

Exact solutions of Einstein's equations in astrophysics and cosmology

Austin Peel
SAp, office 277

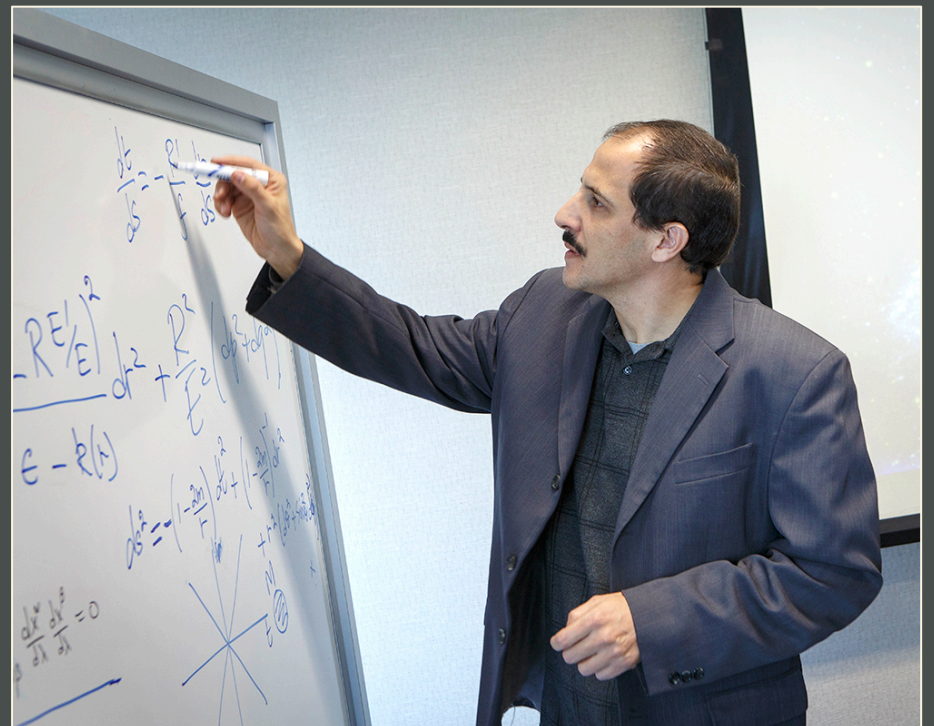
19 Jan, 2016



DALLAS, TEXAS



PhD 2015
supervised by
Mustapha Ishak-Boushaki



Outline

general relativity (GR) in general

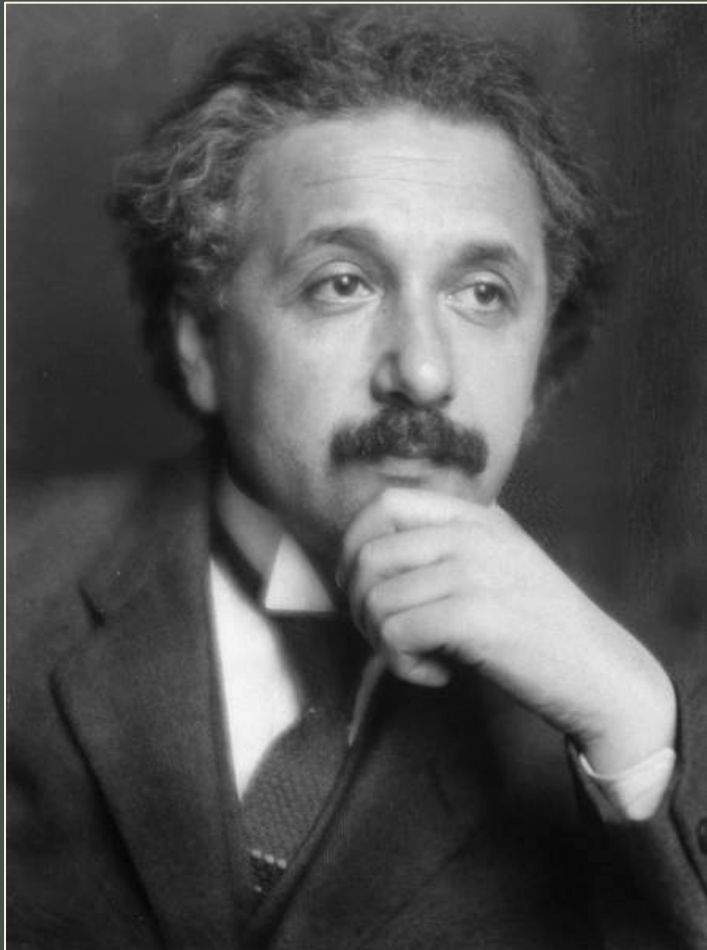
exact solutions

familiar(?) examples

the Szekeres metric

a Swiss-cheese universe model

work at CEA



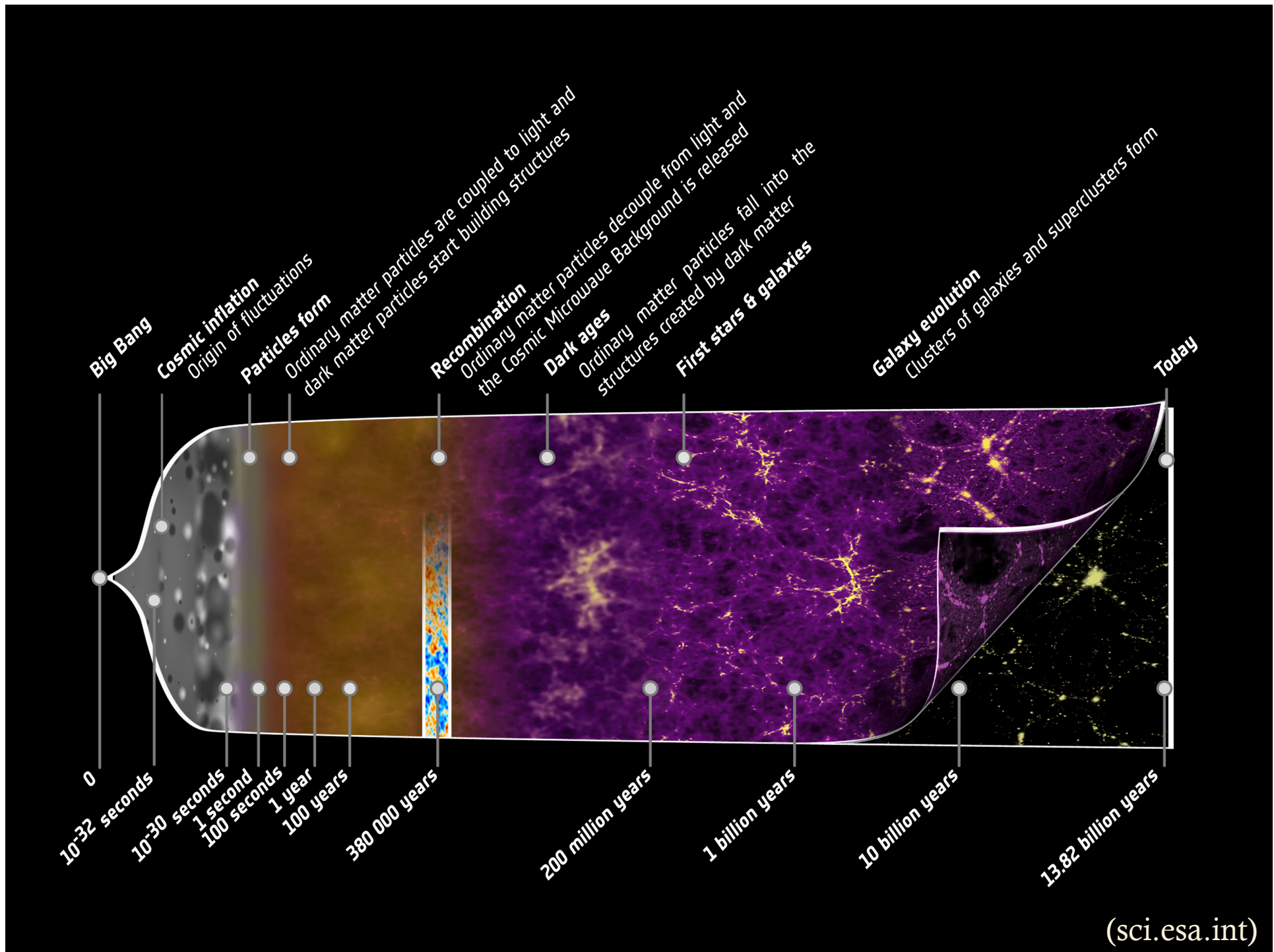
Albert Einstein in 1921

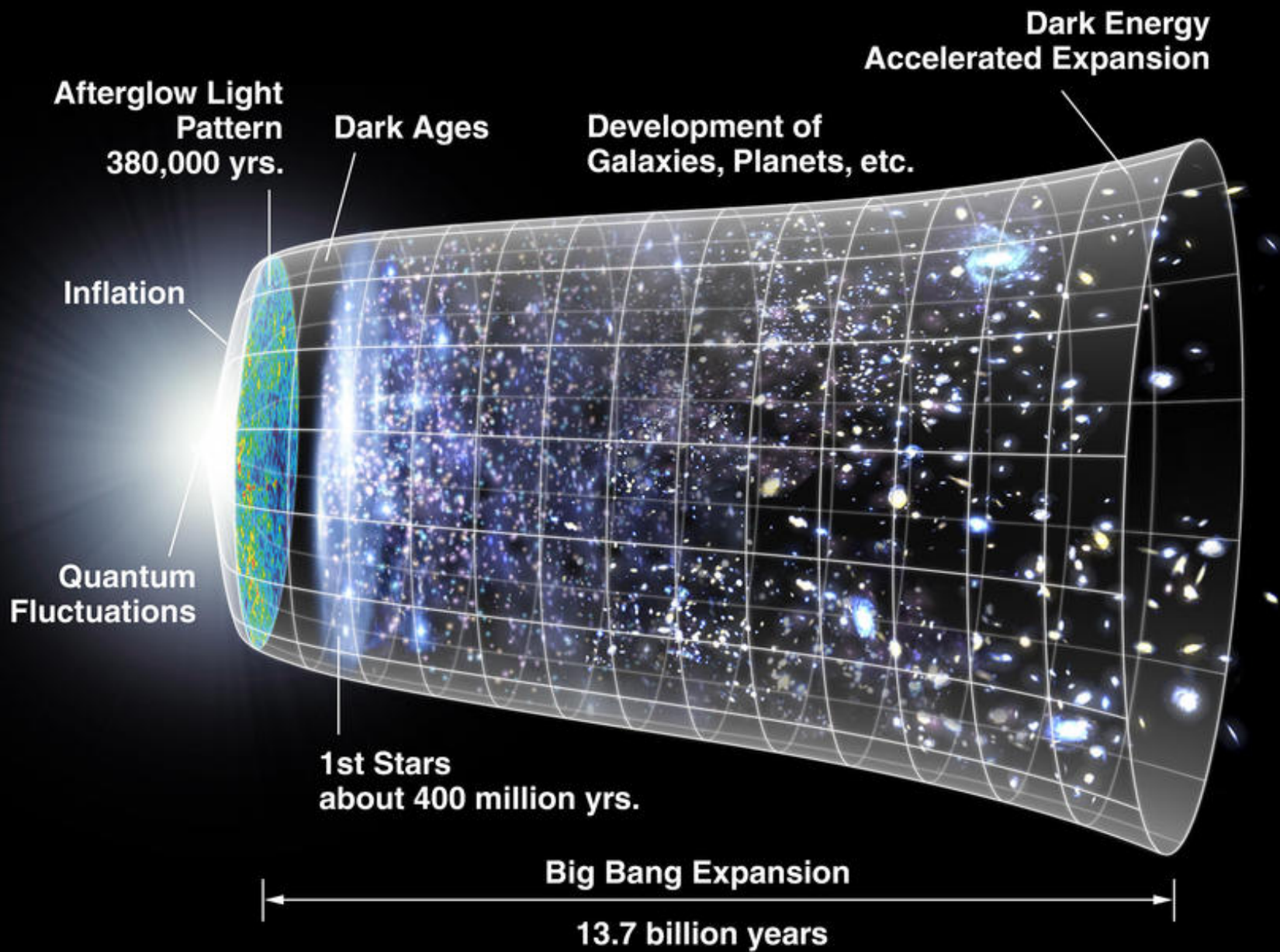
modern theory of gravitation

published by Einstein in 1915

“Spacetime tells matter how to move;
matter tells spacetime how to curve.”

- John A. Wheeler





(NASA/WMAP)



$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(spacetime curvature = energy and momentum)

analogy with electromagnetism

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

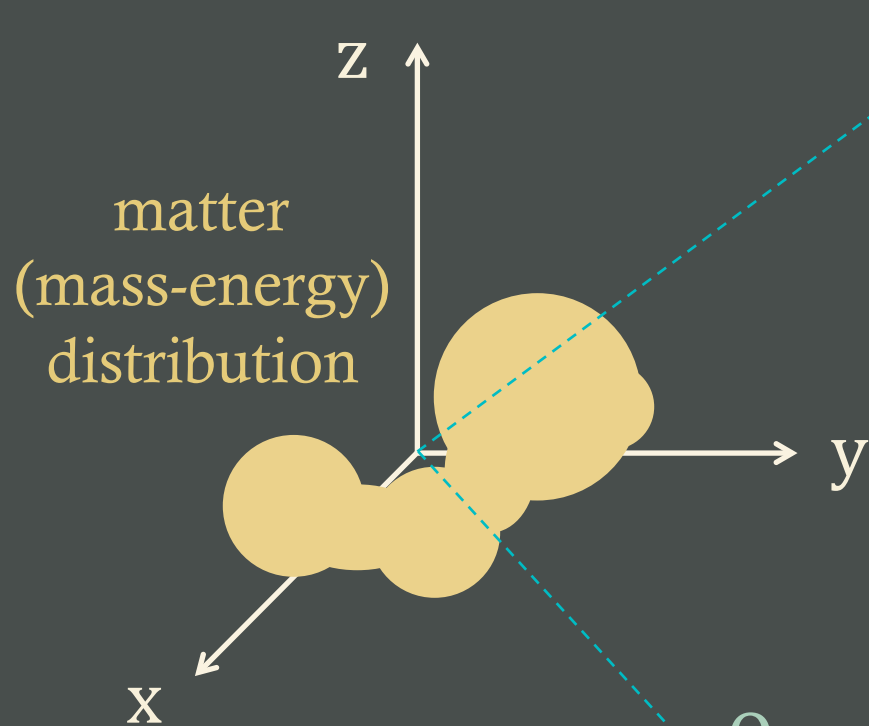
electric field \longleftrightarrow source charges

$$\frac{\partial E_x(x, y, z)}{\partial x} + \frac{\partial E_y(x, y, z)}{\partial y} + \frac{\partial E_z(x, y, z)}{\partial z} = \frac{\rho(x, y, z)}{\epsilon_0}$$



$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

metric tensor



$$g_{\mu\nu}(P) = \left(16 \text{ numbers} \right)$$

$$g_{\mu\nu}(Q) = \left(16 \text{ (different) numbers} \right)$$



Greek indices

$$\mu, \nu, \text{etc.} \in \{0, 1, 2, 3\}$$

$$x^\mu = (x^0, x^1, x^2, x^3)$$

$$\text{or } (t, x, y, z)$$

$$\text{or } (t, r, \theta, \phi)$$

$$\vdots$$

general tensor field components

$$A^{\alpha\beta} = \begin{pmatrix} A^{00} & \dots & A^{03} \\ \vdots & \ddots & \\ A^{30} & & A^{33} \end{pmatrix}$$



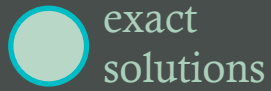
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein tensor $G_{\mu\nu}$ is a complicated nonlinear function of $g_{\mu\nu}$ involving first- and second-order derivatives



$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Energy-momentum tensor $T_{\mu\nu}$ represents the source of the gravitational field (e.g., vacuum, matter fluid, radiation, etc.)



Q. What is an exact solution?

A. $g_{\mu\nu}$ with corresponding $T_{\mu\nu}$ such that ($\kappa \equiv 8\pi G/c^4$)

$$1) \quad G_{00} + \Lambda g_{00} = \kappa T_{00},$$

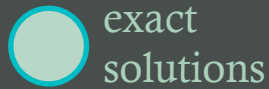
$$2) \quad G_{01} + \Lambda g_{01} = \kappa T_{01},$$

$$\vdots$$
$$\vdots$$

$$6) \quad G_{12} + \Lambda g_{12} = \kappa T_{12},$$

$$\vdots$$
$$\vdots$$

$$10) \quad G_{33} + \Lambda g_{33} = \kappa T_{33} \quad \text{exactly.}$$



Are solutions hard to come by?

“I had not expected that one could formulate the exact solution of the problem in such a simple way.”

- Einstein to Schwarzschild

How many are there?

- more than 1000 independent solutions
- focus now on interpreting existing solutions, rather than deriving new ones



exact
solutions



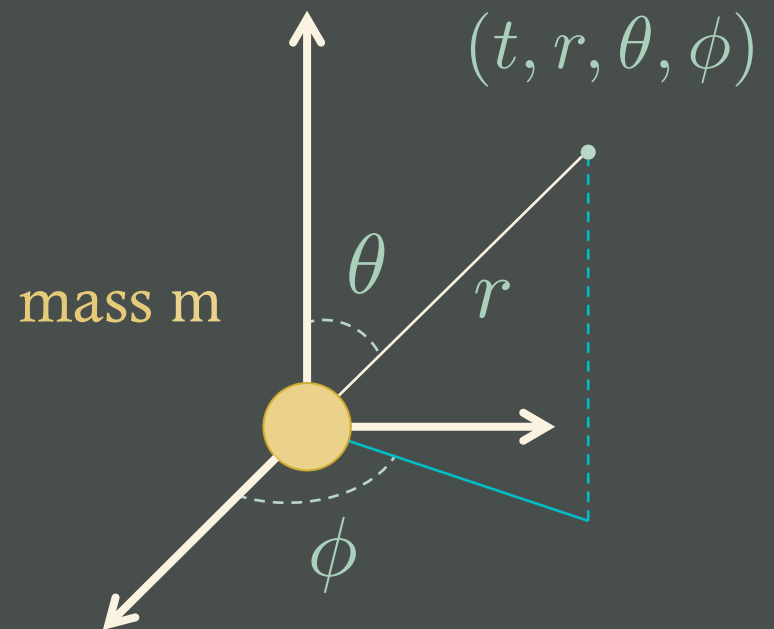
Considerations

- *any* metric tensor is a solution, but...
- usually make *symmetry* assumptions
- metric functions should be expressed in terms of *elementary* or well-known *special functions*
- *interpretation* is often difficult

Astrophysical:

Schwarzschild

- non-rotating **spherically symmetric** object
- *unique* external solution
- idealized **star** or **black hole**





examples



metric / line element

$$ds^2 = -c^2 \left(1 - \frac{2Gm}{c^2 r}\right) dt^2 + \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

metric tensor components

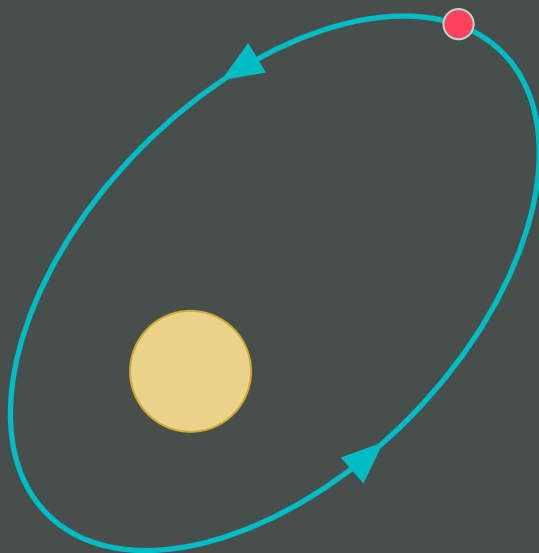
$$g_{\mu\nu} = \begin{pmatrix} -c^2 \left(1 - \frac{2Gm}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$



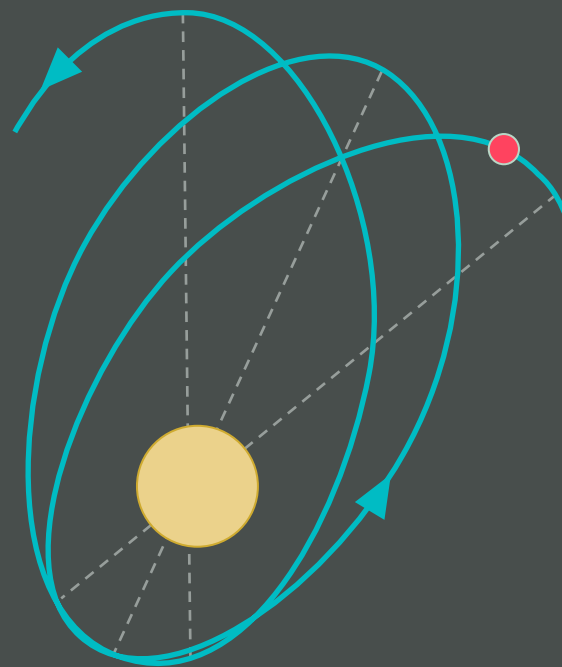
examples



Newton



Einstein



precession

a linguistic sidebar...

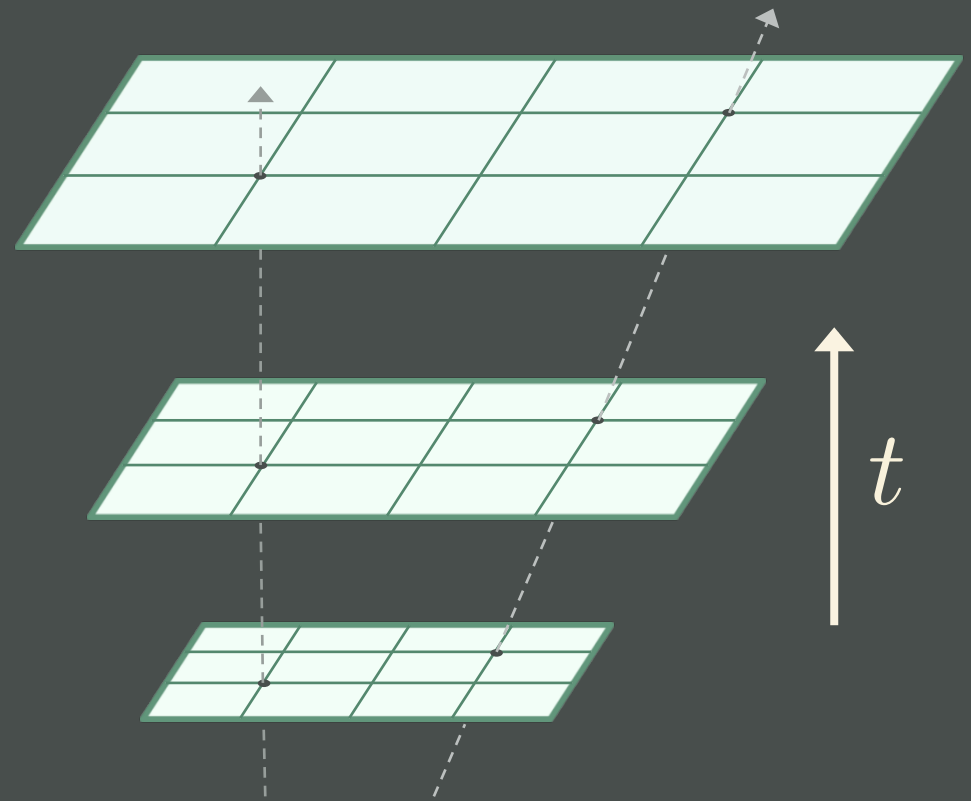
Schwarzs|child ✗

Schwarz|schild ✓

Cosmological exact solutions

Friedmann-Lemaître-Robertson-Walker (FLRW)

- math of cosmology's standard model
- spatially homogeneous and isotropic
- sourced by uniform perfect fluid

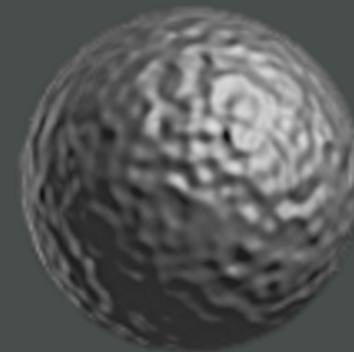
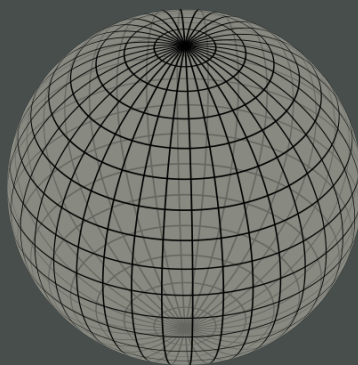




examples



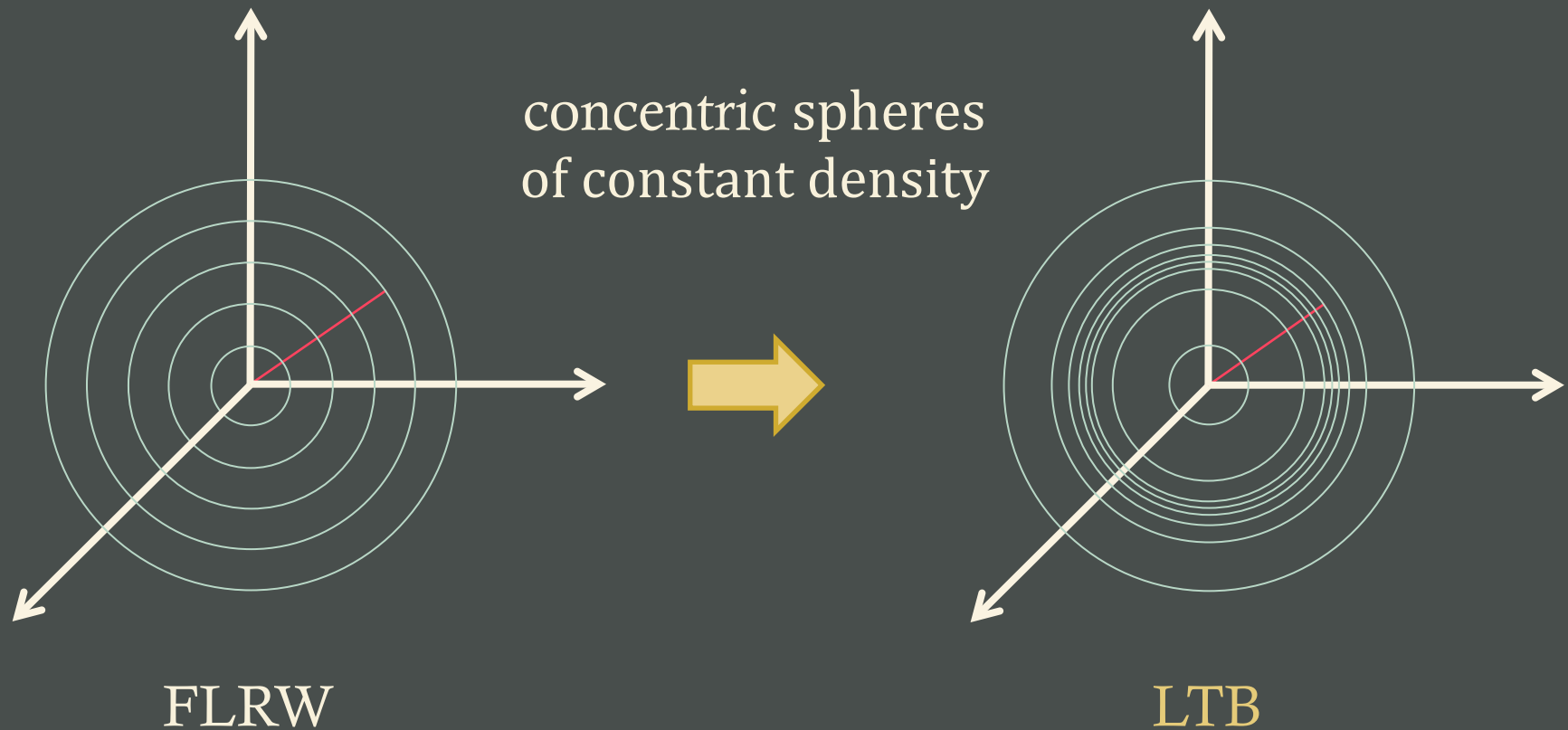
assume spherical ~~cos~~ Earth...



“FLRW stage”

“FLRW +
perturbations”

Lemaître-Tolman-Bondi (LTB)





examples



LTB cosmology

- variable expansion rate mimics cosmic acceleration
- “giant void” models explain away **dark energy**

We can't rule out LTB models.

“Inhomogeneous alternative to dark energy?” A. Håvard, M. Amarzguioui, Ø. Grøn (2006)

“Confronting Lemaître-Tolman-Bondi models with Observational Cosmology”
J. Garcia-Bellido, T. Haugbølle (2008)

Yes we can.

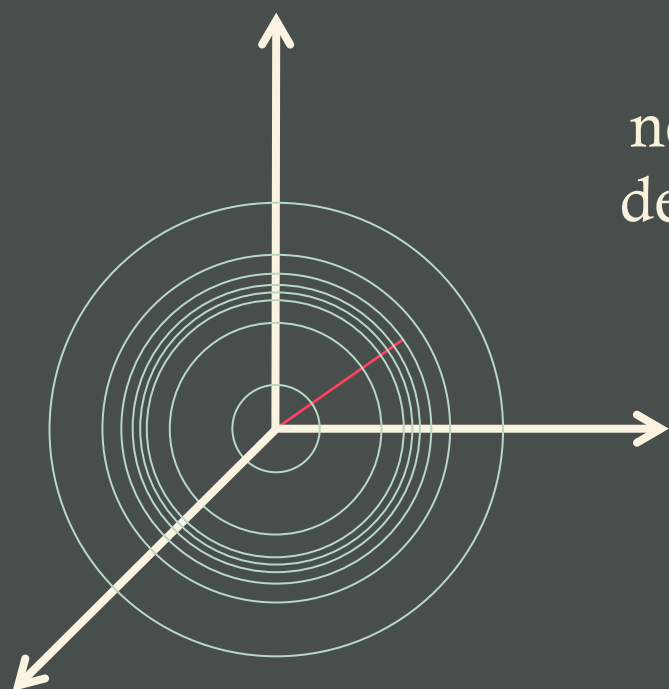
“Precision cosmology defeats void models for acceleration” A. Moss, J. Zibin, D. Scott (2010)



Szekeres

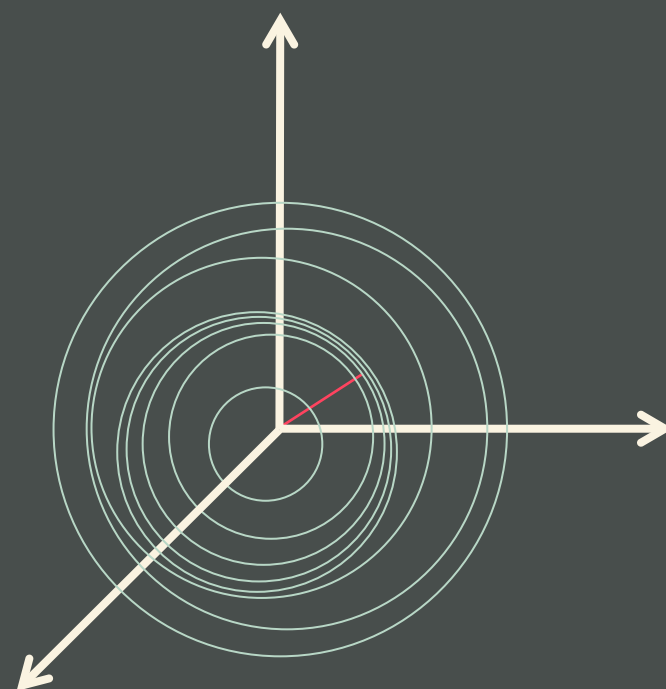


Szekeres (enfin!)



LTB

non-concentric
density spheres



Szekeres

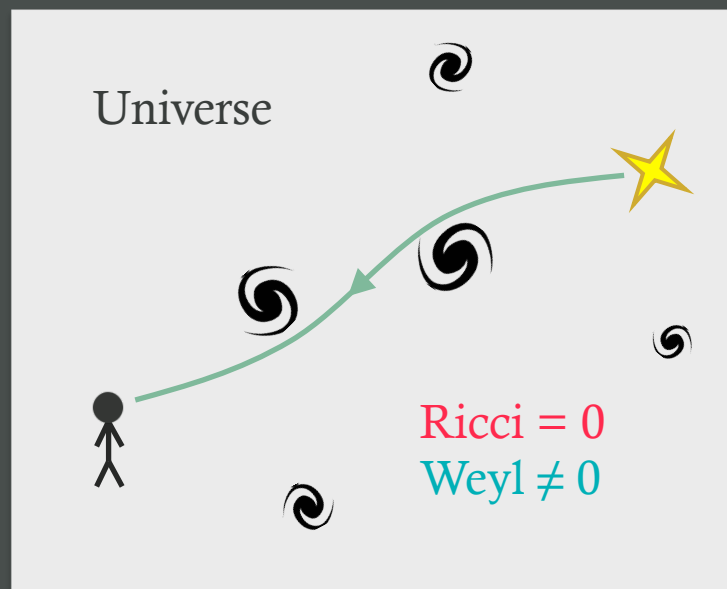
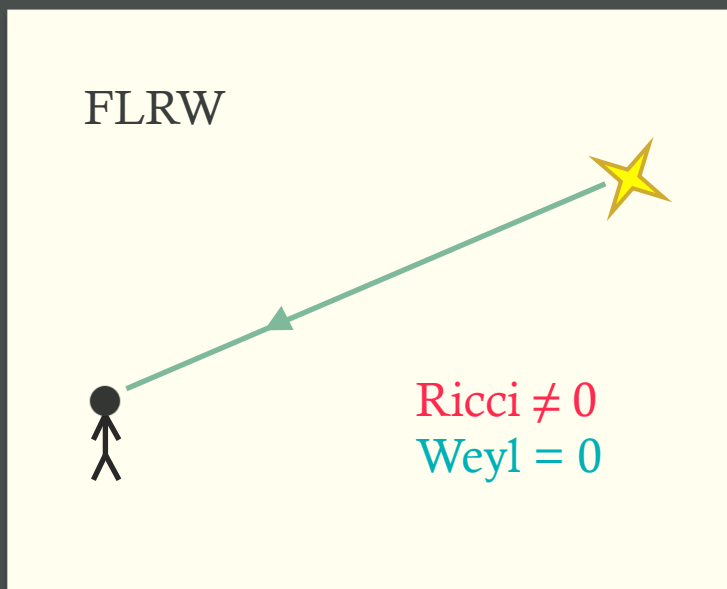


Swiss-cheese



Light propagation

different types of spacetime curvature affect properties of light beams differently



full (Riemann) curvature = Ricci curvature + Weyl curvature



Swiss-cheese



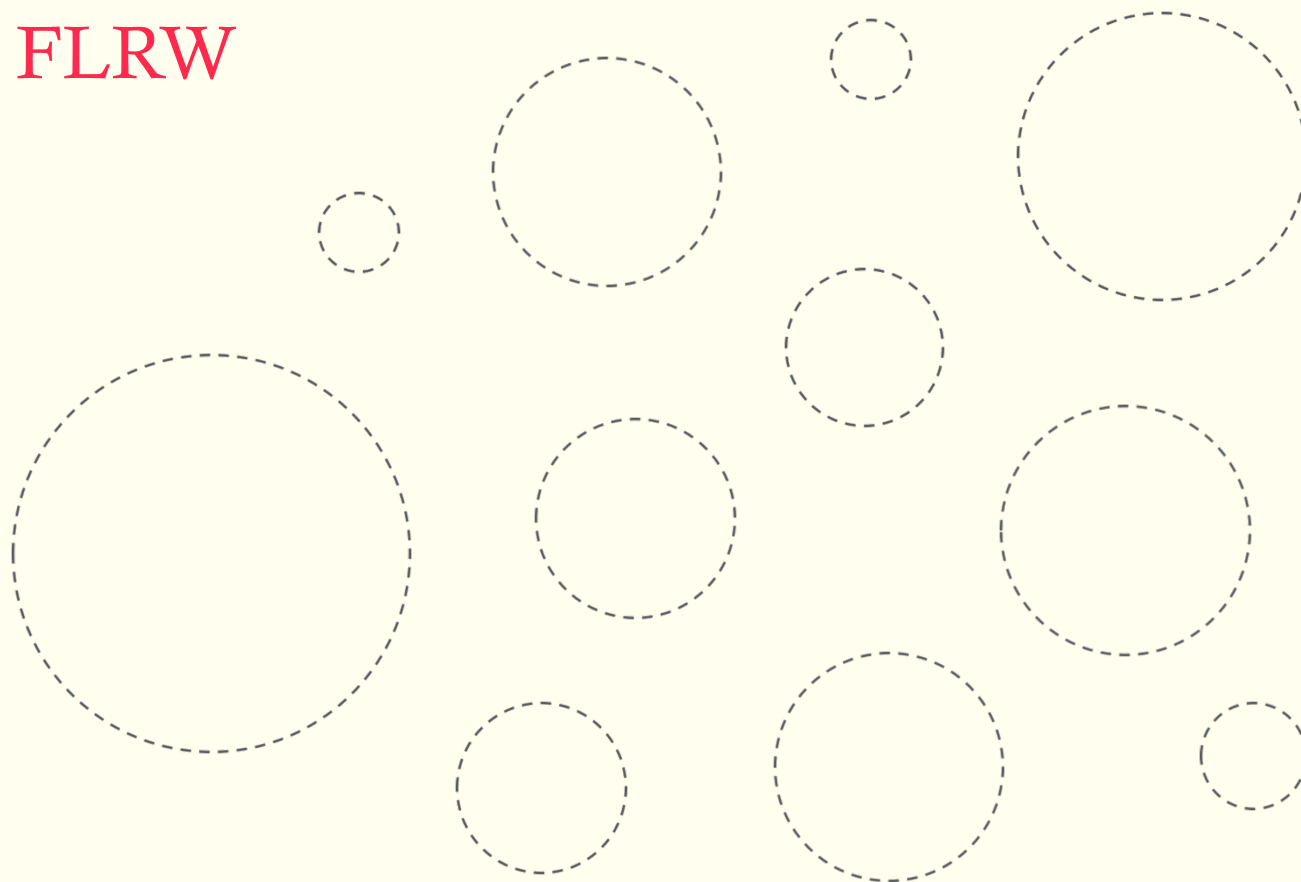
FLRW



Swiss-cheese



FLRW

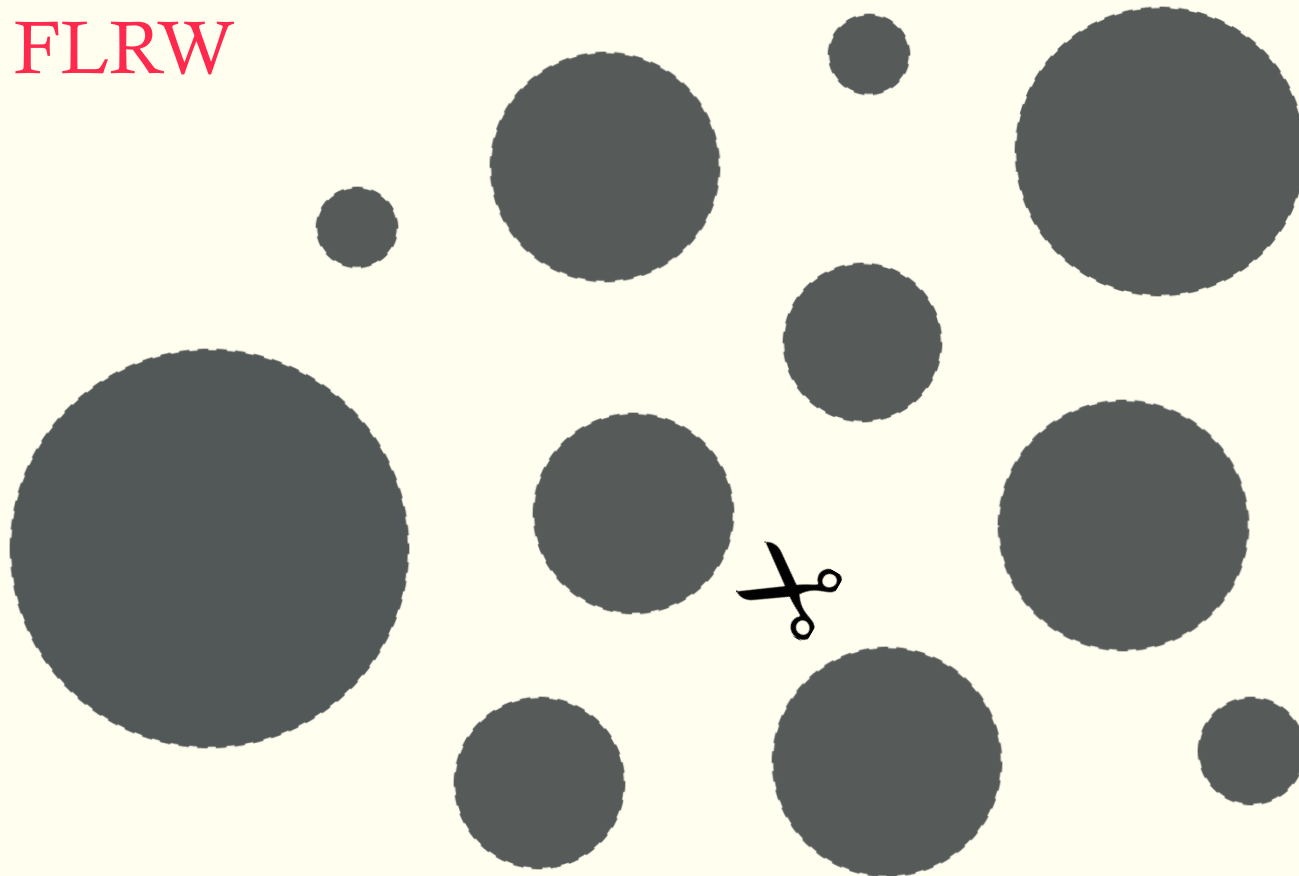




Swiss-cheese



FLRW





Swiss-cheese

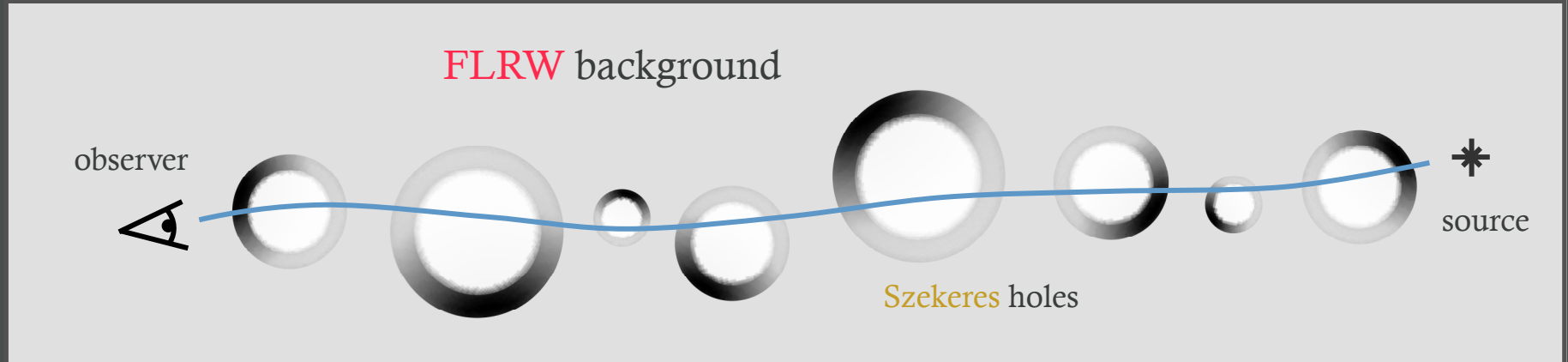


FLRW





Swiss-cheese



- exact inhomogeneous universe with standard model background dynamics
- Einstein's equations continuous throughout
- lines of sight primarily underdense
- overdense matter shells mimic filamentary cosmic web
- incorporates nonlinear GR effects



Swiss-cheese



results summary

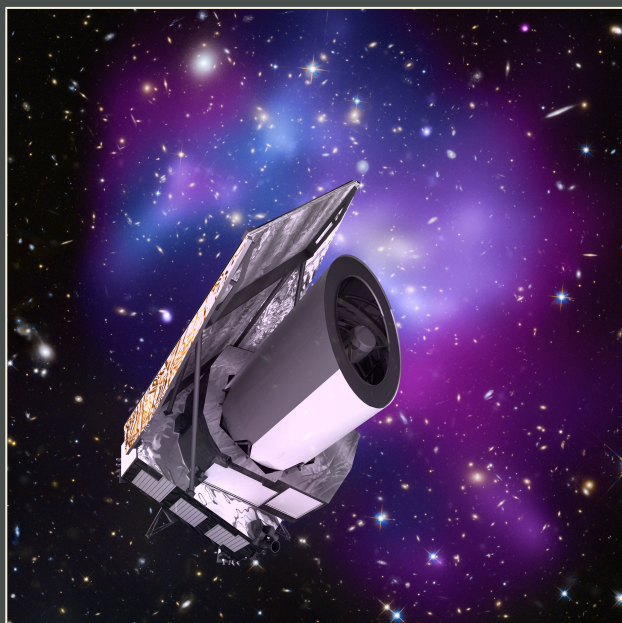


- effect on **inferred distance** potentially substantial for a single line of sight
- some sources seem **closer**, others **farther** away
- averaging over **many sources** nullifies the effect
- small additional **scatter**



now

work at CEA



Weak gravitational lensing mass maps for Euclid mission

measure slightly distorted galaxy shapes



determine (dark) matter distribution



build catalog of galaxy clusters



constrain cosmological models

Thank you.