Star Planet Interaction and the Habitable zone,
Saclay, 18-21 November 2014

TIDES OF ENCOUNTERING PLANETESIMALS

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OUTLINE

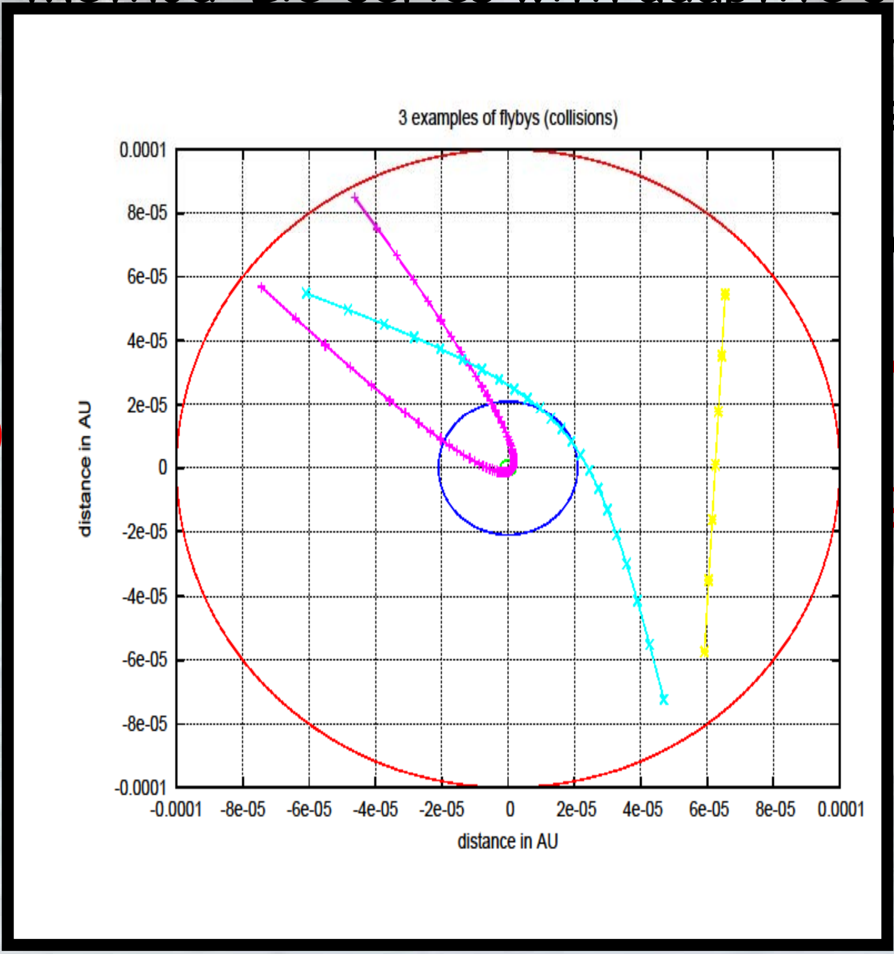
- The important Role of Collisions of bodies in the early Solar System
- Smooth Particle Hydrodynamics: basic ideas
- Numerical results about collisions (Simulations)
- Numerical results of tide interactions (Simulations)
- Appendix: The Formation of ‘our’ habitable world

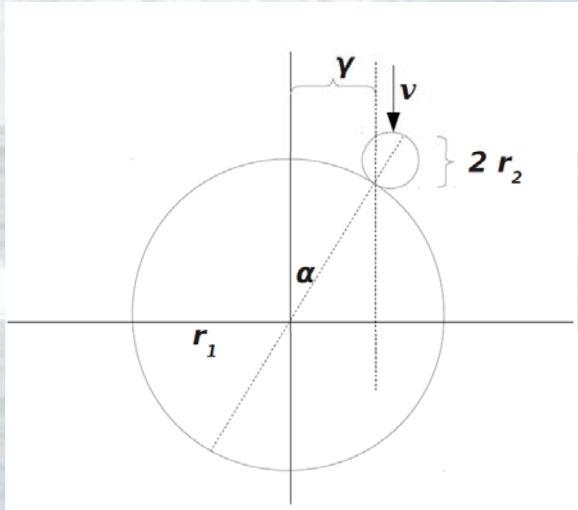
COLLISION SCENARIA OF SMALL BODIES

-> Numerical Integrations in the Newtonian framework
-> Bodies as mass points
-> Method: Lie-series with adaptive step size
->
-> ...se as 0.0001AU

→ ...heres

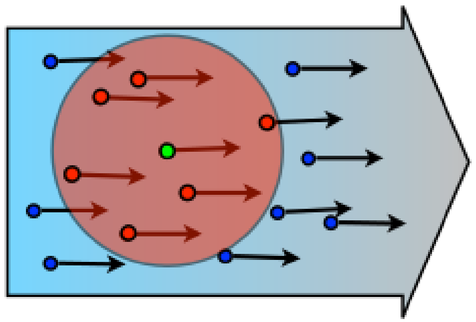
a) ... merging to 1 body
b)
c) ...g of tides





Collision Simulations

- Objectives
  - Elasto-plastic solid state mechanics with brittle failure/fragmentation
  - Self-gravity
  - Track different materials (e.g., water/ice, basalt) and their distribution before, during, and after the impact
- Chosen method: smoothed particle hydrodynamics (SPH)
  - Lagrangian method
  - “Particles” of different material
  - Full elasto-plastic continuum mechanics including brittle failure
  - Tensorial correction (Speith 2007) for first-order consistency
  - New custom-developed code in cooperation with University of Tübingen



Speith (2012)

SPH: Solid state elastic dynamics

Continuity equation (mass conservation):

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho \frac{\partial v^\alpha}{\partial x^\alpha}$$

EOM (conservation of momentum):

$$\frac{\mathrm{d}v^\alpha}{\mathrm{d}t} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta} - \frac{\partial \Phi}{\partial x^\alpha}, \qquad \text{stress tensor } \sigma^{\alpha\beta} = -p\,\delta^{\alpha\beta} + S^{\alpha\beta}$$

Energy conservation:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{p}{\rho}\frac{\partial v^\alpha}{\partial x^\alpha} + \frac{1}{\rho} S^{\alpha\beta}\dot{\varepsilon}^{\alpha\beta}, \qquad \text{strain rate tensor } \dot{\varepsilon}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha}\right)$$

Constitutive equation:

$$\frac{\mathrm{d}S^{\alpha\beta}}{\mathrm{d}t} = 2\mu\left(\dot{\varepsilon}^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\varepsilon}^{\gamma\gamma}\right) + S^{\alpha\gamma}R^{\gamma\beta} + S^{\beta\gamma}R^{\gamma\alpha}$$

with rotation rate tensor $R^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} - \frac{\partial v^\beta}{\partial x^\alpha}\right)$

EOS: $\quad p = p(\rho, u)$

SPH: equation of state (EOS)

- Connects the thermodynamic variables *ρ*, *p*, and *u* to close the set of equations
- Several analytical and semi-empirical approaches exist, e.g.,
  - Murnaghan EOS (isothermal only)

$$p = \frac{K_0}{n}\left[\left(\frac{\rho}{\rho_0}\right)^n - 1\right]$$

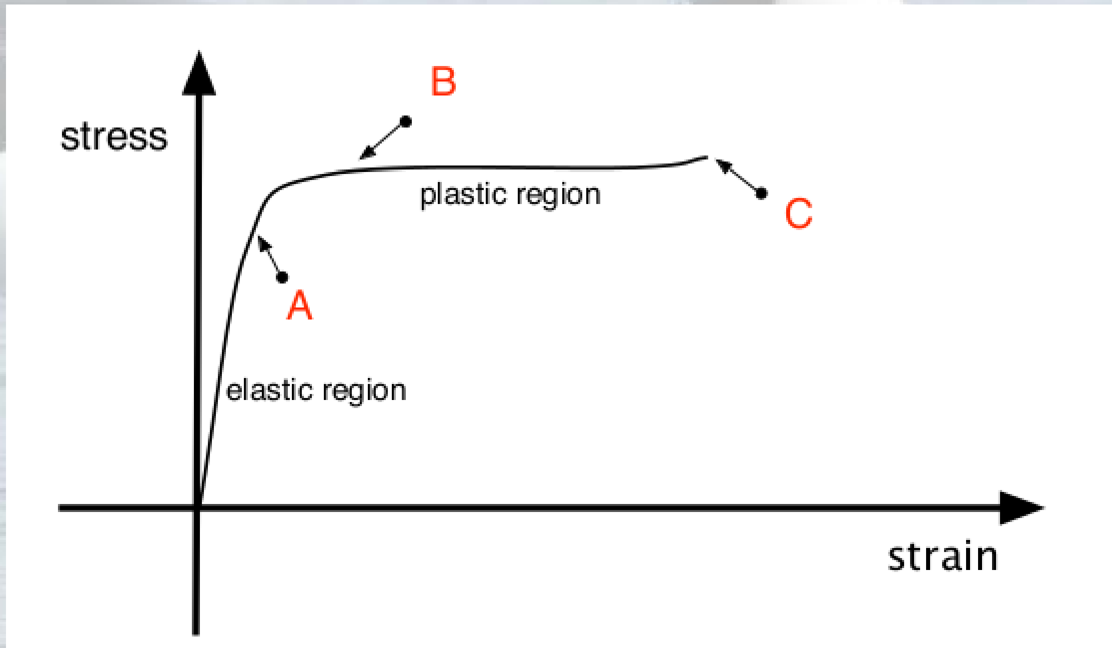
  - Tillotson (1962) EOS

$$p = \left(a + \frac{b}{\frac{u}{u_0\eta^2} + 1}\right)\rho u + A\mu + B\mu^2,\ \eta = \frac{\rho}{\rho_0},\ \mu = \eta - 1 \qquad (u < u_{iv})$$

$$p = a\rho u + \left[\frac{b\rho u}{\frac{u}{u_0\eta^2} + 1} + A\mu e^{-\beta\left(\frac{\rho_0}{\rho} - 1\right)}\right] e^{-\alpha\left(\frac{\rho_0}{\rho} - 1\right)} \qquad (u > u_{cv})$$

  - ANEOS EOS (semi-analytical, not freely available)

SPH solid state elasto-plastic dynamics



A … Hooke's law: elasticity, deviatoric stress rate proportional to strain rate
→ elastic dynamics

B … Yielding relations: plasticity by modifying stresses beyond the elastic limit
→ von Mises yielding criterion

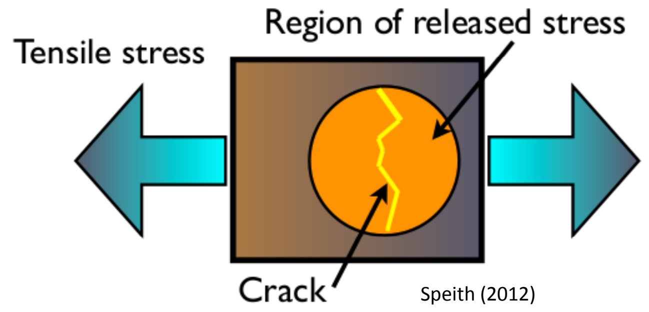
C … Damage and brittle failure for tensile stresses beyond material strength
→ Grady & Kipp fracture model

SPH: plasticity and brittle failure

- von Mises yielding criterion
  - Limit deviatoric stress tensor by a factor of *f* depending on the material yield stress Y_0

$$S^{\alpha\beta} = f\, S^{\alpha\beta} \qquad f = \min\left[\frac{Y_0^2}{3\,J_2}, 1\right], \qquad J_2 = \frac{1}{2} S^{\alpha\beta} S^{\alpha\beta}$$

- Fracture model (Grady & Kipp 1980)
  - Large enough local strain causes flaws to develop into cracks that grow at half the speed of sound until local stress is relieved
  - Number of flaws *n* per unit volume with activation threshold < ε are Weibull (1938) distributed: $n(\varepsilon) = k\, \varepsilon^m$
  - Stress proportional to (1 – D), D … damage, 0 ≤ D ≤ 1
  - Modified stress tensor:



$$\sigma_{\alpha\beta}^{\text{damaged}} = \begin{cases} -p\delta_{\alpha\beta} + (1-D)S_{\alpha\beta} & , \; p \geq 0 \; \text{(compression)} \\ -(1-D)p\delta_{\alpha\beta} + (1-D)S_{\alpha\beta} & , \; p < 0 \; \text{(tension)} \end{cases}$$

Full material equations vs. strengthless material

Continuity equation (mass conservation):

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho \frac{\partial v^\alpha}{\partial x^\alpha}$$

EOM (conservation of momentum):

$$\frac{\mathrm{d}v^\alpha}{\mathrm{d}t} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta} - \frac{\partial \Phi}{\partial x^\alpha}, \qquad \text{stress tensor } \sigma^{\alpha\beta} = -p\,\delta^{\alpha\beta} + \cancel{S^{\alpha\beta}}$$

Energy conservation:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{p}{\rho}\frac{\partial v^\alpha}{\partial x^\alpha} + \cancel{\frac{1}{\rho} S^{\alpha\beta}\dot{\varepsilon}^{\alpha\beta}}, \qquad \text{strain rate tensor } \cancel{\dot{\varepsilon}^{\alpha\beta}} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha}\right)$$

Constitutive equation:

$$\cancel{\frac{\mathrm{d}S^{\alpha\beta}}{\mathrm{d}t} = 2\mu\left(\dot{\varepsilon}^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\varepsilon}^{\gamma\gamma}\right) + S^{\alpha\gamma}R^{\gamma\beta} + S^{\beta\gamma}R^{\gamma\alpha}}$$

$$\cancel{\text{with rotation rate tensor } R^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} - \frac{\partial v^\beta}{\partial x^\alpha}\right)}$$

Hooke's law

EOS: $\quad p = p(\rho, u)$

Plasticity and brittle failure

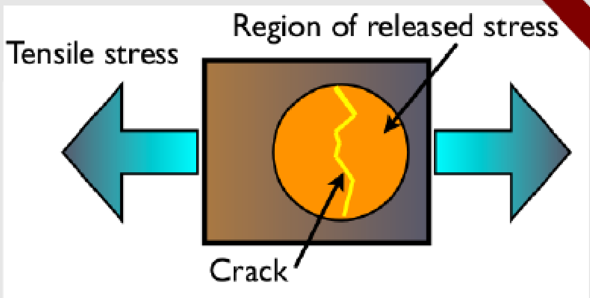
hydro

- Plastic behavior: von Mises yielding criterion
  - Limits deviatoric stress beyond material dependent yield stress Y_0

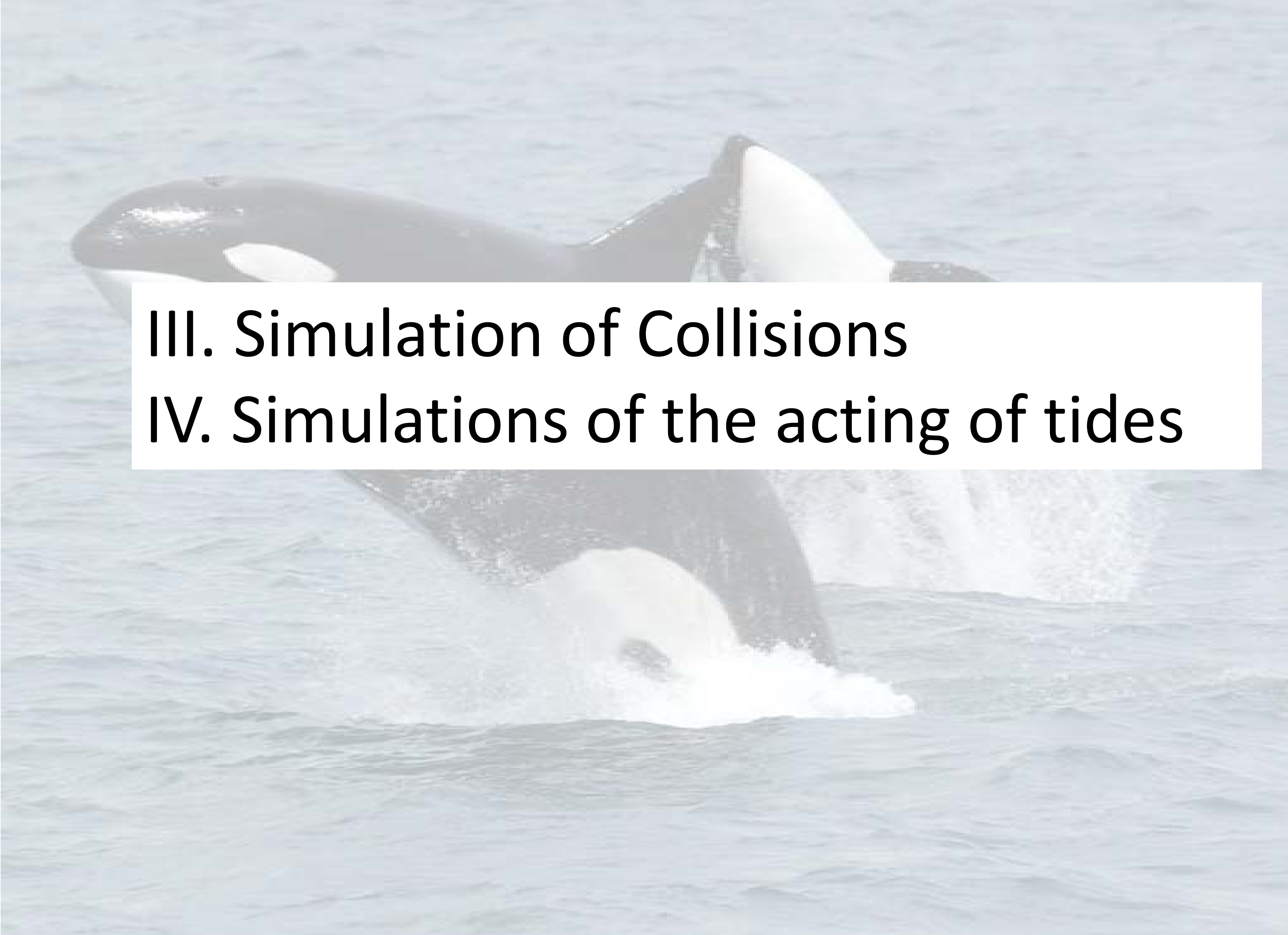
$$S^{\alpha\beta} = f\, S^{\alpha\beta}$$

$$f = \min\left[\frac{Y_0^2}{3\,J_2}, 1\right], \qquad J_2 = \frac{1}{2} S^{\alpha\beta} S^{\alpha\beta}$$

- Fracture: Grady & Kipp (1980) damage model
  - Stress proportional (1 – D), damage D, 0 ≤ D ≤ 1
  - Statistically distributed flaws develop into cracks beyond their strain threshold
  - The crack/flaw ratio determines a volume element's damage
  - Cracks grow at half the speed of sound until the local stress is relieved



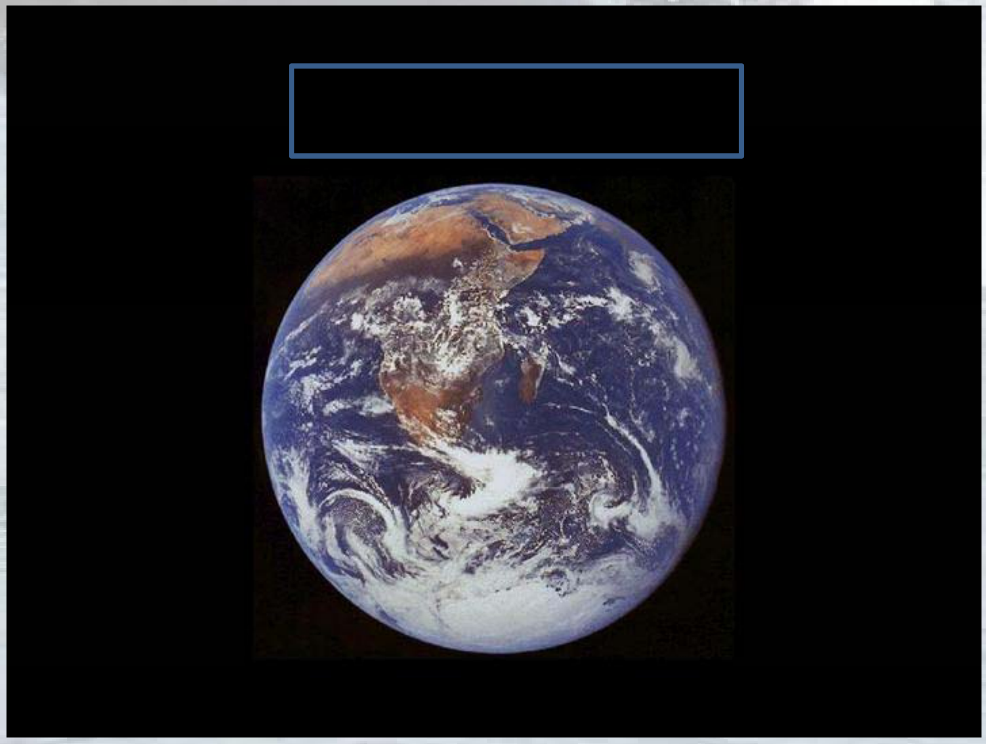
Speith (2012)



III. Simulation of Collisions
IV. Simulations of the acting of tides

APPENDIX

THE FORMATION OF OUR HABITABLE WORLD



The Formation of Planets in the habitable zone with water after the Grand Tack

A status report

→Gas Giants form when there is still the gaseous disk

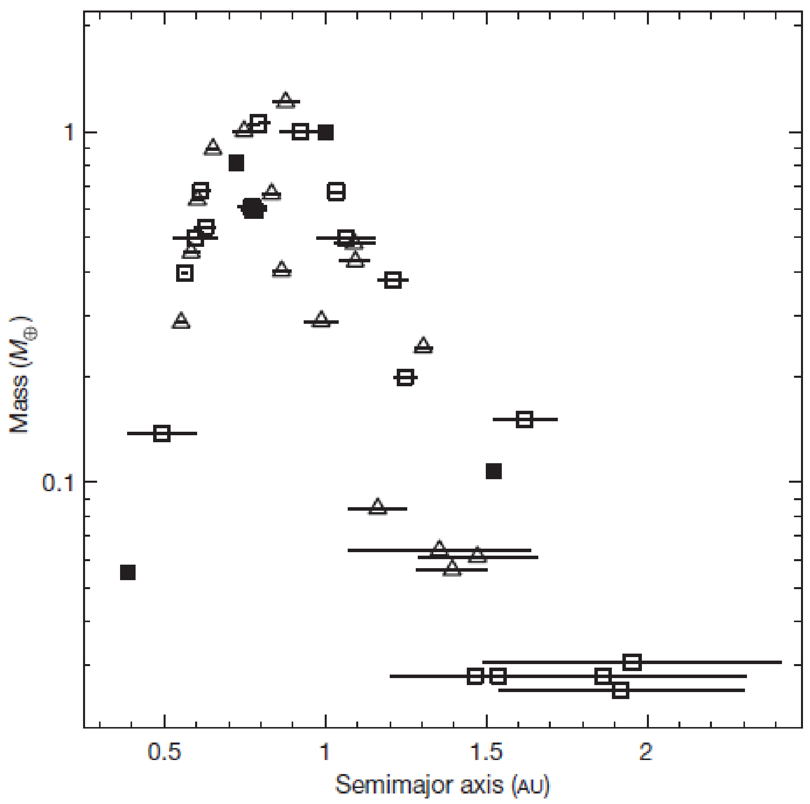
→A planet like Jupiter produces an annular gap and migrates inward on the local viscous time scale

→A Saturn mass planet migrates faster because of strong gravitational feedback during the disk clearing

→ASSUMING J underwent rapid gas accretion before S then (hydrodynamic code results) J migrated slow, but S still accreted gas and migrated quickly

→ And S was caught by J up to the trapping in the 2:3 MMR

→THEN the direction when J was at 1.5 AU the inward migration was reversed (J 'tacked') up to the moment when no more gas was present and J and S were again trapped in a MMR



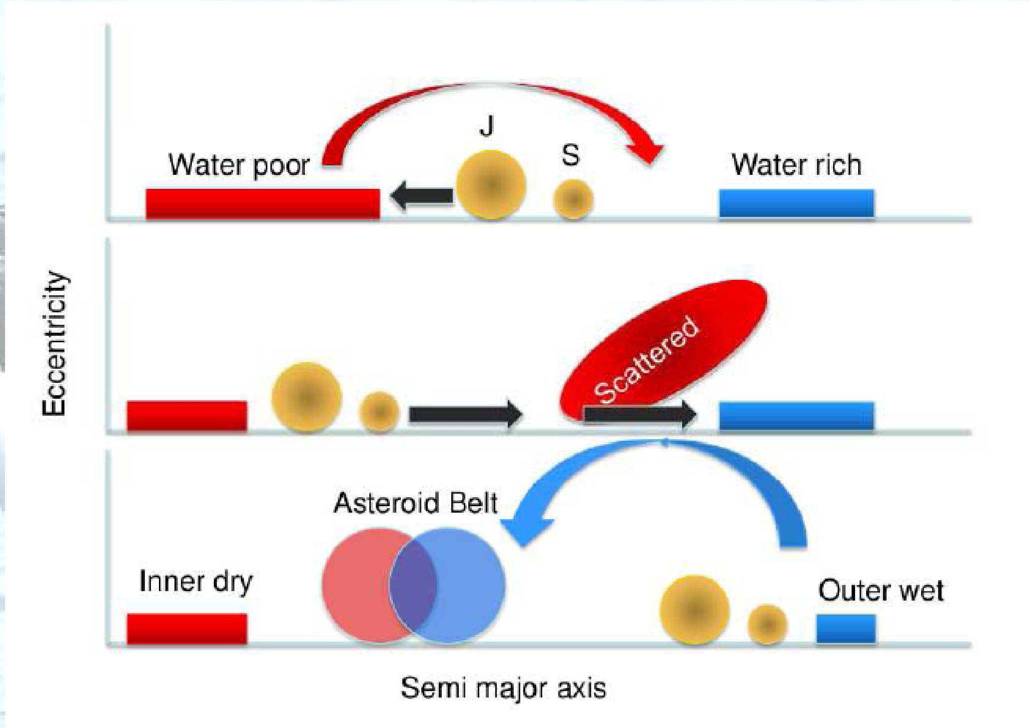
LETTER

doi:10.1038/nature10201

A low mass for Mars from Jupiter's early gas-driven migration

Kevin J. Walsh[1,2], Alessandro Morbidelli[1], Sean N. Raymond[3,4], David P. O'Brien[5] & Avi M. Mandell[6]

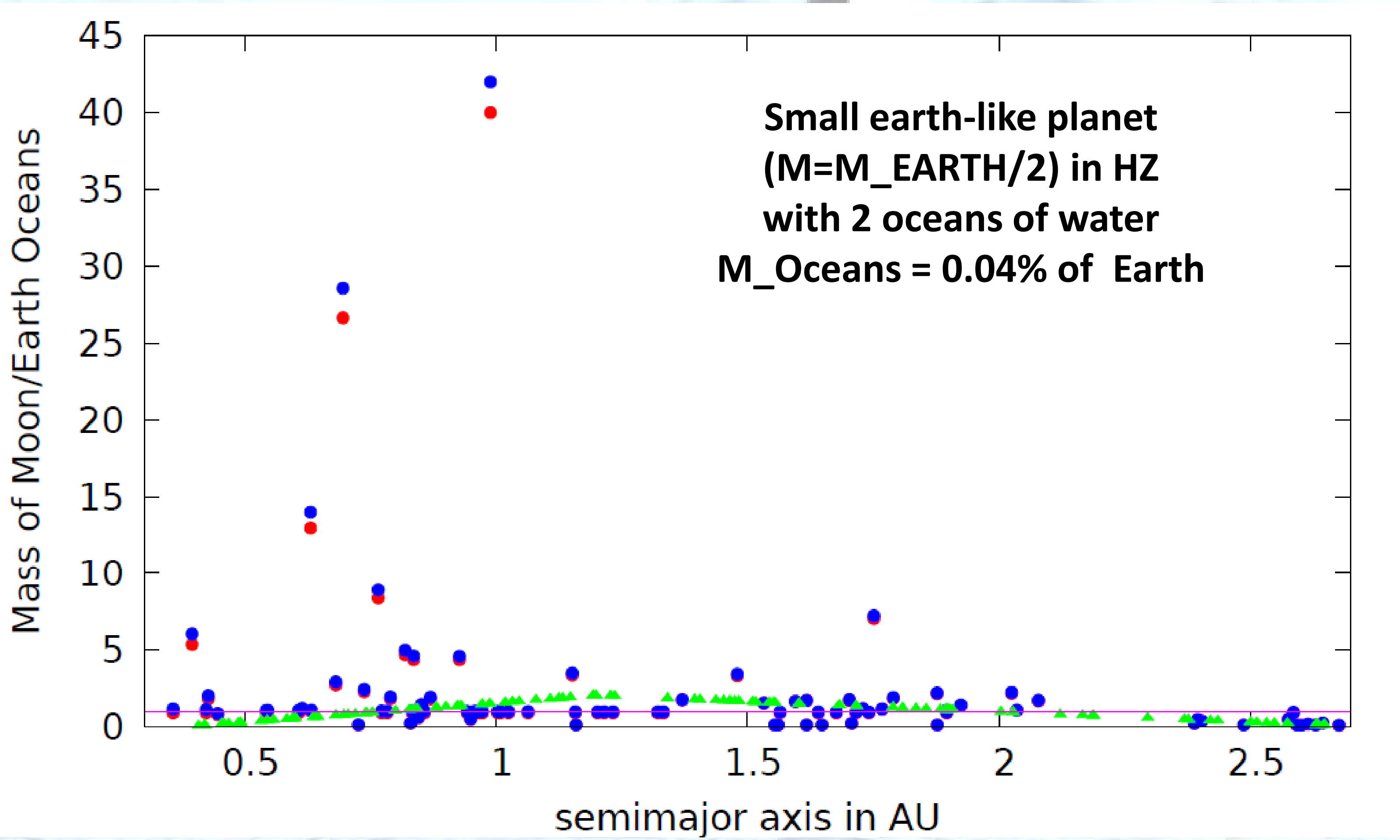
as compared to 89.9 for the current Solar System.

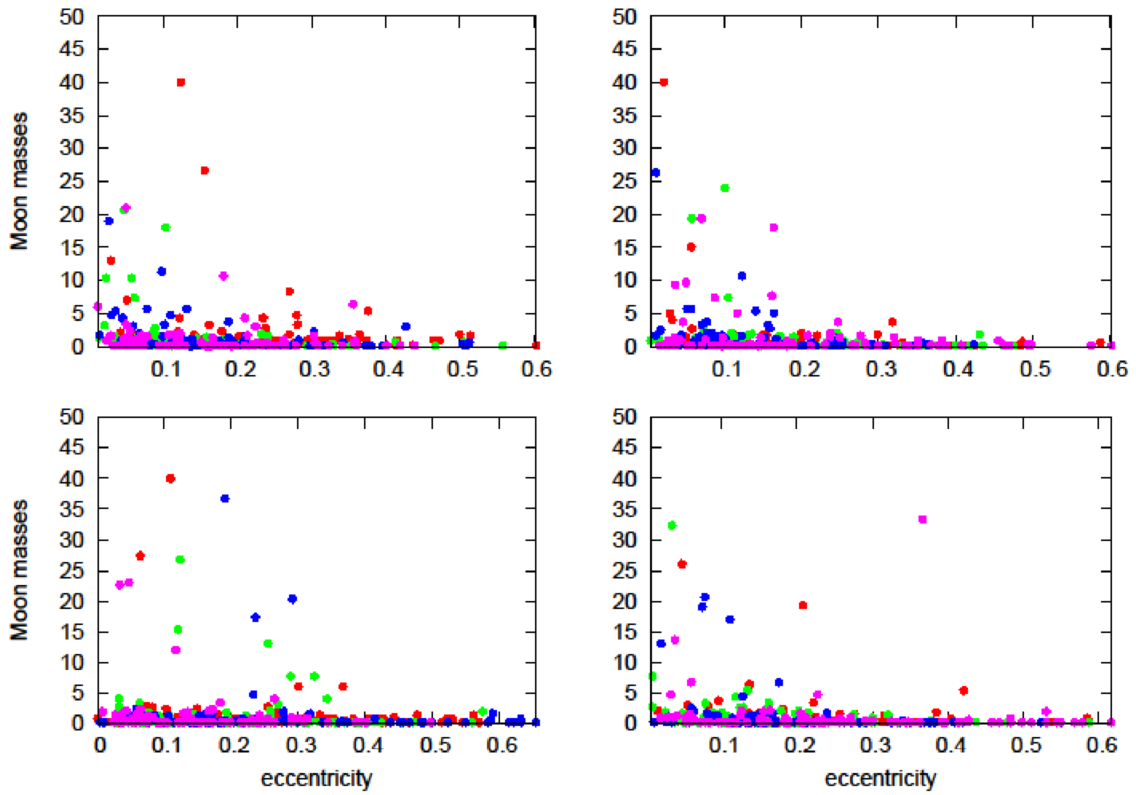


End of GT scenario is the starting point of this new integration:

→Lie with 100 dry planetesimals (~m_Moon) 0.5AU < a < 2.5, e,i small
→When 2 bodies merge a new small body from a=2.6AU (wet region) is added
→The number of massive bodies is thus always kept to N=100
→Water is only added from the new bodies (m=M/10) at a=2.6 AU and e,i
→Scenario A: only with Jupiter
→Scenario B: with Jupiter and Saturn

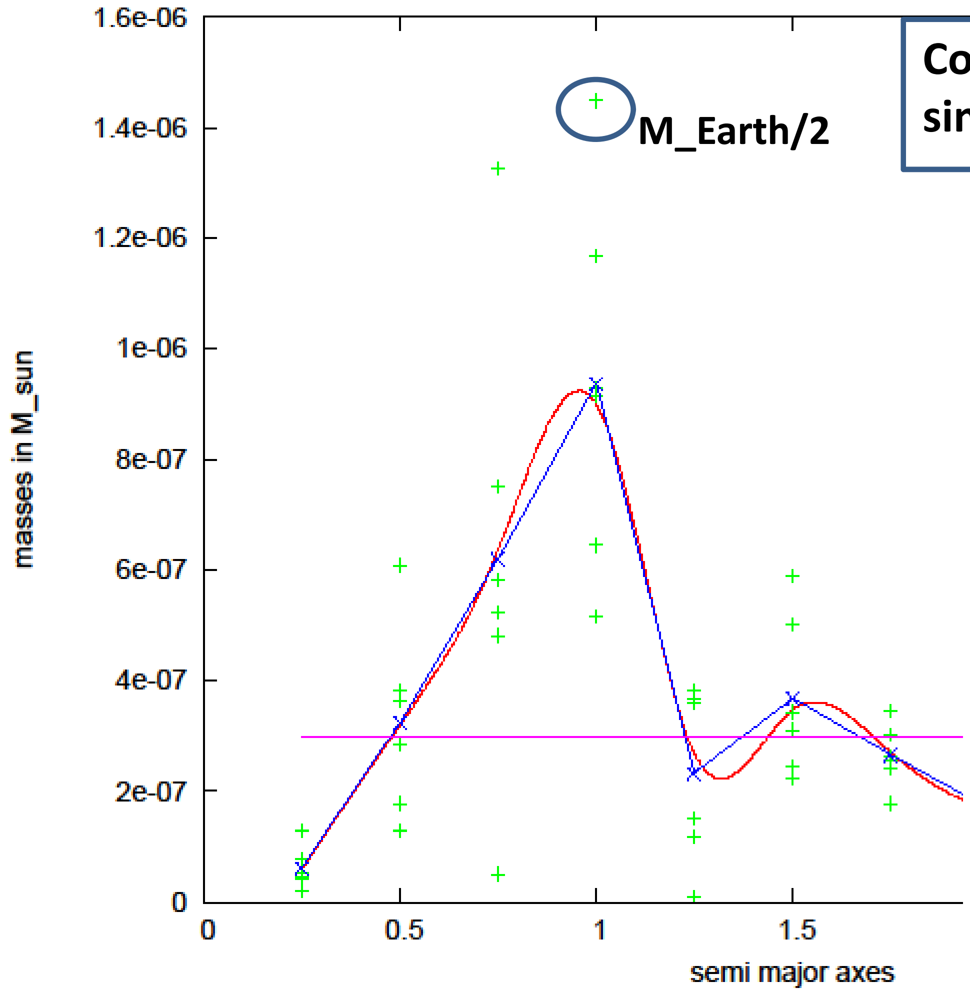
1 Simulation out of 24 for ~ 2 Myrs





Bigger planets have orbits with significant small eccentricities

Combined results of 6 simulations



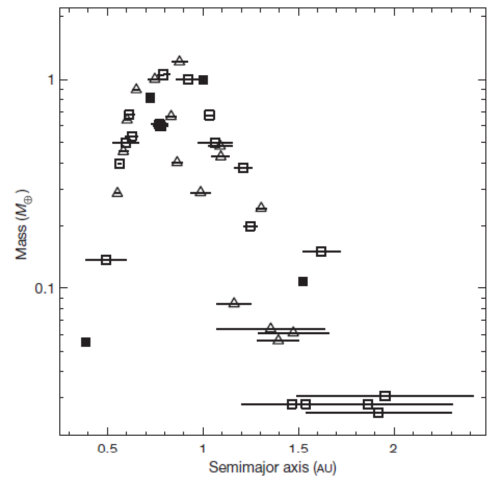


Figure 4 | Results of the eight terrestrial planet simulations. The mass versus semimajor axis of the synthetic planets (open symbols) is compared to the real planets (filled squares). The triangles refer to simulations starting with 40 embryos of $\sim 0.051\ M_{\oplus}$, and squares to simulations from 80 embryos of $\sim 0.026\ M_{\oplus}$. The horizontal error bars show the perihelion–aphelion excursion of each planet along their orbits. The initial planetesimal disk had an inner edge at 0.7 AU to replicate previous work[8], and an outer edge at ~1.0 AU owing to the truncation caused by the inward and outward migration of the giant planets as described in the text. Half of the original mass of the disk interior to Jupiter ($1.85 M_{\oplus}$) was in ~727 planetesimals. At the end of giant planet migration, the evolution of all objects inward of 2 AU was continued for 150 Myr, still accounting for the influence from Jupiter and Saturn. Collisions of embryos with each other and with planetesimals were assumed fully accretional. For this set of eight simulations, the average normalized angular momentum deficit[30] was 0.0011 ± 0.0006, as compared to 0.0018 for the current Solar System. Similarly, the radial mass concentration[30] was 83.8 ± 12.8 as compared to 89.9 for the current Solar System.

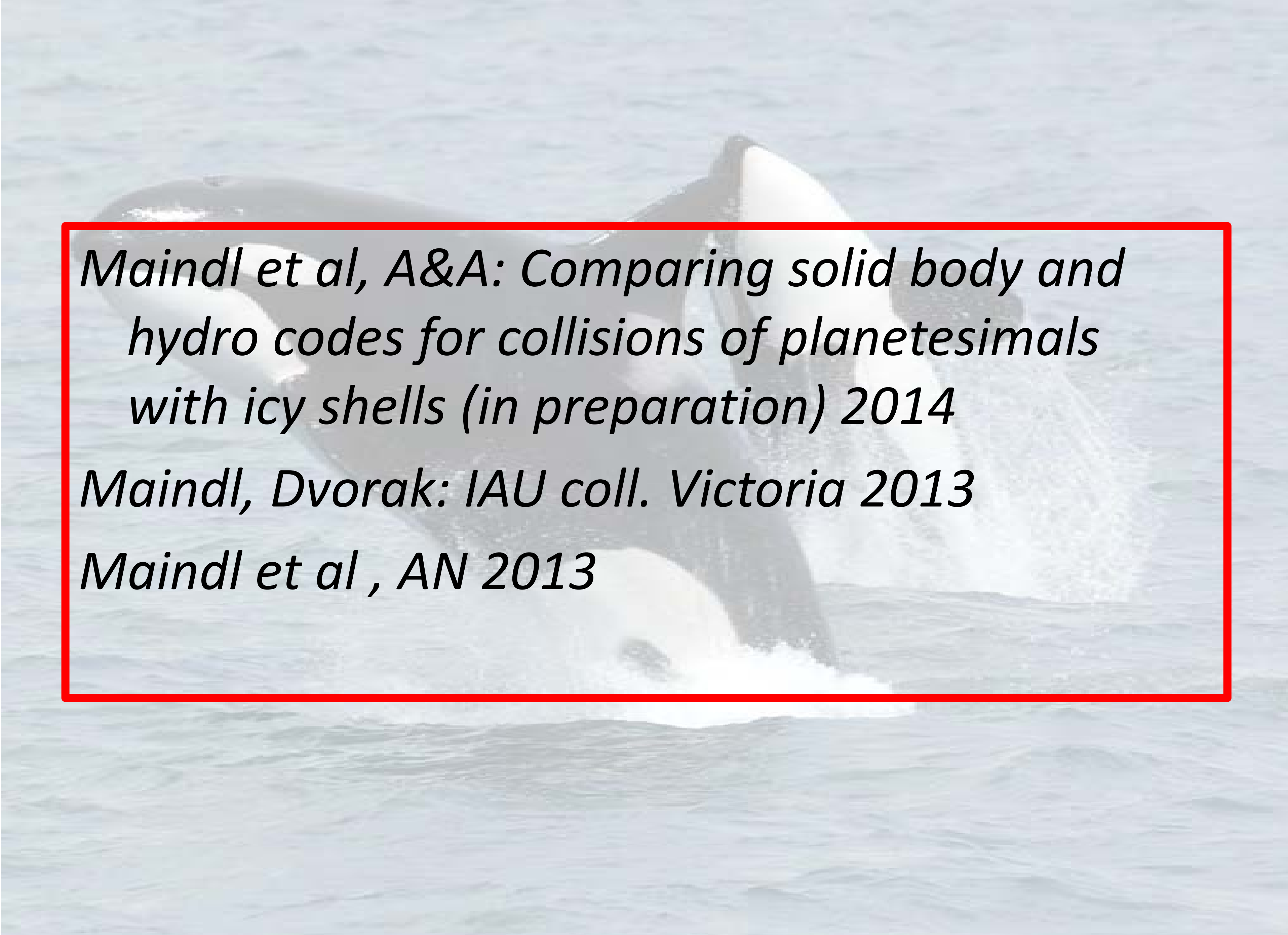
Some results

100 dry bodies (~m_Moon) distributed according to GT
After a merging → new wet smaller body from outside
Critical role of the collision parameter 1<d<8
When do we have the merging (r_1+r_2)*d
Collision process is different in reality (fragmentation..TIDES)

Statistically the probability of forming a terrestrial planet in the habitable zone is relatively large (after Grand Tack initial conditions)
Water is transported in quantities comparable to Earthwater

INCLUSION OF SPH RESULTS INTO THE NUMERICAL CODES is next step


MERCI! THANK YOU!



Maindl et al, A&A: Comparing solid body and hydro codes for collisions of planetesimals with icy shells (in preparation) 2014

Maindl, Dvorak: IAU coll. Victoria 2013

Maindl et al , AN 2013