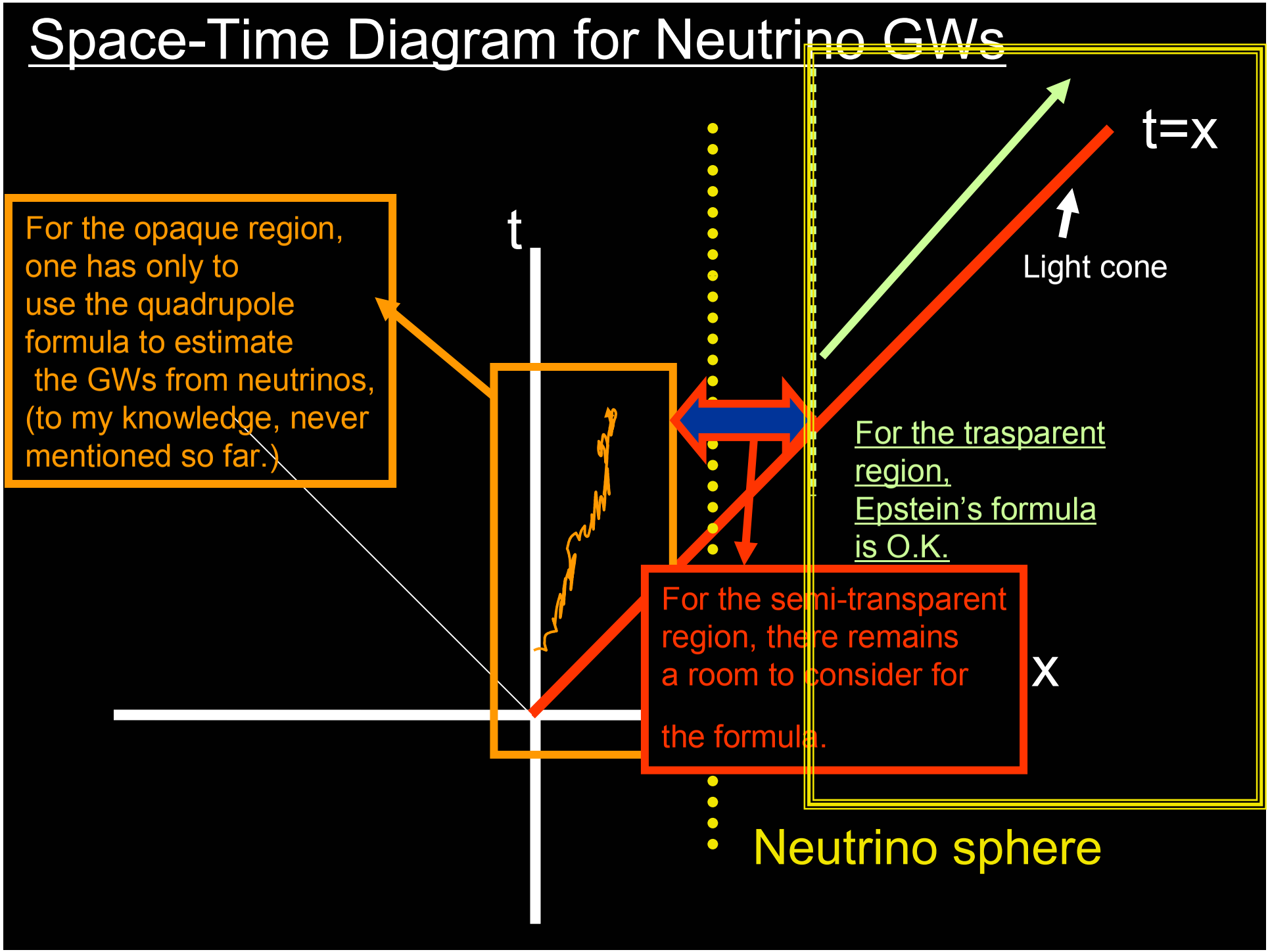
$$\int f(\Omega,t)d\Omega = 1$$

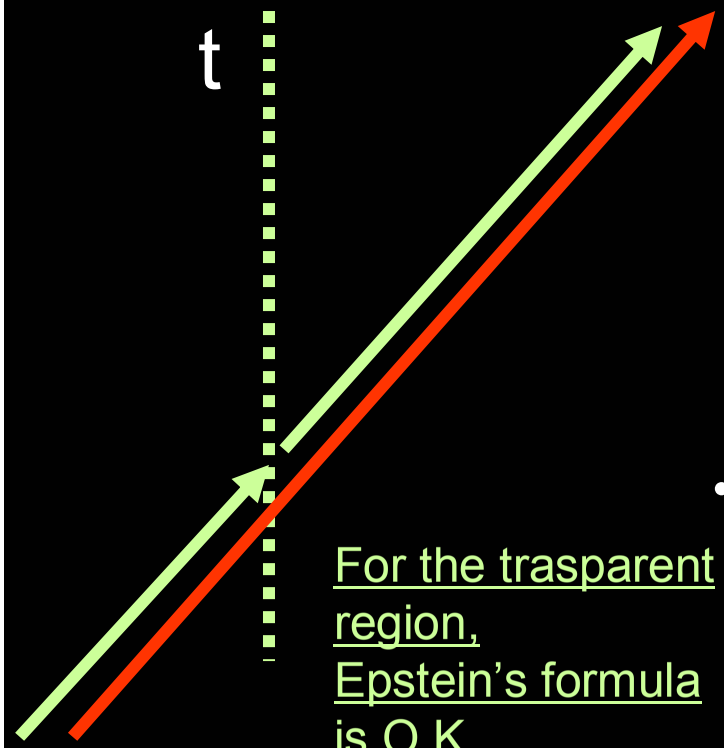
the angular distribution of neutrino radiation

$$r = |\boldsymbol{x}|$$

This form expresses the radiation field from a point source, namely emitted at x = 0.
This is valid for the distant observer from the source.

Space-Time Diagram for Neutrino GWs





For the trasparent region, Epstein's formula is O.K.

Why would neutrinos traveling on flat-space geodesics radiate GWs ??

- The **energy-momentum** tensor of GWs is quadratic w.r.t h,

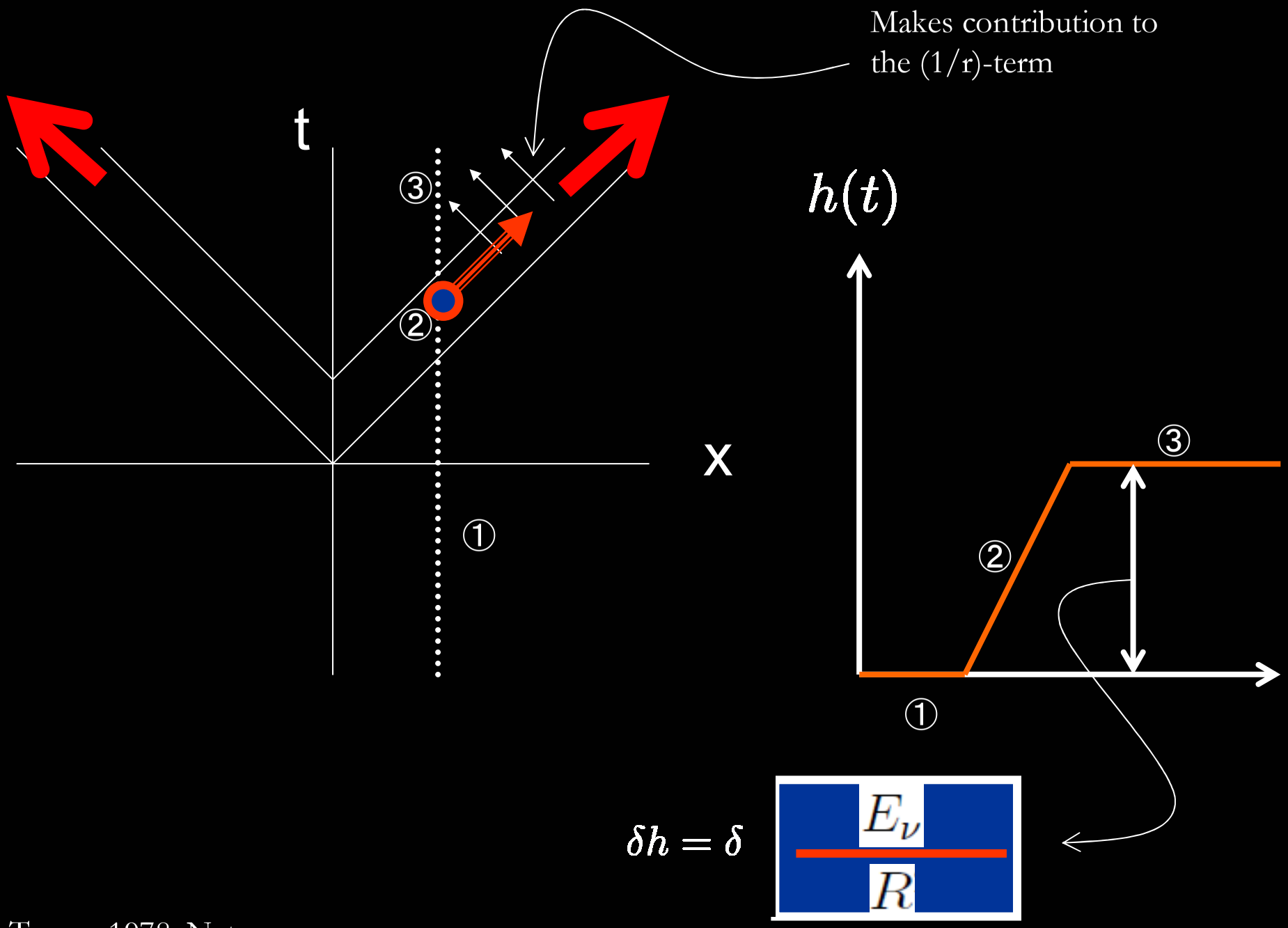
$$T^{\rm GW}_{\mu\nu} = \frac{1}{32\pi}\left\langle h_{\alpha\beta,\mu}h^{\alpha\beta}{}_{,\mu} - \frac{1}{2}h_{,\mu}h_{,\mu} - h^{\alpha\beta}{}_{,\beta}h_{\alpha\mu,\nu} - h^{\alpha\beta}{}_{,\beta}h_{\alpha\nu,\mu}\right\rangle$$

・The energy carried off by GWs can be given by computing surface volume and time integral

$$\delta E^{\rm GW} = \int dS \int dt \; T^{\rm GW}_{\mu\nu}$$

Non-vanishing memory term @ infinity should be $|h| \propto r^{-1}$ as $r \to \infty$

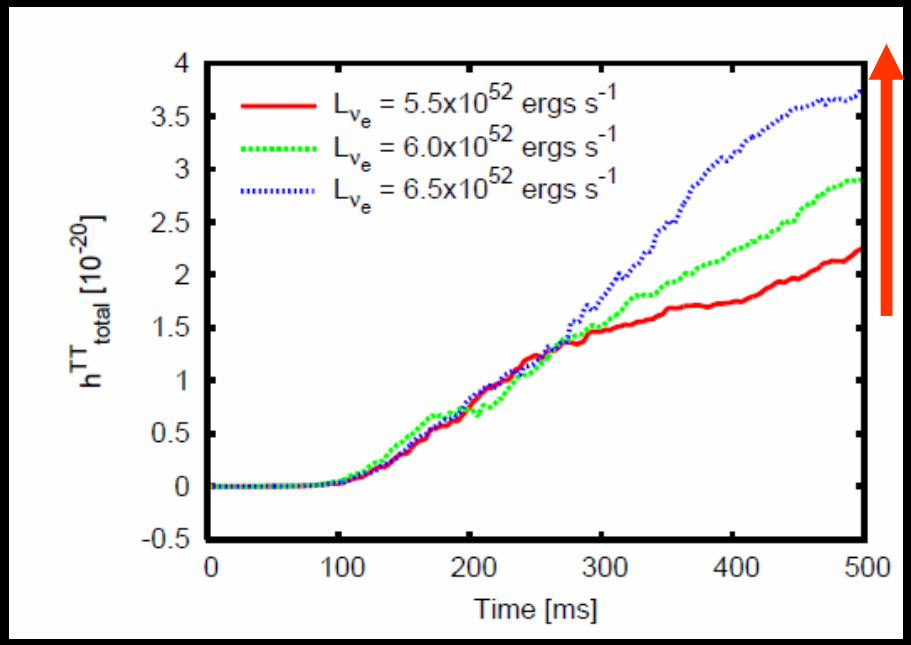
If neutrinos travel from t = - ∞ to t =∞, no GWs will be emitted.



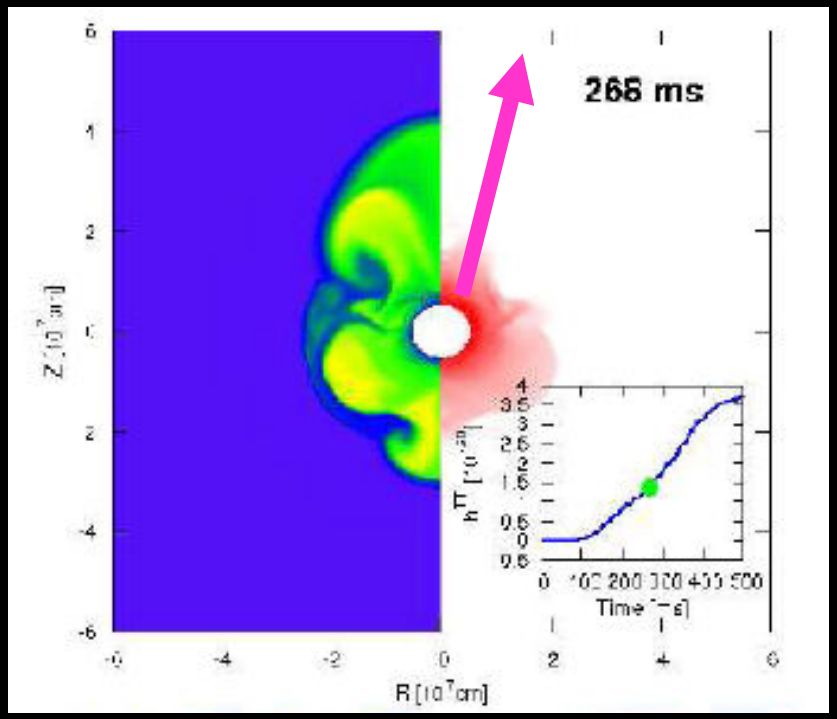
Turner 1978, Nature

Features of neutrino-originated GWs in 2D

waveform



KK et al. ApJ (2007)

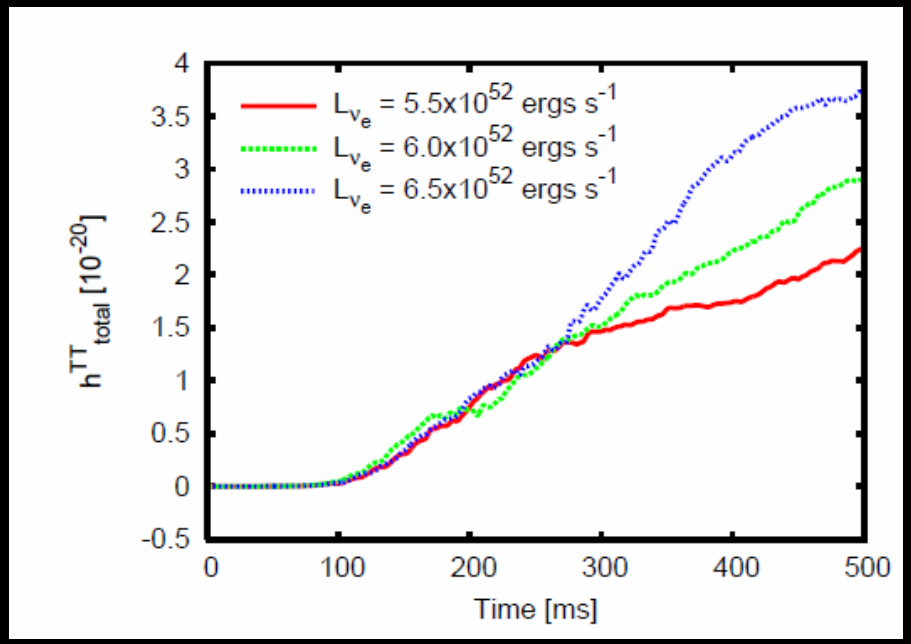


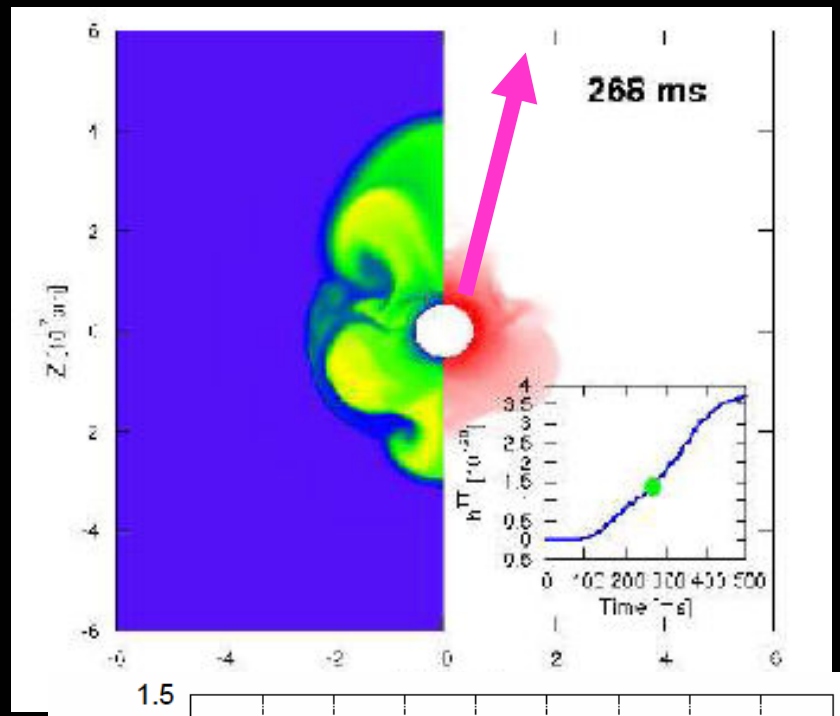
- GWs from neutrinos almost monotonically increase with time.
- For the higher luminosity, the amplitudes become higher.

Features of neutrino-originated GWs

KK et al. ApJ (2007)

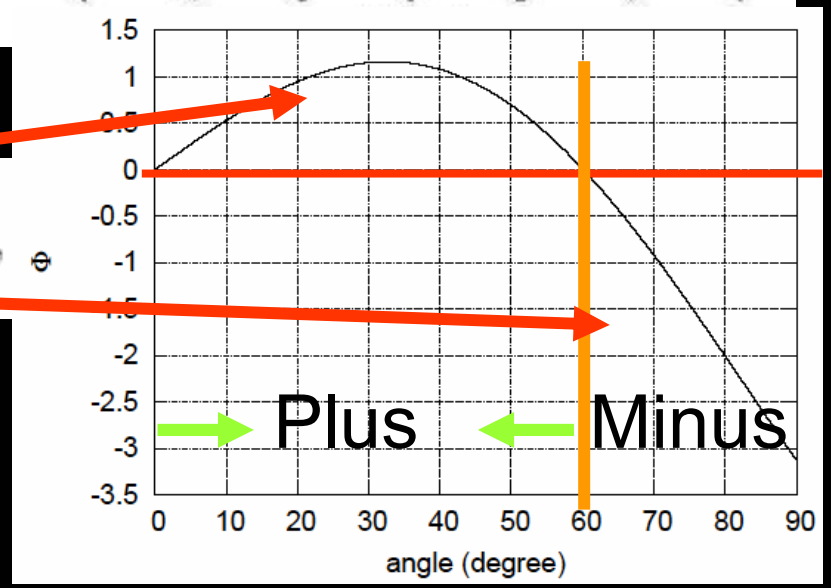
waveform





GWs from neutrinos in 2D,

$$h_\nu^{TT} = \frac{8G}{c^4 R}\int_{-\infty}^{t-R/c} dt' \int_0^{\pi/2} d\theta' \, \Phi(\theta') \, \frac{dL_\nu(\theta', t')}{d\Omega'},$$



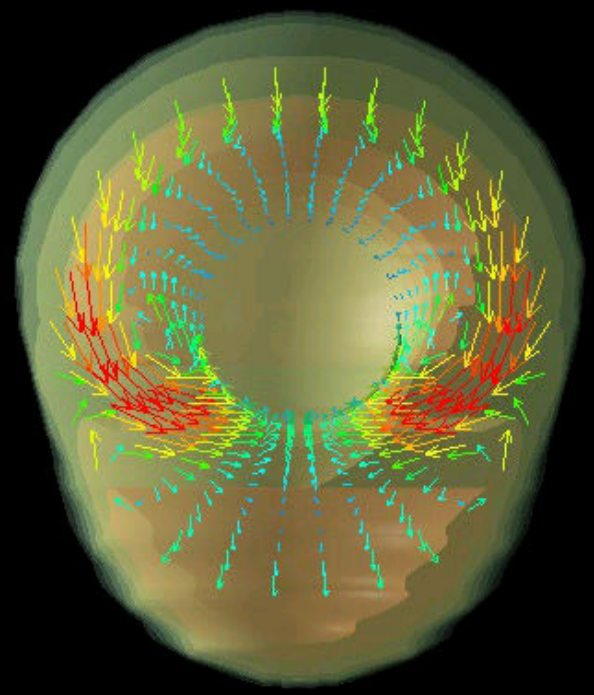
- angle dependent factor * neutrino luminosity per solid angle
- angle dependent factor is a function of the angle from the symmetry axis.

3D case

Iwakami, KK, Ohnishi, Yamada ApJ (2008)

Non-exploding model

Meridian plane



No elongation along the symmetry axis like 2D.

3D detailed simulations of SASI

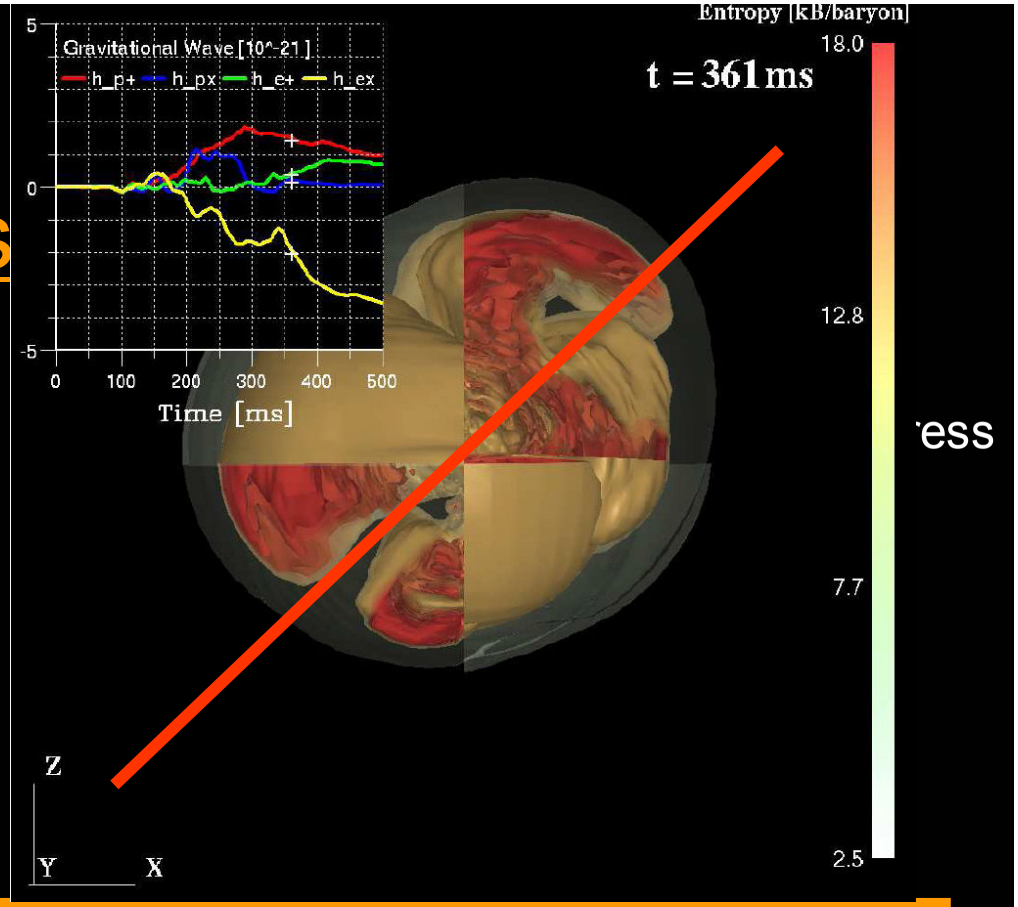
Exploding model

Iwakami, KK, Ohnishi, Yamada,ApJ (2008)

Animation !

First 3D detailed s

Exploding model



ess

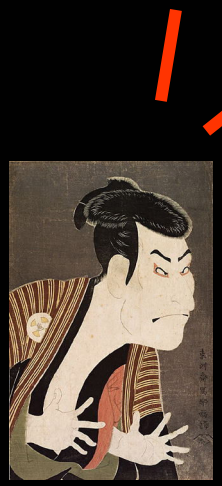
In contrast to 2D,

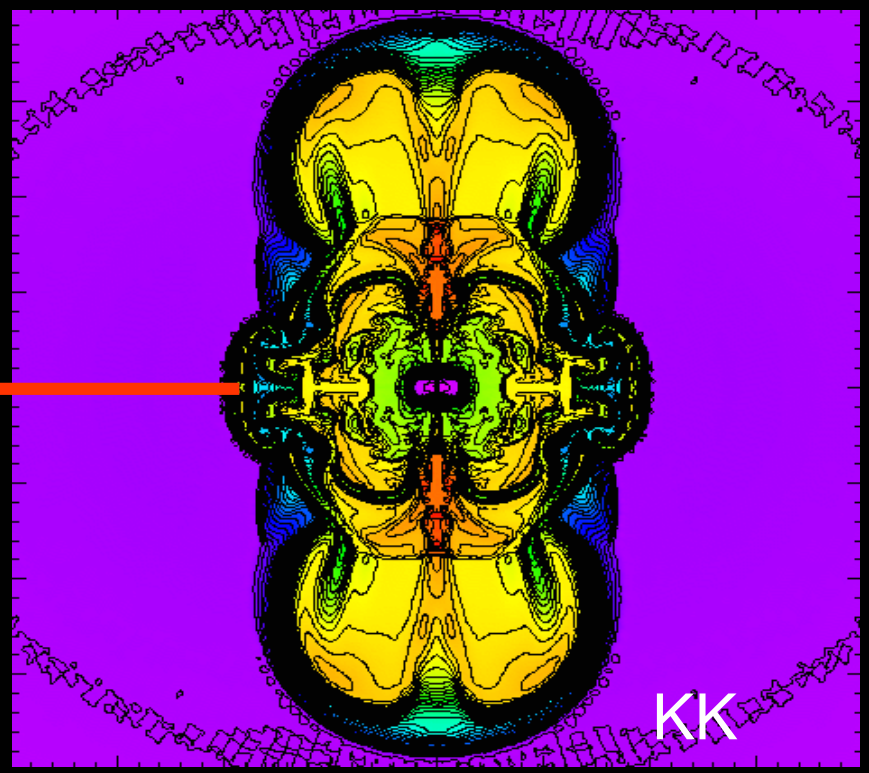
・The major axis is not necessary aligned with the coordinate symmetry axis.

・The flow inside the standing shock is not symmetric with respect to the major axis.

In 2D,

No GWs are observed seen from the rotational (symmetry) axis.

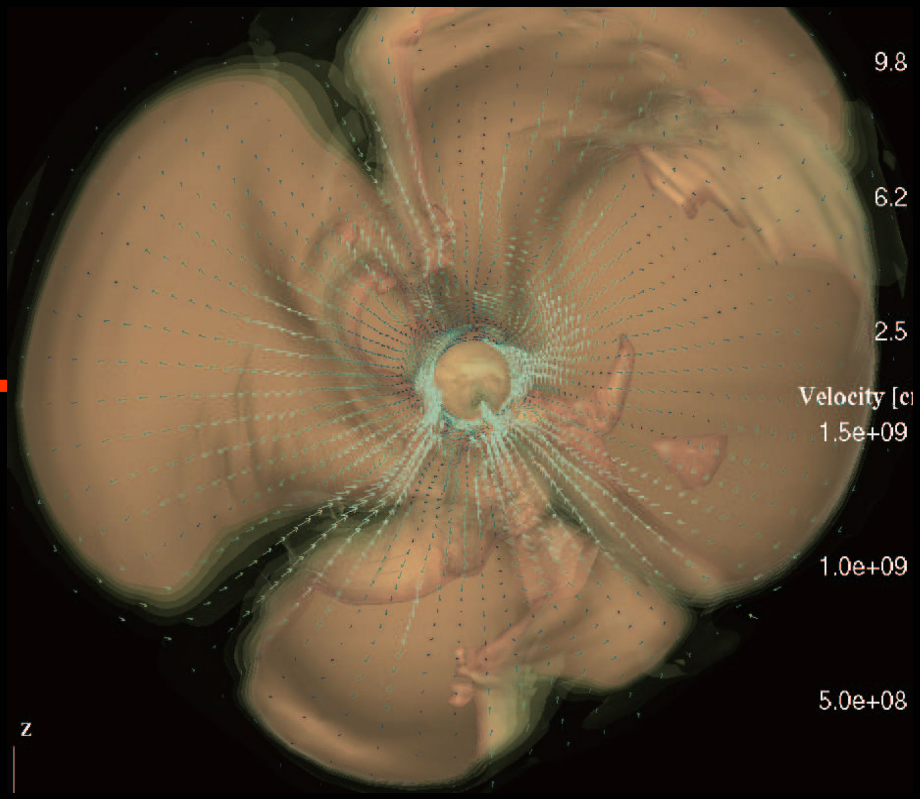




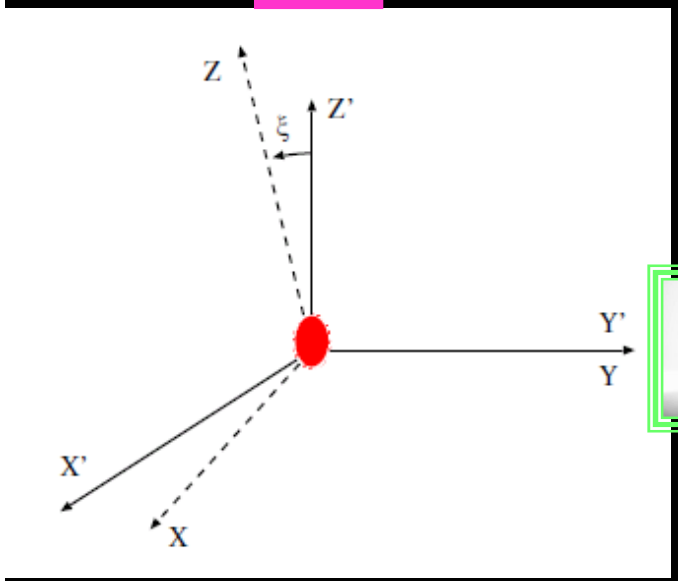
In 3D,

Pole

Equator



Closed Formulae for computing neutrino GWs in 3D



$$h_+^p = \frac{2G}{c^4 R}\int_0^t \int_{4\pi} d\Omega' \; \Phi_+^p(\theta',\phi') \frac{dl_\nu(\Omega',t')}{d\Omega'},$$

$$\Phi_+^p(\theta',\phi') \equiv (1+\cos\theta') \; \cos 2\phi' \; ,$$

$$h_\times^p = \frac{2G}{c^4 R}\int_0^t \int_{4\pi} d\Omega' \; \Phi_\times^p(\theta',\phi') \frac{dl_\nu(\Omega',t')}{d\Omega'},,$$

$$\Phi_\times^p(\theta',\phi') = (1+\cos\theta') \; \sin 2\phi' \; .$$

Mueller & Janka '97

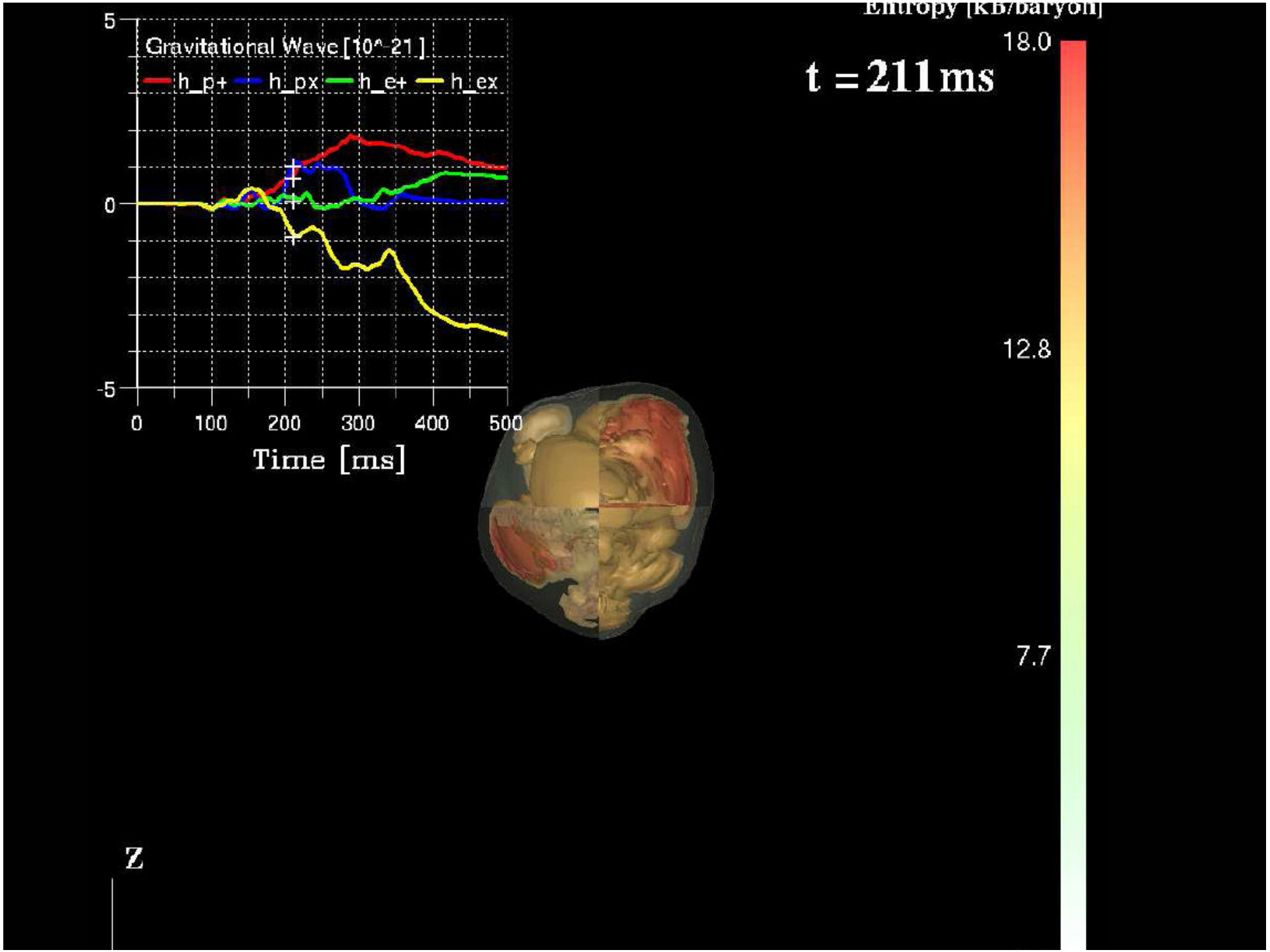
$$h_+^e = \frac{2G}{c^4 R}\int_0^t \int_{4\pi} d\Omega' \; \Phi_+^e(\theta',\phi') \frac{dl_\nu(\Omega',t')}{d\Omega'},$$

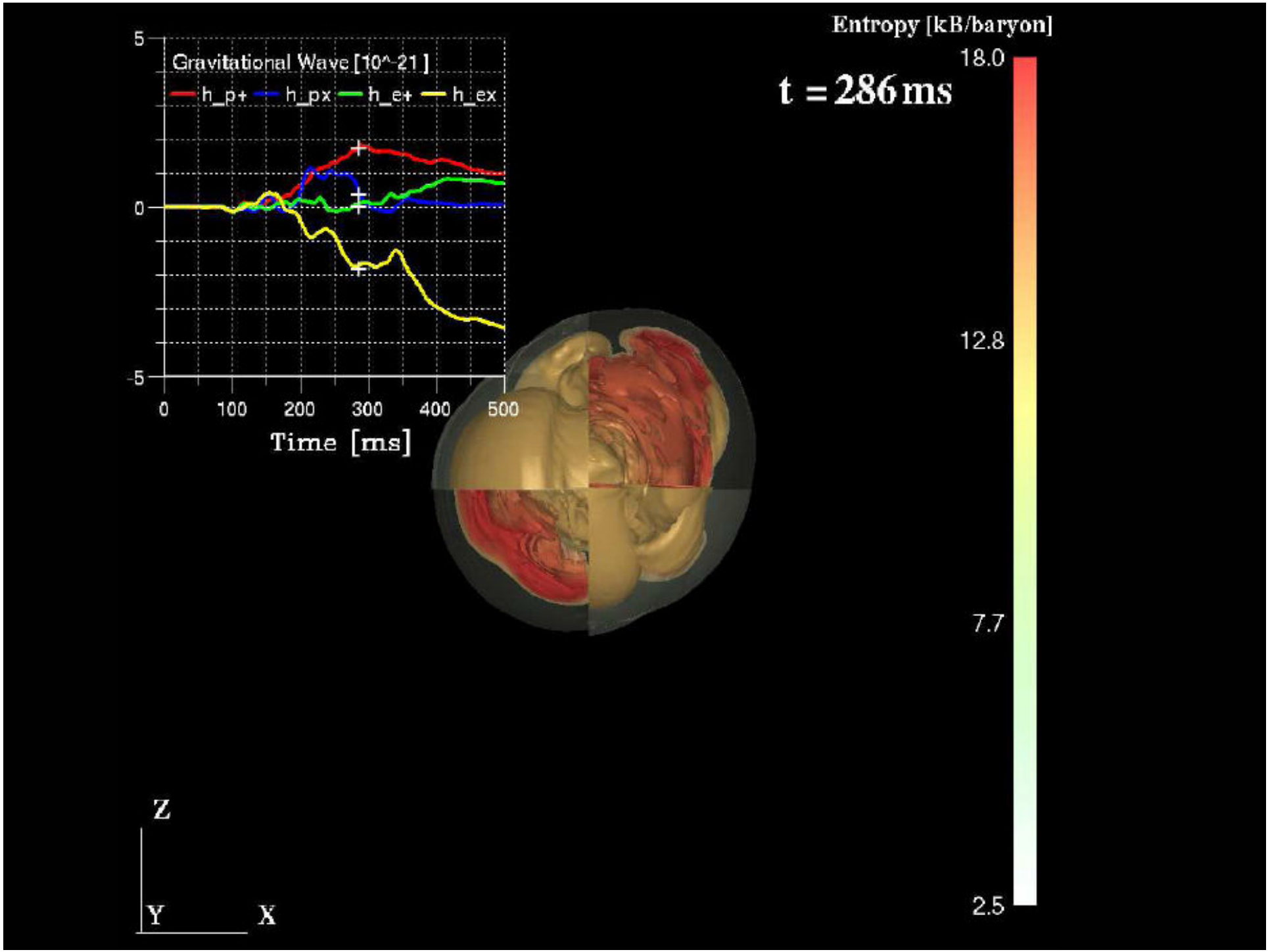
$$\Phi_+^e(\theta',\phi') \equiv (1+\sin\theta'\cos\phi') \frac{\cos^2\theta' - \sin^2\theta'\sin^2\phi'}{\cos^2\theta' + \sin^2\theta'\sin^2\phi'}$$

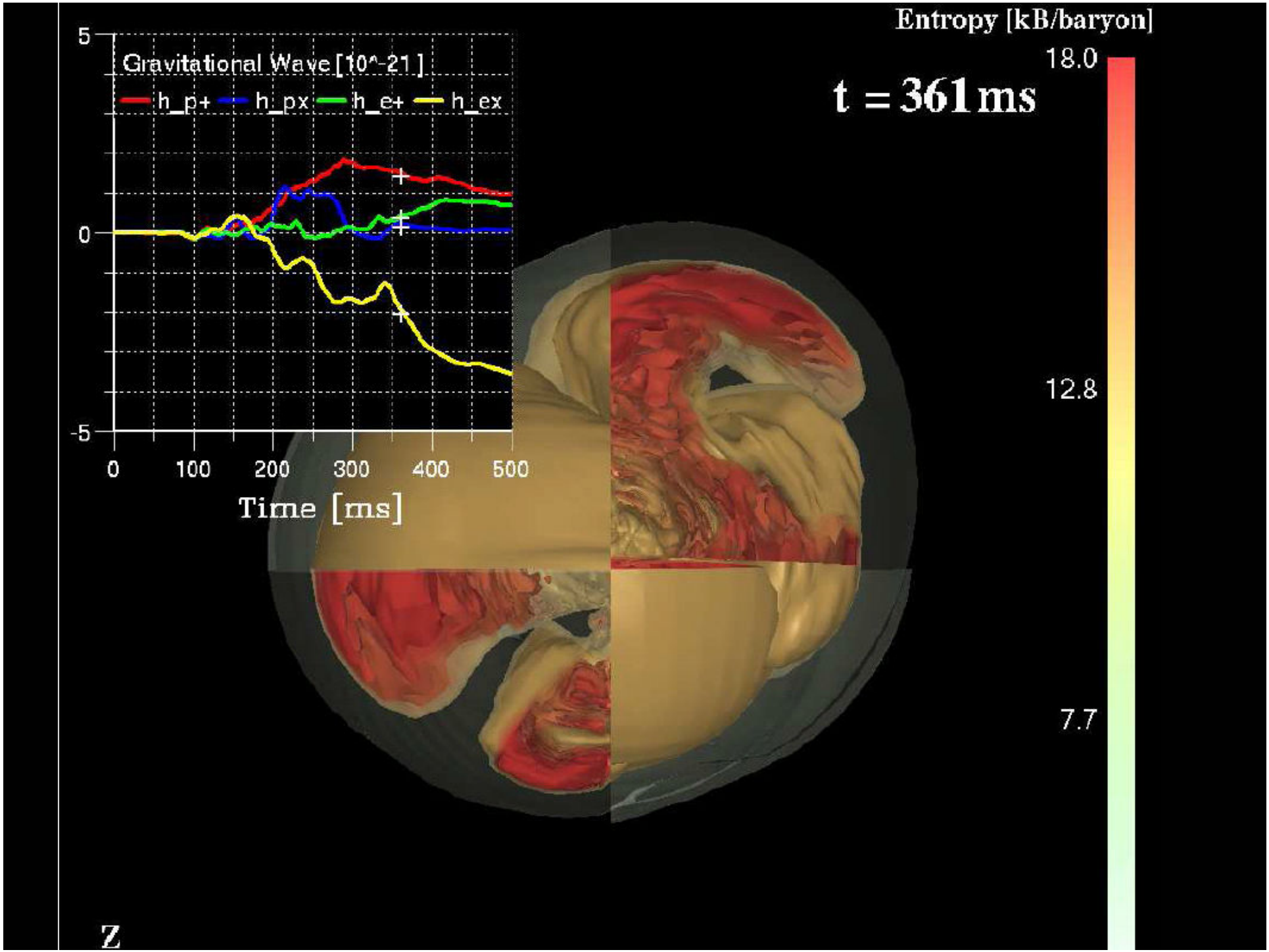
$$h_\times^e = \frac{2G}{c^4 R}\int_0^t \int_{4\pi} d\Omega' \; \Phi_\times^e(\theta',\phi') \frac{dl_\nu(\Omega',t')}{d\Omega'},$$

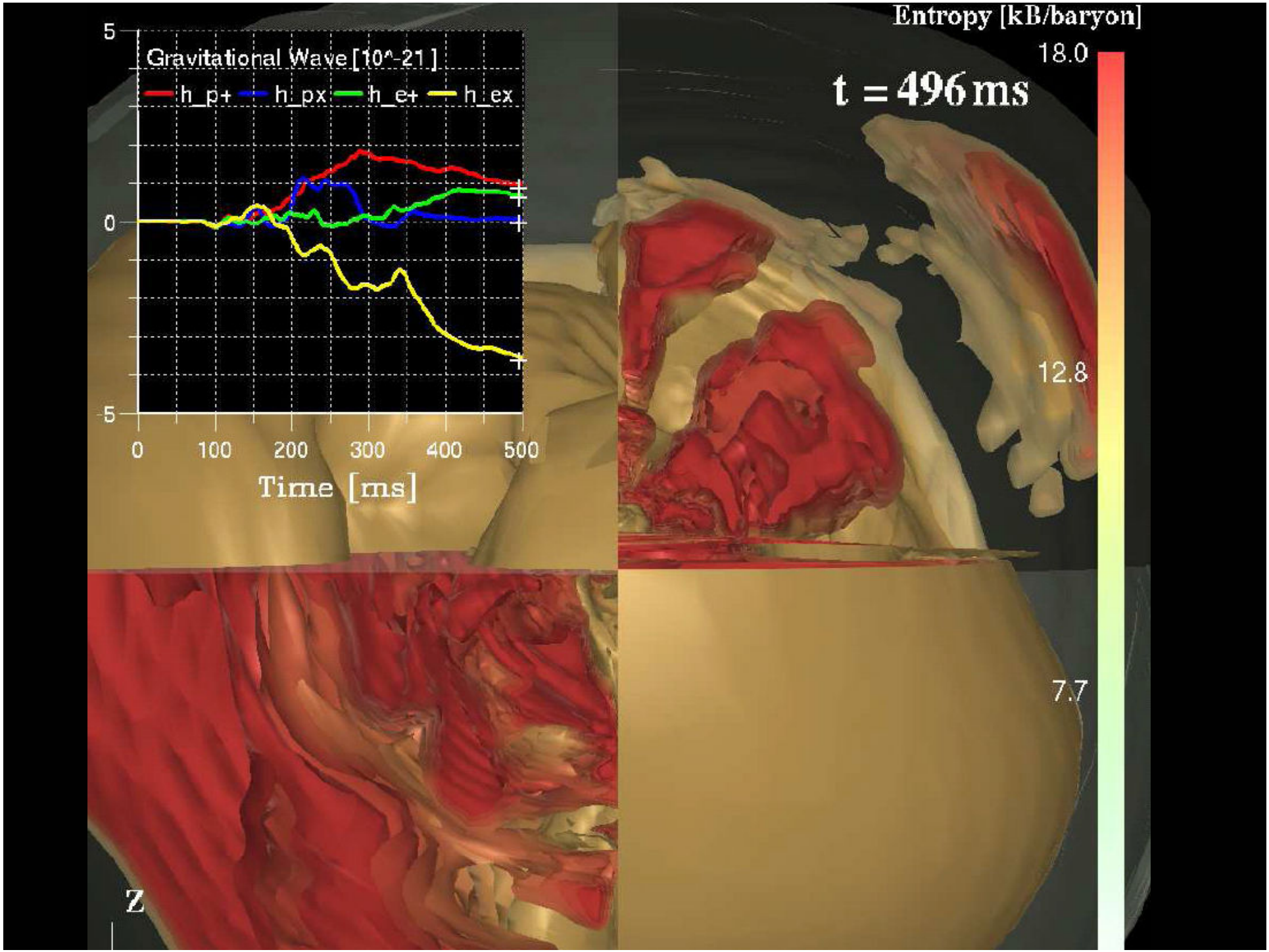
$$\Phi_\times^e(\theta',\phi') \equiv (1+\sin\theta'\cos\phi') \frac{\sin 2\theta' \sin\phi'}{\cos^2\theta' + \sin^2\theta'\sin^2\phi'},$$

KK et al. 07.08


Gravitational Wave [10^-21]
h_p+
h_px
h_e+
h_ex
5
0
-5
0
100
200
300
400
500
Time [ms]
Entropy [kB/baryon]
18.0
12.8
7.7
t = 211 ms
Z


Gravitational Wave [10^-21]
h_p+
h_px
h_e+
h_ex
5
0
-5
0
100
200
300
400
500
Time [ms]
Entropy [kB/baryon]
18.0
12.8
7.7
2.5
t = 286 ms
Z
Y
X

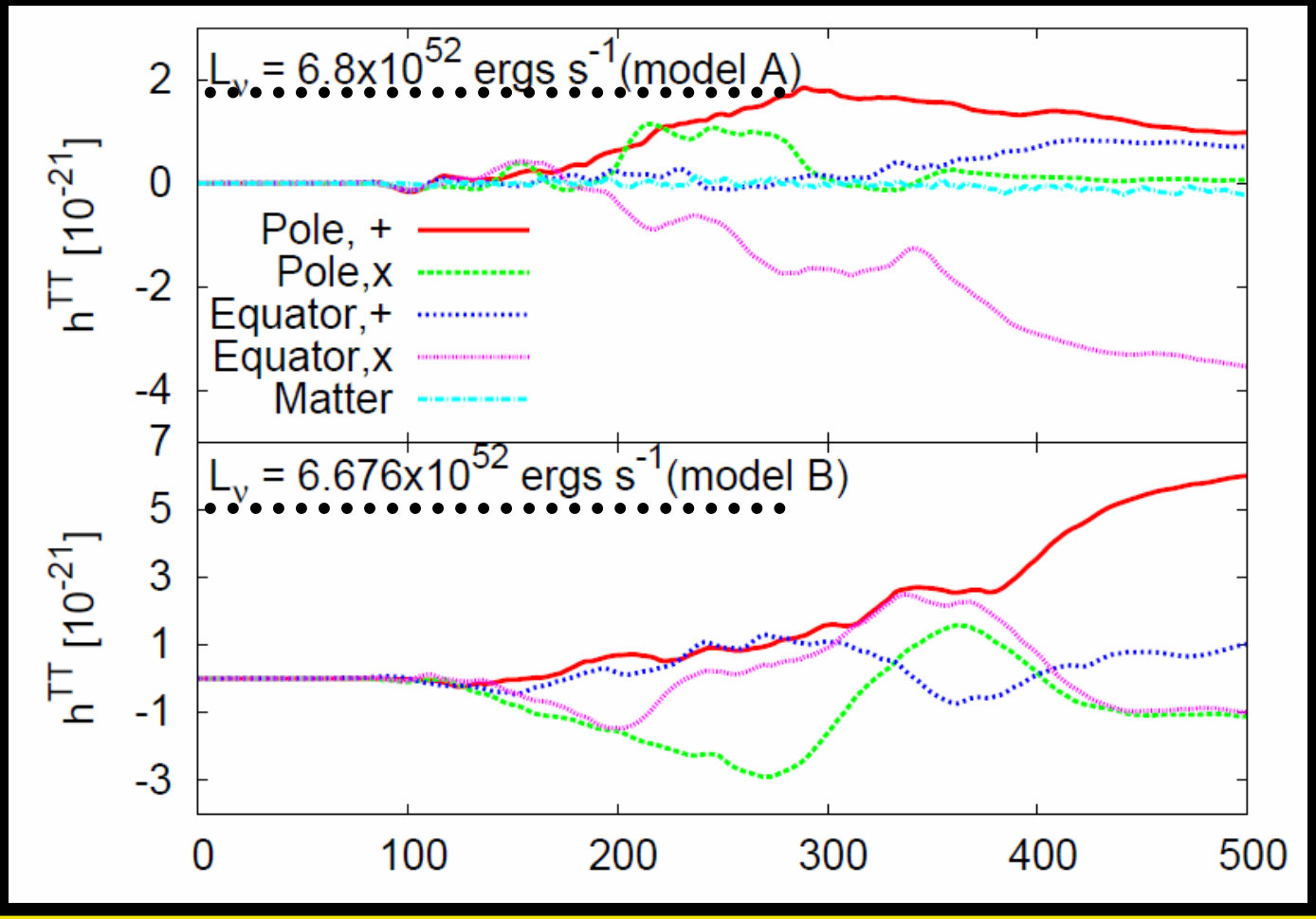

Gravitational Wave [10^-21]
h_p+
h_px
h_e+
h_ex
5
0
-5
0
100
200
300
400
500
Time [ms]
Entropy [kB/baryon]
18.0
12.8
7.7
t = 361 ms
Z


Gravitational Wave [10^-21]
h_p+
h_px
h_e+
h_ex
5
0
-5
0
100
200
300
400
500
Time [ms]
Entropy [kB/baryon]
18.0
12.8
7.7
t = 496 ms
Z

Details in the Waveforms in 3D (1)

The input luminosity differs only 0.5 %.

KK et al. in prep

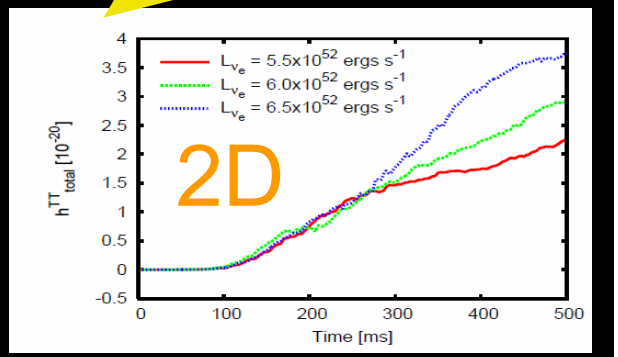


The waveforms vary stochastically due to SASI, which grows chaotically in all the directions.

Details in the Waveforms in 3D (2)

L_ν (10foe erg/s)	Δt (ms)	$\lvert h^{\rm p}_{\rm max}\rvert$ (10^{-21})	$\lvert h^{\rm e}_{\rm max}\rvert$ (10^{-21})	$E_{{\rm GW},\nu}$ ($10^{-11} M_\odot c^2$)	$E_{\rm GW,matter}$ ($10^{-11} M_\odot c^2$)	$E_{\rm GW}$ ($10^{-11} M_\odot c^2$)
6.8	507	1.85(+)	3.57(×)	0.95	3.45	4.4
6.766	512	6.04(+)	2.49(×)	1.50	7.42	8.92
6.7	532	3.21(+)	2.78(×)	1.23	7.49	8.72
6.6	667	9.10(×)	3.77(×)	2.48	10.86	13.34
6.4	795	2.16(+)	3.92(+)	2.01	7.74	9.75
6.85	429	3.45(×)	4.03(×)	1.04	5.63	6.67
6.8	547	—	20.03(+)	1.11	13.1	14.21

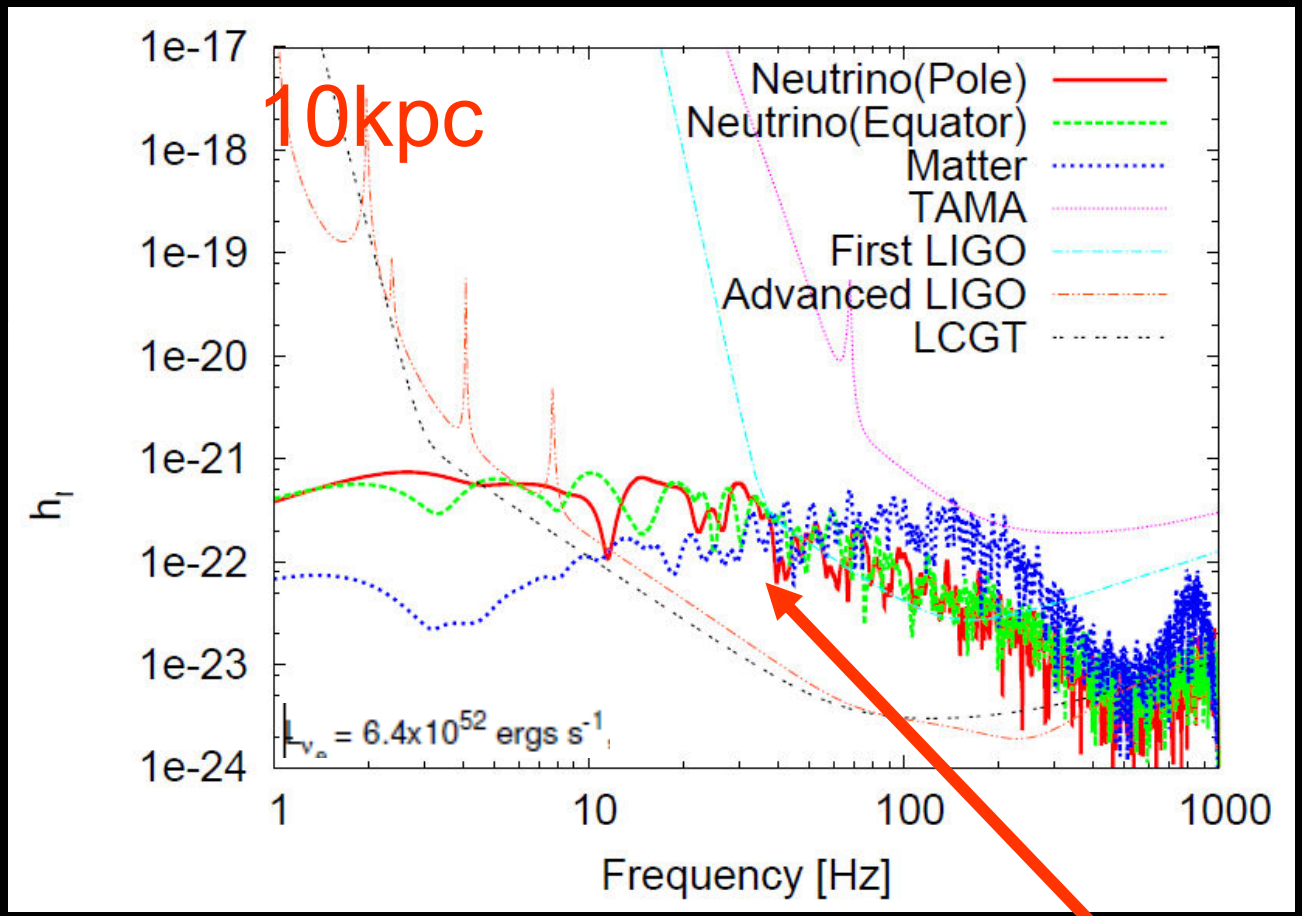
- Maximum GW amplitude is obtained for the moderate luminosity model.
- Neither systematic dependence of the input luminosity on the maximum amplitudes nor on the radiated GW energy are found.



- Despite a variety of the waveforms, their values are found to show little variability among the models.

Detectability

KK et al. in prep



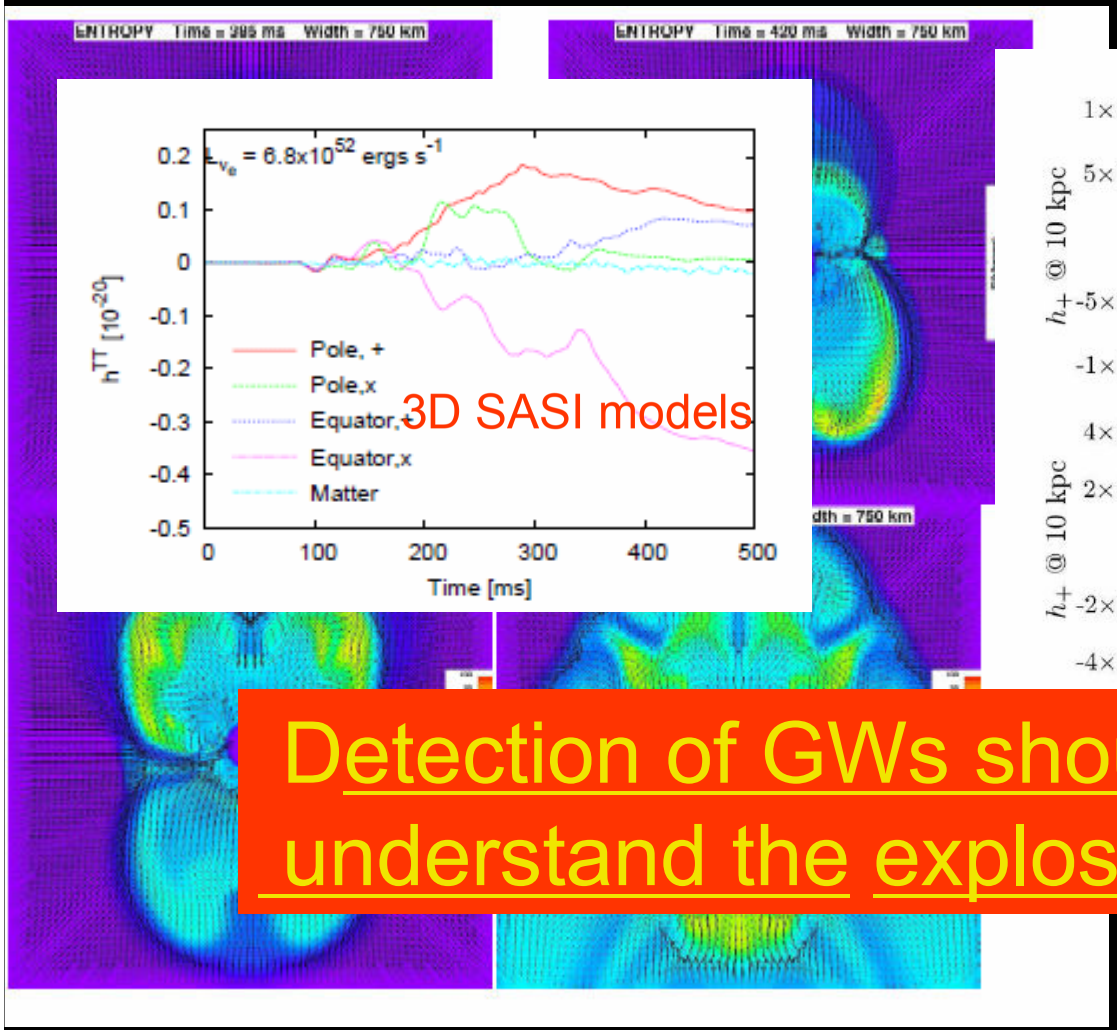
- GWs from neutrinos generically dominate over the ones from the matter motions below ~100 Hz.
- They may be visible to the next generation detectors for a galactic SN. (Of course, not certain at all due to the idealized computations.)

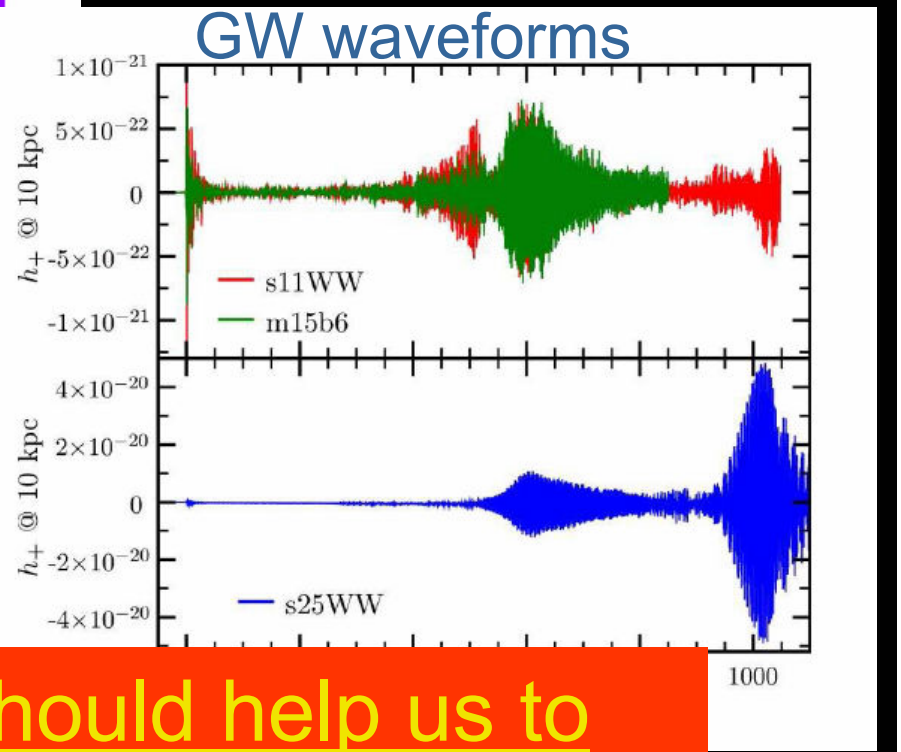
Acoustic Mechanism

Burrows et al. 2006,7

Exploding of 11Msolar at ~600 msec after bounce

Ott et al. 2007





Detection of GWs should help us to understand the explosion mechanism.

Summary

If core-collapse supernovae are triggered by the neutrino heating mechanism aided by SASI/convection (Bethe '90, Marek &Janka) in 3D,

☆ Gravitational waveforms change stochastically due to the chaotically and non-locally growing SASI.

☆ GWs from neutrinos generically dominate over the ones from matter motions below 100 Hz, which are visible to the next-generation detectors (KK et al. 07, 08).

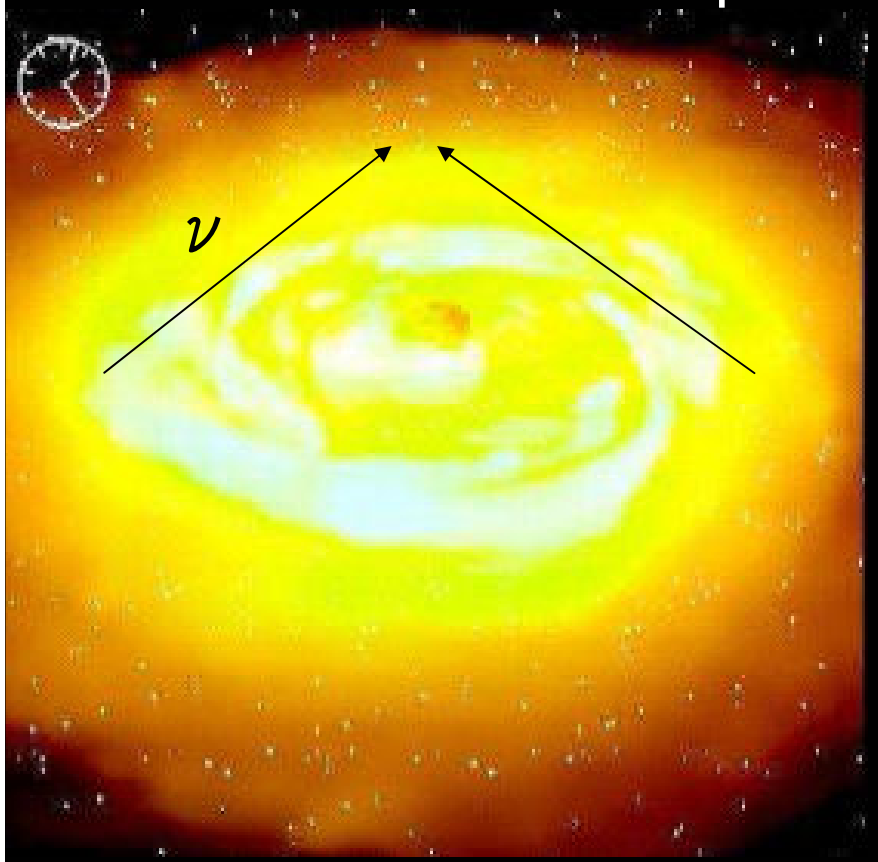
Thank you very much !

For detection, the predictions of GWs in more accurate 3D models are indispensable.

GWs from neutrinos from Neutrino-Driven GRBs

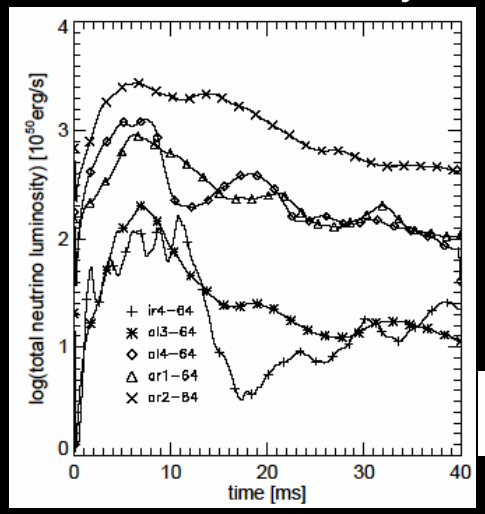
Hiramatsu, KK, Kudoh, Taruya, MNRAS (2006)

3D simulations of collapsar



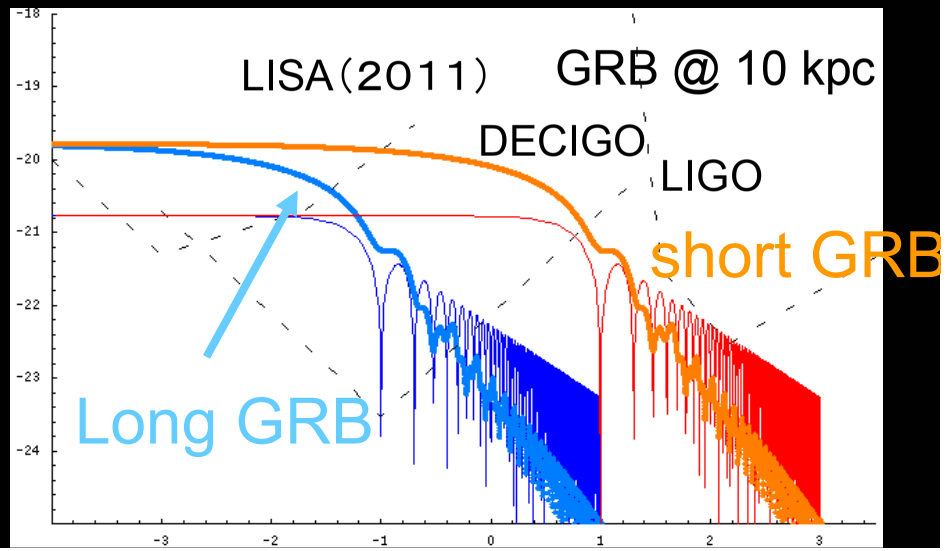
Setiawan et al. (2004)

Neutrino Luminosity



$$h_\nu^{\rm TT} = \frac{8G}{c^4 R}\int_{-\infty}^{t-R/c} dt' \int_0^{\pi/2} d\theta' \; \Phi(\theta') \; \frac{dL_\nu(\theta', t')}{d\Omega'},$$

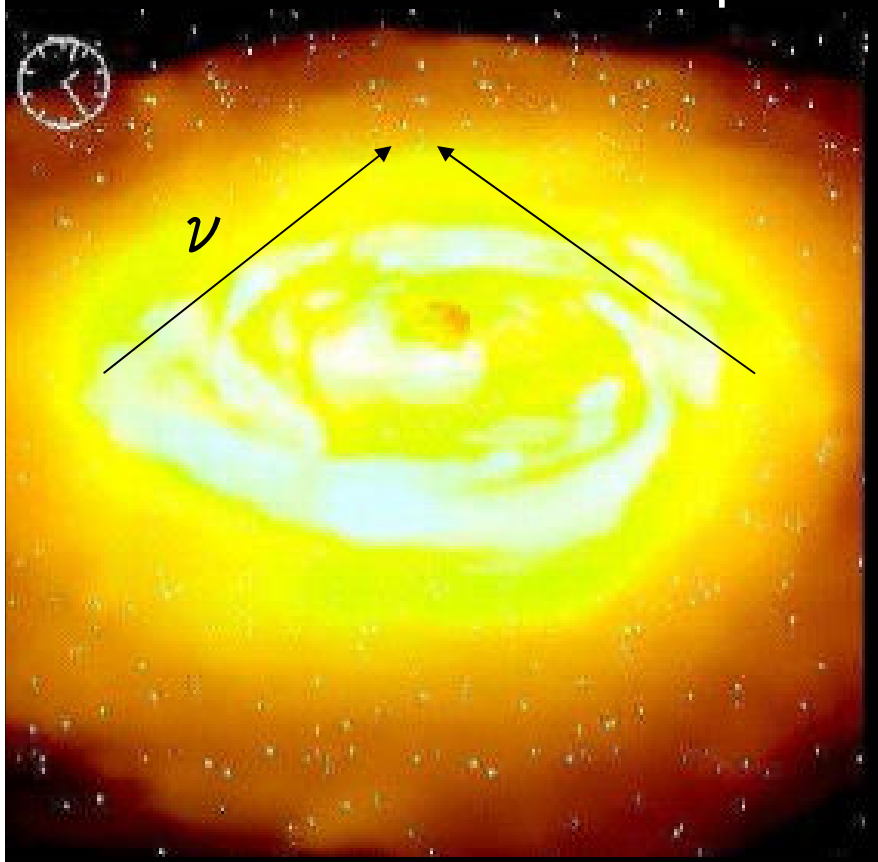
GWs from neutrinos (per GRB)



GWs from neutrinos from Neutrino-Driven GRBs

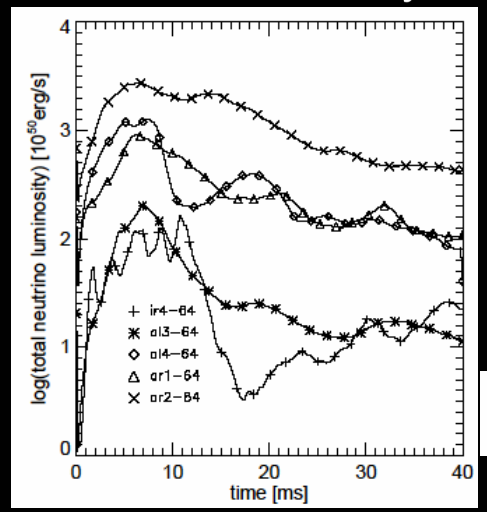
Hiramatsu, KK, Kudoh, Taruya, MNRAS (2006)

3D simulations of collapsar



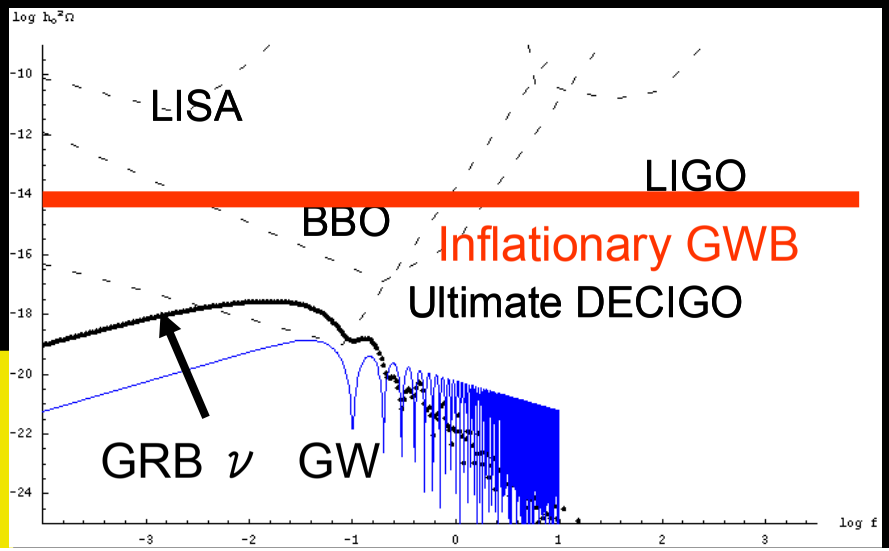
Setiawan et al. (2004)

Neutrino Luminosity



$$h_\nu^{\rm TT} = \frac{8G}{c^4 R}\int_{-\infty}^{t-R/c} dt' \int_0^{\pi/2} d\theta'\ \Phi(\theta')\ \frac{dL_\nu(\theta',t')}{d\Omega'},$$

GW background



GWBs from GRBs are
Well below the inflationary GWBs