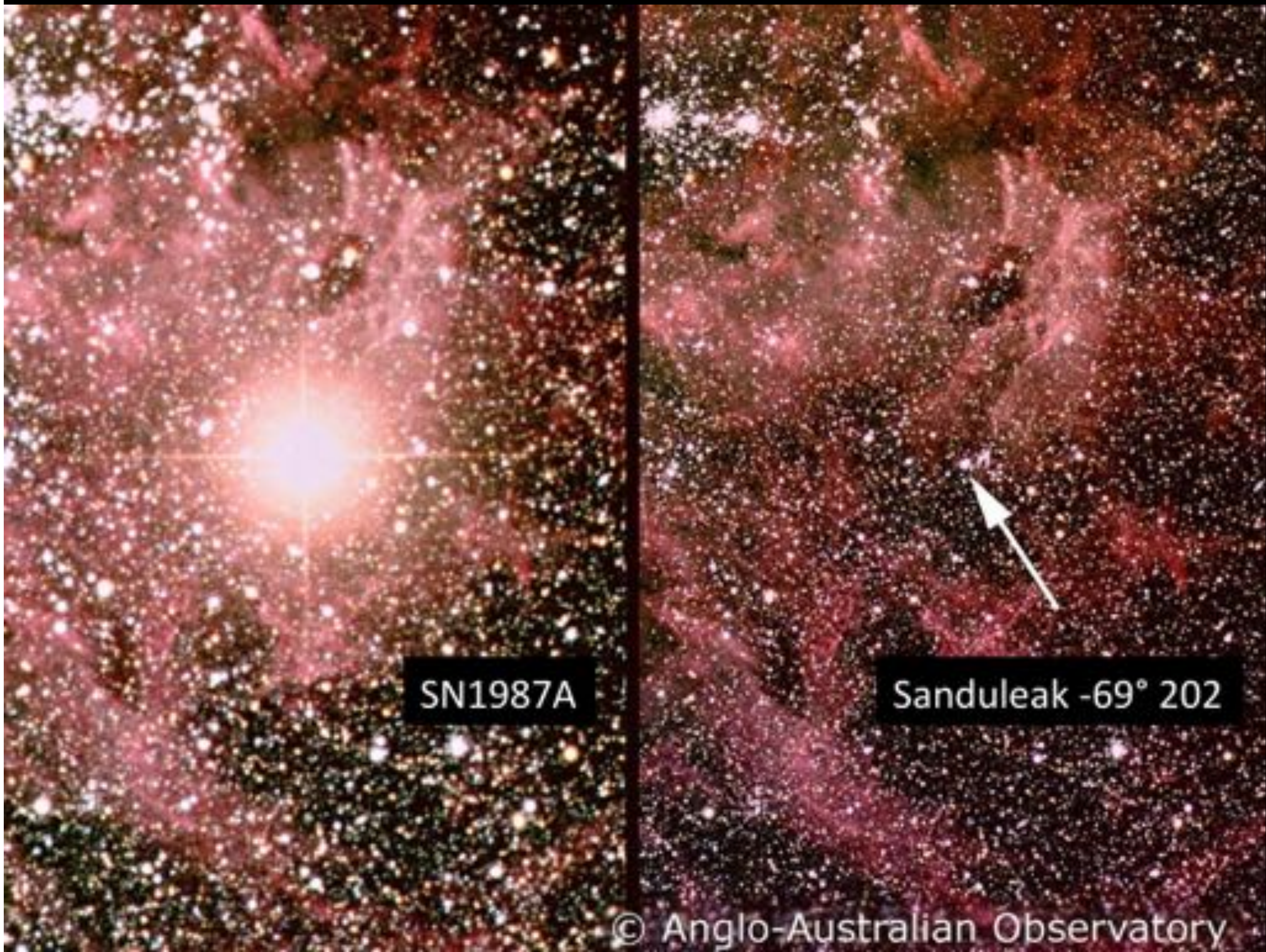
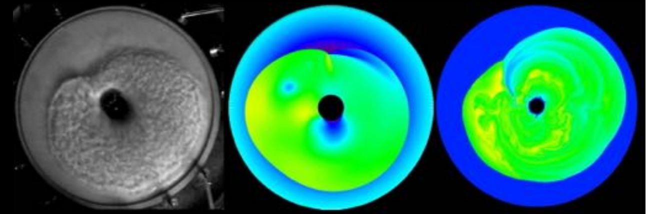




5 lectures on

The Physics of Core-Collapse Supernovae

Outline of lecture 2

Introduction to supernovae: following our common sense

- can a collapse bounce into an explosion ?
- the basics of shock waves

The framework of delayed neutrino driven explosions

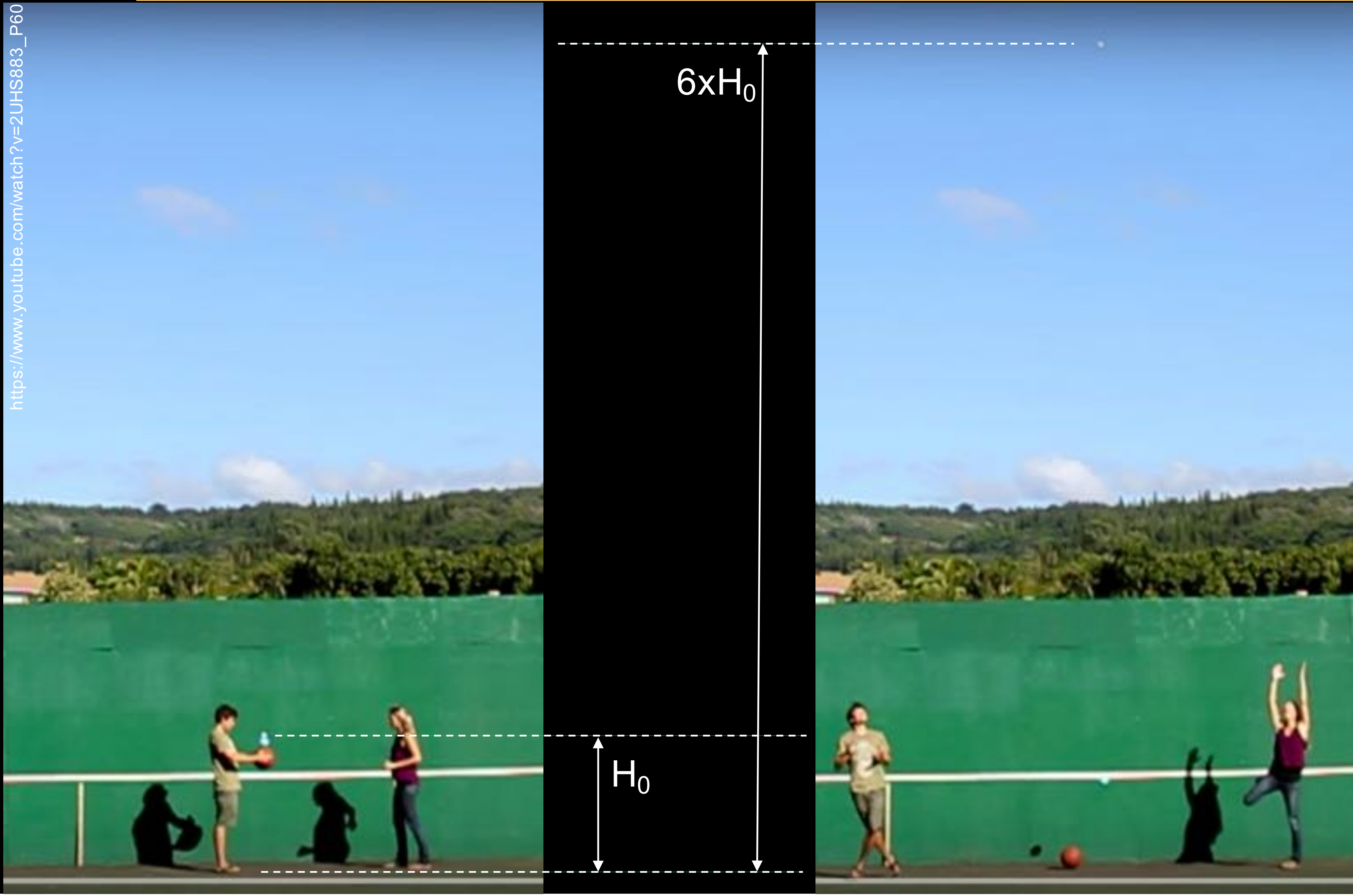
- the 5 zones of the model by Betthe & Wilson
- the spherical explosion of $10M_{sol}$
- the puzzle of more massive progenitors

Some observational clues and puzzles

- the hints for asymmetric explosions
- constraints from the progenitor, the ejecta and the neutron star

How can a collapse turn into an explosion?

Physics girl/
stacked ball drop

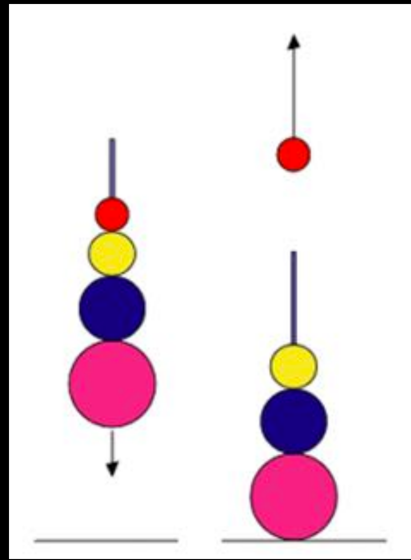


https://www.youtube.com/watch?v=2UHS883_P60

How can a collapse turn into an explosion?

stacked bouncing balls

Consider two elastic balls of mass $m_1 > m_2$
dropped in free fall from a height H_0
reaching a hard surface



AstroBlaster
invented by
Stirling Colgate

From energy conservation $\frac{v^2}{2} + gH = gH_0$

their velocity just before bounce (H=0) is $-v_0$ with $v_0 \equiv (2GH_0)^{\frac{1}{2}}$

The velocity of the first ball is symmetric upon an elastic bounce is $v_1 = +v_0$
The second ball collides the first one with a velocity $v_2 = -v_0$

Momentum conservation $m_2 v_2' + m_1 v_1' = (m_1 - m_2) v_0$
Energy conservation $m_2 v_2'^2 + m_1 v_1'^2 = (m_1 + m_2) v_0^2$

so
$m_2 (v_2' + v_0) = -m_1 (v_1' - v_0)$
$v_2' = v_1' + 2v_0$

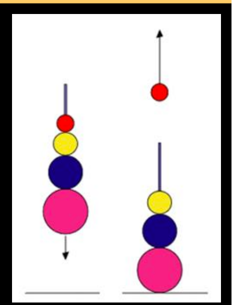
$$\frac{v_2'}{v_0} = \frac{3 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} \qquad \frac{v_1'}{v_0} = \frac{1 - 3\frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} \qquad \frac{H_2'}{H_0} = 9\left(\frac{1 - \frac{m_2}{3m_1}}{1 + \frac{m_2}{m_1}}\right)^2$$

Note that

- $v_1' = 0$ if $m_1 = 3m_2$. In this case, $v_2' = 2v_0$ & $H_2 = 4H_0$
- if $m_1 = m_2$, then $v_2' = -v_1' = v_0$: the lower ball "bounces a second time" and follows the first one

How can a collapse turn into an explosion?

stacked bouncing balls



If the balls are partially elastic

v_a, v_b : pre-bounce, v_a', v_b' post-bounce

Momentum conservation $\quad m_a v_a' + m_b v_b' = m_a v_a + m_b v_b$

anelastic collision $\quad v_b' - v_a' = \varepsilon(v_a - v_b)$

$\varepsilon=1$: elastic bounce

$\varepsilon=0$: balls are stuck together ($v_b'=v_a'$)

solving this system

$(m_a+m_b)v_a'=(m_a-\varepsilon m_b)v_a+(1+\varepsilon)m_b v_b$

$(m_a+m_b)v_b'=(1+\varepsilon)m_a v_a+(m_b-\varepsilon m_a)v_b$

$$v_{\mathrm{a}}' = \frac{m_{\mathrm{a}} - \epsilon m_{\mathrm{b}}}{m_{\mathrm{a}} + m_{\mathrm{b}}} v_{\mathrm{a}} + \frac{(1+\epsilon) m_{\mathrm{b}}}{m_{\mathrm{a}} + m_{\mathrm{b}}} v_{\mathrm{b}}$$

$$v_{\mathrm{b}}' = \frac{(1+\epsilon) m_{\mathrm{a}}}{m_{\mathrm{a}} + m_{\mathrm{b}}} v_{\mathrm{a}} + \frac{m_{\mathrm{b}} - \epsilon m_{\mathrm{a}}}{m_{\mathrm{a}} + m_{\mathrm{b}}} v_{\mathrm{b}}$$

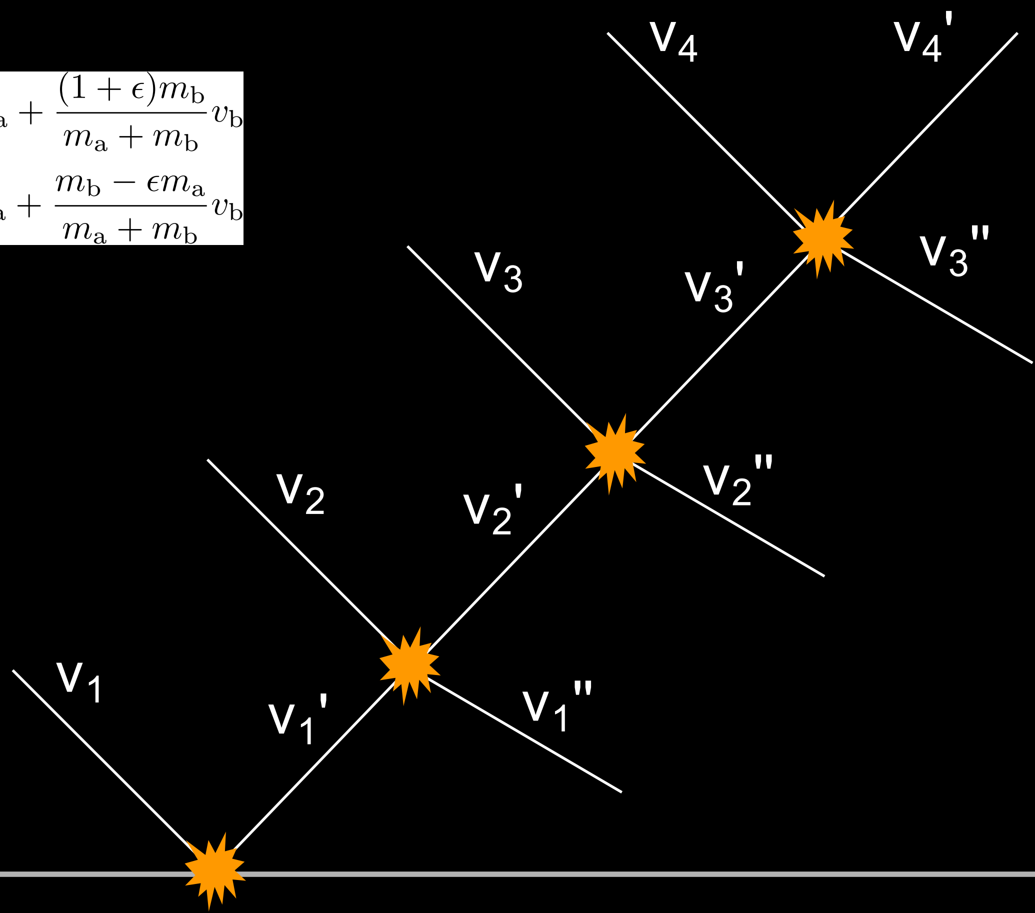
Energy loss

$m_a v_a'^2 + m_b v_b'^2 - (m_a v_a^2 + m_b v_b^2) = 2\Delta E$

$2(m_a+m_b)^2 \Delta E = m_a\,((m_a-\varepsilon m_b)v_a+(1+\varepsilon)m_b v_b)^2$

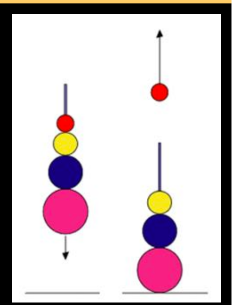
$+m_b((1+\varepsilon)m_a v_a+(m_b-\varepsilon m_a)v_b)^2-(m_a+m_b)^2\,(m_a v_a^2+m_b v_b^2)$

$$\Delta E = -\frac{1-\epsilon^2}{2} \frac{m_{\mathrm{a}} m_{\mathrm{b}}}{m_{\mathrm{a}} + m_{\mathrm{b}}} (v_{\mathrm{a}} - v_{\mathrm{b}})^2$$



How can a collapse turn into an explosion?

stacked bouncing balls



Ball 1: mirror symmetric

$v_a=-v_0,\ v_b=v_0$
$m_b=m_a=m_1$

$$v'_1 = \epsilon v_0$$

Ball 2

$v_a=-v_0,\ v_b=\varepsilon v_0$
$m_a=m_2,\ m_b=m_1$

$$v'_2 = [(2+\epsilon)m_1\epsilon - m_2]\,\frac{v_0}{m_1+m_2}$$

Ball 3

$v_a=-v_0,\ v_b=v_2'$
$m_a=m_3,\ m_b=m_2$

$$v'_3 = \left\{m_2\left[(\epsilon^2+3\epsilon+3)\epsilon m_1 - m_2 - m_3\right] - m_1 m_3\right\}\frac{v_0}{(m_3+m_2)(m_1+m_2)}$$

Ball 4

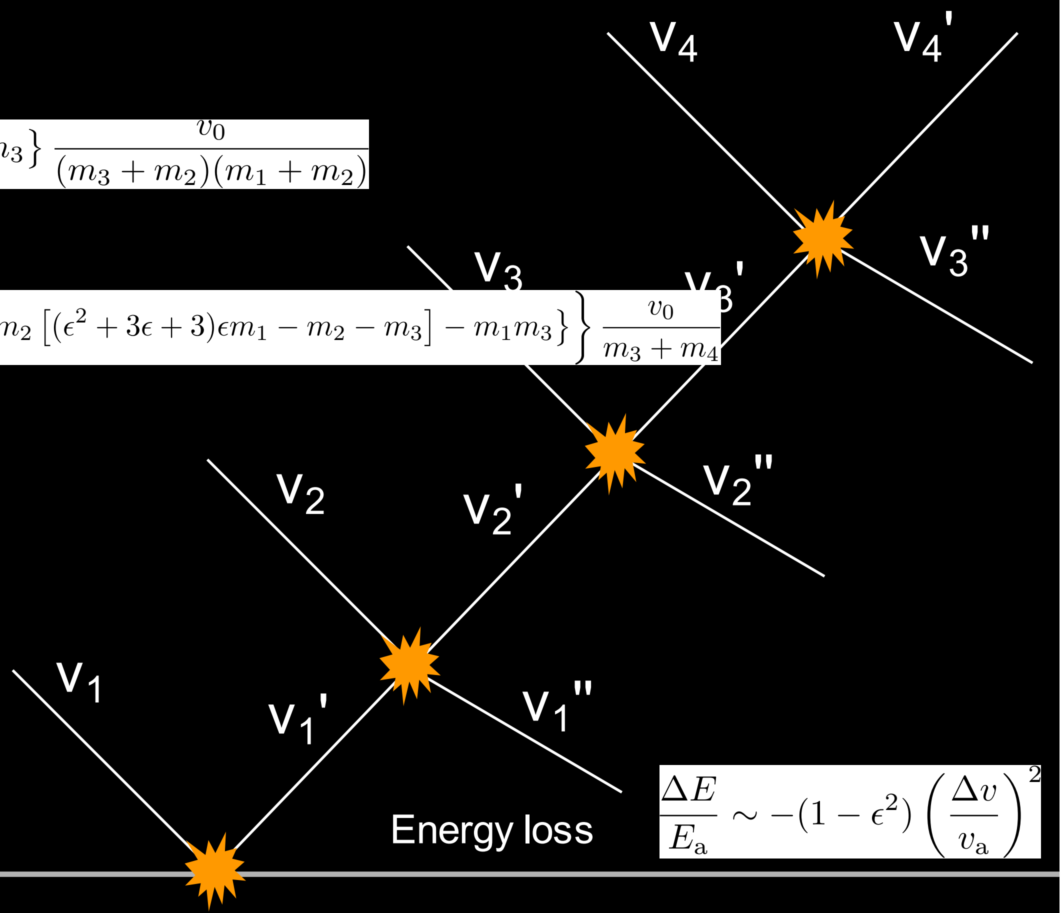
$v_a=-v_0,\ v_b=v_3'$
$m_a=m_4,\ m_b=m_3$

$$v'_4 = \left\{-m_4+\epsilon m_3+(1+\epsilon)\frac{m_3}{(m_3+m_2)(m_1+m_2)}\left\{m_2\left[(\epsilon^2+3\epsilon+3)\epsilon m_1 - m_2 - m_3\right] - m_1 m_3\right\}\right\}\frac{v_0}{m_3+m_4}$$

If $m_4 \ll m_3 \ll m_2 \ll m_1$

$$\begin{aligned}
\frac{v'_1}{v_0} &= \epsilon \le 1,\\
\frac{v'_2}{v_0} &= (\epsilon+2)\epsilon \le 3,\\
\frac{v'_3}{v_0} &= (\epsilon^2+3\epsilon+3)\epsilon \le 7,\\
\frac{v'_4}{v_0} &= (\epsilon^3+4\epsilon^2+6\epsilon+4)\epsilon \le 15.
\end{aligned}$$

$$\begin{aligned}
\frac{H'_1}{H_0} &= \epsilon^2 \le 1,\\
\frac{H'_2}{H_0} &= (\epsilon+2)^2\epsilon^2 \le 9,\\
\frac{H'_3}{H_0} &= (\epsilon^2+3\epsilon+3)^2\epsilon^2 \le 49,\\
\frac{H'_4}{H_0} &= (\epsilon^3+4\epsilon^2+6\epsilon+4)^2\epsilon^2 \le 225.
\end{aligned}$$



Energy loss

$$\frac{\Delta E}{E_a} \sim -(1-\epsilon^2)\left(\frac{\Delta v}{v_a}\right)^2$$

The bouncing height of the n-th ball exceeds $H_0 \times (n\varepsilon)^2$

The long way from bouncing balls to supernova physics

Idealized radial collapse: tranverse motions are difficult to avoid in the experiment

The formalism of point like balls ignores the adiabatic storage of energy into elasticity
The time delay of elastic contraction would translate into a delayed bounce

$$\frac{v^2}{2} + \frac{E_{\rm K}(H)}{m} + gH = gH_0$$

In the collapsing stellar envelope the energy density is written as the Bernoulli parameter, which would be conserved along stationary flow lines if the flow were adiabatic

$$\frac{v^2}{2} + \frac{c^2}{\gamma - 1} - \frac{GM}{r} = \frac{c_0^2}{\gamma - 1} - \frac{GM}{r_0}$$

Among the many differences between stacked balls and the bounce of the stellar core:

- -a shock forms
- -energy is lost in the dissociation of iron
- -energy is lost in the escape of neutrinos (Baade & Zwicky 34)
- -a fraction of the neutrino energy is re-absorbed (Cogate & White 66)
- -the emission of neutrino is delayed (Bethe & Wilson 85)
- -instabilities introduce transverse motions (Herand+92, Blondin+06)

Equations of fluid mechanics for a perfect gas

adiabatic sound speed

$$c^2 \equiv \frac{\gamma P}{\rho},$$

dimensionless entropy

$$S \equiv \frac{1}{\gamma - 1} \log\left(\frac{P}{\rho^\gamma}\frac{\rho_0^\gamma}{P_0}\right).$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0,$$

mass conservation

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v + \frac{\nabla P}{\rho} + \nabla \Phi = 0,$$

Euler equation

$$\frac{\partial S}{\partial t} + v \cdot \nabla S = \mathcal{L}.$$

entropy equation

using

$$(v \cdot \nabla)v = (\nabla \times v) \times v + \nabla\left(\frac{v^2}{2}\right),$$

$$\frac{\nabla P}{\rho} = \nabla\left(\frac{c^2}{\gamma - 1}\right) - \frac{c^2}{\gamma}\nabla S.$$

The Euler equation can be rewritten as follows:

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla\left(\underbrace{\frac{v^2}{2} + \frac{c^2}{\gamma - 1} + \Phi}_{\text{Bernoulli "constant"}}\right) = \frac{c^2}{\gamma}\nabla S$$

Subsonic to supersonic transition in a gas

Idealized stationary spherical collapse of an adiabatic ideal gas: Bondi accretion (1952)

$$\frac{v^2}{2}+\frac{c^2}{\gamma-1}-\frac{GM}{r}=B,$$
$$\rho v r^2=\frac{\dot{M}}{4\pi}$$

$$S\equiv\log\left[\left(\frac{c}{c_0}\right)^{\frac{2}{\gamma-1}}\frac{\rho_0}{\rho}\right]$$

The system of equations satisfied by v, c is

$$\frac{v^2}{2}+\frac{c^2}{\gamma-1}-\frac{GM}{r}=\frac{\gamma+1}{2(\gamma-1)}c_{\rm son}^2-\frac{GM}{r_{\rm son}}$$
$$c^{\frac{2}{\gamma-1}}vr^2=-c_{\rm son}^{\frac{\gamma+1}{\gamma-1}}r_{\rm son}^2$$

Differentiating this system with respect to r gives the regularity conditions at the sonic point

$$v^2\frac{\dot{v}}{v}+\frac{2c^2}{\gamma-1}\frac{\dot{c}}{c}=-\frac{GM}{r^2}$$
$$\frac{\dot{v}}{v}+\frac{2}{\gamma-1}\frac{\dot{c}}{c}=-\frac{2}{r}$$

$$(c^2-v^2)\frac{\dot{v}}{v}=\frac{GM}{r^2}-\frac{2c^2}{r}$$
$$\frac{2}{\gamma-1}(c^2-v^2)\frac{\dot{c}}{c}=\frac{2v^2}{r}-\frac{GM}{r^2}$$

$$c_{\rm son}^2=\frac{GM}{2r_{\rm son}}$$

The Bernoulli constant relates the sound speeds at the sonic point and at "infinity", and defines the sonic radius:

$$c_{\rm son}=\left(\frac{2}{5-3\gamma}\right)^{\frac{1}{2}}c_\infty$$
$$r_{\rm son}=\frac{5-3\gamma}{4}\frac{GM}{c_\infty^2}$$

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The system of equations satisfied by v, c is

$$\frac{v^2}{2} + \frac{c^2}{\gamma - 1} - \frac{GM}{r} = \frac{\gamma+1}{2(\gamma-1)} c_{\rm son}^2 - \frac{GM}{r_{\rm son}}$$
$$c^{\frac{2}{\gamma-1}} v r^2 = -c_{\rm son}^{\frac{\gamma+1}{\gamma-1}} r_{\rm son}^2$$

Differentiating this system with respect to r gives the regularity conditions at the sonic point

$$v^2 \frac{\dot{v}}{v} + \frac{2c^2}{\gamma-1}\frac{\dot{c}}{c} = -\frac{GM}{r^2}$$
$$\frac{\dot{v}}{v} + \frac{2}{\gamma-1}\frac{\dot{c}}{c} = -\frac{2}{r}$$

$$(c^2 - v^2)\frac{\dot{v}}{v} = \frac{GM}{r^2} - \frac{2c^2}{r}$$
$$\frac{2}{\gamma-1}(c^2 - v^2)\frac{\dot{c}}{c} = \frac{2v^2}{r} - \frac{GM}{r^2}$$

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Subsonic to supersonic transition in a gas

Idealized stationary cylindrical collapse of an adiabatic ideal gas

$$\frac{v^2}{2} + \frac{c^2}{\gamma - 1} - \frac{GM}{r} = B, \qquad \rho v r = \frac{\dot{M}}{4\pi}$$

$$S \equiv \log\left[\left(\frac{c}{c_0}\right)^{\frac{2}{\gamma-1}} \frac{\rho_0}{\rho}\right]$$

The system of equations satisfied by v, c is

$$\frac{v^2}{2} + \frac{c^2}{\gamma - 1} - \frac{GM}{r} = \frac{\gamma+1}{2(\gamma-1)} c_{\rm son}^2 - \frac{GM}{r_{\rm son}}$$

$$c^{\frac{2}{\gamma-1}} v r = -c_{\rm son}^{\frac{\gamma+1}{\gamma-1}} r_{\rm son}$$

Differentiating this system with respect to r gives the regularity conditions at the sonic point

$$v^2 \frac{\dot{v}}{v} + \frac{2c^2}{\gamma-1}\frac{\dot{c}}{c} = -\frac{GM}{r^2}, \qquad \frac{\dot{v}}{v} + \frac{2}{\gamma-1}\frac{\dot{c}}{c} = -\frac{1}{r}$$

$$(c^2 - v^2)\frac{\dot{v}}{v} = \frac{GM}{r^2} - \frac{c^2}{r}, \qquad \frac{2}{\gamma-1}(c^2 - v^2)\frac{\dot{c}}{c} = \frac{v^2}{r} - \frac{GM}{r^2}$$

$$c_{\rm son}^2 = \frac{GM}{r_{\rm son}}$$

The Bernoulli constant relates the sound speeds at the sonic point and at "infinity", and defines the sonic radius:

$$c_{\rm son} = \left(\frac{2}{3-\gamma}\right)^{\frac{1}{2}} c_\infty$$

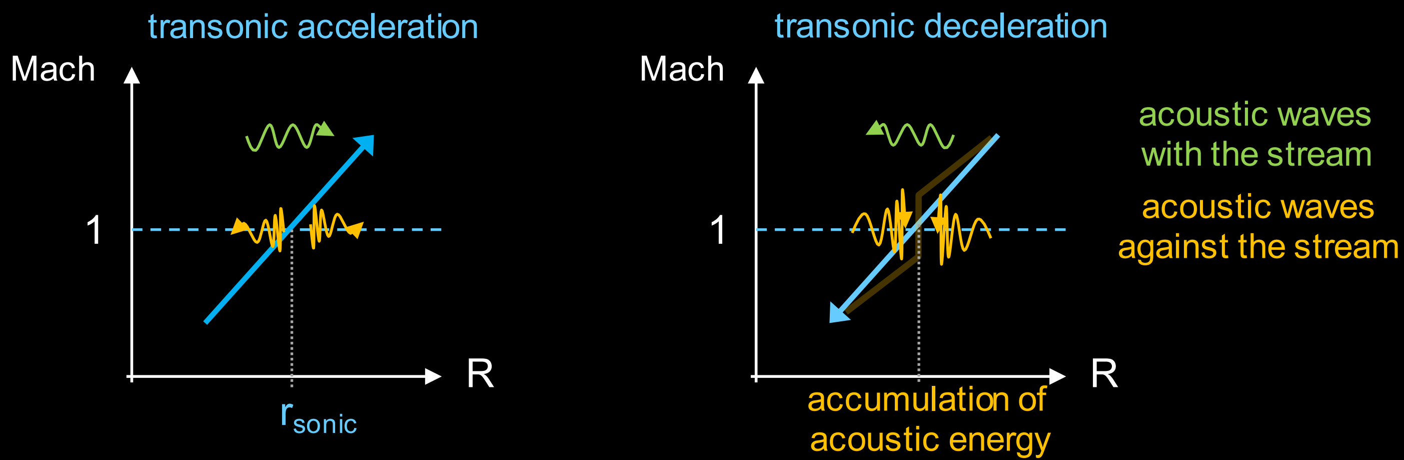
$$r_{\rm son} = \frac{3-\gamma}{2}\frac{GM}{c_\infty^2}$$

Transonic transition in a gas

As the free falling collapsing iron gas approaches the center, the flow has to decelerate from supersonic to subsonic velocities.

A flow can accelerate continuously from subsonic to supersonic: the nozzle of a rocket, Bondi accretion onto a black hole or stellar winds are examples of transonic acceleration.

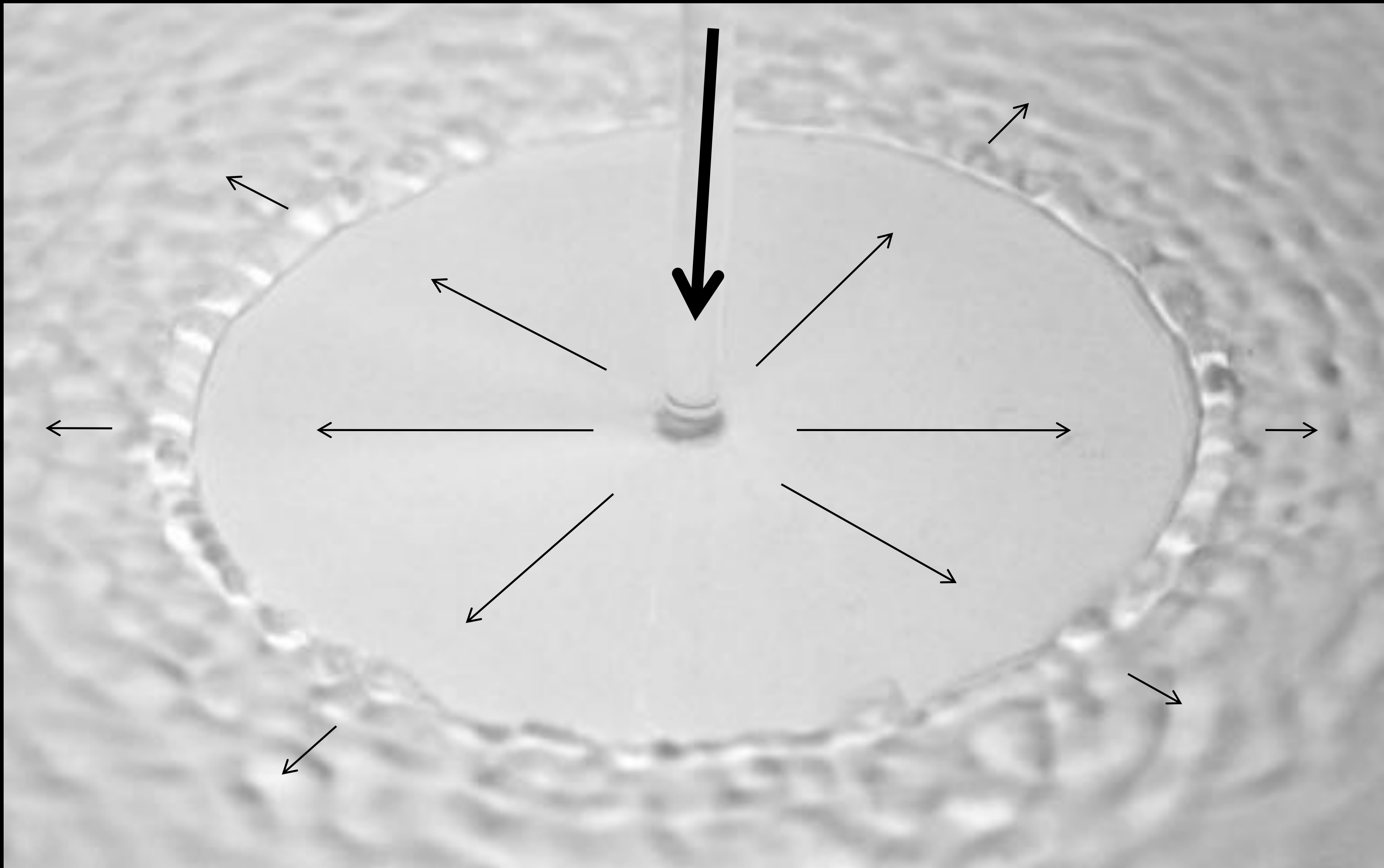
By contrast, the reverse solution ($v \rightarrow -v$) of a transonic deceleration cannot exist without forming a shock because of the accumulation of acoustic energy at the sonic point



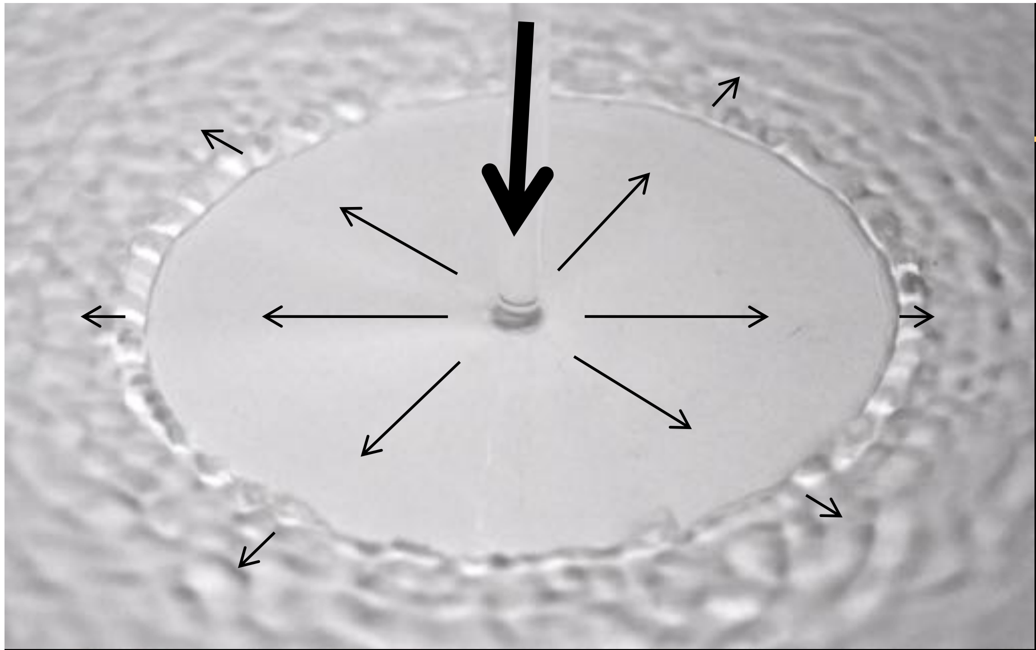
A shock is an abrupt conversion of kinetic energy into enthalpy.
In a gas, the large scale kinetic energy is converted into small scale kinetic energy (heat)

Hydraulic jumps in shallow water are analogous to shocks.
The large scale kinetic energy is converted into potential energy

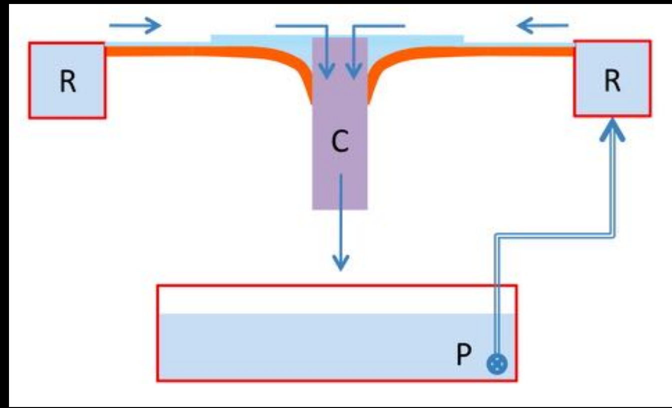
Hydraulic jumps and shock waves

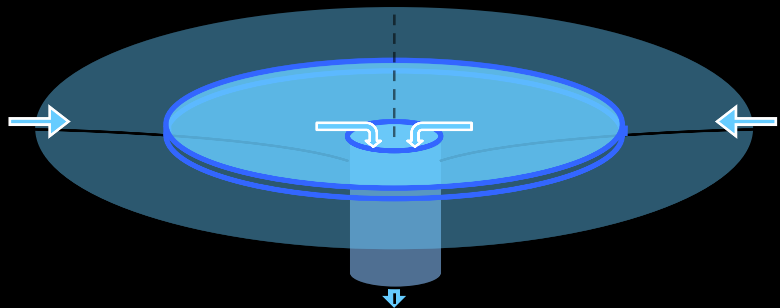


Like the deceleration shock of stellar winds, the circular hydraulic jump marks the transition between a fast and shallow inner flow and a slower deeper outer flow



Analogy between hydraulic jumps and shock





acoustic waves
shock wave
pressure

↔

surface waves
hydraulic jump
depth

Derivation of the Rankine Hugoniot jump conditions

The jump conditions across a shock are deduced from the conservation of

- the mass flux: ρ v
- the momentum flux: P + ρ v^2
- the energy density flux: ρ v B

The jump conditions depend on the strength of the shock, measured by the incident Mach number $M_1=v_1/c_1$

$$\boxed{\begin{aligned} \rho_1 v_1 &= \rho_2 v_2, \\ P_1 + \rho_1 v_1^2 &= P_2 + \rho_2 v_2^2, \\ \frac{v_1^2}{2} + \frac{c_1^2}{\gamma-1} + \Phi_1 &= \frac{v_2^2}{2} + \frac{c_2^2}{\gamma-1} + \Phi_2. \end{aligned}}$$

eliminating $P=\rho c^2/\gamma$

$$\begin{aligned} \rho_1 v_1 &= \rho_2 v_2, \\ \rho_1 c_1^2 \left(1+\gamma M_1^2\right) &= \rho_2 c_2^2 \left(1+\gamma M_2^2\right), \\ c_1^2 \left[2+(\gamma-1)M_1^2\right] &= c_2^2 \left[2+(\gamma-1)M_2^2\right]. \end{aligned}$$

$$\begin{aligned} \rho_1 v_1 &= \rho_2 v_2, \\ v_1 \left(\frac{1}{M_1^2}+\gamma\right) &= v_2 \left(\frac{1}{M_2^2}+\gamma\right), \\ v_1^2 \left(\frac{2}{M_1^2}+\gamma-1\right) &= v_2^2 \left(\frac{2}{M_2^2}+\gamma-1\right). \end{aligned}$$

a polynomial of order 2 in M_1^2 and M_2^2 is obtained by eliminating v_1/v_2 and $c_2/c_1=(v_2/v_1)(M_1/M_2)$

$$\left[2+(\gamma-1)M_1^2\right] M_1^2 \left(1+\gamma M_2^2\right)^2 = \left[2+(\gamma-1)M_2^2\right] M_2^2 \left(1+\gamma M_1^2\right)^2$$

the trivial solution $M_1^2=M_2^2$ can be factorized $\left(M_2^2-M_1^2\right)\left[-M_2^2(2\gamma M_1^2-\gamma+1)+2+(\gamma-1)M_1^2\right]=0$

2 important equations
$$\boxed{M_2^2 = \frac{2+(\gamma-1)M_1^2}{2\gamma M_1^2-\gamma+1}} \qquad \boxed{\frac{v_2}{v_1} = \frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}}$$

the other jump conditions are easily deduced:

$$\begin{aligned} \frac{\rho_2}{\rho_1} &= \left(\frac{v_2}{v_1}\right)^{-1}, \\ \frac{c_2^2}{c_1^2} &= \left(\frac{v_2}{v_1}\right)^2 \frac{M_1^2}{M_2^2} = \frac{(2+(\gamma-1)M_1^2)(2\gamma M_1^2-\gamma+1)}{(\gamma+1)^2 M_1^2}, \\ S_2 - S_1 &= \frac{1}{\gamma-1}\left[\log\left(\frac{M_1^2}{M_2^2}\right)+(\gamma+1)\log\left(\frac{v_2}{v_1}\right)\right]. \end{aligned}$$

Derivation of the Rankine Hugoniot jump conditions

-Entropy is produced across a shock: non reversible process

$$S_2 - S_1 \equiv \frac{1}{\gamma - 1} \log \left[\frac{P_2}{P_1} \left(\frac{\rho_1}{\rho_2} \right)^\gamma \right] \sim \frac{2}{\gamma - 1} \log M_1$$

temporal variations of shock strength produce entropy gradients, which are a source of convective instability

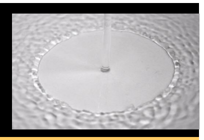
-the compression ratio ρ_2/ρ_1 reaches $(\gamma+1)/(\gamma-1)$ for a strong shock

$\gamma=5/3$:	$\rho_2/\rho_1 \leq 4$
$\gamma=4/3$:	$\rho_2/\rho_1 \leq 7$
isothermal gas ($\gamma=1$):	$\rho_2/\rho_1 = M_1^2 = 1/M_2^2$

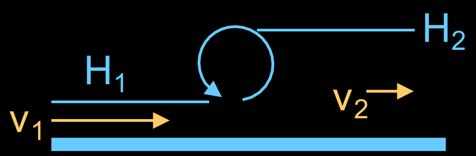
A isothermal gas is characterized by only 2 physical quantities: e.g. velocity and density

A isothermal shock cannot conserve the energy flux: energy is implicitely radiated away

Hydraulic jump conditions



The shallow water flow is also described by 2 physical quantities: velocity and depth (no entropy analogue).
Depth plays the same role as the compressibility of a gas (i.e. surface density).
The jump conditions for a hydraulic jump are deduced from the conservation of mass flux and momentum flux.
Energy is dissipated in a viscous roller within the width of the hydraulic jump.



$$H_1 v_1 = H_2 v_2,$$
$$\frac{gH_1^2}{2} + H_1 v_1^2 = \frac{gH_2^2}{2} + H_2 v_2^2.$$

The Froude number is analogous to the Mach number $\mathrm{Fr} \equiv \frac{v}{(gH)^{\frac{1}{2}}}$

$$\mathrm{Fr}_1 H_1^{\frac{3}{2}} = \mathrm{Fr}_2 H_2^{\frac{3}{2}},$$
$$H_1^2(1+2\mathrm{Fr}_1^2) = H_2^2(1+2\mathrm{Fr}_2^2).$$

$$\left(1+2\mathrm{Fr}_1^2\right)\mathrm{Fr}_2^{\frac{4}{3}} = \left(1+2\mathrm{Fr}_2^2\right)\mathrm{Fr}_1^{\frac{4}{3}}$$

This polynomial of order 3 in $\mathrm{Fr}^{3/2}$ can be factorized by $(\mathrm{Fr}_1^{3/2}-\mathrm{Fr}_2^{3/2})$

$$(\mathrm{Fr}_2^{\frac{2}{3}} - \mathrm{Fr}_1^{\frac{2}{3}})(-2\mathrm{Fr}_2^{\frac{4}{3}}\mathrm{Fr}_1^{\frac{4}{3}} + \mathrm{Fr}_2^{\frac{2}{3}} + \mathrm{Fr}_1^{\frac{2}{3}}) = 0$$

$\mathrm{Fr}_2^{3/2}$ is thus a root of a second order polynomial $2\mathrm{Fr}_2^{\frac{4}{3}}\mathrm{Fr}_1^{\frac{4}{3}} - \mathrm{Fr}_2^{\frac{2}{3}} - \mathrm{Fr}_1^{\frac{2}{3}} = 0$

$$\mathrm{Fr}_2 = \frac{1}{8\mathrm{Fr}_1^2}\left[1+(1+8\mathrm{Fr}_1^2)^{\frac{1}{2}}\right]^{\frac{3}{2}}$$

$$\frac{v_2}{v_1} = \frac{H_1}{H_2} = \left(\frac{\mathrm{Fr}_2}{\mathrm{Fr}_1}\right)^{\frac{2}{3}} = \frac{1}{4\mathrm{Fr}_1^2}\left[1+(1+8\mathrm{Fr}_1^2)^{\frac{1}{2}}\right]$$

The jump conditions for hydraulic jumps differ slightly from the gas

For a strong jump: $\frac{v_2}{v_1} \propto 2^{-\frac{1}{2}}\mathrm{Fr}_1^{-1}, \quad \mathrm{Fr}_2 \propto 2^{-\frac{3}{4}}\mathrm{Fr}_1^{-\frac{1}{2}}.$

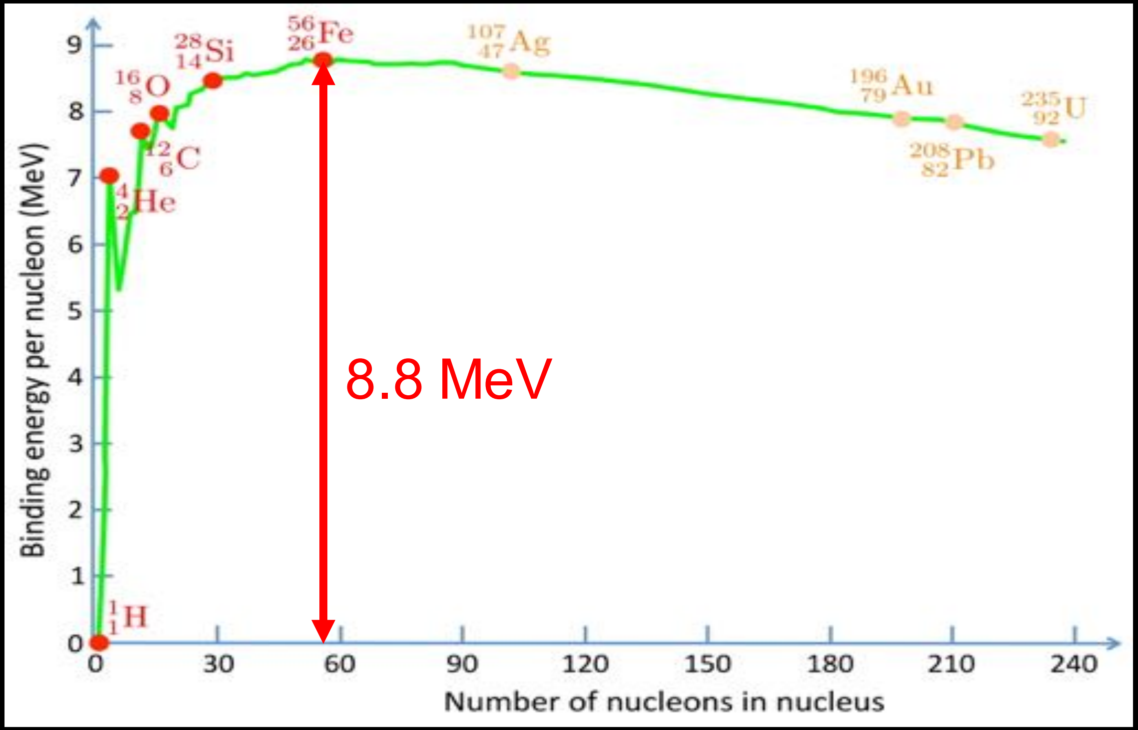
Isothermal shock: $\frac{v_2}{v_1} = M_1^{-2}, \quad M_2 = M_1^{-1}.$

Dissociation of nuclei after the shock

The binding energy of iron is 8.8 MeV/nucleon

The kinetic energy of free fall in the gravitational potential of the proto-neutron star is sufficient to dissociate the iron nuclei into alpha particles, protons and neutrons if the shock radius is smaller than 220km:

$$\frac{\frac{1}{2}m_{\rm p}v_{\rm ff}^2}{8.8{\rm MeV}} \sim \frac{220{\rm km}}{r}\left(\frac{M_{\rm ns}}{1.4M_{\rm sol}}\right)$$



-For a shock radius ~150km, the full dissociation of ^{56}Fe would absorb 150/220=68% of the gravitational energy

-The nucleons which are not accreted onto the neutron star may return a fraction of this energy upon expansion, if the shock is launched

Structure of the accretion flow as the shock stalls

The composition of the infalling gas changes:

-across the shock, heavy nuclei are dissociated into nucleons

$$^{56}\mathrm{Fe} + 56 \times 8.8\mathrm{MeV} \rightarrow 26p + 30n + 26e$$

-in the gain region, neutrons and protons intercept some neutrinos

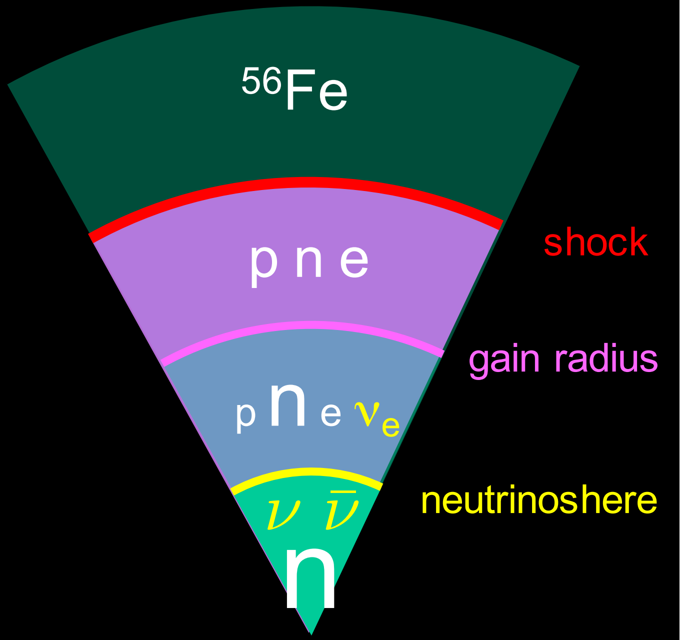
$$n + \nu_\mathrm{e} \rightarrow p^+ + e^-$$
$$p^+ + \bar{\nu}_\mathrm{e} \rightarrow n + e^+$$

-below the gain radius, protons & electrons turn into to neutrons & neutrinos near the proto-neutron star,

$$p^+ + e^- \rightarrow n + \nu_\mathrm{e}$$

- inside the neutrinosphere, a thermal bath of neutrons, neutrinos and anti-neutrinos

$$e^- + e^+ \leftrightarrow \nu_\mathrm{e} + \bar{\nu}_\mathrm{e}$$



neutrino interactions

Neutrino Reactions in Supernovae

Beta processes:	$e^- + p \rightleftharpoons n + \nu_e$
	$e^+ + n \rightleftharpoons p + \bar{\nu}_e$
	$e^- + A \rightleftharpoons \nu_e + A^*$
Neutrino scattering:	$\nu + n, p \rightleftharpoons \nu + n, p$
	$\nu + A \rightleftharpoons \nu + A$
	$\nu + e^\pm \rightleftharpoons \nu + e^\pm$
Thermal pair processes:	$N + N \rightleftharpoons N + N + \nu + \bar{\nu}$
	$e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$
Neutrino-neutrino reactions:	$\nu_x + \nu_e, \bar{\nu}_e \rightleftharpoons \nu_x + \nu_e, \bar{\nu}_e$ ($\nu_x = \nu_\mu, \bar{\nu}_\mu, \nu_\tau$, or $\bar{\nu}_\tau$)
	$\nu_e + \bar{\nu}_e \rightleftharpoons \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$

Dominant heating and cooling reactions

Heating by neutrino absorption:

$$n + \nu_e \to p + e^-$$
$$p + \bar{\nu}_e \to n + e^+$$

$Q_{heat} \sim L_\nu / R^2$

Cooling by electron capture:

$$p + e \to n + \nu_e$$

$Q_{cool} \sim T^6 \sim 1/R^6$

-cooling is dominant near the NS surface
-cooling decreases radially faster than heating

the gain radius R_{gain} (Bethe & Wilson 85) is defined by the balance between cooling and heating

$R_{gain} \sim (L_\nu)^{1/4}$

Neutrino interactions with nuclei are sensitive to the accuracy of their description in the equation of state

1D structure of the accretion flow as the shock stalls

$15M_{sol}$
Marek & Janka 09

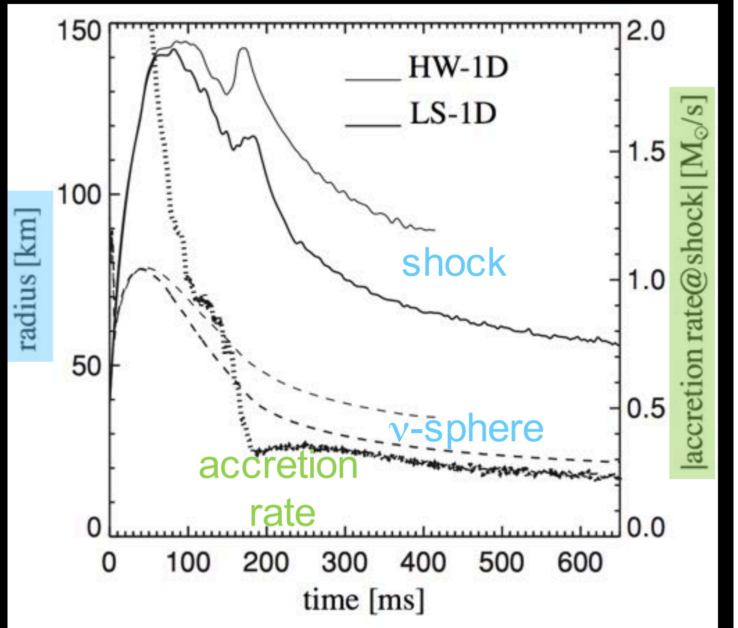
The size of the neutrinosphere depends on the nuclear equation of state.
It shrinks as neutrino energy diffuses out.

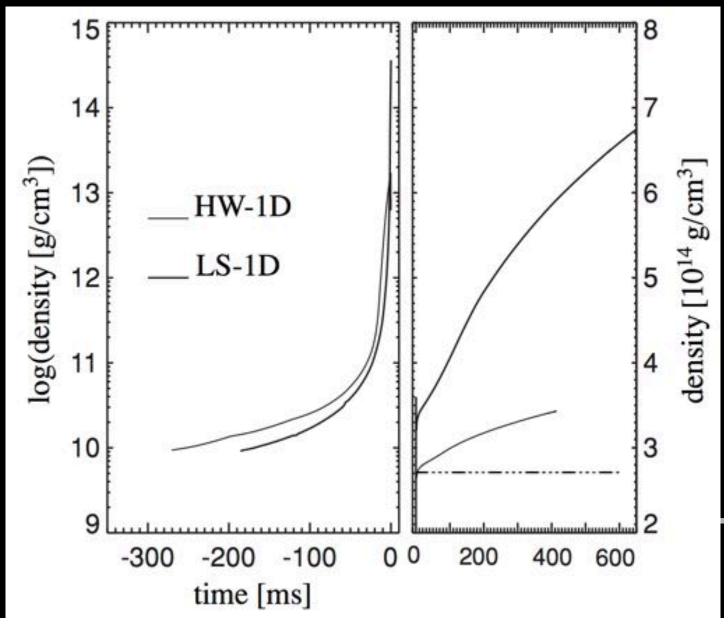
The neutrino energy and luminosity are larger if the neutrinosphere is deeper in the gravitational potential.

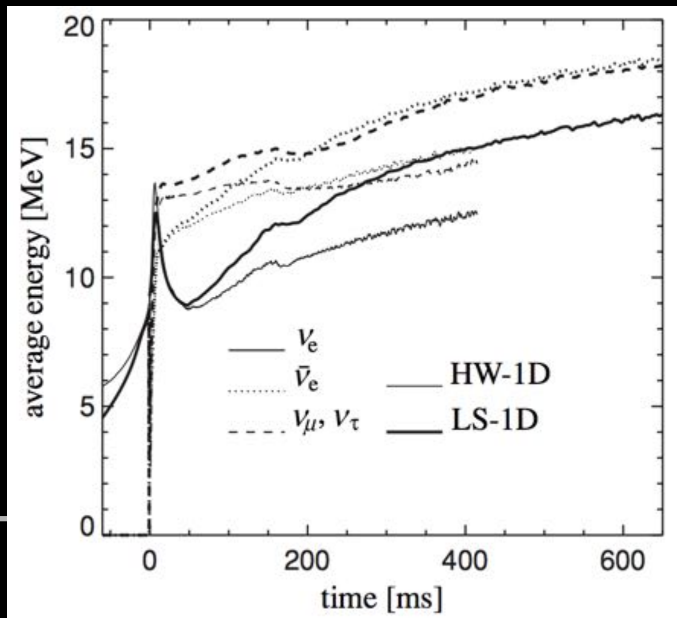
-The gravitational accretion power of the collapse decreases dramatically as the oxygen rich layer reaches the neutrinosphere, (from $2M_{sol}/s$ to $0.3M_{sol}/s$):

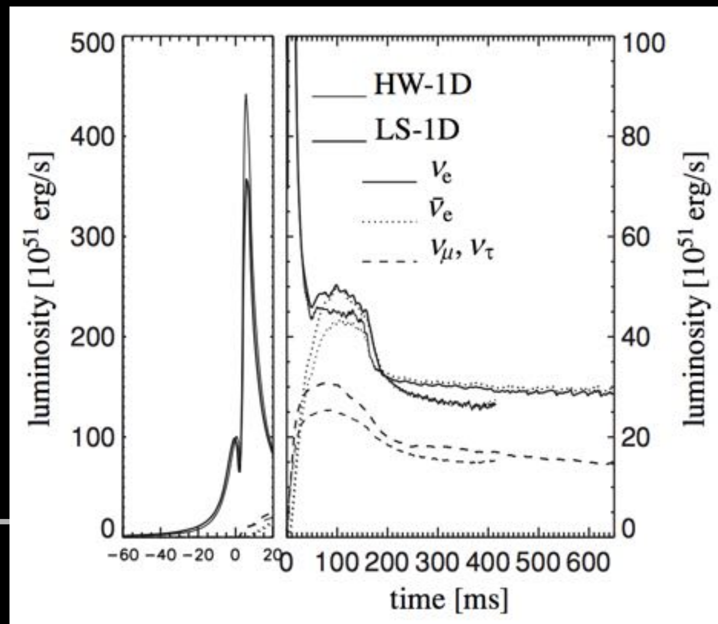
$$\frac{G\dot{M}M}{R} = 6.5\times10^{52}\left(\frac{M}{1.4M_{\rm sol}}\right)\left(\frac{\dot{M}}{0.35M_{\rm sol}{\rm s}^{-1}}\right)\left(\frac{R}{20{\rm km}}\right)^{-1} {\rm erg\ s}^{-1}$$

-The total neutrino luminosity decreases from 15×10^{52} erg/s to 8×10^{52} erg/s in the first 200ms after bounce, exceeding the incoming gravitational accretion power of the collapse for t>200ms









neutrino transport approximations

The time (t) dependent transport of neutrinos in each of point of the 3D space (x,y,z) requires an integration of all incoming particles from every direction (θ,ϕ), for every neutrino species for any energy (E): this calculation in 6+1 dimensions is beyond the power of existing supercomputers

-adiabatic simulations neglect neutrino heating and evacuate matter at the inner boundary
(Blondin & Mezzacappa 07, Foglizzo+12, Endeve+12)

-light bulb+heating/cooling functions (everyone)

-leakage scheme (Ott, O'Connor, Couch) estimate an optical depth to mimick neutrino losses

-multi group flux limited diffusion=MGFLD energy E, flux F
(Burrows)

-Isotropic Diffusion Source Approximation=IDSA: 2 ν-distributions, diffusive and free streaming, 2D
(Liebendörfer+09, Takiwaki, Suwa)

-M1 closure: radiative energie E, radiative flux F, radiation pressure P, closure P=D E
the Eddington tensor D(F/cE) defines an interpolation between the diffusion and the transport limits
(Audit+02, Obergaulinger & Janka, Kuroda, Kotake & Takiwaki, Skinner & Burrows)

-ray by ray plus (Janka, Müller, Mezzacappa): Boltzmann transport along rays

-full Boltzmann 6+1 (Sumiyoshi+15): only on short timescales so far

Nuclear Equation of State

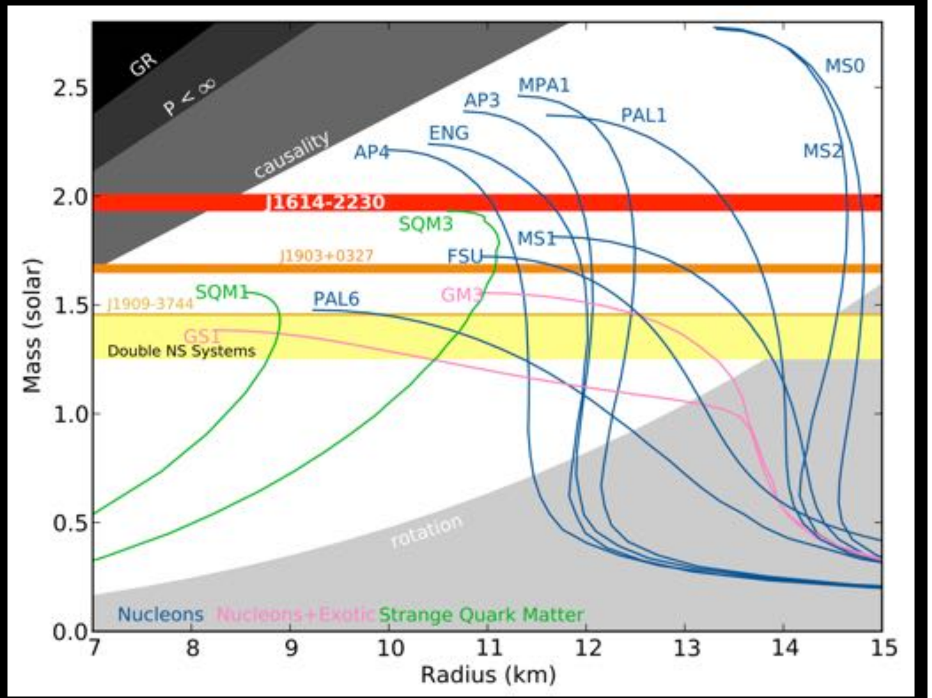
Numericl simulations of core-collapse supernovae almost always use a restricted set of EOS: Lattimer & Swesty 91, and H. Shen+98 with one degree of freedom, their compressibility or "softness"

Softer EOS lead to easier explosions: more compressible =deeper gravitational potential.

Some recent updates G. Shen+11, Hempel+11

Updated modern EOS are freely available through the internet, e.g. the Compstar database Compose (Typel+15) compose.obspm.fr

Improved EOS include:
-intermediate nuclei: a single heavy nucleus or an explicit distribution
-light nuclei in addition to p, n, alpha: deuteron, triton...
-exotic particles such as pions, hyperons (Oertel+12) or even a quark phase (Sagert+09, Fisher+11)



Demorest+10

Softness is limited by the observed mass of neutron stars.
The parameter space shrinked in 2010 (Demorest+10) with M_{NS}~2 M_{sol}
Gravitational waves from coalescing NS may further constrain the EOS (Bauswein+14)

PSR J1614-2230 — $M=1.97+-0.04M_{sol}$ (Demorest+10) updated to $M=1.928+-0.017M_{sol}$ (Fonseca+16)
PSR J0348+0432 — $M=2.01+-0.04\ M_{sol}$ (Antoniadis+12)

The uncertainties associated to the initial conditions

Progress in supernova theory is limited by the progress in stellar structure and stellar evolution

Most of the progenitors are computed by very few groups: Woosley & Weaver (1995), Woosley, Heger & Weaver (2002), Woosley & Heger (2007)

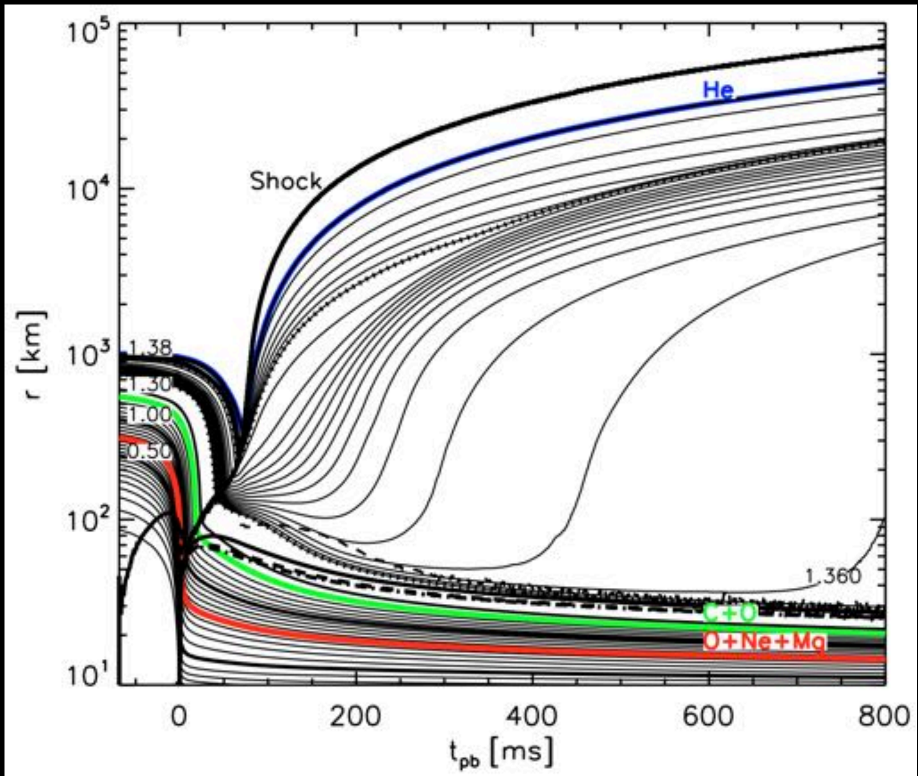
The mass on the main sequence is a very indirect and non monotonous tag for the structure of the progenitor (mass loss and unstable burning history). The impact of the diversity of radial structures on the explosion mechanism has been discovered only recently (O'Connor & Ott 11, Ugliano+12, Sukhbold & Woosley 14)

The stellar structure of the progenitor is spherical, resulting from stellar evolution with 1D presciptions for transport processes (e.g. Mixing Length Theory) with uncertain distribution of angular momentum and magnetic fieds.

The turbulent convective structure of the oxygen, silicon layers was ignored until recently (Arnett & Meakin 11, Arnett+15, Couch & Ott 13, 15, Müller+16)

The impact of binary interactions is most often ignored (Podsiadlowski+04, Sana+12)

The spherical delayed explosion of a $8.8M_{sol}$ progenitor



Kitaura+06

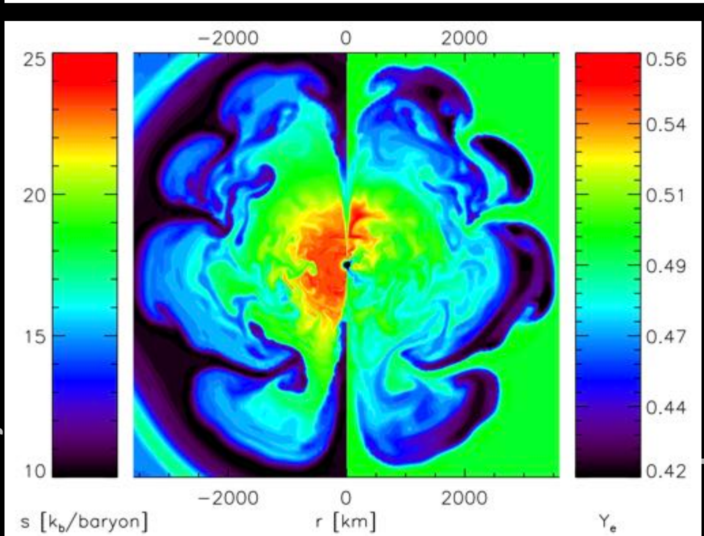
O-Ne-Mg core
τ_{expl}~200ms after bounce

$M(^{56}Ni)<0.015M_{sol}$
$E\sim0.1\times10^{51}$ erg depending on the EOS
$M_{NS}\sim1.36M_{sol}$

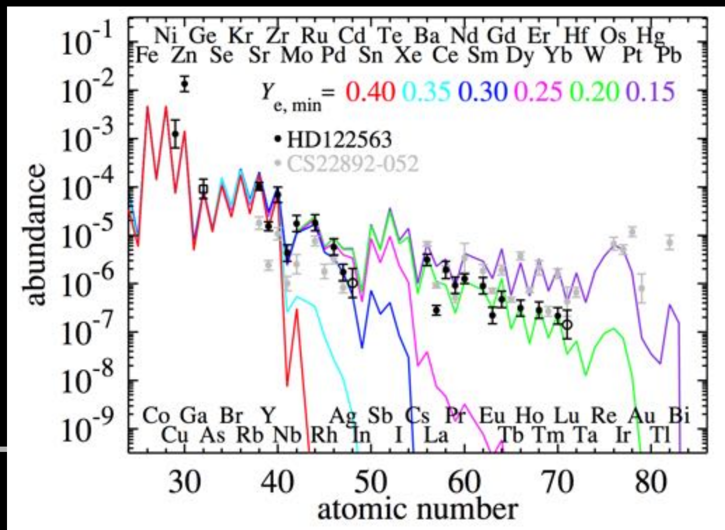
possible model for subluminous SN including the Crab supernova
$E\sim0.6\text{-}1.5\times10^{50}$ erg, $M_{ej}=4.6\pm1.8M_{sol}$

Nucleosynthesis of ^{64}Zn:
<30% of core collapse originate from electron capture SN
(Wanajo+09)

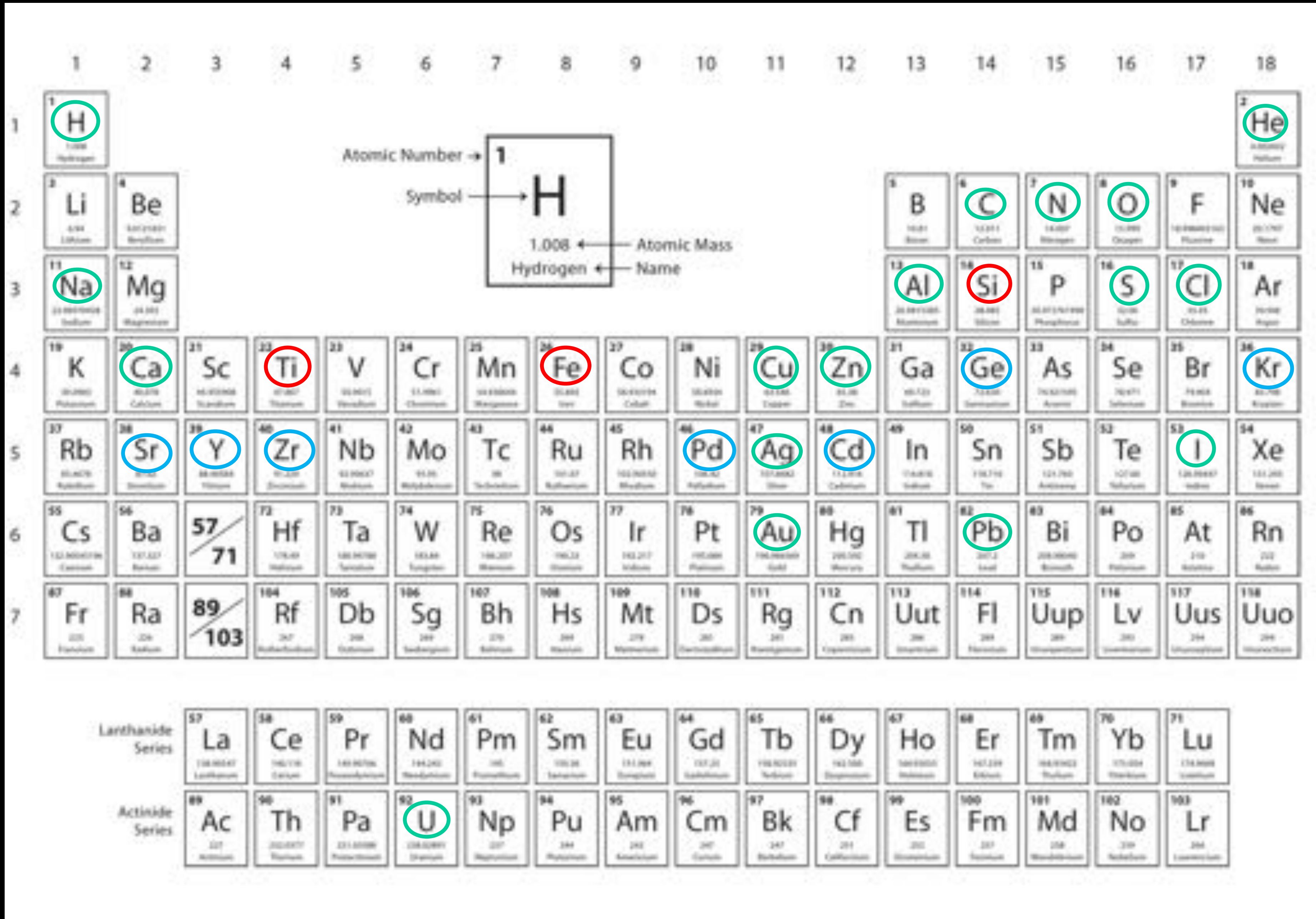
2D update (Wanajo+11): neutron rich convective lumps are favourable to the r-process up to Zr, possibly up to Ag
Ratio $^{86}Kr/^{16}O$→ECSN are 4% of CCSN



Wanajo+11

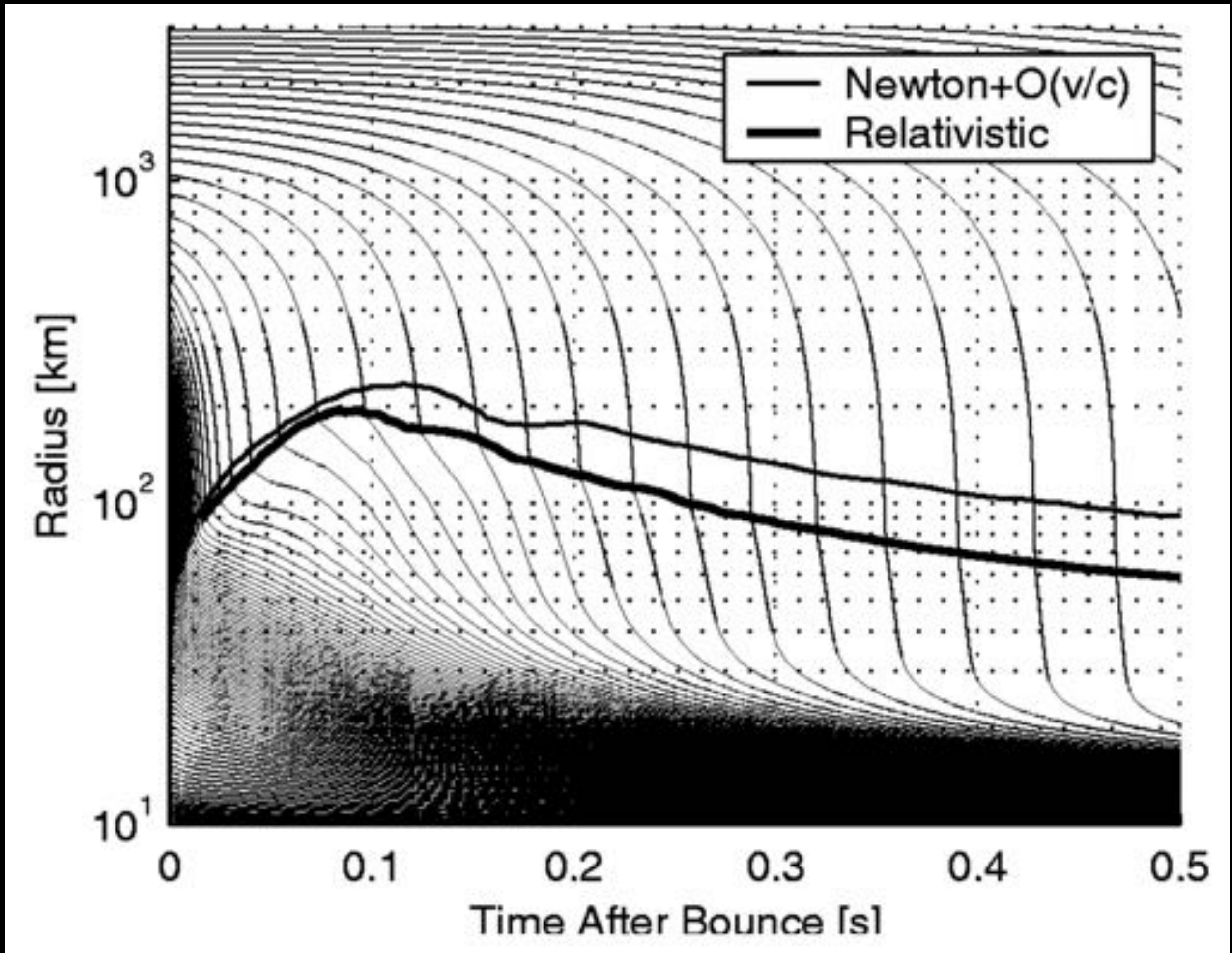


Classification of the elements



The failure of spherical explosions for $M=13M_{sol}$

Liebendörfer+01



full Boltzmann neutrino transport

Lattimer & Swesty EOS

General relativity:

deeper gravitational potential,
smaller proto NS,
smaller shock radius,
higher neutrino luminosity

Outline of lecture 2

Introduction to supernovae: following our common sense

- can a collapse bounce into an explosion ?
- the basics of shock waves

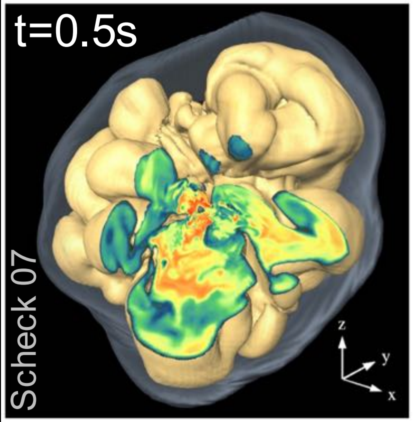
The framework of delayed neutrino driven explosions

- the 5 zones of the model by Betthe & Wilson
- the spherical explosion of $10M_{sol}$
- the puzzle of more massive progenitors

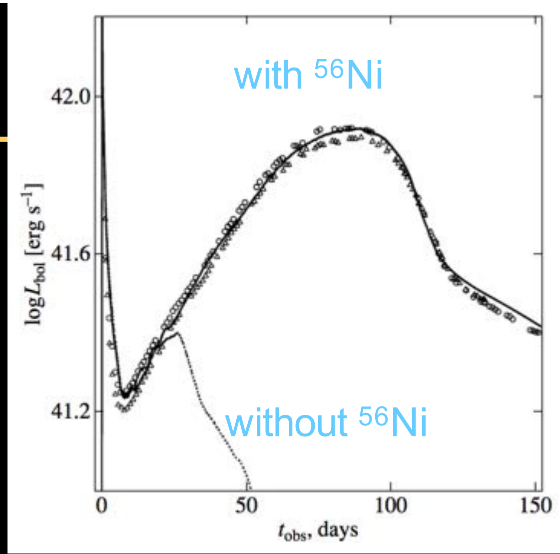
Some observational clues and puzzles

- the hints for asymmetric explosions
- constraints from the progenitor, the ejecta and the neutron star

Hints of aspherical explosion of 1987A

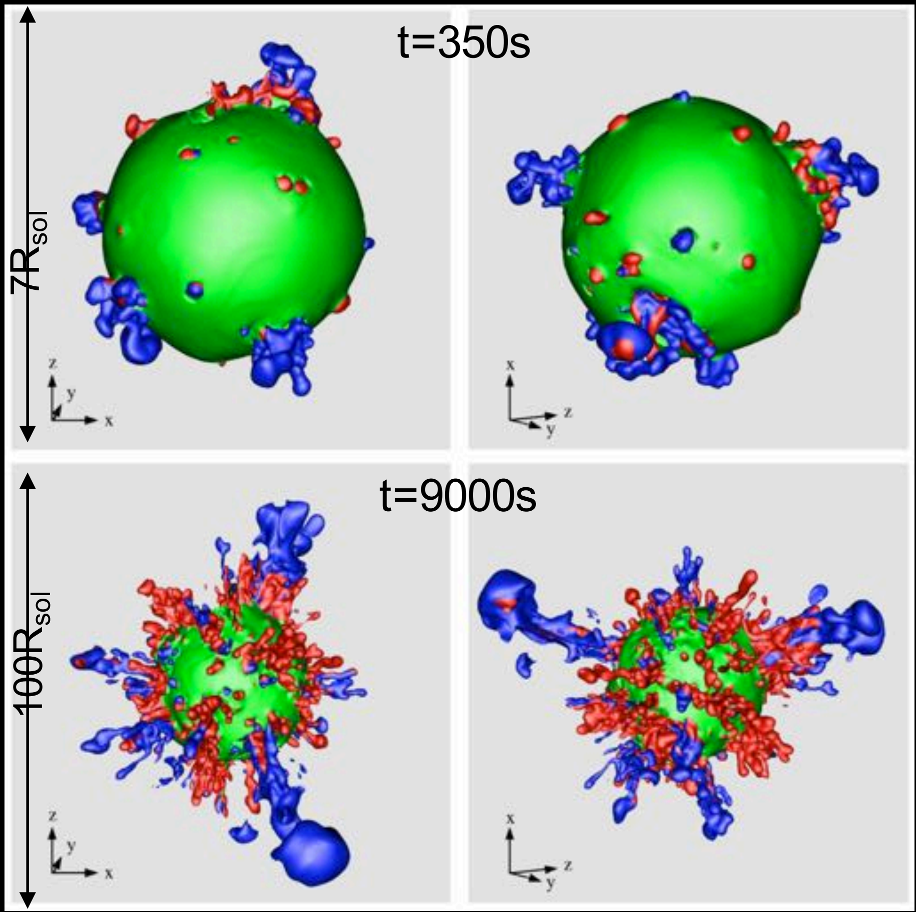


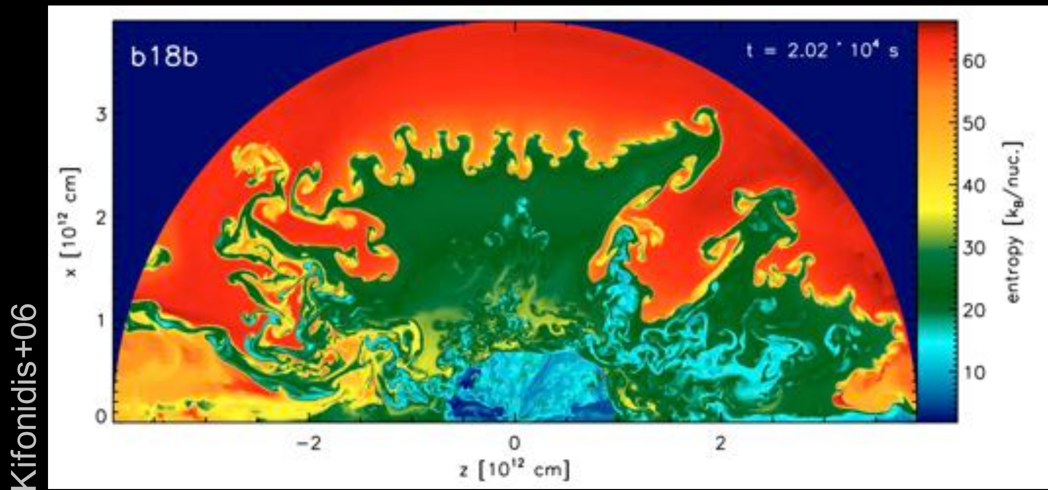
H/He mixing and Ni clumps are required to explain the light curve of 1987A, and the early emergence of X and γ rays (Woosley 88, Utrobin 04)



blue: 7% nickel
green: 3% carbon
red: 3% oxygen

blue: nickel
green: He core
red: H rich



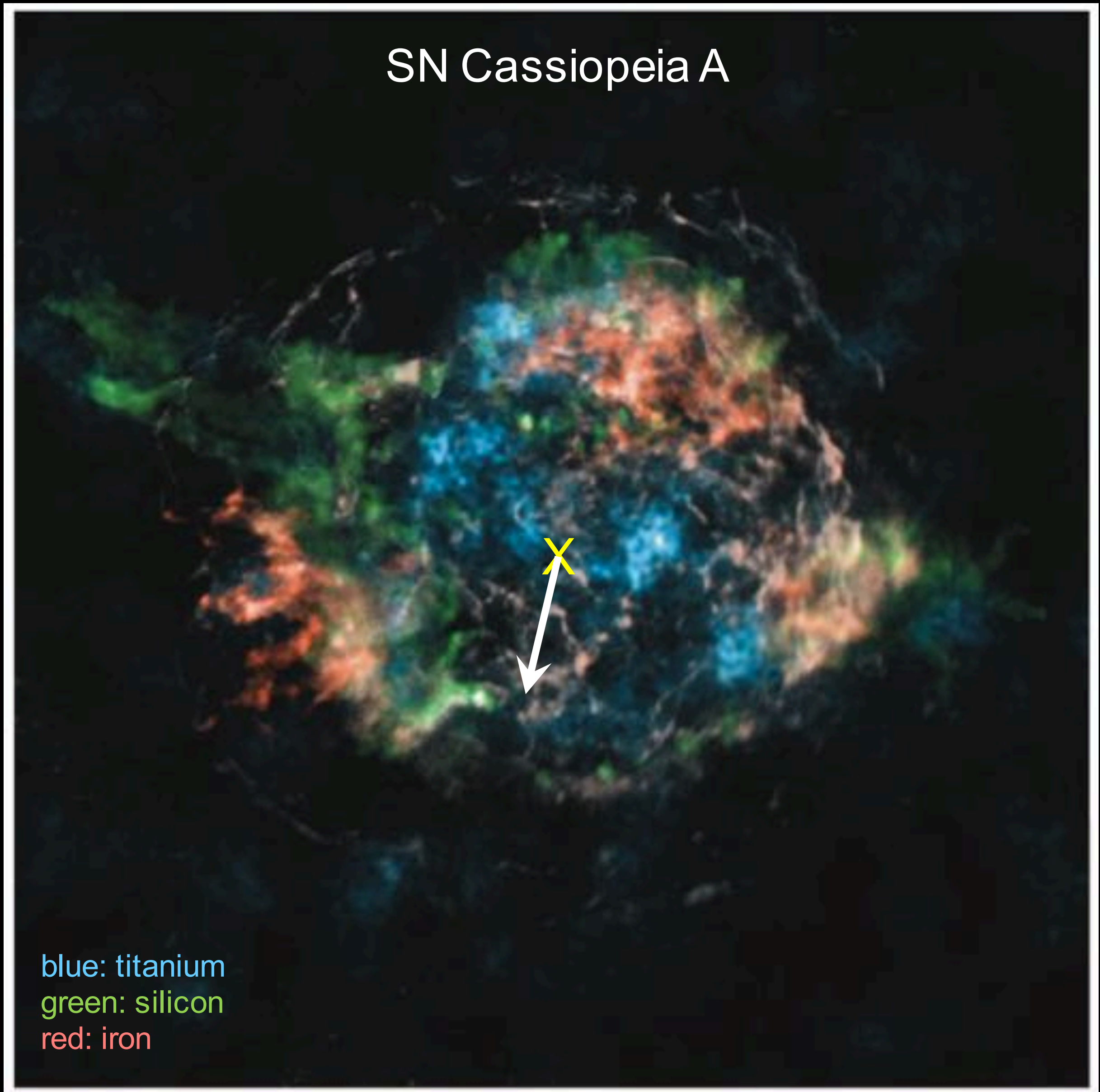


2D simulation with large intial asymmetry (Kifonidis+06)
→efficient H/He mixing by the RMi

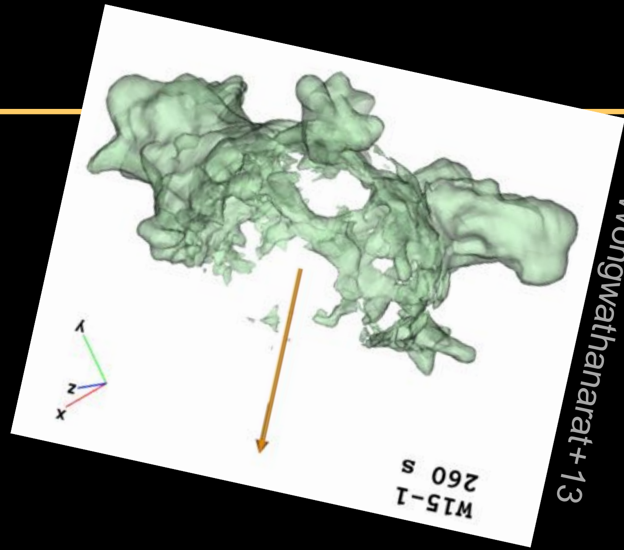
3D simulation with smaller initial asymmetry (Hammer+10)
→high velocity nickel clumps 4500km/s
→more efficient RT mixing than in 2D

However no satisfactory pre-supernova model yet (Utrobin+15)

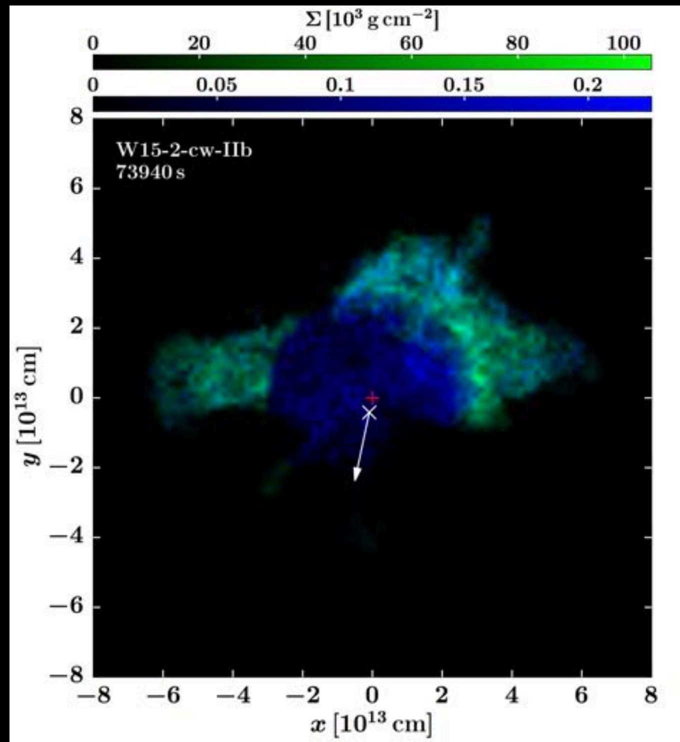
Inhomogeneous nucleosynthesis



Grefenstette+14



Wongwathanarat+13



Wongwathanarat+16

3D structure of stripped supernovae from spectropolarimetry

Tanaka+12

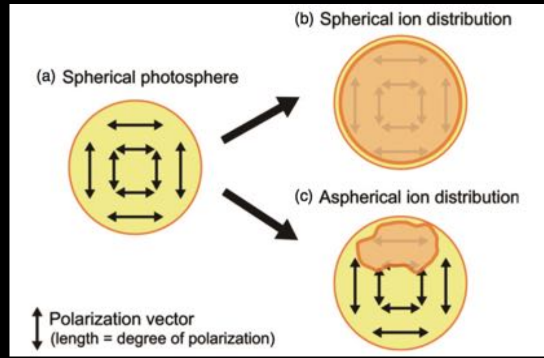
$$Q \equiv \frac{I_0 - I_{90}}{I}$$

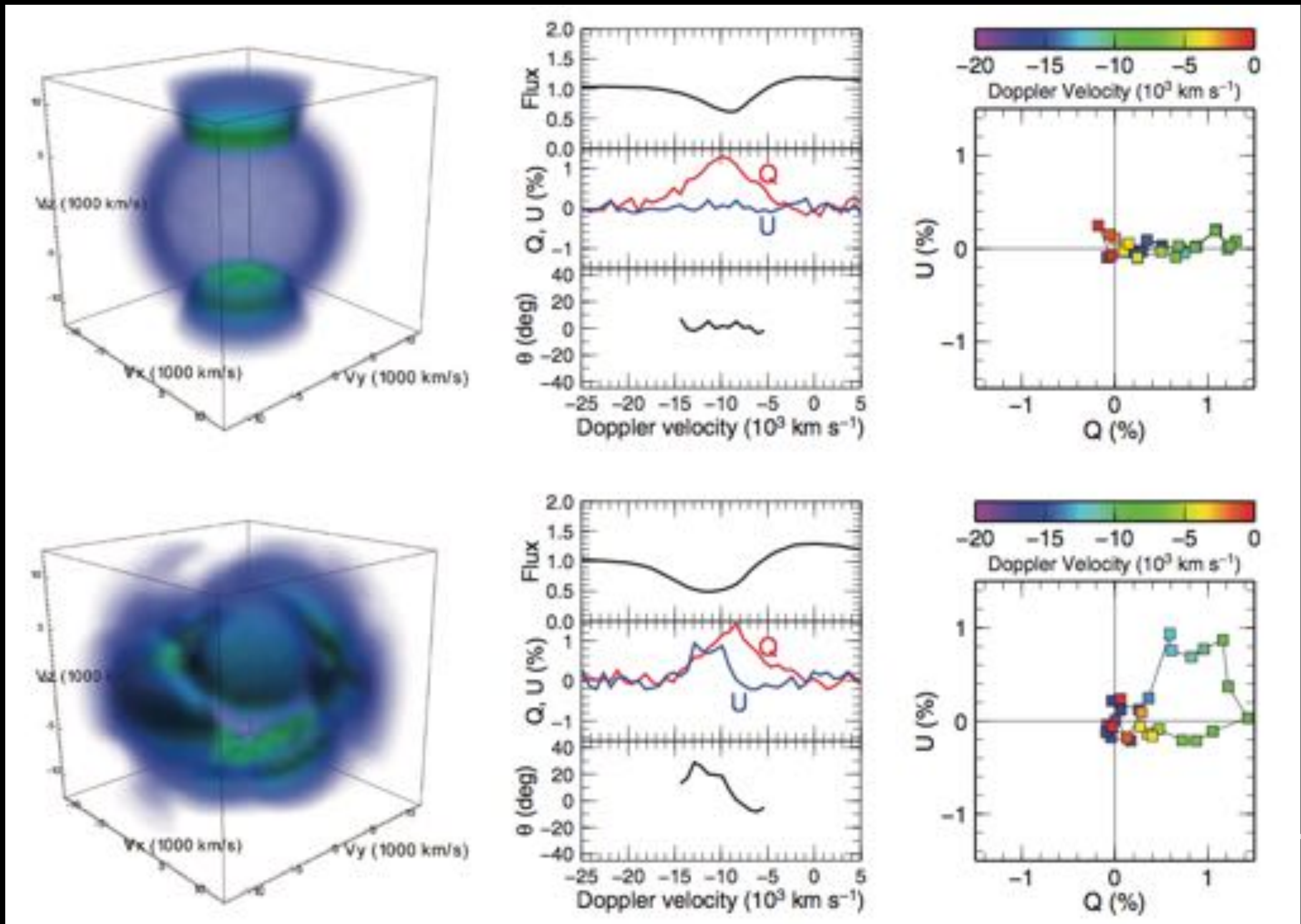
$$U \equiv \frac{I_{45} - I_{135}}{I}$$

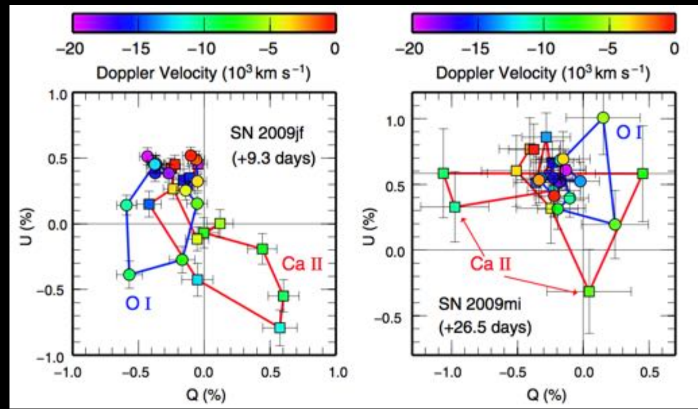
$$2\theta \equiv \mathrm{atan}\left(\frac{U}{Q}\right)$$

An axisymmetric structure transates into
a 1D distribution of the Stokes parameter Q,U

A non axisymmetric structure produces
a loop in the Q,U diagram







Type Ib

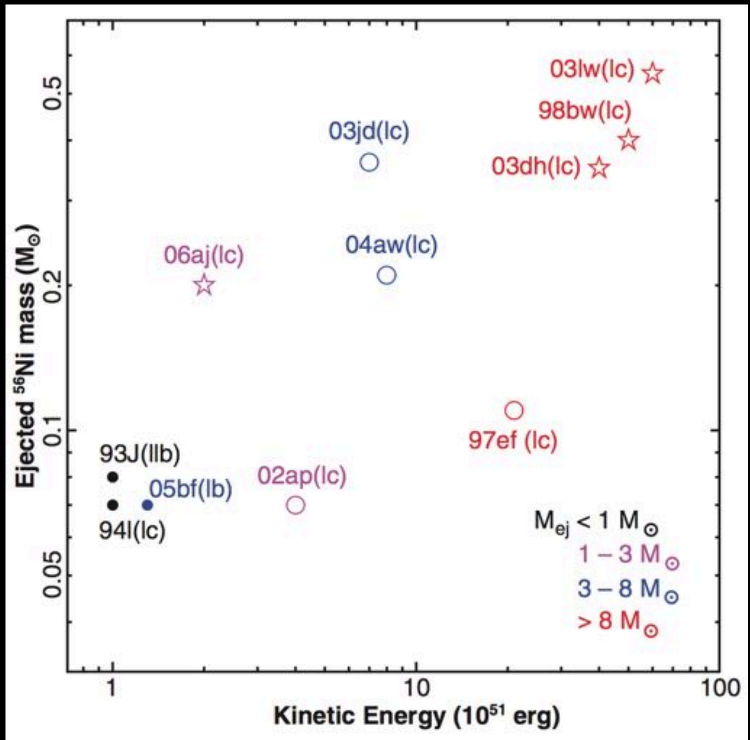
Type Ic

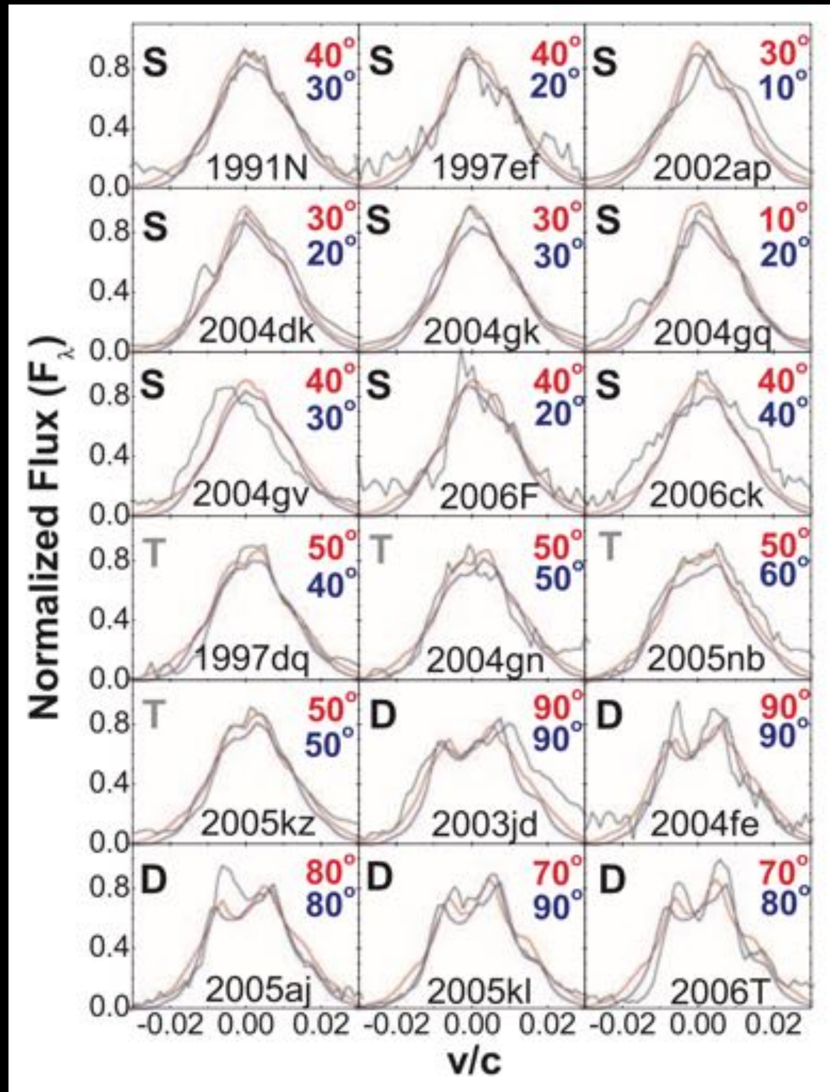
5 out of 6 stripped enveloppe SN
have a 3D non axisymmetric structure

→A 2D theory of stellar explosions may
be insufficient

Aspherical explosions: continuity among stripped SN from spherical to strongly bipolar

Maeda+08

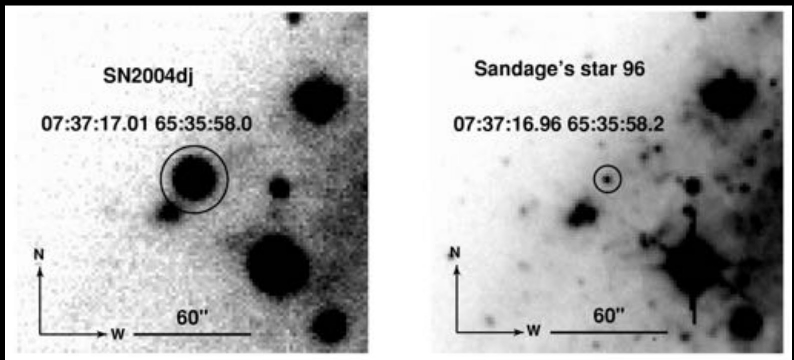




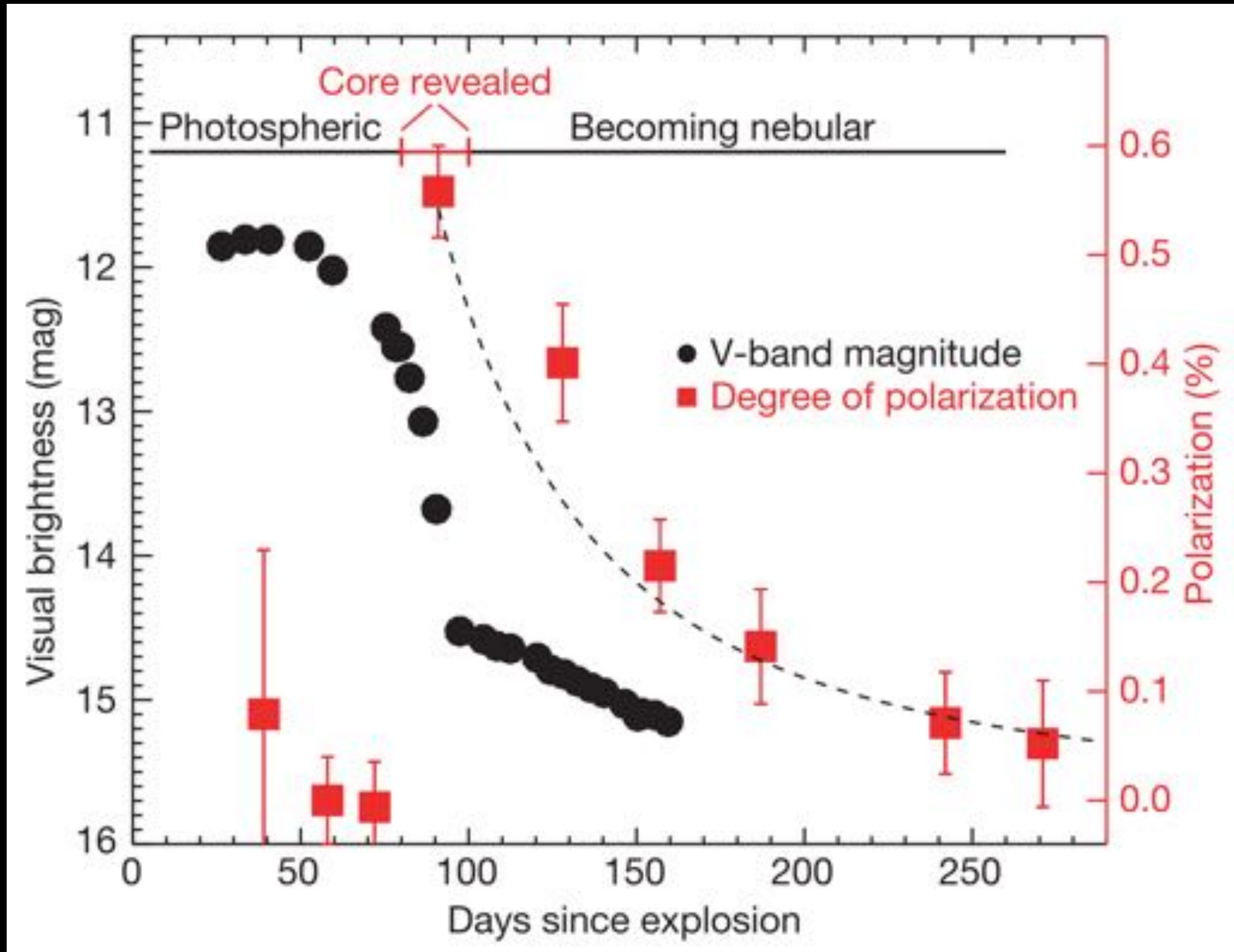
Oxygen doublet 6300, 6363
S: single peaked
T: transition
D: double peaked
red: bipolar model
blue: less aspherical model

Hints of aspherical explosion of SNIIP from spectropolarimetry

Type IIP SN2004dj (Leonard+06)
in a young compact star cluster S96
~20Myr
progenitor identified as a supergiant star
~12M_{sol} (Wang+05)



11/08/2004 20/12/1995

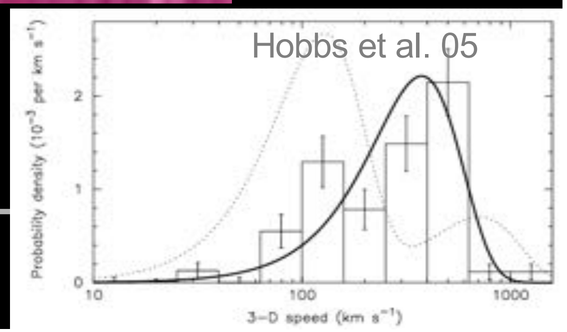


→Even the most "normal" and abundant supernovae have an asymmetric structure

The high velocities of neutron stars suggest an asymmetric supernova explosion

Chatterjee & Cordes+02

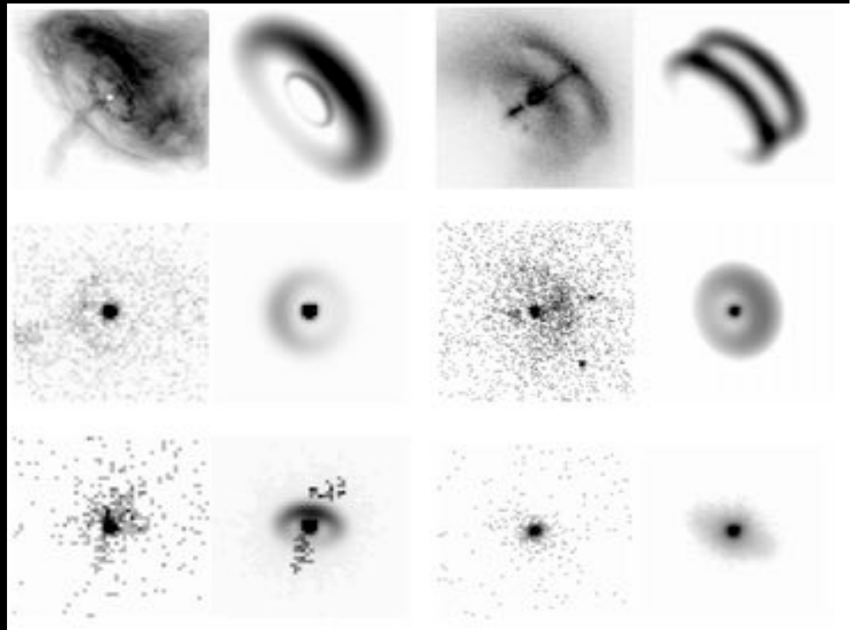
pulsar in the guitar nebula: >1000km/s



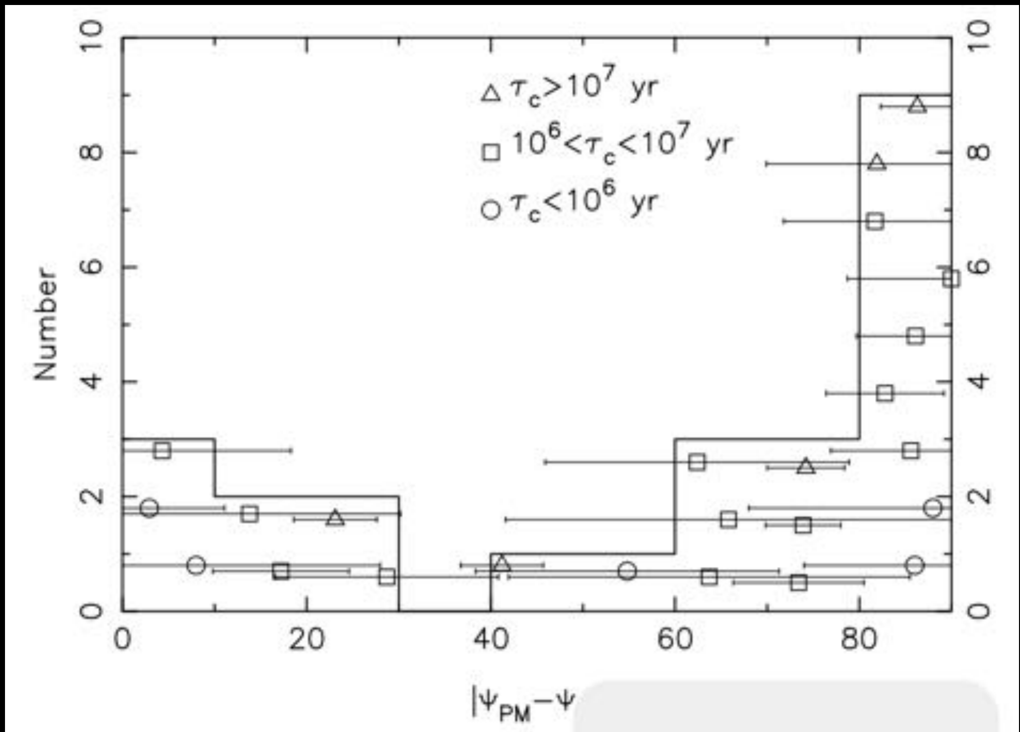
Is there a kick spin correlation?

The spin axis is deduced from 2 methods
polarization from 24 pulsars (Wang+06), 56 pulsars (Noutsos+12)
+-90° uncertainty from polarization measures:
no correlation rejected with 99% confidence

axis of pulsar wind nebulae from 6 pulsars (Ng & Romani 04, Wang+06),
A contribution from binary orbital kicks is expected
The initial kick direction is affected by the galactic potential (Noutsos+13)



Ng & Romani 04

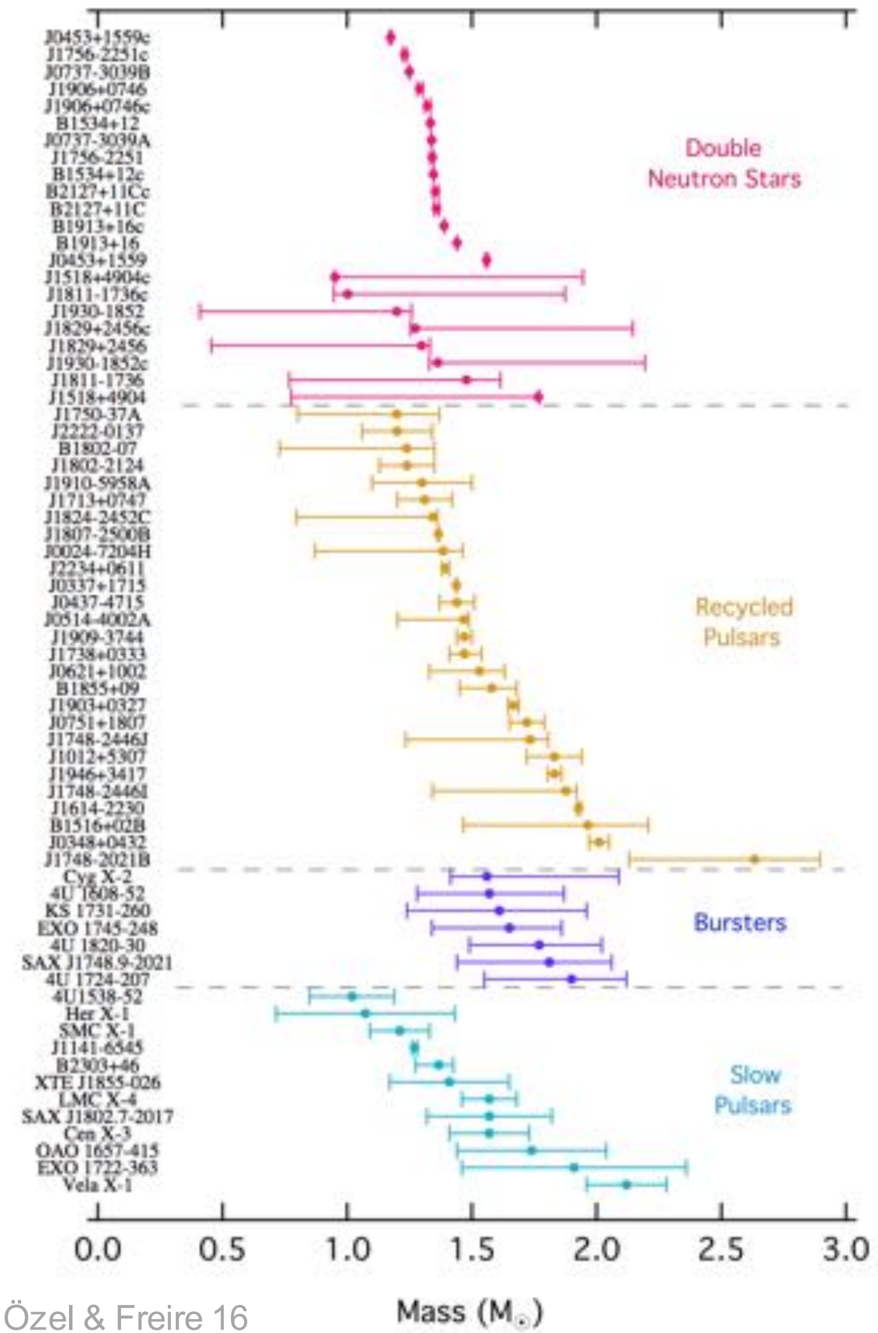


Wang+06

PSR	Ψ_{rot} (deg)	Ψ_{PM} (deg)	$\lvert\Delta\Psi_{\Omega v}\rvert$ (deg)
B0531+21........ (Crab)	124.0 ± 0.1	292 ± 10	12 ± 10
J0538+2817.................	155 ± 8	328 ± 4	7 ± 9
B0833−45.... (Vela)	130.6 ± 0.1	301 ± 2	8.6 ± 4
B1706−44....................	163.6 ± 0.7	160 ± 10	3.6 ± 11
B1951+32....................	85 ± 5	252 ± 7	13 ± 9
J1124−5916.................	...	...	22 ± 7

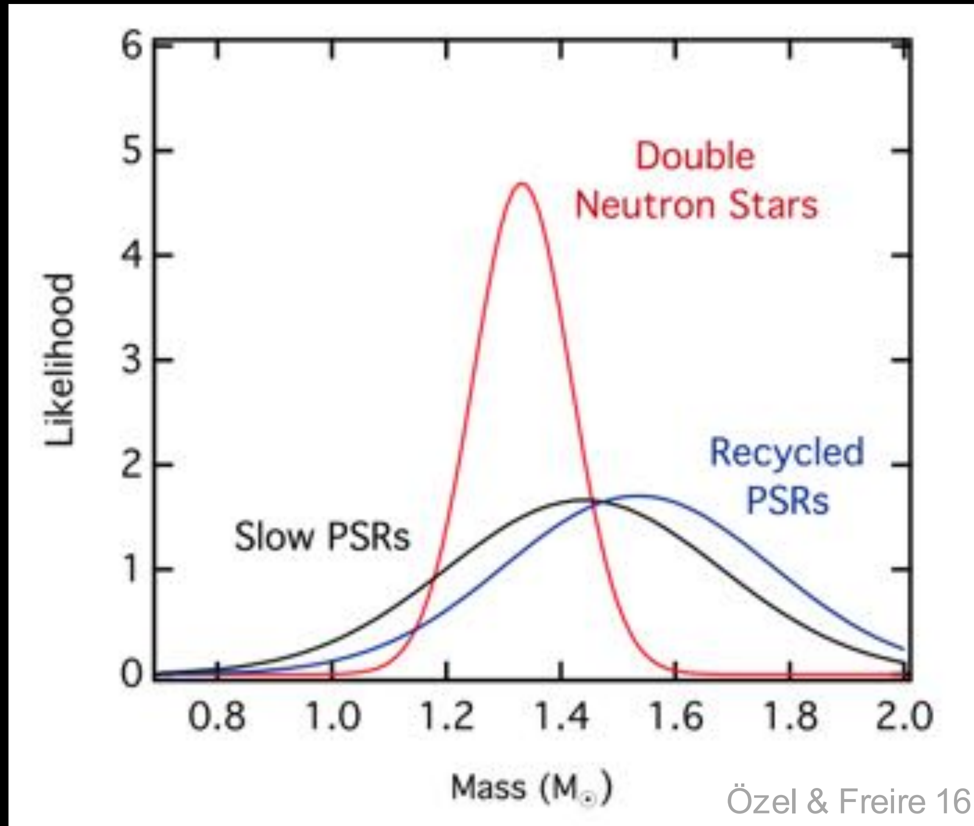
Wang+06

Are neutron star masses clustered around $1.4M_{sol}$?



Özel & Freire 16

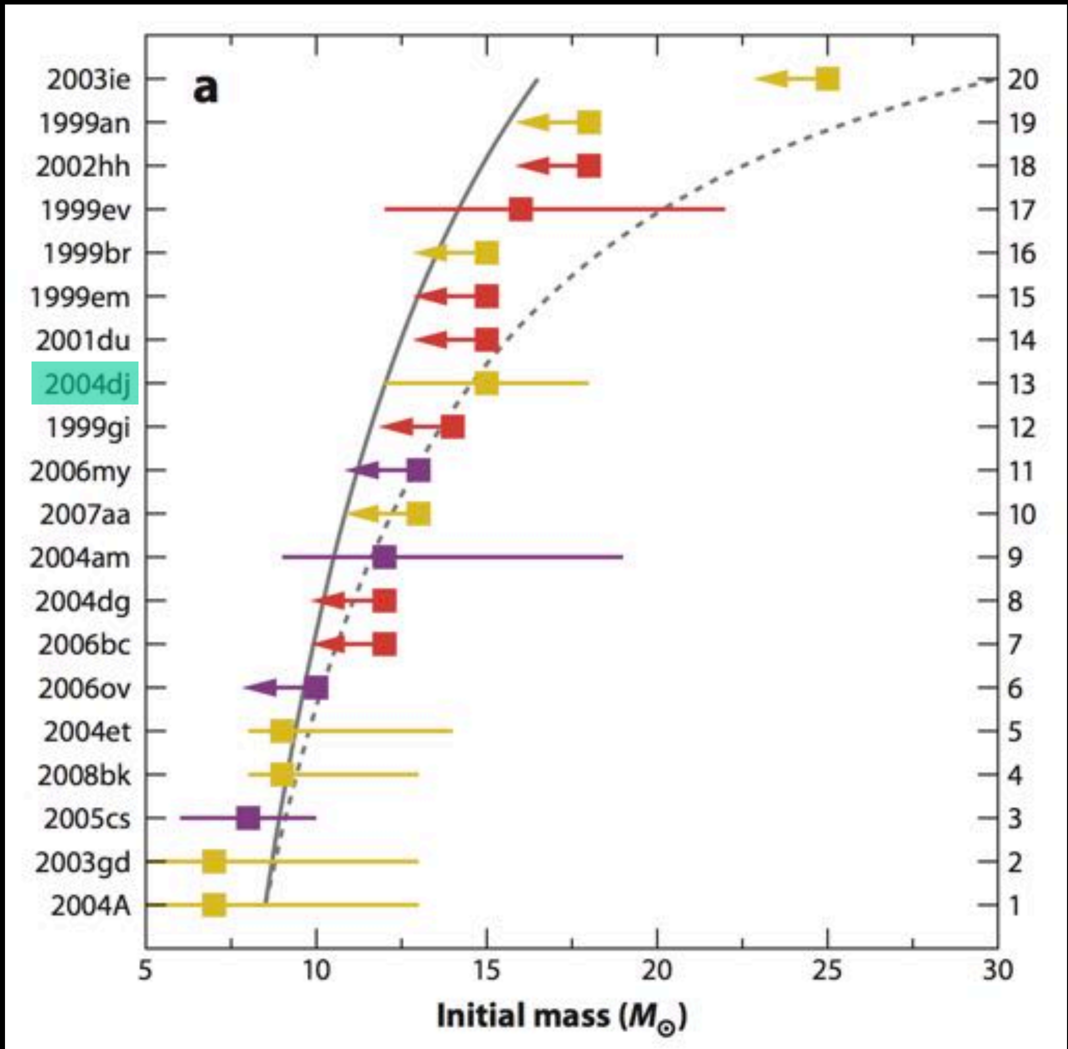
Demorest+10:	PSR J1614-2230	$M=1.97+-0.04M_{sol}$
Fonseca+16:		$M=1.928+-0.017M_{sol}$
Antoniadis+12:	PSR J0348+0432	$M=2.01+-0.04\ M_{sol}$
Barr+17:	PSR J1946+3417	$M=1.83+-0.022\ M_{sol}$
Antoniadis+16:	PSR J1012+5307	$M=1.83+-0.11M_{sol}$

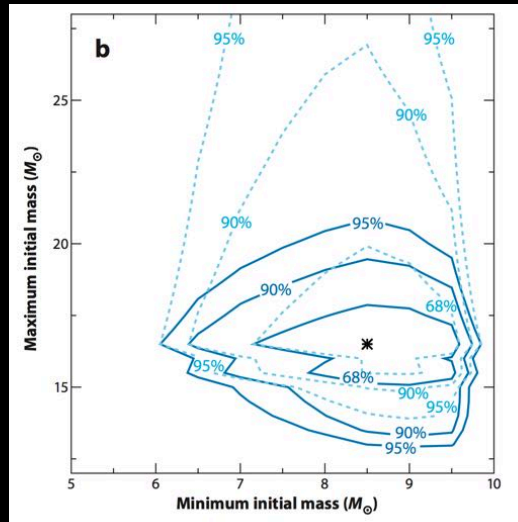


→The mass distribution is broader than previously thought.

The "Red Supergiant Problem" ?

Surprising lack of progenitors with $M>16M_{sol}$ for type IIP SN (Smartt 09)
most probable Salpeter IMF (solid line) α=-2.35, M_{min}=8.5M_{sol}, M_{max}=16.5M_{sol}





Dust production ? (Walmswell & Eldridge 12)
→M_{max}=21M_{sol}
Superwind ? (Yoon & Cantiello 10)
→mass sequence IIP-IIL-IIn-IIb-Ib-Ic
Black holes ? (Kochanek 14)
→from compactness, 18+11-9% BH

Selected milestones of mainstream supernova theory: towards 3D ab-initio models

Baade & Zwicky 1934: coins the term "supernova" and suggests the stellar collapse to a neutron star

Gamow & Schönberg 41: energy removal through neutrinos
Colgate & White 66: energy deposition by neutrinos

Bethe & Wilson 85: delayed neutrino-driven explosion
SN1987A
Herant+92: simulation of neutrino-driven convection in 2D

Liebendörfer+01: failed explosion for 13 M_{sol} 1D ab-initio with Boltzman transport

Scheck+04: pulsar kicks explained by asymmetric explosions in 2D

Kitaura+06: subluminous explosion from a 8-10 M_{sol} 1D ab-initio

Blondin+06: discovery of the Standing Accretion Shock Instability in 2D

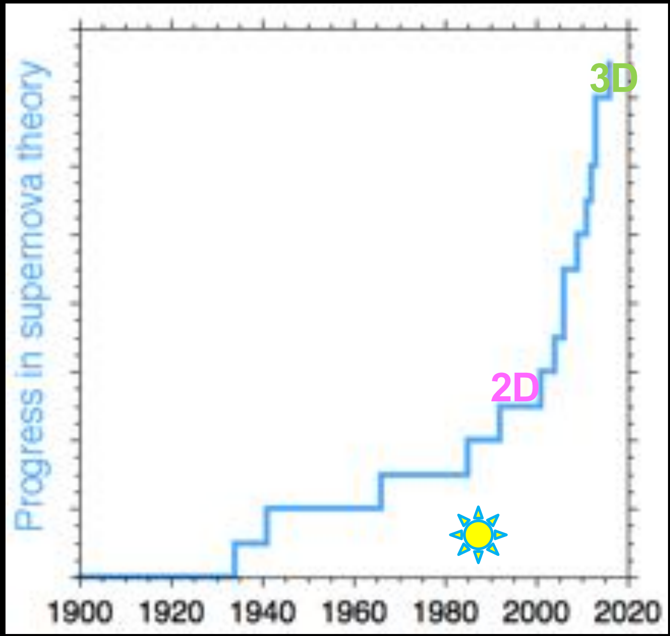
Marek & Janka 09: explosion of 15 M_{sol} 2D ab-initio

O'Connor & Ott 11: impact of the stellar core compactness in 1D

Müller+12: explosion of 27 M_{sol} 2D ab-initio: two different explosion paths

Hanke+13: failed explosion of 27 M_{sol} 3D ab-initio

Couch & Ott 13: impact of precollapse turbulence in 3D



Sukhbold +16: SN outcomes from 9 to 120 M_{sol} in 1D