Shear-induced turbulent transport in stellar radiative zones

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8th November 2016

Resume

2010 – 2013: PhD at IRAP (Toulouse)

- supervisor: François Lignières
- simulations of shear instability
    - thermal diffusion
    - chemical stratification

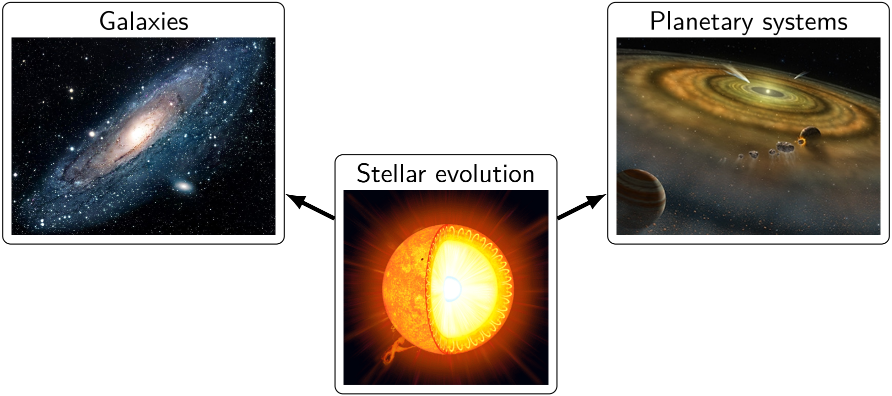
2013 – 2015: fellowship at MPA (Garching, Germany)

- supervisor: Ewald Müller
- effect of viscosity (with Jérôme Guilet & Maxime Viallet)
- other processes (convection and thermohaline convection)

Now: ERC fellowship at CEA

- supervisor: Stéphane Mathis
- effect of rotation + horizontal shear
- gravito-inertial waves + tides

Stellar evolution: a central role in astrophysics

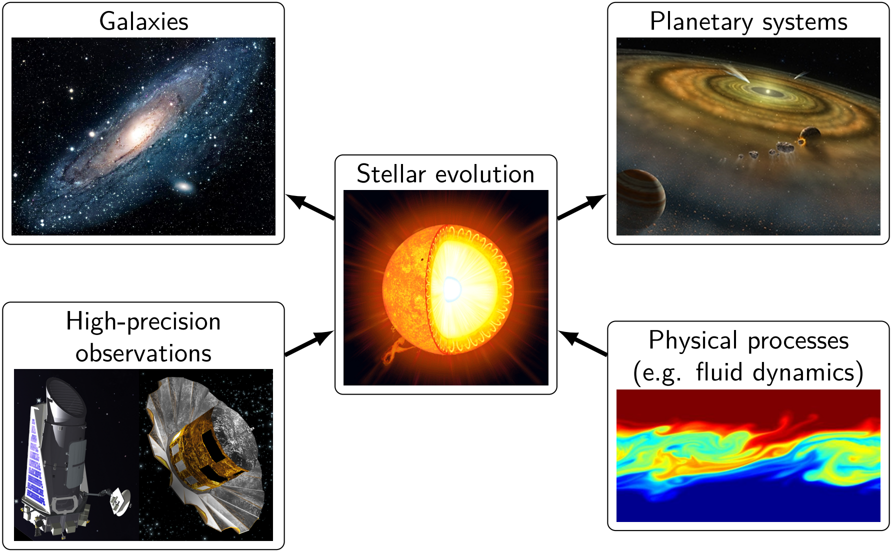


Galaxies

Stellar evolution

Planetary systems

Stellar evolution: a central role in astrophysics



Galaxies

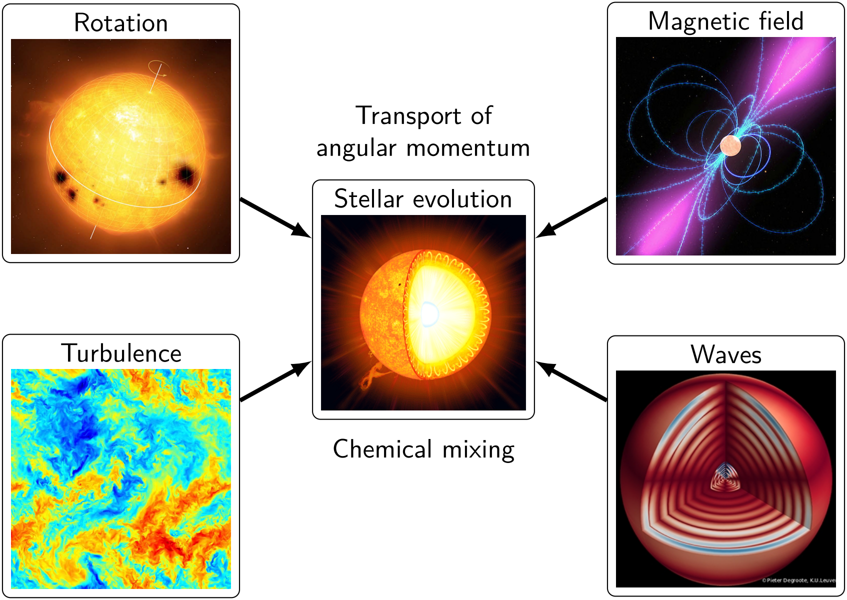
Planetary systems

Stellar evolution

High-precision observations

Physical processes (e.g. fluid dynamics)

Key: dynamical processes



Rotation

Magnetic field

Transport of
angular momentum

Stellar evolution

Chemical mixing

Turbulence

Waves

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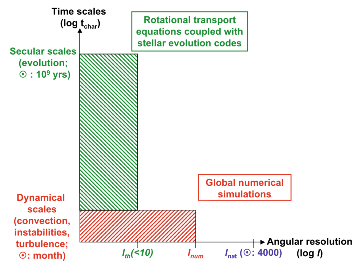
Stellar evolution: 1D *vs* multi-D

Magneto-hydrodynamic processes in stellar evolution

- many fundamentally 2D or 3D processes
- usually very short length and time scales



Decressin et al. (2009)

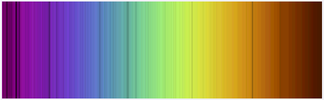
Necessary 1D/2D stellar evolution

- numerically too demanding to do proper 3D stellar evolution
- need to prescribe multi-D processes in secular models
  → uncertainties from models

Need to improve these models

Observational constraints

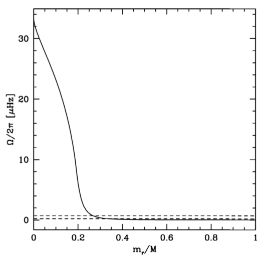
Surface chemical abundances



- profiles of absorption lines → abundances
- models unable to reproduce some observed stellar populations (Brott et al., 2011; Potter et al., 2012)

Asteroseismology

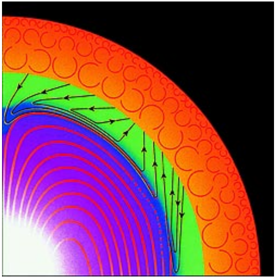
- global seismic parameters → fundamental parameters
- internal structure → distribution of chemicals
- rotation rate: not reproduced with current transport models (Eggenberger et al., 2012; Ceillier et al., 2012; Marques et al., 2013)



Ceillier et al. (2012)

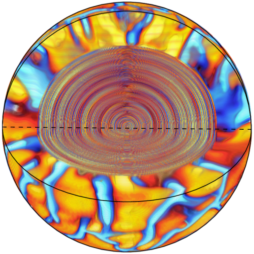
Candidates for efficient transport

Magnetic field



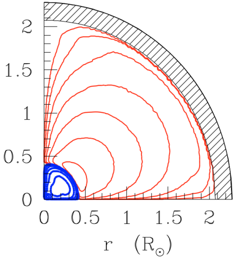
Gough & McIntyre (1998)

Internal gravity waves



Alvan et al. (2014)

Meridional circulation



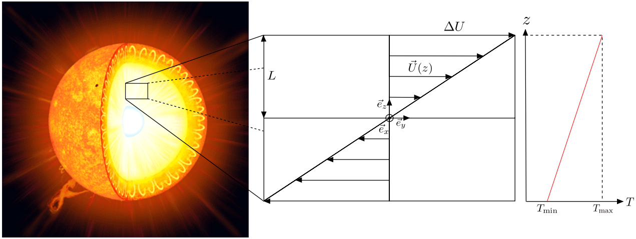
Decressin et al. (2009)

I will focus on the transport induced by the shear instability

Vertical shear mixing

Shear instability in stars

- radiative zone: no convective motions
- velocity shear (S) and stable entropy gradient (N)



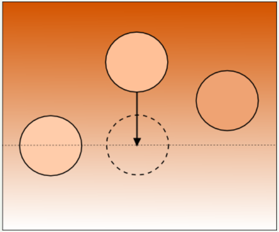
Classical description (geophysics)

- linear instability criterion in the inviscid case: $Ri = (N/S)^2 < Ri_c = 1/4$
- generally $Ri \gg 1$ in stellar radiative zones $\Rightarrow$ no transport

Stellar regime

Effect of thermal diffusion

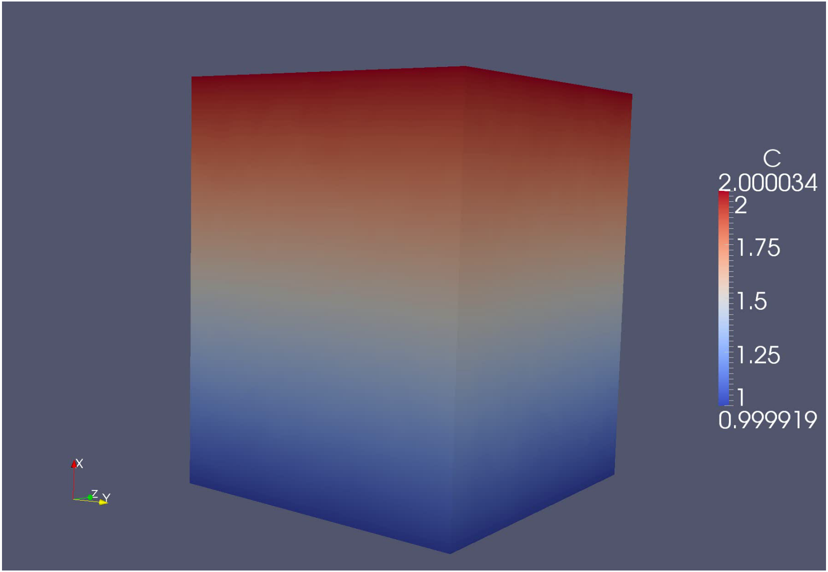
- reduces the amplitude of the buoyancy force
- more efficient at small scales
  ⇒ instability possible only at small scales
- called secular shear instability ($\neq$ dynamical)
- several phenomenological models for turbulent transport



Very high thermal diffusivity

- $\tau_{\mathrm{diff}} \ll \tau_{\mathrm{dyn}} \quad \Rightarrow$ unreachable without approximation
- Small-Péclet-number (SPN) approximation (Lignières, 1999)
  - asymptotic development when advection is much slower than diffusion
  - no more prohibitive numerical cost for high thermal diffusivity

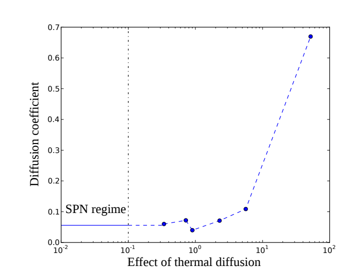
Example of simulation



Numerical simulations (Prat & Lignières, 2013)

Test of the SPN approximation

- different simulations:
  - full Boussinesq simulations at various thermal diffusivities
  - one simulation in the SPN approximation
- convergence at high thermal diffusivity



Meaning for shear mixing

- SPN approximation consistent with Zahn (1992):

$$D_{\mathrm{v}} \propto \kappa \frac{(r \sin\theta \; \mathrm{d}\Omega/\mathrm{d}r)^2}{N^2}$$

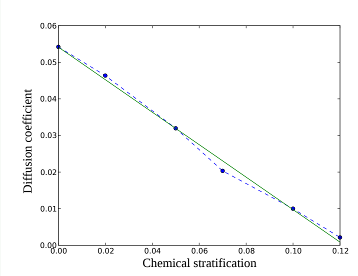
- significantly lower proportionality constant: less transport

Effect of chemical stratification (Prat & Lignières, 2014)

Importance for transport

- strong stable chemical gradients (e.g. boundary of convective core)
- transport in these zones strongly affects evolution

Simulation results (in the SPN approximation)



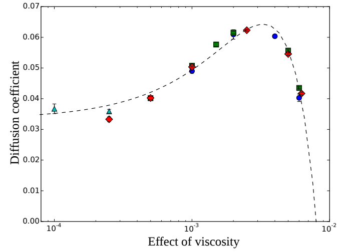
- chemical stratification can completely inhibit mixing
- existence of dead zones without any transport

Effect of viscosity (Prat et al., 2016)

In Zahn's model

- no dependence on viscosity when shear-unstable scales are large enough
- no transport if they are too small

Simulations



- asymptotic regime at low viscosity
- cut-off when viscosity is efficient
  $\Rightarrow$ dead zones

Conclusion on vertical shear mixing

Current description

- complex interplay between stratification (thermal & chemical), thermal diffusion and viscosity
- new empirical prescription that can be implemented in stellar evolution codes

Consequences on evolution

- dead zones because of stratification/viscosity: even less transport than previous models

Does not help to account for observations

- other processes needed
  - magnetic field
  - internal gravity waves
  - anisotropic turbulence?

New theoretical study

Motivation

- turbulence is 3D
- vertical transport generates horizontal transport (and vice-versa)
- horizontal transport enhances meridional circulation
  $\Rightarrow$ more transport of angular momentum
- some models exist but with no simultaneous dependence on stratification and rotation

Objectives

- propose a new prescription including the effect of stable stratification and rotation
- investigate the effect on stellar models

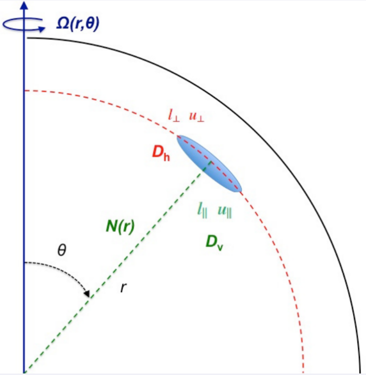
New physical picture (Mathis et al., 2016)

Anisotropy

- vertical scales limited by stable stratification
- horizontal scales limited by Coriolis acceleration
- Kitchatinov & Brandenburg (2012):

$$\frac{u_\perp}{u_\parallel} \propto \frac{N^2\tau}{\Omega}$$

- in stellar radiative zones, usually $N \gg \Omega$
  $\Rightarrow$ strong anisotropy

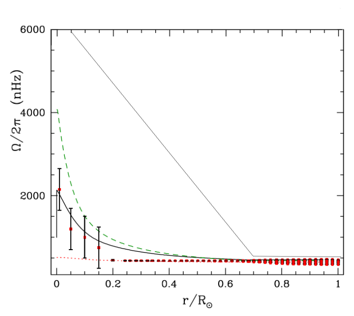


Transport

- very strong horizontal mixing ($D_h \gg D_v$)
- efficient transport of angular momentum by meridional circulation

Impact on stellar evolution

The Sun



- reproduces quite well the rotation profile (at least in order of magnitude)
- was not the case of other phenomenological models of D_h

Sub-giant stars

- helps to flatten the rotation profile
- but not sufficient alone

Conclusion and prospects

Conclusion

- vertical shear mixing is not efficient enough to account for observations
- transport generated by horizontal mixing helps a lot on the main sequence
- but not always enough

Prospects

- test the new prescription with numerical simulations
    - constraints on D_h
    - effect of rotation on D_v
- extend the theory by including missing physical ingredients
    - horizontal shear
    - magnetic field

Thank you for your attention.

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