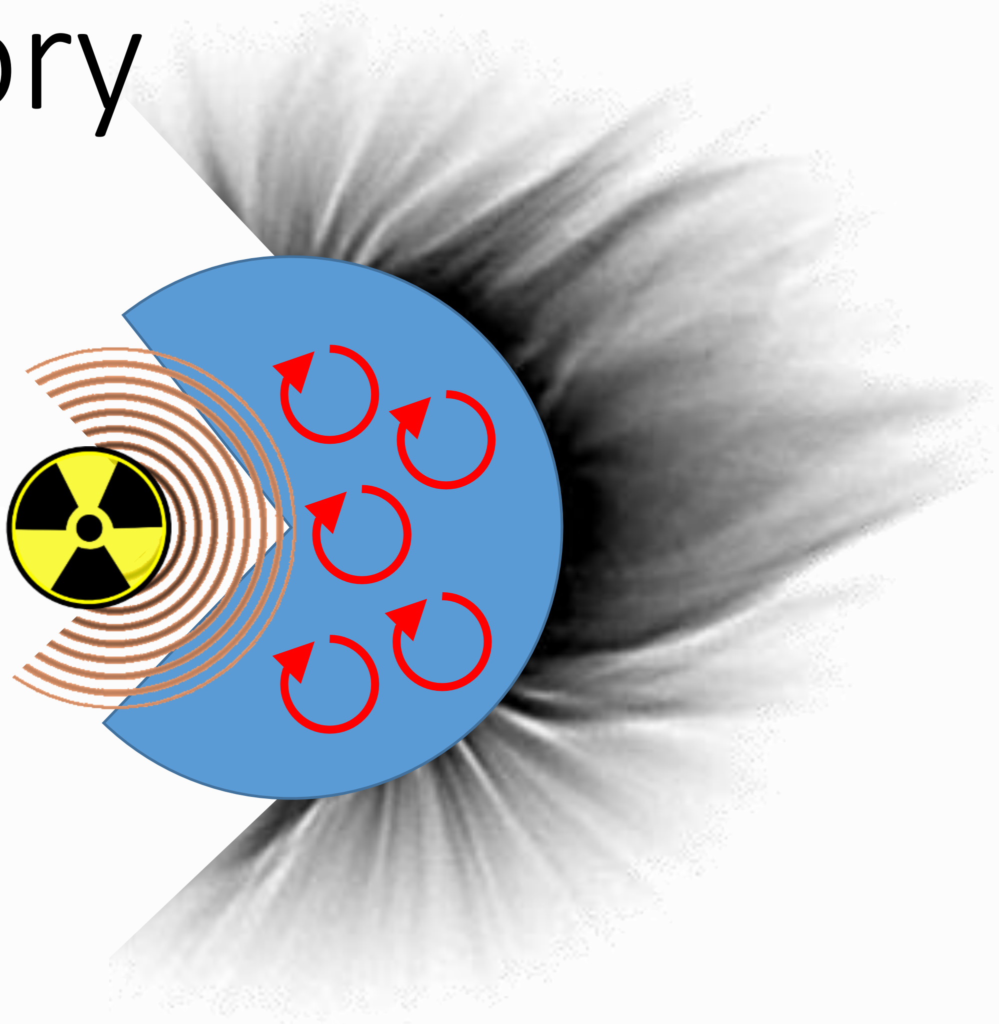
# Stars: A testbed for MHD turbulence theory



Kyle Augustson (Room 266)

SAp Postdoc Seminar

27 September 2016

# Academic Background

- Oregon State University
  - Bachelor's Degrees in
    - Computer Science (Parallel and Threaded Program Design)
    - Mathematics (Fluid Flow in Porous and Permeable Media)
    - Physics (Much Dabbling)

- University of Colorado
  - PhD in Astrophysical and Planetary Sciences
    - Convection and Dynamo Action in Massive Stars

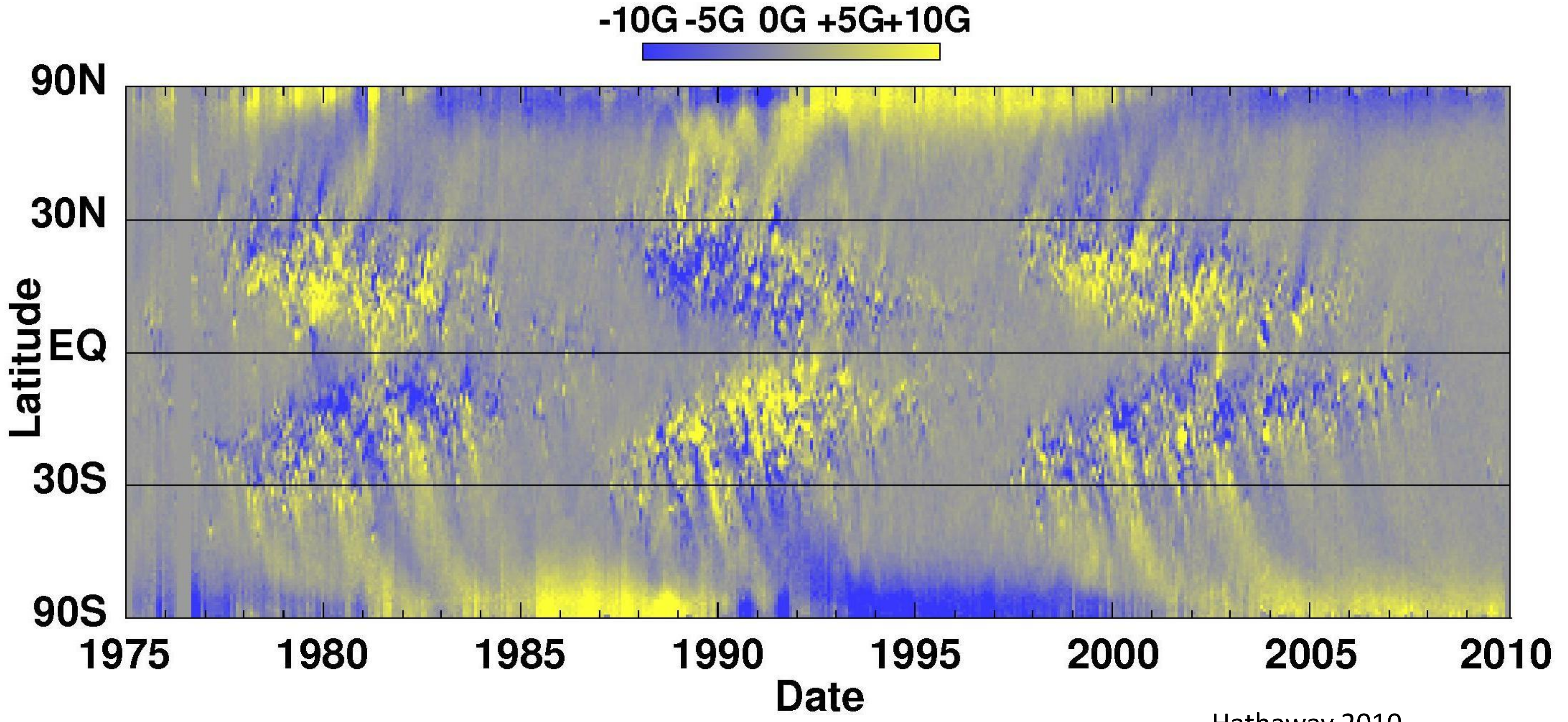
- National Center for Atmospheric Research
  - Advanced Study Postdoctoral Program

# Whence the testbed?

**Which stellar observations currently provide the best constraints on analytical and numerical stellar models?**

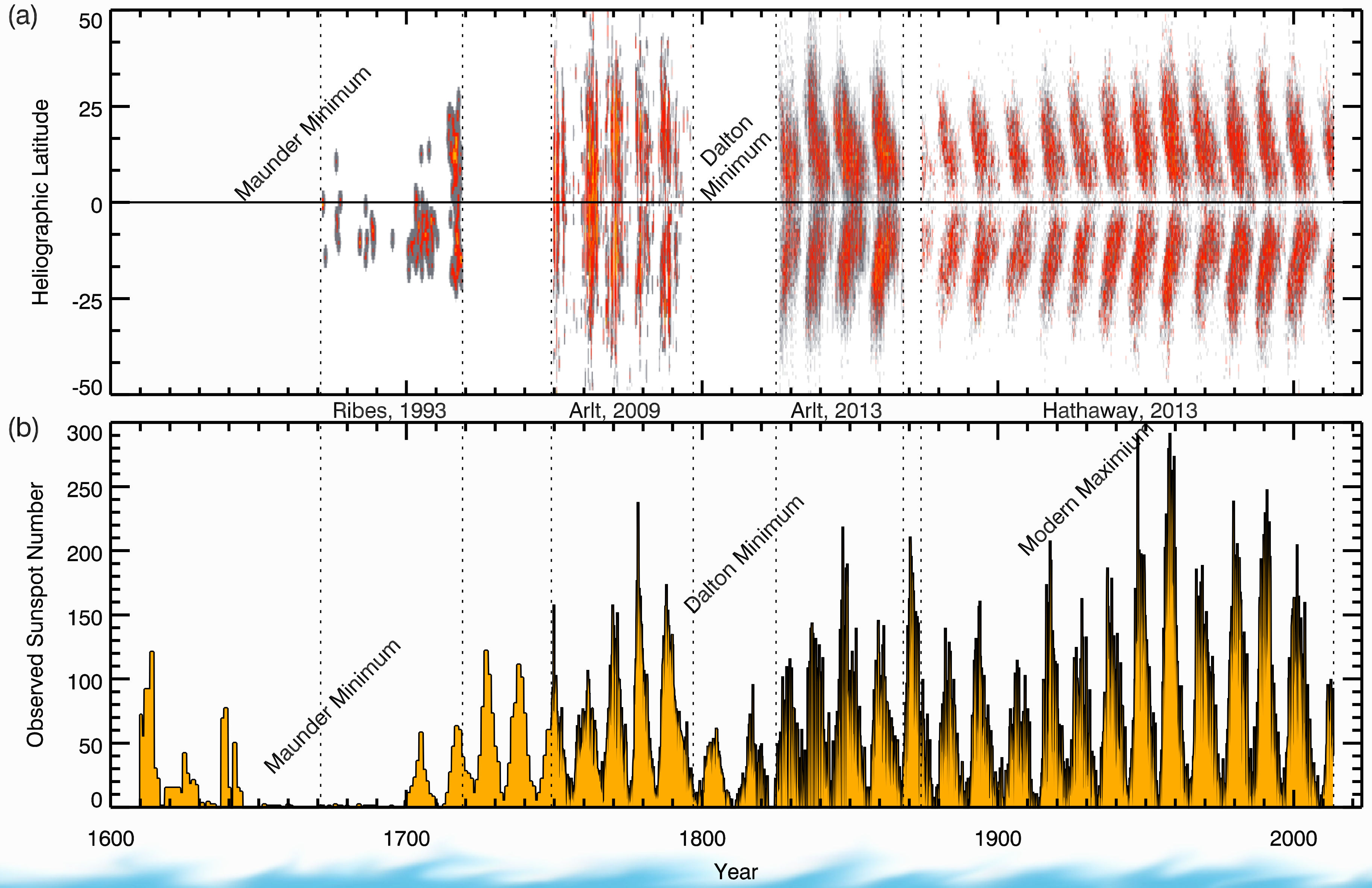
- The Sun
  - Activity cycles, detailed dynamics, helioseismology
- A large selection of Kepler and Mt Wilson stars, among a few others
  - Sensitivity of internal structure and magnetism to fundamental parameters
- Red Giants
  - Convective Core Dynamos, internal structure
- M dwarfs
  - Impact of a stable interior, fully convective dynamics, proxy for pre-main sequence stars

# Basic Aspects of Solar Magnetism

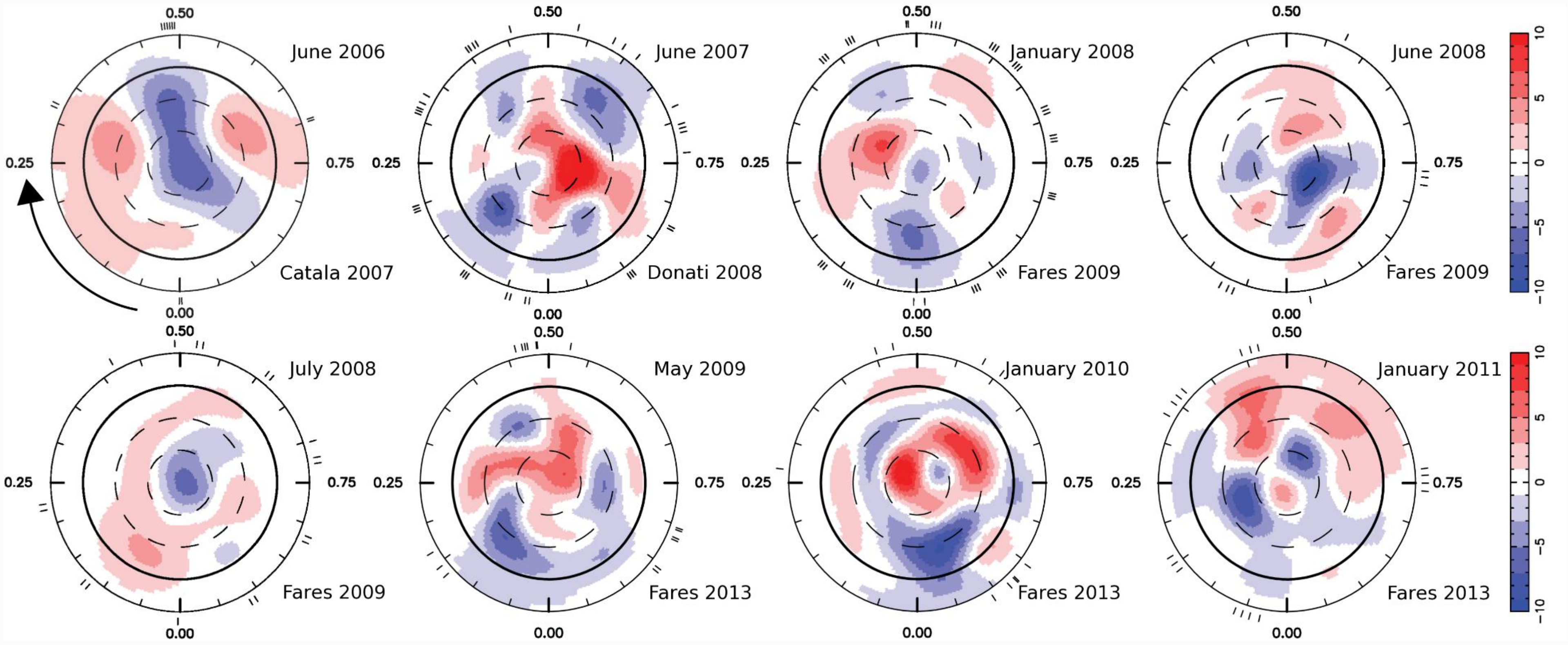



Hathaway 2010

# Basic Aspects of Solar Magnetism



# Stellar Cycles & Magnetism



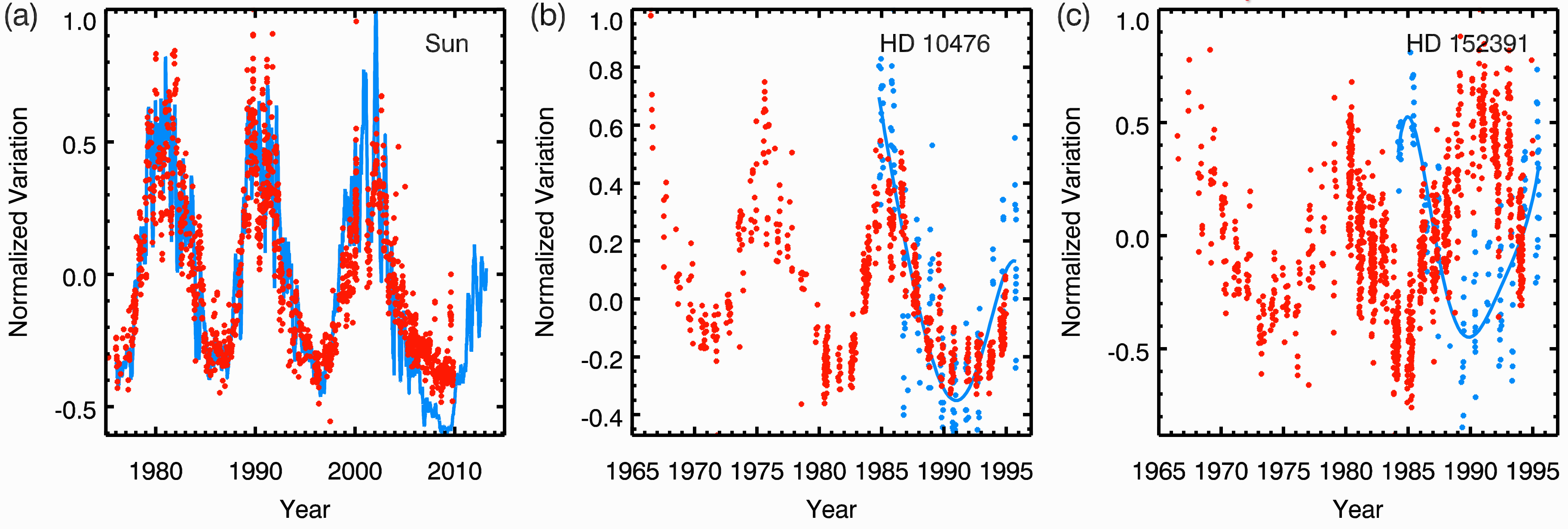

Spectropolarimetry

Tau-Bootis F-type Star

# Stellar Cycles & Magnetism

Faculae Dominated

Spot Dominated



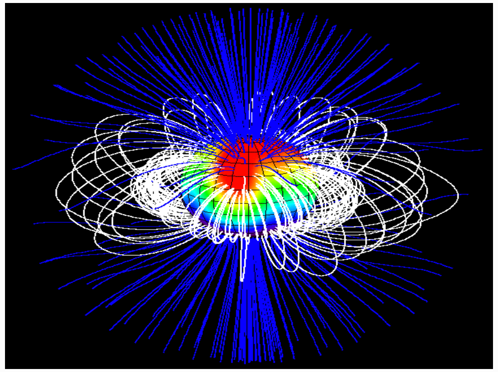

Froehlich 2013,
Livingston 2007

Radick 1998

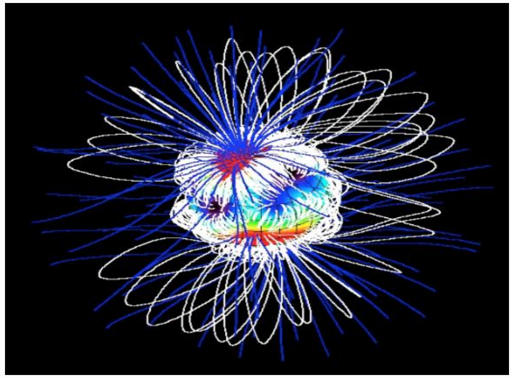
Ca H & K Emission

Long-term Activity Cycles

# Tiny Stars with Strong Fields



ultra-cool star V374 Pegasi
(Donati et al.)



Young star V2129 Oph
(Donati et al.)

*Measurements of stellar magnetic fields*

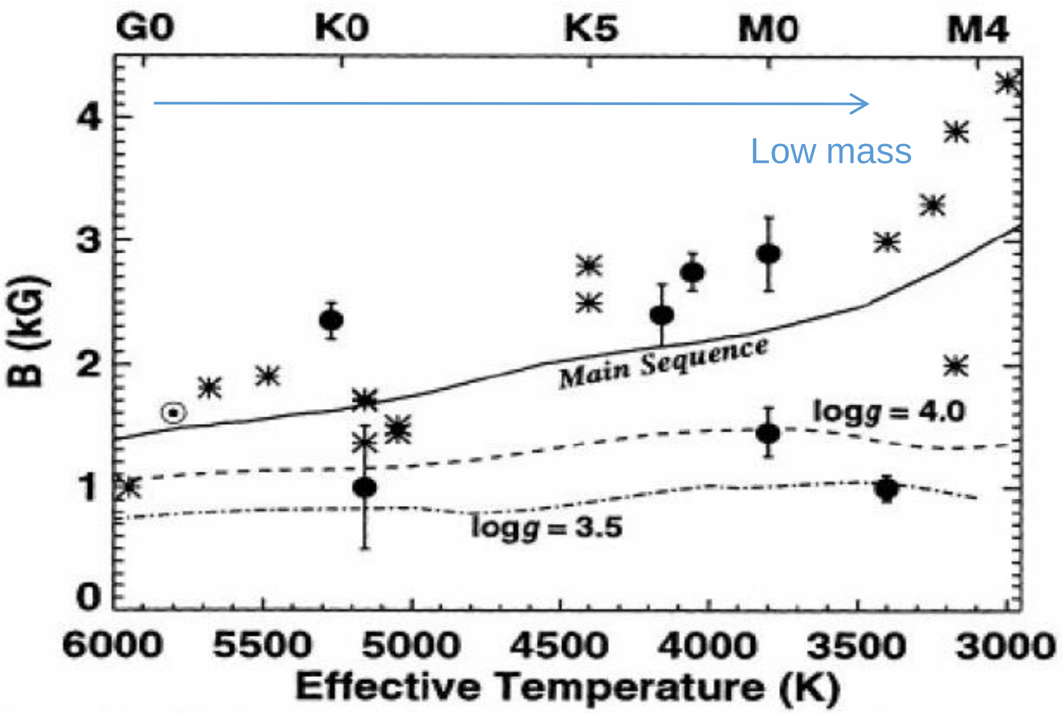
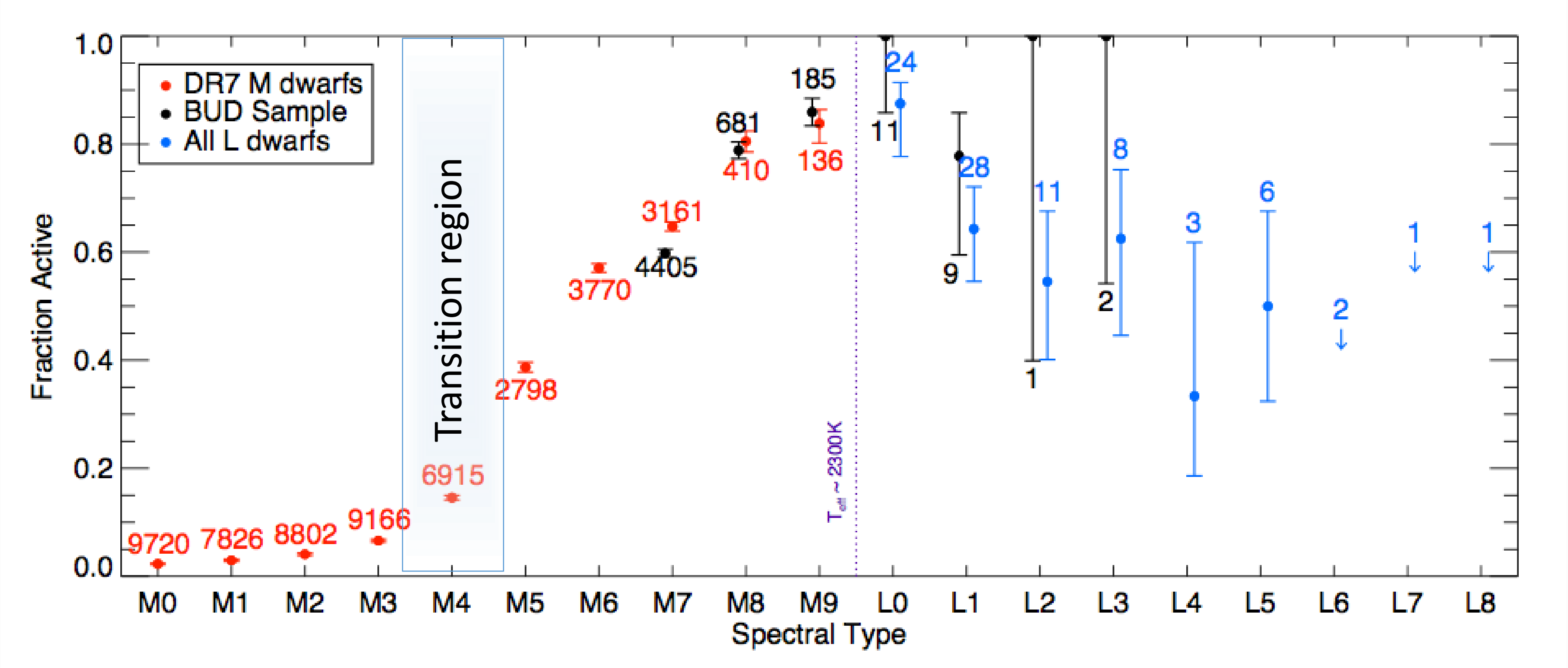



Figure 5. Predicted surface "equipartition" magnetic fields for cool stars. Also shown are measurements of main sequence stars (asterisks) and TTS (solid circles). The sun is shown by an encircled dot.

(J. Schmitt, 2003, IAU S219) From Johns-Krull & Valenti

# Interface Dynamo Transition?




Schmidt 2014

# Beyond just magnetism…

Many features of the stellar interior may exhibit a strong influence upon the observable characteristics of stars:

- Tachoclinic transitions
- Transport by internal waves
- Tidal interactions
- Double diffusive instabilities
- Shear instabilities
- And so on… to greater complexity.

# A small sampling of such puzzles in stellar evolution and magnetism

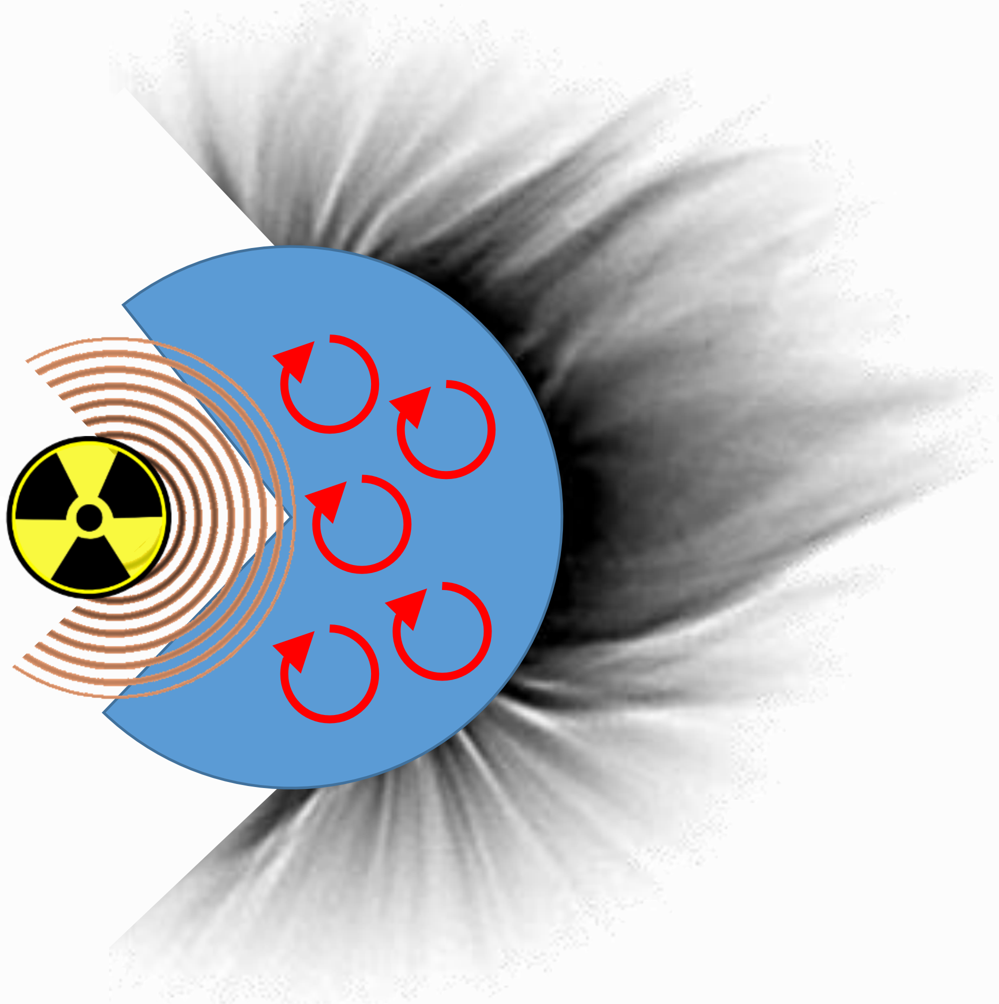
- Can some basic aspects of a star’s magnetic state be determined from it’s global parameters?

- What morphologies of magnetic fields are stable within radiative regions?

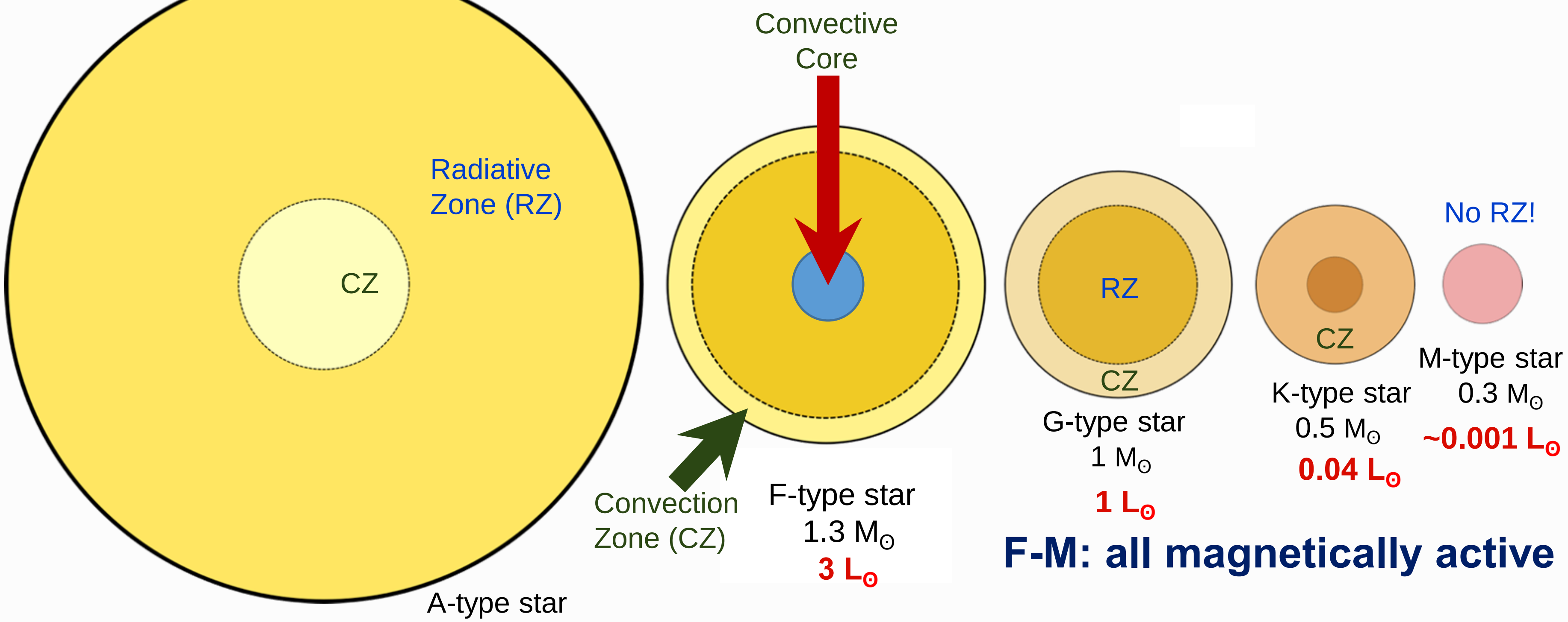
- What is the structure of the interface between convectively stable and unstable regions?  Does it depend upon rotation?

# Basic Stellar Physics – On the main sequence

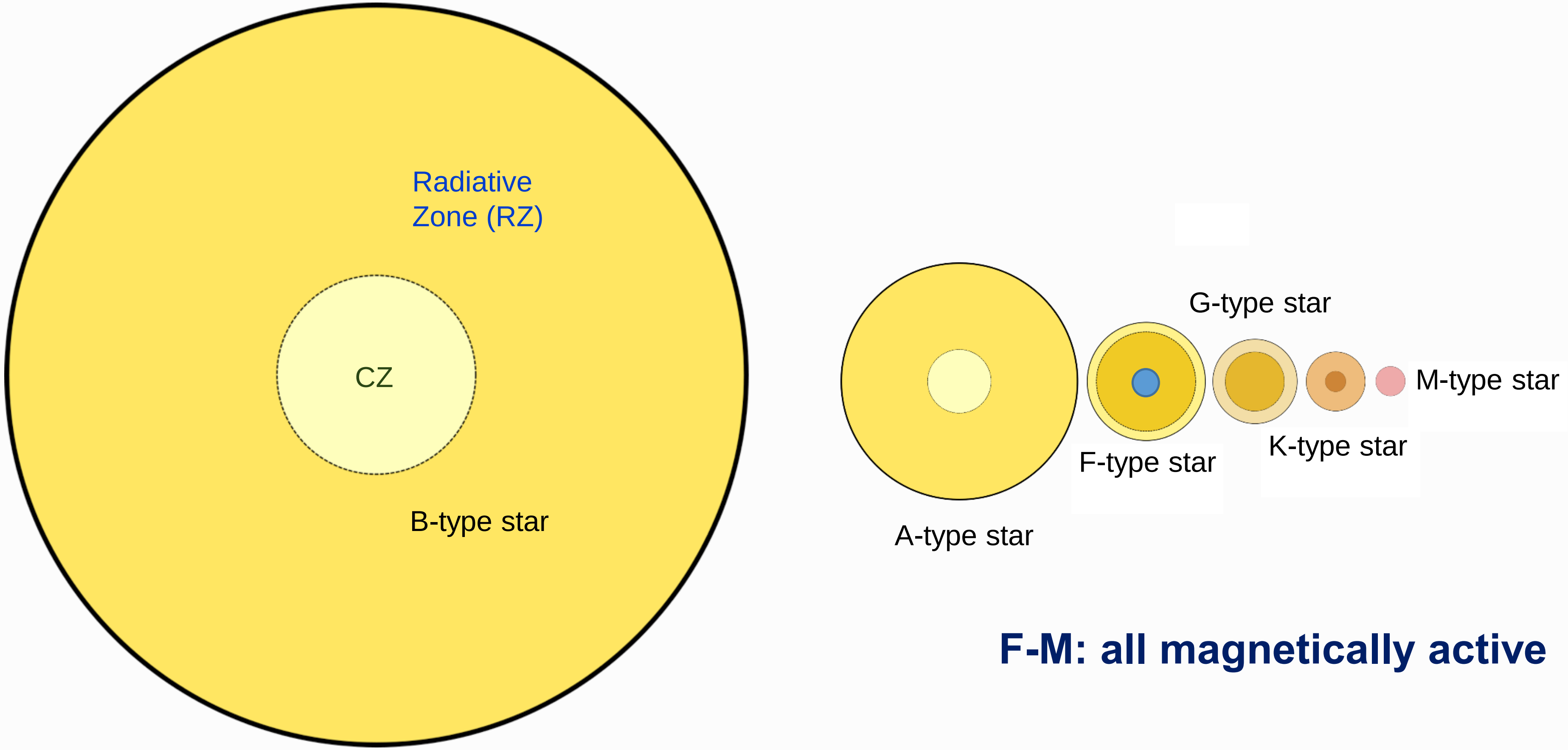
- Fusion in the core – p-p chain and/or CNO cycle
- Radiative envelope, where thermodynamic and compositional properties permit a small opacity and small thermal gradient
- Convective zone(s), where radiation is inefficient at carrying heat (e.g., large opacity and/or steep thermal gradient)
- Exterior winds and corona above photosphere



# Stellar Convection Zones



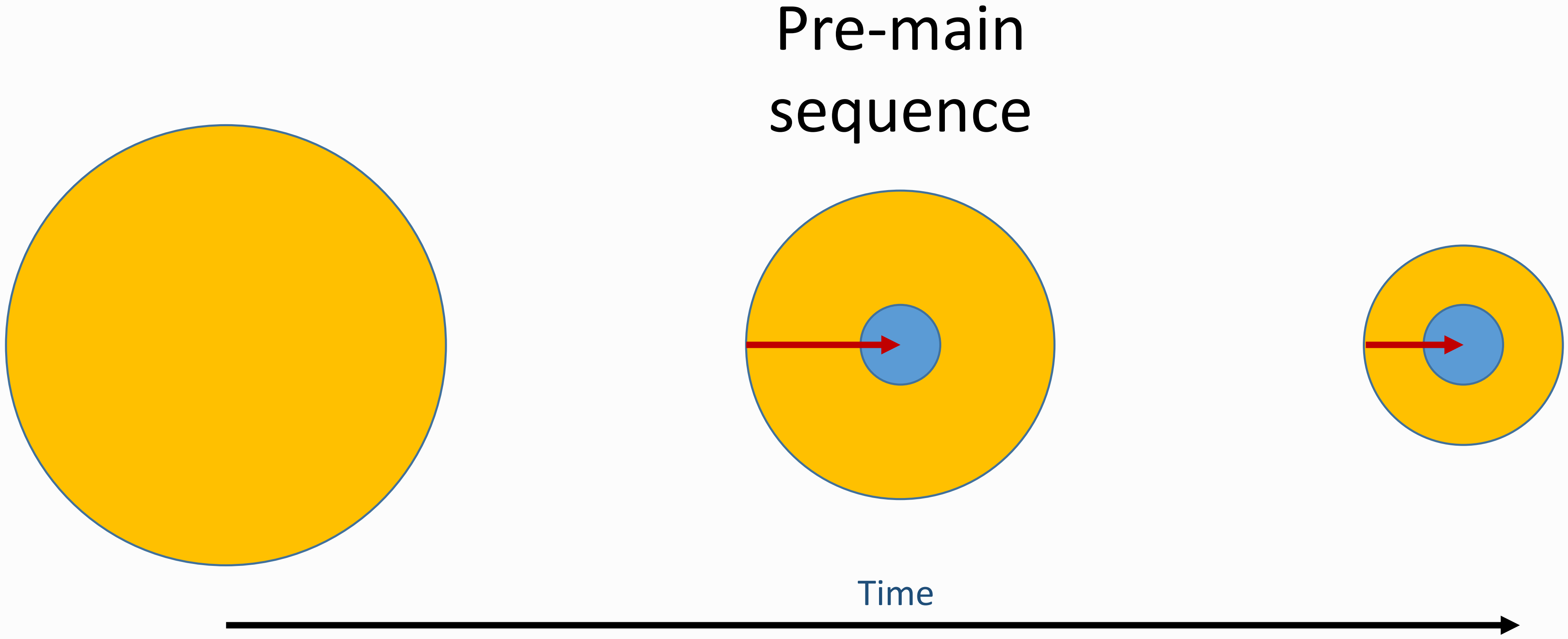
# Stellar Convection Zones




**F-M: all magnetically active**

# What about other evolutionary phases?

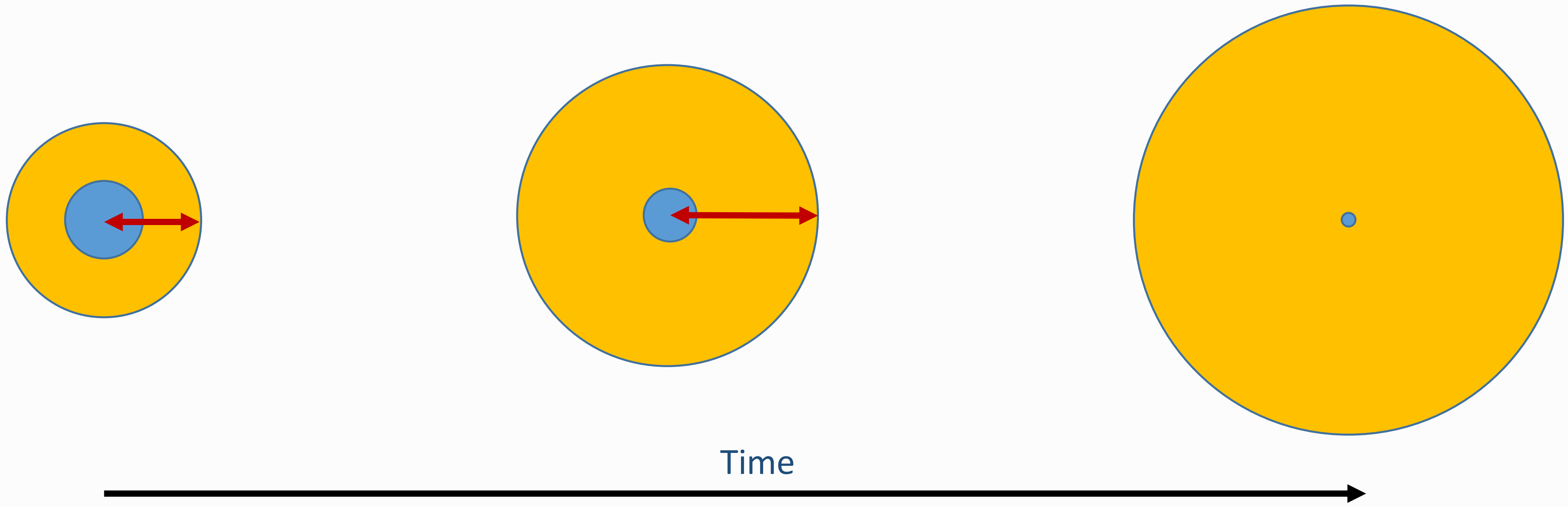
- Since, all phases are interesting from the viewpoint of turbulence and dynamos…



# What about other evolutionary phases?

- Since, all phases are interesting from the viewpoint of turbulence and dynamos…

Sub-Giant to Giant



# So what does this have to do with MHD?

- First, what is MHD?
- Under a double coarse-graining:
  - The first arising from ensemble particle dynamics, to the dynamics of statistical distributions (kinetic theory)
  - And the second assumes charge quasi-neutrality, and that time and length-scales of motion that are much larger than kinetic ones.

# So what does this have to do with MHD?

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$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \rho \mathbf{u} = 0,$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \boldsymbol{\nabla} \cdot \left[ \rho \mathbf{u} \otimes \mathbf{u} + \left( P + \frac{\mathbf{B}^2}{8\pi} \right) \mathbf{I} - \frac{1}{4\pi} \mathbf{B} \otimes \mathbf{B} - \boldsymbol{\mathcal{D}} \right] = \rho \boldsymbol{\nabla} \Phi,$$

$$\frac{\partial E_M}{\partial t} + \boldsymbol{\nabla} \cdot \left[ \mathbf{u}(E_M + P) + \mathbf{q} - \mathbf{u} \cdot \boldsymbol{\mathcal{D}} - \frac{1}{4\pi} \mathbf{u} \cdot \mathbf{B}\mathbf{B} + \frac{\eta}{4\pi} \left( \frac{1}{2} \boldsymbol{\nabla} \mathbf{B}^2 - \mathbf{B} \cdot \boldsymbol{\nabla} \mathbf{B} \right) \right] = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times [\mathbf{u} \times \mathbf{B} - \eta \boldsymbol{\nabla} \times \mathbf{B}],$$

$$E_M = \frac{1}{2} \rho \mathbf{u}^2 + \frac{\mathbf{B}^2}{8\pi} + \epsilon + \rho \Phi,$$

Equations! Panic!!

# Essential Dynamo Processes

## Evolution of Mean Magnetic Fields

$$\frac{\partial A}{\partial t} = \overbrace{\eta\left(\nabla^2 - \frac{1}{\lambda^2}\right)A}^{\text{magnetic diffusion}} - \overbrace{\frac{1}{\lambda}\mathbf{u}_m \cdot \nabla(\lambda A)}^{\text{meridional advection}} + \overbrace{\hat{\phi}\cdot\overline{\mathbf{u}'\times\mathbf{B}'}}^{\text{poloidal generation}}$$

$$\frac{\partial B}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\lambda^2}\right)B}_{\text{magnetic diffusion}} + \underbrace{\frac{1}{\lambda}\frac{\partial\eta}{\partial r}\frac{\partial\lambda B}{\partial r}}_{\text{diffusive transport}} - \underbrace{\lambda\mathbf{u}_m\cdot\nabla\frac{B}{\lambda}}_{\text{meridional advection}}$$

$$- \underbrace{B\nabla\cdot\mathbf{u}_m}_{\text{compression}} + \underbrace{\lambda\nabla\Omega\cdot\nabla\times A\hat{\phi}}_{\text{stretching}} + \underbrace{\hat{\phi}\cdot\nabla\times\overline{\mathbf{u}'\times\mathbf{B}'}}_{\text{toroidal generation}}$$

Turbulent Correlations

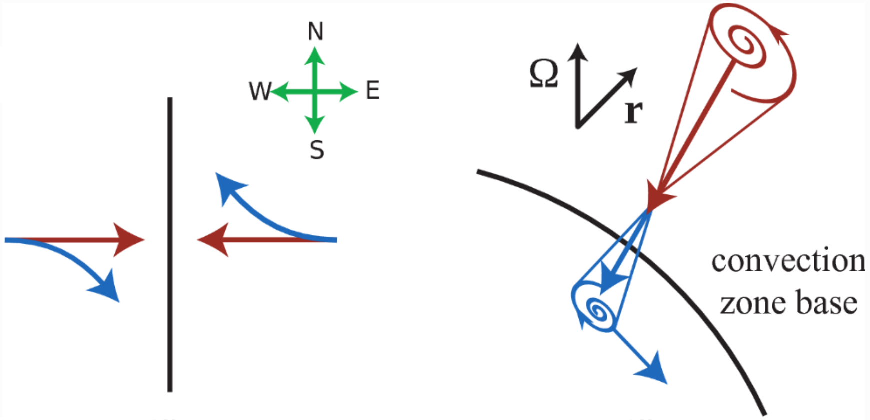
Rotation

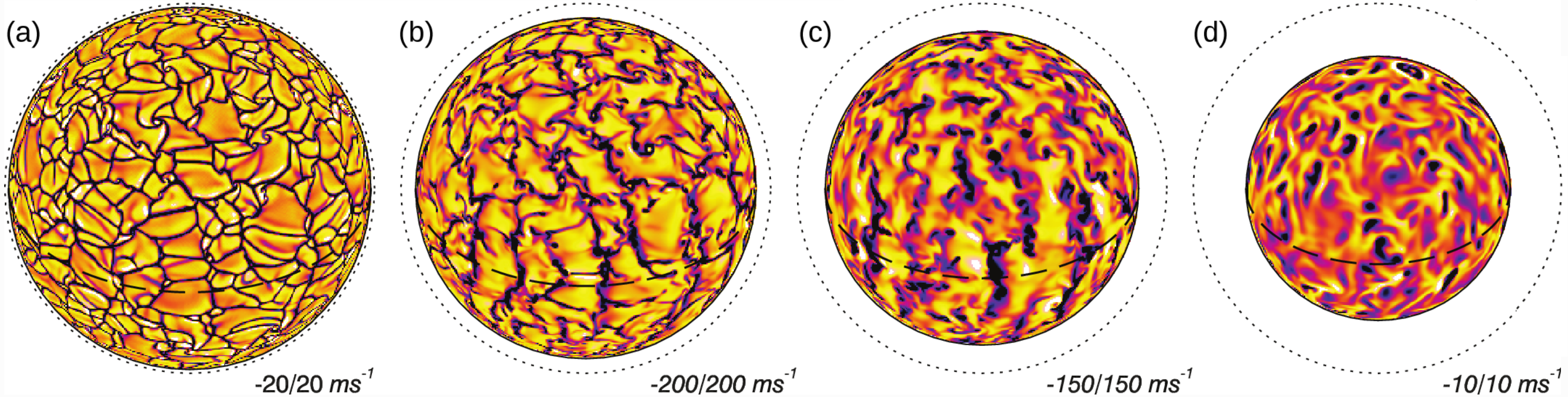
- Fully resolved nonlocal 3D MHD
  Self-consistent flow and field
- Flux-transport dynamo (e.g., BL)
  Prescribed flows & model EMF
- Delta-correlated turbulence (MFT)
  Prescribed flows & model EMF
- Parametrically asymptotic models
  A way of reaching more stellar-like parameter regimes

# Essential Dynamo Processes

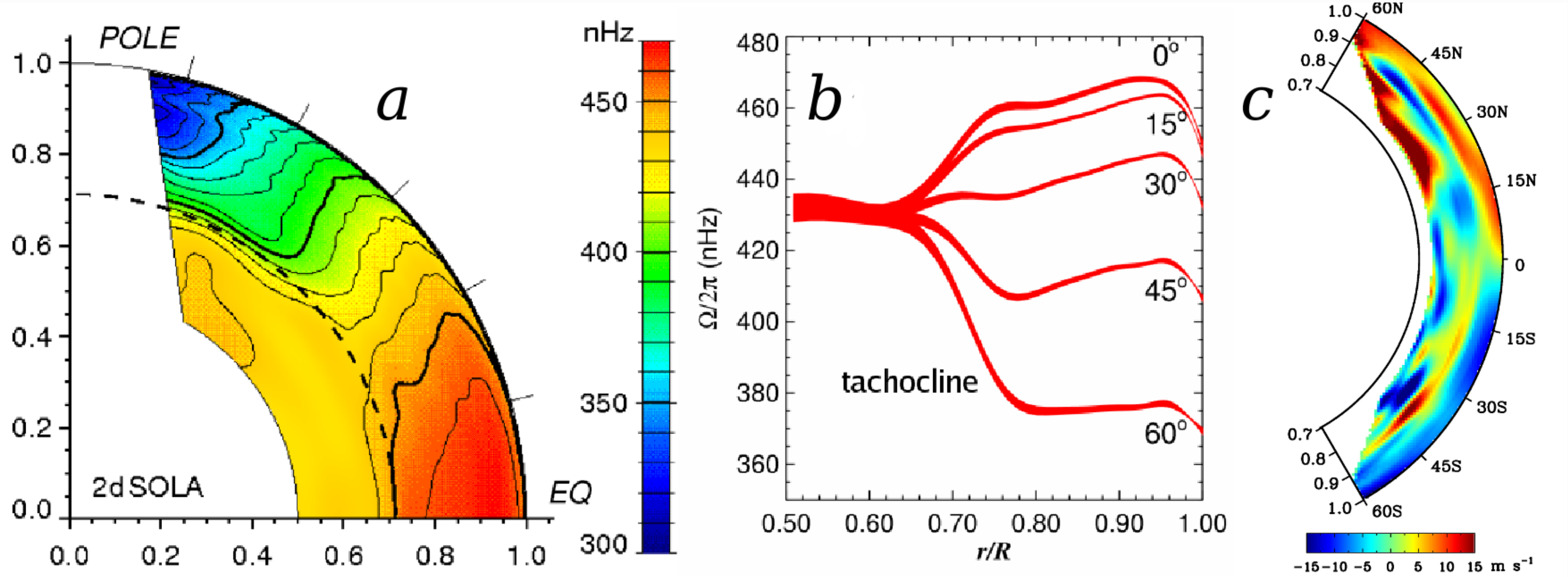
The basic building block of stellar dynamos:

Helical convection



# Essential Dynamo Processes



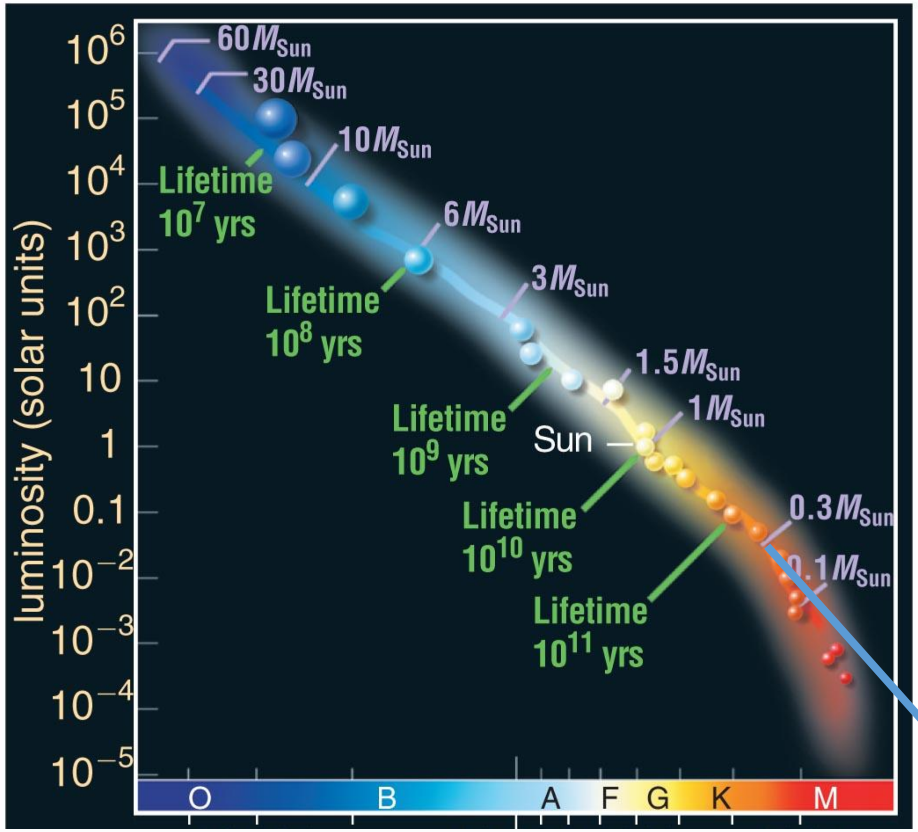

Differential Rotation

Howe 2009

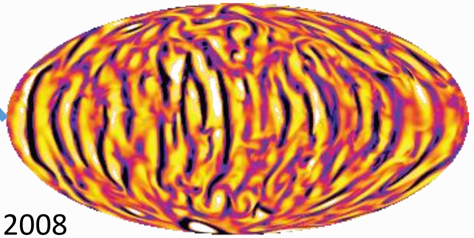
Meridional Circulation

Zhao et al 2013

# Simulating stars along the Main-Sequence

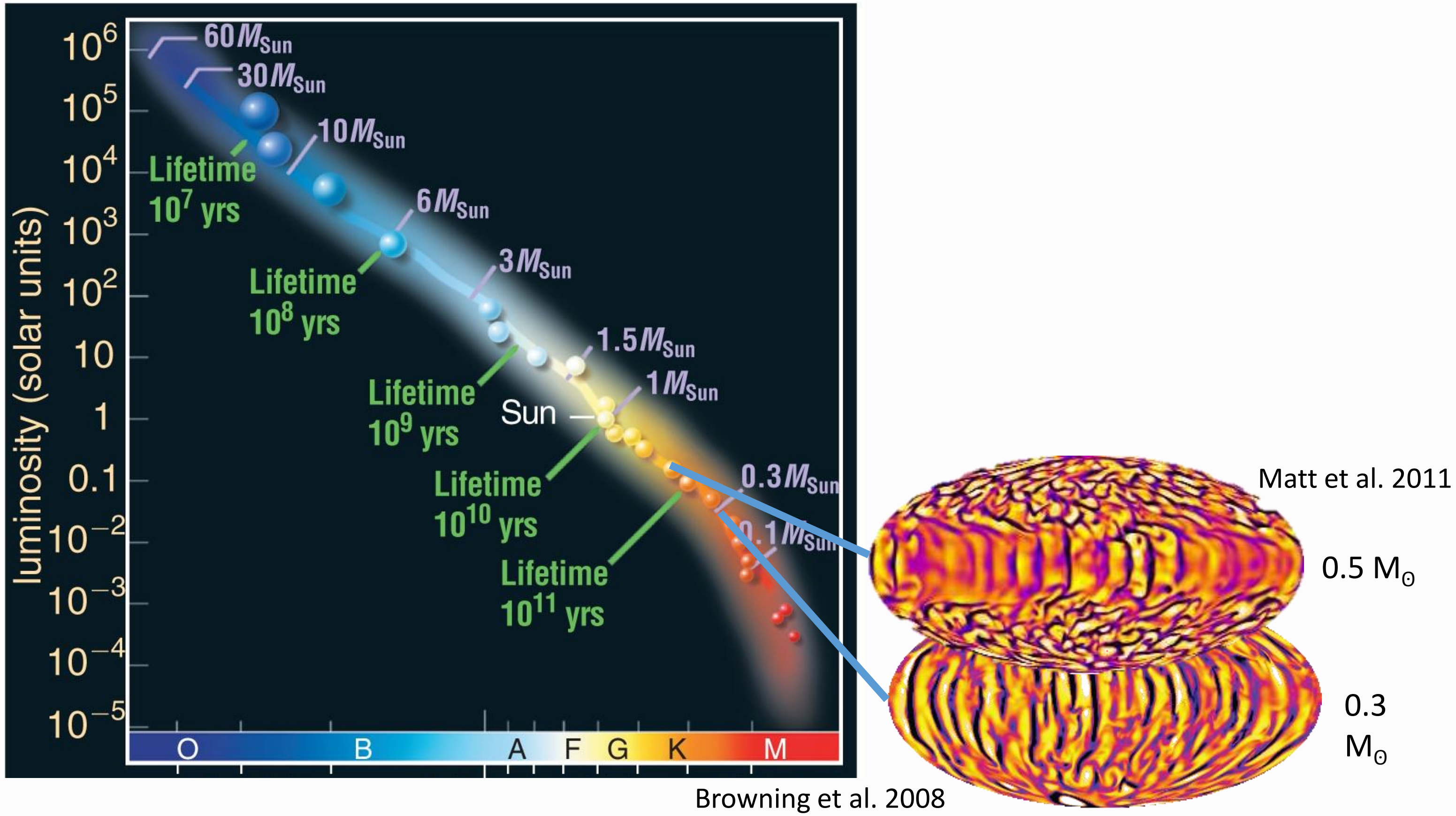



Browning et al. 2008



0.3 $M_{\odot}$

# Simulating stars along the Main-Sequence



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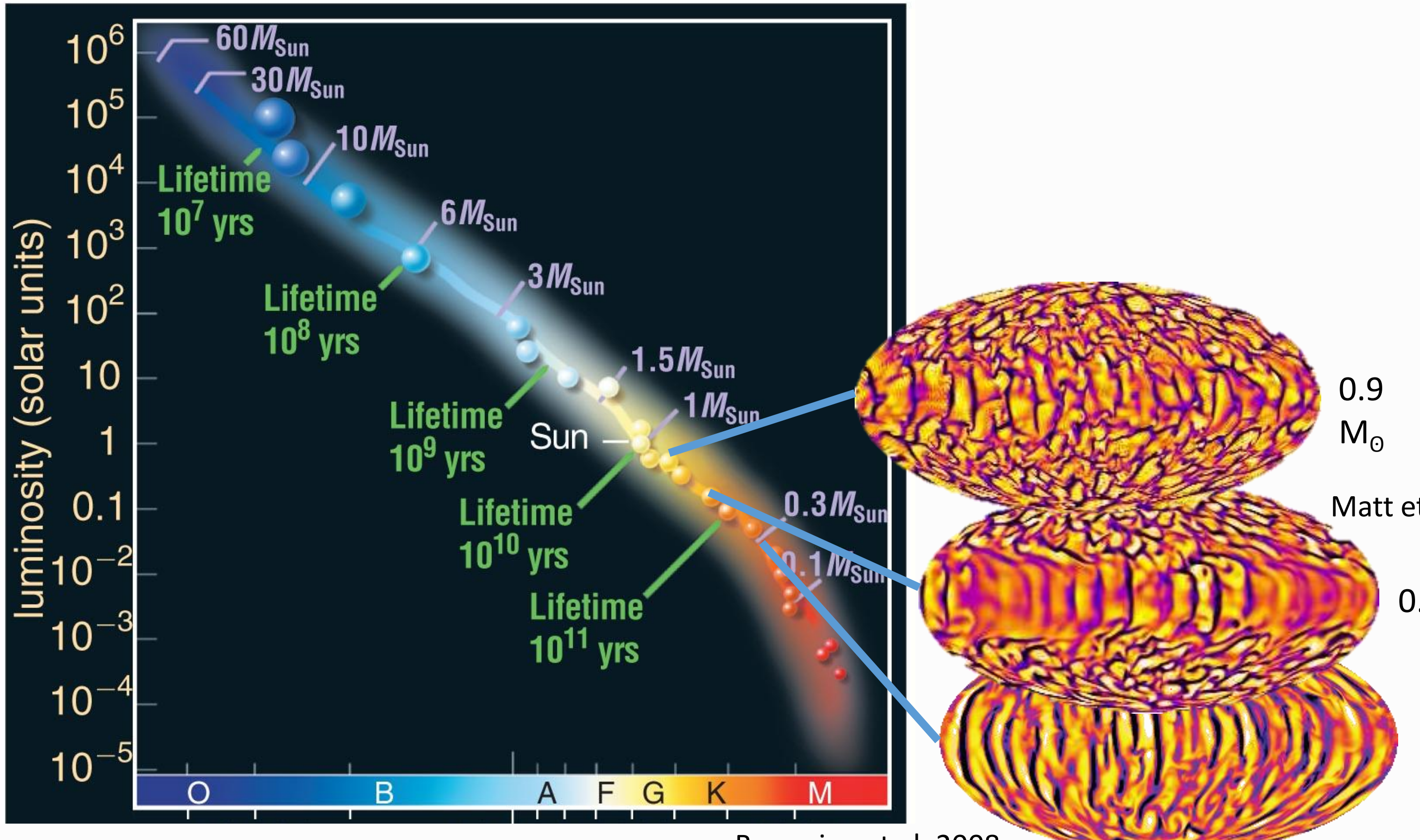

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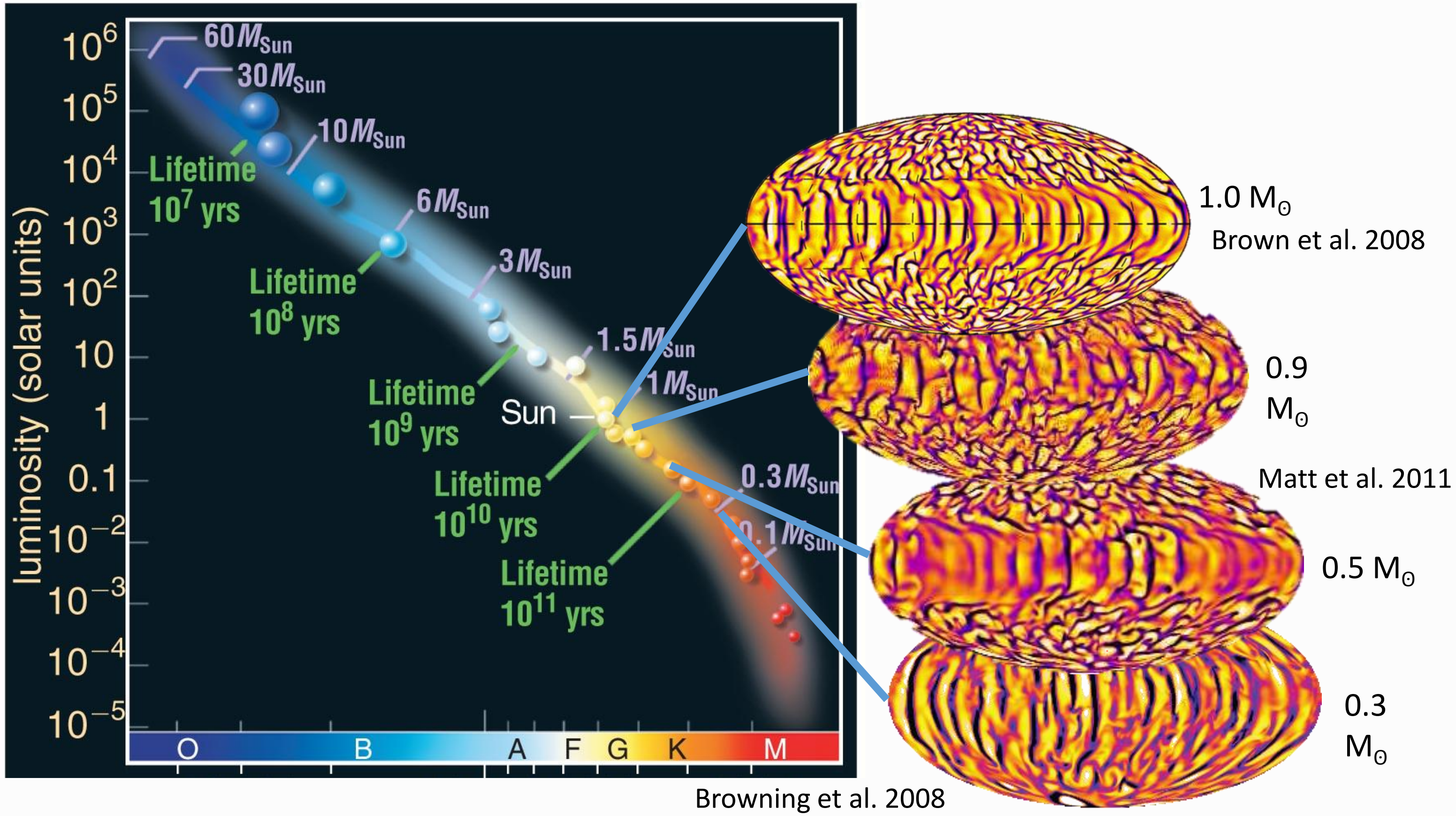

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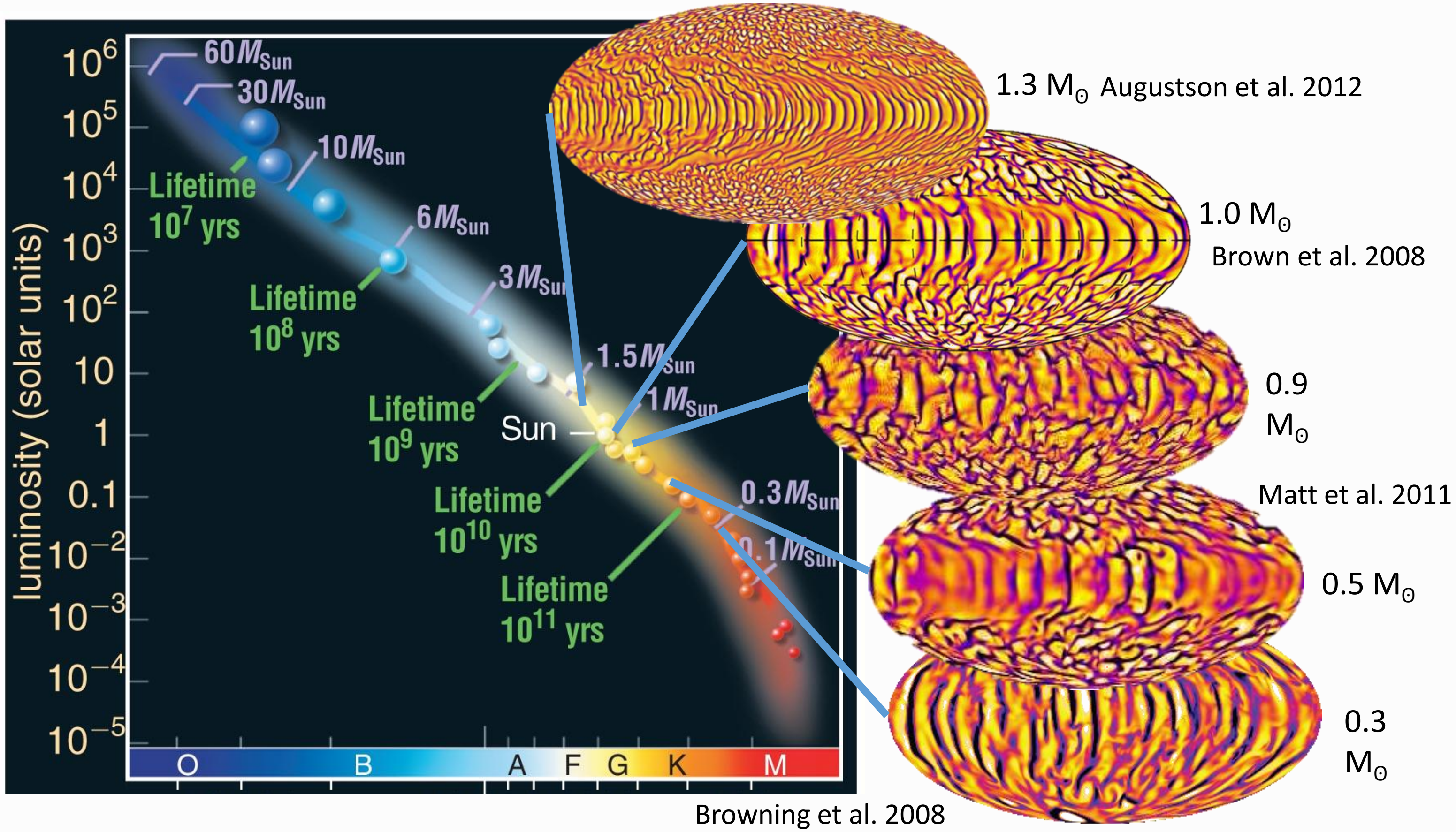

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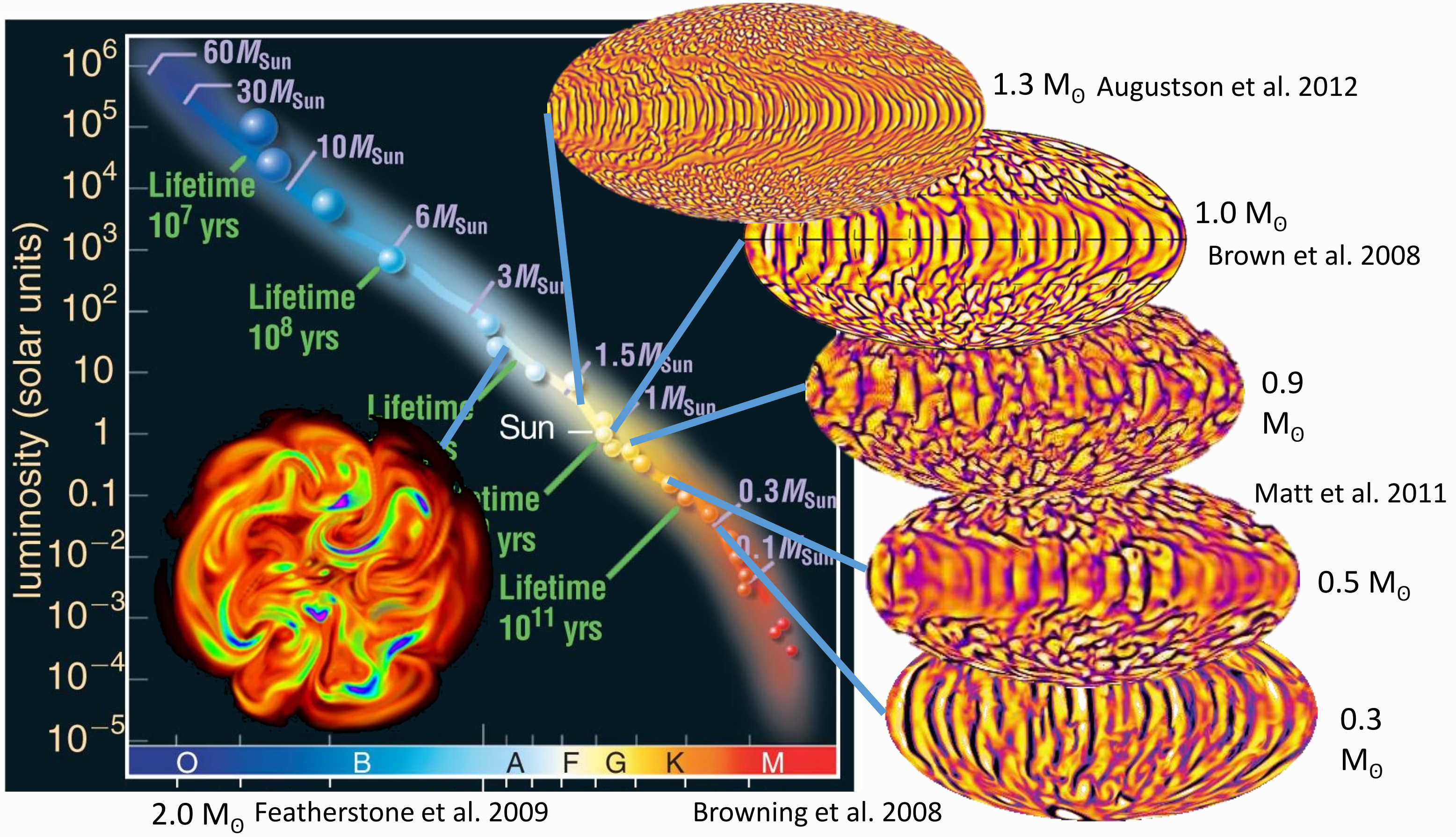

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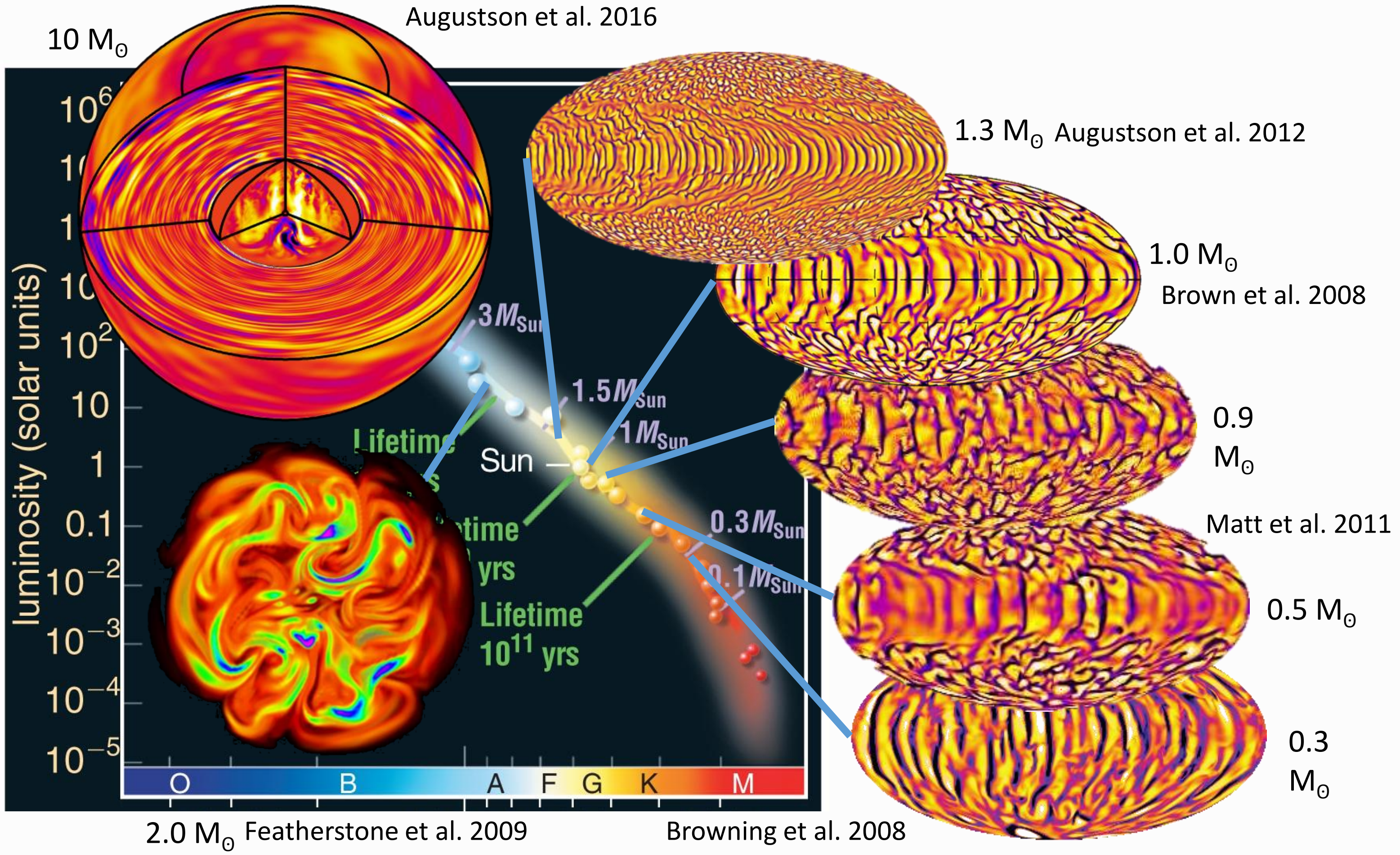
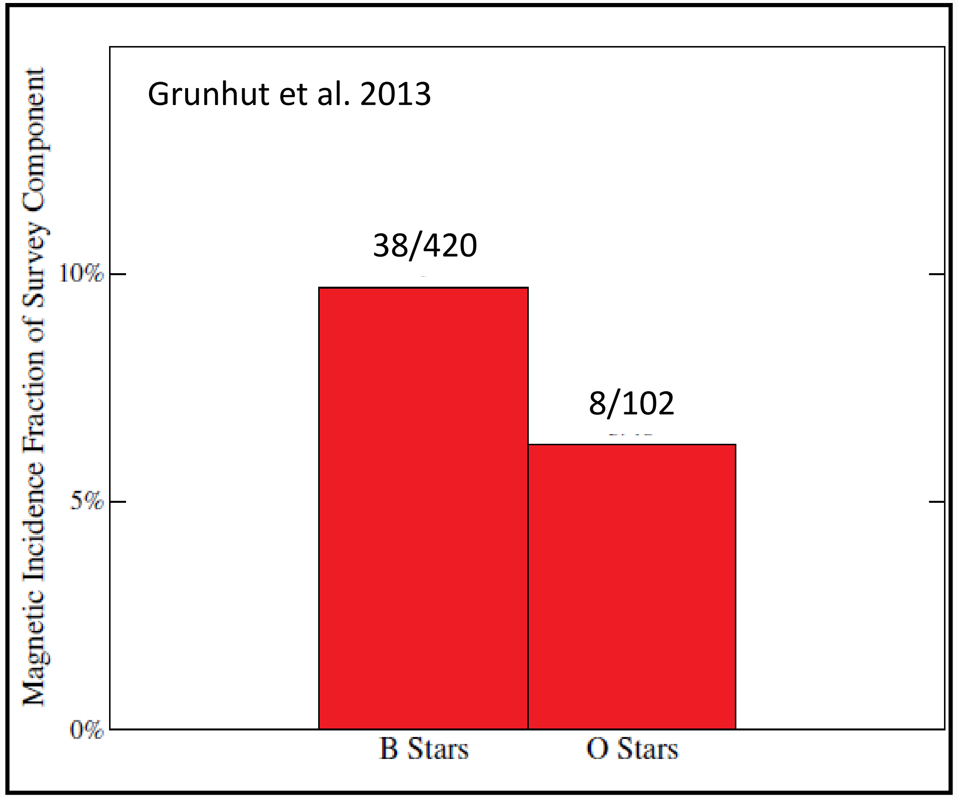
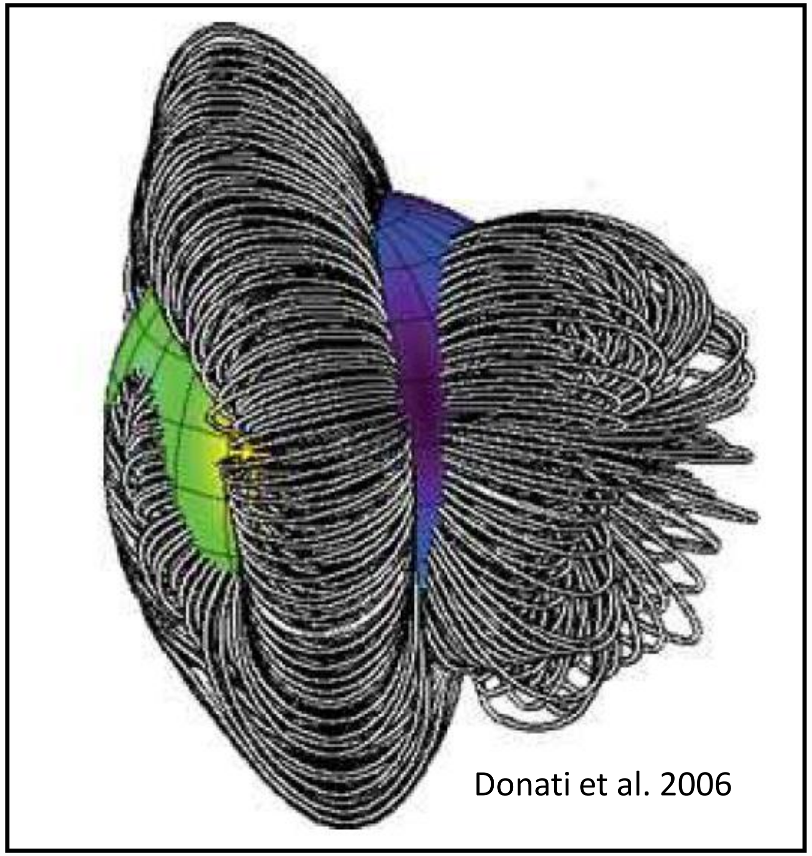


# Massive Star Magnetism



# Origins of Observed Massive Star Magnetism

- Dynamo-generated or frozen-in fields from PMS

  Lead to stable fossil fields in radiative regions

  (e.g. Braithwaite et al. 2006, Duez & Mathis 2010, Emeriau & Mathis 2015)

  Potential instability driving dynamo in radiative regions

  (e.g. Spruit 2002, Mullan et al. 2005)

- Core dynamo-generated fields from convective regions

  Influences later stages of evolution

  (e.g. Moss et al. 1989, Brun et al. 2005, Featherstone et al. 2009)

# So even if the core fields remain hidden…

- Evolution of a Massive Star
  - Pre-Main Sequence
    - Either convective or radiative depending upon mass
  - Main Sequence
    - Convective core and surface region, generate field how does this link to fossil field?
  - Helium Burning
    - Like the main-sequence, with a more compact core, stronger fields (geometry + density)!
  - Mixed Element Burning
    - Shellular burning with an even more compact core -> even stronger fields!
  - Silicon Burning
    - End-stage with shellular burning and extremely compact core -> strongest fields!

- The magnetic field at each stage depends upon the topological evolution of the previous one!

# Some Simple Considerations for Scaling Laws

- Consider a statistically steady state with the following force balance for a non-rotating system:

$$\rho \mathbf{v} \cdot \nabla \mathbf{v} \approx \frac{1}{4\pi} \boldsymbol{\nabla} \times \mathbf{B} \times \mathbf{B}.$$

- Further, let

$$\ell_v = Pm \ell_B$$

- Then, the equipartition magnetic field should roughly be

$$\frac{4\pi \ell_B}{\ell_v} \rho v^2 \approx B^2 \implies B_{\mathrm{eq}} \approx \left[ \frac{4\pi \rho v^2}{Pm} \right]^{1/2}$$

# Some Simple Considerations for Scaling Laws

- Extend this statistically-steady force balance to a rotating system:

$$\alpha\rho\mathbf{v}\cdot\nabla\mathbf{v}+2\rho\mathbf{v}\times\hat{\Omega}_0 \approx \frac{1}{4\pi}\nabla\times\mathbf{B}\times\mathbf{B},$$

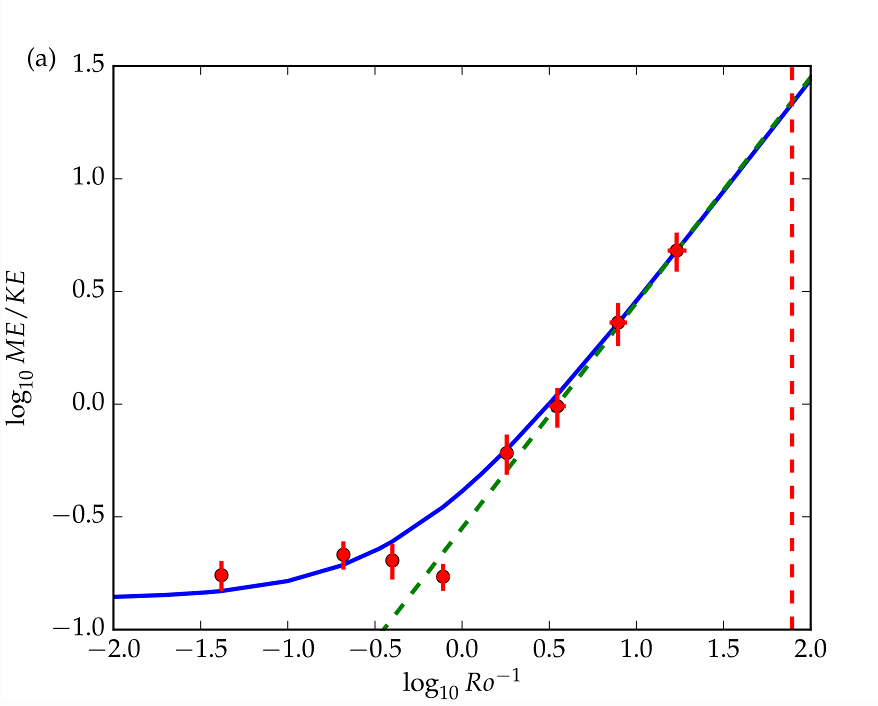
$$\Longrightarrow \frac{\alpha}{\ell}\rho v^2+2\rho v\Omega_0 \approx \frac{B^2}{4\pi\ell},$$

- Then, the super-equipartition magnetic field may scale as

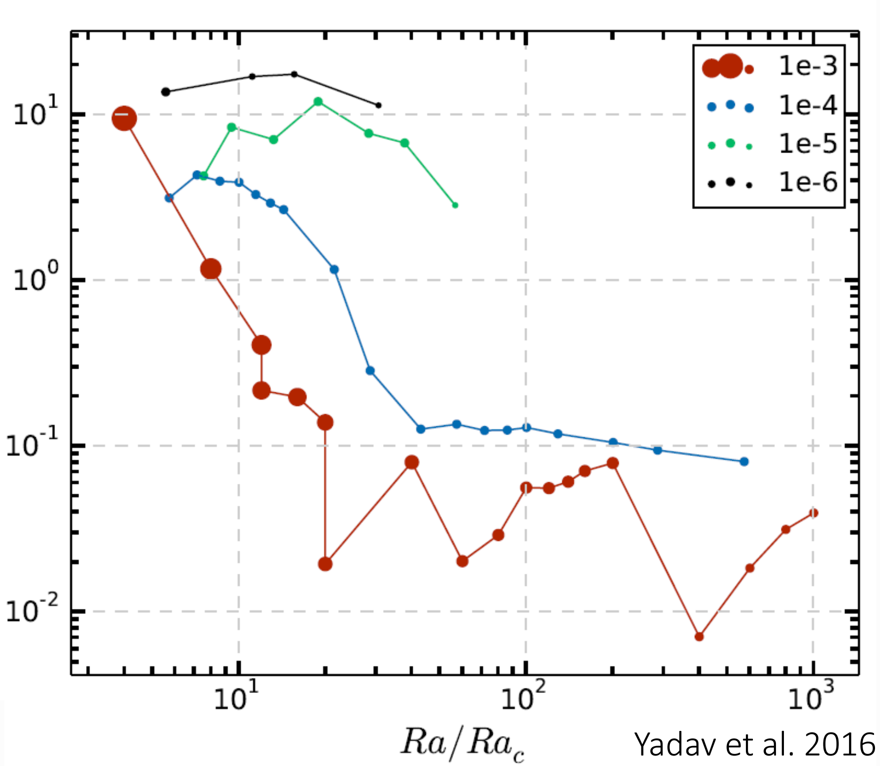
$$\Longrightarrow \frac{B^2}{8\pi} \approx \frac{1}{2}\rho v^2\left(\alpha+2\ell\Omega_0/v\right),$$

$$\Longrightarrow \frac{ME}{KE} \approx \alpha+Ro^{-1}.$$

# Some Simple Considerations for Scaling Laws




Augustson et al. 2016




Yadav et al. 2016

# What about other stars?

- How do the scalings depend upon the diffusive properties of the convective dynamo?
- Consider the MAC balance (for low Ro):

$$\boldsymbol{\Omega} \cdot \nabla \boldsymbol{u} \sim \nabla \times (\beta T' \mathbf{g}) \sim \nabla \times (\boldsymbol{B} \cdot \nabla \boldsymbol{B} / \rho \mu)$$

- The other basic consideration (for low Pm):

$$\int_{R_C} \beta \overline{T' \boldsymbol{u}} \cdot \mathbf{g} dV + \int_{R_C} \overline{\boldsymbol{J}^2} / \rho \sigma dV = 0.$$

# What about other stars?

- Together these imply that or

$$\frac{\ell_\perp}{\ell_{//}} \sim \left(\frac{\mathrm{P}}{\Omega^3 \ell_{//}^2}\right)^{1/9},$$

$$\frac{B^2/\rho\mu}{u^2} \sim \left(\frac{\mathrm{P}}{\Omega^3 \ell_{//}^2}\right)^{-2/9},$$

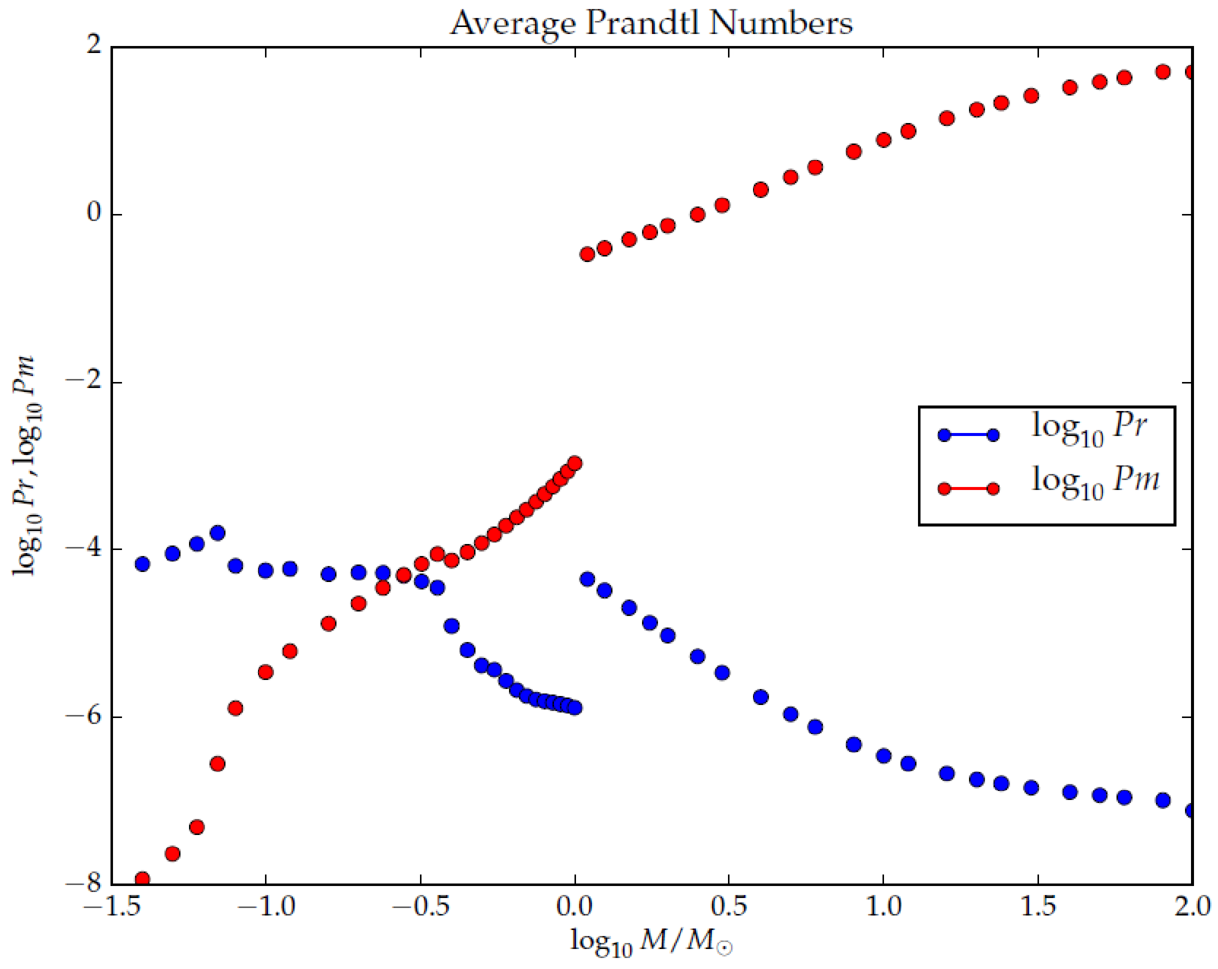
$$R_m = \frac{u\ell_{//}}{\lambda} \sim \frac{\sigma B^2}{\rho\Omega}\left(\frac{\mathrm{P}}{\Omega^3 \ell_{//}^2}\right)^{-2/9} = \Lambda\left(\frac{\mathrm{P}}{\Omega^3 \ell_{//}^2}\right)^{-2/9},$$

and

$$Ro = \frac{u}{\Omega\ell_{//}} \sim \left(\frac{\mathrm{P}}{\Omega^3 \ell_{//}^2}\right)^{4/9} = \left(\frac{(g\beta/\rho c_p)\, q_T}{\Omega^3 \ell_{//}^2}\right)^{4/9}.$$

$$\Longrightarrow \frac{ME}{KE} \approx \alpha + Ro^{-1/2}.$$

# What about other stars?



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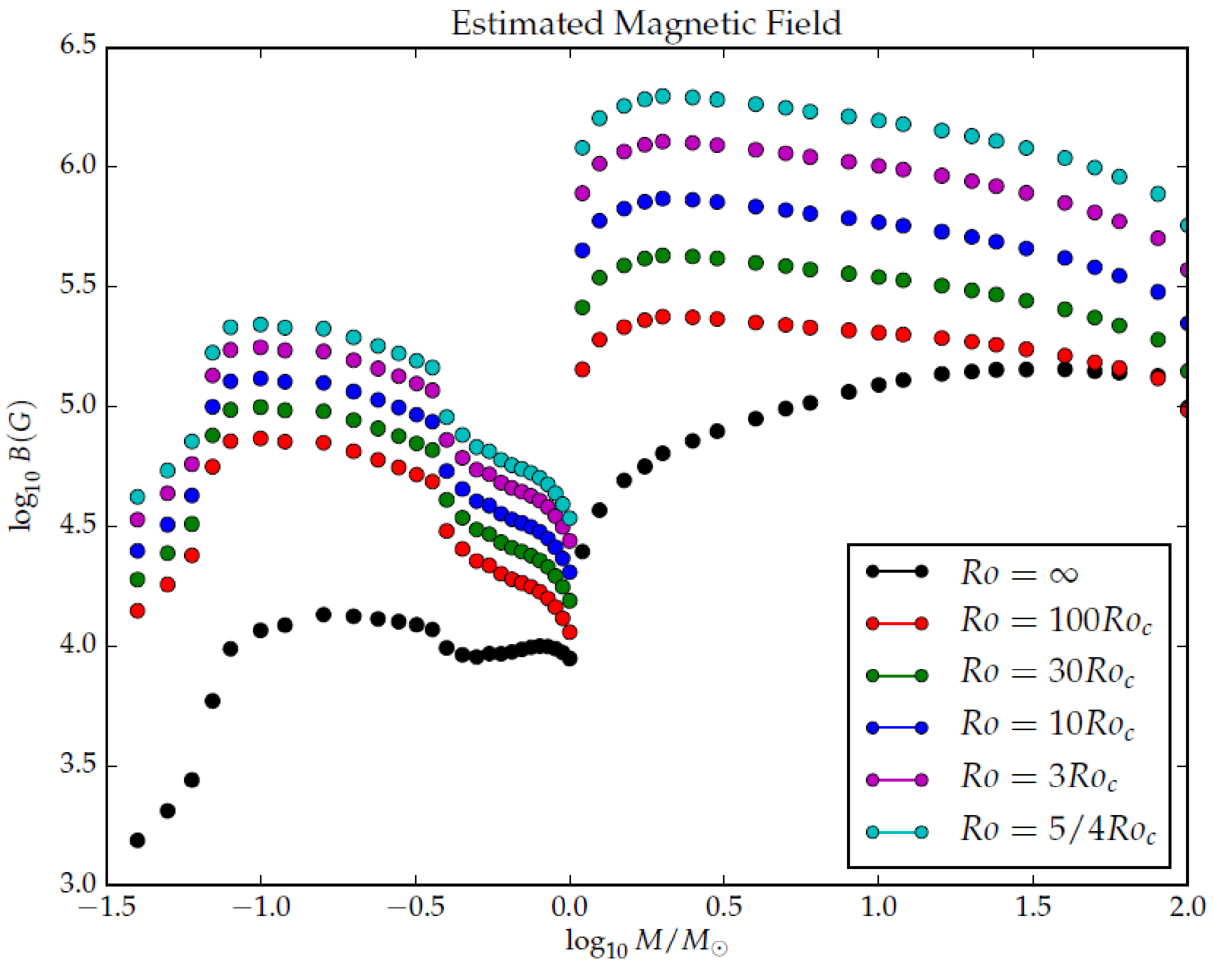
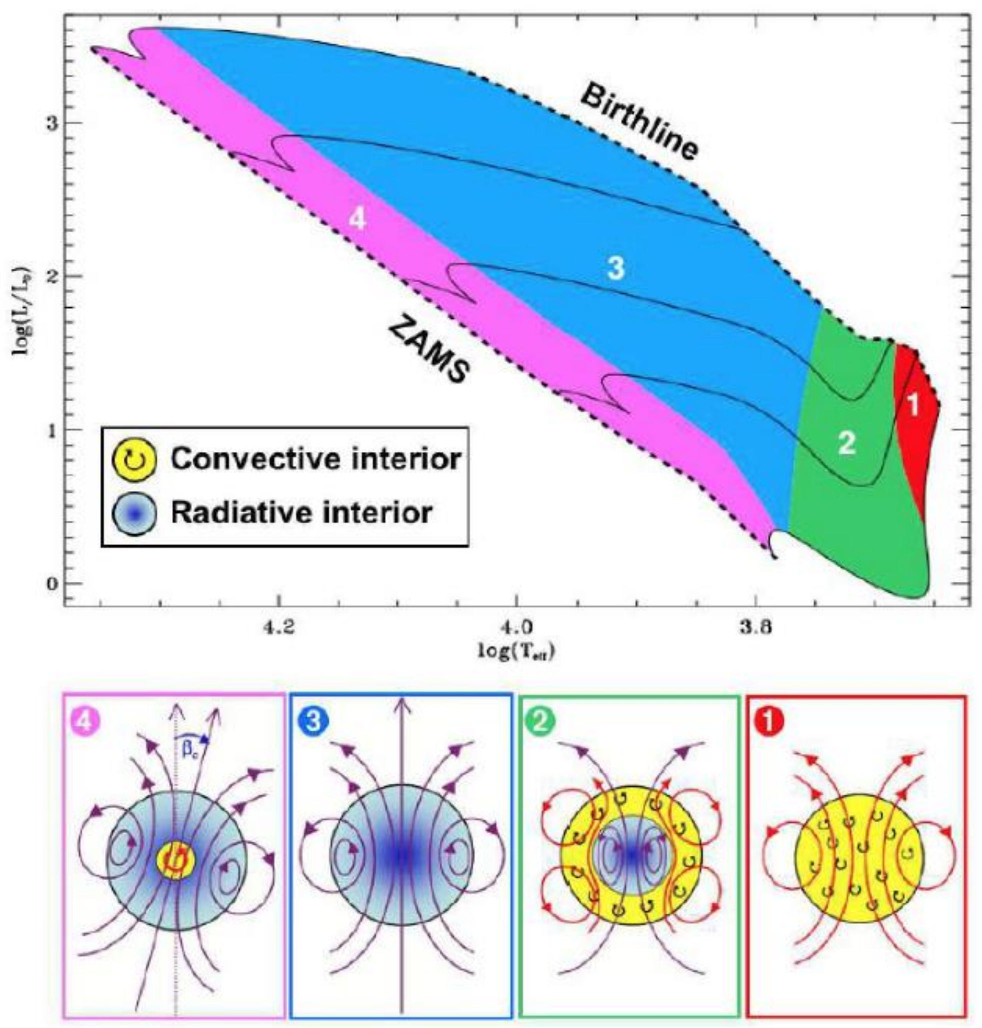


# The stability of magnetic field configurations

- Consider the formation of a star:
- It is convecting, rapidly rotating, and most likely differentially rotating
- So, DYNAMO!

- But what happens once the radiative region begins to form?



# The stability of magnetic field configurations

- With the tangled complex field remaining from the convective dynamo:
- What results after Ohmic decay and instabilities set in?
- The simplest story:
- With nothing to sustain them, large-scale flows are quenched on an Alfven time scale (fast compared to evolutionary scales).
- So, if in a stable configuration, the remaining magnetic field will slowly diffuse away.

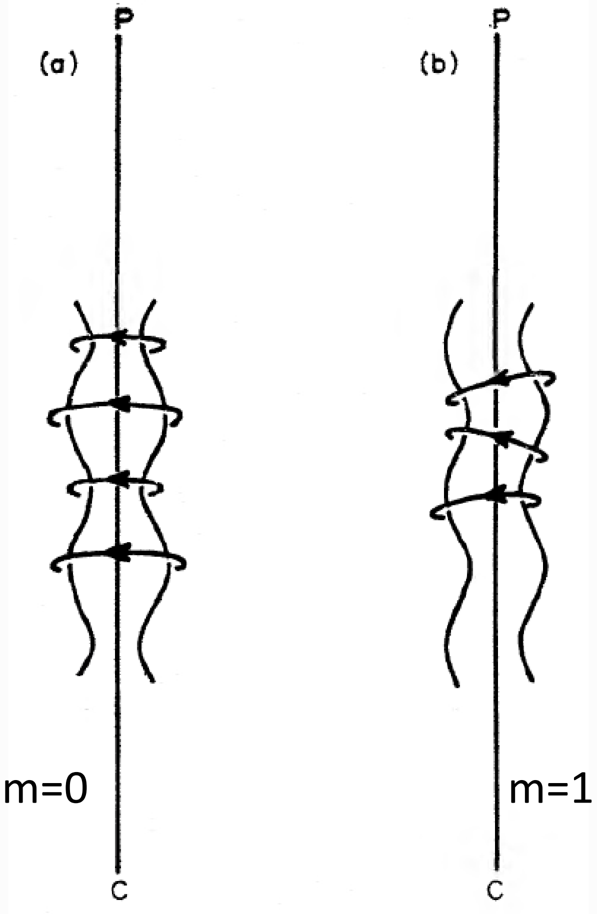
# The stability of magnetic field configurations

- This requires a stable magnetic field configuration!
- Yet there are possible instabilities, leading to a further loss of magnetic energy,
- e.g., the Tayler instability.

$$\omega^2 = -\frac{\langle \psi, \mathcal{F}[\psi]\rangle}{\langle \psi, \psi\rangle} = \frac{2\Delta W}{\langle \psi, \psi\rangle}.$$

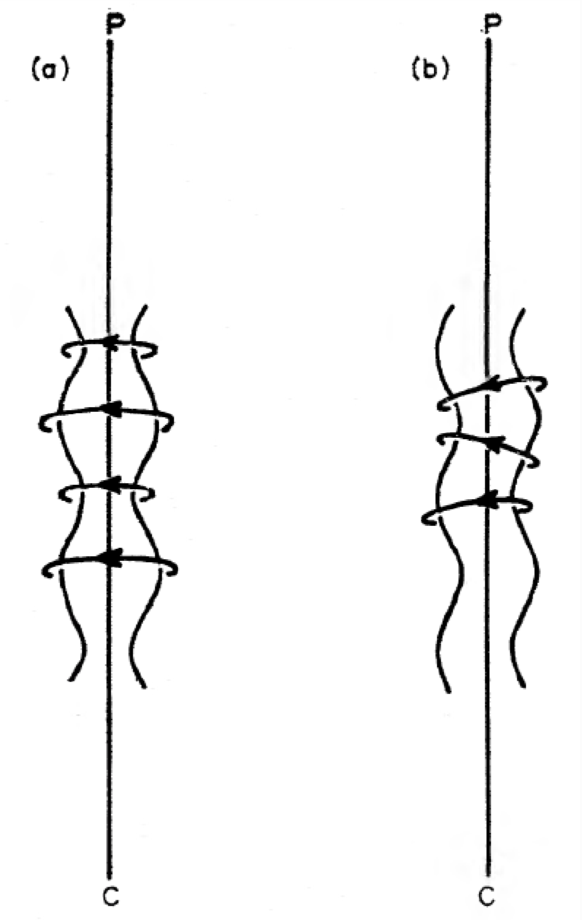
$$g_\varpi \frac{\partial \rho}{\partial \varpi} - \frac{(\rho g_\varpi - 2B^2/\varpi)^2}{B^2 + \gamma P} - \frac{2B}{\varpi}\frac{\partial B}{\partial \varpi} + \frac{2B^2}{\varpi^2} > 0,$$ m=0

$$g_\varpi \frac{\partial \rho}{\partial \varpi} - \frac{\rho^2 g_\varpi{}^2}{\gamma P} - \frac{B^2}{\varpi^2} - \frac{2B}{\varpi}\frac{\partial B}{\partial \varpi} > 0$$ m=1



# The stability of magnetic field configurations

- The Tayler instability, predicts that flows can develop if
- Certain magnitudes of the magnetic field are reached,
- or he field has a certain type of radial dependence.
- The flows instabilities can be suppressed if
- Bp ~ Bphi



# The stability of magnetic field configurations

- What remains unknown is what happens if the background magnetic field is non-axisymmetric!
- This is a tricky problem… requiring the minimization of $\Delta W = \Delta W_L + \Delta W_B + \Delta W_P$
- Which in a sphere looks like:

$$\Delta W_L = \int dr r^2 \sum_{\substack{\ell,m,\nu,\mu,\lambda \\ \ell_1,m_1,\nu_1 \\ \ell_2,m_2,\nu_2 \\ \ell_3,m_3,\nu_3 \\ \ell_4,m_4,\nu_4}} \left[ {}_{\mu,\lambda}L^{m_1,m_2,m_3,m_4}_{\substack{\ell_1,\ell_2,\ell_3,\ell_4 \\ \nu_1,\nu_2,\nu_3,\nu_4}} \left( \xi^{m_3}_{\ell_3,\nu_3} B^{m_4}_{\ell_4,\nu_4} \right) \xi^{*m}_{\ell,\nu} \mathcal{J}^{\ell_2,m_2,\lambda}_{\substack{\ell_3,\nu_3,m_3 \\ \ell_4,\nu_4,m_4}} \mathcal{J}^{\ell,m,\nu}_{\substack{\ell_1,\nu_1,m_1 \\ \ell_2,\nu_2,m_2}} \right].$$

$$\begin{aligned} {}_{\lambda,\mu}L^{m_1,m_2,m_3,m_4}_{\substack{\ell_1,\ell_2,\ell_3,\ell_4 \\ \nu_1,\nu_2,\nu_3,\nu_4}} (\cdot) = & \\ B^{m_1}_{\ell_1,\nu_1} & \left[ E^{\ell_2,m_2}_{\nu_2,\lambda} \frac{\partial^2 (\cdot)}{\partial r^2} + \frac{F^{\ell_2,m_2}_{\nu_2,\lambda}}{r} \frac{\partial (\cdot)}{\partial r} + \frac{G^{\ell_2,m_2}_{\nu_2,\lambda} (\cdot)}{r^2} \right] \mathcal{I}_{\lambda,\mu} \\ & - \left[ D^{\ell_1,m_1}_{\nu_1,\lambda} \frac{\partial B^{m_1}_{\ell_1,\lambda}}{\partial r} + C^{\ell_1,m_1}_{\nu_1,\lambda} \frac{B^{m_1}_{\ell_1,\lambda}}{r} \right] \left[ D^{\ell_2,m_2}_{\nu_2,\mu} \frac{\partial (\cdot)}{\partial r} + \frac{C^{\ell_2,m_2}_{\nu_2,\mu} (\cdot)}{r} \right], \end{aligned}$$

$$\begin{aligned} \Delta W_B = \sum_{\substack{\ell,m,\ell_1 \\ \ell_2,m_2,\nu_2}} \int dr \frac{(-1)^{m_2} g r^2}{2\ell + 1} \left( \sqrt{\ell + 1} \xi^{*m}_{\ell,1} - \sqrt{\ell} \xi^{*m}_{\ell,-1} \right) & \\ \left[ \sqrt{\ell + 1} \left( \frac{\partial}{\partial r} + \frac{\ell + 2}{r} \right) \mathcal{K}^{m,m_2}_{\ell_1;\substack{\ell,1 \\ \ell_2,\nu_2}} \right. & \\ \left. - \sqrt{\ell} \left( \frac{\partial}{\partial r} - \frac{\ell - 1}{r} \right) \mathcal{K}^{m,m_2}_{\ell_1;\substack{\ell,-1 \\ \ell_2,\nu_2}} \right] & \rho^{m_2 - m}_{\ell_1} \xi^{m_2}_{\ell_2,\nu_2}. \end{aligned}$$

$$\begin{aligned} \Delta W_P = -\sum_{\substack{\ell,m,\ell_1 \\ \ell_2,m_2}} \int dr \frac{r^2}{\sqrt{(2\ell + 1)(2\ell_2 + 1)}} & \\ \left[ \sqrt{\ell + 1} \left( \frac{\partial}{\partial r} + \frac{\ell + 2}{r} \right) \xi^{*m+m_2}_{\ell,1} + \sqrt{\ell} \left( -\frac{\partial}{\partial r} + \frac{\ell - 1}{r} \right) \xi^{*m+m_2}_{\ell,-1} \right] & \\ \left\{ \sum_{\nu_1} \xi^{m}_{\ell_1,\nu_1} \left[ \sqrt{\ell_2 + 1} \left( \frac{\partial}{\partial r} - \frac{\ell_2}{r} \right) \mathcal{K}^{m_1,m_2}_{\ell;\substack{\ell_1,\nu_1 \\ \ell_2,1}} \right. \right. & \\ \left. - \sqrt{\ell_2} \left( \frac{\partial}{\partial r} + \frac{\ell_2 + 1}{r} \right) \mathcal{K}^{m_1,m_2}_{\ell;\substack{\ell_1,\nu_1 \\ \ell_2,-1}} \right] P^{m_2}_{\ell_2} & \\ + (-1)^{m+m_2} \gamma \mathcal{H}_{\substack{\ell_1,\ell,\ell_2 \\ m,m_2}} P^{m}_{\ell} \left[ \sqrt{\ell_2 + 1} \left( \frac{\partial}{\partial r} + \frac{\ell_2 + 2}{r} \right) \xi^{m_2}_{\ell_2,1} \right. & \\ \left. \left. + \sqrt{\ell_2} \left( -\frac{\partial}{\partial r} + \frac{\ell_2 - 1}{r} \right) \xi^{m_2}_{\ell_2,-1} \right] \right\}. & \end{aligned}$$

# Conclusions

- Convection = Dynamo,
- Convection + Rotation = Even better Dynamo,
- Scaling laws have (a likely limited, but nevertheless interesting) power to estimate the potential magnetic field strength in a range of stars.
- Pre-main sequence stars will build magnetic fields, as they develop convectively stable regions, those fields will decay toward a stable configuration.
- Certain instabilities may govern which of those remain, and hence what might be observable.