Tidally driven flows and magnetic fields due to the elliptical instability

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ISTerre
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Star-Planet Interactions and the Habitable Zone (2014)

Saclay, Tuesday 18-21 November 2014, 17:15

Introduction
Method, approach

- **Modelization :** **a rotating fluid**
- **Fluid mechanics point of view**
- **Incremental approach ⇒ understanding the role of each process**

Credit : ESA/C Carreau

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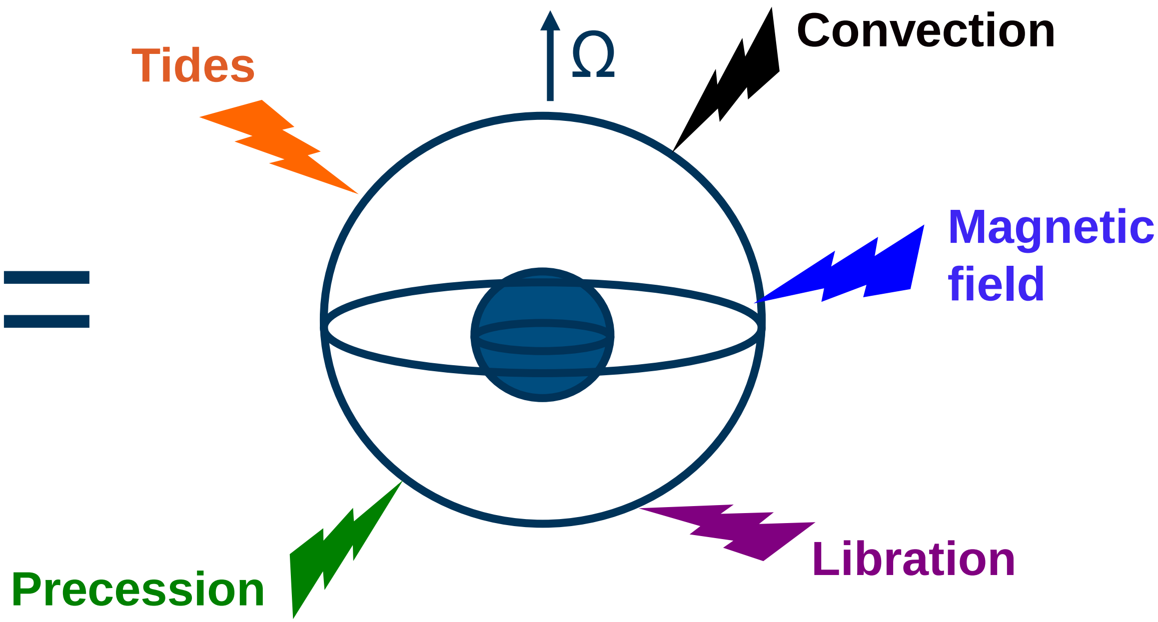
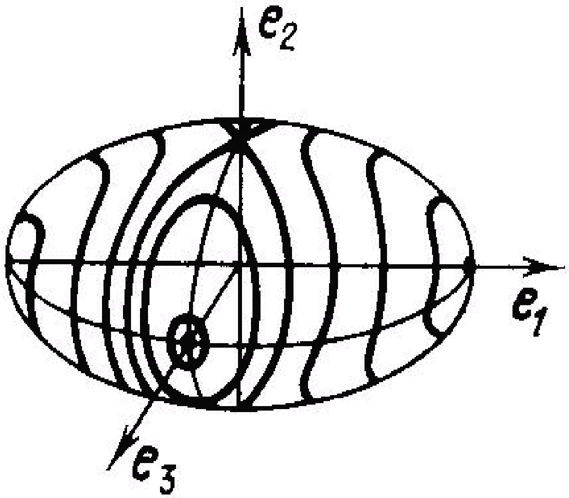


Introduction – The tidal instability

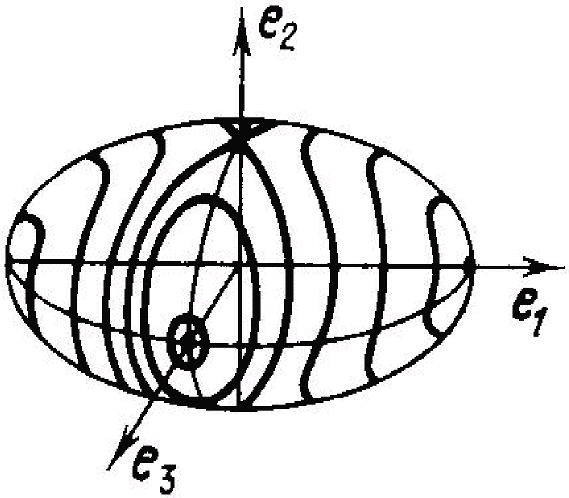
Inertial instability in rotating solids



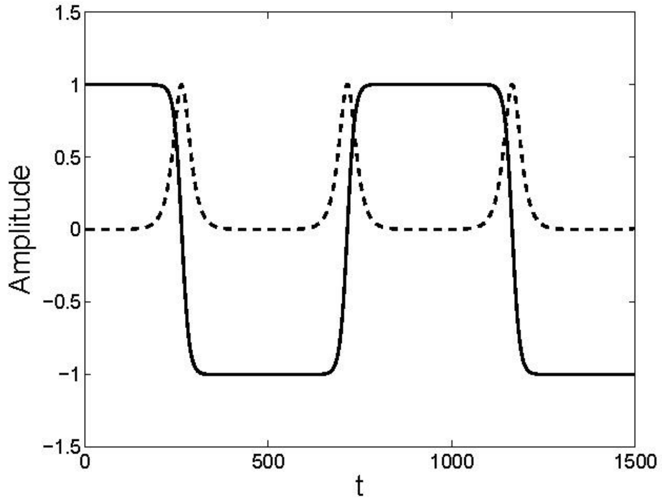
An initial rotation around the middle inertia axis is unstable!

Introduction – The tidal instability

Inertial instability in rotating solids



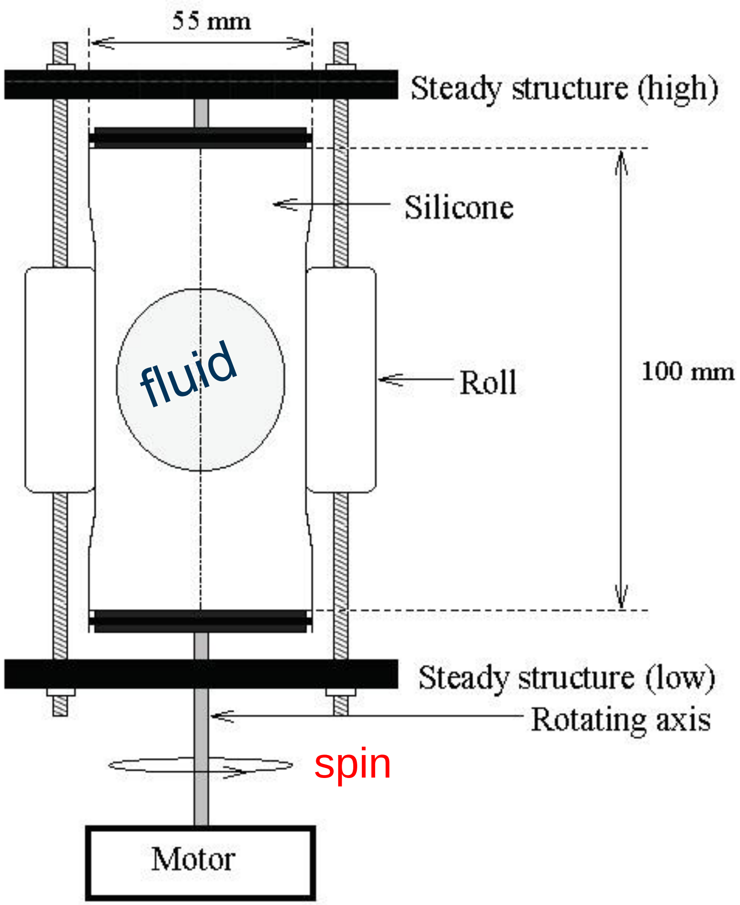
$$\begin{cases} I_1\, d\Omega_1/dt + (I_3-I_2)\,\Omega_2\Omega_3 = 0 \\ I_2\, d\Omega_2/dt + (I_1-I_3)\,\Omega_3\Omega_1 = 0 \\ I_3\, d\Omega_3/dt + (I_2-I_1)\,\Omega_1\Omega_2 = 0 \end{cases}$$

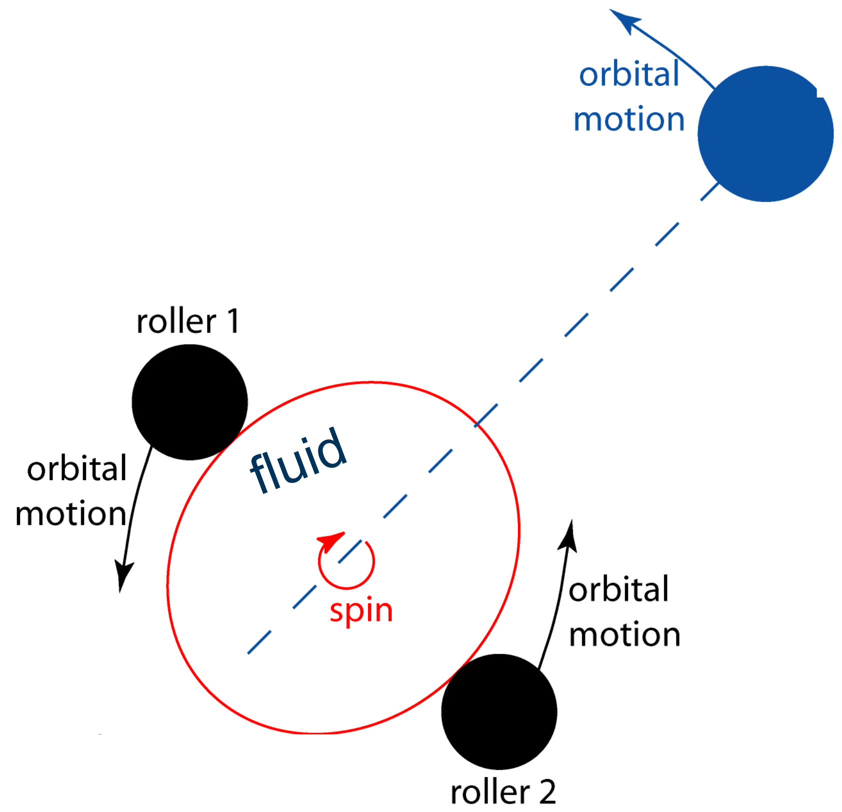


An initial rotation around the middle inertia axis is unstable!

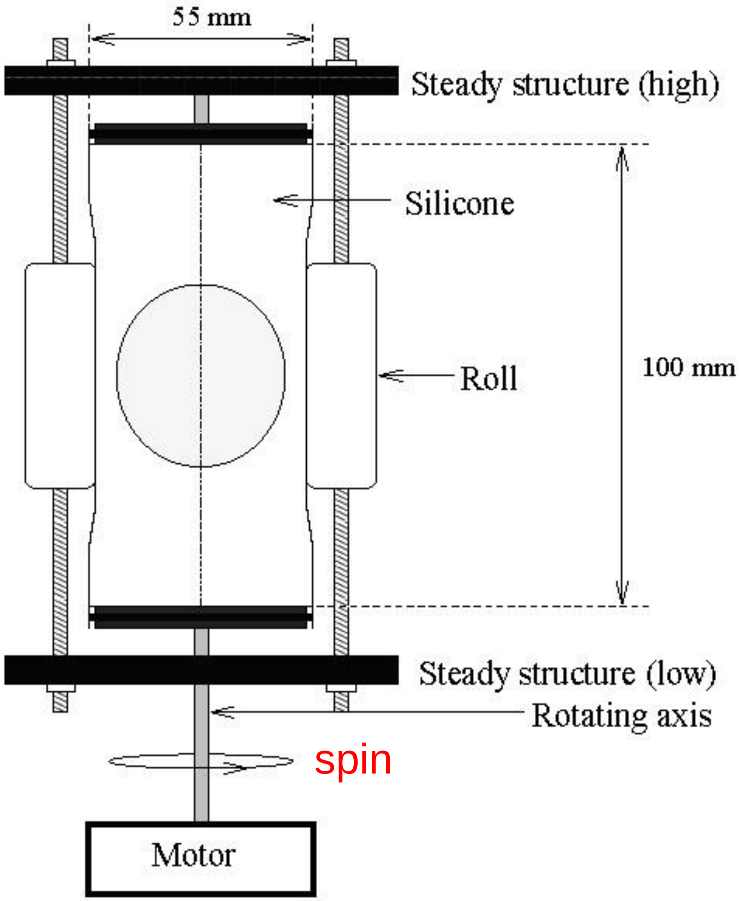
Introduction – The tidal instability

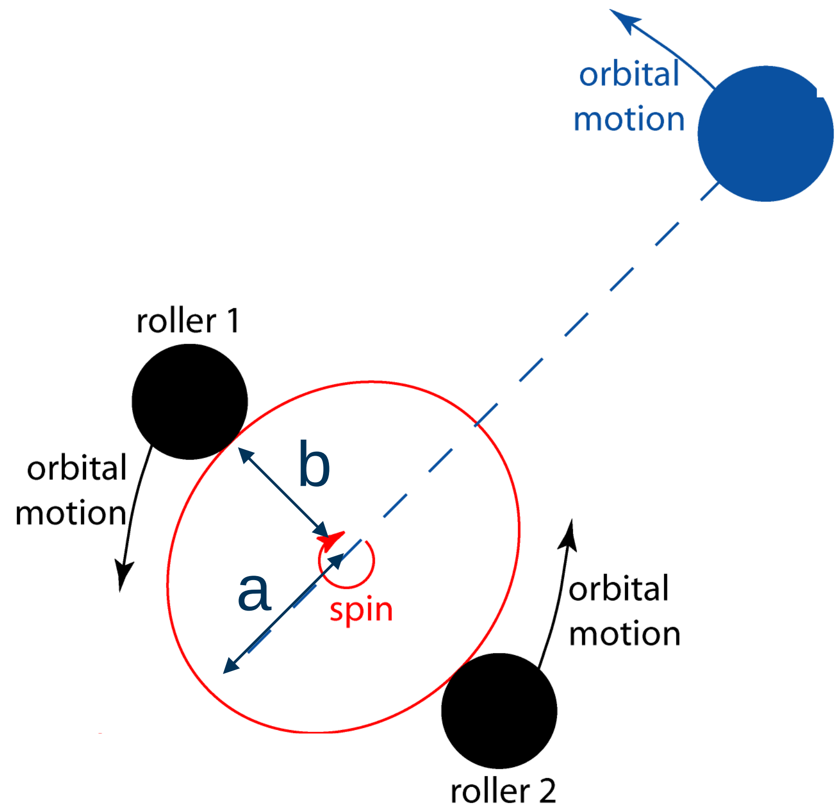
Toy experiment representing a tidally deformed spinning body





Introduction – The tidal instability
Toy experiment representing a tidally deformed spinning body





2 dimensionless numbers: β **and** $E = \nu/\Omega^F R^2 = Re^{-1}$

$$\beta = \frac{a^2 - b^2}{a^2 + b^2}$$

Introduction – The tidal instability
Non linear viscous modelling (Lacaze et al. , 2004)

$$\dot{\Omega}_1 = -\frac{\varepsilon}{(2-\varepsilon)}(1+\Omega_3)\Omega_2 + \nu_{SO}\Omega_1,$$

$$\dot{\Omega}_2 = -\frac{\varepsilon}{(2+\varepsilon)}(1+\Omega_3)\Omega_1 + \nu_{SO}\Omega_2,$$

$$\dot{\Omega}_3 = \varepsilon\Omega_1\Omega_2 + \nu_{EC}\Omega_3 + \nu_{NL}\left(\Omega_1^2 + \Omega_2^2\right).$$

ν 's effects calculated from boundary layers analyses
(Greenspan, Kerswell)

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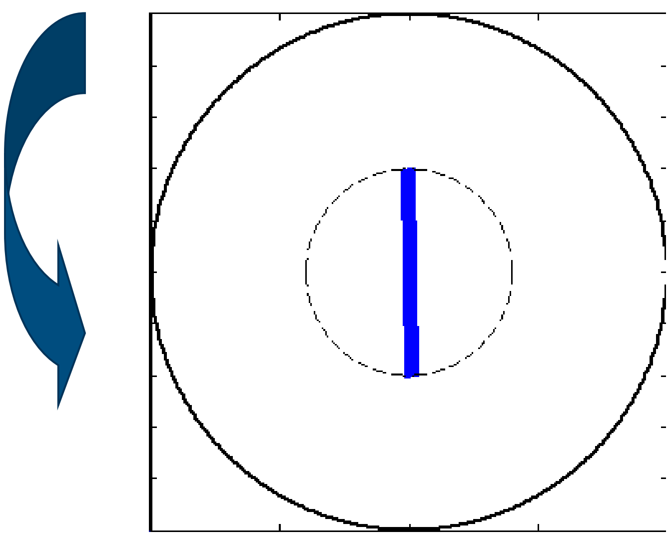
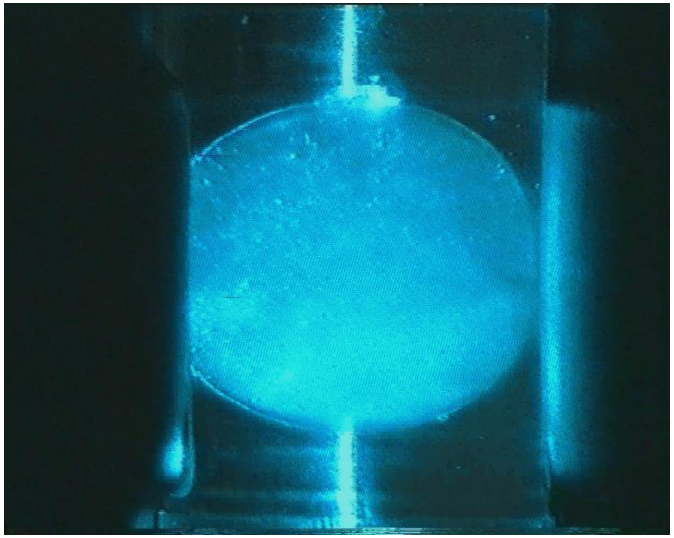
β **&** E = $\nu/\Omega^F R^2$ = Re^{-1}





Introduction – The tidal instability
Non linear viscous modelling

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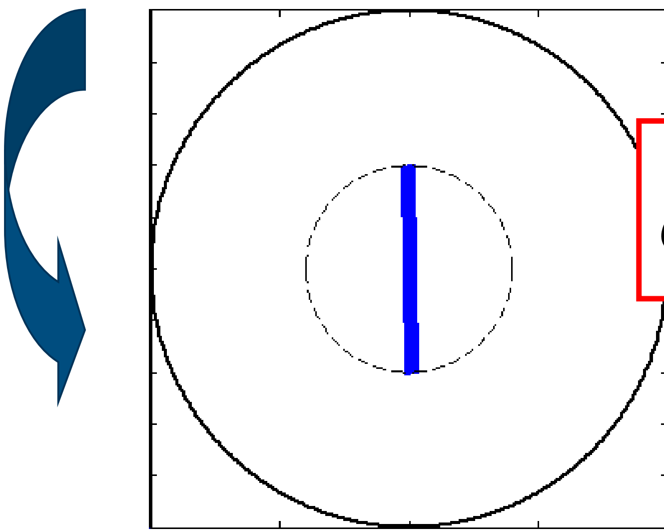
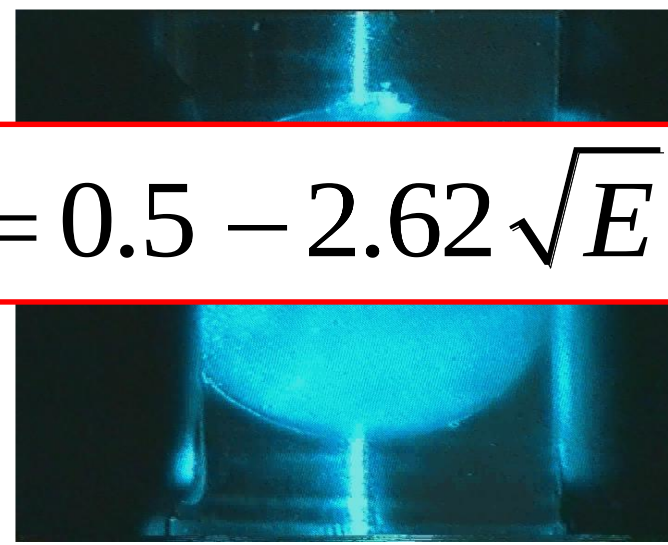


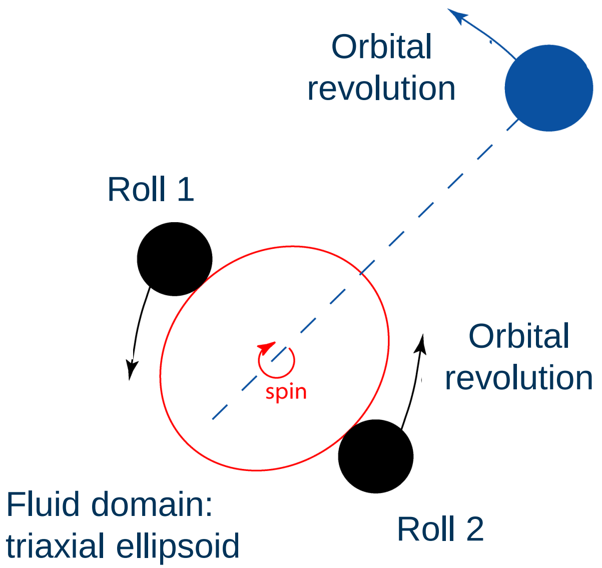


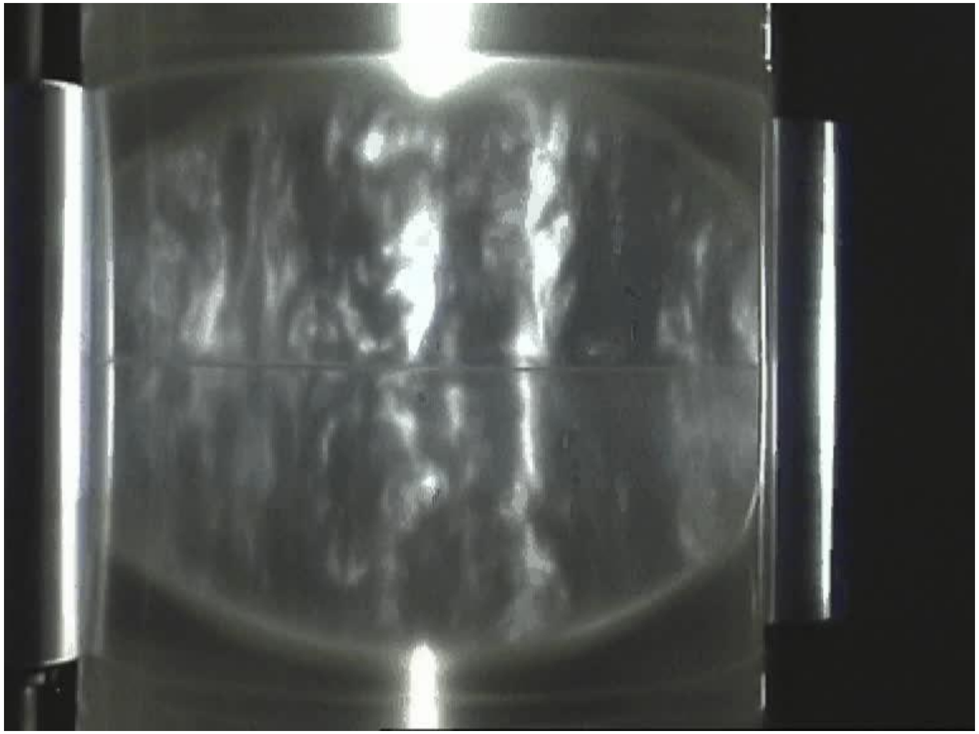
$$\sigma / \beta = 0.5 - 2.62\sqrt{E} / \beta$$

Exact theoretical expression!

An experimental evidence

The tidal instability, a kind of inertial instability





Cycles, turbulence, relaminarisation, …

An experimental evidence
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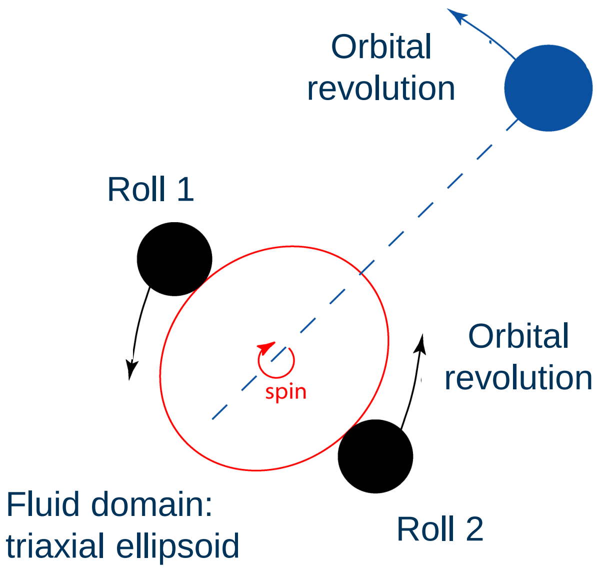
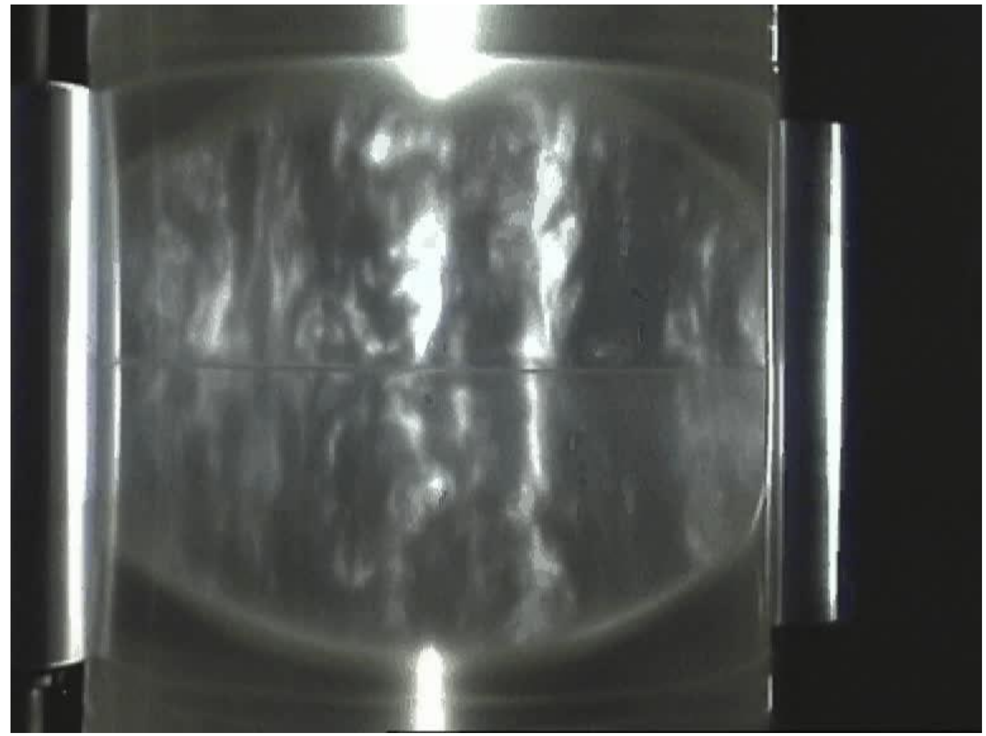
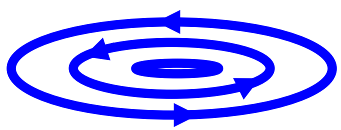




Cycles, turbulence, relaminarisation, …

Local origin



Vortex interactions, shears, etc.

Very small deformation ⇒ 1st order consequences for the flow

Introduction

An parametric instability : triadic resonance of inertial waves

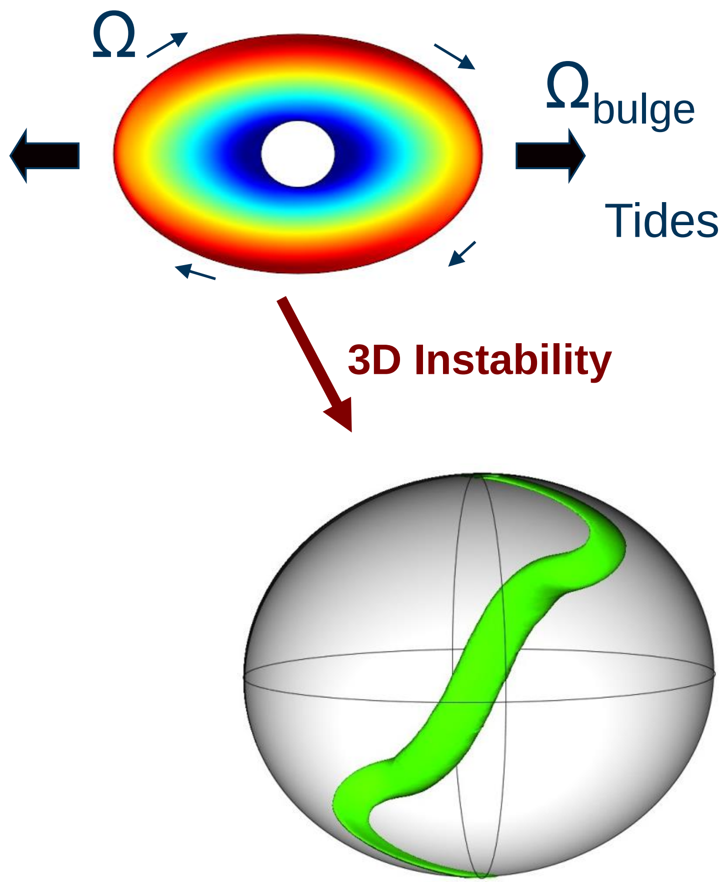
- **Instability under two conditions**

If tidal deformation is large enough $\beta \gg E$

If $\Omega \neq \Omega_{bulge}$

- Stars : Non-synchronized body (TDEI)

- Planets : Synchronized body with librations due to elliptic orbit (LDEI)



Tilted solid body rotation
= (Tidal) spinover mode

Cébron et al., *Phys. Earth Planet. Int.*, **182**, 119-128, 2010

Introduction

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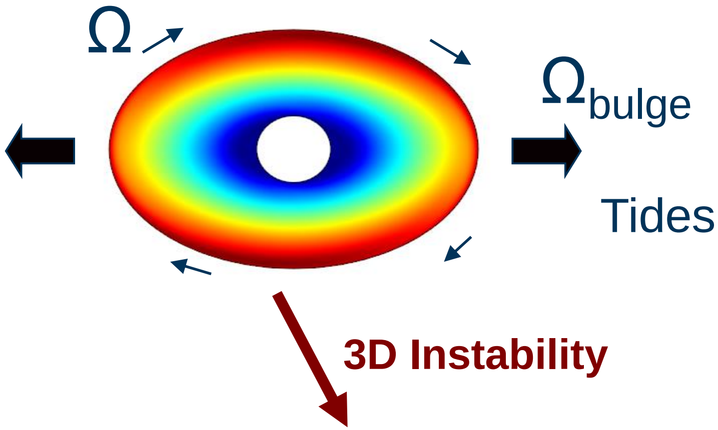

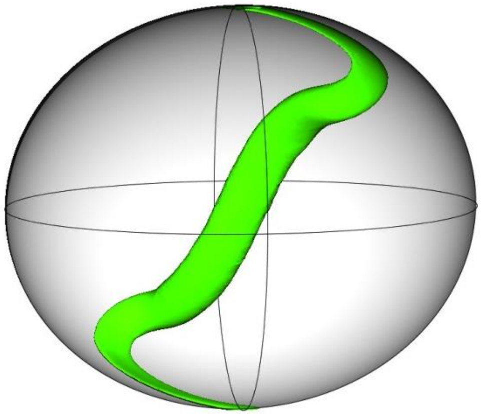

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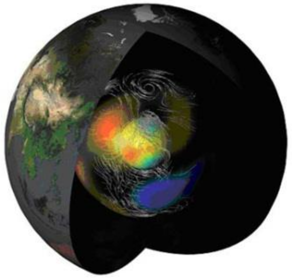
- **Parametric resonance of (magneto-gravito-) inertial waves**

Remains valid in presence of T & B fields

Cébron et al., *Phys. Earth Planet. Int.*, **182**, 119-128, 2010

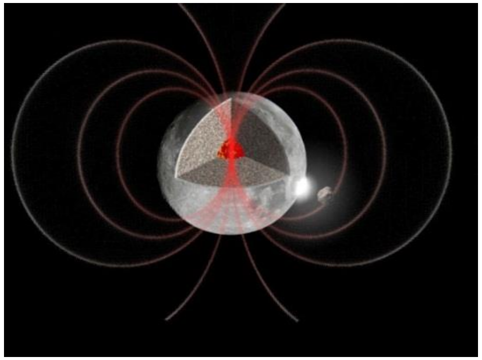
Context - Some issues
Earth-Moon system issues



IPGP (simulation)

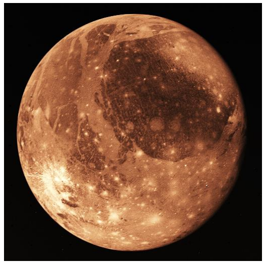
Early geodynamo?

Pozzo et al. *Nature* 2012 : a sufficient thermosolutal flux?

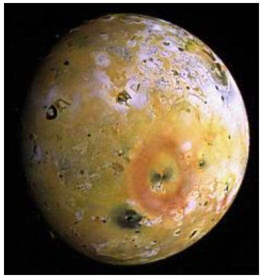


Origin of the lunar magnetic field?

Early dynamo magnetic field? How?



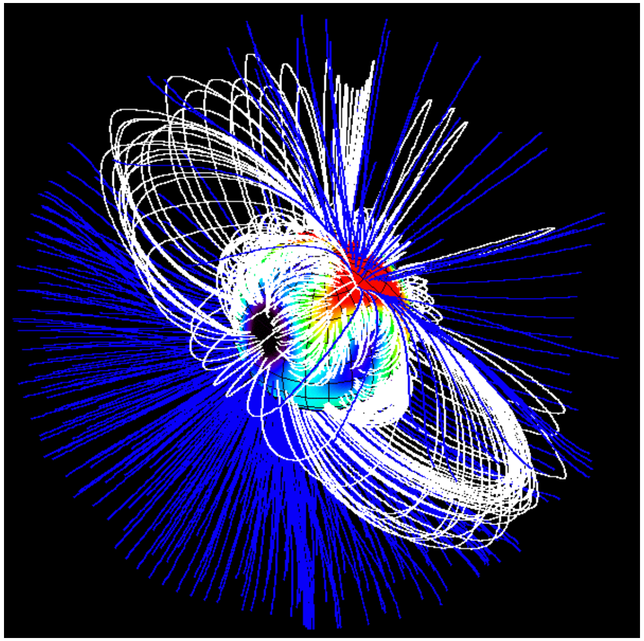
Ganymède: **only** moon with a magnetosphere…
Origin?



Io: ~1000nT of induced field variation…
Flows in the liquid core?

Context - Some issues
In Hot-jupiter systems?

- **Origin of fast magnetic field reversals ?**
  Tau-boo : magnetic cycles of 800 days!



Tau-boo magnetic field
(Donati & Jardine)

Cébron et al., *Icarus*, **226**, 2, 1642-1653, 2013.

Context - Some issues
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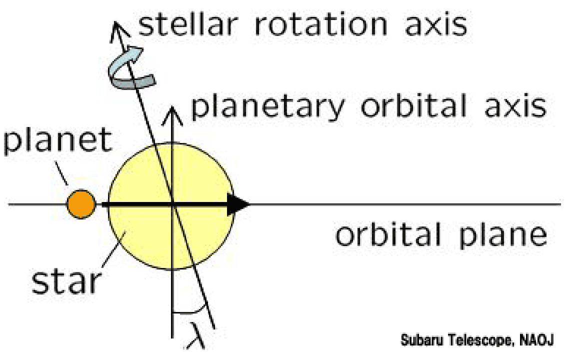
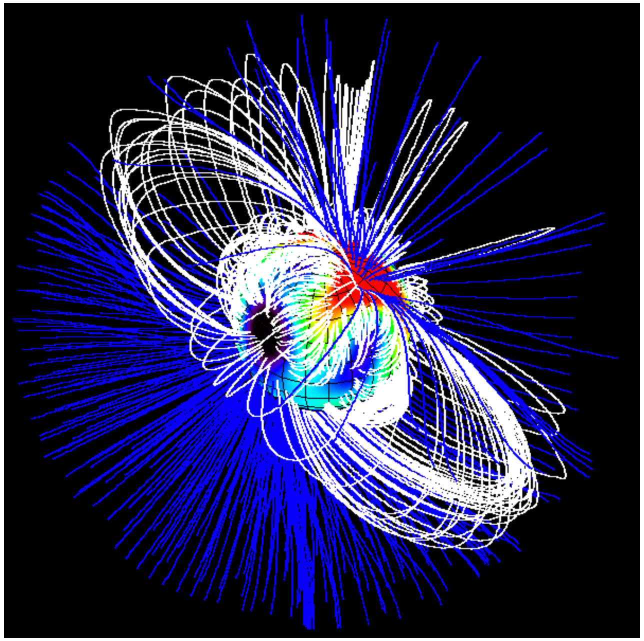
- **Origin of orbital specificities ?**
  e.g. spin-orbit misalignment





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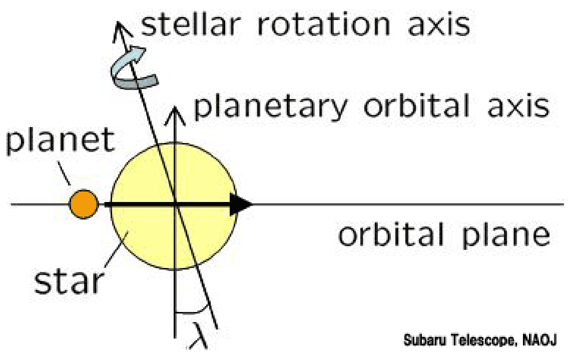
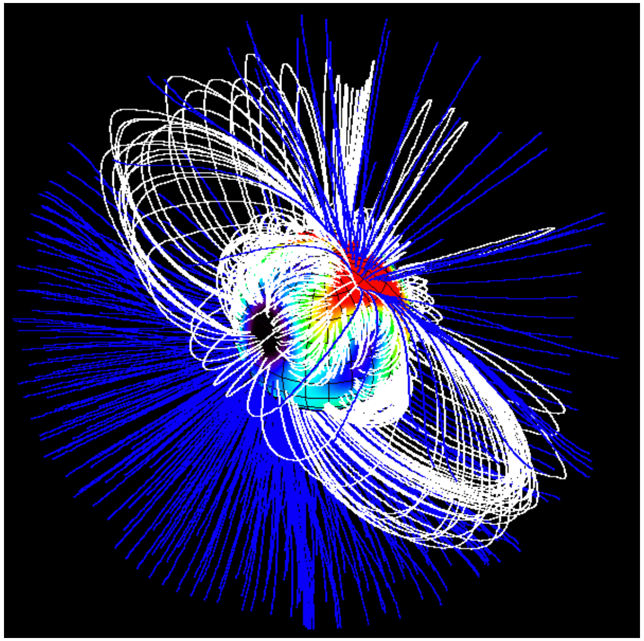




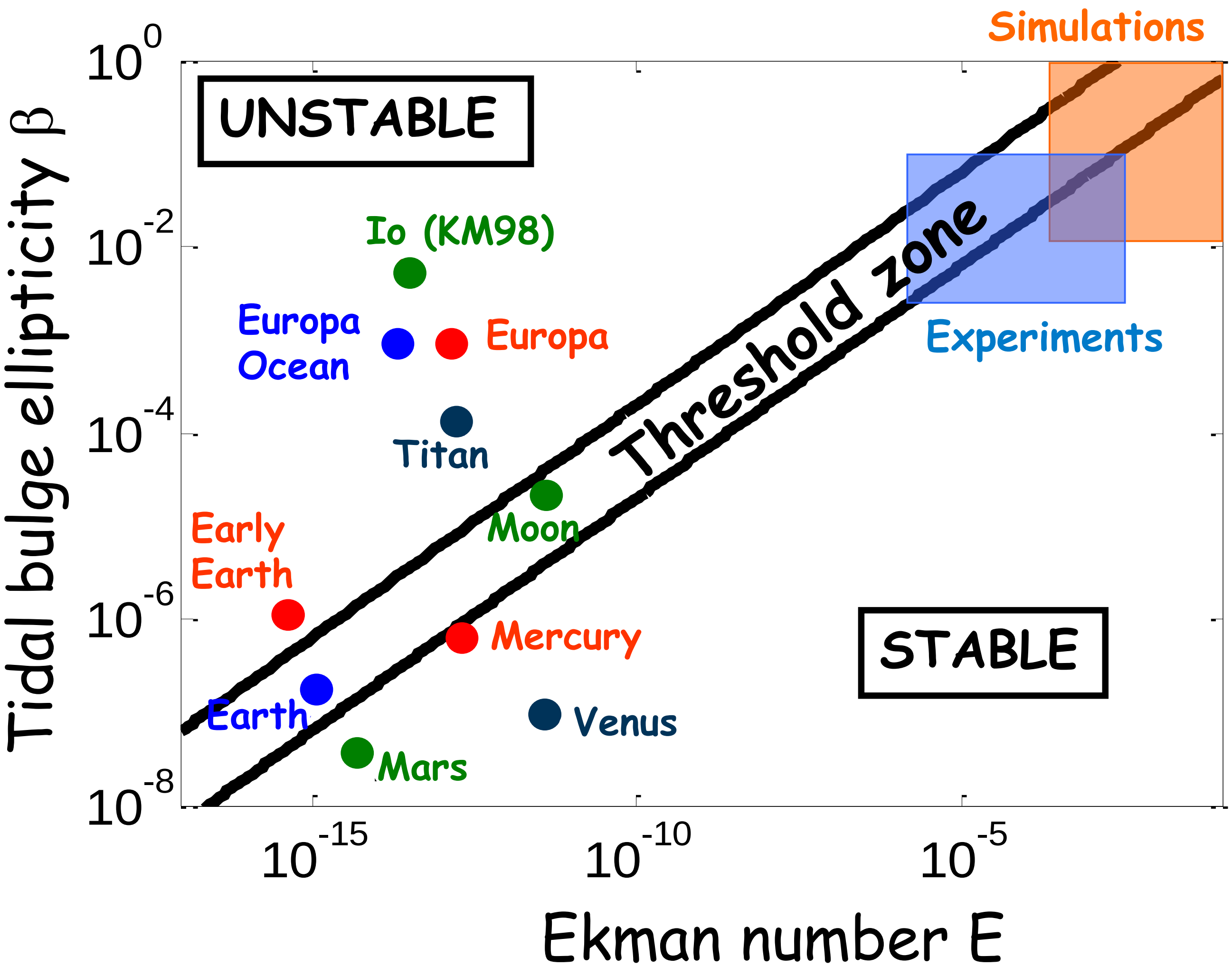
Tau-boo magnetic field
(Donati & Jardine)

- **Bloated Hot-Jupiters ?** (a dissipation source is lacking!)

Cébron et al., *Icarus*, **226**, 2, 1642-1653, 2013.

Applications

From the lab' to the planets and stars



Threshold
$\beta = O(1\text{-}10)\, E^{1/2}$

Early Moon closer from the Earth
⇒ **Ellipticity larger**
⇒ **Unstable**

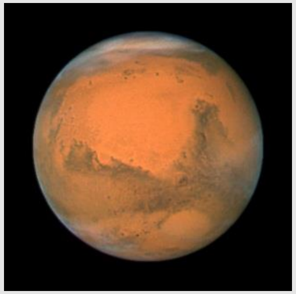
Cébron et al., *A& A*, , **539**, A78, 1-16, *2012.*

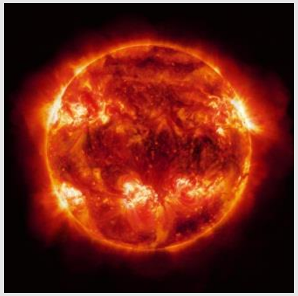
Applications

From the lab' to the planets and stars

Non-synchronized body

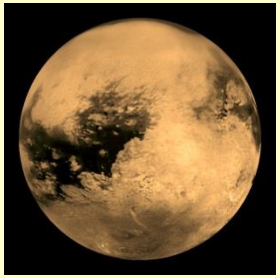
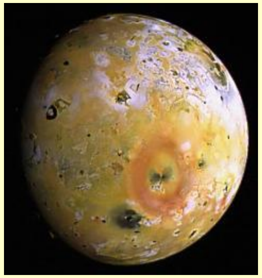
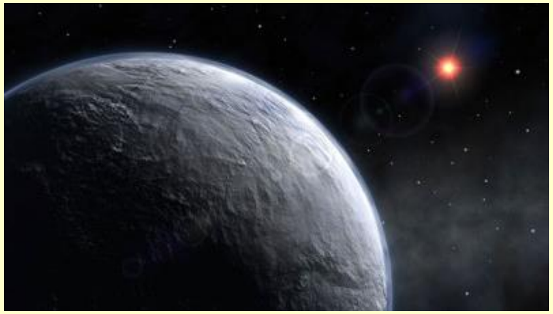
- Planets, stars
- Early moons



Librating synchronized body

- Most of the moons
- Super-Earths

Applications
From the lab' to the planets and stars

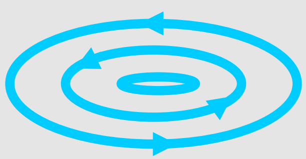
Non-synchronized body

- Planets, stars
- Early moons
- **Constant** differential rotation

$$\Omega_{diff} = \Omega - \Omega_{def} = cte \neq 0$$

- Elliptical instability : **TDEI**

$$\frac{\Omega_{def}}{\Omega}$$



- β > Dissipation f(E)

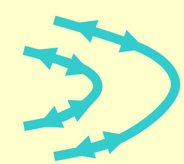
Synchronized body

- Most of the moons
- Super-Earths
- **Oscillating** differential rotation

$$\Omega_{diff} = \Omega - \Omega_{def} = K_{lib} \sin\left(\omega_{lib}\, t\right)$$

- Elliptical instability : **LDEI**

$$\varepsilon = \frac{K_{lib}}{\Omega} \qquad \omega = \frac{\omega_{lib}}{\Omega}$$



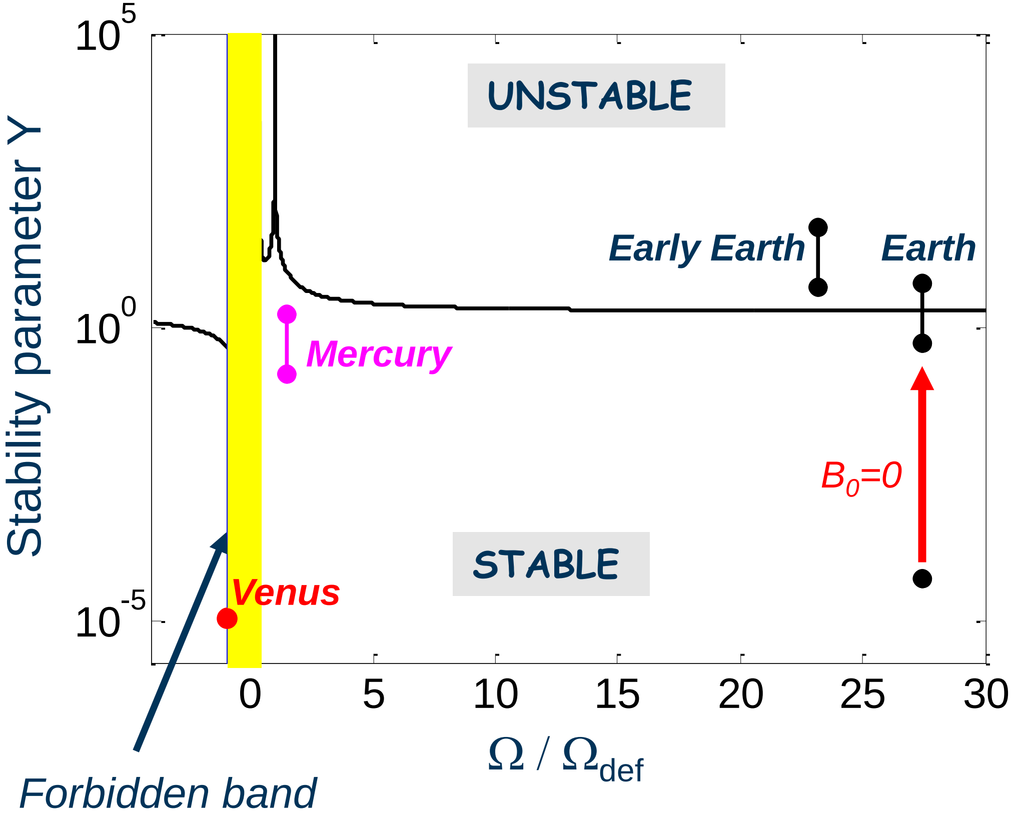
- β ε > Dissipation f(E)

Cébron et al., *A& A*, , **539**, A78, 1-16, *2012.*

Applications

From the lab' to the planets and stars

$$\sigma = \frac{\left(3\Omega - \Omega_{def}\right)^2}{16\,|\Omega|^3}\beta - \frac{\Lambda}{4|\Omega|^3} - \alpha\sqrt{E} \geq 0 \quad \Rightarrow \quad Y = \beta\left(\alpha\sqrt{E} + \frac{\Lambda}{4|\Omega|^3}\right)^{-1} \geq \frac{16\,|\Omega|^3}{\left(3\Omega - \Omega_{def}\right)^2}$$



Early Moon *~2 times closer : smaller E, & larger β*

If the magnetic field of the Earth is due to the instability : ***B_0=0***

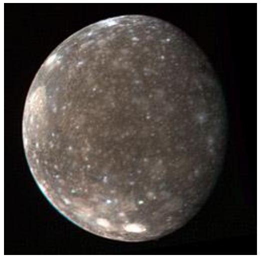
Dynamo ***on-off*** *?*

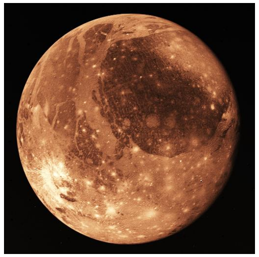
Cébron et al., *A& A*, , **539**, A78, 1-16, *2012.*

Applications
From the lab' to the planets and stars

$$\sigma = \frac{16+\omega^2}{64}\beta\varepsilon - \frac{\omega^2}{16}\Lambda - \alpha\sqrt{E} \;\geq\; 0$$

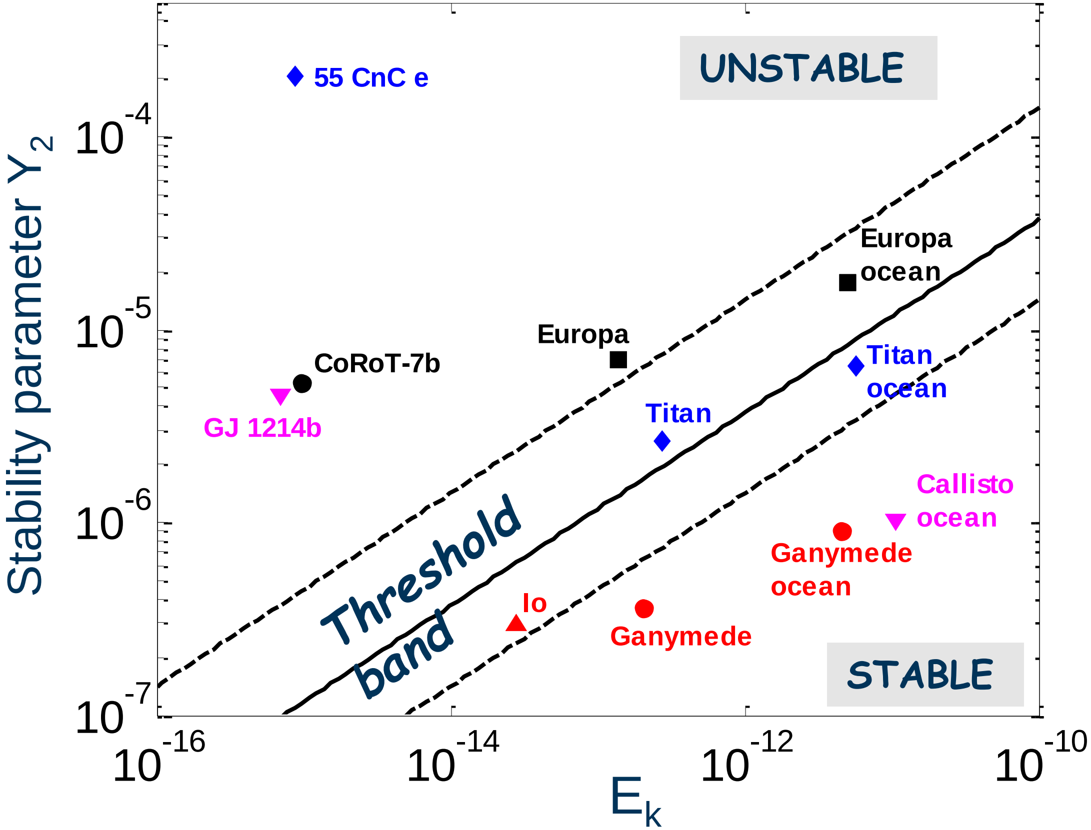
$$\Rightarrow \quad Y_2 = \left[\beta\varepsilon - \frac{4}{17}\Lambda\right] \geq \frac{64}{17}\alpha\sqrt{E}$$


Callisto


Ganymede


CoRoT-7b

Stability diagram (most destabilizing case of libration)



E_k is the Ekman number based on the fluid layer depth rather than the external radius.

Hot-jupiter systems
Fluid mechanic particularities ?

- **Tidal deformation calculated**

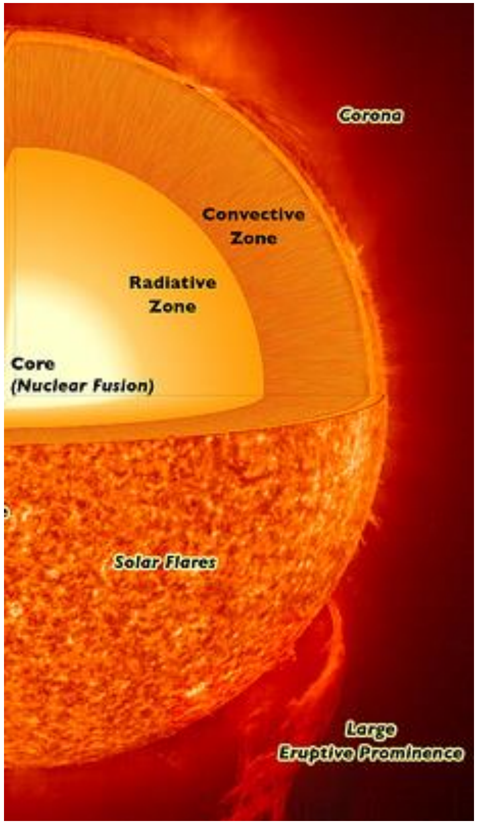
- **Particularities**
  - **Very thin shell or not**
  - **Strong convection**
  - **Differential rotation**
  - **Compressibility**
  - **Free surface (SPH)**

Tackled with

- Lab. experiments

- Theory

- Num. MHD simulations



Credit : www.ualberta.ca

$$\boxed{\frac{\sigma}{\beta} = f_2\left(\frac{\Omega_{spin}}{\Omega_{orb}}\right)}$$

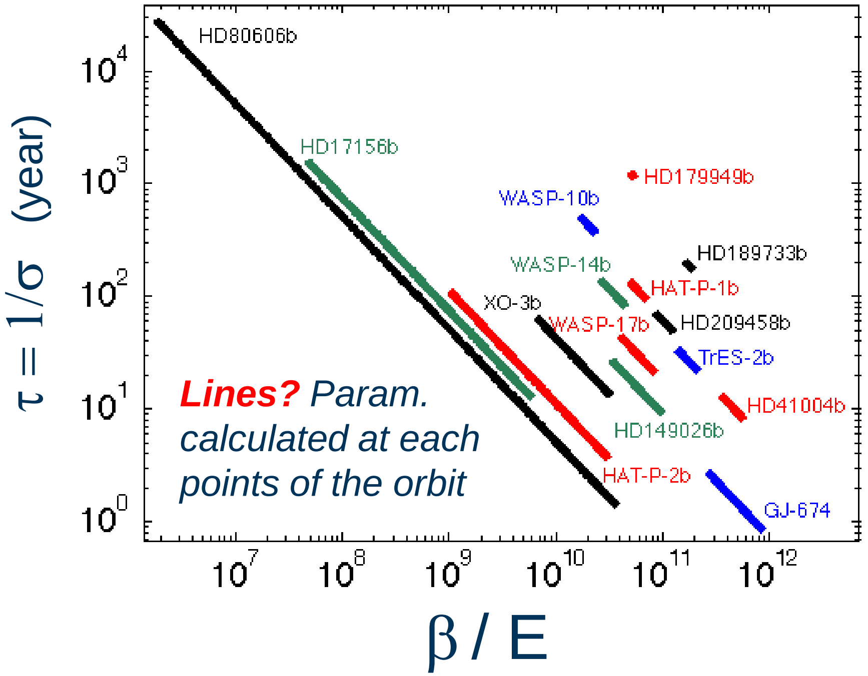
(WKBJ analysis)

f_2 known analytically

Cébron et al., *Icarus*, **226**, 2, 1642-1653, 2013.

Observational signatures?
Considering the Hot-jupiter planet

Presence in the Hot-Jupiter?



- Very short typical growth times τ
- **Vigorous instabilities?**

Collaboration : J. Leconte (CITA / LMD)

Observational signatures?

Considering the Hot-jupiter planet

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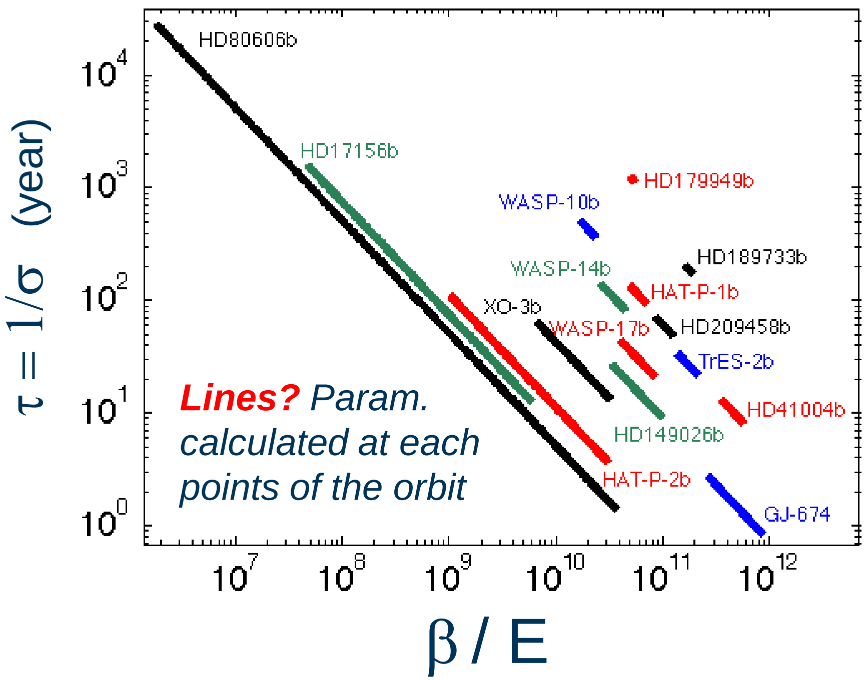
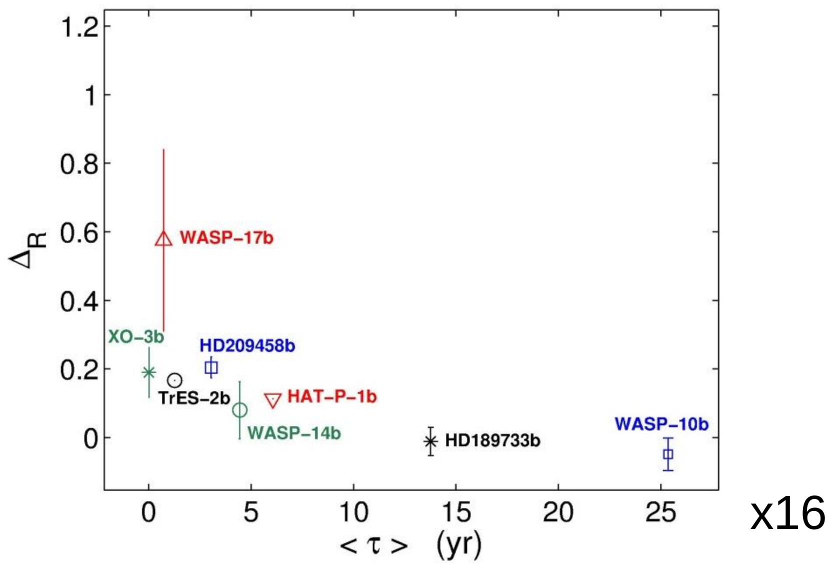


- Very short typical growth times τ
- **Vigorous instabilities?**

Planetary proxy : radius anomaly?

Evolution model + Mass + EOS
$\Rightarrow$ Theoretical **prediction** of the **radius**

Difference with measures **: tidal instability?**



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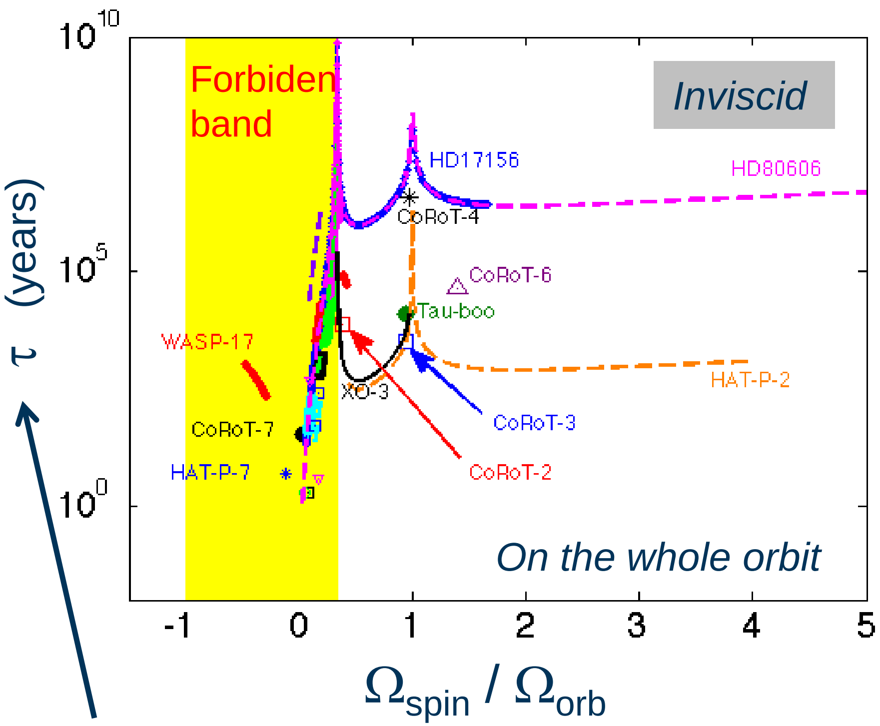
Observational signatures?
Considering the star

Presence in the star?



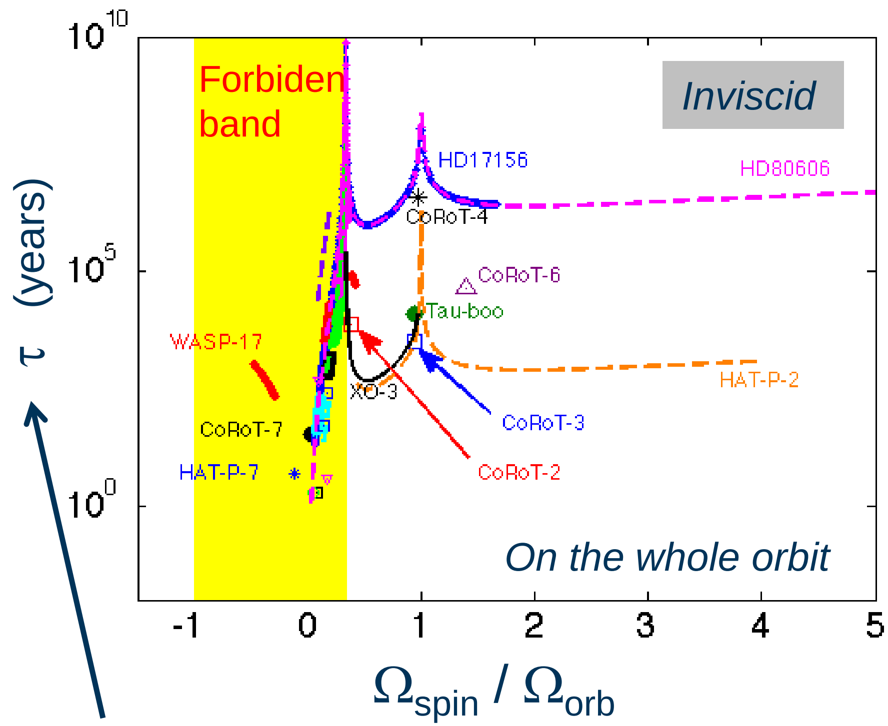
Instability typical growth time

- **Tau-boo** : magnetic cycle of 800d…

Collaboration : Claire Moutou (OAMP, CFHT)

Observational signatures?
Considering the star

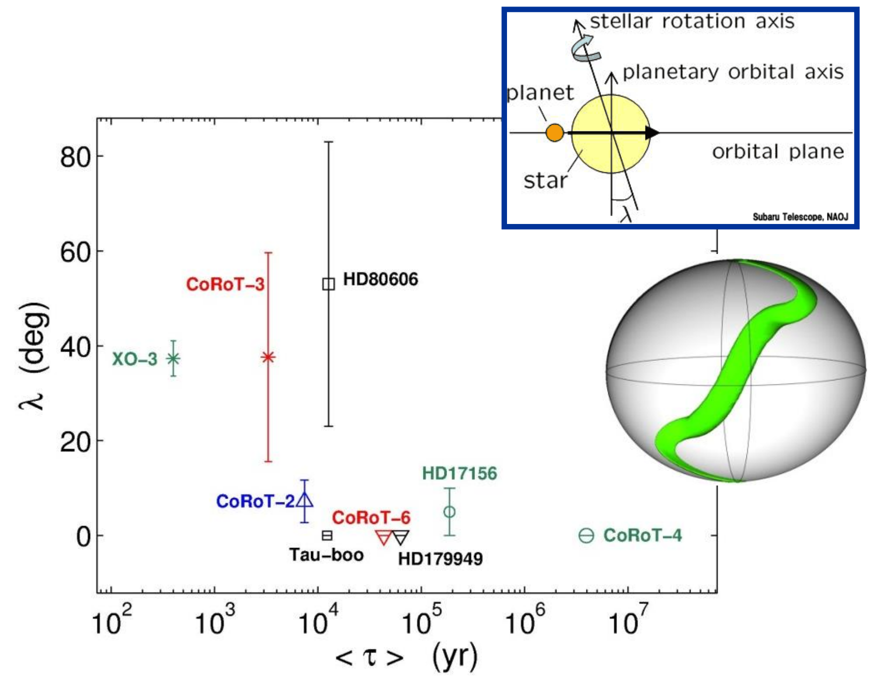
Presence in the star?



Instability typical growth time

- **Tau-boo** : magnetic cycle of 800d…

Stellar proxy : the angle spin-orbit?



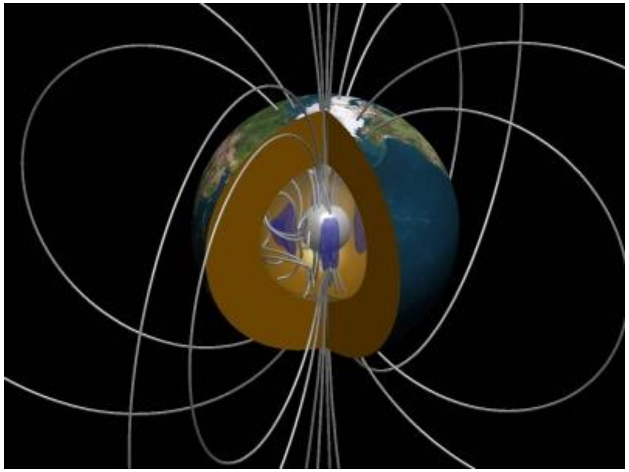
Stellar proxy : the magnetic field

Exemple : a **tides driven dynamo** on **τ-boo?**

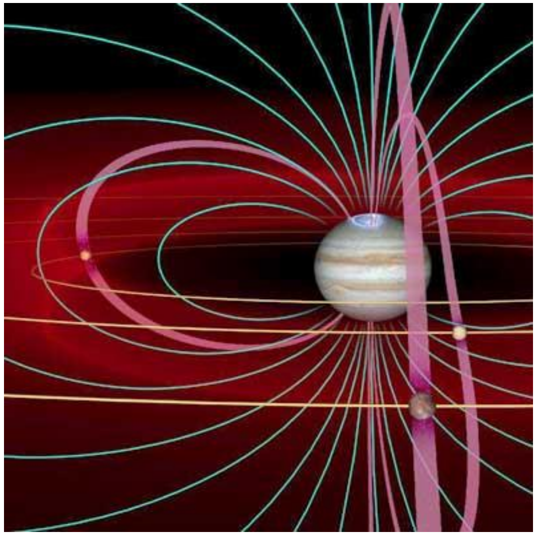
Collaboration : Claire Moutou (OAMP, CFHT)

MHD tidally driven flows & instabilities
Origin of natural dynamos?

- **Prevalent model:** magnetic field generated by **thermochemical convective motions** within an electrically conductive fluid layer



Terrestrial planets like the Earth…

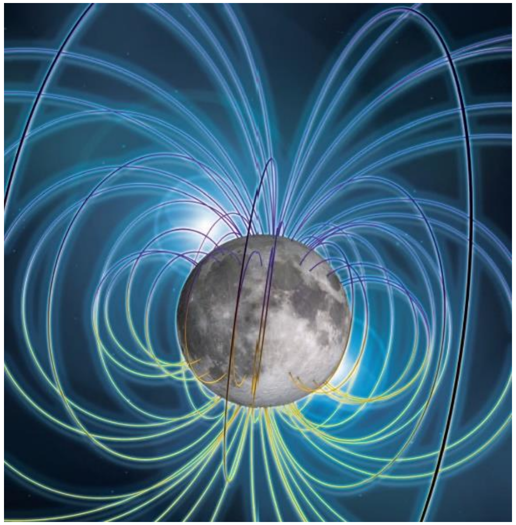


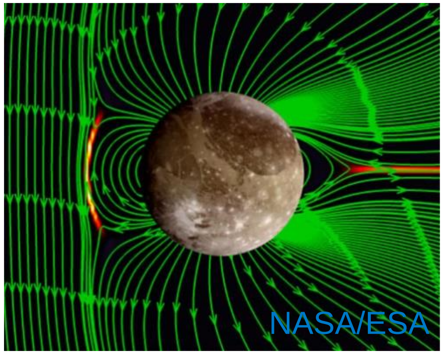
Gaseous planets like Jupiter…

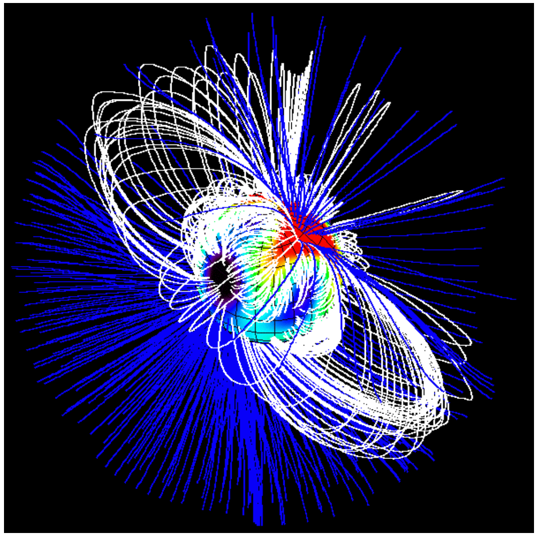
Stars like the Sun…

MHD tidally driven flows & instabilities
Origin of planetary core flows and dynamos?

- **Prevalent model:** magnetic field generated by **thermochemical convective motions** within an electrically conductive fluid layer







Tau-boo magnetic field (Donati & Jardine)

- but **Early Earth? Moon? Ganymede? Mars? Mercury? Hot-Jupiter/binary systems?** *(Jones 2011, Fares et al. 2009, Cébron et al. 2013, etc.)*

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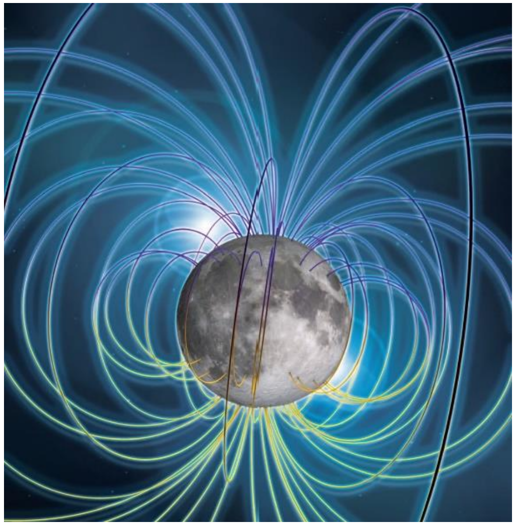

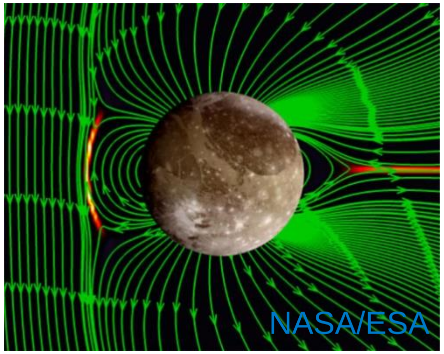

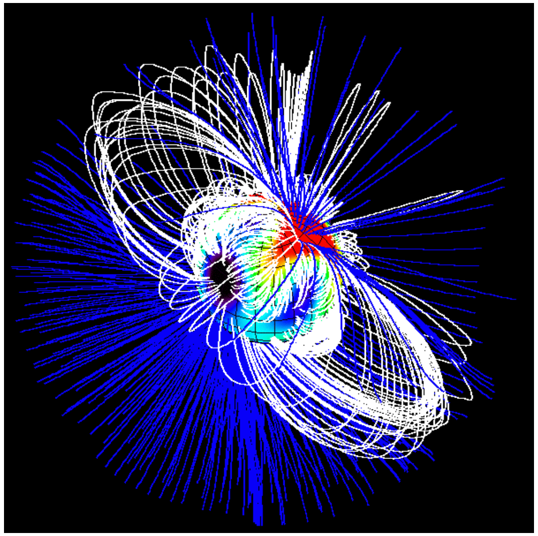

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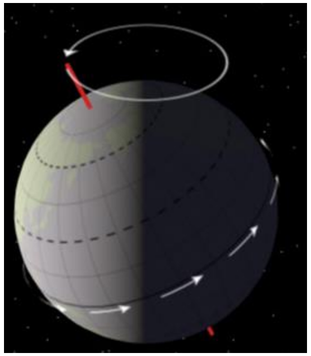
Besides, even if dynamo of convective origin, role of other driving mechanisms in the fluid motions and dynamo effect?

MHD tidally driven flows & instabilities
Alternative dynamo forcings?

- **Precession (Tilgner 2005, 2007) : dynamo capable!**
  ”precession of the rotation axis of the Earth and tidal deformation of the CMB are two effects of which we know that they exist, whereas we do not know with certainty whether the core is convecting, which makes it ***indispensable to study the response of the rotating core to precession and tides****”*



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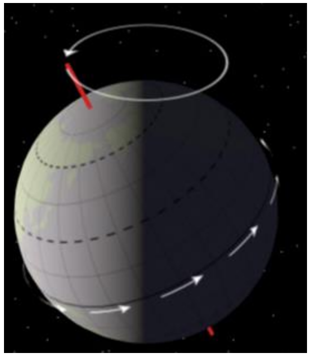
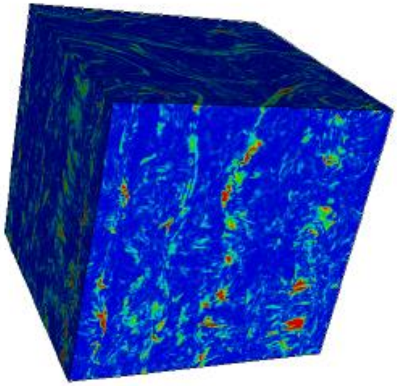


- **Tides ?**
  Barker & Lithwick, MNRAS 2014 : tidally driven small-scale dynamos in a periodic box (w/ hyperdiffusion)



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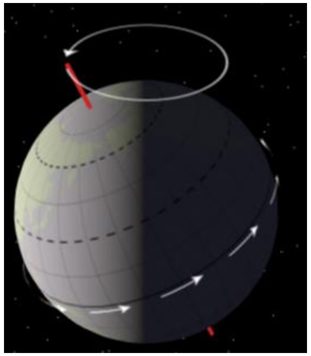

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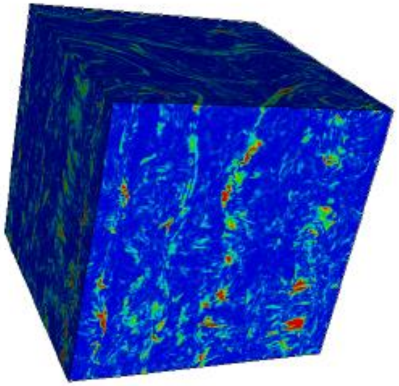
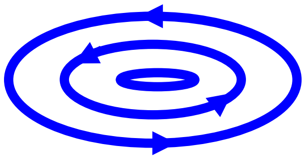
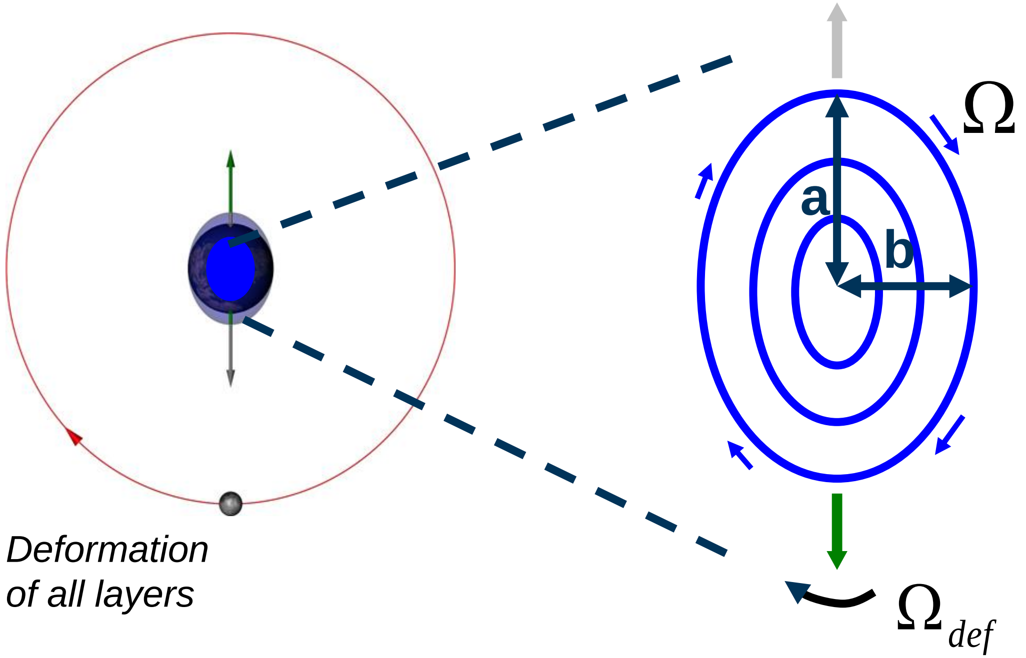


⇒ **Tidally driven large-scale dynamos?**

proposed for ***Mars*** *(Arkani-Hamed 2008,2009)*
*the **Early Moon** (Le Bars et al. 2011)*
Hot-Jupiter or binary stars *(Fares et al. 2009)*
…

Tides → How to simulate elliptical streamlines?



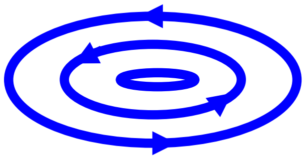


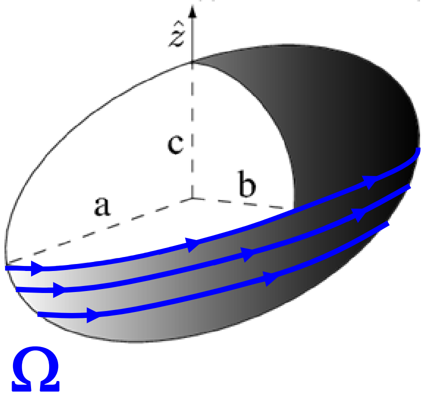
Deformation of all layers

Cébron et al., *Geophys. Astrophys. Fluid Dyn*., 106, 4-5, 524-546, 2012.

MHD tidally driven flows & instabilities
Simulations of tidally driven flows?

Tides → How to simulate elliptical streamlines?





- **Method 1 : non-axisymmetric** ellipsoid
  local method (FV as YALES2, FE as COMSOL),
  but **huge CPU cost** for **dynamo problems**!

Cébron et al., *Geophys. Astrophys. Fluid Dyn*., 106, 4-5, 524-546, 2012.

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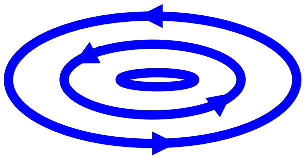

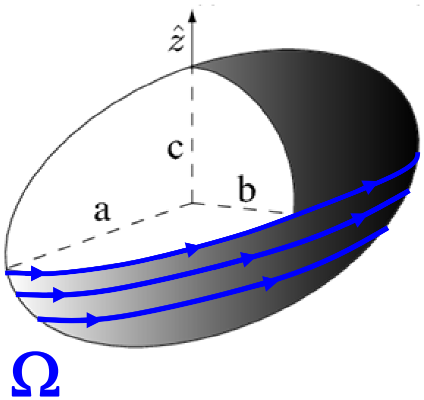

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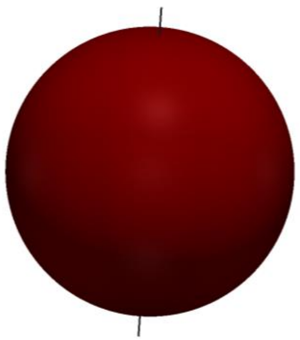
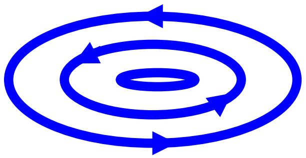
  => **tricks** to benefit from **spectral codes efficiency**

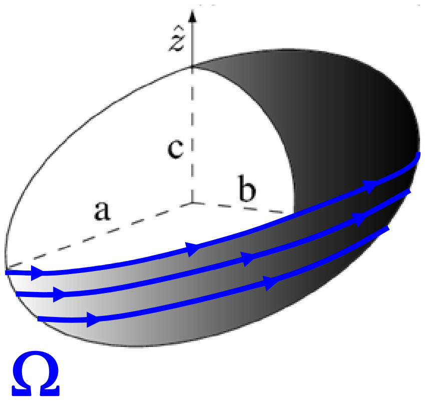


Method	Spectral	Finite-Volume	Finite-element
Formulation	Toroidal/Poloidal	**B**, div-form	**A**
Unknowns	81(rad)x42x42	1.5M	4k
CPUh (τ_η)	300	15k	9k

Simulations of tidally driven flows?

Tides → How to simulate elliptical streamlines?





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  1) Imposing **boundary injection/suction**? Favier et al., MNRAS 2014
  Issues for the **dynamo problem**.

  2) ***Using an appropriate force…*** ***Cébron & Hollerbach, ApJ, 789, L25, 2014***

MHD tidally driven flows & instabilities
Mathematical description of the problem?

- **Scales:** R, 1/Ω, R Ω $(\mu\rho)^{1/2}$

- **Equations**

$$\frac{\partial \vec{B}}{\partial t} = \frac{E}{Pm}\nabla^2 \vec{B} + \nabla \times \left(\vec{u} \times \vec{B}\right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u} = -\nabla p + E\nabla^2\vec{u} + \vec{F}_0 + \left\{\left(\nabla \times \vec{B}\right)\times \vec{B}\right\}$$

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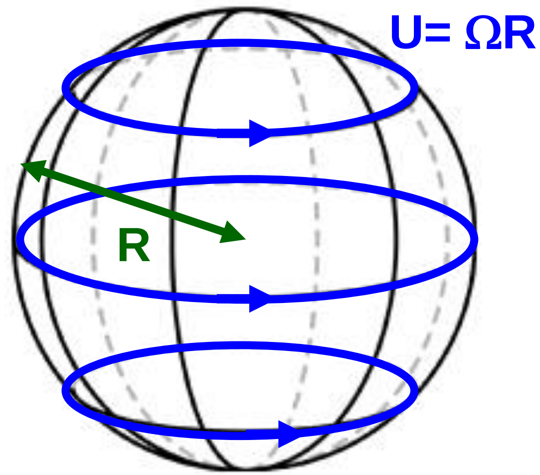

Sphere *of fluid (density ρ, kin. viscosity ν, permeability μ, conductivity σ)*

$$E = \frac{\nu}{\Omega R^2} \qquad Pm = \sigma\mu\nu$$

- **Force F_0: non-conservative!**

$$\boxed{\vec{F}_0 = \varepsilon\left(r\sin\theta\right)^3\left(1-r^2\right)\cos 2\phi \;\; \vec{e}_s}$$

See Lewis & Bellan (1990)

Keep things simple: *tidal field rotation neglected!*

MHD tidally driven flows & instabilities
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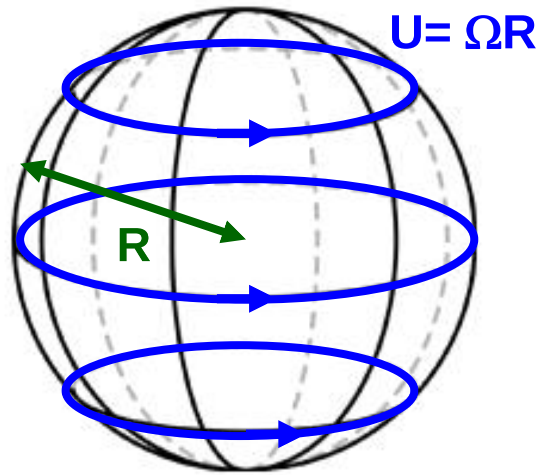

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3 control parameters

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See Lewis & Bellan (1990)

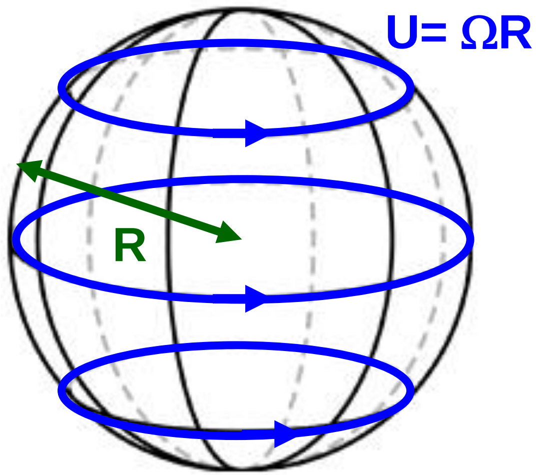
Keep things simple: *tidal field rotation neglected!*

MHD tidally driven flows & instabilities
Details on numerical simulation?

- **Code:** spectral code **H2000**
  Hollerbach 2000; Hollerbach et al. 2013

- **Solve:** departure **u*** from solid body rotation

$$\vec{u} = \underbrace{r \sin\theta\, \vec{e}_\phi}_{\text{solid body rotation}} + \vec{u}\,*$$



***Sphere** of fluid (density ρ, kin. viscosity ν, permeability μ, conductivity σ)*

- **Boundary conditions on u*:**
  zero angular momentum & stress-free B.C
  (i.e. non-zero angular momentum & stress-free B.C. on **u**)

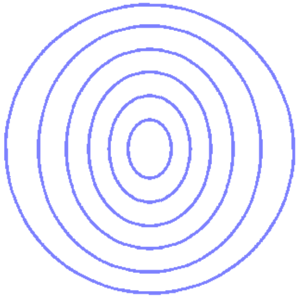
- **Fixed Ekman number here:** **E = 5.10^{-3} (non-parallel code)**

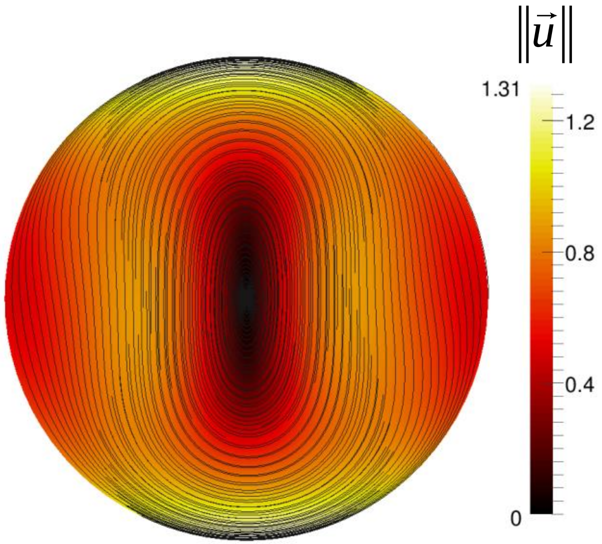
MHD tidally driven flows & instabilities

Forced basic flow

- **Basic flow:**
  Steady
  even azimuthal wavenumbers m

  Streamlines: ellipses of long (resp. short) axis a (resp. b)





Equatorial plane (ε=10, **B**=**0)**

- **Basic flow:**
  Steady
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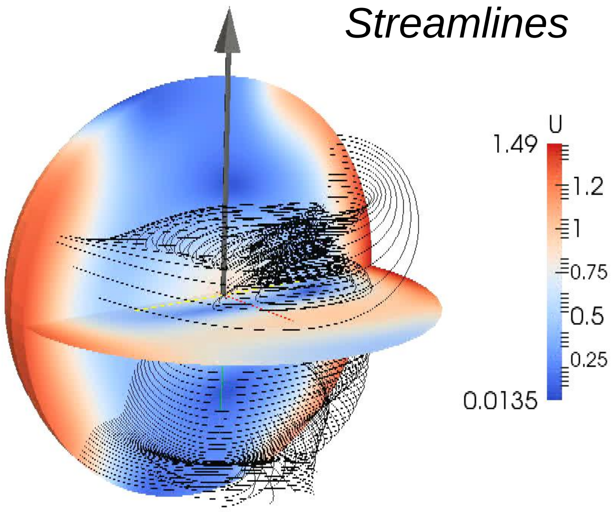
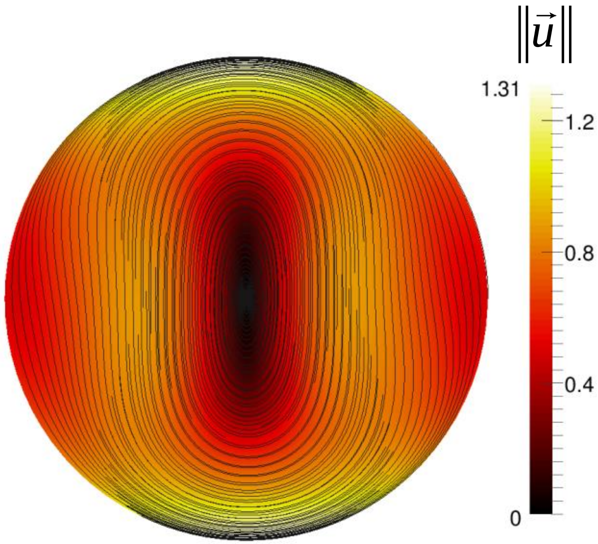
Streamlines





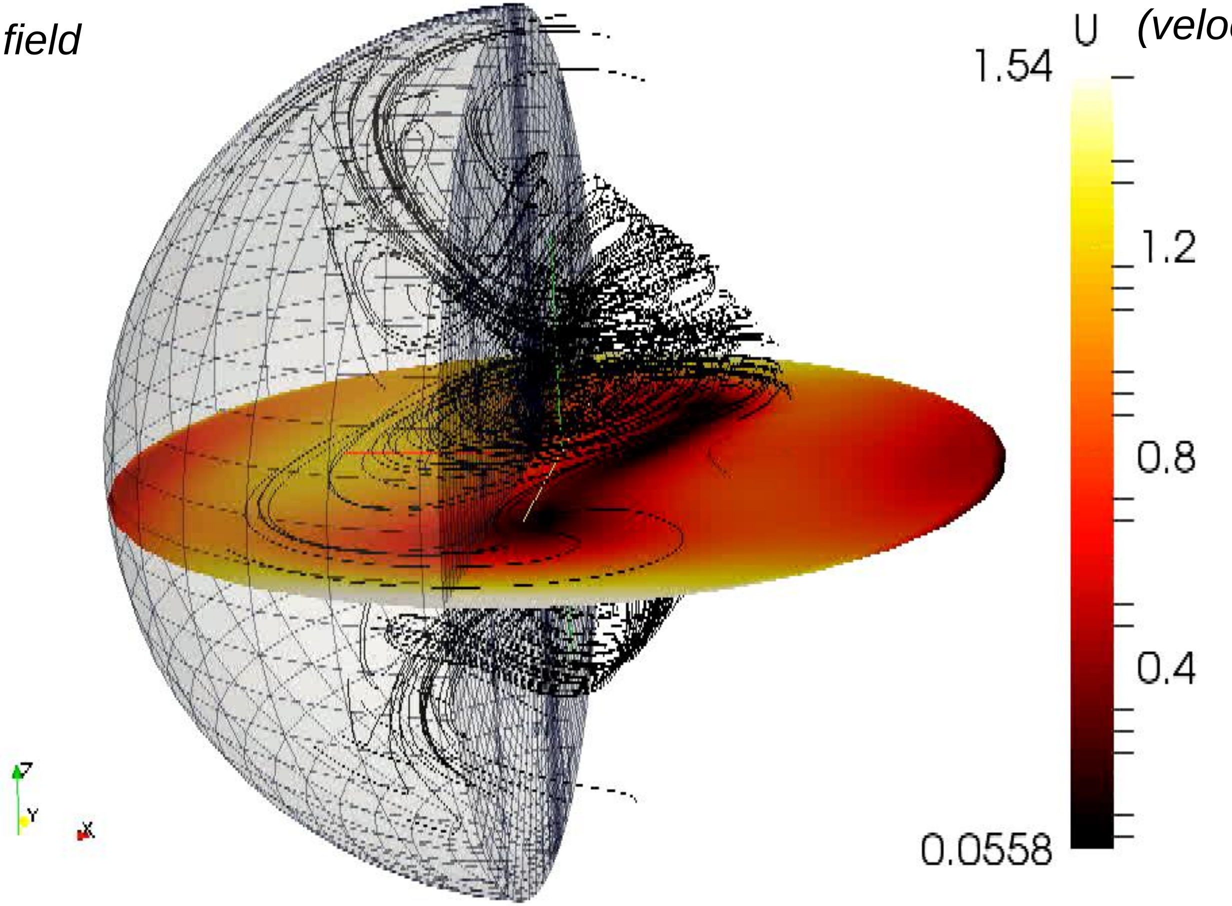
Equatorial plane (ε=10, **B**=**0**)

For ε>10.4, an instability generates an <u>**equatorially symmetric flow, oscillating at ω~1**</u>

MHD tidally driven flows & instabilities

Self-consistent dynamos (here ε=13, Pm=5)

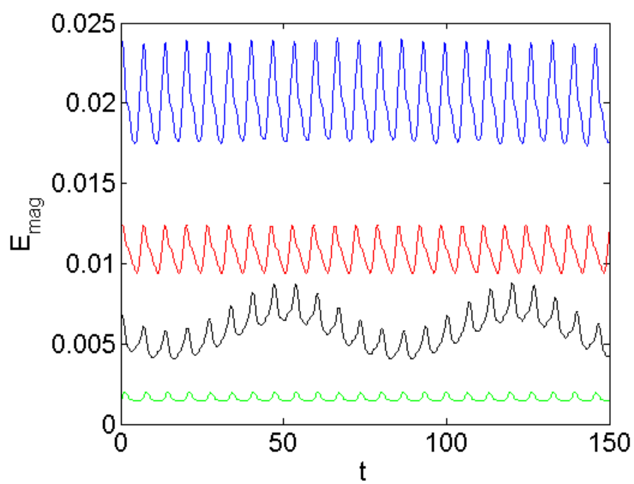
Magnetic field lines



(velocity magnitude)

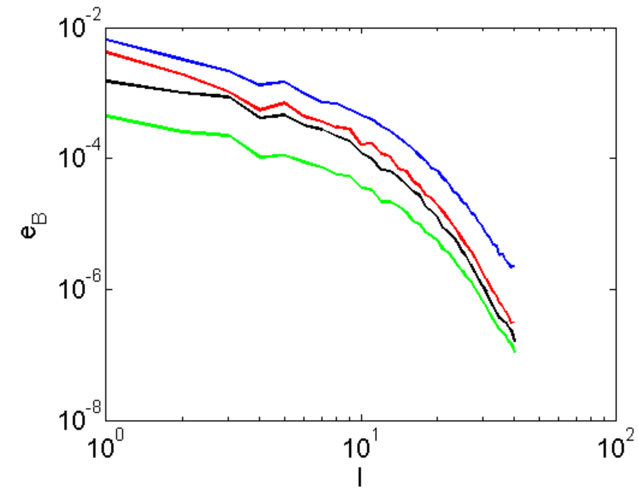
MHD tidally driven flows & instabilities

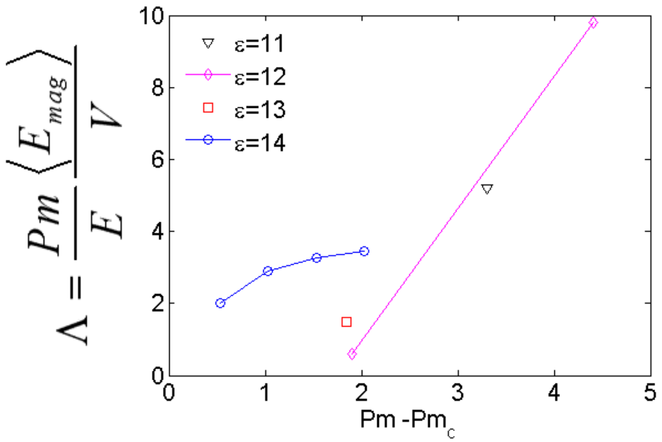
Self-consistent dynamos (slightly dipole dominated)



(ε, Pm)=

(12, 10)
(11, 10)
(13, 5)
(12, 7.5)





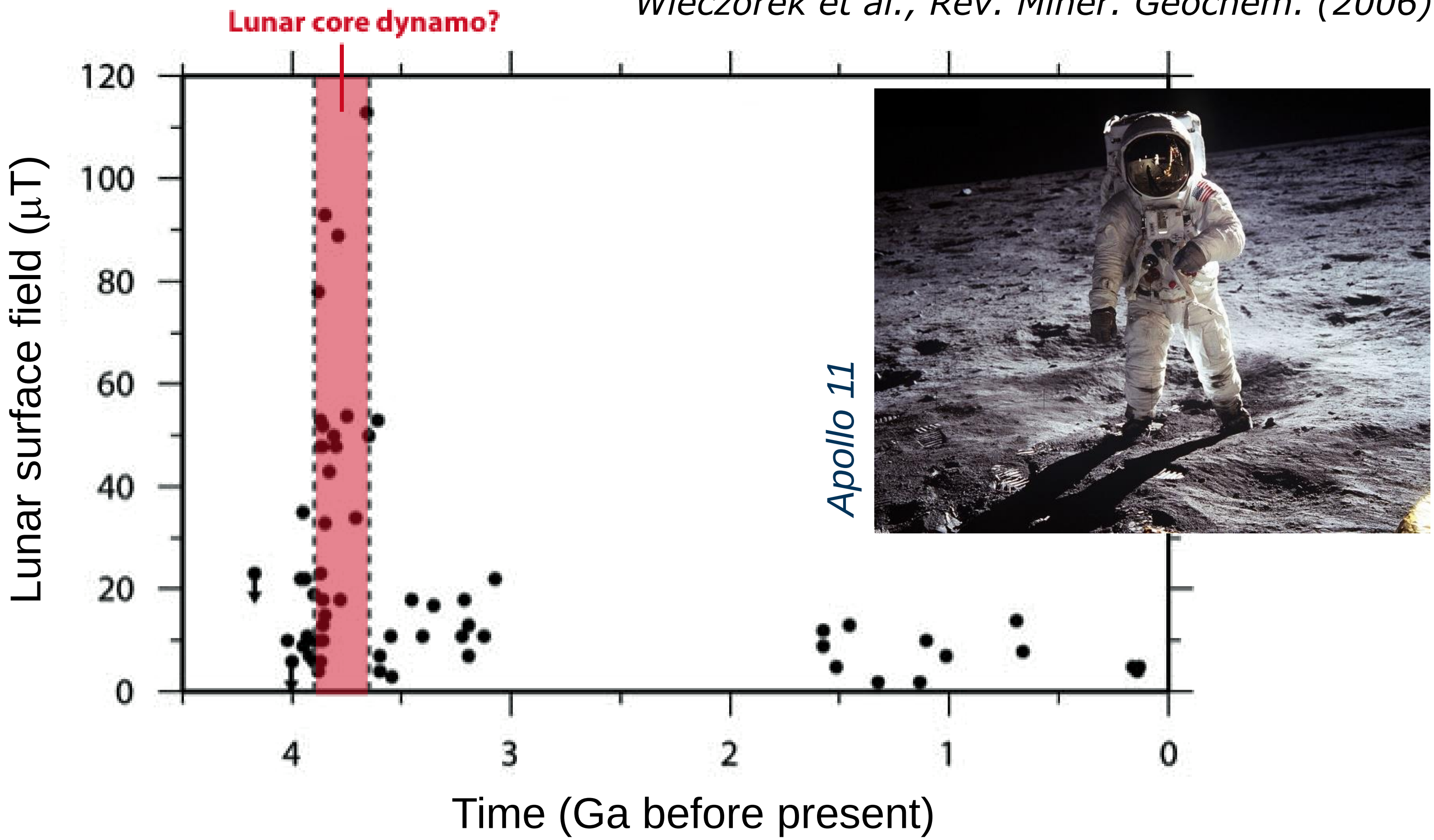
- The periodic solutions of kinematic dynamos can become **quasi-periodic**
- **Diurnal** time scale (ω~1)
- Same spectra than kinematic dynamos
- $\mathbf{E_{mag}/E_{kin} < 0.1}$

Cébron & Hollerbach, *Astrophys. J. Let.*, **789**, L25, 2014.

Early lunar dynamo

Paleomagnetic analyse of Apollo rocks

Fuller & Cisowski, Geomagnetism (1987)
Wieczorek et al., Rev. Miner. Geochem. (2006)



Apollo 11

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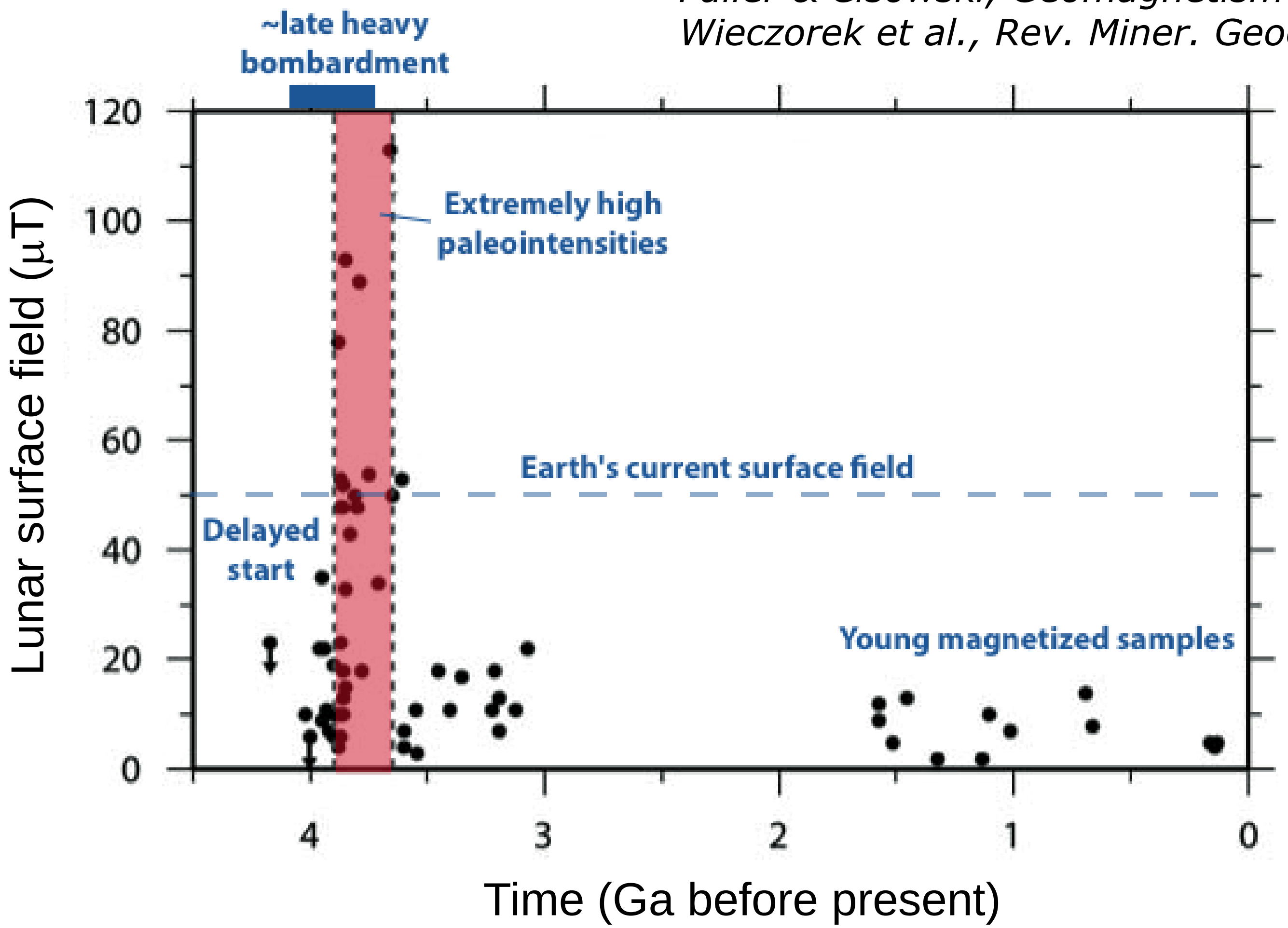
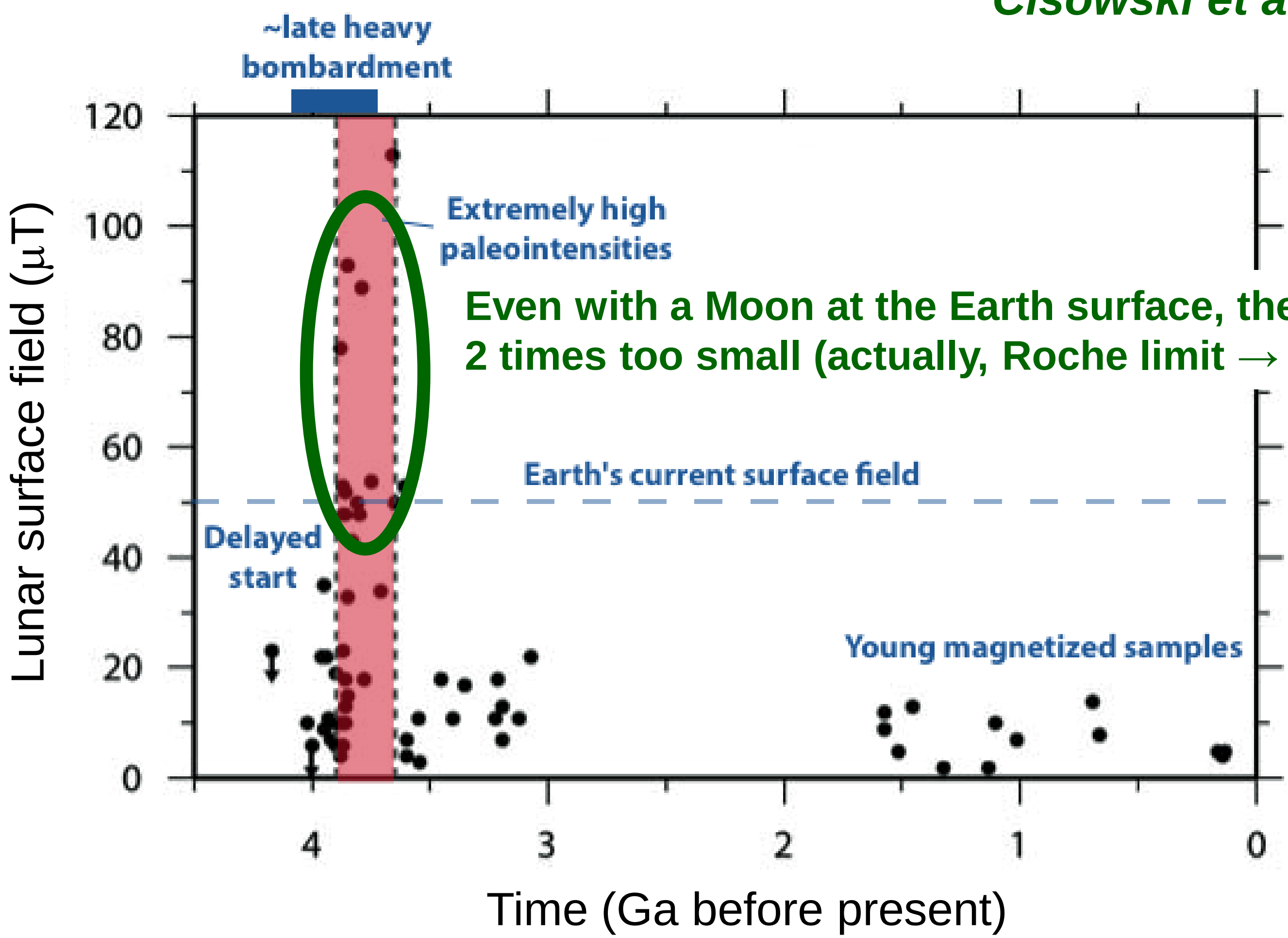


Early lunar dynamo
Paleomagnetic analyse of Apollo rocks

Cisowski et al., JGR (1983)



Early lunar dynamo

Dynamo driven by mechanical forcings?

Planetary **Rotation**
=> Core Rotation

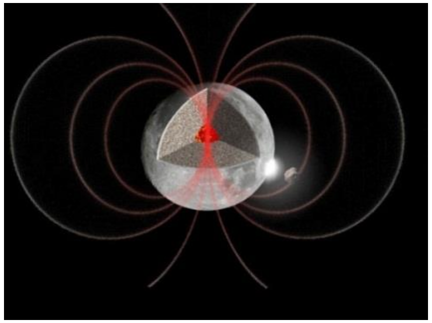
Impacts

Elliptical instability?

Strong & complex
3D **flows**

?

Dynamo effect
generation



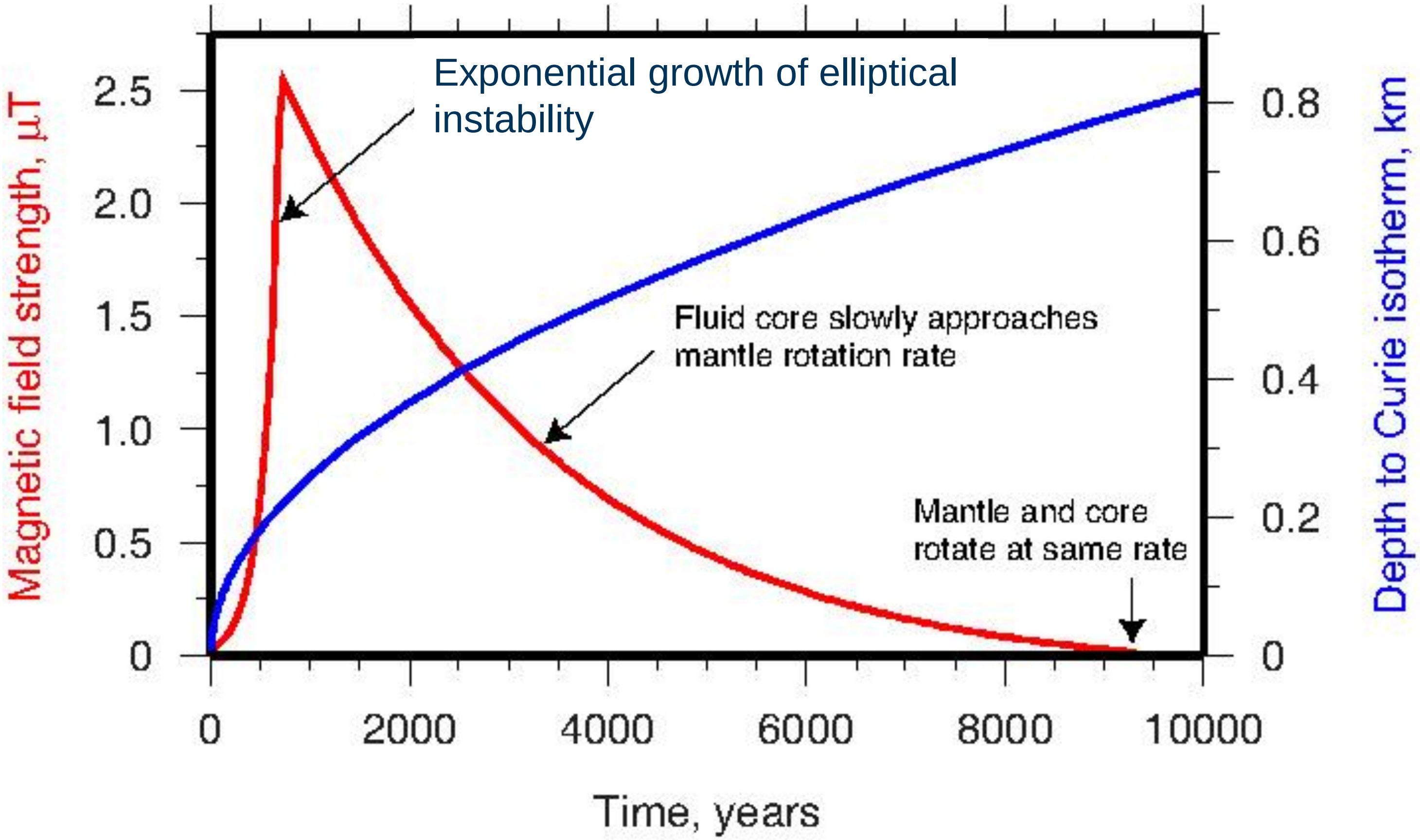
Impacts…

Model: core dynamo driven by mechanical stirring following an impact that either unlocked the Moon from synchronous rotation, and/or set up large amplitude librations

- **Energy source:** impact induced rotation of the Moon
- **Instability:** <u>elliptical instability</u>

Le Bars et al., *Nature*, **479**, 215-218, 2011.

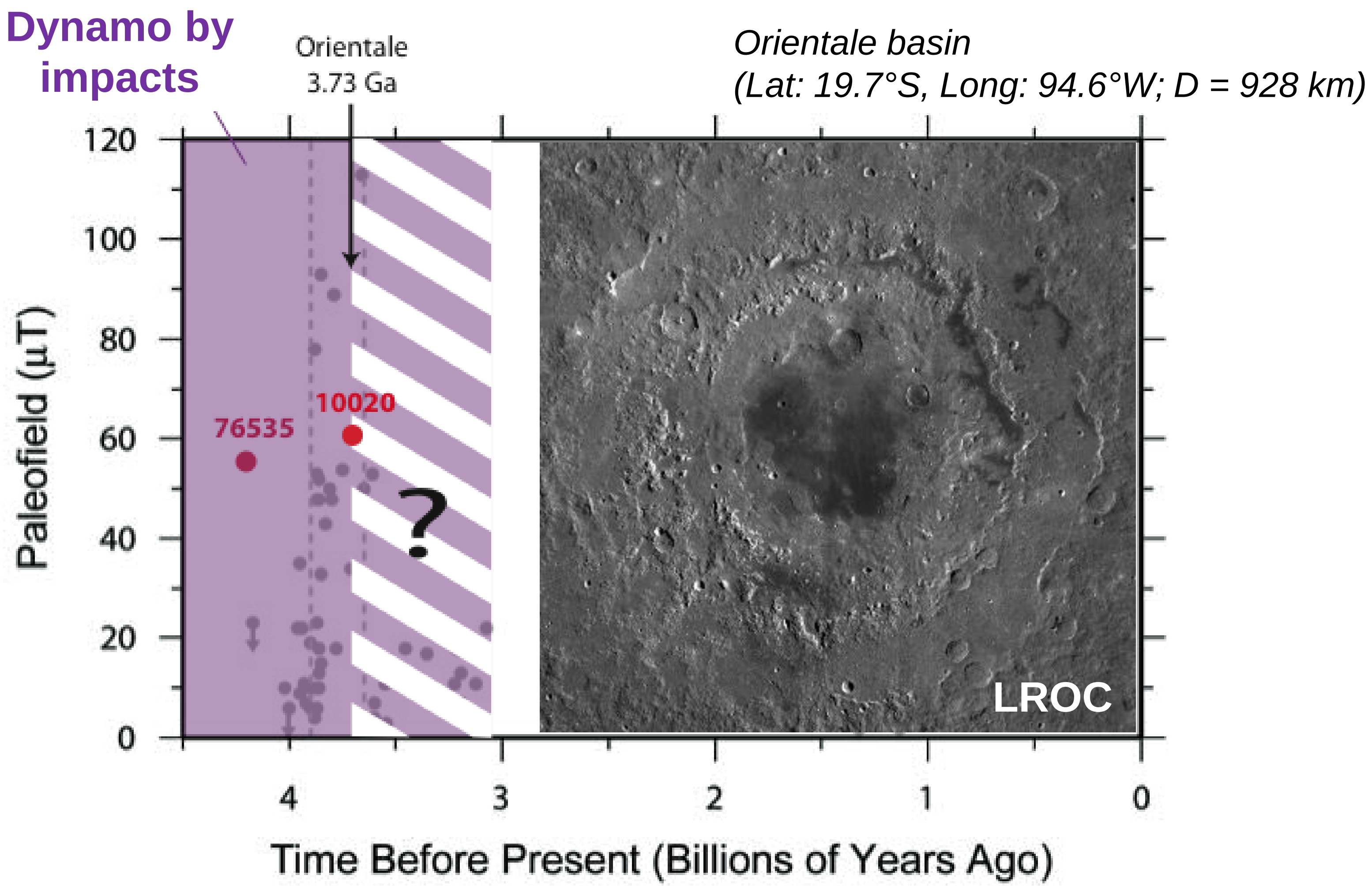
Early lunar dynamo
Results



The typical timescales and magnetic field amplitudes of this scenario are in good agreement with available paleomagnetic data

Le Bars et al., *Nature*, **479**, 215-218, 2011.

Early lunar dynamo
Comparison with data



Conclusion
Take home message

(even small) tidal deformation + rotation

⬇

Elliptical instability not inhibited by convection $\Rightarrow$ **convective** zone

destabilized by a stratification $\Rightarrow$ **radiative** zone

⬇

1) **Very strong** tidal/elliptical instability **dissipation**
- **Synchronization process** modified (${k_2}$ / Q calculated)
- **Tilted rotation** axis (internal torques induced)

2) **Dynamo effect** or **Induction** effect: e.g. τ-boo

3) **Heat** flux, **Angular** momentum, **Scalar transport** very **modified**

Conclusion

Take home message

(even small) tidal deformation + rotation

Elliptical instability not inhibited by convection $\Rightarrow$ **convective** zone

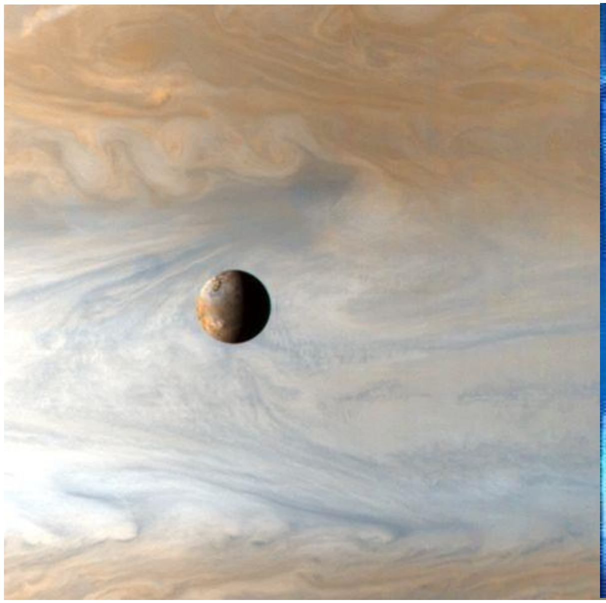
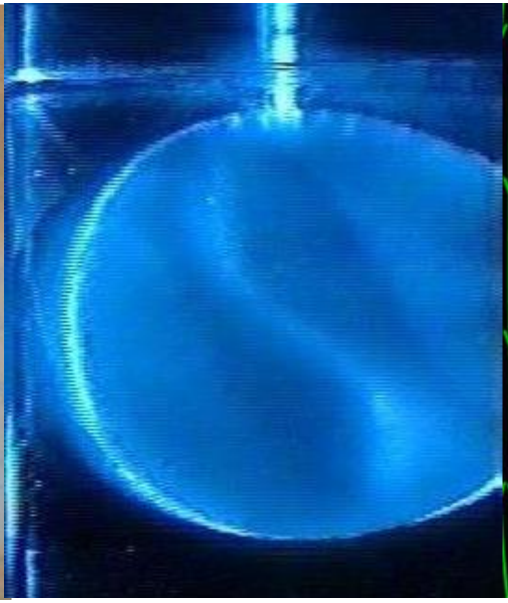
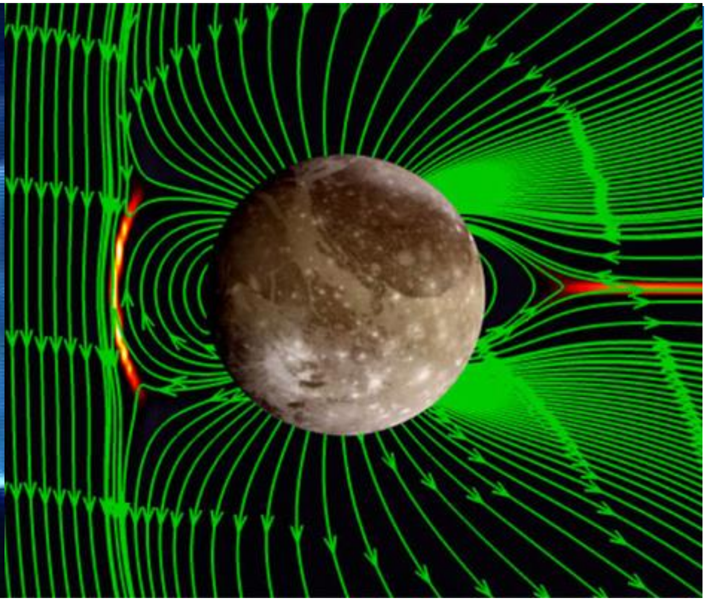
destabilized by a stratification $\Rightarrow$ **radiative** zone

1) On no account, core fluid motions & dynamo systematically mean convection…

2) On no account, a stratified layer (radiative zone) is a problem for instabilities, flows or dynamo (gravito-inertial instabilities dynamos)

3) Not necessarily a m=2 component or an orbital frequency in the magnetic field of a tides driven dynamo/induction (if tidal instability)

Thank you for your attention

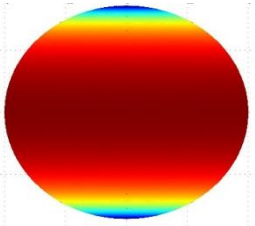


TIDAL INSTABILITY DRIVEN FLOWS

Differential rotation, polytrope

Natural complexities
Influence of a stellar differential rotation

- **Incompressible fluid**
- **Latitudinal differential rotation** $\Omega_{(\theta)} = 1 - \alpha \sin^2 \theta$



Ω

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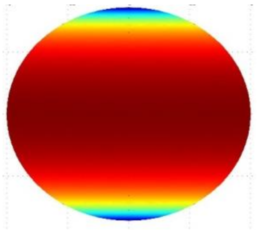
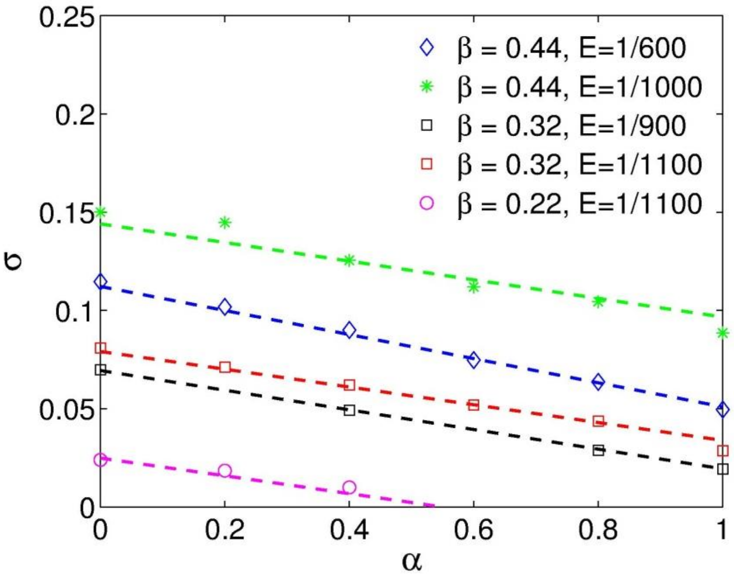


Ω

<u>Case 1</u> : Without synchronized latitude



$$\frac{\sigma}{\beta} = \frac{1}{2} - 2.62 \frac{\sqrt{E}}{\beta} \Rightarrow \boxed{\frac{\sigma}{\beta} = \frac{1}{2} - (2.62 + 1.5\alpha) \frac{\sqrt{E}}{\beta}}$$

Differential rotation

$\Rightarrow$ At 1st order, only change **the viscous damping**

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Influence of a stellar differential rotation

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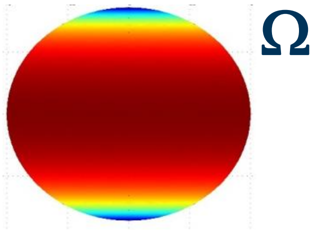
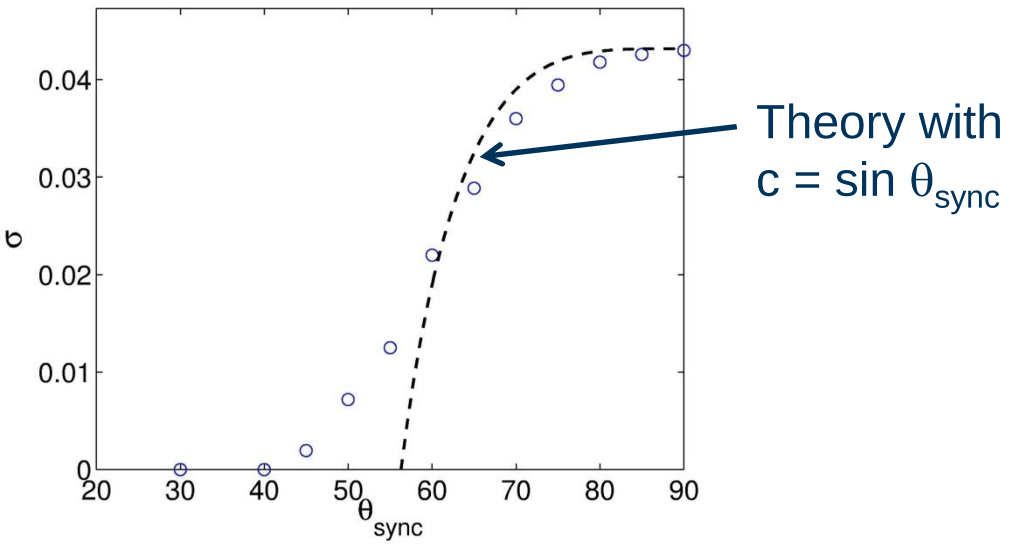
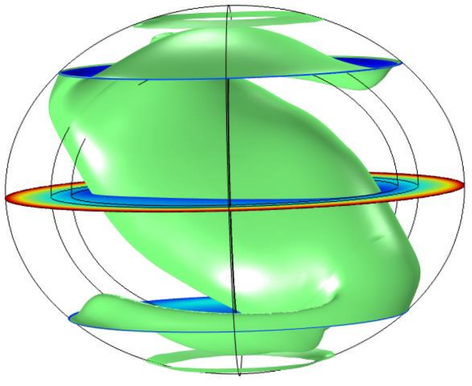


<u>Case 2</u> : With a synchronized latitude





$\Rightarrow$ At 1st order, only change **the effective polar radius**

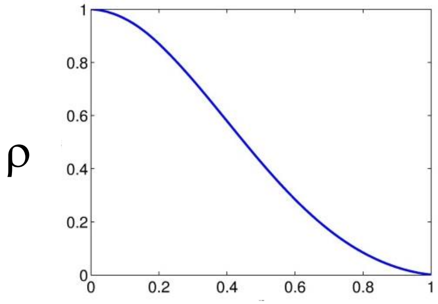
A numerical approach
Influence of a density profile

- **Polytropic** fluid (imposed density profile)
- **Split :** u = u_b + u*
  (u_b = basic flow ie. elliptical streamlines**)**
- **Solving u*** with **stress-free** conditions

$$\frac{\partial u^*}{\partial t} + u^* \cdot \nabla u^* - u_b \cdot \nabla u^* - u^* \cdot \nabla u_b = -\nabla p^* + E \nabla^2 u^*$$

$$\nabla \cdot \left(\rho u^* \right) = 0$$

Energy cost for radial motions



$$r = \sqrt{\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2}$$

Natural complexities

Influence of a density profile

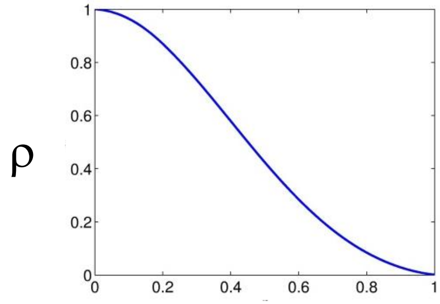
- **Polytropic** fluid (imposed density profile)
- **Split :** $u = u_b + u^*$
  (u_b = basic flow ie. elliptical streamlines**)**
- **Solving u*** with **stress-free** conditions

$$\frac{\partial u^*}{\partial t} + u^* \cdot \nabla u^* - u_b \cdot \nabla u^* - u^* \cdot \nabla u_b = -\nabla p^* + E \nabla^2 u^*$$

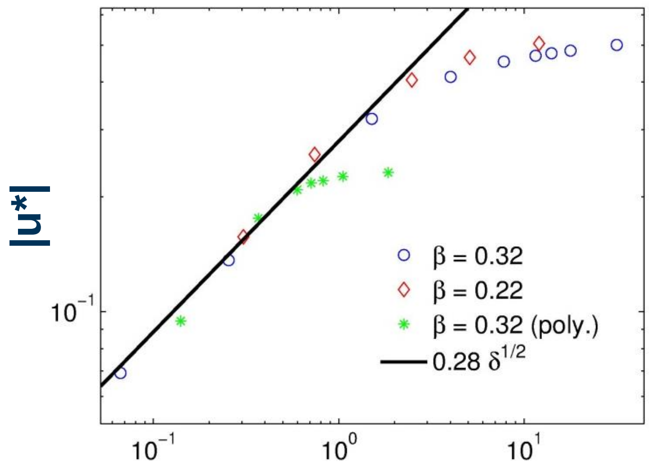
$$\nabla \cdot \left(\rho u^* \right) = 0$$

Energy cost for radial motions

Similar to incompressible flow with a factor 2 on the amplitude of the driven flow



$$r = \sqrt{\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2}$$



Normalized distance to the onset