The high-dimensional and multi-modal Bayesian inference code

DIAMONDS

Application to Asteroseismology

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Earlier Career

- PhD 2009 - 2012 in Asteroseismology
  Department of Physics and Astronomy University of Catania (**Italy**)
  Sydney Institute for Astronomy, University of Sydney (**Australia**)

Supervisors:
Dr. Alfio Bonanno
Dr. Dennis Stello

THE UNIVERSITY OF SYDNEY

- Postdoc 2012 - 2014
  Instituut voor Sterrenkunde - Katholieke Universiteit Leuven (**Belgium**)

Collaborator: **Dr. Joris De Ridder**

KATHOLIEKE UNIVERSITEIT LEUVEN

CEA

- Postdoc since October 2014
  Collaborator: **Dr. (HDR) Rafael A. García**
- **Speciality**: Asteroseismology of solar-like stars (from main sequence to red giants) using Bayesian techniques with DIAMONDS

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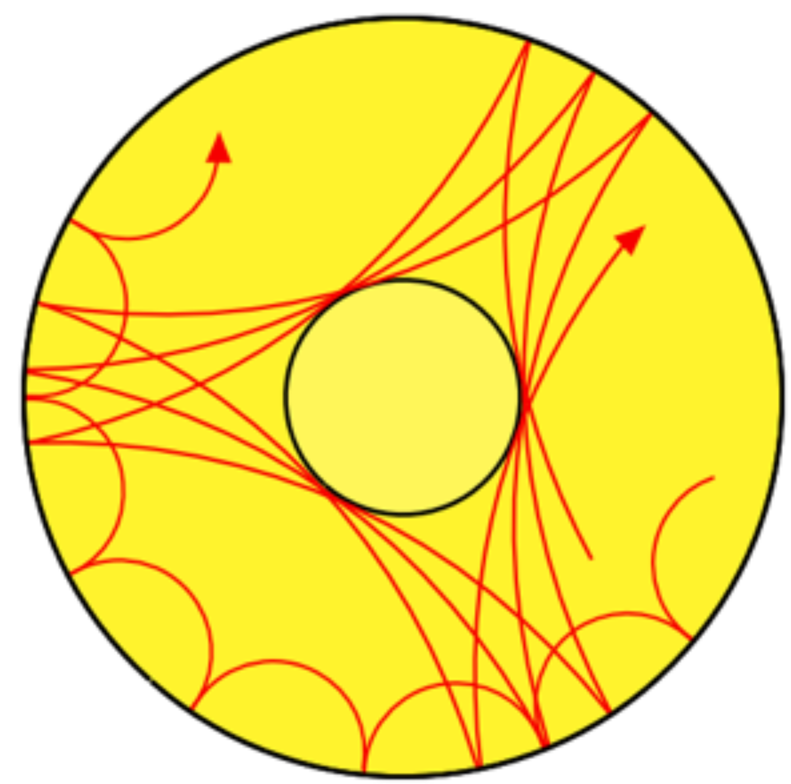

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What is DIAMONDS ?

high-DImensional And multi-MOdal NesteD Sampling

Corsaro & De Ridder 2014 A&A, 571, 71

- C++11 code for **Bayesian inference** problems:
  - Dataset to fit
  - Model to test
  - Estimate the free parameters of the model

What is DIAMONDS ?

high-DImensional And multi-MOdal NesteD Sampling

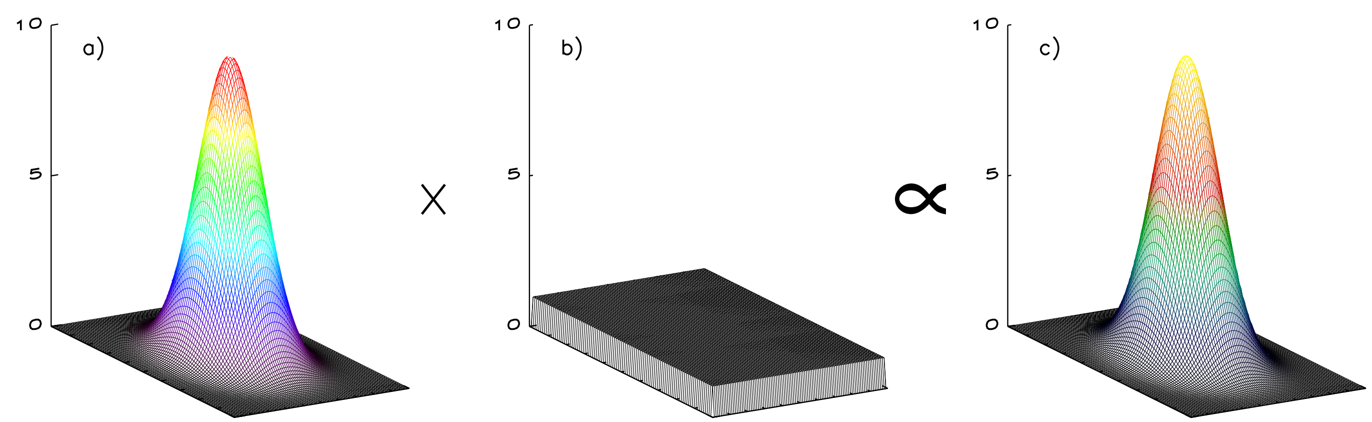
Corsaro & De Ridder 2014 A&A, 571, 71

- C++11 code for **Bayesian inference** problems:
  - Dataset to fit (**Likelihood**)
  - Model to test (**Prior**)
  - Estimate the free parameters of the model (**Posterior**)

Likelihood **Prior** **Posterior**

Bayes' Theorem

$$p(\boldsymbol{\theta}) = \frac{\mathcal{L}(\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})}{\mathcal{E}}$$



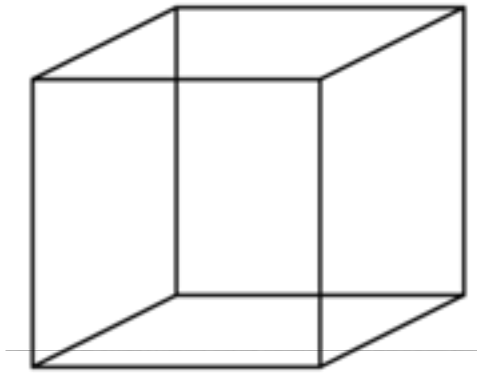
The basic algorithm

- **Nested Sampling Monte Carlo**
  Skilling 2004
  - For k free parameters to estimate, Bayesian Evidence is a k-dimensional integral

$$\mathcal{E} = \int \mathcal{L}(\boldsymbol{\theta})\, \pi(\boldsymbol{\theta})\, d\boldsymbol{\theta}$$

Bayes' Theorem

$$p(\boldsymbol{\theta}) = \frac{\mathcal{L}(\boldsymbol{\theta})\, \pi(\boldsymbol{\theta})}{\mathcal{E}}$$



The basic algorithm

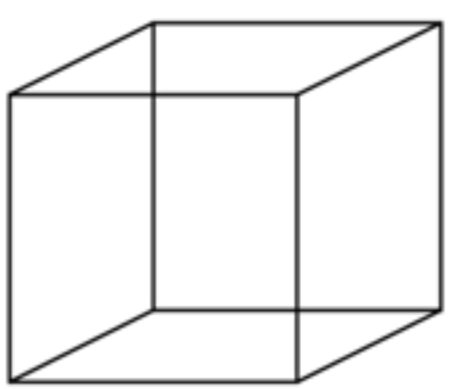
- **Nested Sampling Monte Carlo**
  Skilling 2004
  - For k free parameters to estimate, Bayesian Evidence is a k-dimensional integral

$$\mathcal{E} = \int \mathcal{L}\left(\boldsymbol{\theta}\right) \pi\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta}$$

Bayes' Theorem

$$p\left(\boldsymbol{\theta}\right) = \frac{\mathcal{L}\left(\boldsymbol{\theta}\right) \pi\left(\boldsymbol{\theta}\right)}{\mathcal{E}}$$

  - Convert evidence into a one-dimensional integral



$$\mathcal{E} = \int_0^1 \mathcal{L}\left(X\right) dX$$

$$dX = \pi\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta}$$

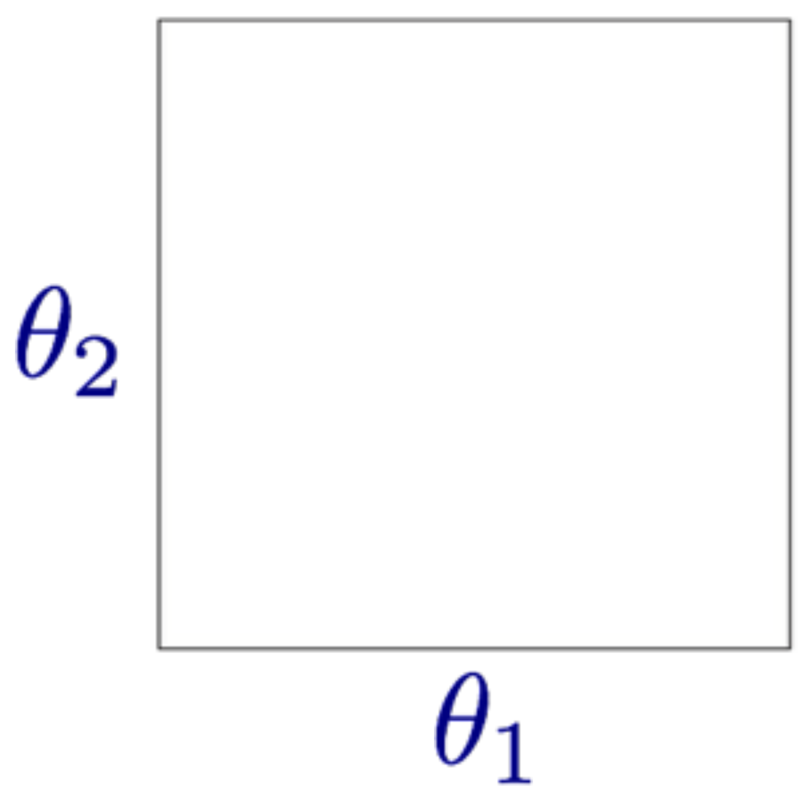
small portion of prior volume (prior mass)

Nested Sampling

in a nutshell…

Bayesian Evidence

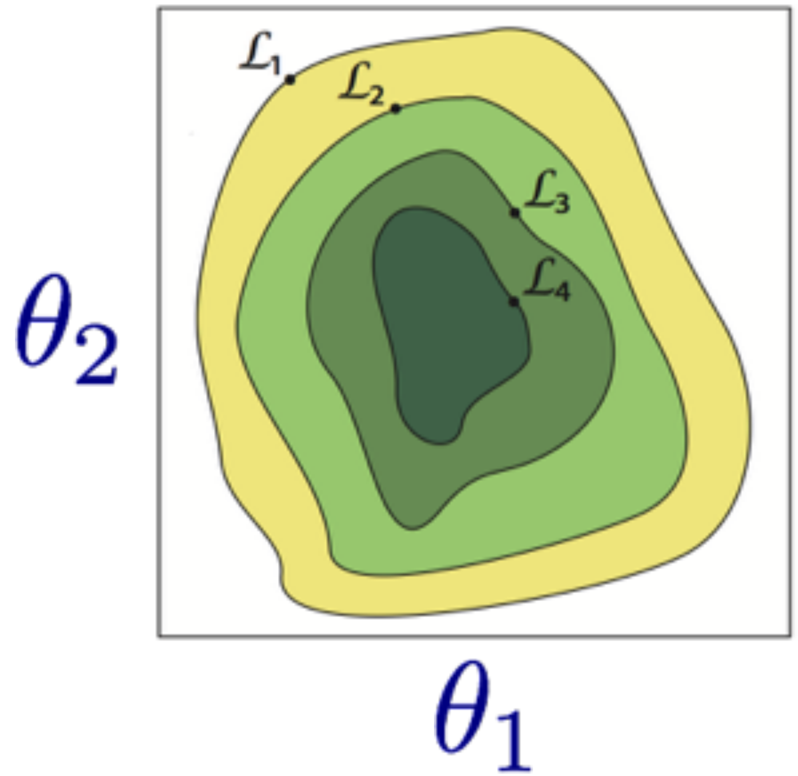
$$\mathcal{E} = \int_0^1 \mathcal{L}\left(X\right) dX$$



Nested Sampling
in a nutshell…

Bayesian Evidence

$$\mathcal{E} = \int_0^1 \mathcal{L}(X)\, dX$$

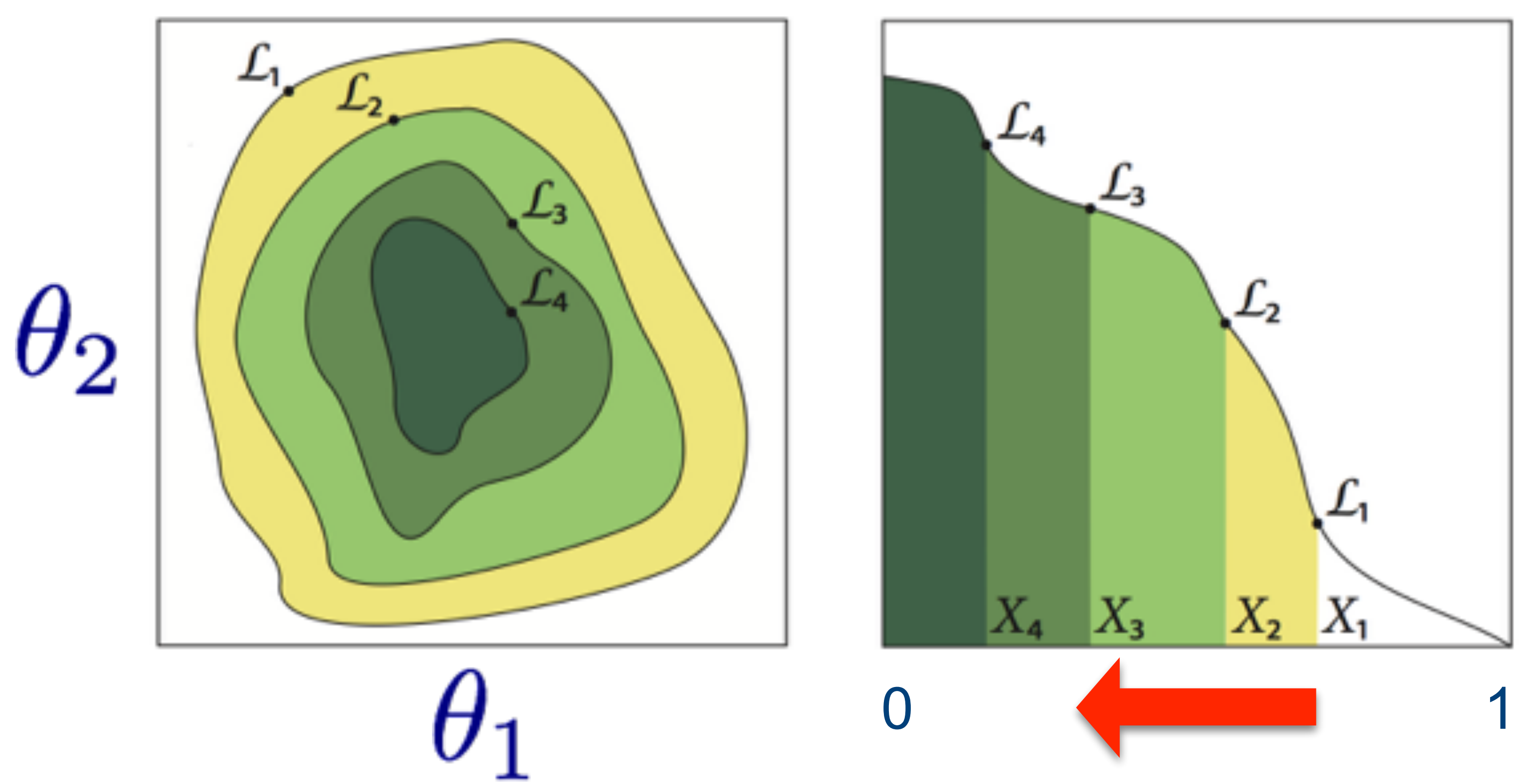


Nested Sampling

in a nutshell…

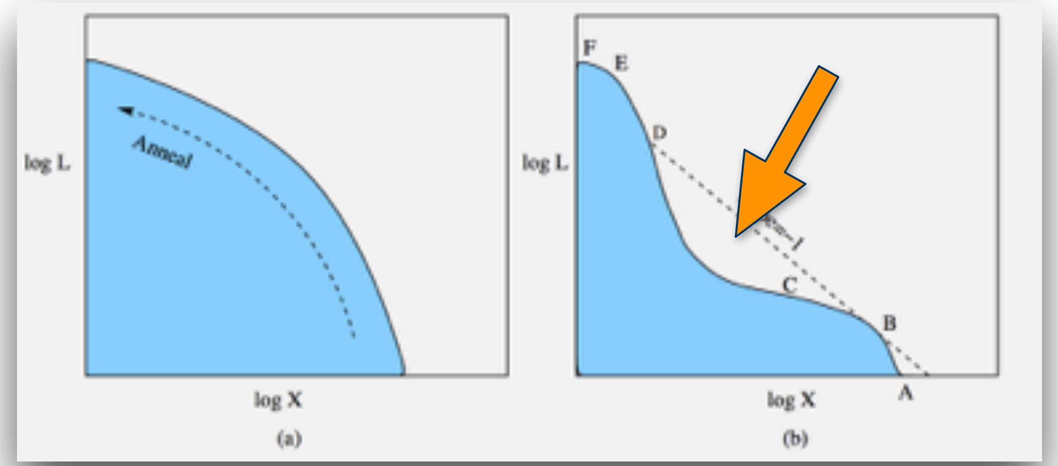
Bayesian Evidence

$$\mathcal{E} = \int_0^1 \mathcal{L}(X)\, dX \quad \longrightarrow \quad \mathcal{E} = \sum_{i=0}^{M} \mathcal{L}_i \Delta X_i$$



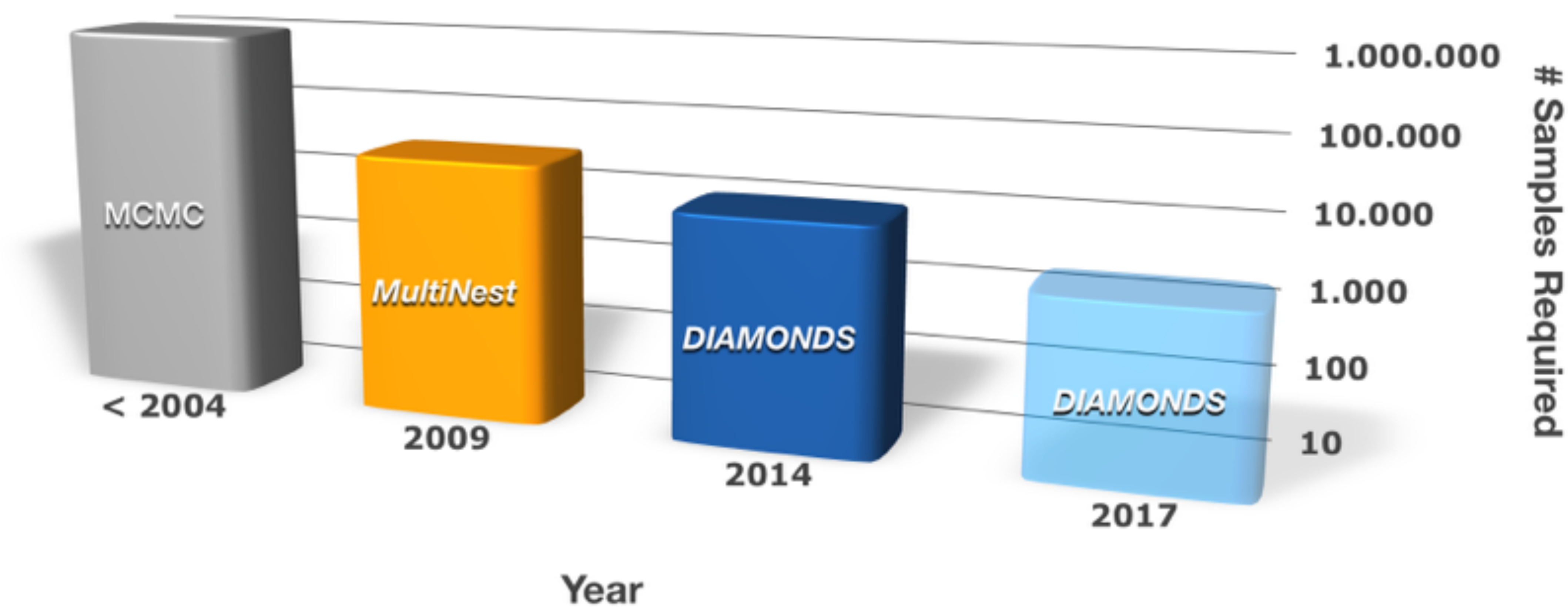
Nested Sampling
in a nutshell…

- **ADVANTAGES** with respect to Markov chain Monte Carlo:
  1. Typically **~100 times fewer** samples than thermodynamic integration to calculate evidence to same accuracy + error bar
  2. **Direct** solution to model comparison problems
  3. No troubles with phase changes in likelihood (**multi modal distributions**)



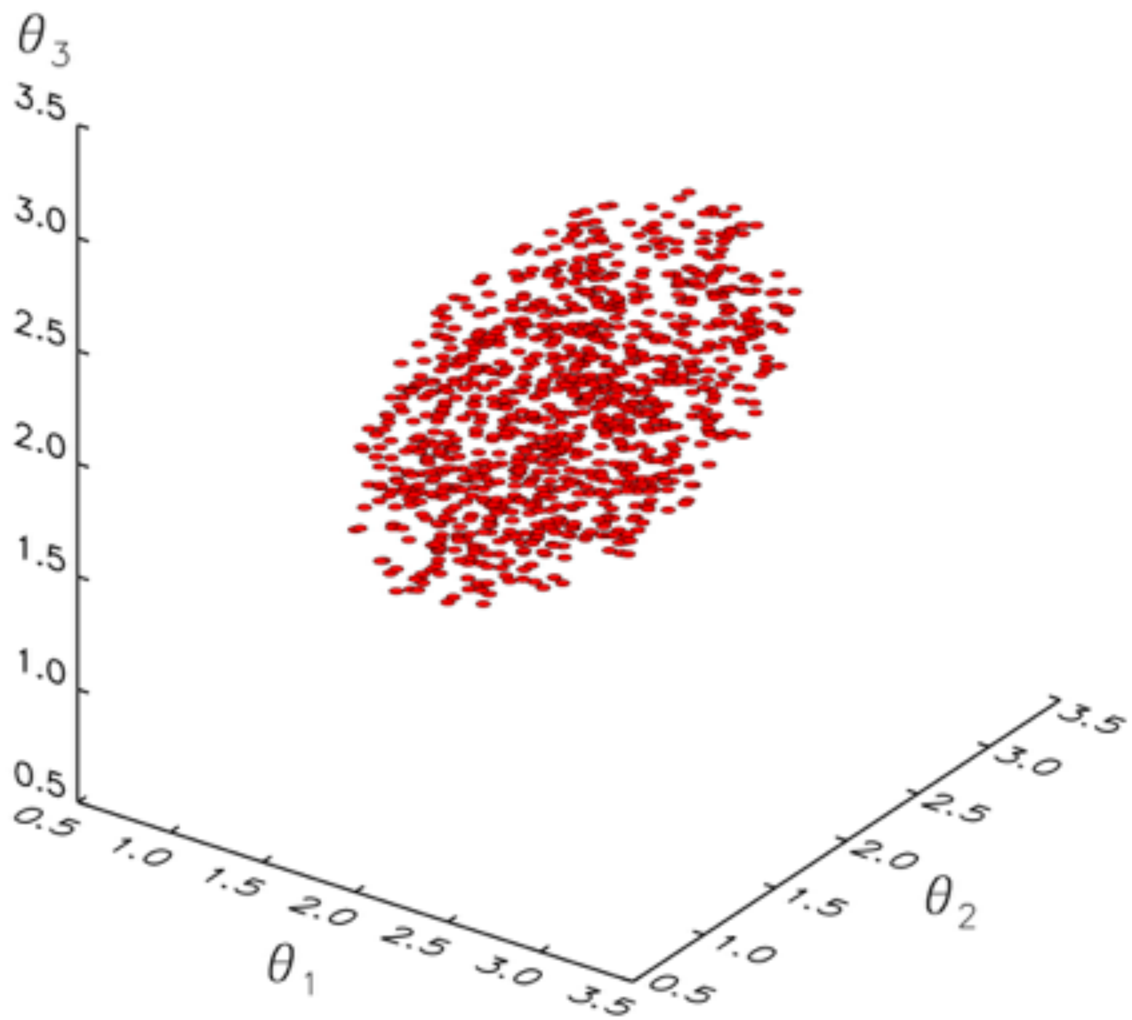
What makes DIAMONDS so appealing?

- Basic core **public** available
- General for **any** application involving Bayesian Inference
- Code implementation is **flexible** and easy to upgrade
- Different types of **prior** distributions and **likelihood** functions already provided
- **Overtakes** other existing MC, NS codes
- Attracted already more than **40 users** from many world's institutions and different fields of physics



Prior distributions

3D Uniform



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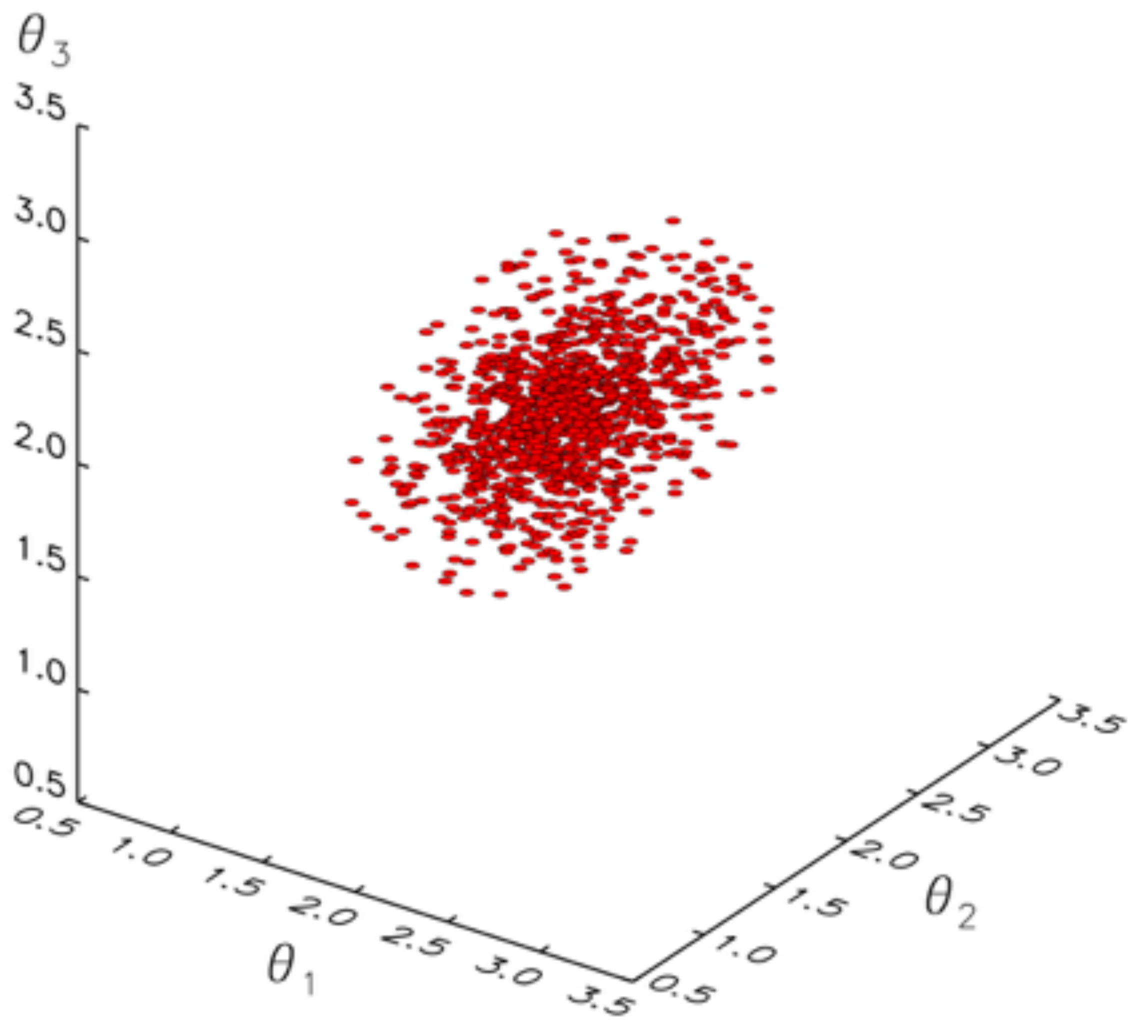
Prior distributions

3D Gaussian



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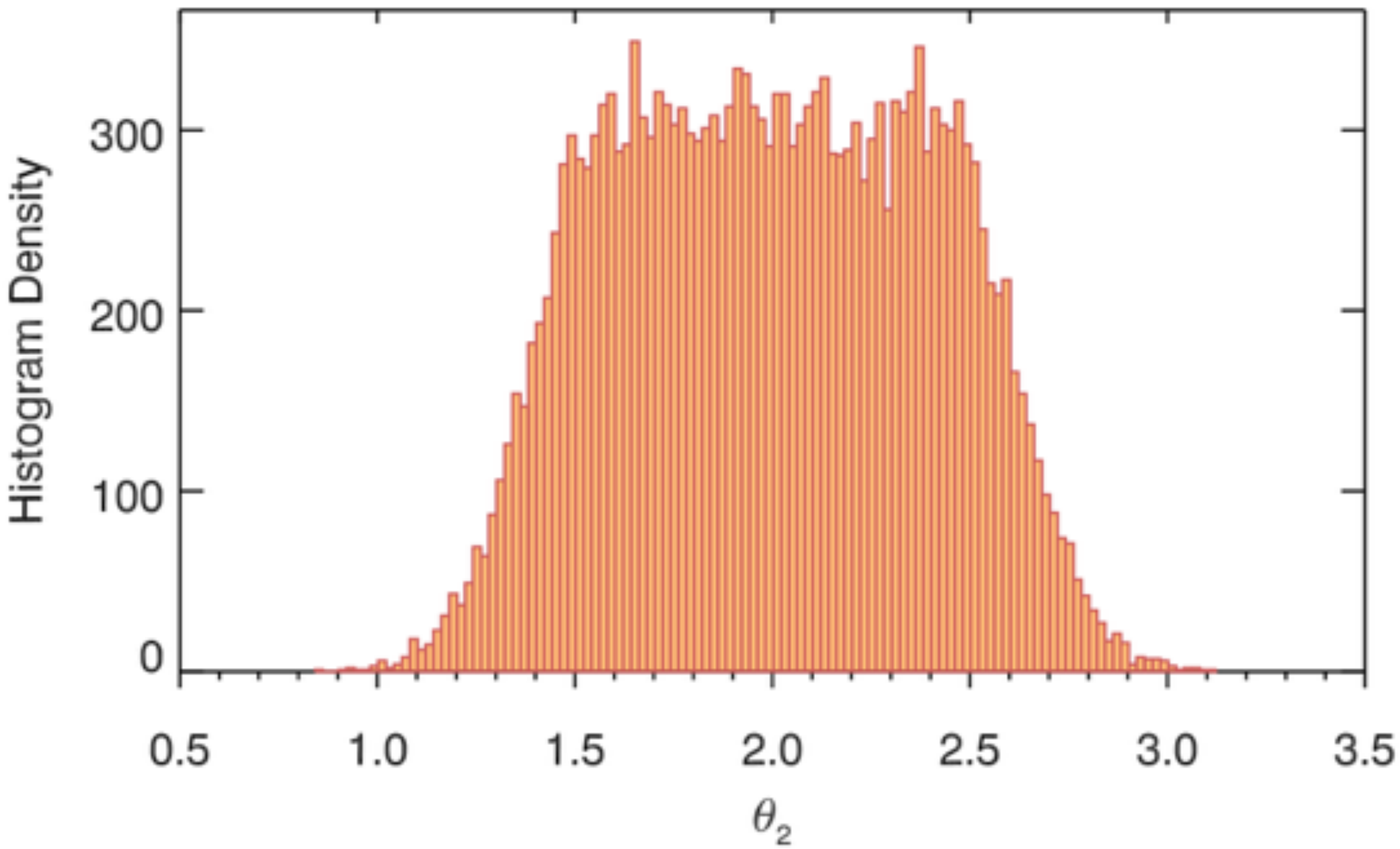
Prior distributions



1D Super Gaussian

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Sampling efficiency demos

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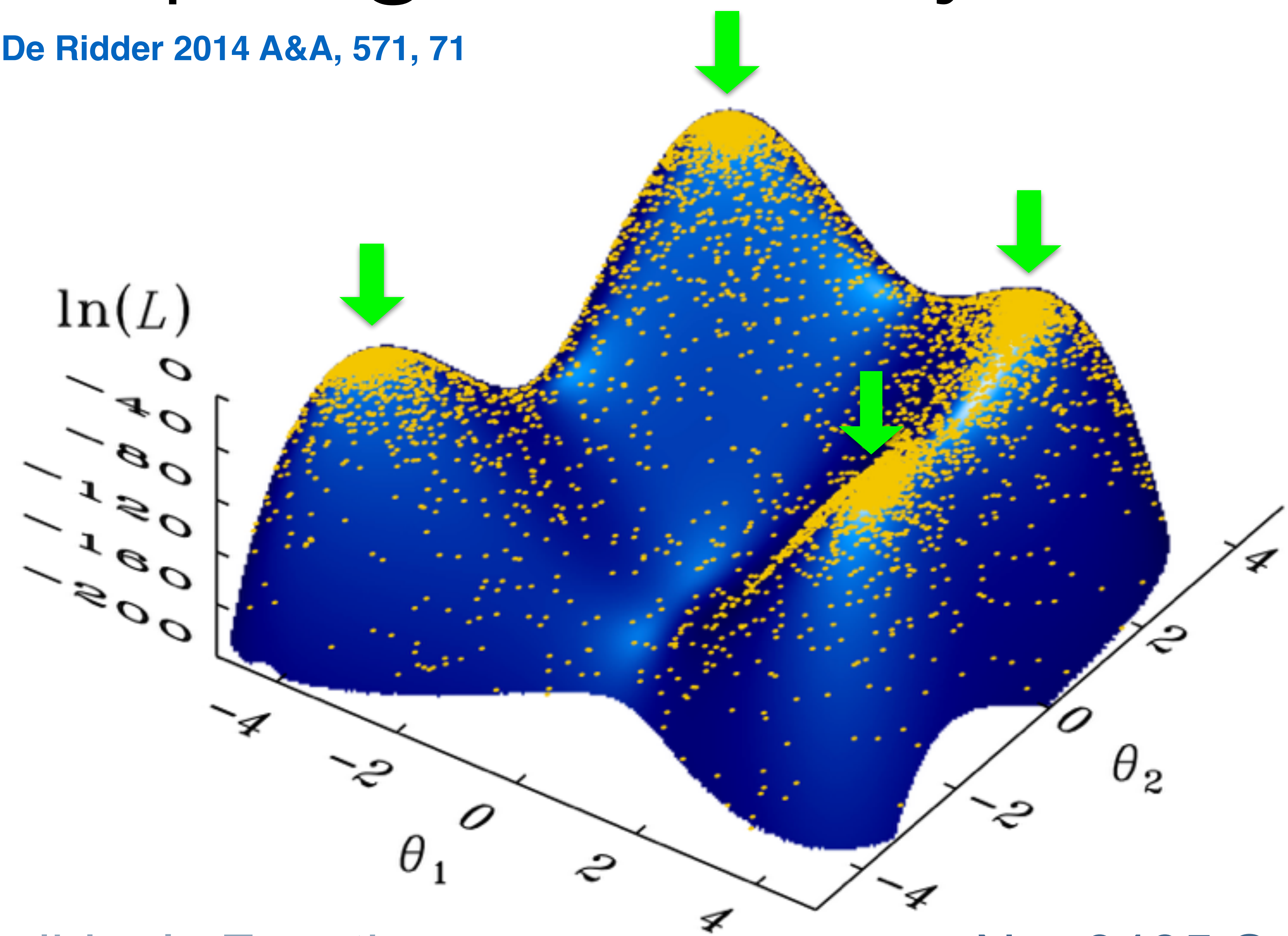

Himmelblau's Function

N = 8485 Samples

Sampling efficiency demos

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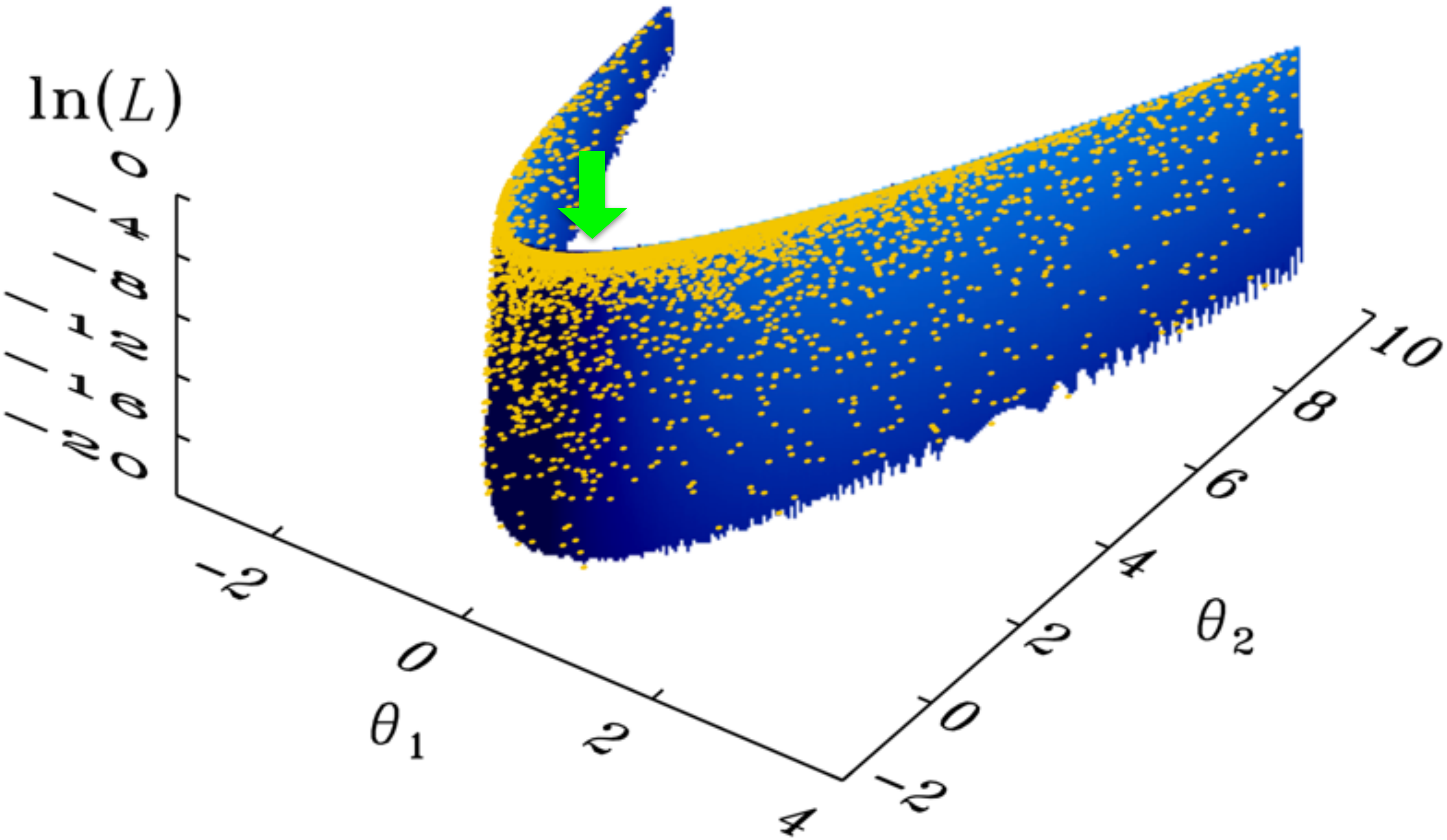

Rosenbrock's Function

N = 8558 Samples

Sampling efficiency demos

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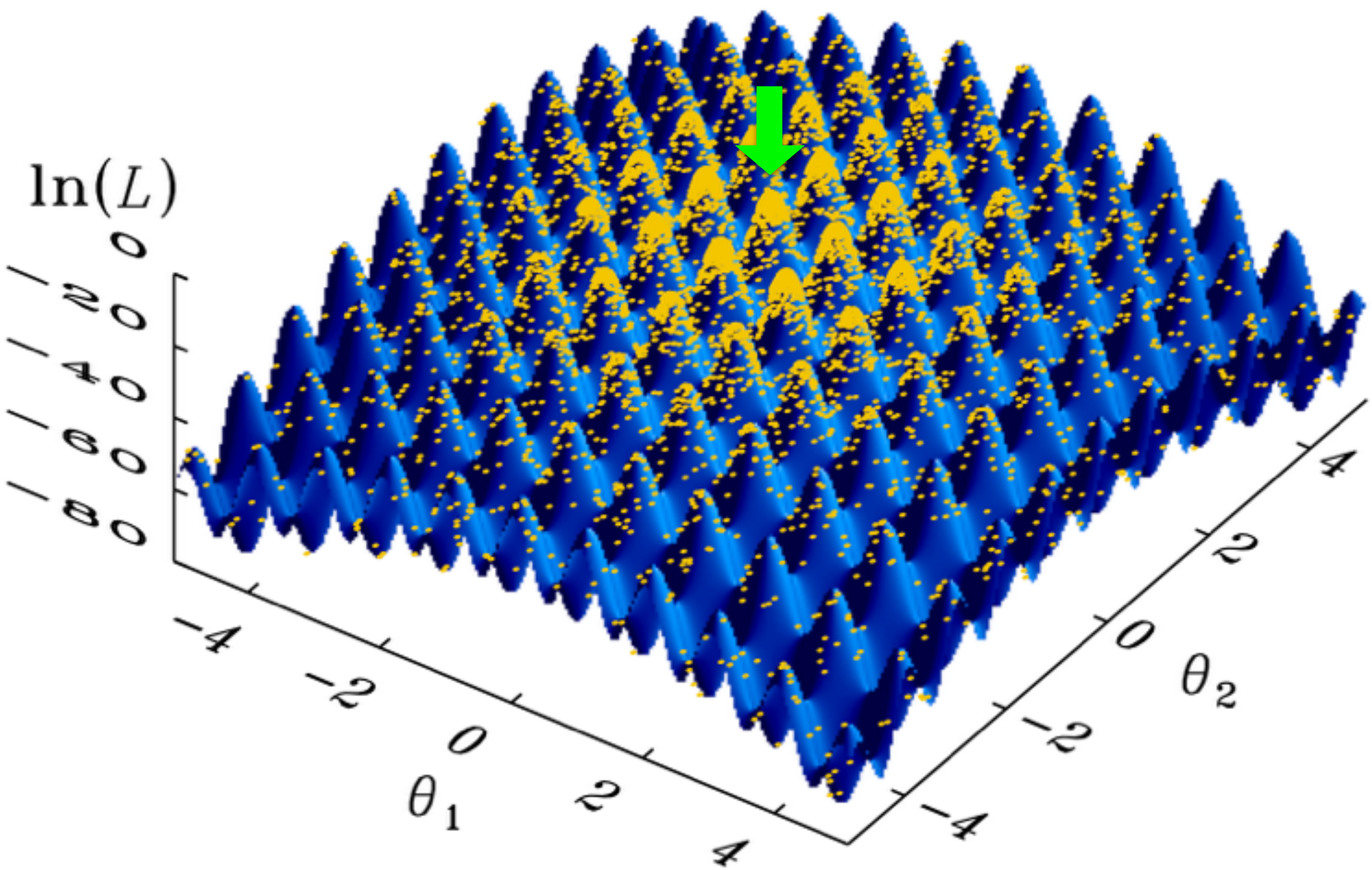

Rastrigin's Function

N = 10648 Samples

Sampling efficiency demos

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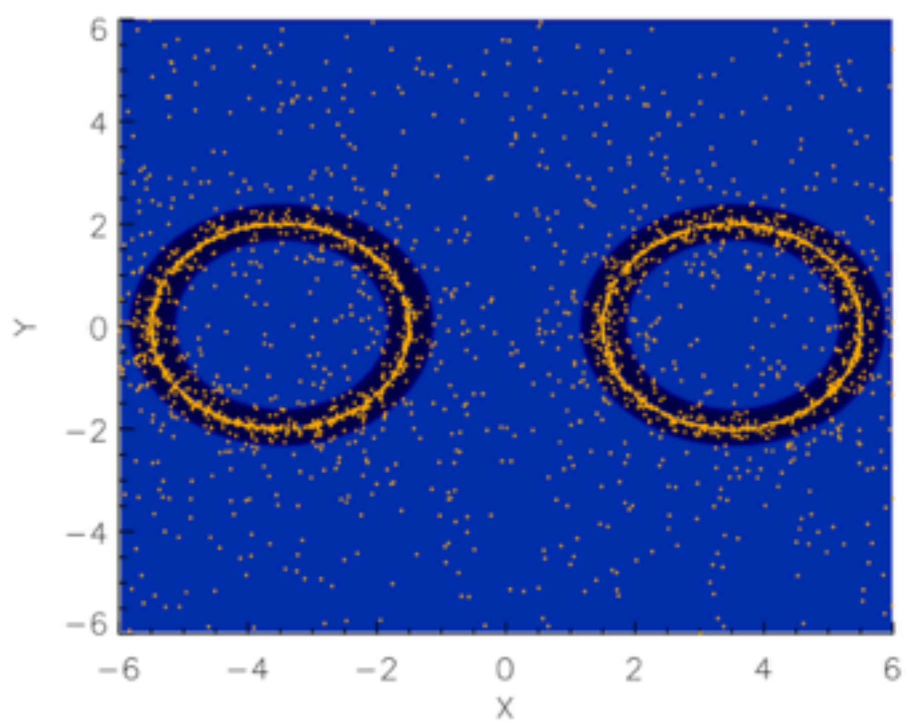

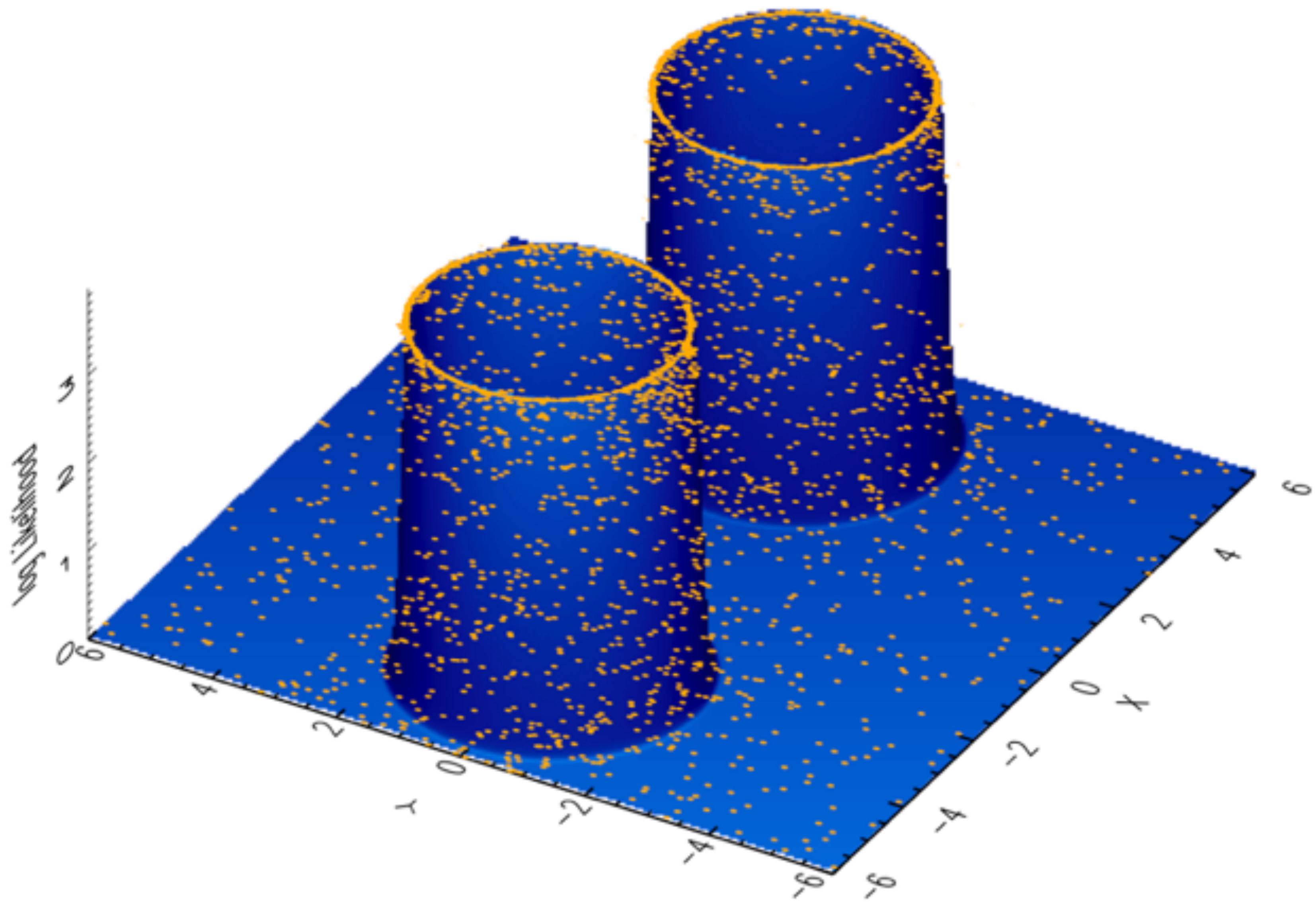

Gaussian Shell Function

N = 3100 Samples

Sampling efficiency demos

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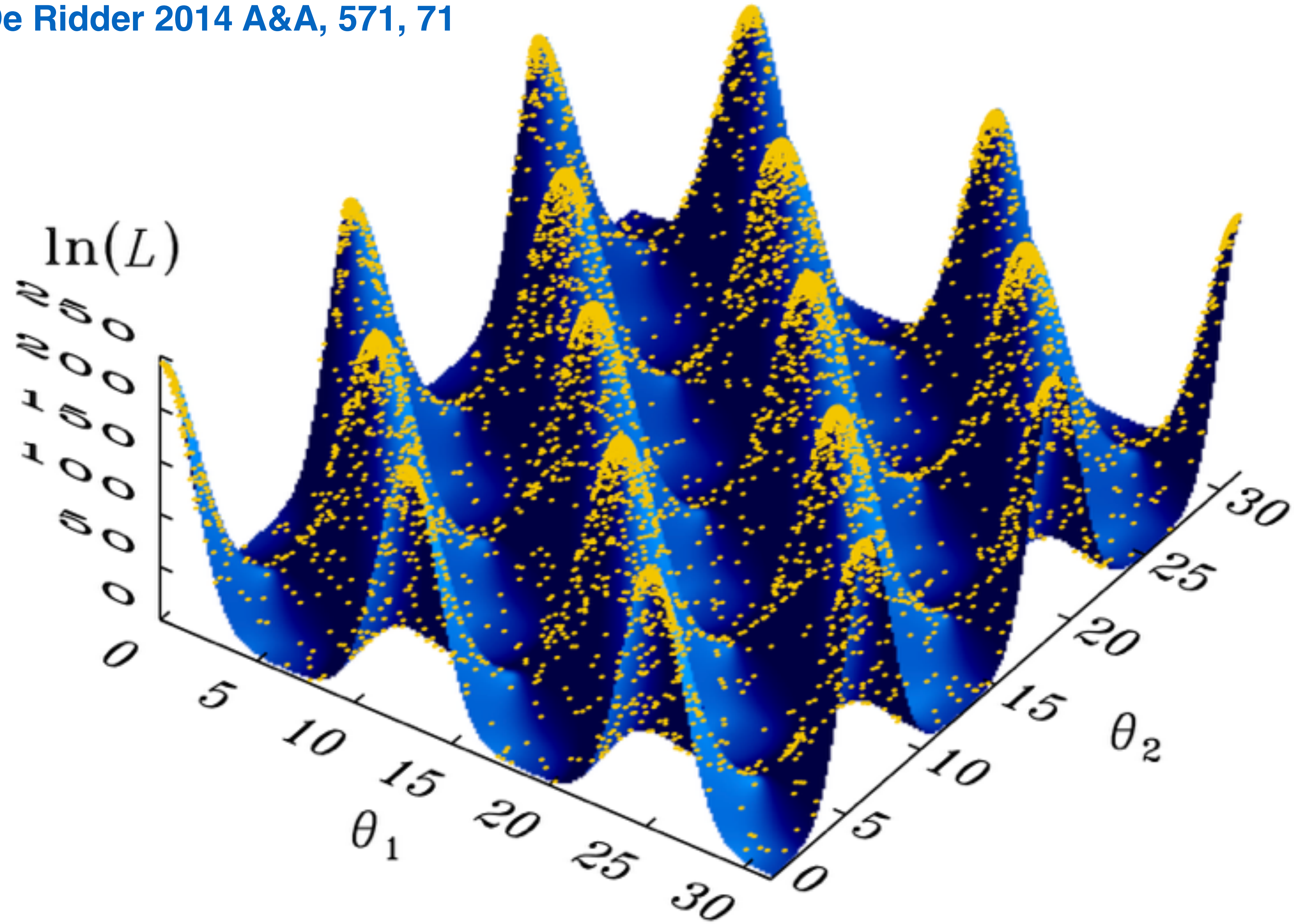


Eggbox Function

N = 8207 Samples

Application

Why do we need DIAMONDS?

1. Tackling **high-dimensional** and/or **multi-modal** fitting problems at high speed (otherwise very difficult, if not impossible, to solve with standard methods and available computational power)

2. Easy and direct solution to model comparison problems

For example?

Asteroseismology!

What is Asteroseismology?

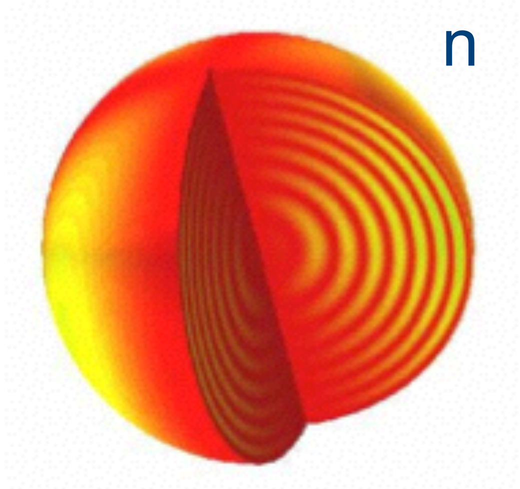
The analysis of stellar oscillations to probe stellar structure, dynamics, and evolution

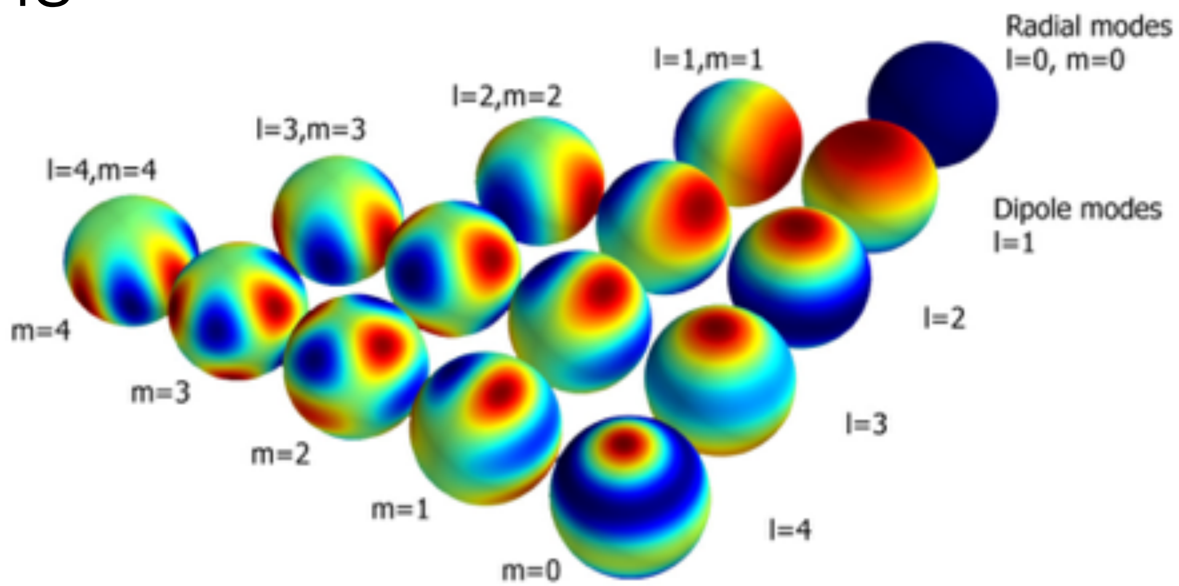
- The **most powerful** available approach to look inside the stars!
- Our main example: **the Sun** (helioseismology)
- Many stars oscillate similarly to the Sun (**solar-like**): about **40,000 known to date** and growing every year

Solar-like oscillations

- Acoustic waves from convection in envelope of low- and intermediate-mass stars (**p modes**)
- Produce tiny brightness variations on the surface (**10^{-6} - 10^{-3} mag**)
- Each oscillation mode can be identified by three quantum numbers

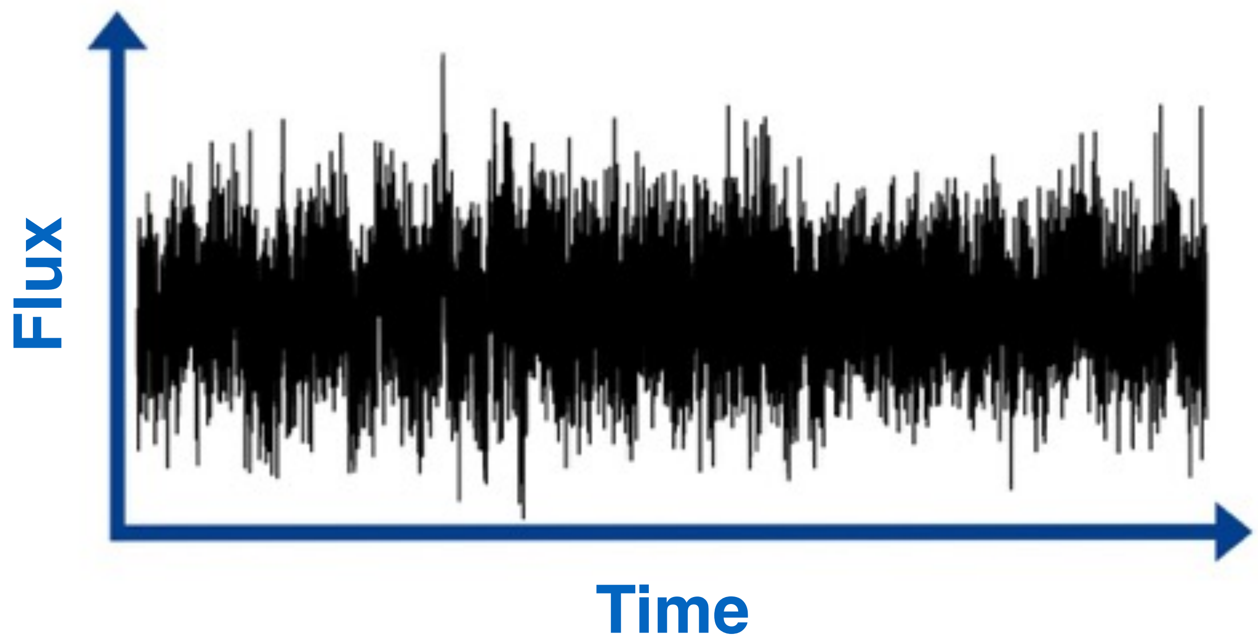
$$\nu_{\mathrm{n,l,m}}$$





Beck & Kallinger S&W 2013

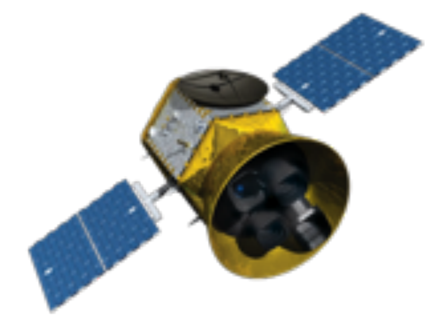
Solar-like oscillations



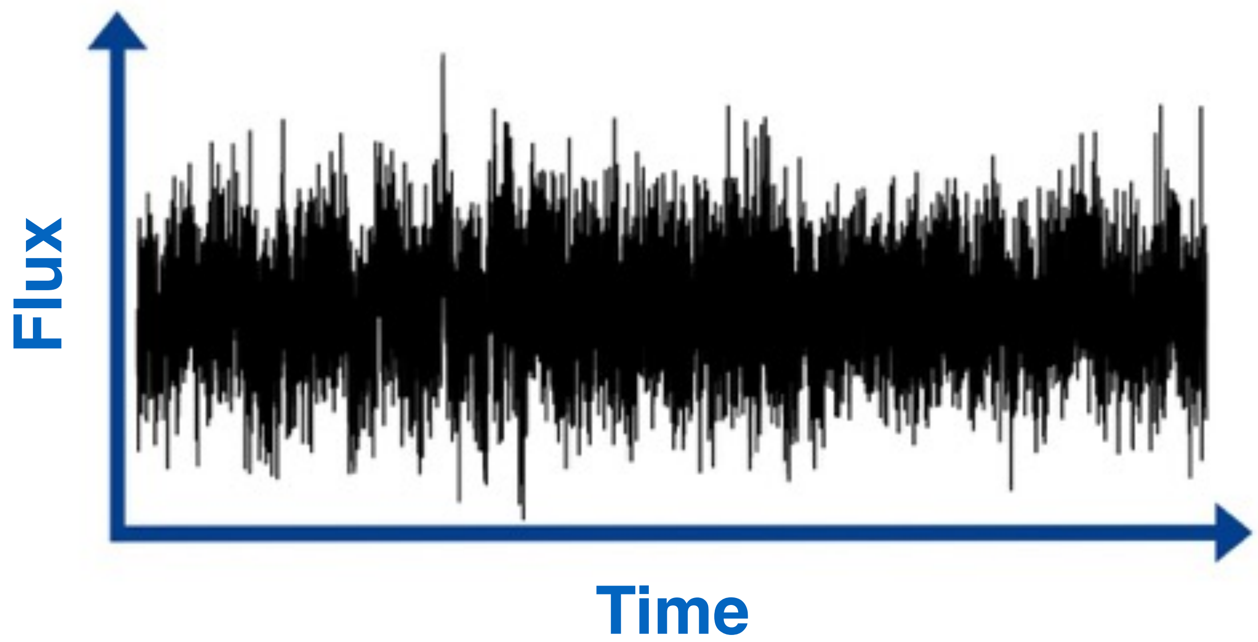
CoRoT

Kepler, K2

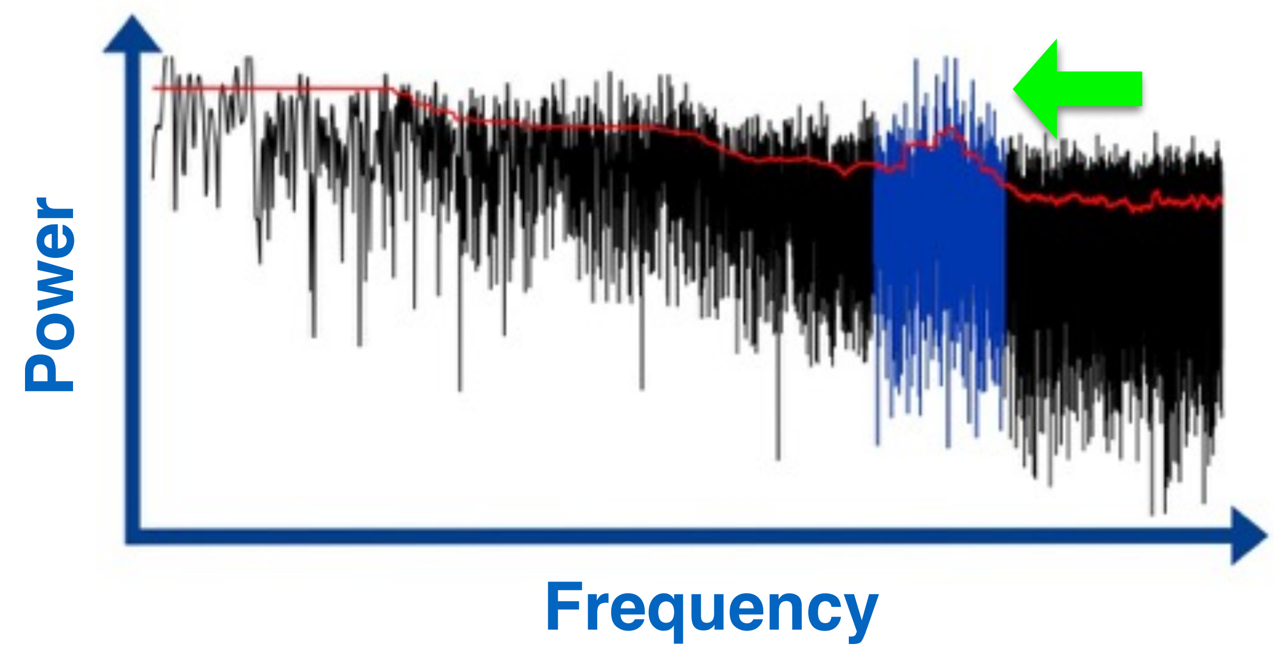
TESS



Solar-like oscillations

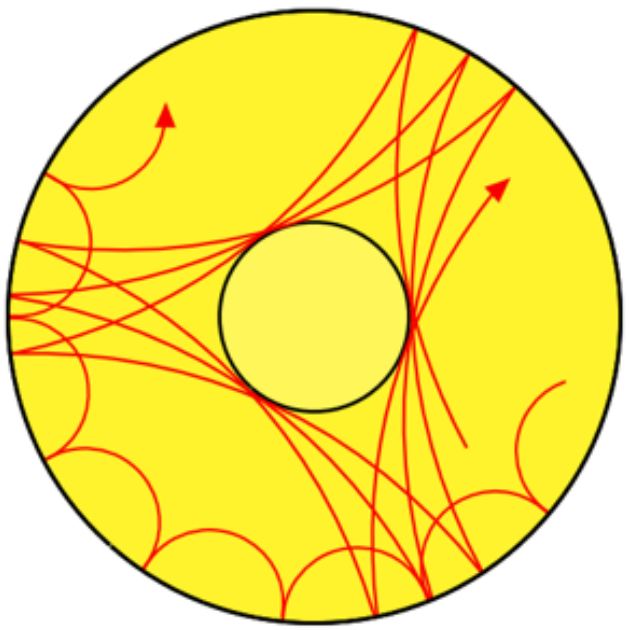


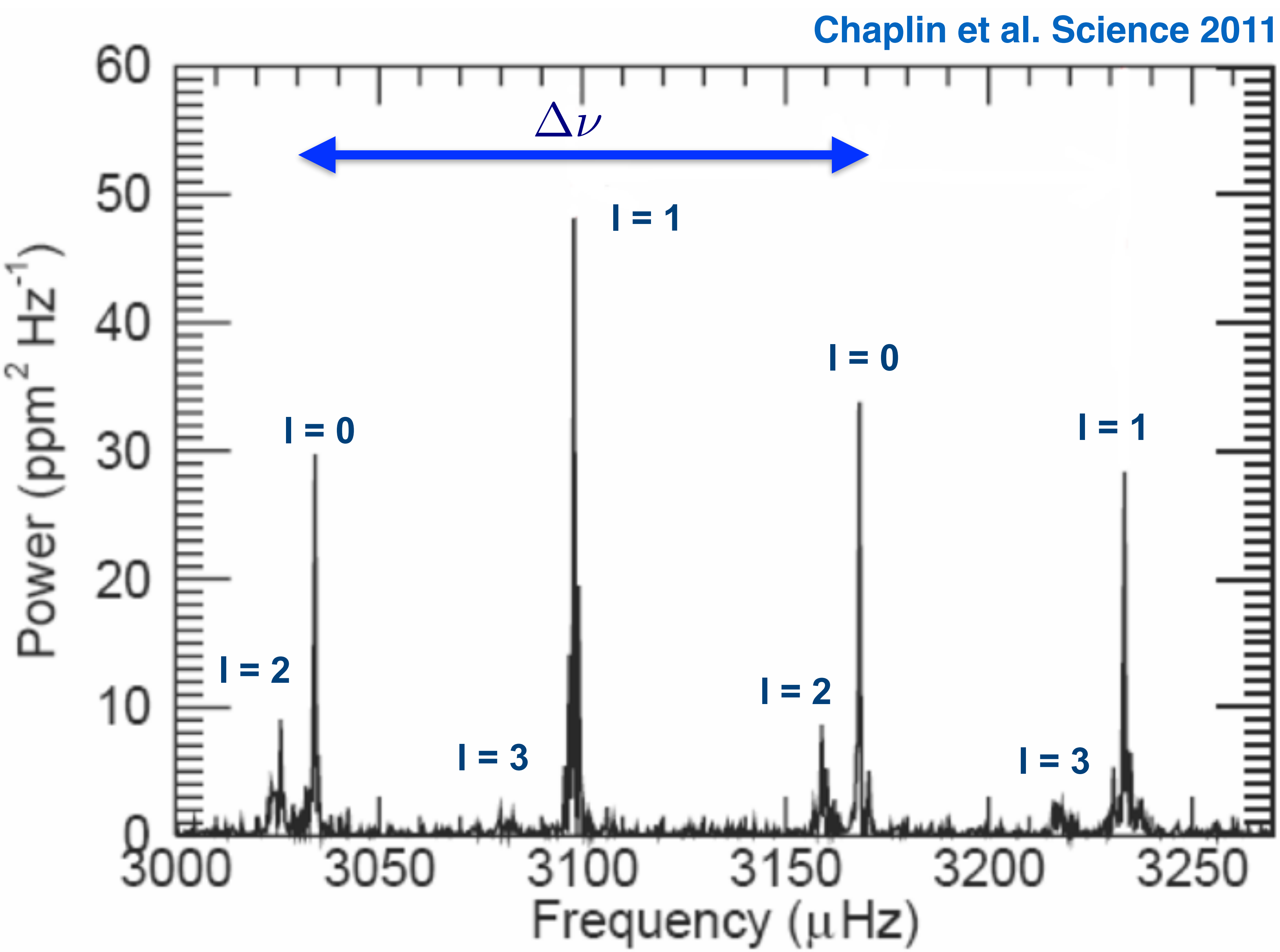
Fourier Transform

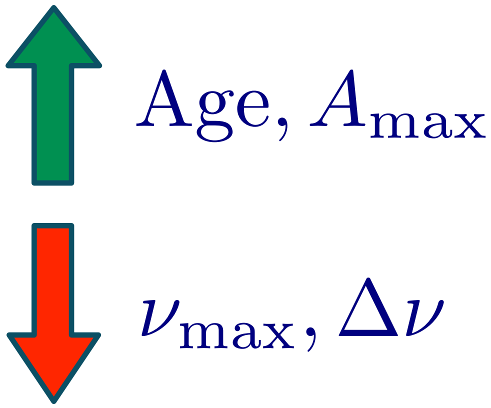


Fine-structure of p modes

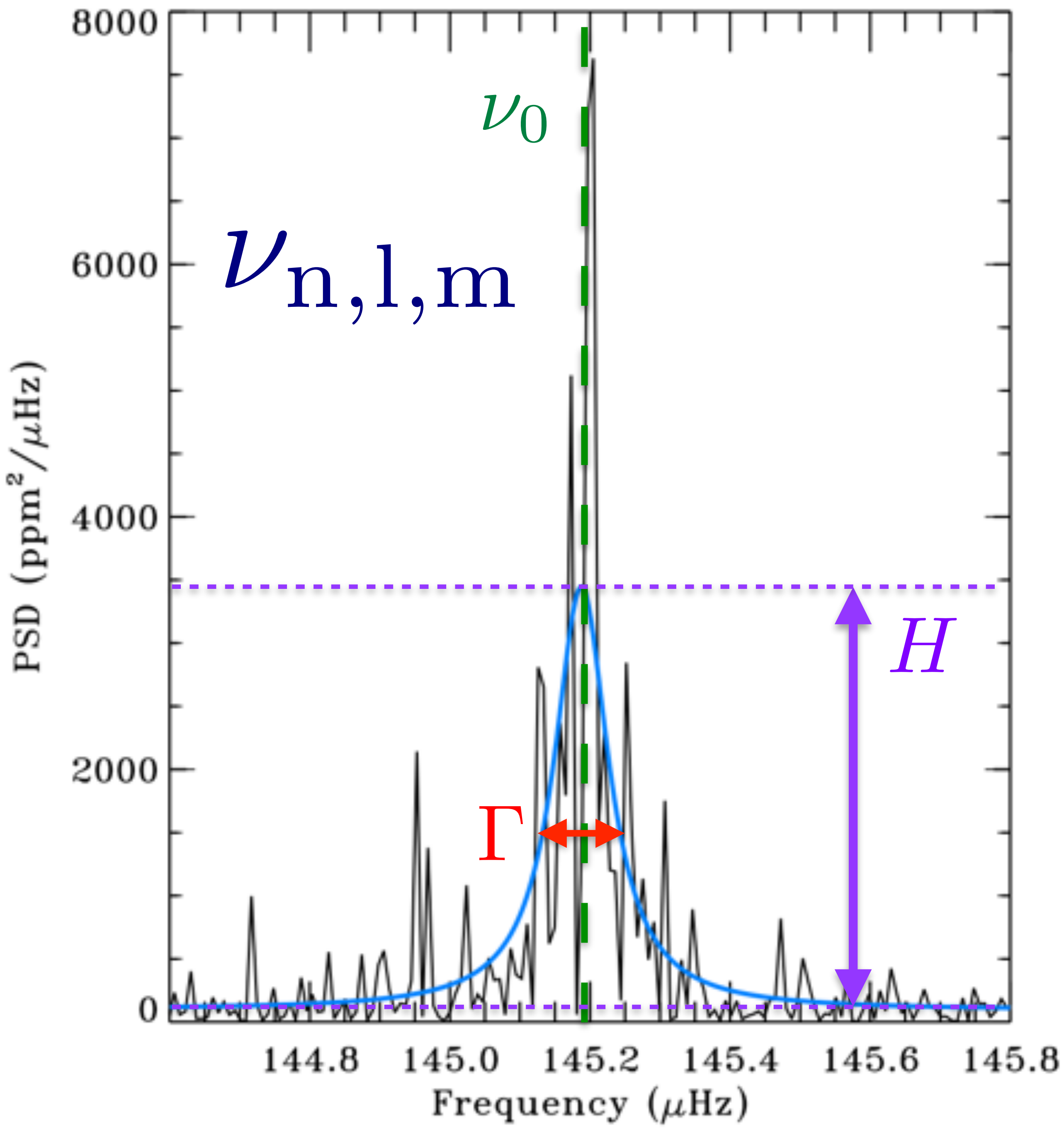
$\Delta\nu \propto \bar{\rho}$







Fine-structure of p modes



Damped oscillation

Lorentzian profile

$$T_{\mathrm{obs}} \gg \tau$$

$$\Gamma \propto \tau^{-1}$$

$$A^2 = \pi H \Gamma$$

$$\nu_0, \Gamma, A$$

Why do we need this?

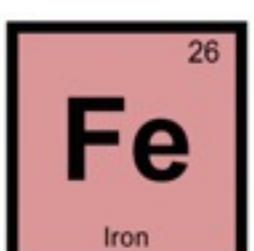
Constrain and understand to the best level possible

+ Spectroscopy

Physical Properties & Internal Structure

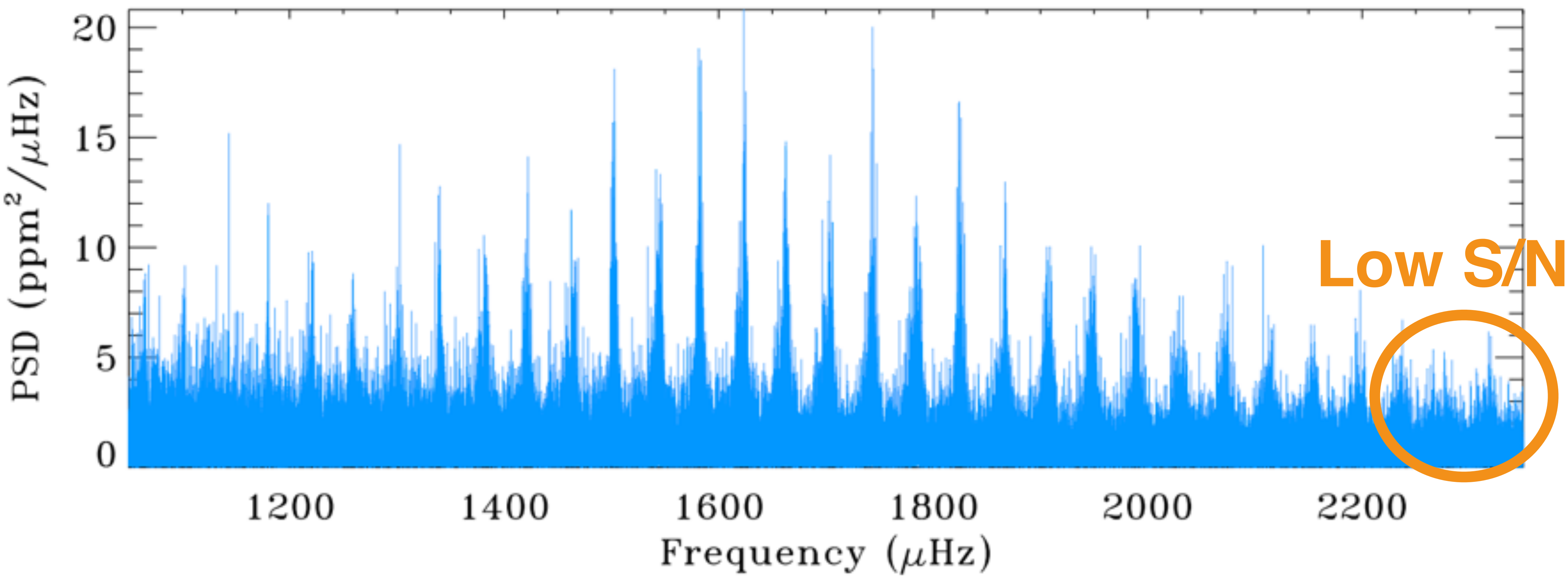
Mass, Radius

Position of BCZ, HeII
Evolutionary stage



Metallicity effect

Problems



- **Problem 1**: big dataset + fitting numerous oscillation modes (peaks) per star (can be more than 100)
- **Problem 2**: testing if a peak is real or not (noise)

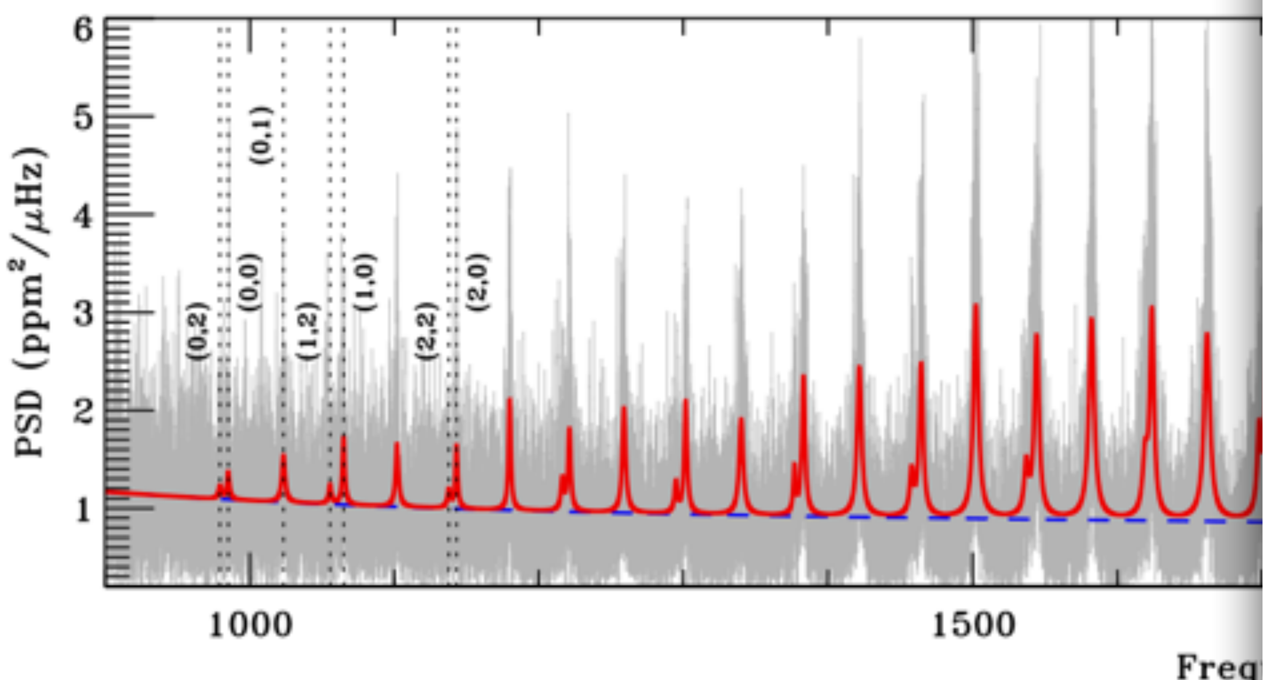
Problem 1

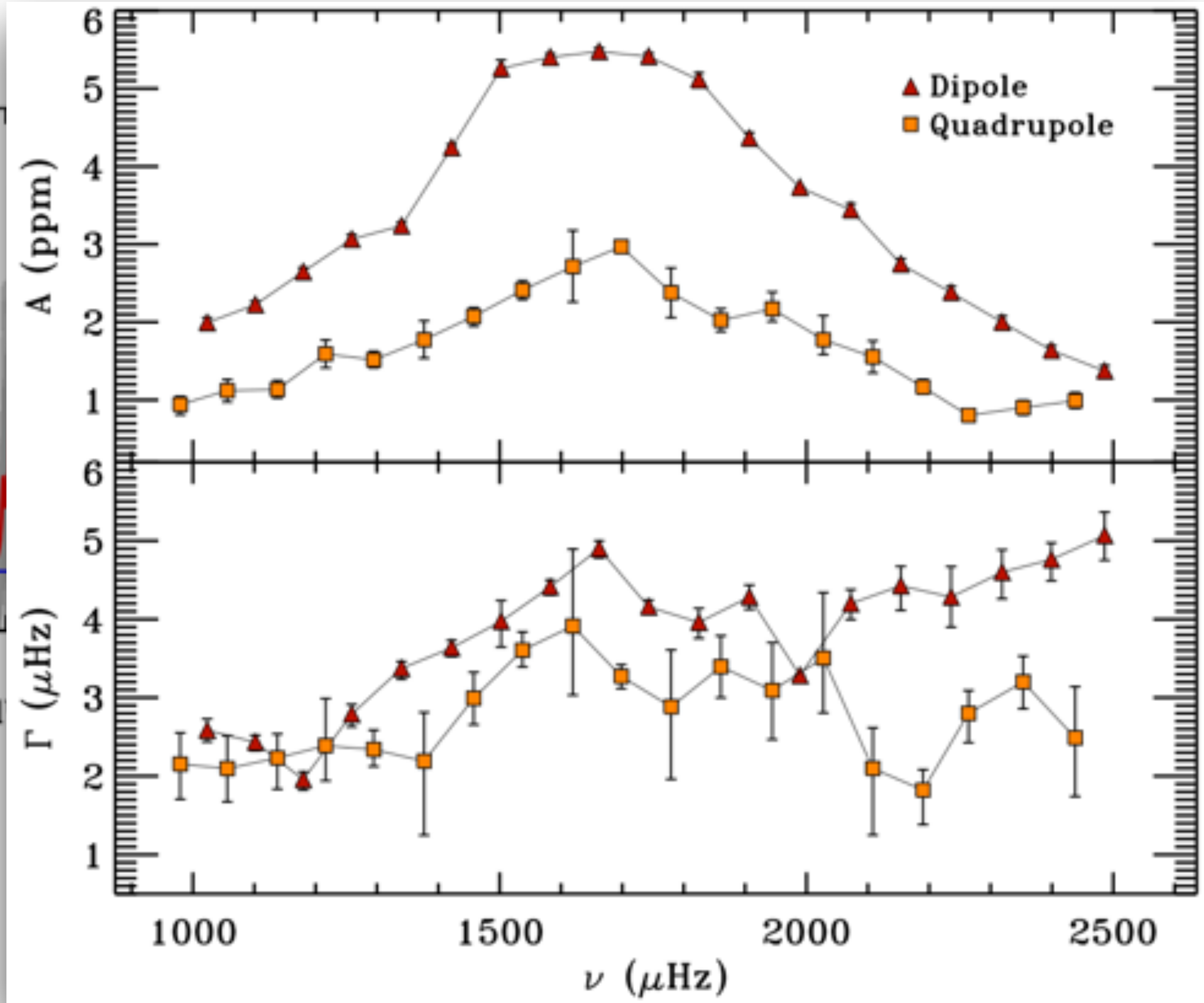
Solving a high-dimensional fitting problem

High-dimensional Model

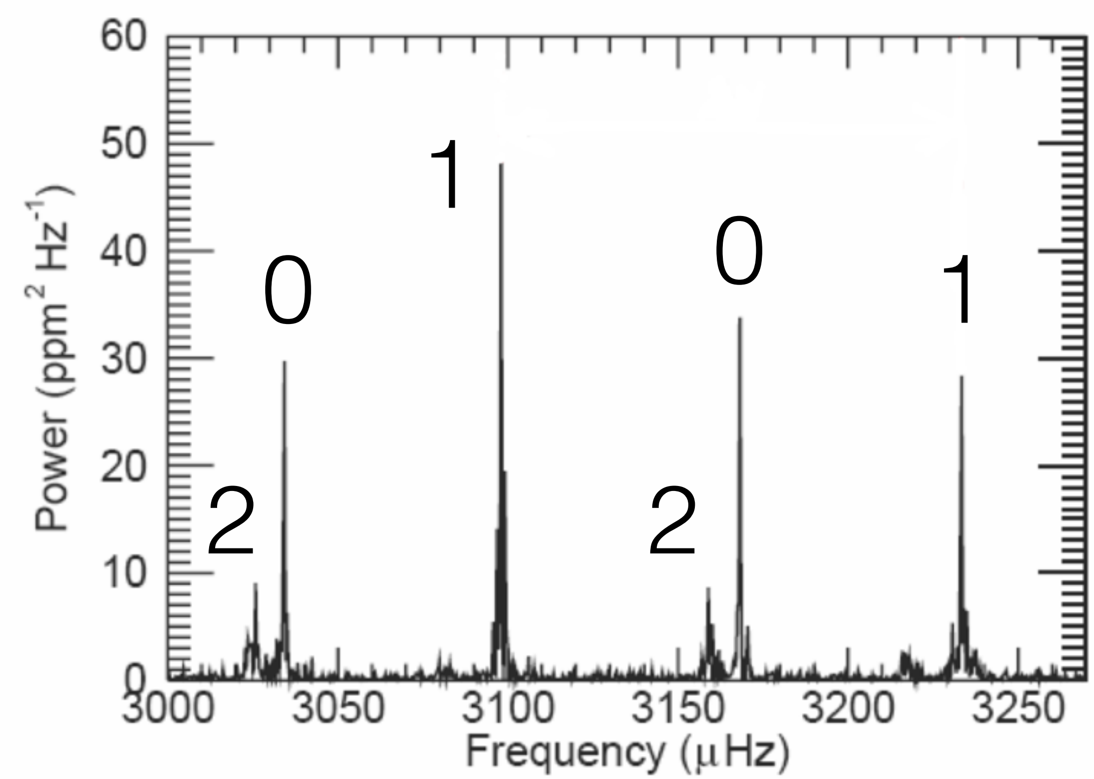
About 180 free parameters!
Computational time increases a lot

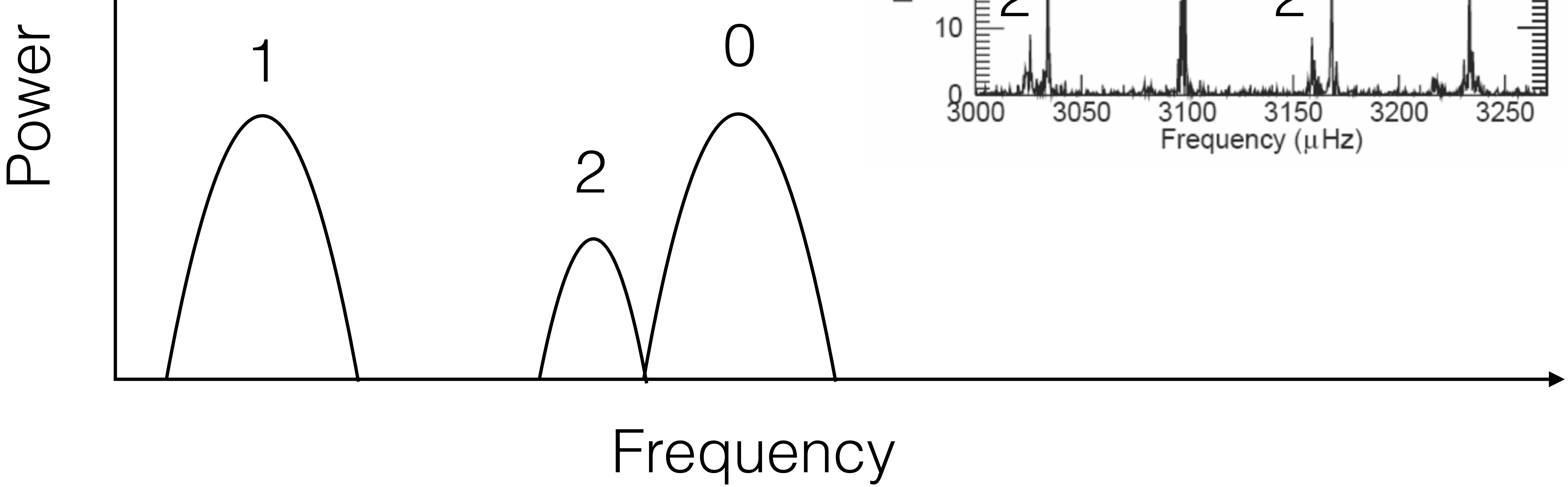
KIC 9139163



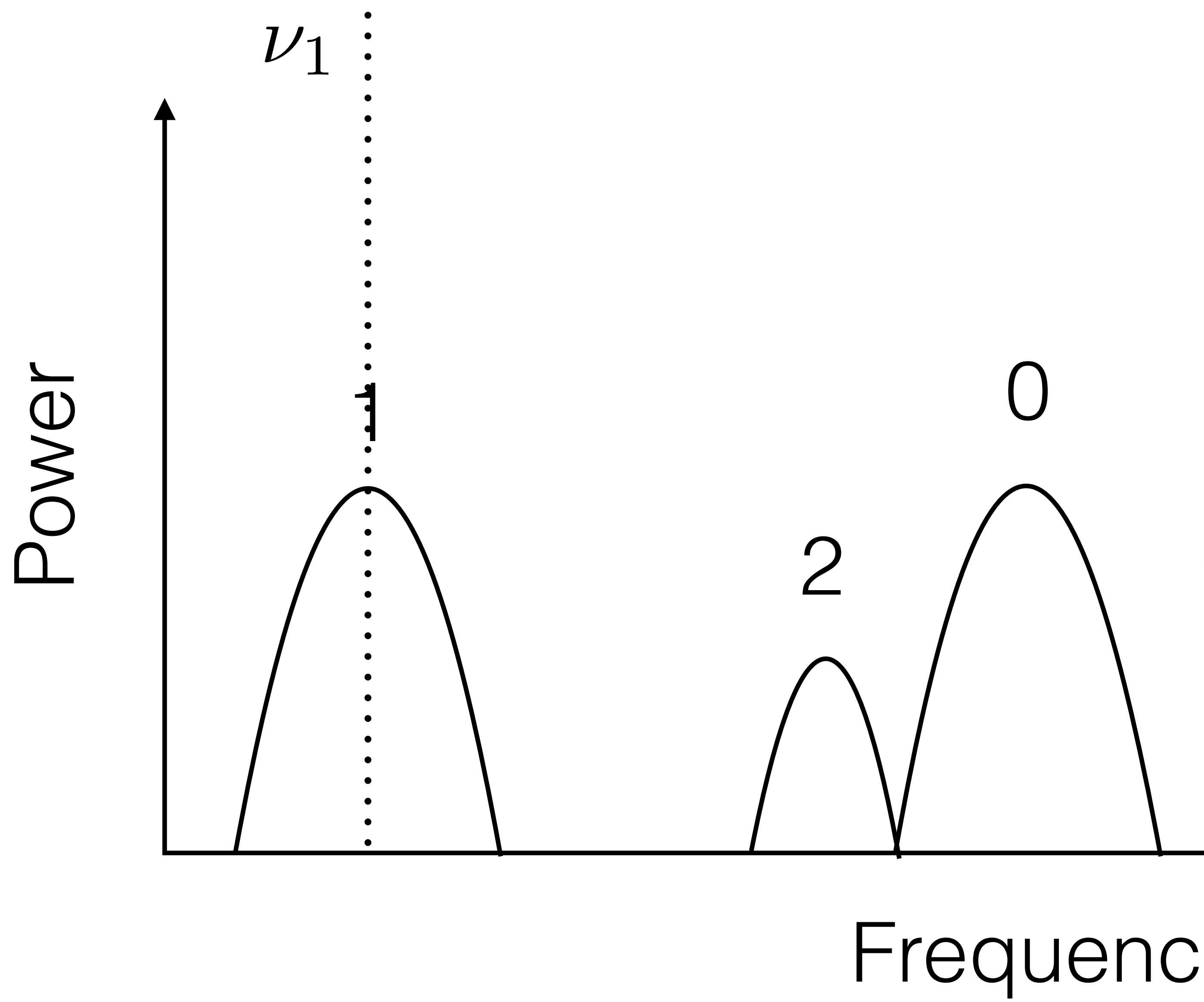


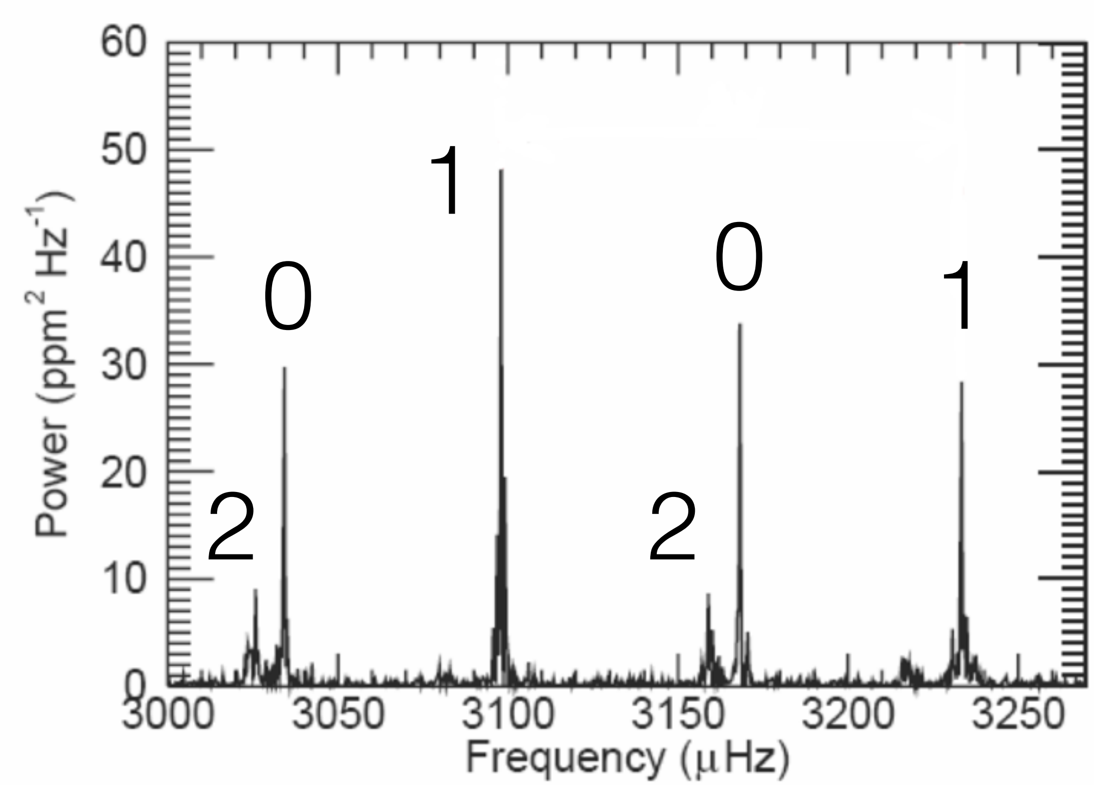
Multi-modal Model



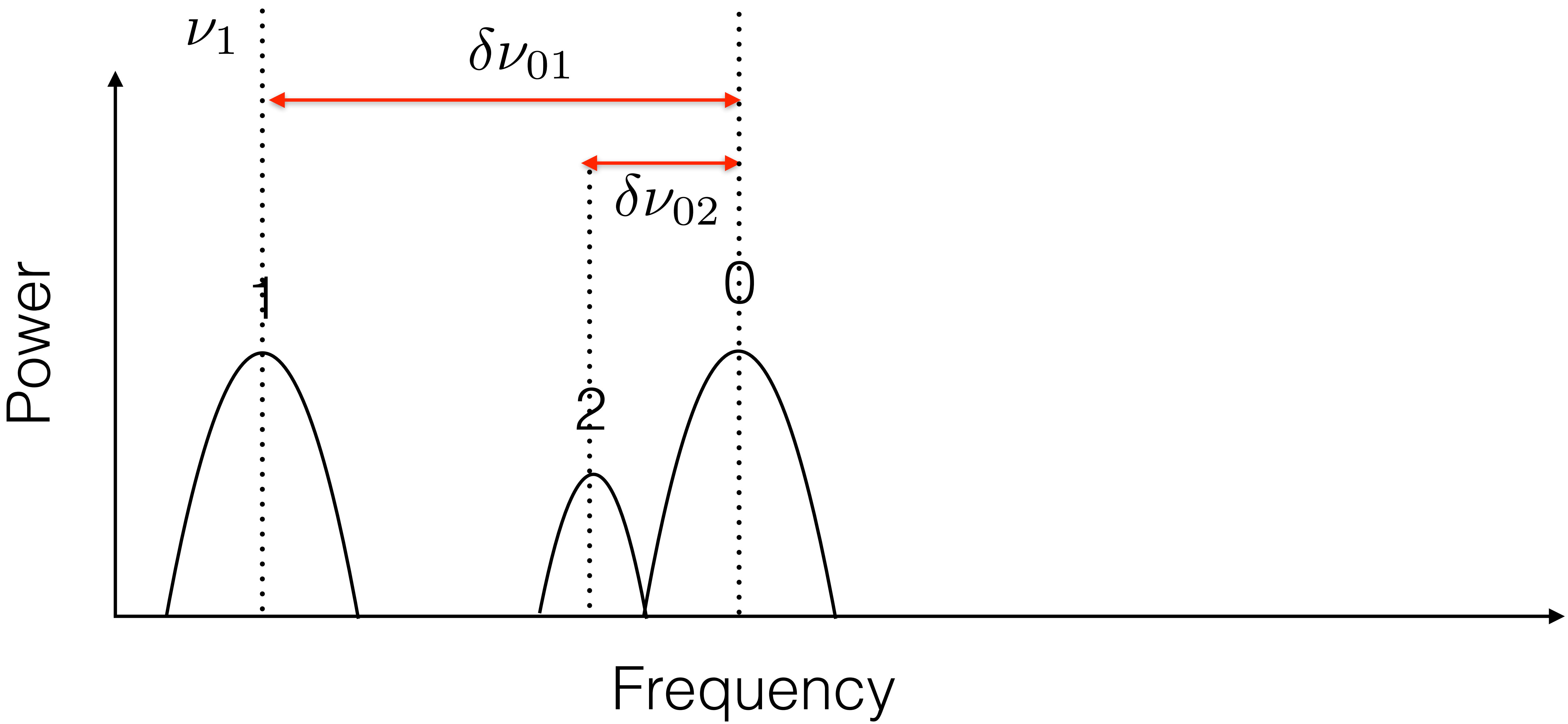


Multi-modal Model

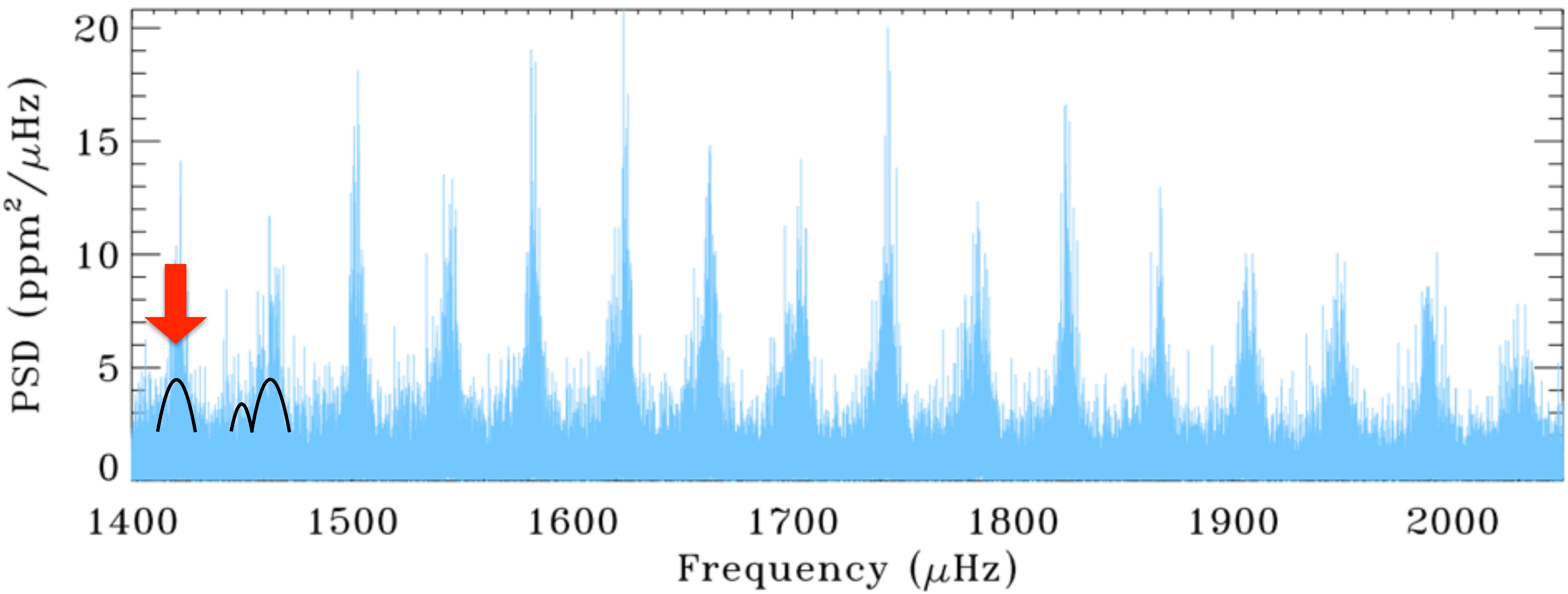




Multi-modal Model



Multi-modal Model

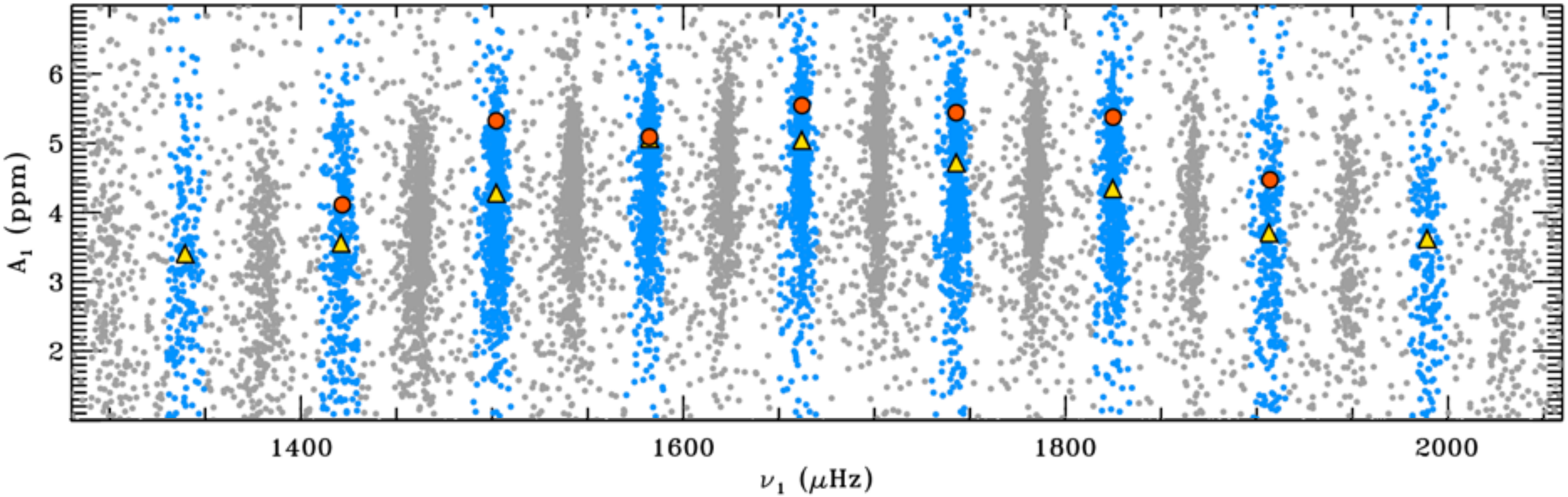


Corsaro & De Ridder 2014 A&A, 571, 71

Results

Multi-modal inference problem on 9 consecutive radial orders (27 peaks)

Only 9 free parameters!



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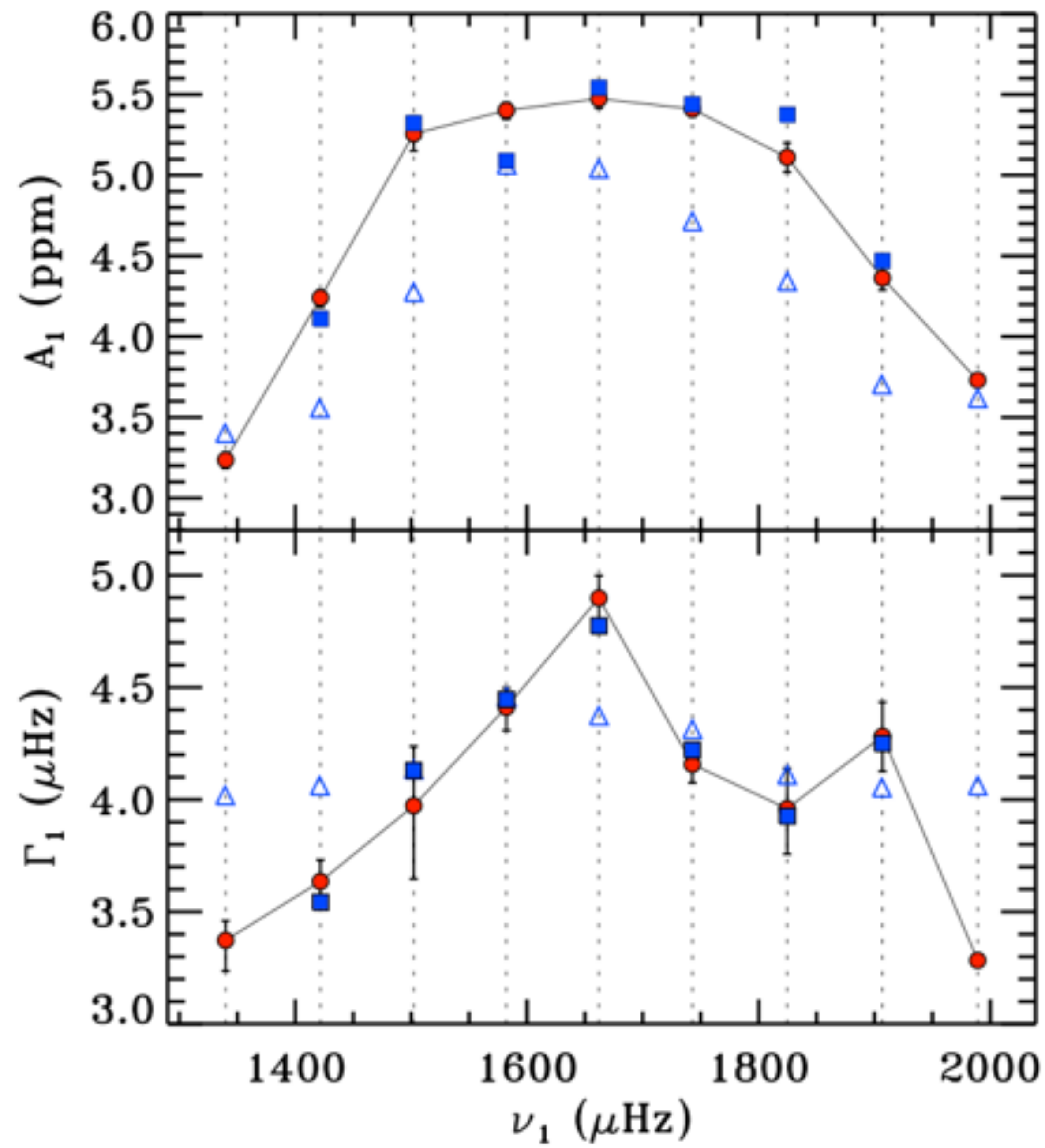
Comparison

Red: uni-modal fit
Blue: multi-modal fit



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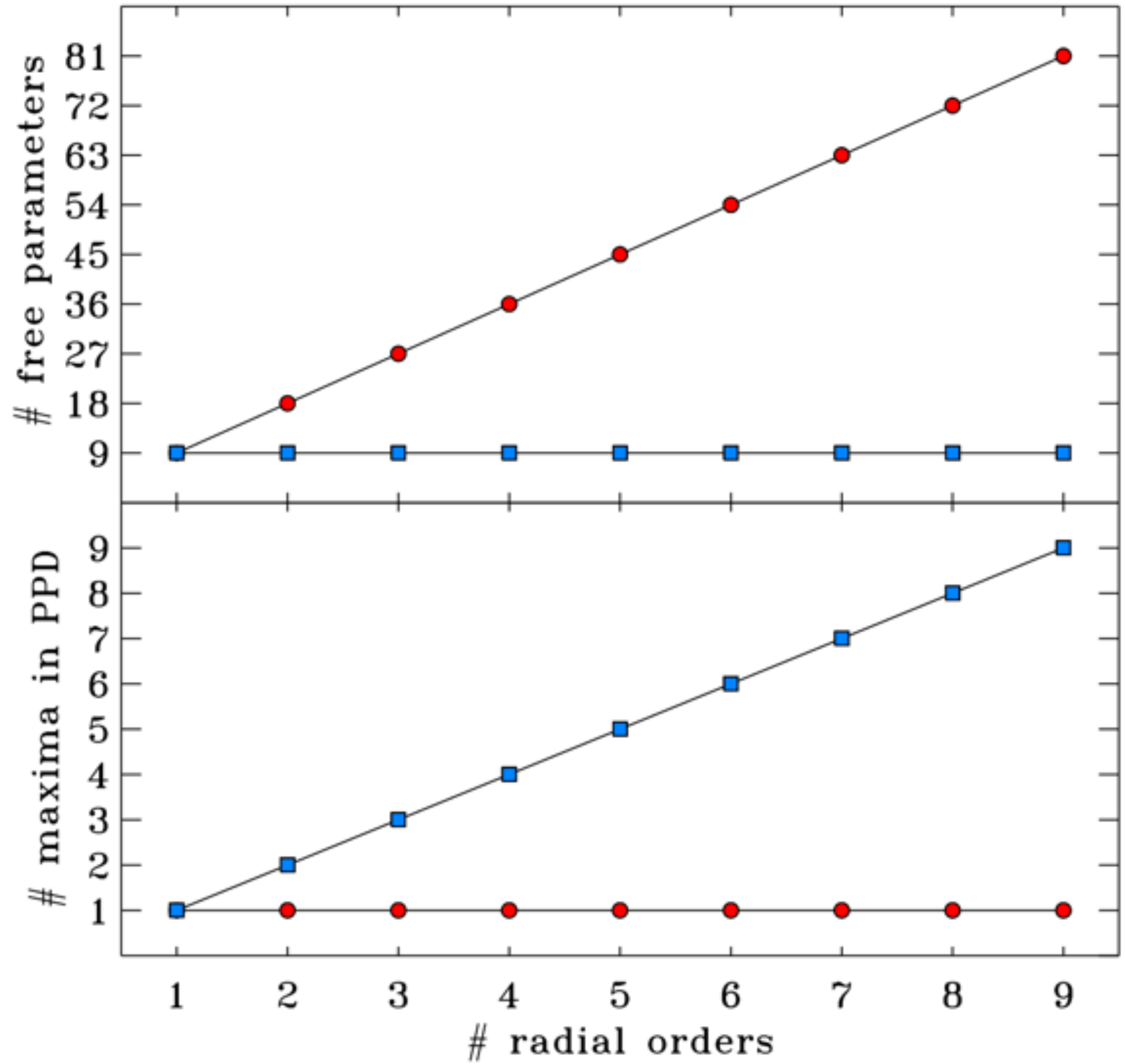

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Problem 2

Test the significance of an oscillation peak

Bayesian Model Comparison

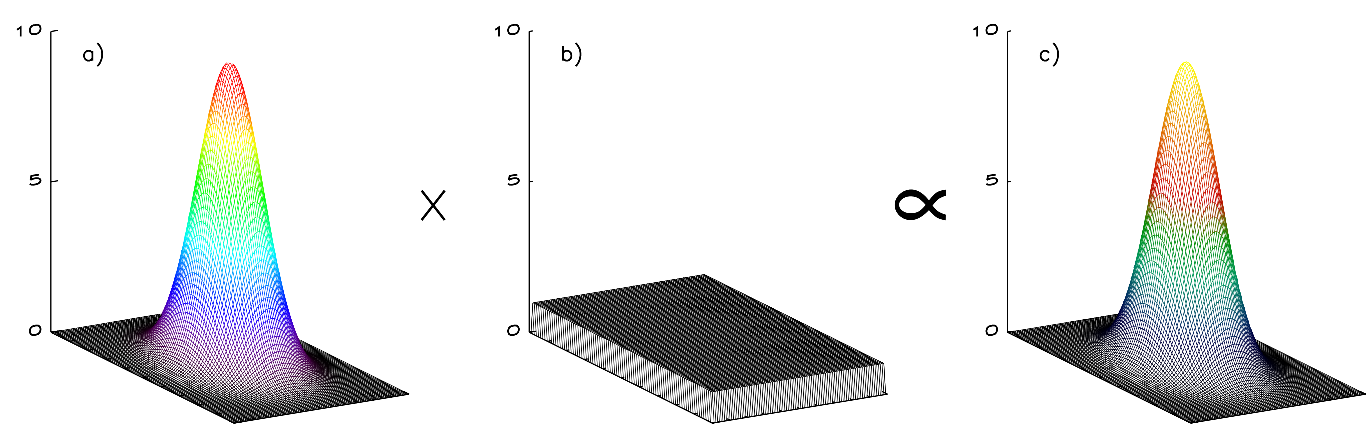
Bayesian Evidence

$\mathcal{E}$ ↑ Fit quality

↓ # free parameters

WEIGHT: simple models are preferred (Occam's razor)

Likelihood **Prior** **Posterior**

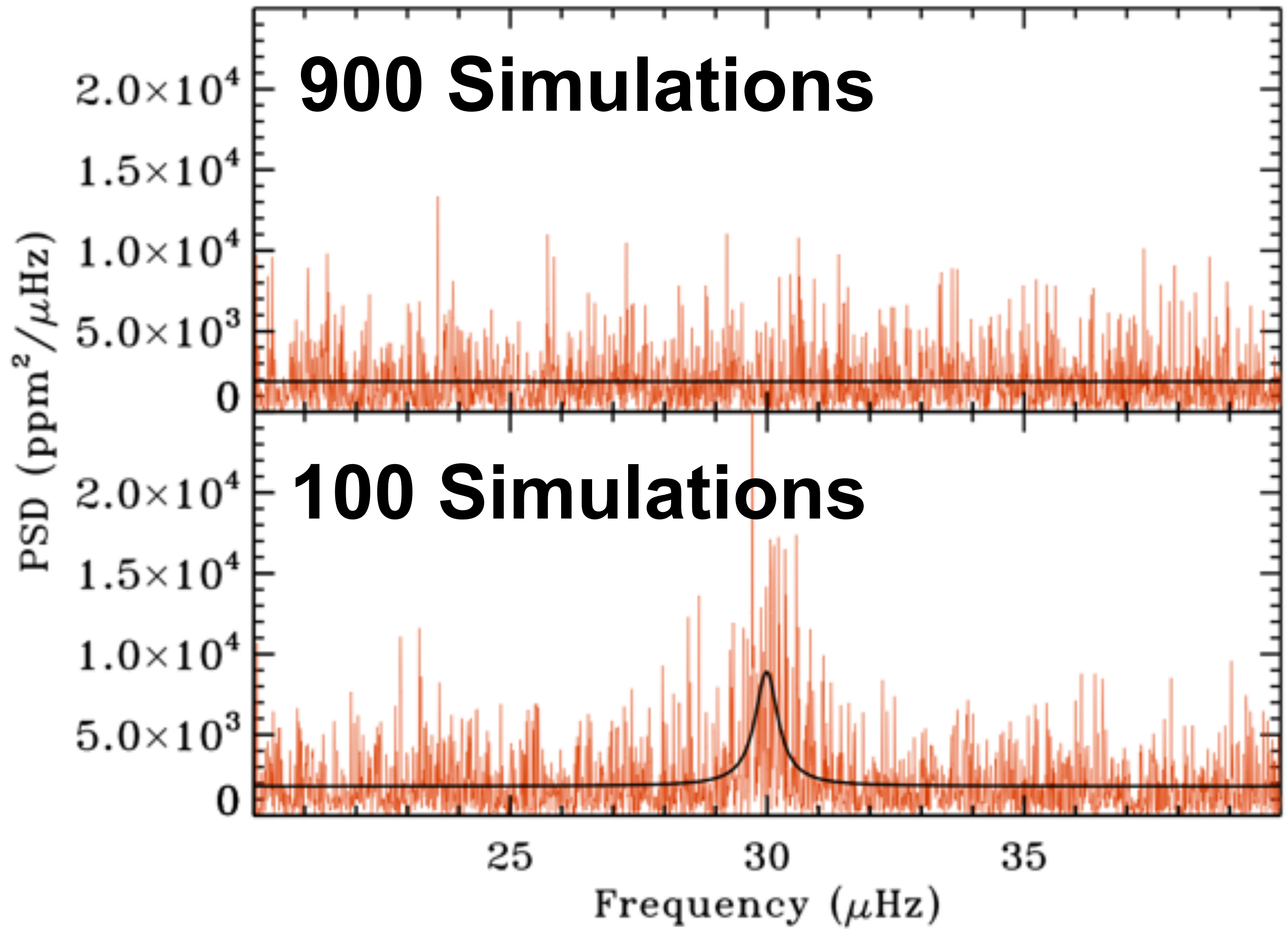


Bayes' Theorem

$$p(\boldsymbol{\theta}) = \frac{\mathcal{L}(\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})}{\mathcal{E}}$$

Peak Significance Criterion

- Simulations test
- **1000** artificial chunks of PSD
- Blind search for those with a peak



Bayes' factor

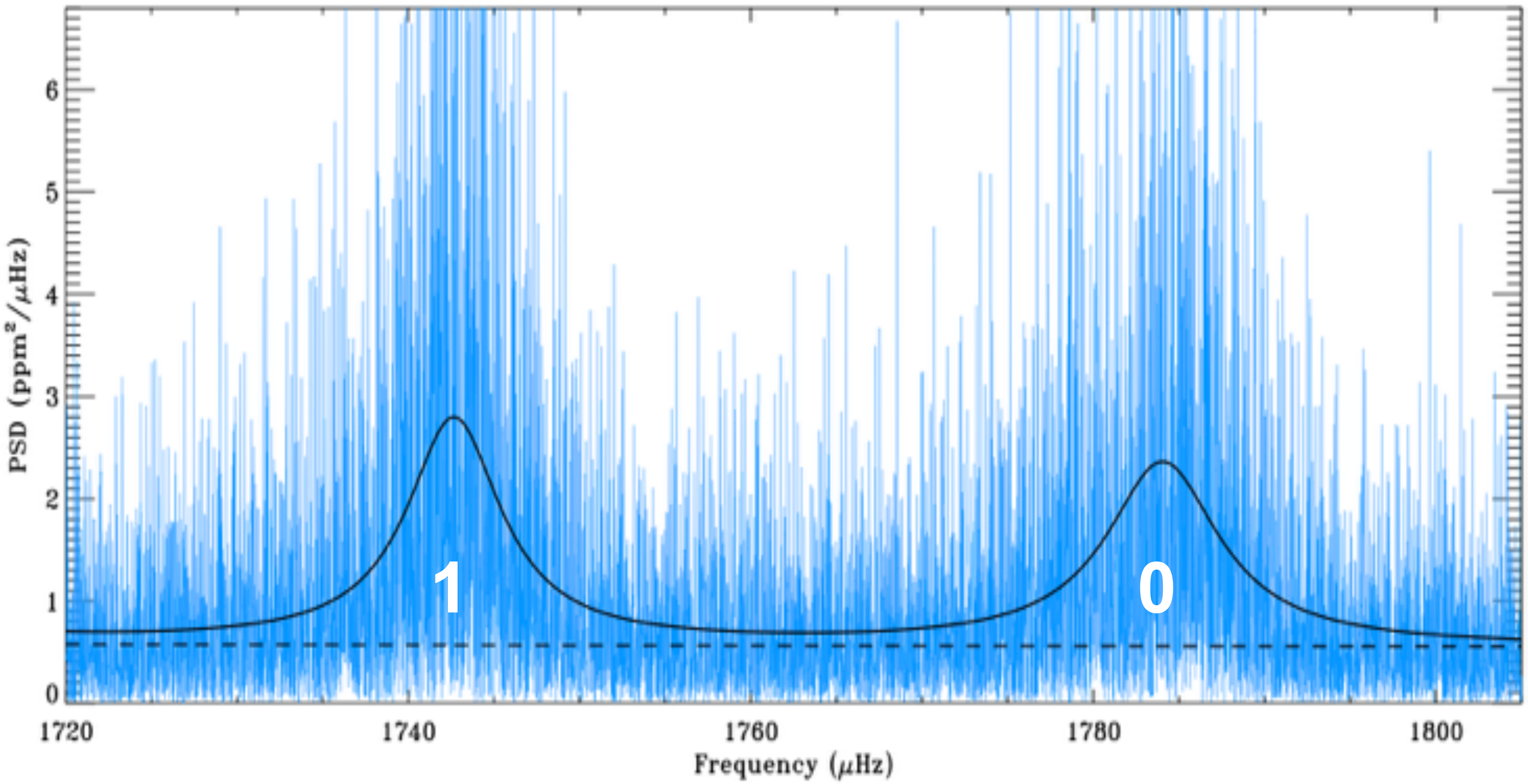
$$B_{yes,no} = \frac{\mathcal{E}_{yes}}{\mathcal{E}_{no}}$$

$$B_{yes,no} \sim 150$$

Strong Evidence (Jeffreys' scale)

All peaks found!

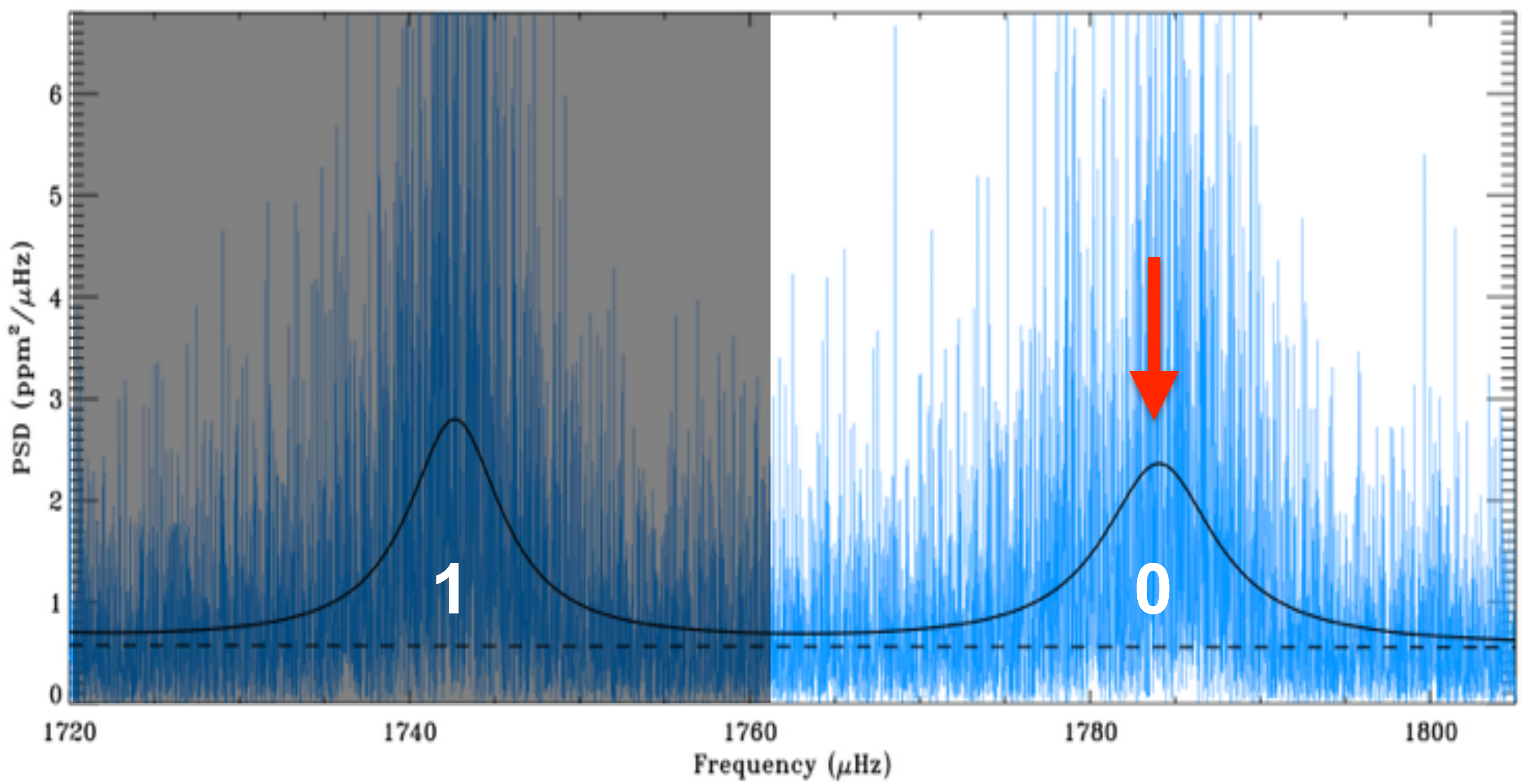
Peak significance



Peak significance

$\mathcal{M}_{\ell=0}$ **Only ℓ = 0**

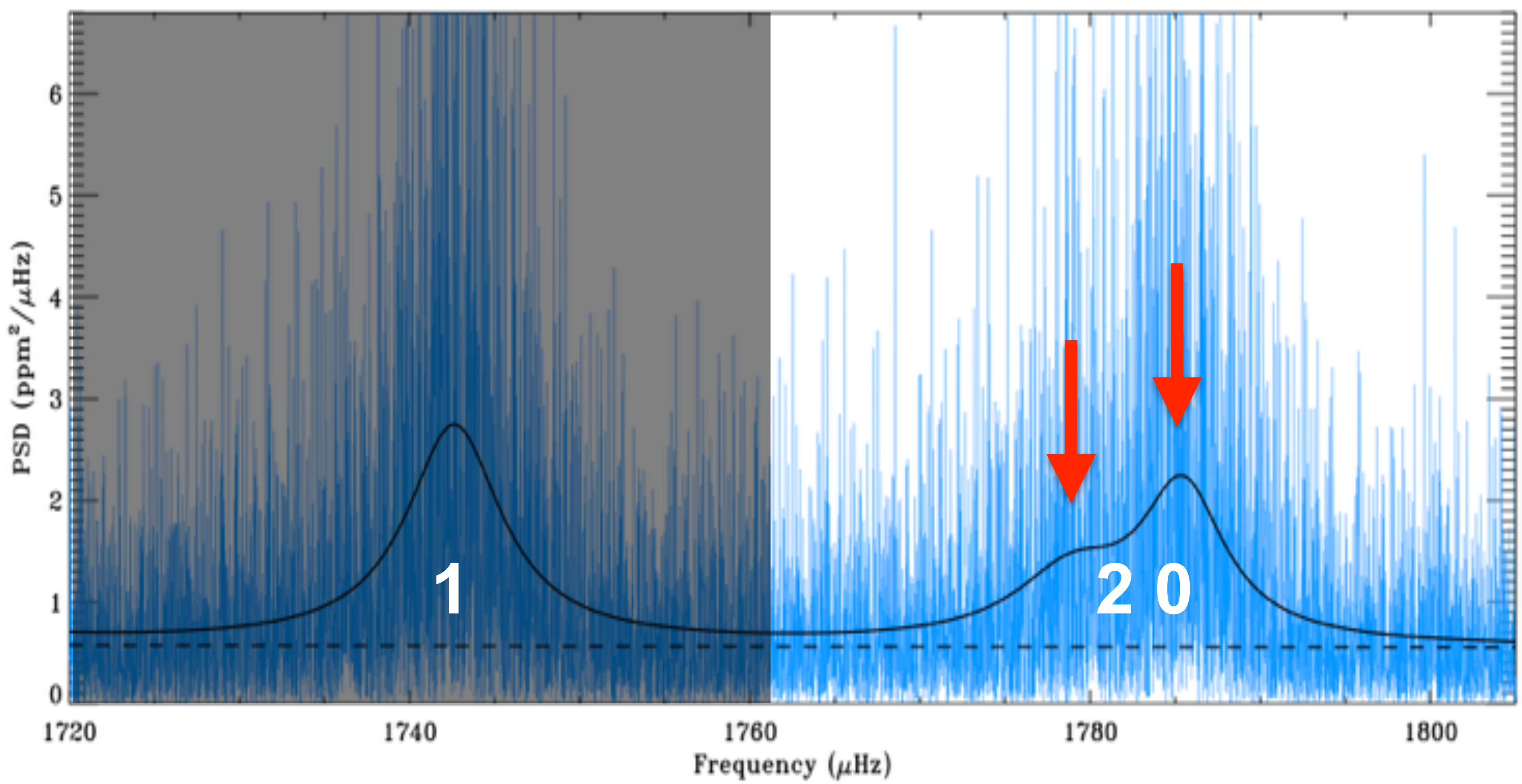
$\mathcal{E}_{\ell=0}$ **Bayesian Evidence**



Peak significance

$\mathcal{M}_{\ell=2}$ **Both ℓ = 2 and ℓ = 0**

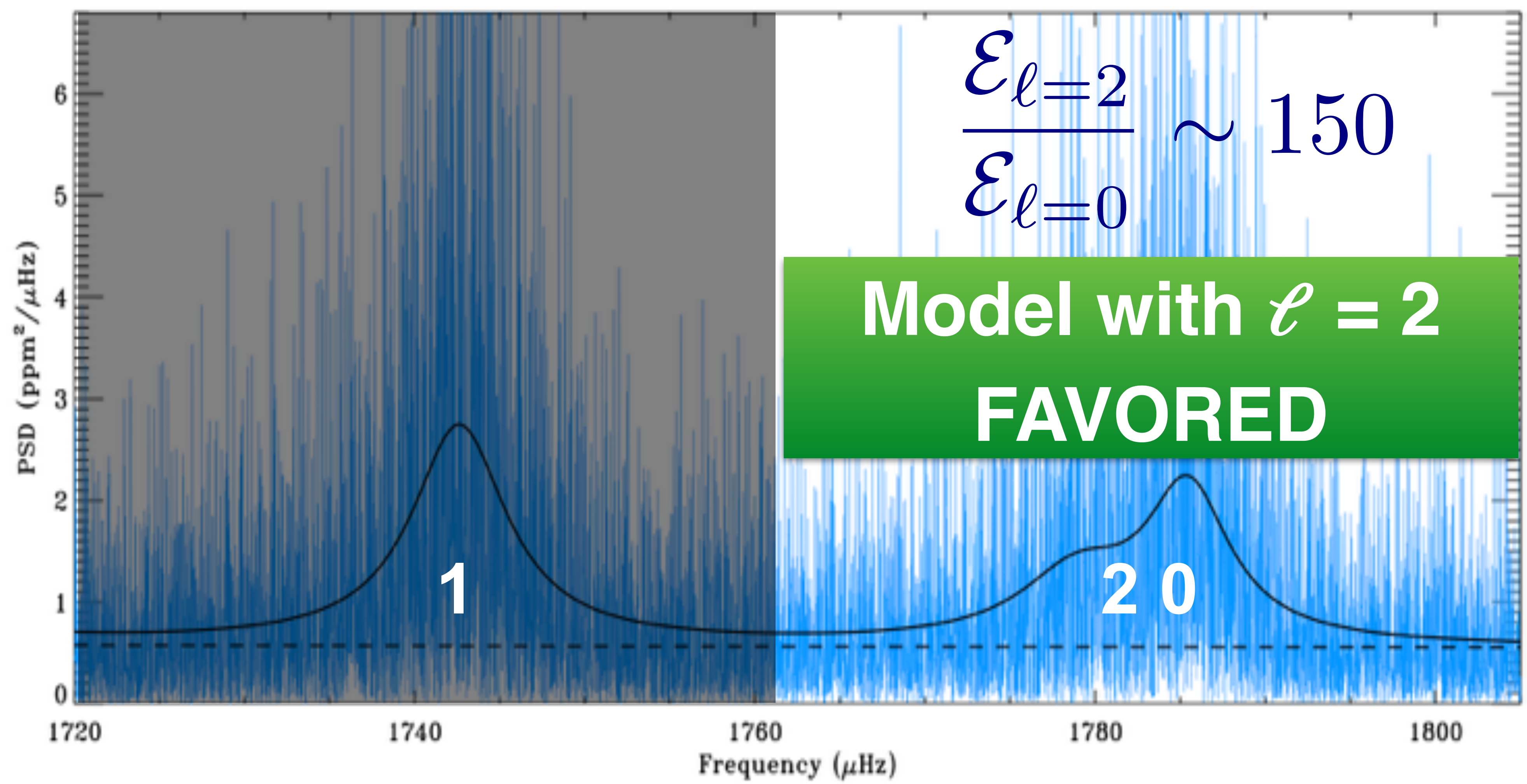
$\mathcal{E}_{\ell=2}$ **Bayesian Evidence**



Peak significance

$\mathcal{M}_{\ell=2}$ **Both ℓ = 2 and ℓ = 0**

$\mathcal{E}_{\ell=2}$ **Bayesian Evidence**



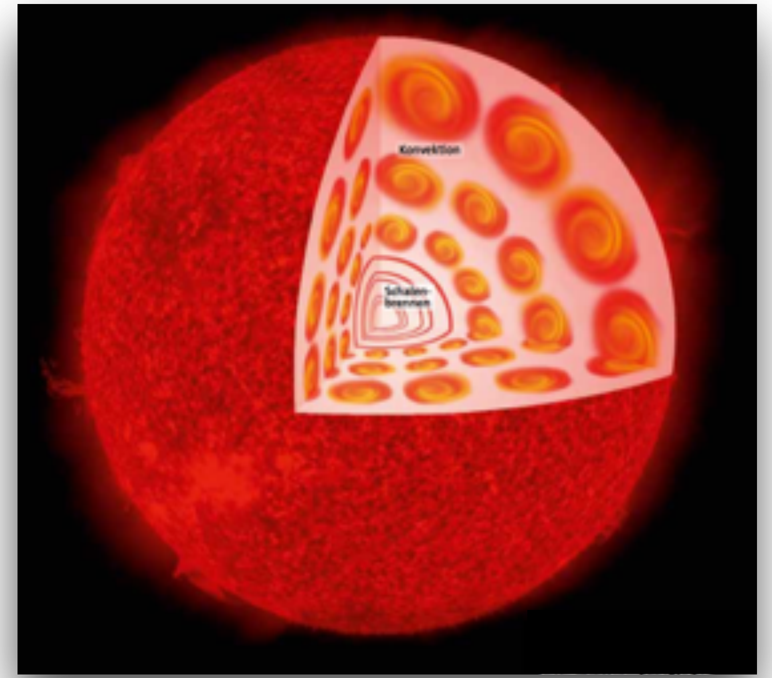
Past research @ CEA

- Detailed analysis of more than 1600 oscillation modes in red giant stars observed with NASA Kepler

  Corsaro, De Ridder, García 2015 A&A, 579, 83

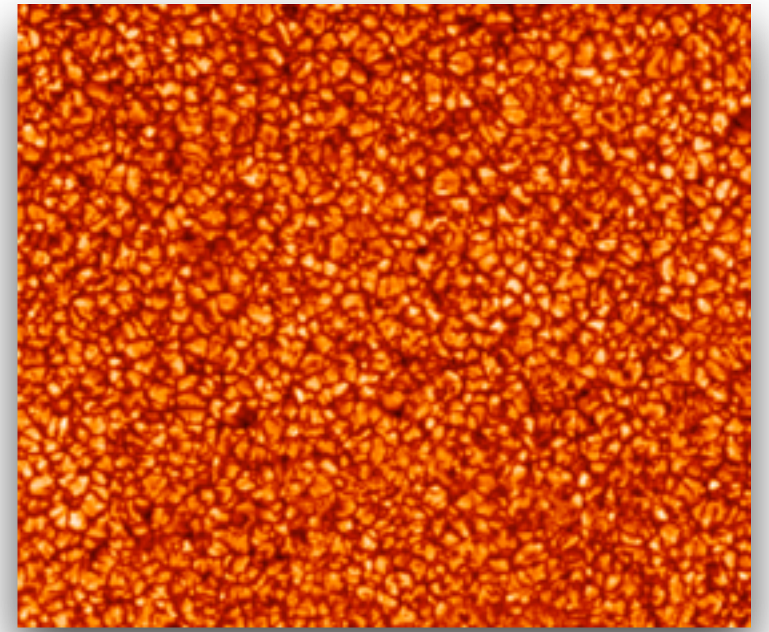
- Constraining position of zones of sharp structure variation inside stars up to 2% precision

  Corsaro, De Ridder, García 2015 A&A, 578, 76



Ongoing research @ CEA

- Analysis of granulation and oscillation properties in the Sun with GOLF & VIRGO + correlation with magnetic activity



- Full characterization of red giant stars in Kepler open clusters NGC 6791, NGC 6811, NGC 6819

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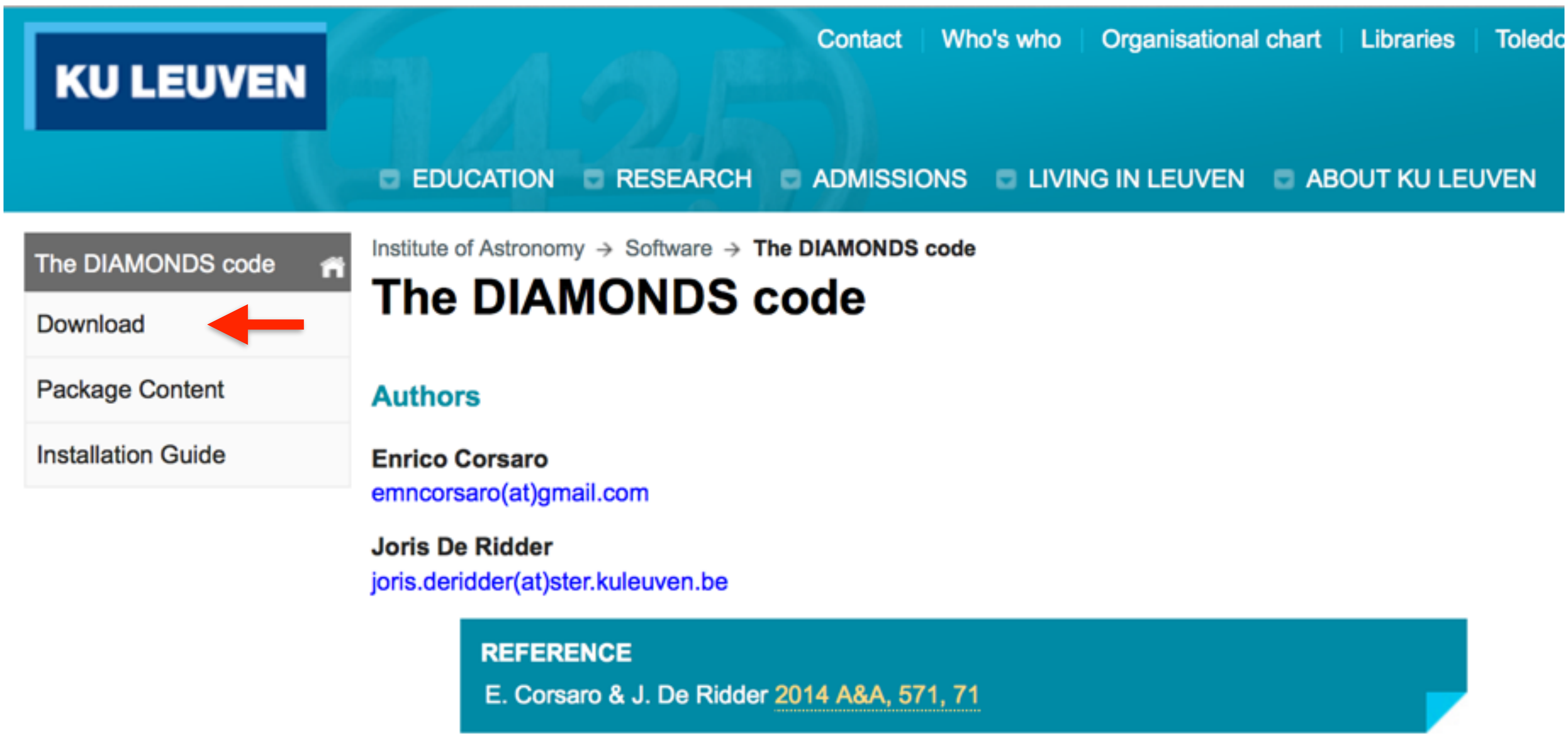

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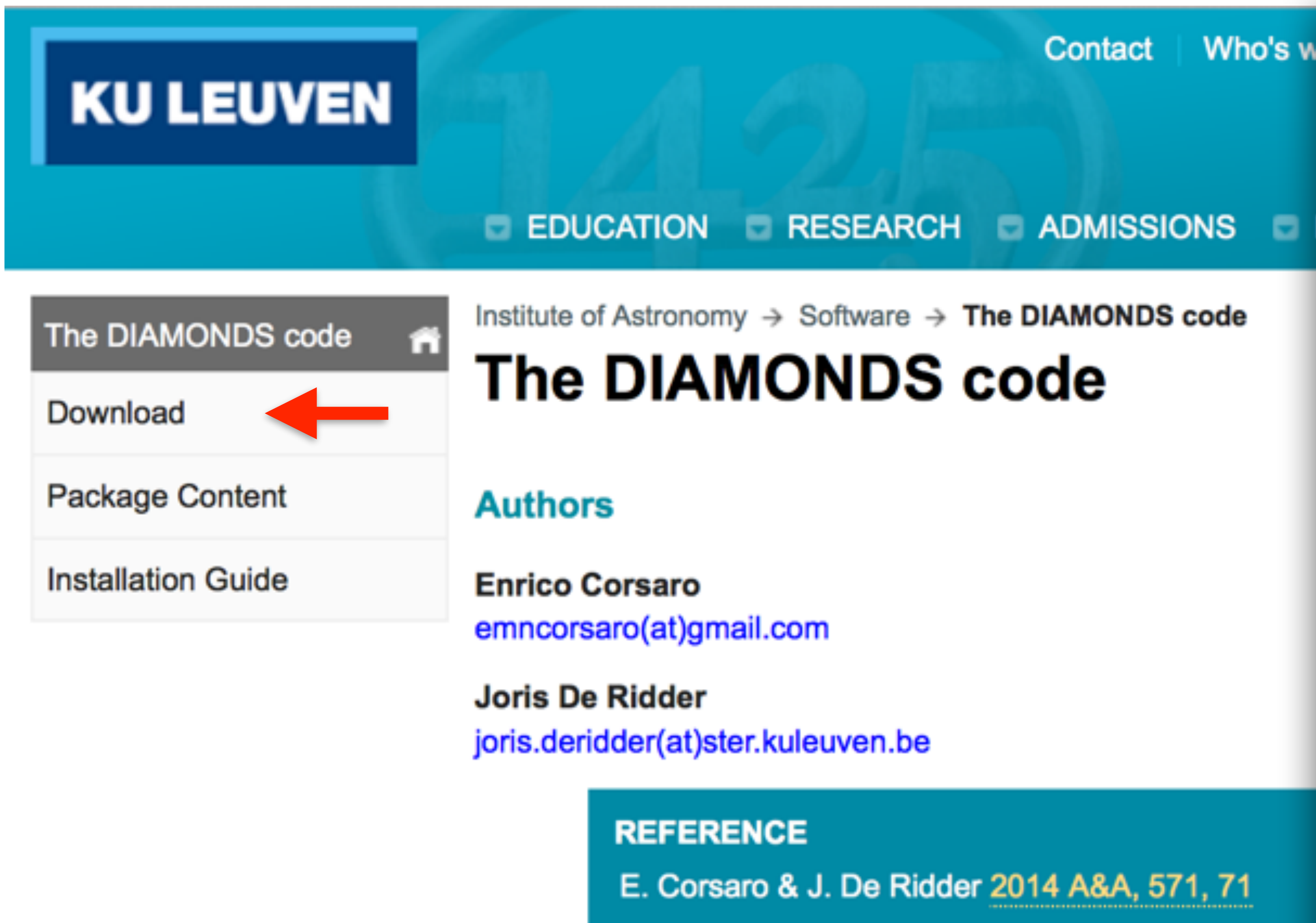
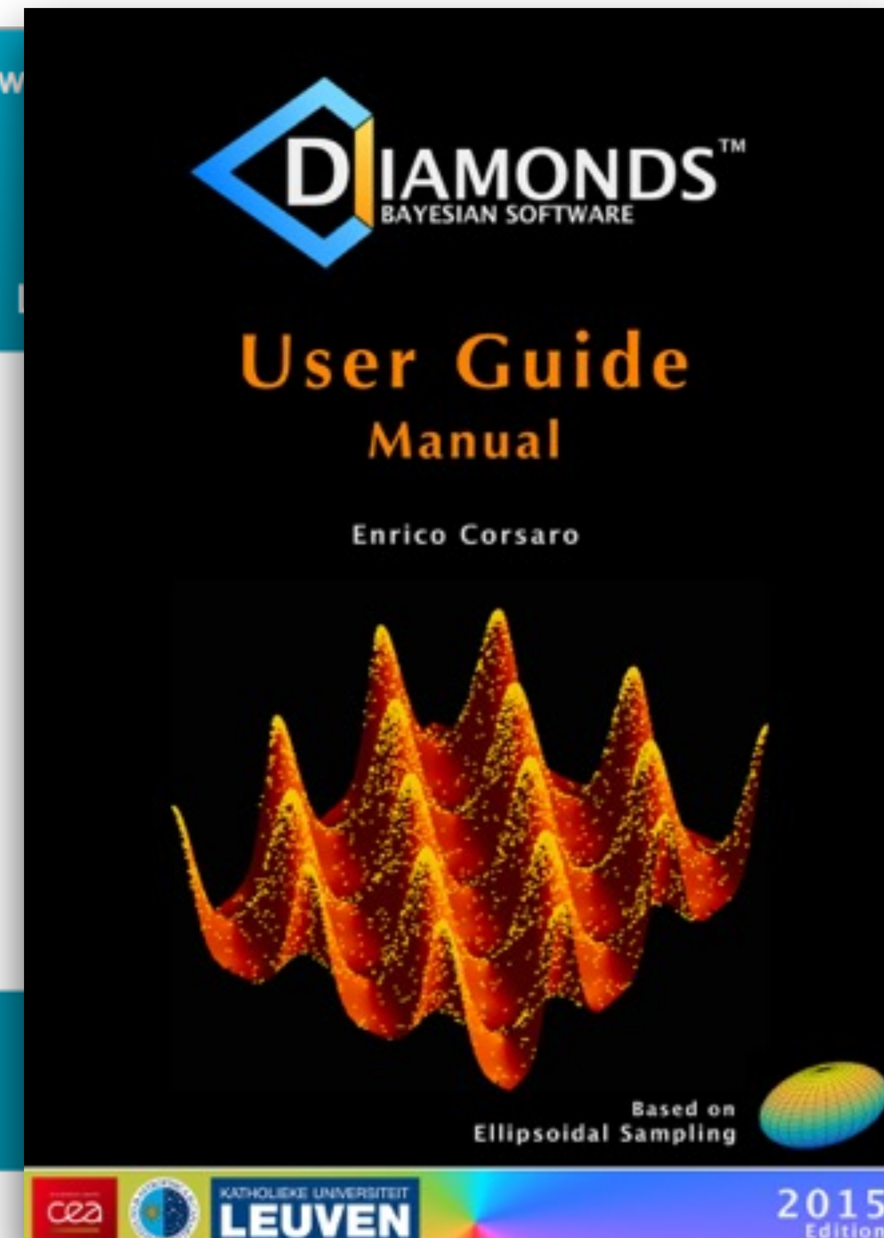
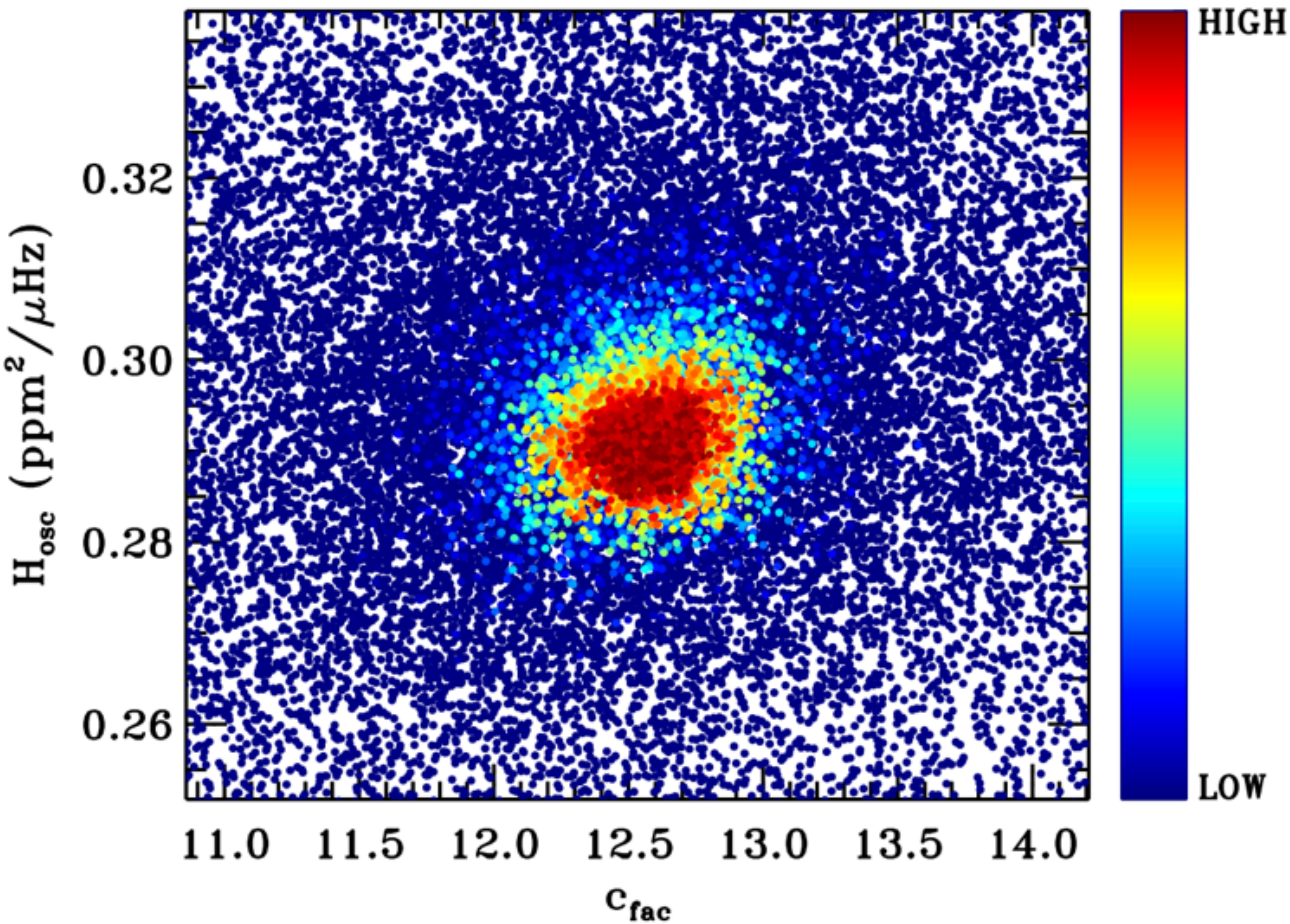
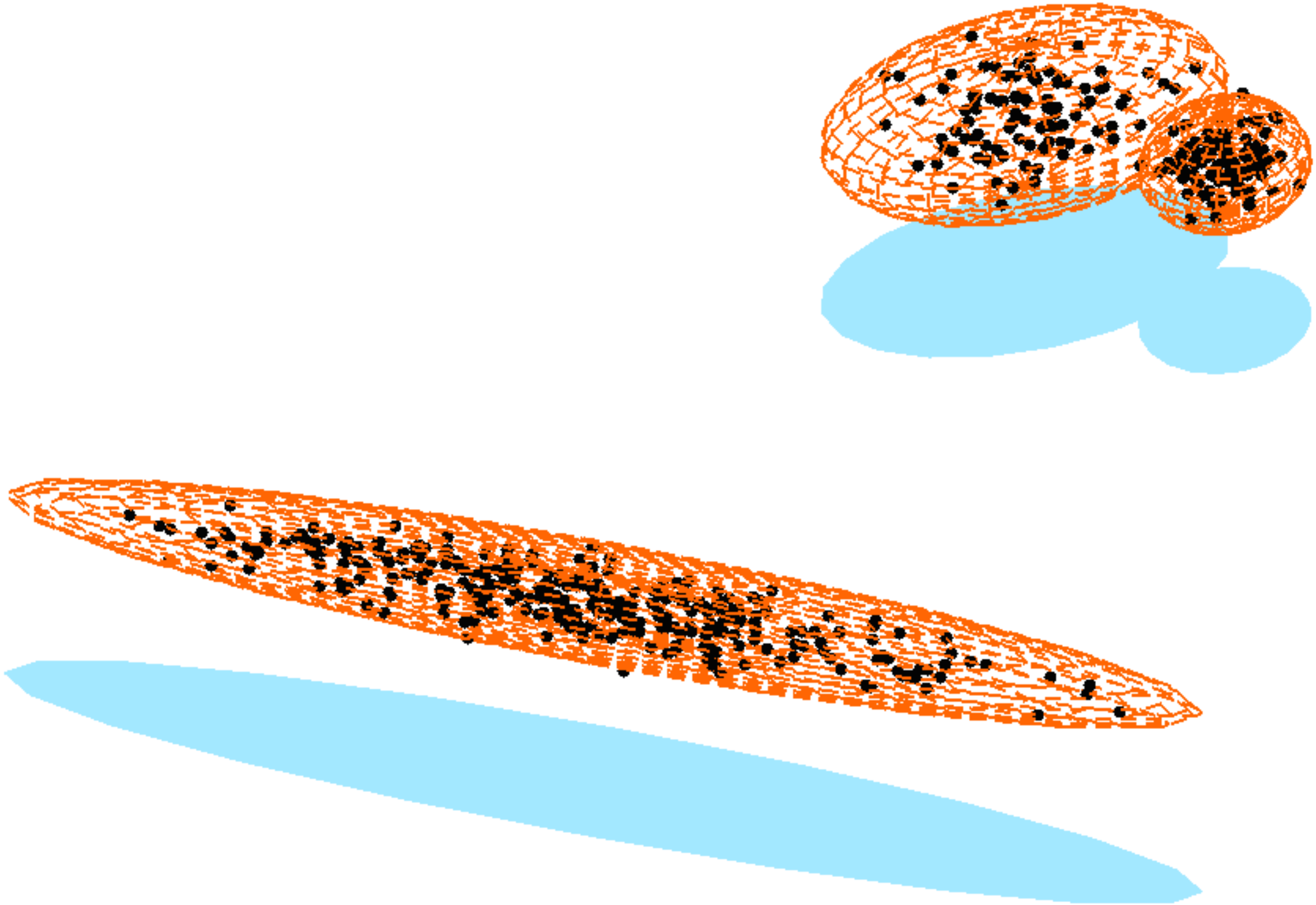


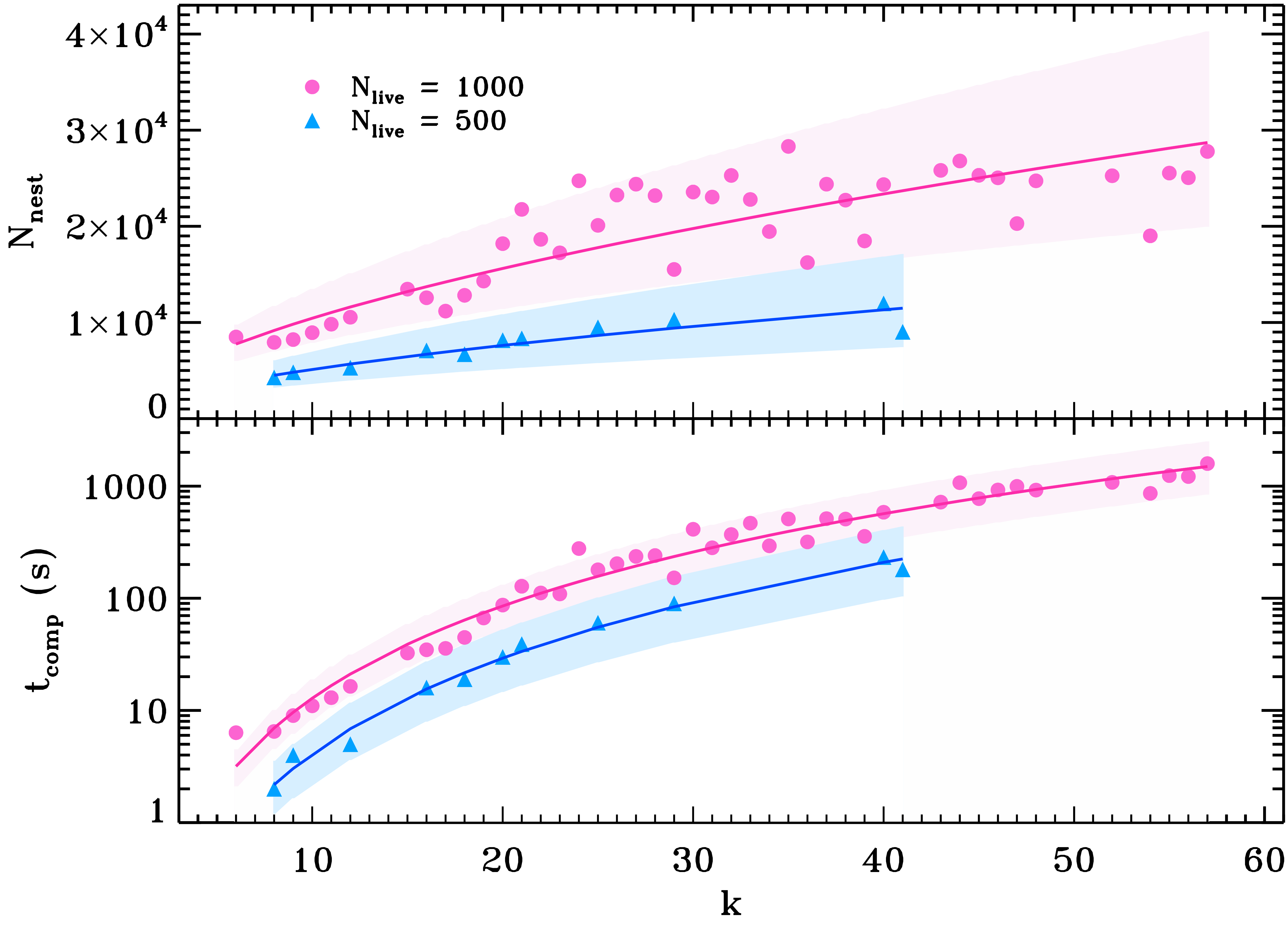


Thank you!



HIGH
LOW
0.32
0.30
0.28
0.26
H_{osc} (ppm^2/μHz)
11.0
11.5
12.0
12.5
13.0
13.5
14.0
c_{fac}





N_{live} = 1000
N_{live} = 500
N_{nest}
4×10^4
3×10^4
2×10^4
1×10^4
0
t_{comp} (s)
1000
100
10
1
10
20
30
40
50
60
k