

“Tidal evolution of close-in planets using a Maxwell visco-elastic rheology”

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CEA, Saclay

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Tidal effect



Global Picture:

Poincaré (1898)

Landau & Lifshitz (1960)

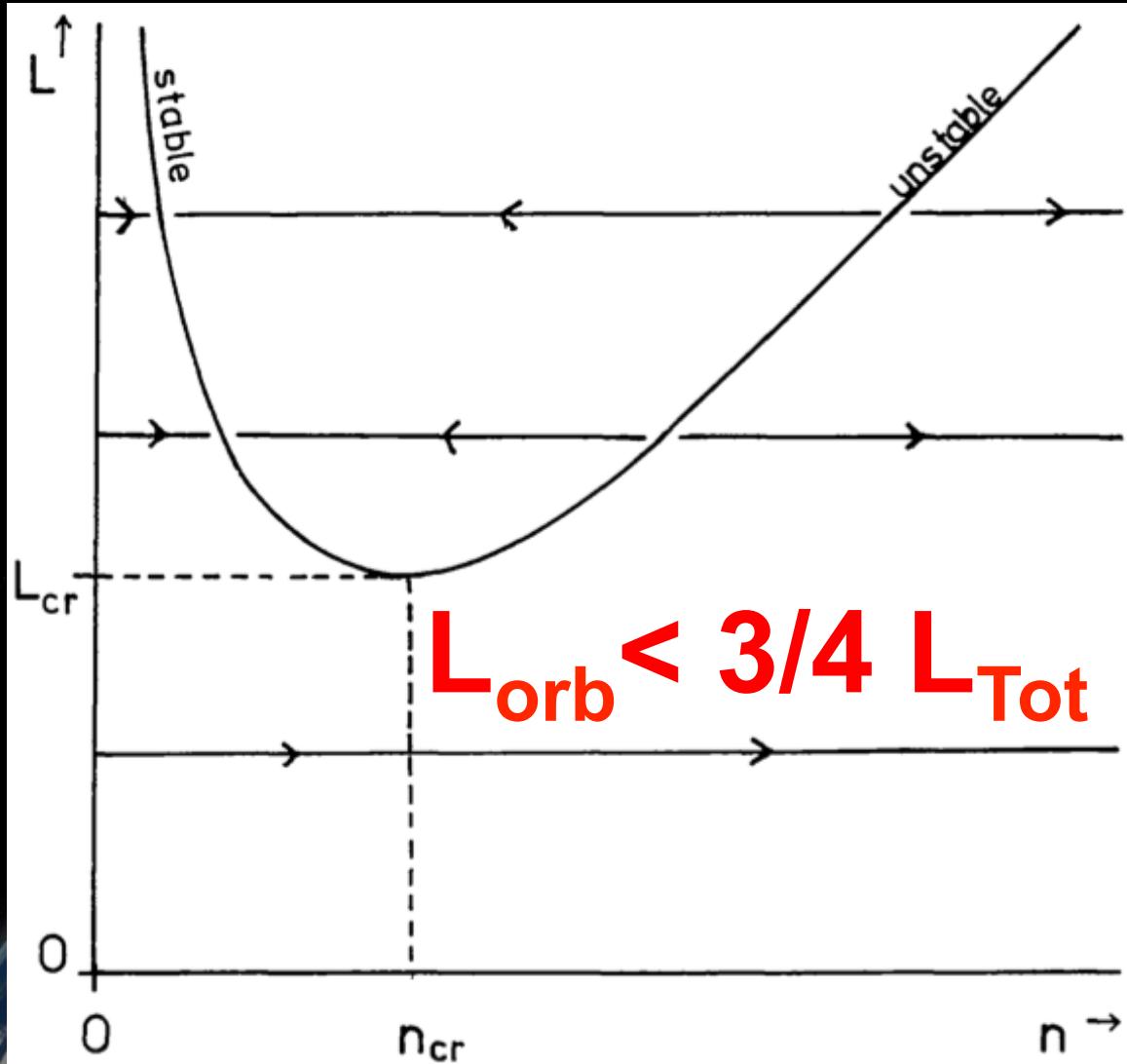
Hut (1980)

$P_{\text{orb}} = P_{\text{rot}}$
perpendicular

axis ($\varepsilon=0$)

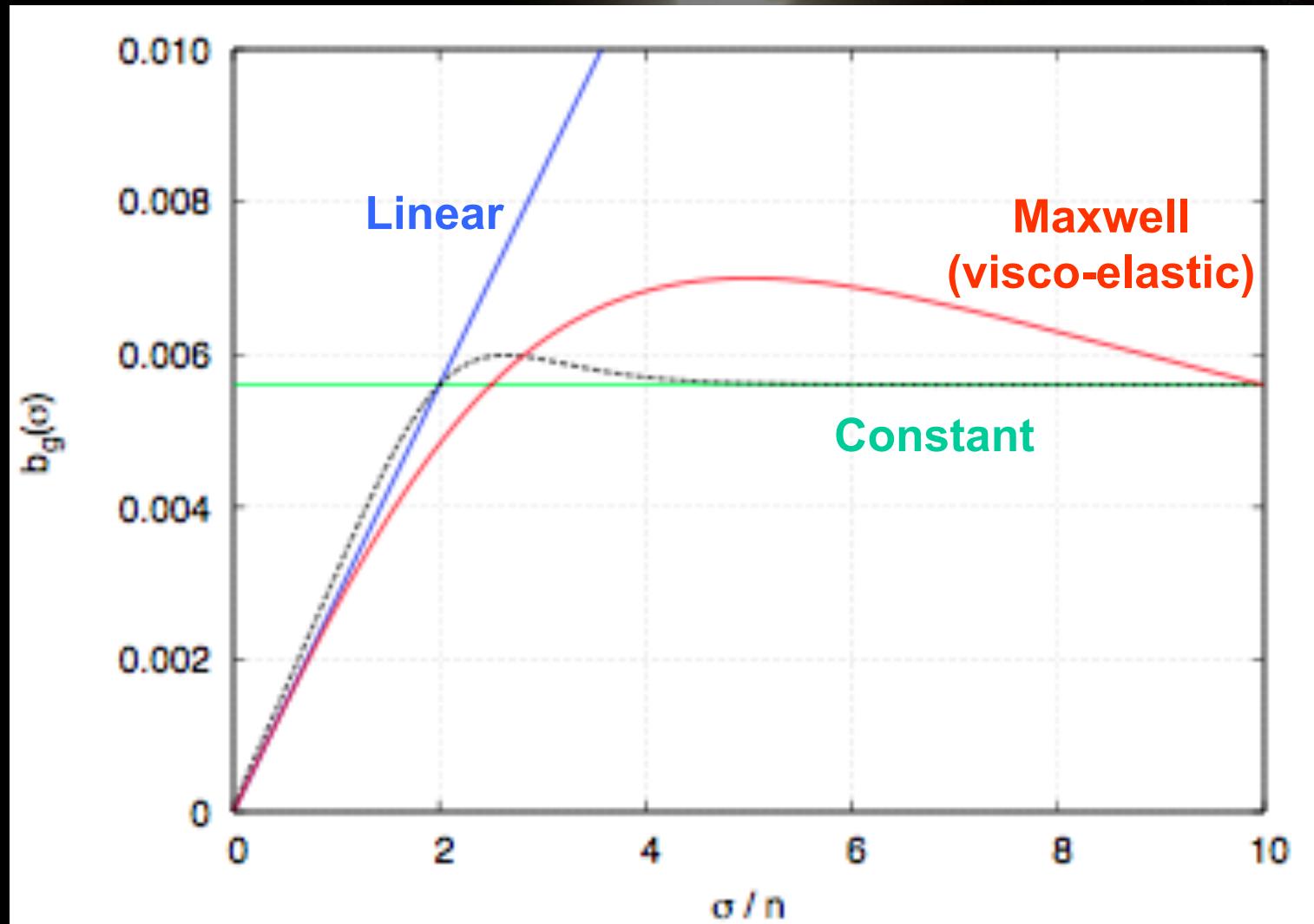
circular

orbits ($e=0$)

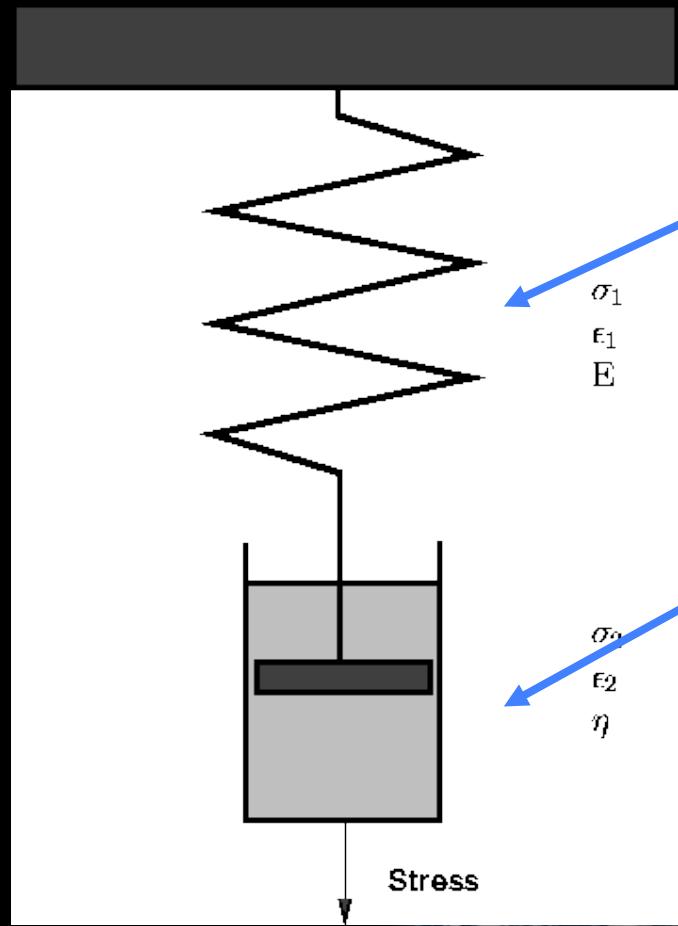


$$L_{\text{cr}} = 4 \left\{ \frac{1}{27} G \frac{M^3 m^3}{M + m} (I_1 + I_2) \right\}^{1/4}$$

Tidal models



Maxwell model (1867)



elastic

$$\sigma = E \epsilon$$

strain

viscous

$$\sigma = \eta d\epsilon/dt$$

$$\frac{d\epsilon_{\text{Total}}}{dt} = \frac{d\epsilon_D}{dt} + \frac{d\epsilon_S}{dt} = \frac{\sigma}{\eta} + \frac{1}{E} \frac{d\sigma}{dt}$$

$$\frac{1}{E} \frac{d\sigma}{dt} + \frac{\sigma}{\eta} = \frac{d\epsilon}{dt}$$

Fourier Domain

$$\hat{V}'(\mathbf{R}, \omega) = \hat{k}_2(\omega) \hat{V}_p(\mathbf{R}, \omega)$$

$$(1 + i\omega\tau) \hat{V}'(\mathbf{R}, \omega) = (1 + i\omega\tau_e) k_f \hat{V}_p(\mathbf{R}, \omega)$$

$$\hat{k}_2(\omega) = k_f \frac{1 + i\omega\tau_e}{1 + i\omega\tau}$$

$$\tau_e = \eta/E \quad ; \quad \tau = \eta(1 + E)/E$$

Fourier Domain

$$\hat{V}'(\mathbf{R}, \omega) = \hat{k}_2(\omega) \hat{V}_p(\mathbf{R}, \omega)$$

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Time Domain

Correia et al.
A&A 571 (2014)

$$V' + \tau \dot{V}' = k_f (V_p + \tau_e \dot{V}_p)$$

$$\tau_e = \eta/E \quad ; \quad \tau = \eta(1+E)/E$$

General solution

$$V(t) = k_f \frac{\tau_e}{\tau} V_p(t) + \frac{k_f}{\tau} \left(1 - \frac{\tau_e}{\tau}\right) \int_0^t V_p(t') e^{(t'-t)/\tau} dt' + C e^{-t/\tau}$$

$$V_p(t) = \sum_k \beta_k e^{i\omega_k t} \Rightarrow V(t) = \sum_k k_f \frac{1 + i\tau_e \omega_k}{1 + i\tau \omega_k} \beta_k e^{i\omega_k t} + C e^{-t/\tau}$$

Time Domain

Correia et al.
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$$V' + \tau \dot{V}' = k_f (V_p + \tau_e \dot{V}_p)$$

$$\tau_e = \eta/E \quad ; \quad \tau = \eta(1+E)/E$$

Equations of Motion

$$V(\mathbf{r}) = -\frac{Gm}{r} - \frac{GmR^2 J_2}{2r^3} - \frac{3GmR^2}{r^3} (C_{22} \cos 2\gamma + S_{22} \sin 2\gamma)$$

$$V' + \tau \dot{V}' = k_f (V_p + \tau_e \dot{V}_p)$$

Correia et al.
A&A 571 (2014)

$$J_2 + \tau \dot{J}_2 = J_2^e + \tau_e \dot{J}_2^e ,$$

$$J_2^e = k_f \left[\frac{\Omega^2 R^3}{3Gm} + \frac{1}{2} \frac{m_0}{m} \left(\frac{R}{r} \right)^3 \right] ,$$

$$C_{22} + \tau \dot{C}_{22} = C_{22}^e + \tau_e \dot{C}_{22}^e ,$$

$$C_{22}^e = \frac{k_f}{4} \frac{m_0}{m} \left(\frac{R}{r} \right)^3 \cos 2\gamma ,$$

$$S_{22} + \tau \dot{S}_{22} = S_{22}^e + \tau_e \dot{S}_{22}^e .$$

$$S_{22}^e = -\frac{k_f}{4} \frac{m_0}{m} \left(\frac{R}{r} \right)^3 \sin 2\gamma ,$$

Equations of Motion

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$$\begin{aligned} \ddot{\mathbf{r}} = & -\frac{\mu_0}{r^2} \hat{\mathbf{r}} - \frac{3\mu_0 R^2}{2r^4} J_2 \hat{\mathbf{r}} - \frac{9\mu_0 R^2}{r^4} [C_{22} \cos 2\gamma - S_{22} \sin 2\gamma] \hat{\mathbf{r}} \\ & + \frac{6\mu_0 R^2}{r^4} [C_{22} \sin 2\gamma + S_{22} \cos 2\gamma] \mathbf{K} \times \hat{\mathbf{r}}, \end{aligned} \quad (15)$$

and

Correia et al.
A&A **571** (2014)

$$\ddot{\theta} = -\frac{6Gm_0 m R^2}{Cr^3} [C_{22} \sin 2\gamma + S_{22} \cos 2\gamma], \quad (16)$$

$$J_2 + \tau \dot{J}_2 = J_2^e + \tau_e \dot{J}_2^e ,$$

$$J_2^e = k_f \left[\frac{\Omega^2 R^3}{3Gm} + \frac{1}{2} \frac{m_0}{m} \left(\frac{R}{r} \right)^3 \right] ,$$

$$C_{22} + \tau \dot{C}_{22} = C_{22}^e + \tau_e \dot{C}_{22}^e ,$$

$$C_{22}^e = \frac{k_f}{4} \frac{m_0}{m} \left(\frac{R}{r} \right)^3 \cos 2\gamma ,$$

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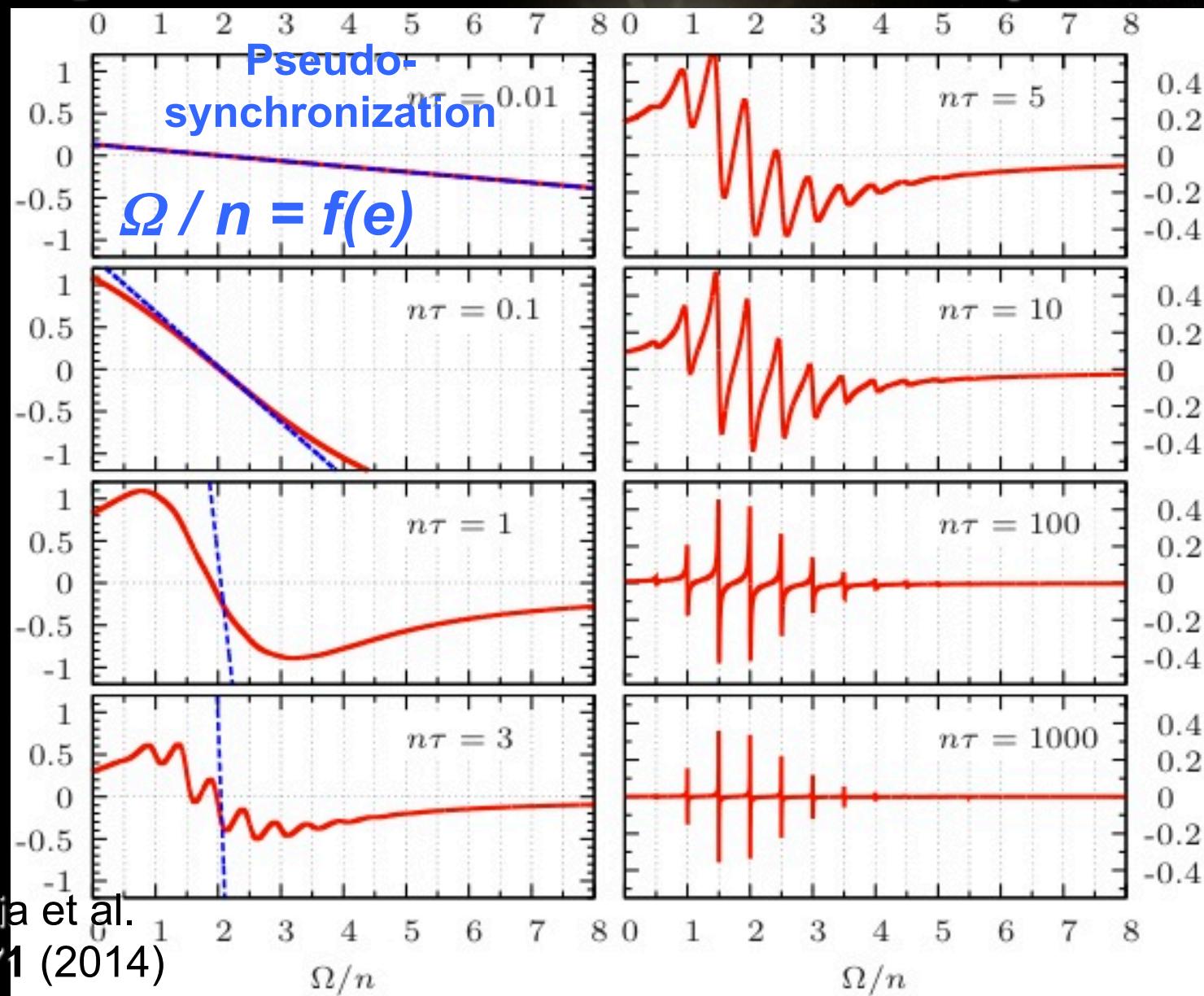
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Correia et al.
A&A 571 (2014)

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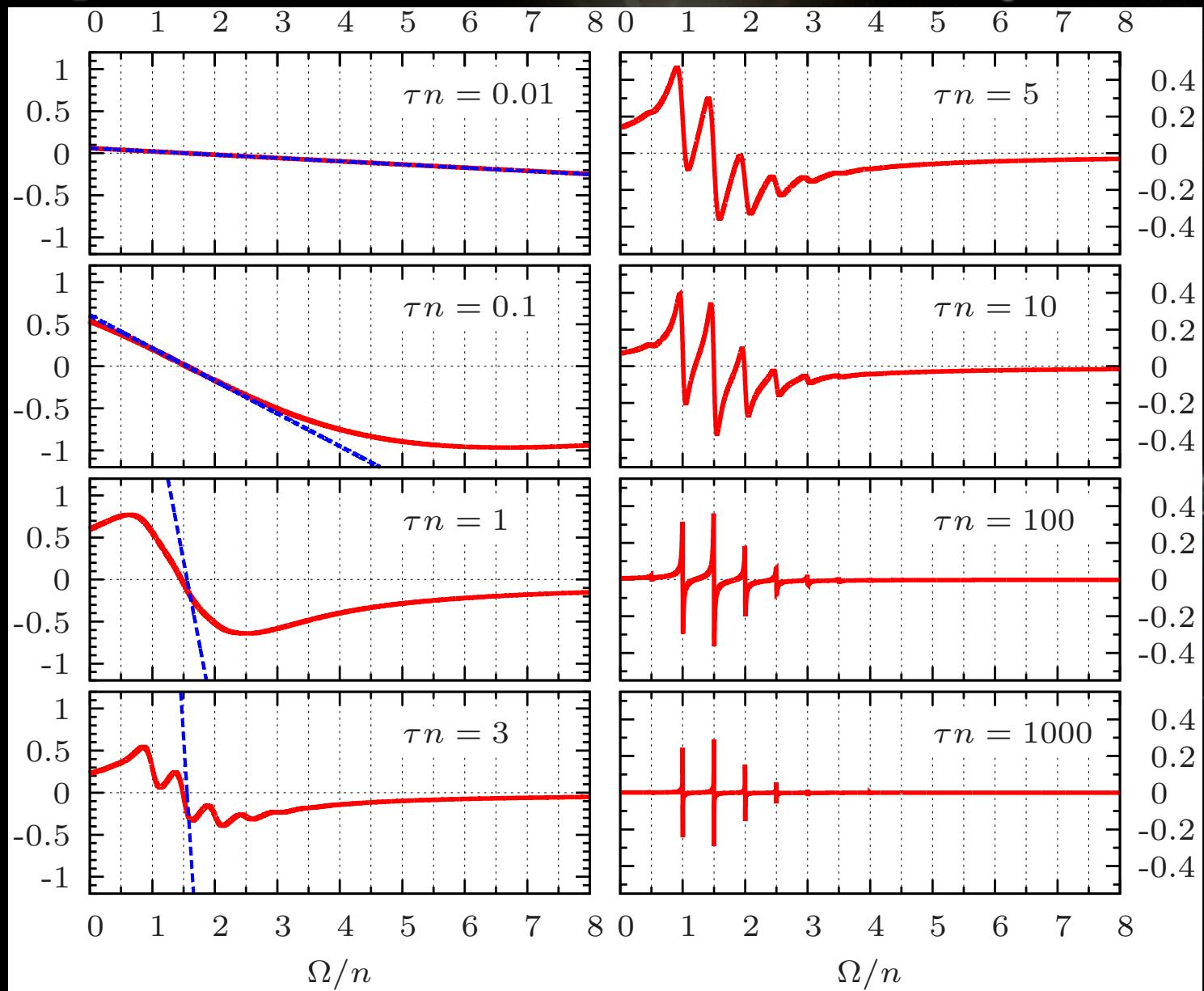
$$\ddot{\theta} = -\frac{6Gm_0mR^2}{Cr^3} \left[C_{22} \sin 2\gamma + S_{22} \cos 2\gamma \right] , \quad (16)$$

Equilibrium Rotations ($e=0.4$)

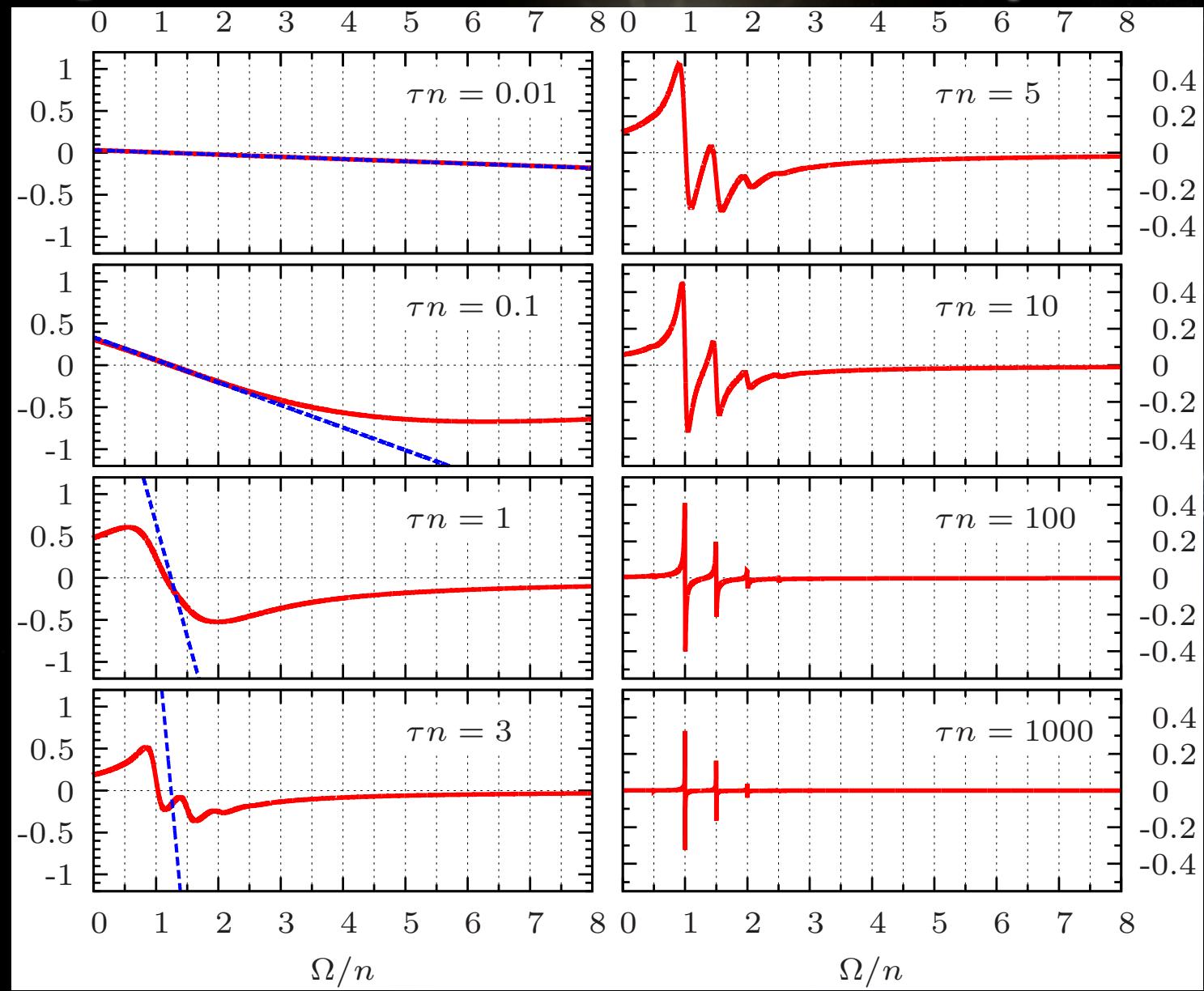


Correia et al.
A&A 571 (2014)

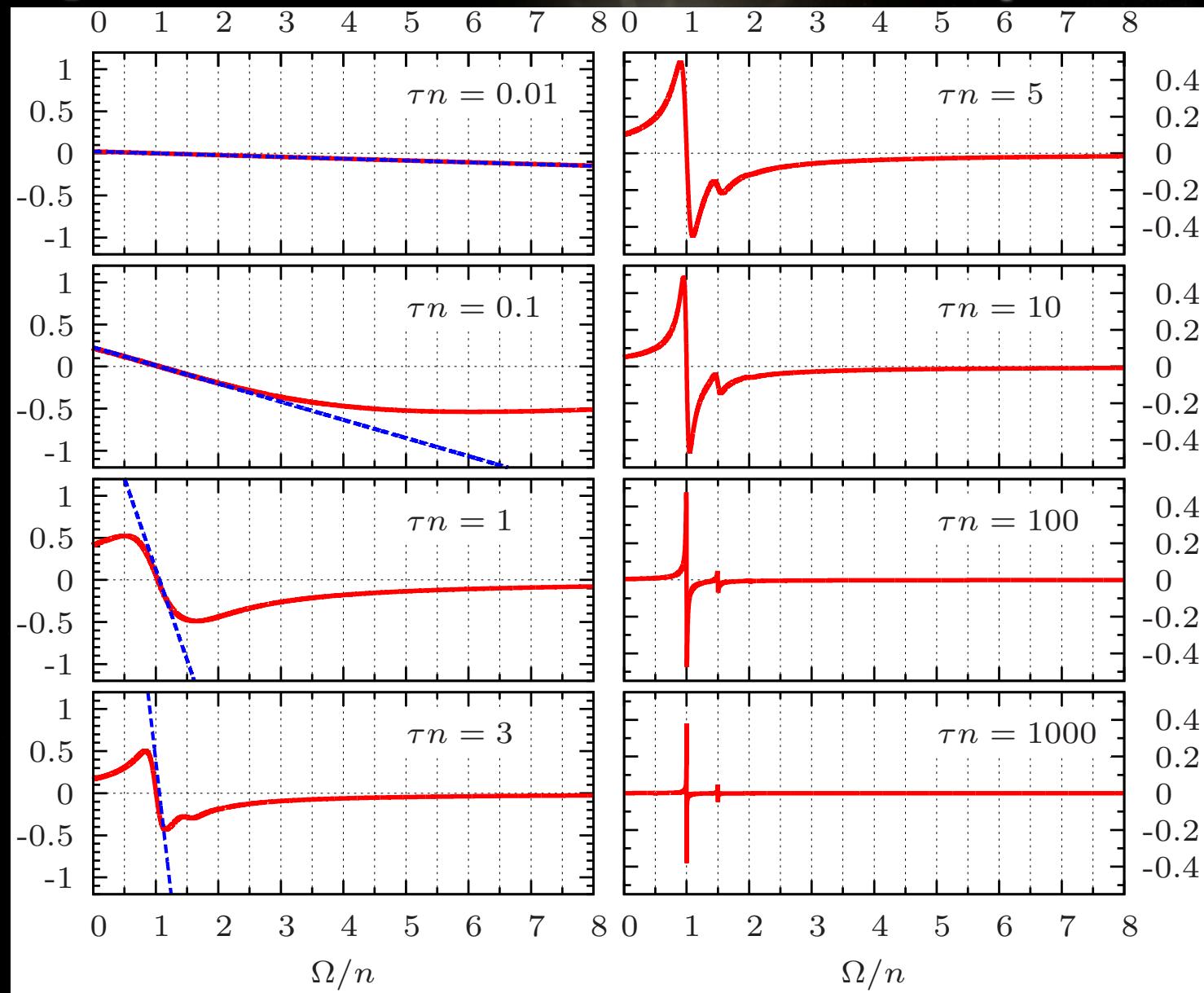
Equilibrium Rotations ($e=0.3$)



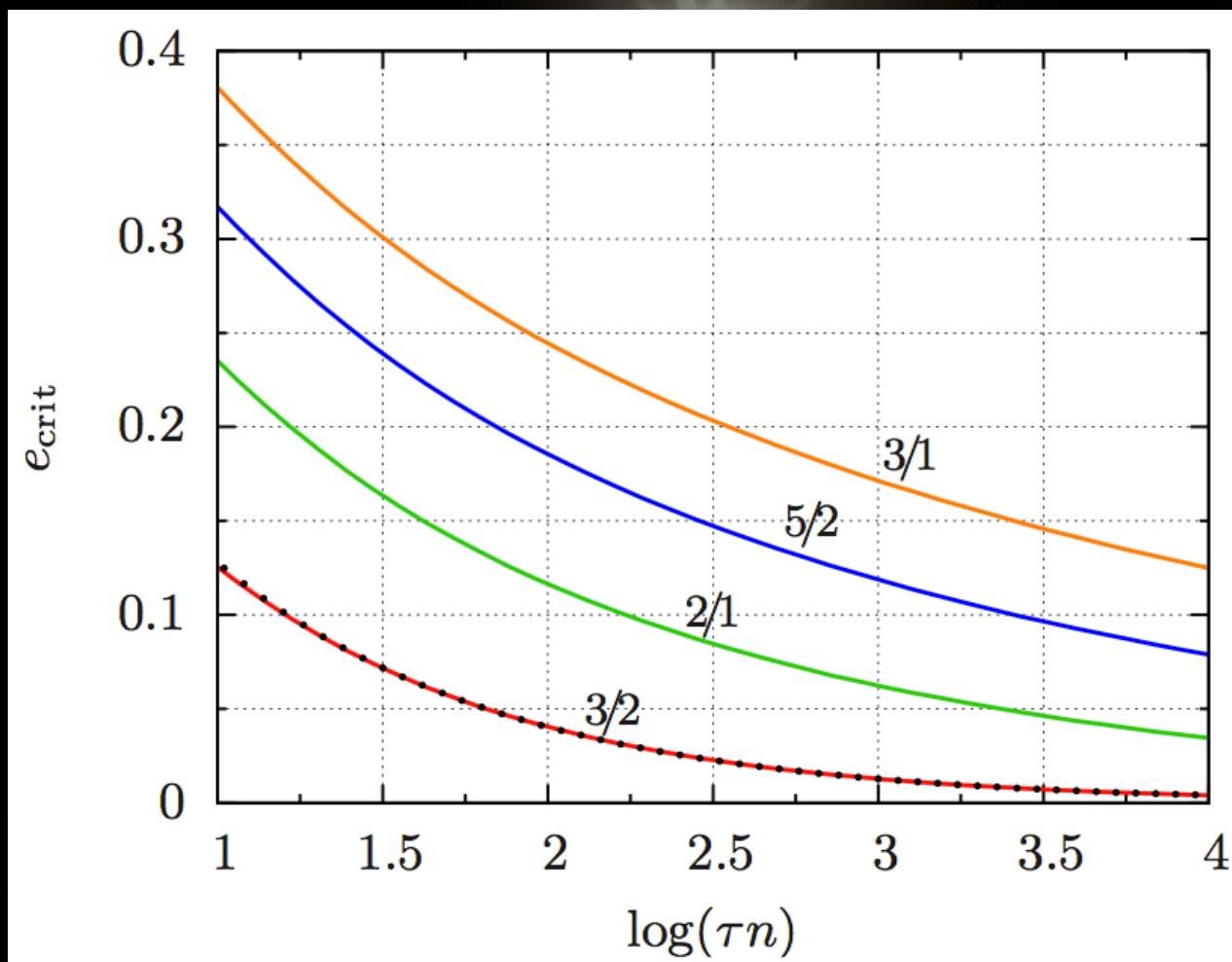
Equilibrium Rotations ($e=0.2$)



Equilibrium Rotations ($e=0.1$)



critical eccentricities

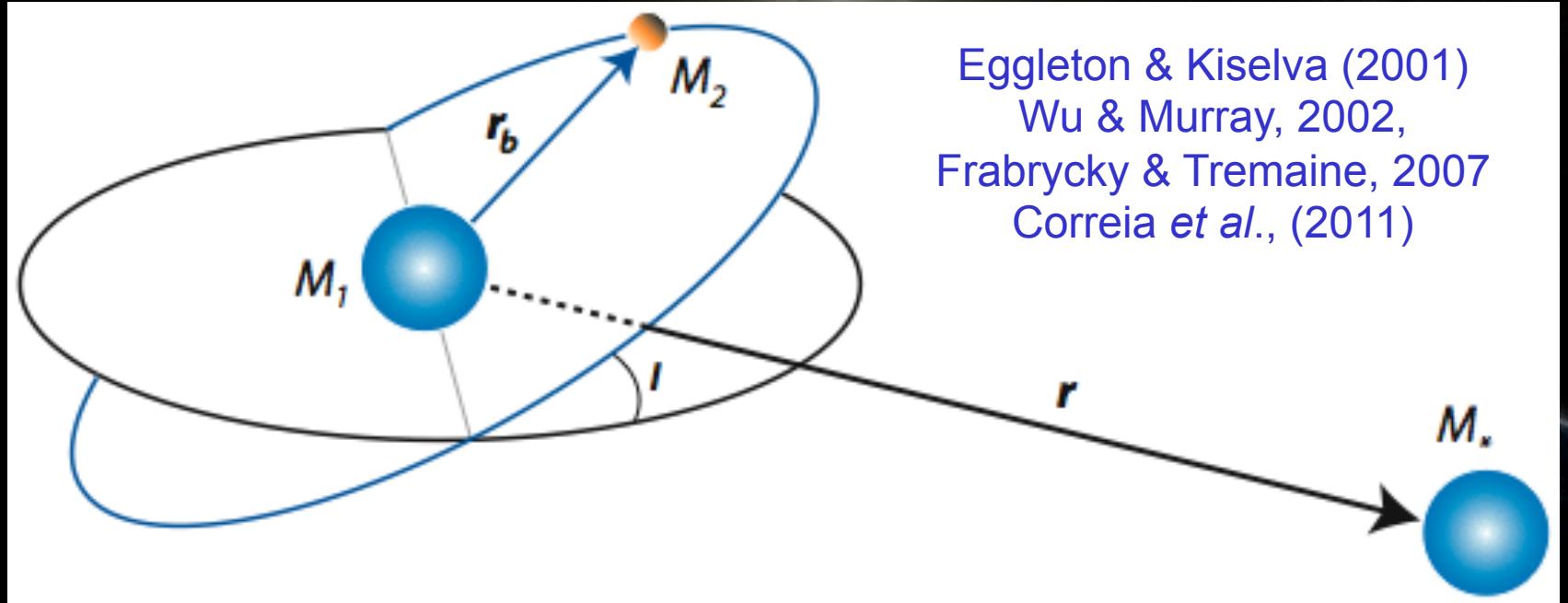


Correia et al.
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HD 80606

Naef et al, 2001

($a_p = 0.45$ AU, $e_p = 0.92$)

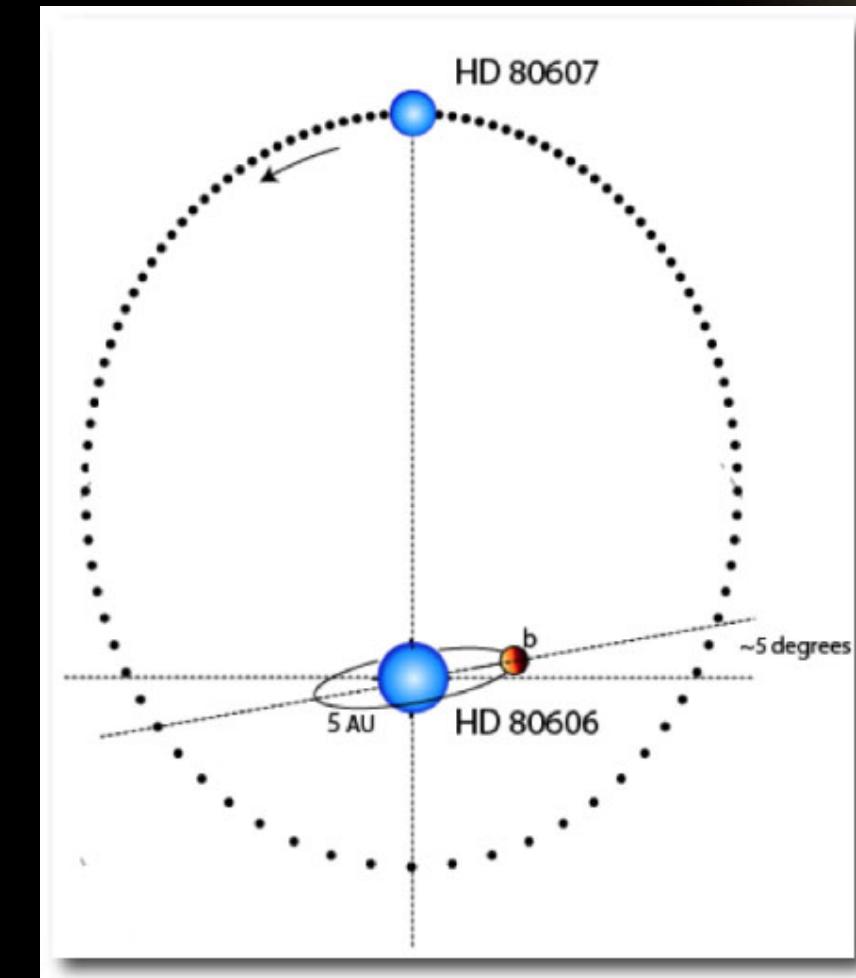


Kozai effect:
($I > 39^\circ$)

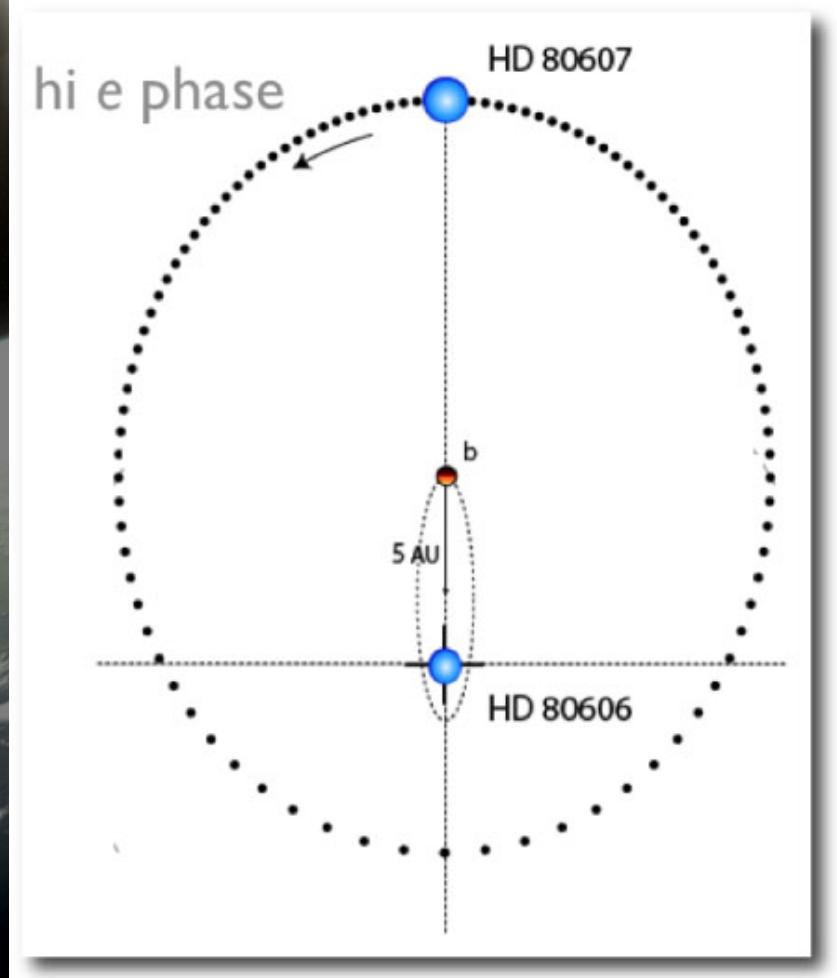
$$\sqrt{1 - e_1^2} \cos I = h_1 = Cte$$

HD 80606

$i \sim 85^\circ, e \sim 0$



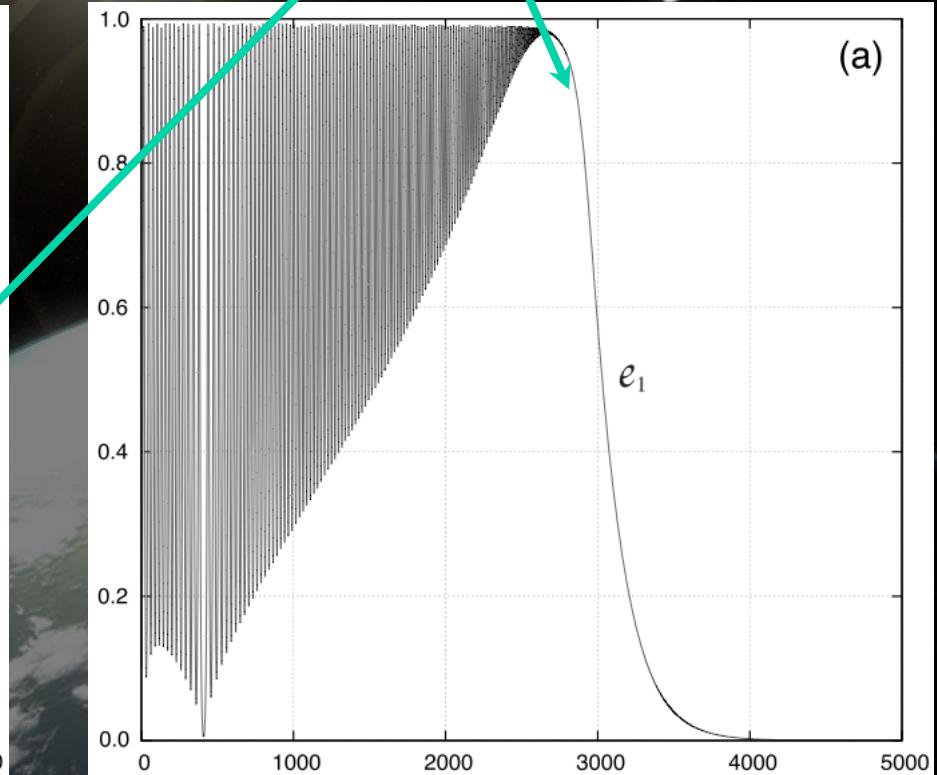
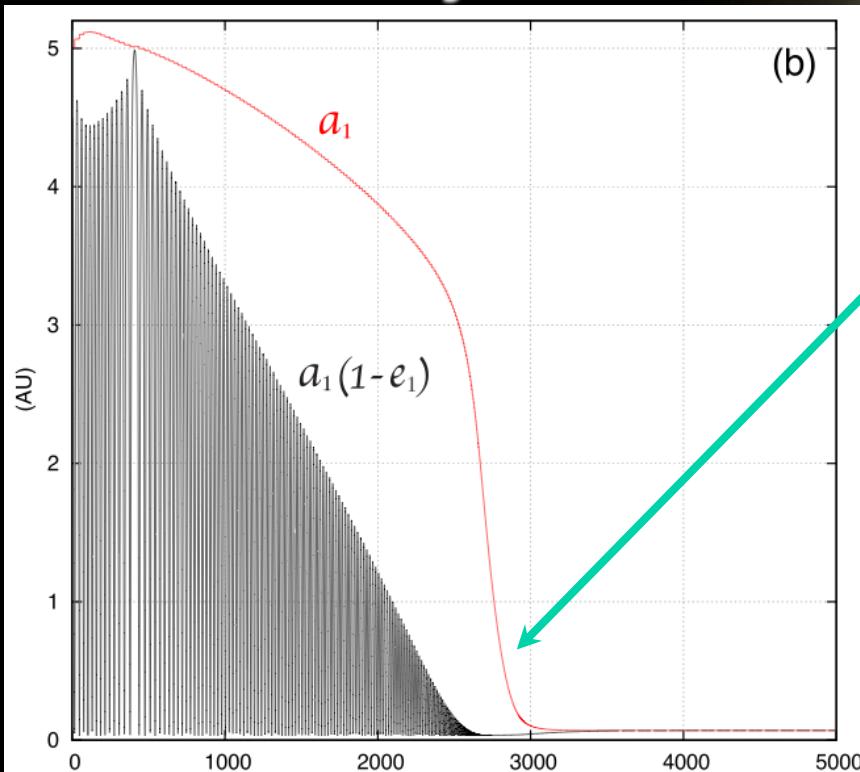
$i \sim 45^\circ, e \sim 0.99$



HD 80606

$(a_p = 0.45 \text{ AU}, e_p = 0.92)$

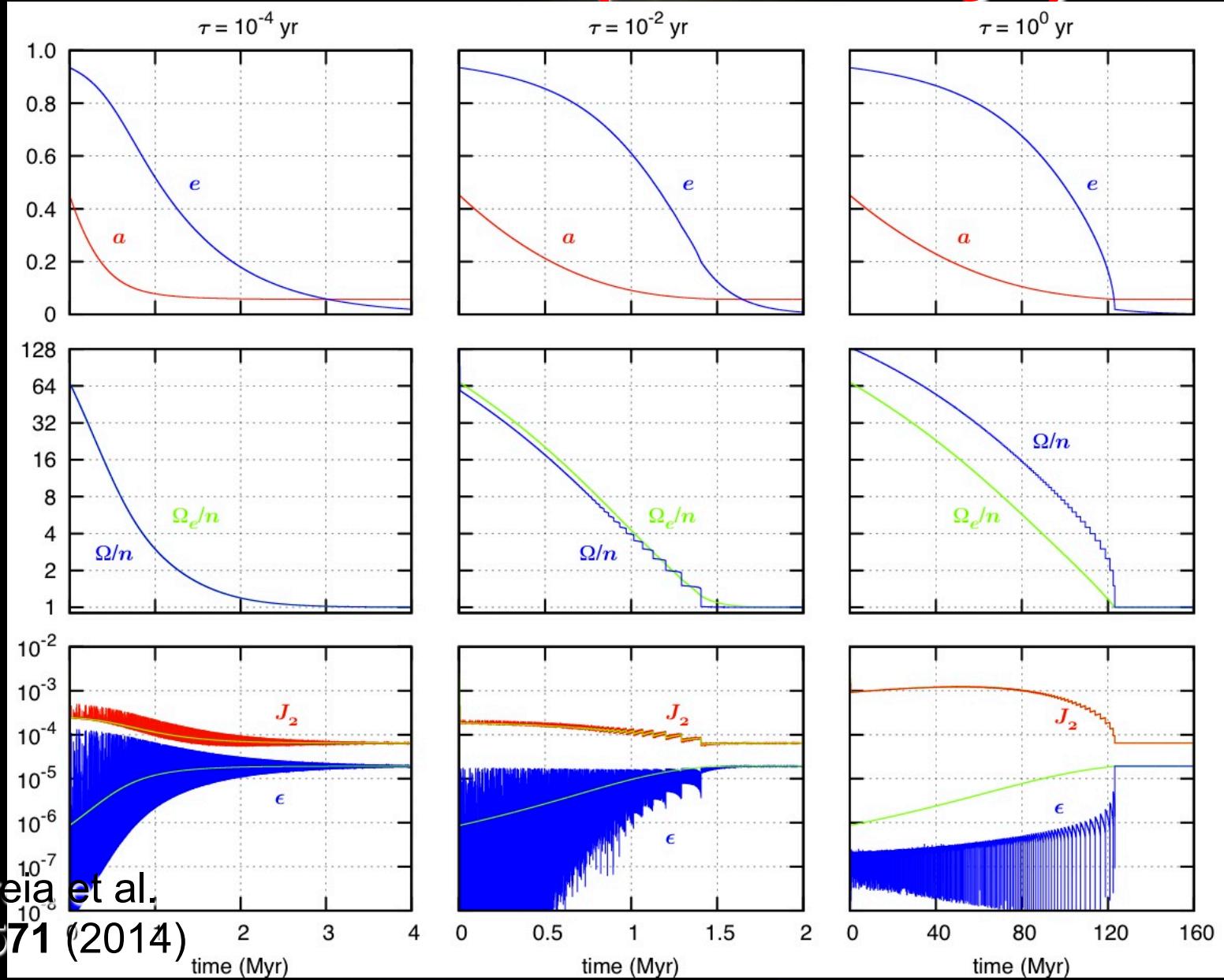
semi-major axis



Wu & Murray (2002)
Fabrycky & Tremaine (2007)
Correia et al. (2011)

time (Myr)

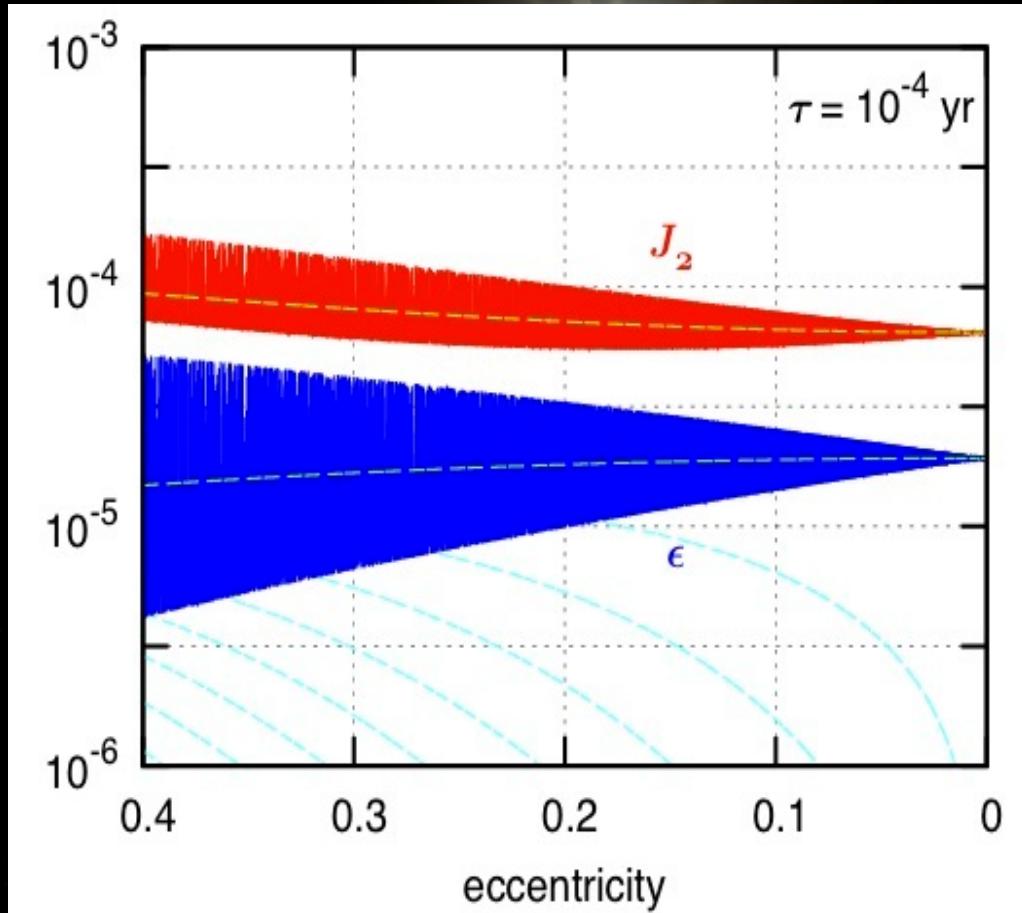
HD 80606 ($n \sim 10^{-2}$ yr)



Correia et al.

A&A 571 (2014)

HD 80606 - deformation ($\tau \ll 1/n$)

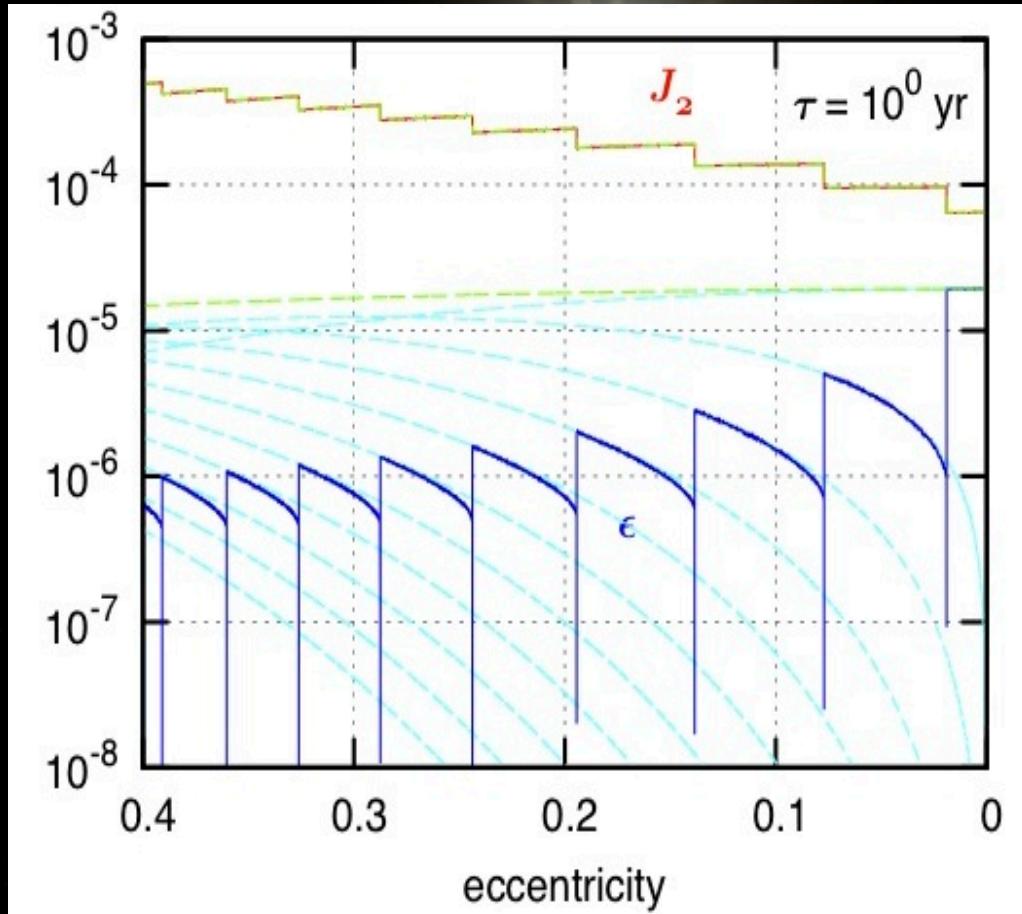


Correia et al.
A&A 571 (2014)

$$\langle \epsilon \rangle_M = \frac{k_f}{4} \frac{m_0}{m} \left(\frac{R}{a} \right)^3 (1 - e^2)^{-3/2}$$

$$\langle \epsilon_p \rangle_M = \beta_{2p} = \frac{k_f}{4} \frac{m_0}{m} \left(\frac{R}{a} \right)^3 X_{2p}^{-3,2}(e)$$

HD 80606 - deformation ($\tau \gg 1/n$)

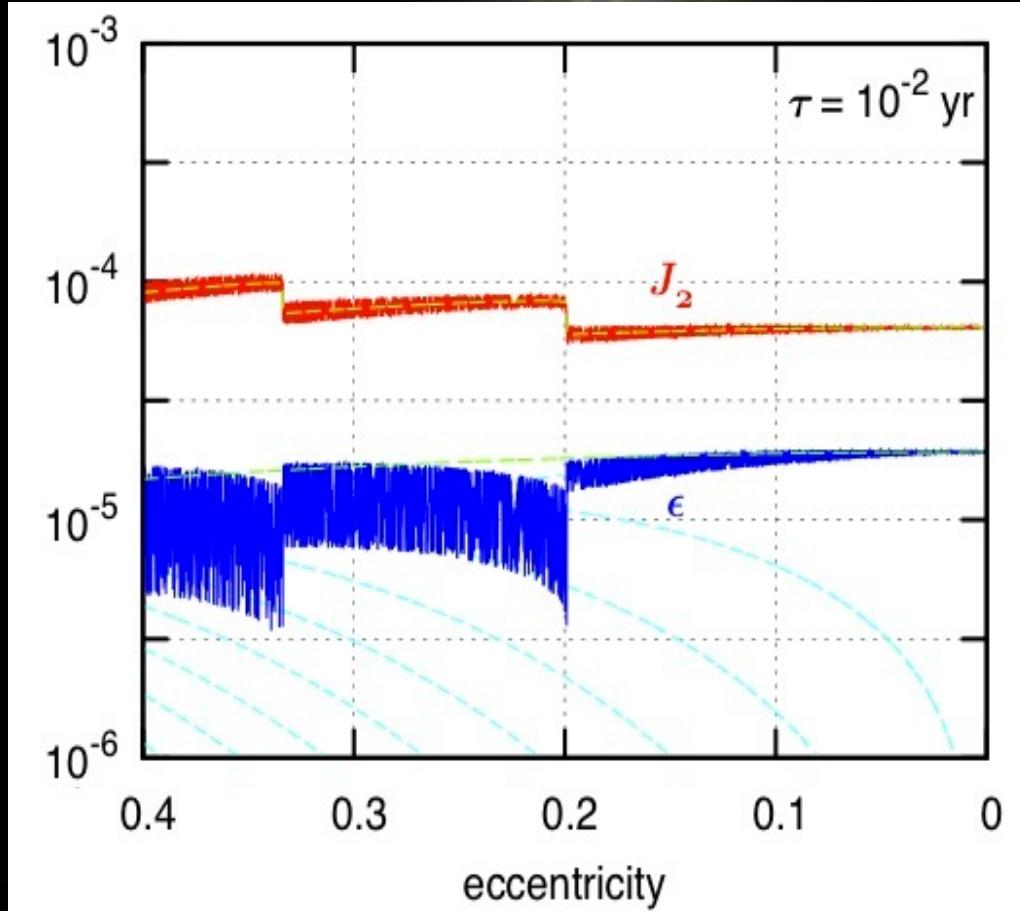


Correia et al.
A&A 571 (2014)

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HD 80606 - deformation ($\tau \sim 1/n$)



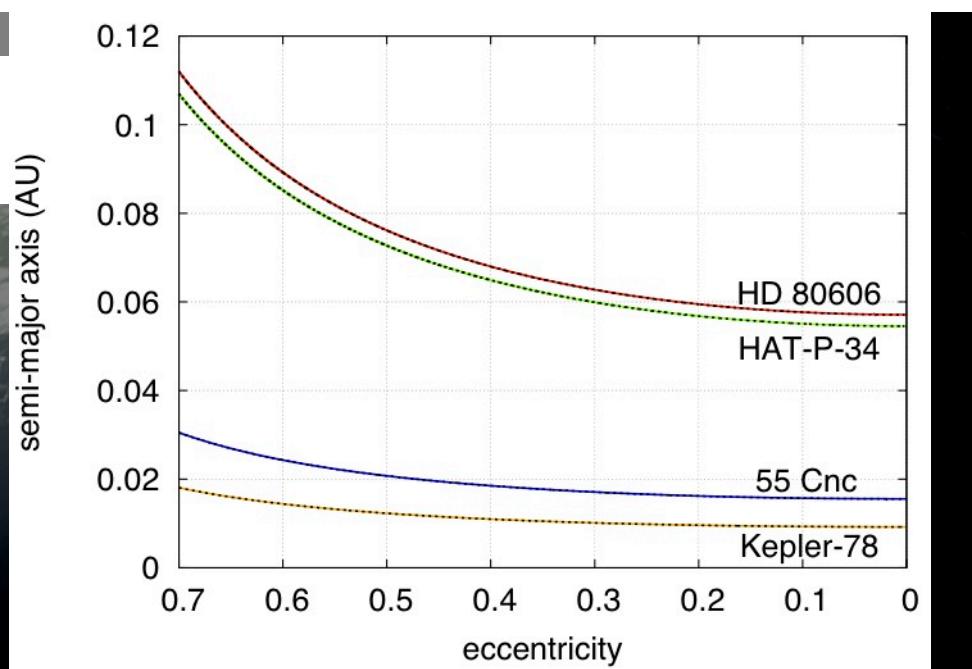
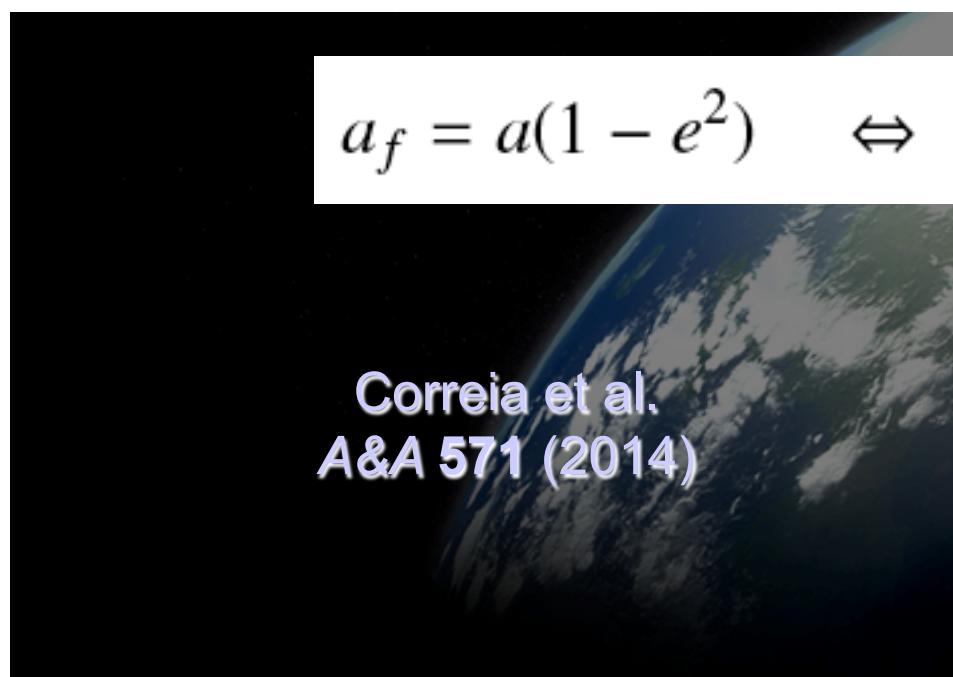
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A&A 571 (2014)

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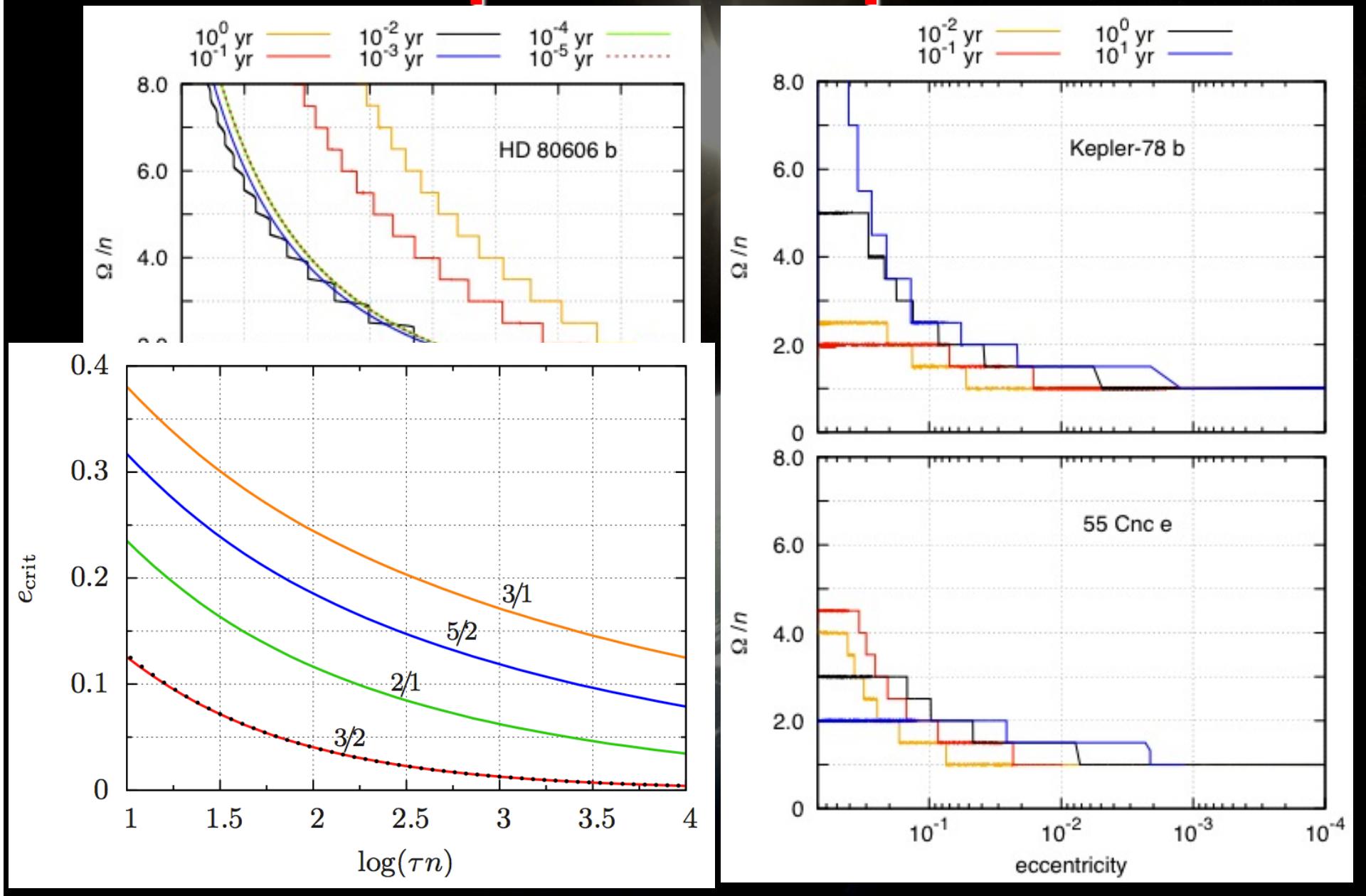
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Application to close-in planets

Param.	HD 80606 b	HAT-P-34 b	55 Cnc e	Kepler-78 b
$m_0 [M_\odot]$	1.01	1.392	0.905	0.83
$m [M_\oplus]$	1297.	1058.	8.63	1.69
$R [R_\oplus]$	10.75	13.12	2.21	1.20
$P [\text{day}]$	111.437	5.4527	0.73654	0.35501
$a [\text{au}]$	0.455	0.0677	0.0156	0.00922
e	0.9330	0.441	0.057	0 (fixed)



Hot-Jupiters / super-Earths



Conclusions

Correia, Boué, Laskar, Rodríguez
Astron. & Astrophys. **571**, A50, (2014)

- We replaced a Fourier series by a rheological law of deformation of the potential. This allow us to simultaneously take into account deformation and dissipation.
- We no longer need to truncate the series for high eccentricities and we avoid a large number of terms. The deformation is also valid for all tidal regimes.
- Spin-orbit resonances arise naturally when the deformation time-scale is longer than the orbital period. The stability increases with the deformation time-scale.
- For rocky planets, the spin-orbit resonances are possible for very low values of the eccentricity, so we expect that some of these planets are not synchronous with the star.