Observational constraints on the tidal parameters k_2 and Q in the solar system

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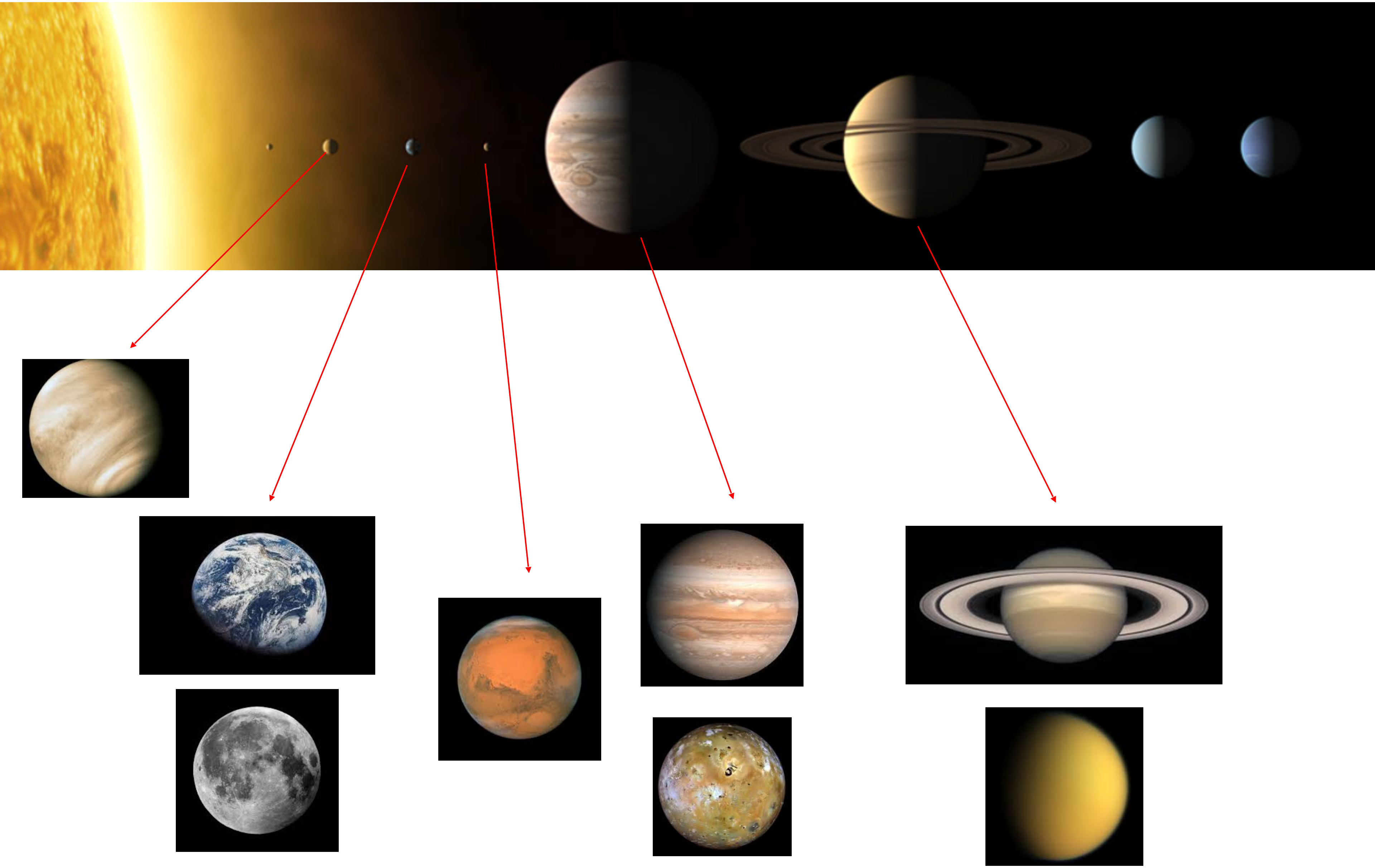
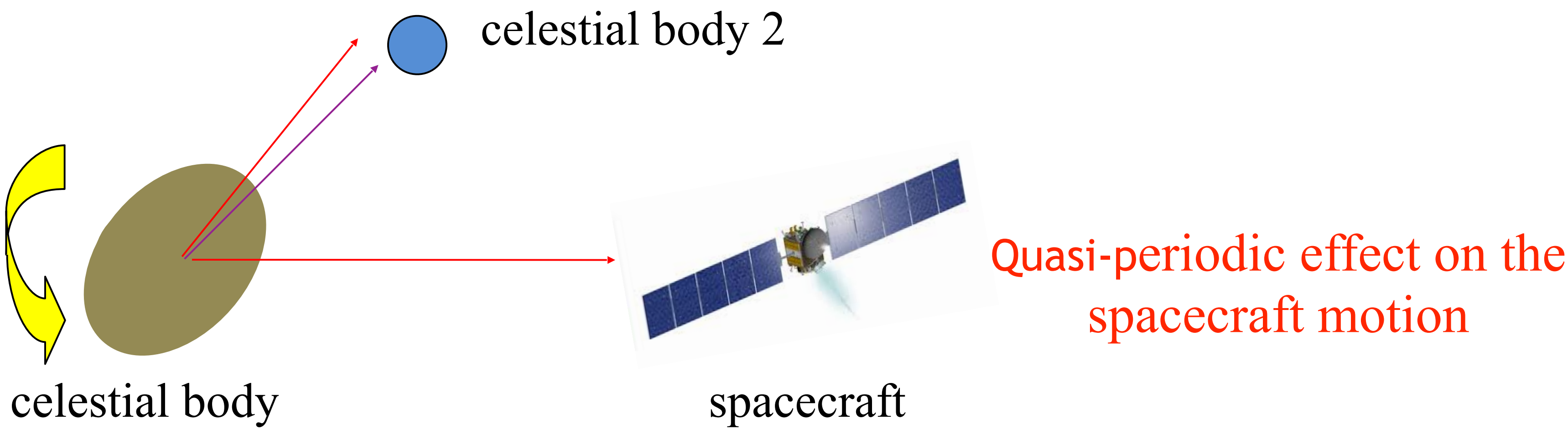
Solar system bodies with tidal parameters known from observations



Determination of tidal parameters

Space geodesy:



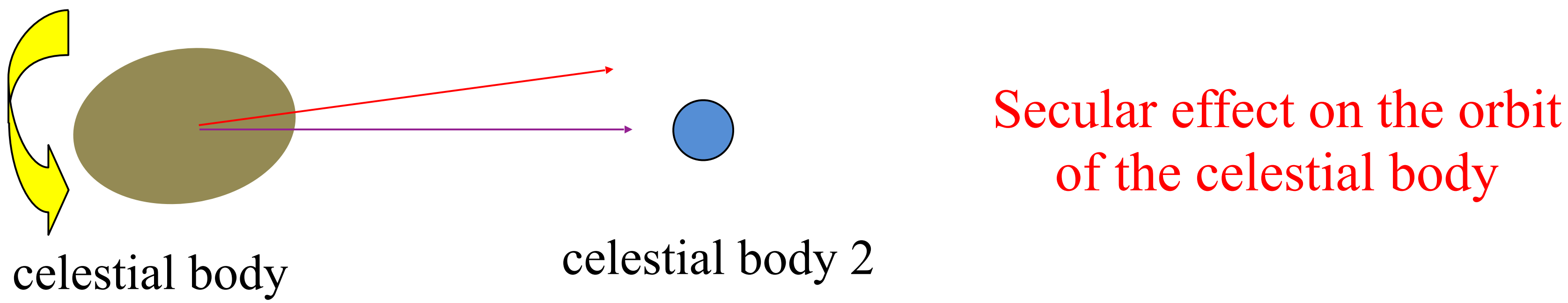
This method benefits from the accuracy of geodesic technics (SLR/GPS/altimetry/radio-science…)

Tidal lag is a small angle, hence difficult to catch…

Requires a spacecraft!

Determination of tidal parameters

Astrometry:

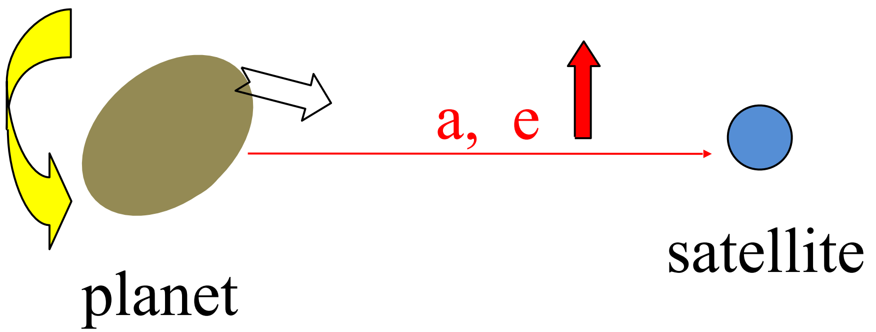


The astrometric observations are much less accurate than geodesic ones (except for the LLR) but can be done from ground « rather » easily

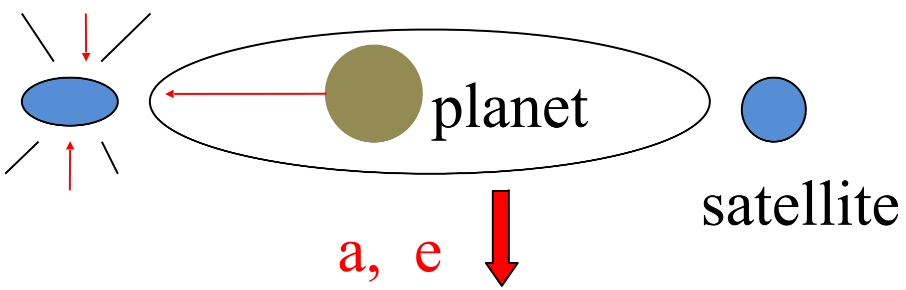
Pretty hard to separate k_2 and Q

Correlations arise when tidal dissipation arises in both celestial bodies

Competition between tidal dissipation effects



Secular deceleration on the mean motion



Secular acceleration on the mean motion

Orbit determination of natural satellites and spacecraft

NB: *Orbit determination of natural satellites and spacecraft follow exactly the same methodology…*

Method in three steps:

1- modeling of the dynamical system

2- gathering the observations

3- fitting the model to the observations

Today, this kind of work is done completely numerically

S/C: GINS, DPODP, GEODYN, …

SAT: NOE, ODYSSEY…

Orbit determination of natural satellites

Step 1: Modeling of the dynamical system

Equations of motion

$$\ddot{\vec{r}}_i = -G(m_0 + m_i)\left(\frac{\vec{r}_i}{r_i^{\,3}} - \nabla_i U_{\bar{i}\hat{0}} + \nabla_0 U_{\bar{0}\hat{i}}\right) + \sum_{j=1,\, j\neq i}^{N} G m_j \left(\frac{\vec{r}_j - \vec{r}_i}{r_{ij}^{\,3}} - \frac{\vec{r}_j}{r_j^{\,3}} - \nabla_j U_{\bar{j}\hat{i}} + \nabla_i U_{\bar{i}\hat{j}} + \nabla_j U_{\bar{j}\hat{0}} - \nabla_0 U_{\bar{0}\hat{j}}\right)$$

$$+\frac{(m_0 + m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}^{\;T} - \vec{F}_{\bar{0}\hat{i}}^{\;T}\right) - \frac{1}{m_0}\sum_{j=1,\, j\neq i}^{N}\left(\vec{F}_{\bar{j}\hat{0}}^{\;T} - \vec{F}_{\bar{0}\hat{j}}^{\;T}\right) + RT$$

Orbit determination of natural satellites

Step 1: Modeling of the dynamical system

Equations of motion

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$$+\frac{(m_0+m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}^{\;T} - \vec{F}_{\bar{0}\hat{i}}^{\;T}\right) - \frac{1}{m_0}\sum_{j=1,\, j\neq i}^{N}\left(\vec{F}_{\bar{j}\hat{0}}^{\;T} - \vec{F}_{\bar{0}\hat{j}}^{\;T}\right) + RT$$

N-body problem

Orbit determination of natural satellites

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N-body problem

Extended gravity fields

Orbit determination of natural satellites

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N-body problem

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Orbit determination of natural satellites

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N-body problem
Extended gravity fields
Tidal effects
Relativistic terms

Orbit determination of natural satellites

Step 1: Modeling of the dynamical system

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N-body problem
Extended gravity fields
Tidal effects
Relativistic terms

Initial conditions are required to solve for equations of motion…

Problem: How will we find the « real » initial conditions of the system we consider?

Orbit determination of natural satellites

Problem: How can we find the « real » initial conditions of the system we consider?

Clearly we need to know at every observation time *t* the quantity $\left.\frac{\partial \vec{r}_i}{\partial c_l}\right|_t$

Orbit determination of natural satellites

Problem: How can we find the « real » initial conditions of the system we consider?

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Variational equations

$$\frac{d^2 \vec{r}_i}{dt^2} = \frac{1}{m_i} \vec{F}_i$$

Orbit determination of natural satellites

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Variational equations

$$\frac{d^2\vec{r}_i}{dt^2} = \frac{1}{m_i}\vec{F}_i \quad \Rightarrow \quad \frac{\partial}{\partial c_l}\left(\frac{d^2\vec{r}_i}{dt^2}\right) = \frac{1}{m_i}\frac{\partial \vec{F}_i}{\partial c_l}$$

Orbit determination of natural satellites

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Clearly we need to know at every observation time *t* the quantity $\left.\dfrac{\partial \vec{r}_i}{\partial c_l}\right|_t$

Variational equations

$$\frac{d^2\vec{r}_i}{dt^2} = \frac{1}{m_i}\vec{F}_i \quad \Longrightarrow \quad \frac{\partial}{\partial c_l}\left(\frac{d^2\vec{r}_i}{dt^2}\right) = \frac{1}{m_i}\frac{\partial \vec{F}_i}{\partial c_l}$$

$$\frac{\partial}{\partial c_l}\left(\frac{d^2\vec{r}_i}{dt^2}\right) = \frac{1}{m_i}\left[\sum_j\left(\frac{\partial \vec{F}_i}{\partial \vec{r}_j}\frac{\partial \vec{r}_j}{\partial c_l} + \frac{\partial \vec{F}_i}{\partial \dot{\vec{r}}_j}\frac{\partial \dot{\vec{r}}_j}{\partial c_l}\right) + \frac{\partial \vec{F}_i}{\partial c_l}\right]$$

Orbit determination of natural satellites

Problem: How can we find the « real » initial conditions of the system we consider?

Clearly we need to know at every observation time t the quantity $\left.\dfrac{\partial \vec{r}_i}{\partial c_l}\right|_t$

Variational equations

$$\frac{d^2\vec{r}_i}{dt^2} = \frac{1}{m_i}\vec{F}_i \qquad \Longrightarrow \qquad \frac{\partial}{\partial c_l}\left(\frac{d^2\vec{r}_i}{dt^2}\right) = \frac{1}{m_i}\frac{\partial \vec{F}_i}{\partial c_l}$$

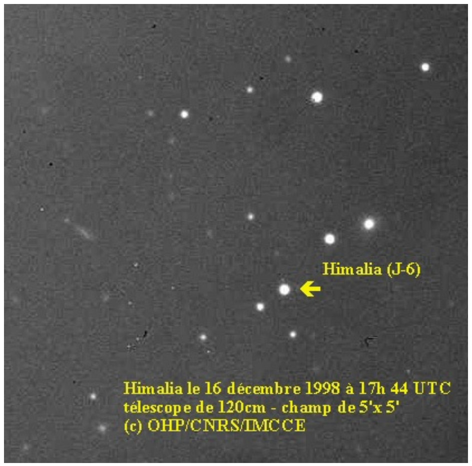
$$\frac{\partial}{\partial c_l}\left(\frac{d^2\vec{r}_i}{dt^2}\right) = \frac{1}{m_i}\left[\sum_j\left(\frac{\partial \vec{F}_i}{\partial \vec{r}_j}\frac{\partial \vec{r}_j}{\partial c_l} + \frac{\partial \vec{F}_i}{\partial \dot{\vec{r}}_j}\frac{\partial \dot{\vec{r}}_j}{\partial c_l}\right) + \frac{\partial \vec{F}_i}{\partial c_l}\right]$$

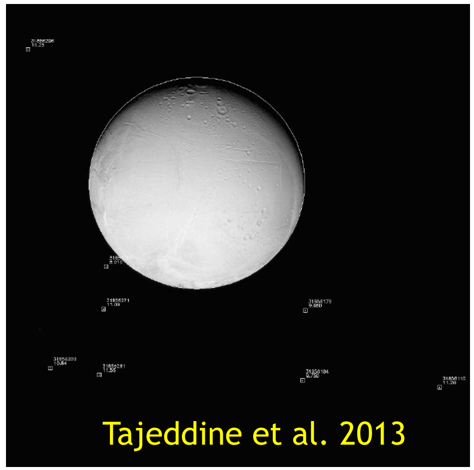
Computation of variational equations can be time consuming and requires a lot of development time!

Orbit determination of natural satellites

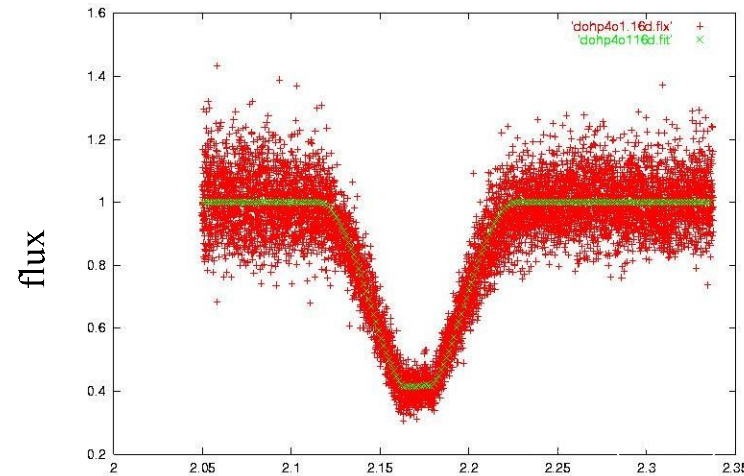
Step 2: gathering the observations

Direct astrometric measurement

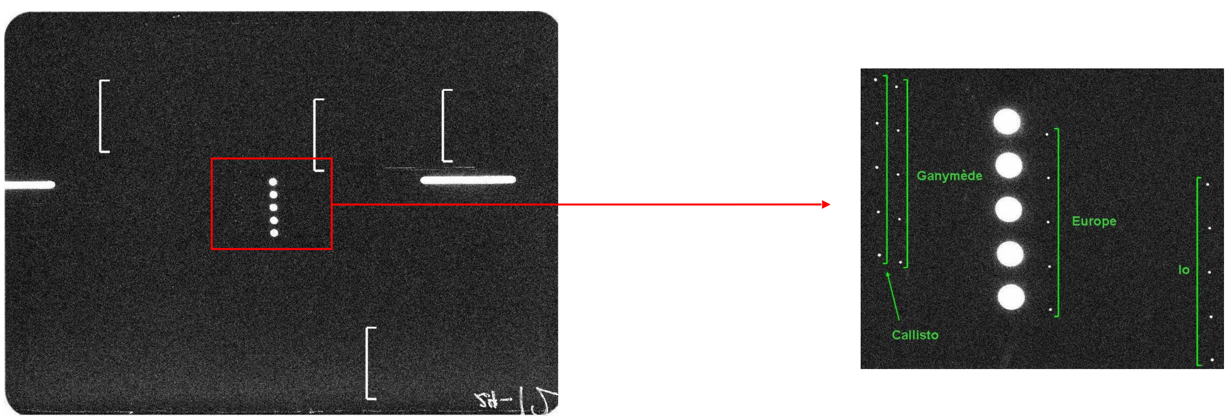




Undirect astrometric measurement (photometry)



Astrometric remeasurement (benefit from modern scanning machine)



<u>Orbit determination of natural satellites</u>

Step 3: Fitting the model to the observations

Step 1 (Integration of the equations of motion + variational equations)

Step 2 (observations)

Step 3 (fitting the model)
Approximation by a linear system
=> *least squares method*

$$(O - C) = \sum_{l=1}^{N} A_i(c_l)\,\Delta c_l \cancel{+ O((\Delta c_l)^2)}$$

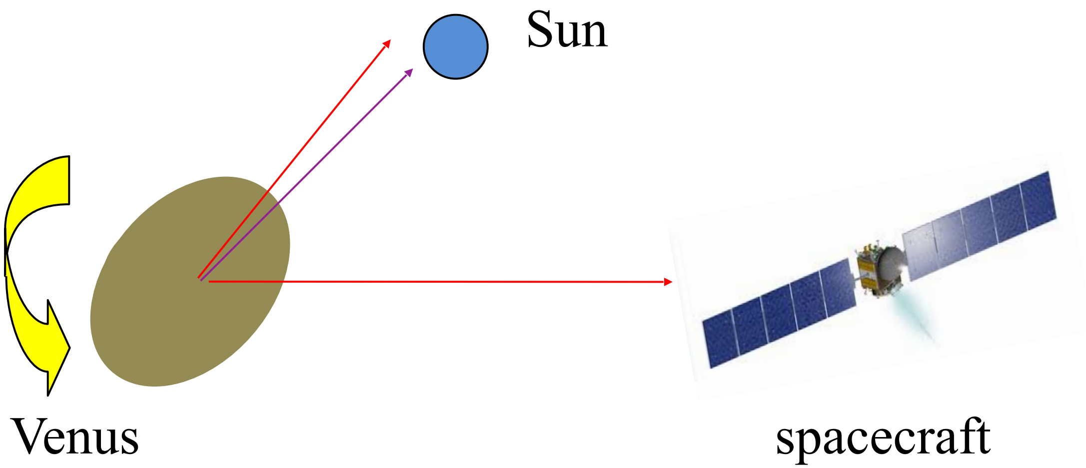
unknown

Venus

Venera

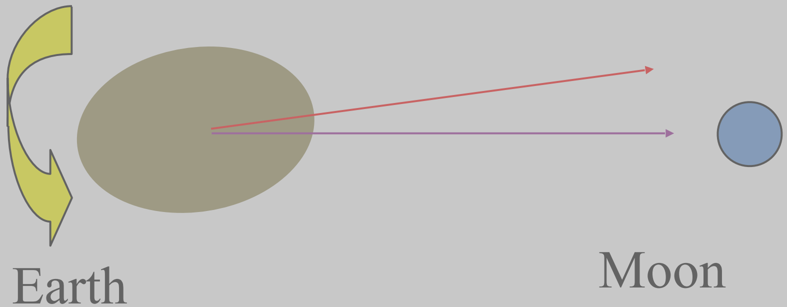
Venus

Space geodesy:



Allows the determination of Venus: k_2

Astrometry:

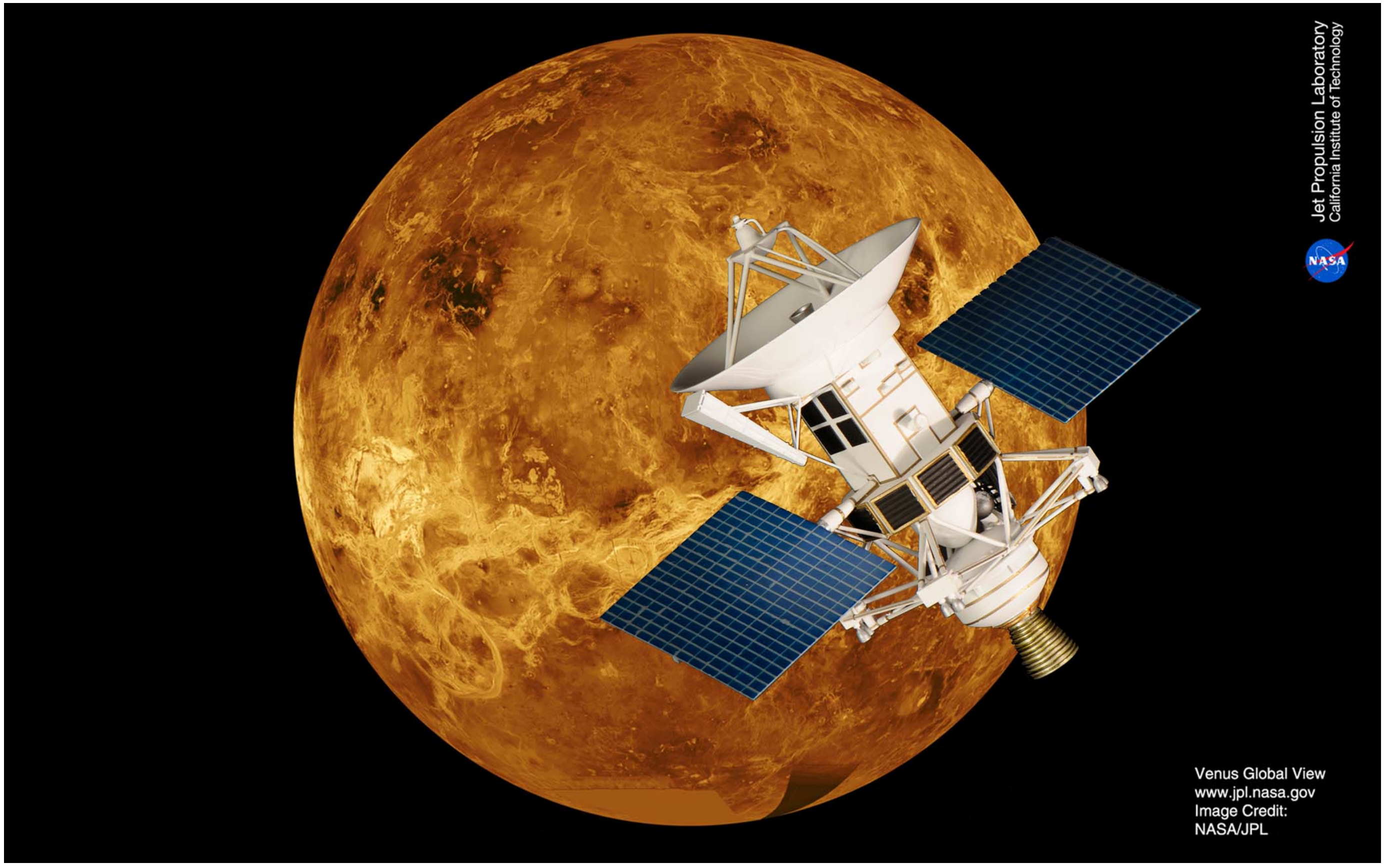


Allows the determination of
Earth: k_2/Q (ocean tide)
Moon: k_2 and Q

Venus

The only measurement so far comes from Magellan and Pioneer Venus Orbiter spacecraft

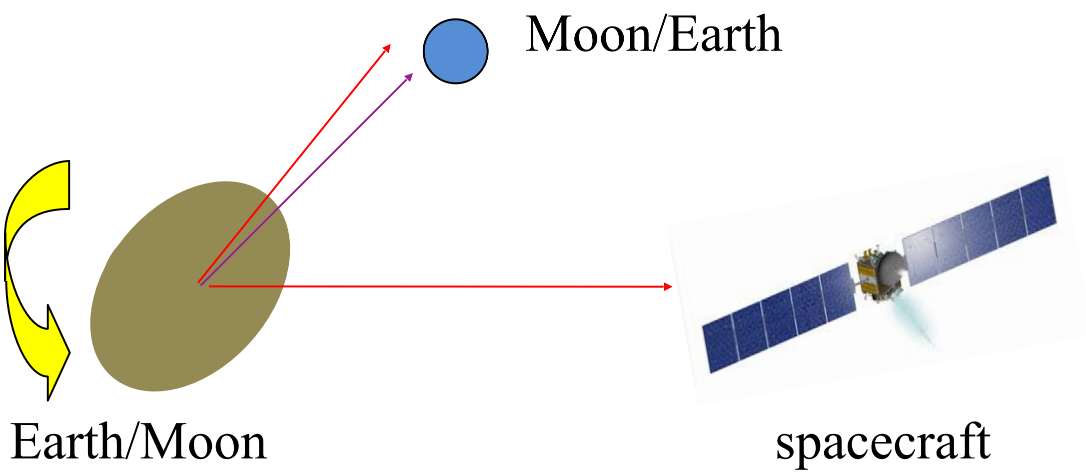
$k_2 = 0.295 +/- 0.066$ ($2\text{-}\sigma$) (Konopliv and Yoder 1996)



The Earth-Moon system

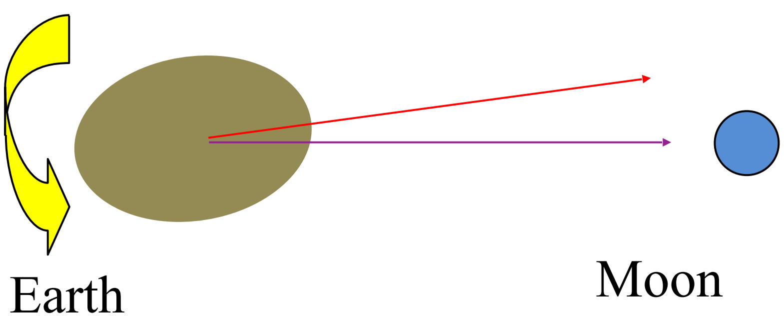
The Earth-Moon system

Space geodesy:



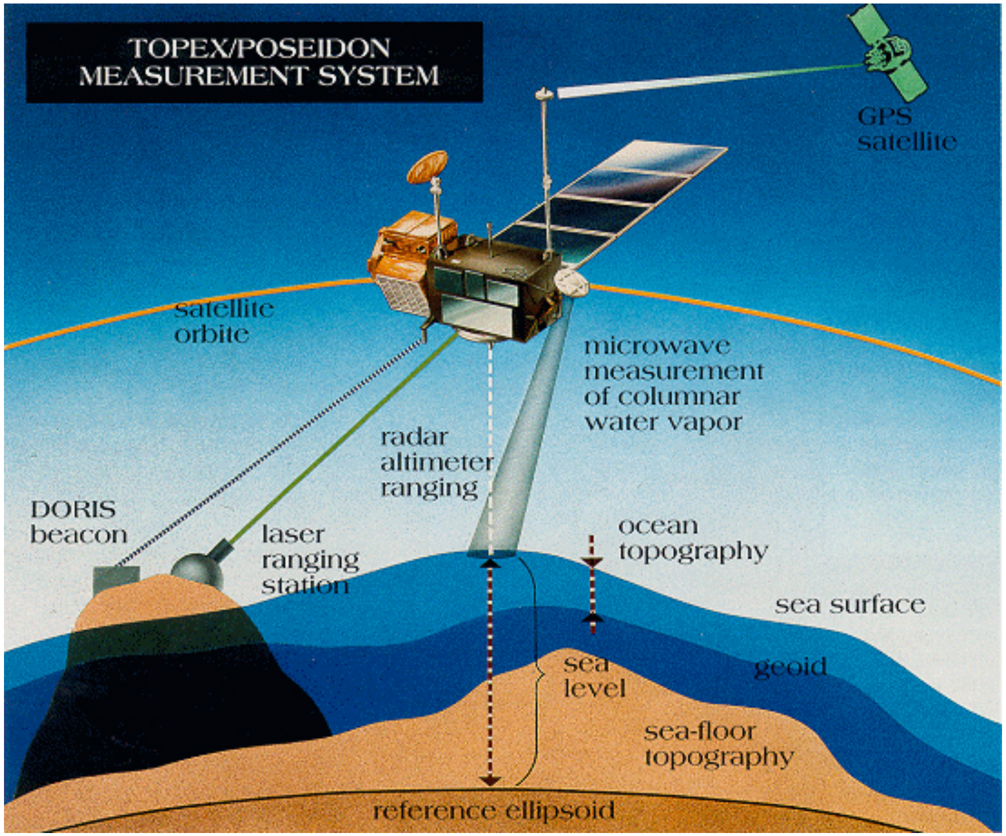
Allows the determination of
Earth: k_2 and Q (both ocean and terrestrial tide)
Moon: k_2

Astrometry:



Allows the determination of
Earth: k_2/Q (ocean tide)
Moon: k_2 and Q

The Earth-Moon system



The first reliable estimate of the Earth's Q associated to terrestrial tide is from Ray et al. Nature 1996

$$Q = 370\ (200,\ 800)$$

later improved by Ray et al. 2001

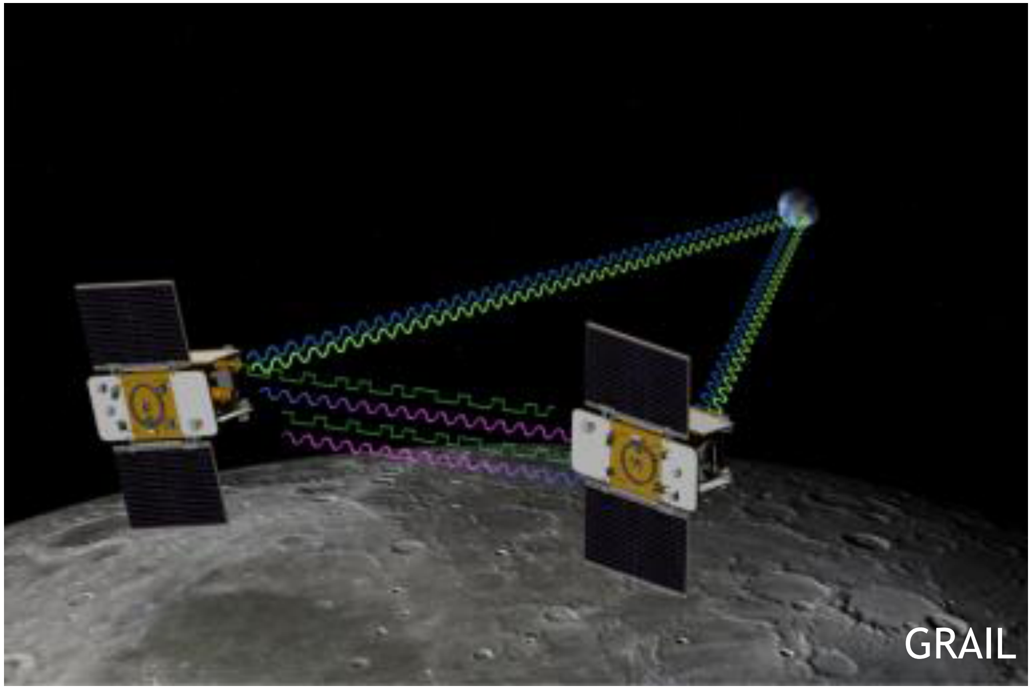
$$Q = 280\ (230,\ 360)$$

using k_2=0.302 (Lerch et al. 1992; Lemoine et al. 1998)

A real challenge: separating the terrestrial tide from the ocean tide (much more significant)

The Earth-Moon system

The GRAIL mission allows an unprecedented accuracy in our knowledge of the Lunar gravity field.

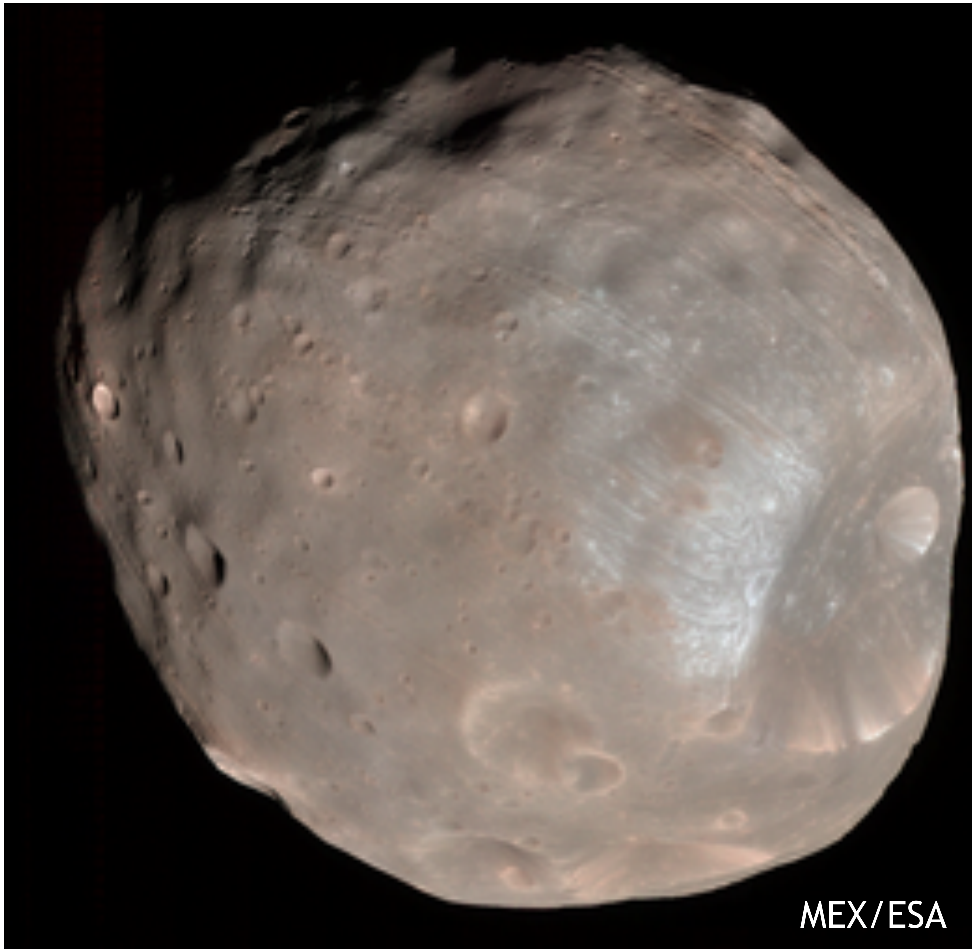


Combined with Lunar Laser Ranging (LLR), a full solution including gravity harmonics up to degree and order 420.

From LLR the Lunar Q is estimated to be Q=37.5 +/- 4 (Williams et al. JGR 2014)

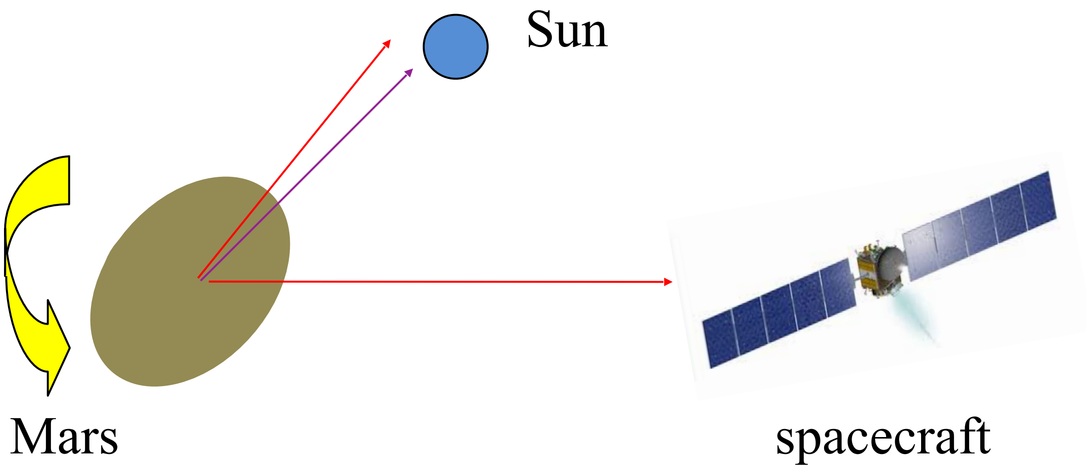
Parameter	GL0660B	GRGM660PRIM
R	1738.0 km	1738.0 km
k_{20}	0.02408 ± 0.00045	0.024165 ± 0.00228
k_{21}	0.02414 ± 0.00025	0.023915 ± 0.00033
k_{22}	0.02394 ± 0.00028	0.024852 ± 0.00042
k_2	0.02405 ± 0.000176	0.02427 ± 0.00026
k_{30}		0.00734 ± 0.00375
k_3	0.0089 ± 0.0021	

The Mars system



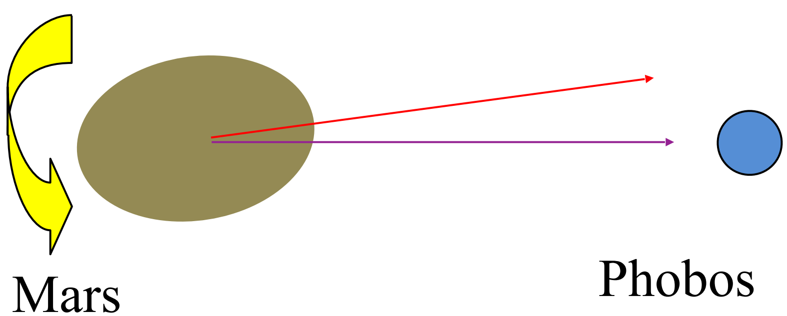
Example of the Mars system

Space geodesy:



Allows the determination of k_2

Astrometry:



Allows the determination of k_2/Q

Example of the Mars system

Estimation of the Mars Love number from Mars' spacecraft

Spacecraft	$\mathrm{Re}(k_2)$, $\mathrm{Im}(k_2) = 0$	$\mathrm{Re}(k_2)$, $\mathrm{Im}(k_2) = 0.01$	$\mathrm{Re}(k_2)$, $\mathrm{Im}(k_2)$ est.	Notes
MGS	0.173 ± 0.009	0.168 ± 0.009	0.159 ± 0.016 0.023 ± 0.025	Best overall solution this paper $\mathrm{Im}(k_2) = 0$, $5\times$ formal σ
MGS	0.153 ± 0.017			Data to April 14, 2002, Yoder et al. (2003)
MGS	0.166 ± 0.011			Data to December 5, 2004, Konopliv et al. (2006)
ODY	0.172 ± 0.014	0.185 ± 0.014	0.167 ± 0.025 -0.004 ± 0.016	Without arcs affected by dust, best Odyssey solution $\mathrm{Im}(k_2) = 0$, $10\times$ formal σ
ODY	0.104 ± 0.013		0.015 ± 0.021 -0.076 ± 0.014	All arcs but with no dust model
ODY	0.161 ± 0.013	0.173 ± 0.013	0.131 ± 0.022 -0.025 ± 0.014	All arcs but with dust model to 30–40 km
ODY	0.172 ± 0.013	0.184 ± 0.013	0.154 ± 0.022 -0.015 ± 0.014	All arcs but with dust model to 30–50 km
MRO	0.175 ± 0.010	0.175 ± 0.010	0.176 ± 0.010 0.036 ± 0.040	$10\times$ formal σ
Other determinations				
MGS	0.201 ± 0.059 0.163 ± 0.056			Bills et al. (2005)
MGS	0.176 ± 0.041			Lemoine et al. (2006)
MGS	0.130 ± 0.030			Balmino et al. (2005) (see Marty et al., 2009)
MGS + ODY	0.120 ± 0.003			Marty et al. (2009)

Konopliv et al. (2011) provides k_2=0.164 +/- 0.009

The various estimation of the tidal Love number k_2 have changed much in the last decade.

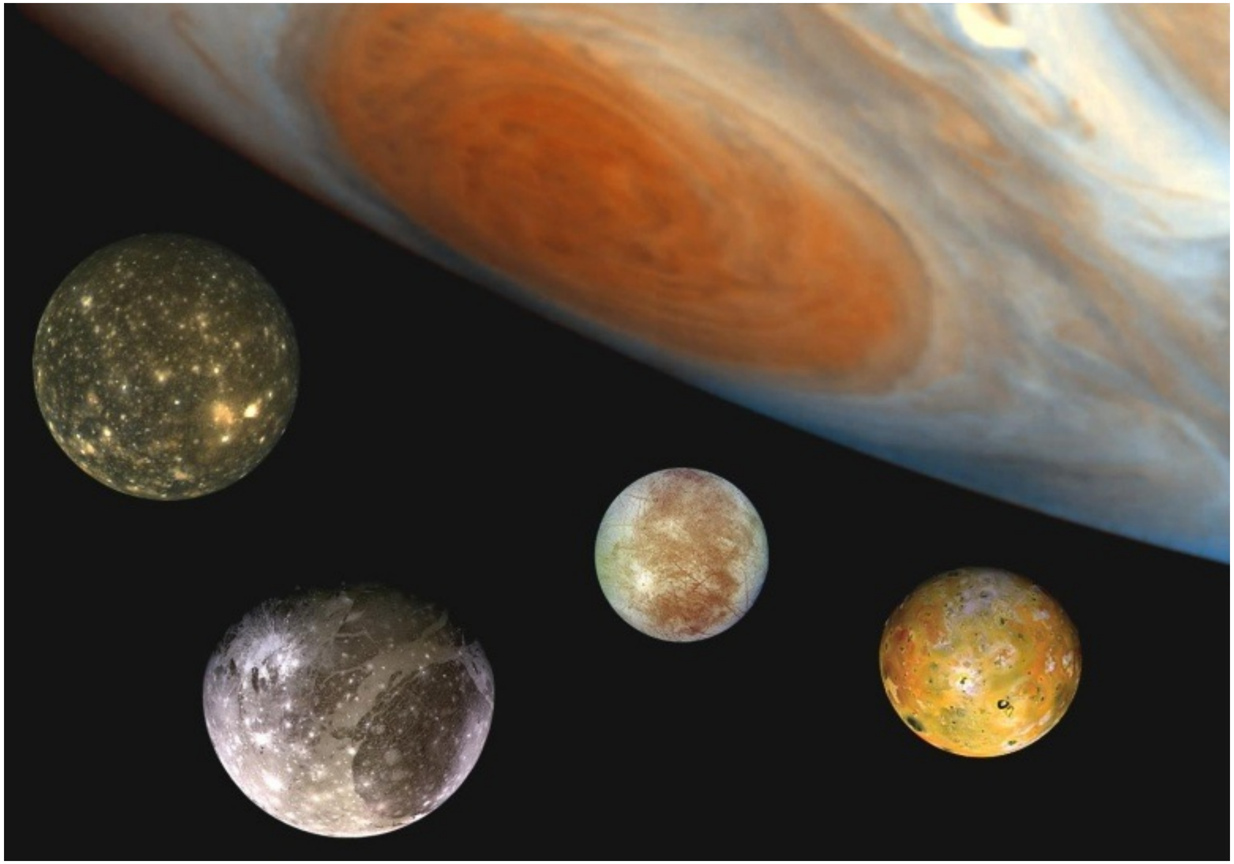
Example of the Mars system

Estimation of Phobos tidal acceleration over time (Jacobson 2010):

Reference	$s \times 10^{-3}$ (deg yr^{-2})	κ_2	Q	γ (deg)
Sharpless (1945)	1.882 ± 0.171			
Shor (1975)	1.427 ± 0.147			
Sinclair (1978)	1.326 ± 0.118			
Jacobson et al. (1989)	1.249 ± 0.018			
Chapront-Touzé (1990)	1.270 ± 0.008			
Emelyanov et al. (1993)	1.290 ± 0.010			
Bills et al. (2005)	1.367 ± 0.006	0.163	85.6 ± 0.4	$0.^\circ3346 \pm 0.^\circ0014$
Rainey & Aharonson (2006)	1.334 ± 0.006	0.153	78.6 ± 0.8	$0.^\circ3645 \pm 0.^\circ0039$
Lainey et al. (2007)	1.270 ± 0.015	0.152	79.9 ± 0.7	$0.^\circ3585 \pm 0.^\circ0031$
Current	1.270 ± 0.003	0.152	82.8 ± 0.2	$0.^\circ3458 \pm 0.^\circ0009$

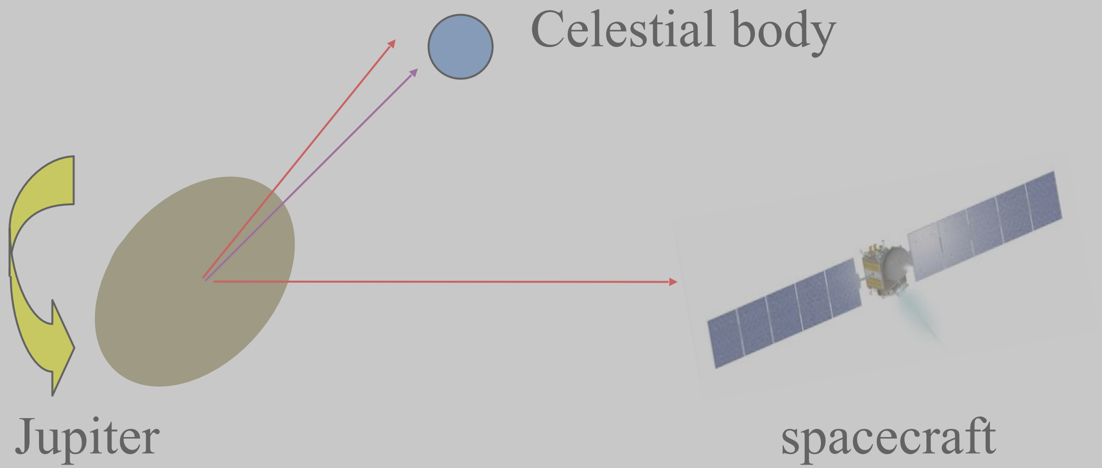
Pretty good agreement since decades!

The Jovian system

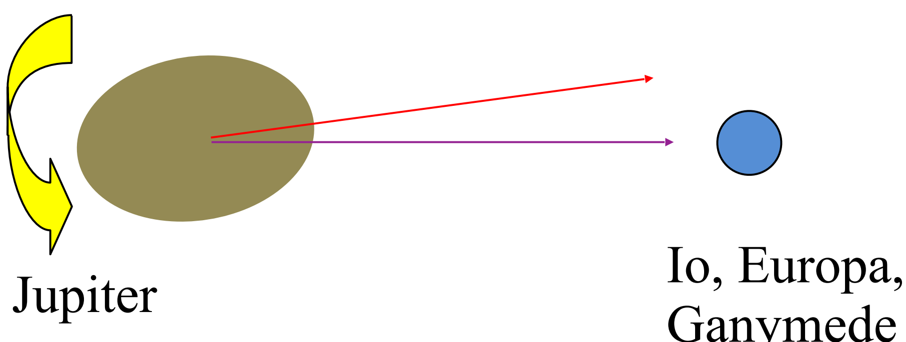


The Jovian system

Space geodesy:



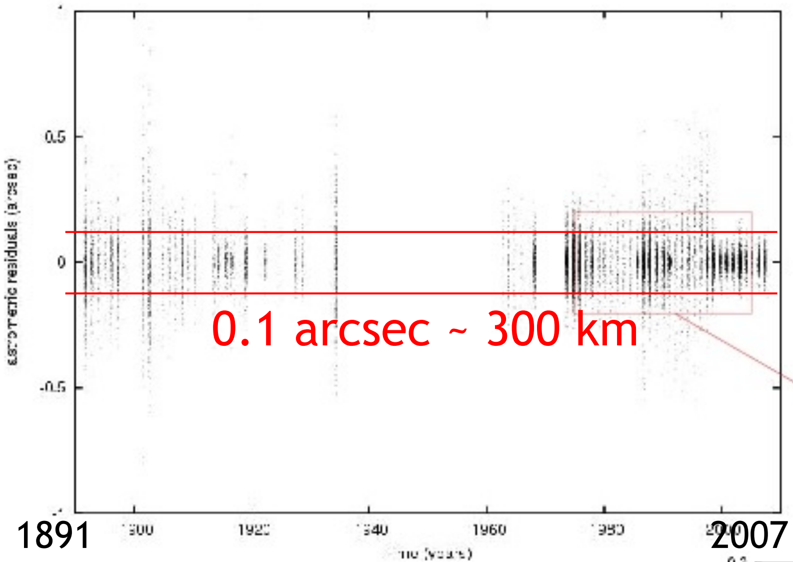
Astrometry:

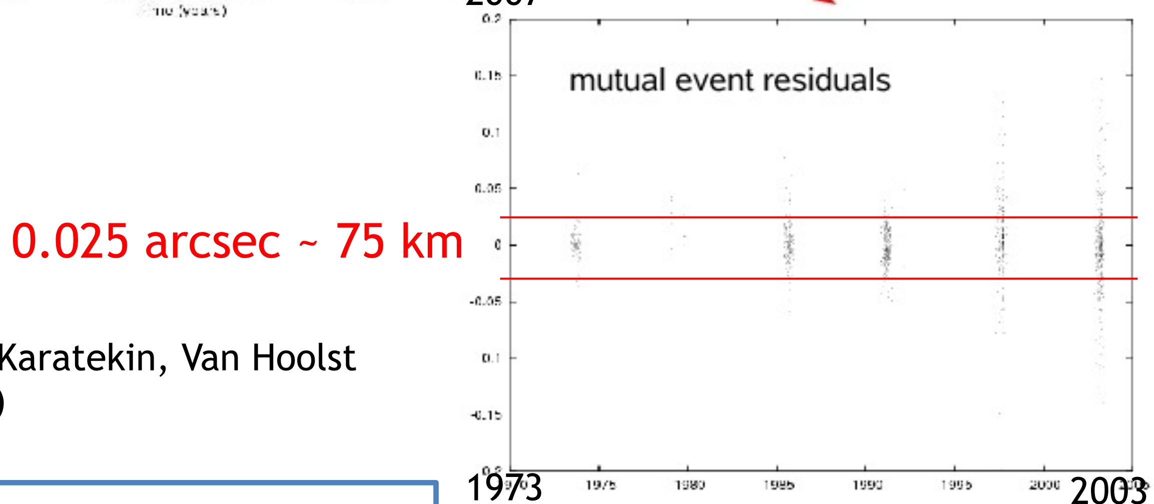


Allows the determination of k_2/Q in Jupiter and Io

The Jovian system

→Residuals after fitting the initial state vectors of all the Galilean moons and the ratios k_2/Q inside Io and Jupiter



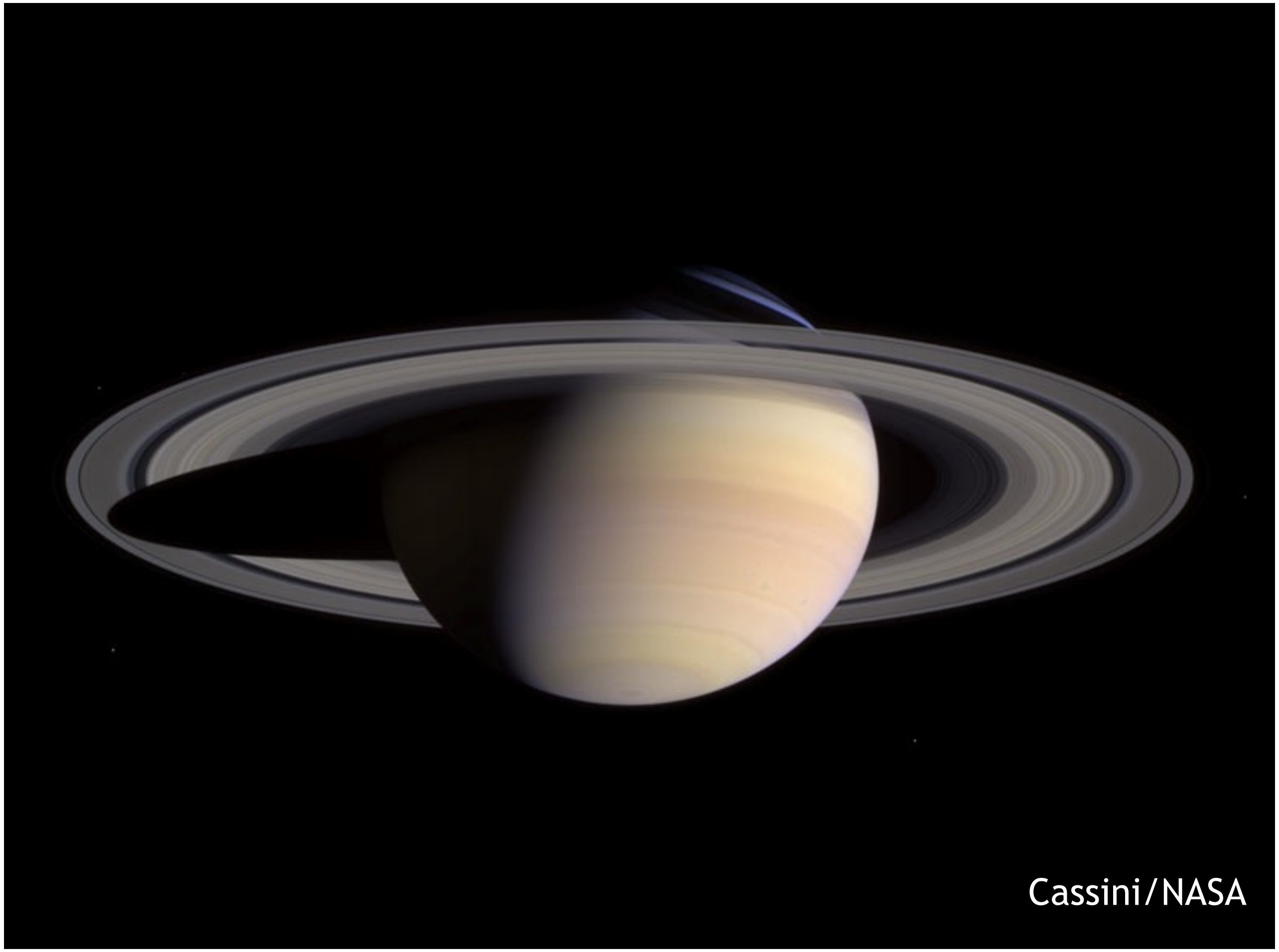


Lainey, Arlot, Karatekin, Van Hoolst (Nature, 2009)

Io: $k_2/Q = 0.015 \pm 0.003$

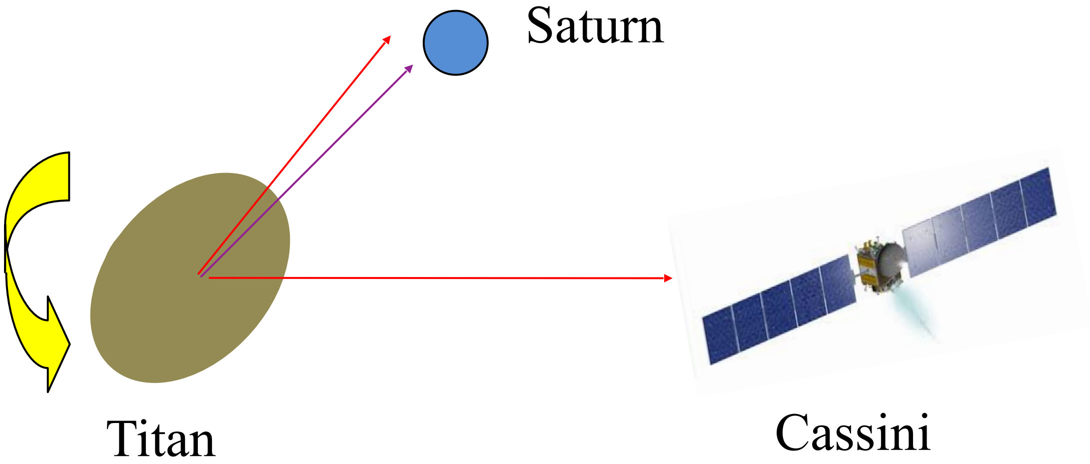
Jupiter: $k_2/Q = (1.102 \pm 0.203) \times 10^{-5}$

The Saturnian system



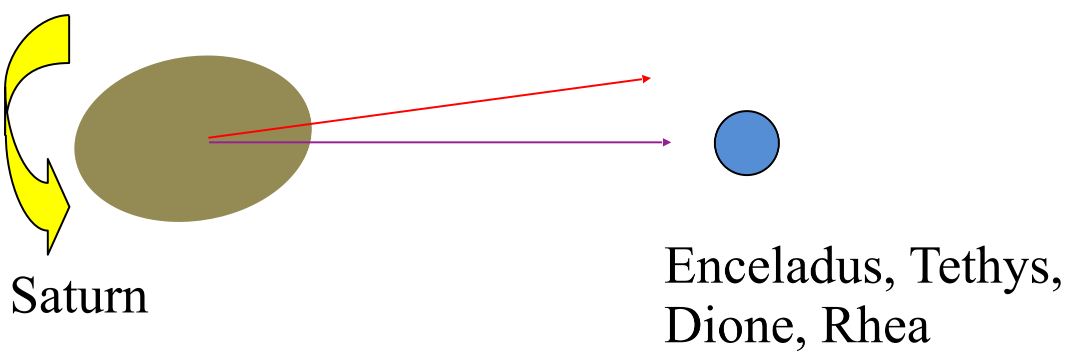
The Saturn's system

Space geodesy:



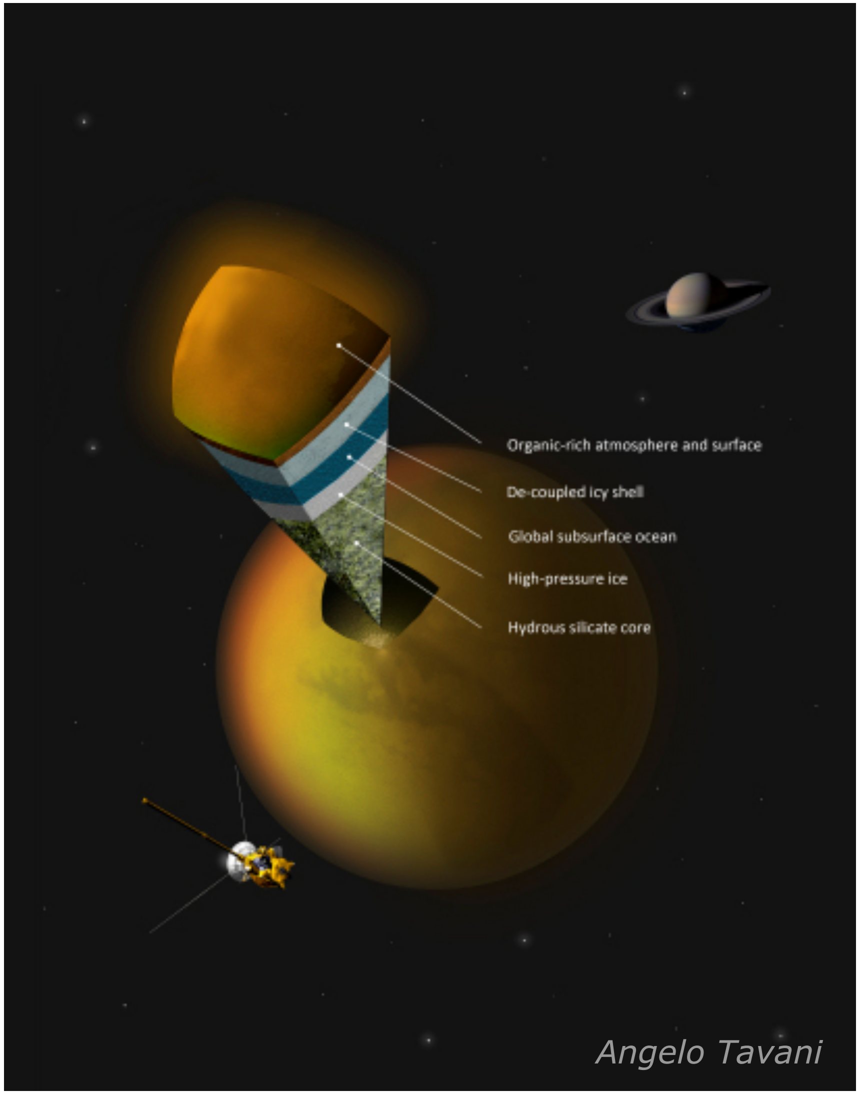
Allows the determination of Titan's k_2

Astrometry:



Allows the determination of k_2/Q in Saturn

The Saturn's system



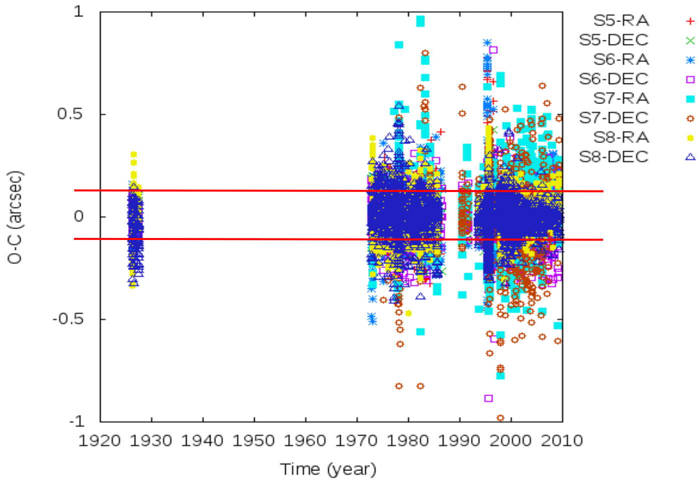
Using six flybys of Titan by Cassini (devoted to radio-science experiment) allowed to quantify Titan's gravity field and Love number k_2

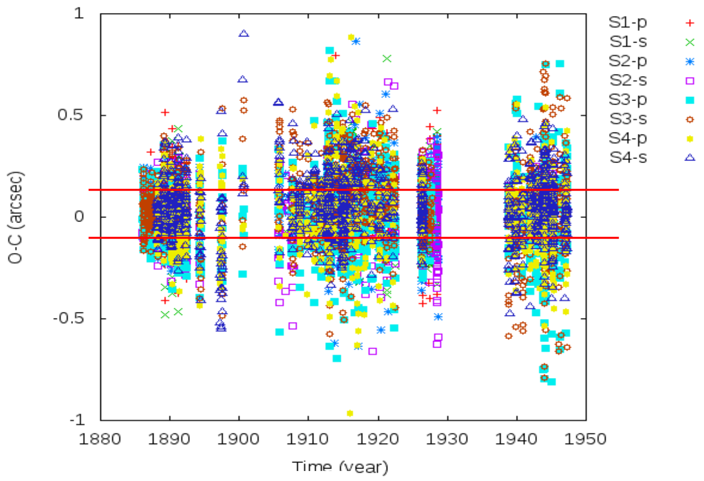
$$k_2 = 0.589 \pm 0.150$$

Iess et al. Science 2012

The Saturn's system

→Residuals after fitting the initial state vectors of all the eight main Saturn moons, the ratio k_2/Q inside Saturn.

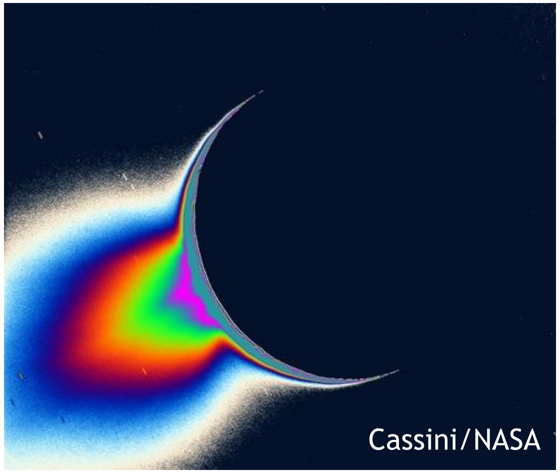




0.1 arcsec ~ 600 km

Lainey et al. (2012) finds a much higher value for k_2/Q in Saturn than expected…

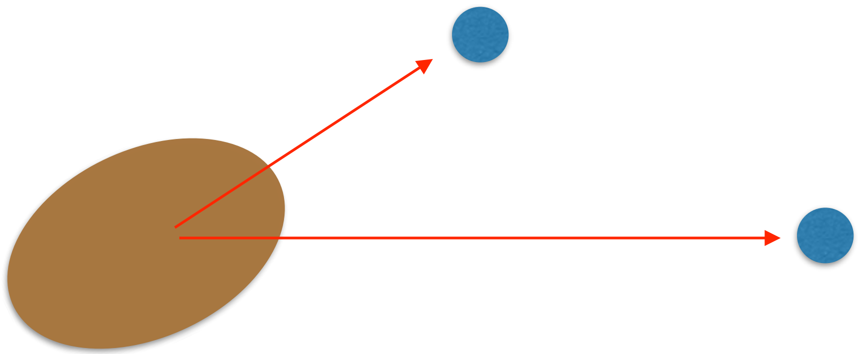
$$k_2/Q = (2.3 \pm 0.7) \times 10^{-4}$$



New results…

New result…

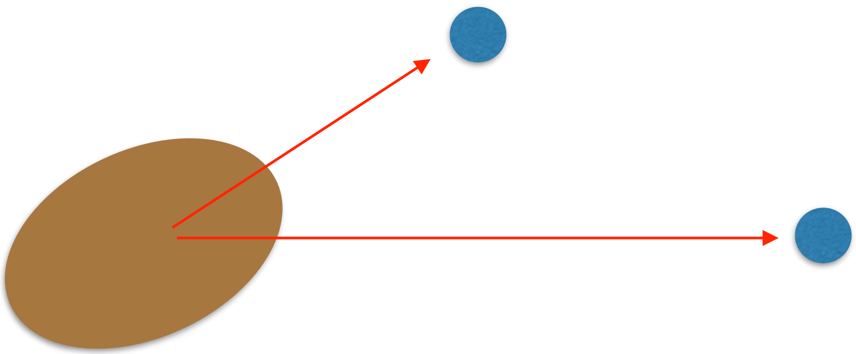
Separating k_2 and Q!



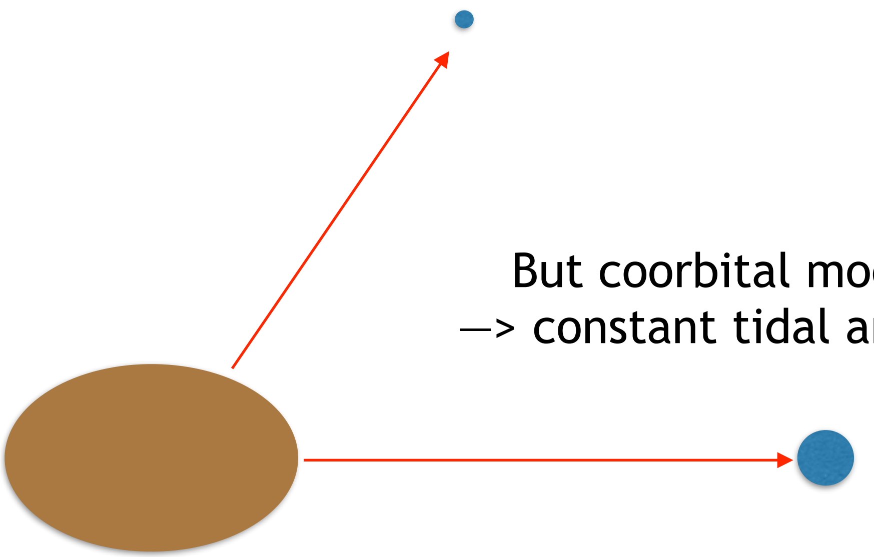
Interaction of mutual tidal bulges is a quasi-periodic effect, and so, is hard to detect from astrometry…

New result…

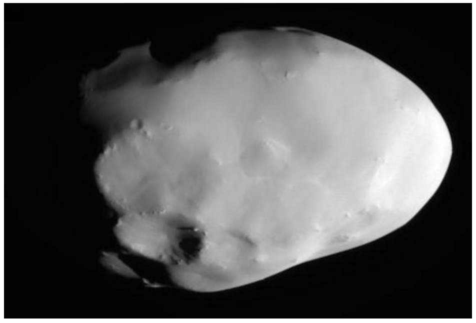
Separating k_2 and Q!



Interaction of mutual tidal bulges is a quasi-periodic effect, and so, is hard to detect from astrometry…

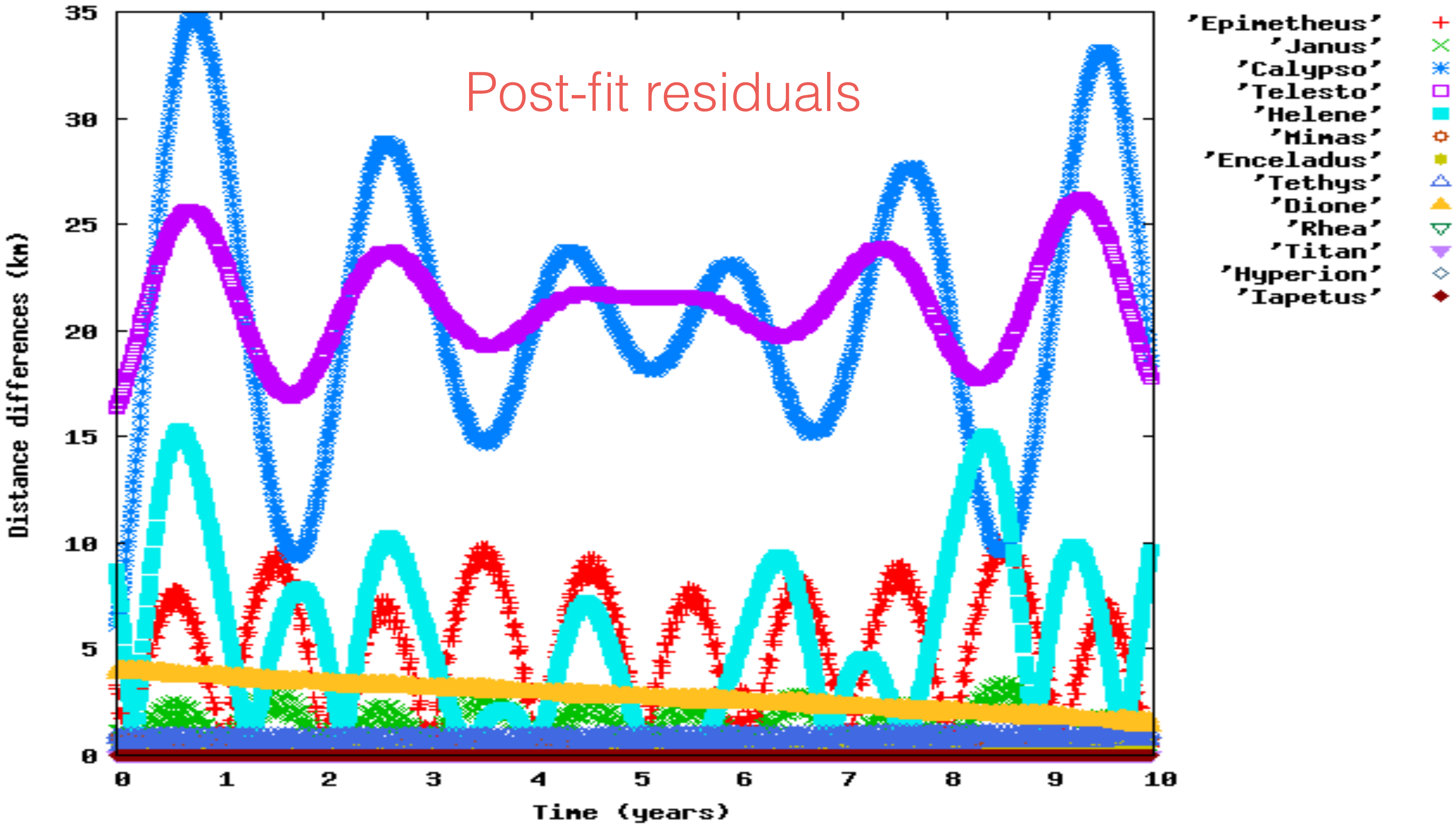


But coorbital moons
—> constant tidal angle!!



<u>New result…</u>

Separating k_2 and Q!



Thanks to coorbital satellites, Saturn's k_2 and Q can be separated!

Conclusion:

1- There are not so many objects for which we have measured k_2 and Q

2- Space geodesy and astrometry are two complementary techniques to characterize the tidal dissipation and Love numbers inside a solar system body

3- Cassini imaging data opens the door to a lot of exciting possible new results (direct quantification of tidal dissipation in Enceladus, separation of k_2 and Q in Saturn…)

4- Future missions like JUNO and JUICE or projects like Europa clipper, PhoDex and many more will for sure increase our knowledge of tides in solar system bodies