DECONVOLUTION OF GALAXY IMAGES WITH A EUCLID-LIKE PSF

Samuel Farrens

Office: **279**
Ext: **8377**

29th March 2016

OVERVIEW

A Brief History of Me

Project Involvement

PSF & Image Reconstruction

A Brief History of Me

The Daily Globe

Free Beer

See terms and conditions.

0,0€

A Brief History of Me

September 5th 1983

Up in the Éire

Samuel Farrens was born on the emerald isle in 1983 and lived there for eight eventful years before moving with his family to the land of opportunity.

Though his early education included classes in Irish rumour has it he cannot remember a thing.

Baby Born on Island

Down South

Sam would spend the next six years of his life in Charleston, South Carolina. One of the original thirteen colonies of the United States of America.

During this period he discovered Stephen Hawking and he got his first taste of astrophysics.

THE GLOBE

The Daily Globe

Free Beer

See terms and conditions.

0,0€

A Brief History of Me

July 12th 1998

Banana Republic

At age fourteen Sam and his family moved to a small town called Atenas (the Spanish name for Athens) in Costa Rica.

Here Sam attended high school, learned to speak a new language and focused on his objective of becoming and astrophysicist.

Man Attends University

University College London

Dr. Filipe Abdalla and man who works on large scale structure, weak lensing, dark energy, cosmological neutrinos and photometric redshifts.

London Calling

After finishing his secondary education, Sam moved to London and spent a year attending night courses on maths and physics at Birkbeck College. He was then admitted to University College London where he would remain for the next seven years and obtain his PhD. in astrophysics.

THE GLOBE

The Daily Globe

Free Beer

See terms and conditions.

0,0€

A Brief History of Me

October 1st 2015

Gaudi Style

Sam's first postdoctoral position was at the Institut de Ciències de l'Espai (ICE) in Barcelona.

Following Joyce

Sam's second postdoctoral position was at the Osservatorio Astronomico di Trieste (OATs).

You're Hired!

Pardon my French

Sam's is currently working at the Commissariat à l'Énergie Atomique (CEA) in Saclay and will remain there for the next 3 years. Sources have reported that so far he is enjoying the work he is doing and the people he works with. The Daily Globe will be keeping a close eye on his progress for future editions.

OVERVIEW

A Brief History of Me

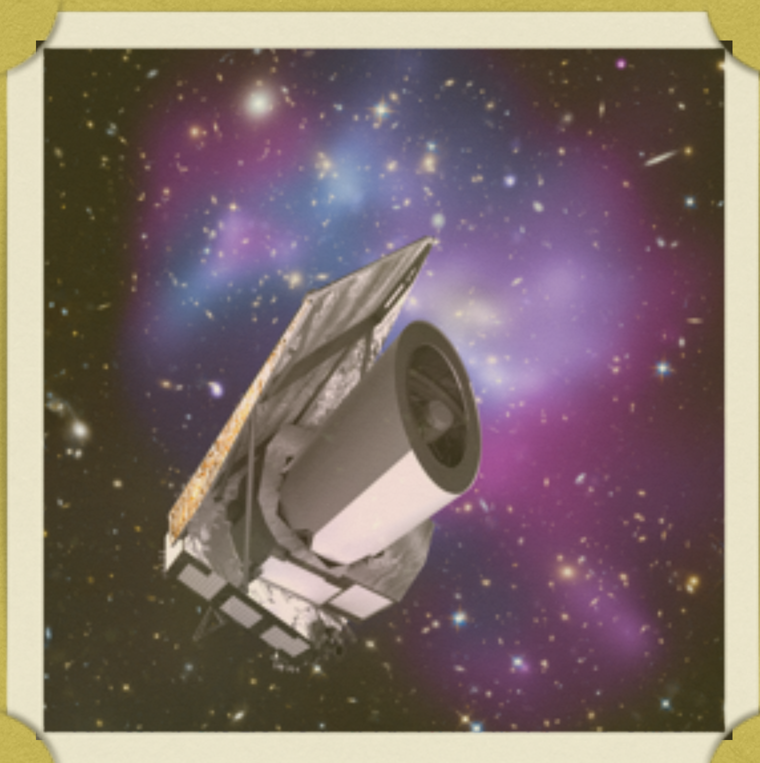
Project Involvement

PSF & Image Reconstruction

Project Involvement

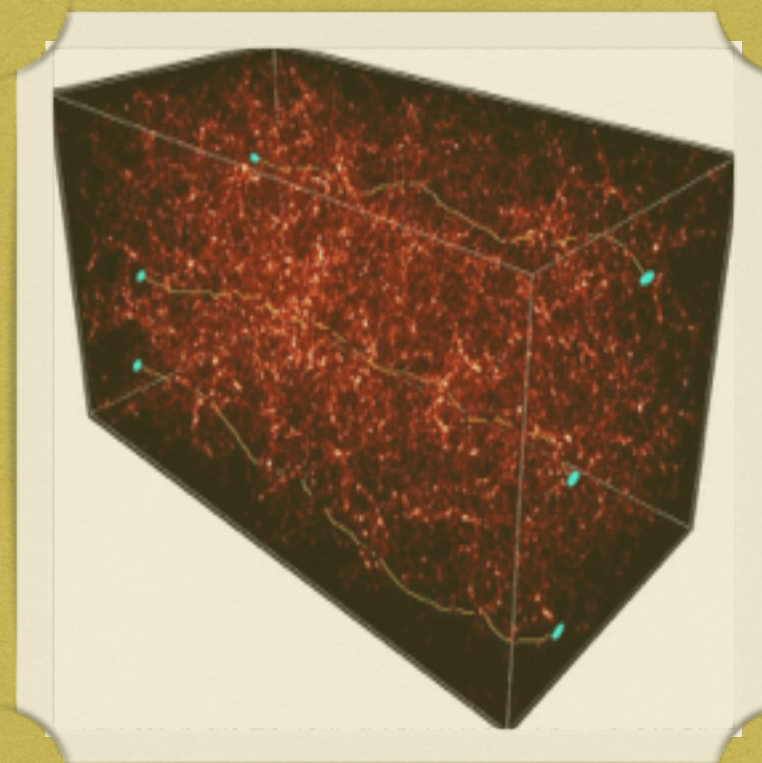
EUCLID

SPACE MISSION



IR imager and slitless spectrograph.

WEAK LENSING



Accurate shape measurements for probing cosmology.

GALAXY CLUSTERS

Large sample of “high” redshift galaxies

euclid consortium

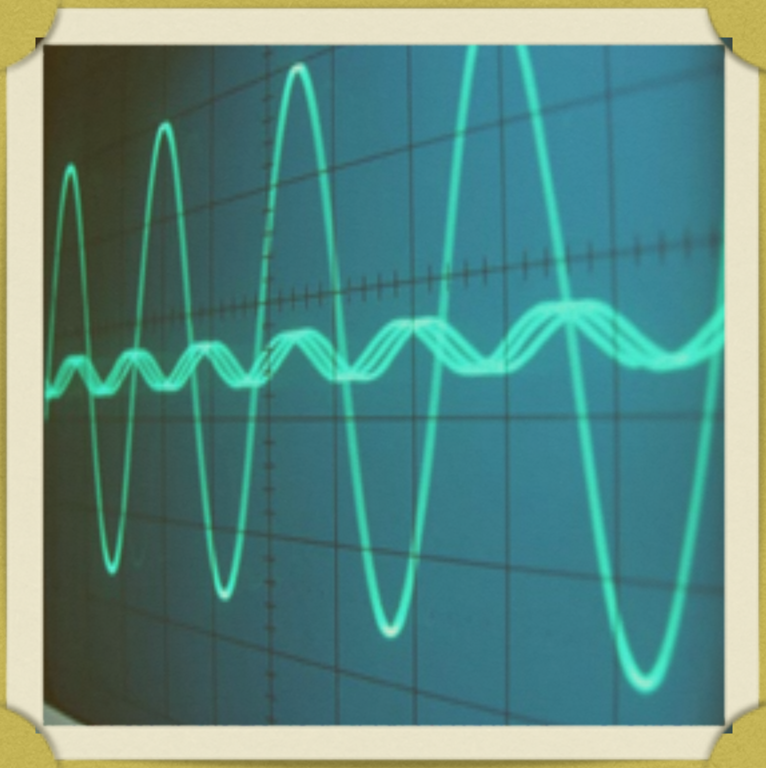
http://www.euclid-ec.org/

DEDALE

DATA DRIVEN

Data Learning on Manifolds and Future Analaysis

SIGNAL PROCESSING



PSF modelling and shape measurement.

COSMOLOGY

Spectroscopic and photometric redshift estimation for Euclid

http://dedale.cosmostat.org/

OVERVIEW

A Brief History of Me

Project Involvement

PSF & Image Reconstruction

PSF & Image Reconstruction

COLLABORATORS

Jean-Luc Starck
Office: 275

Fred Ngolè
Office: 279

INVERSE PROBLEM

Data → Model Parameters

$$Y = MX$$

$$X = M^{-1}Y$$

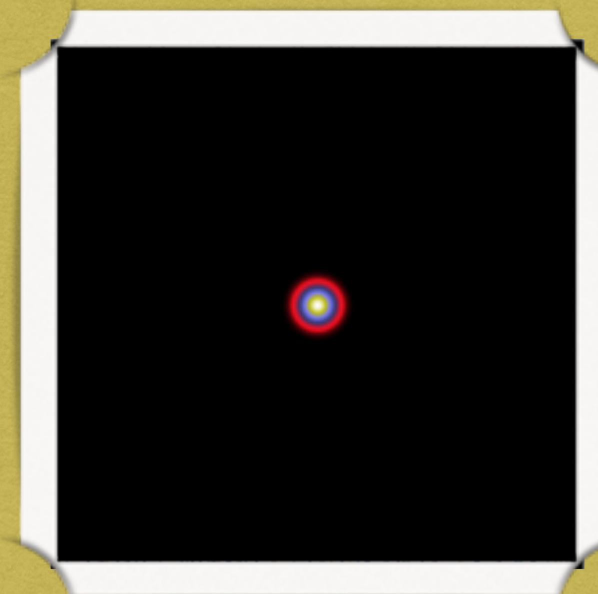
$$X = (M^T M)^{-1} M^T Y$$

Can only be solved analytically if M or $M^T M$ is nonsingular.

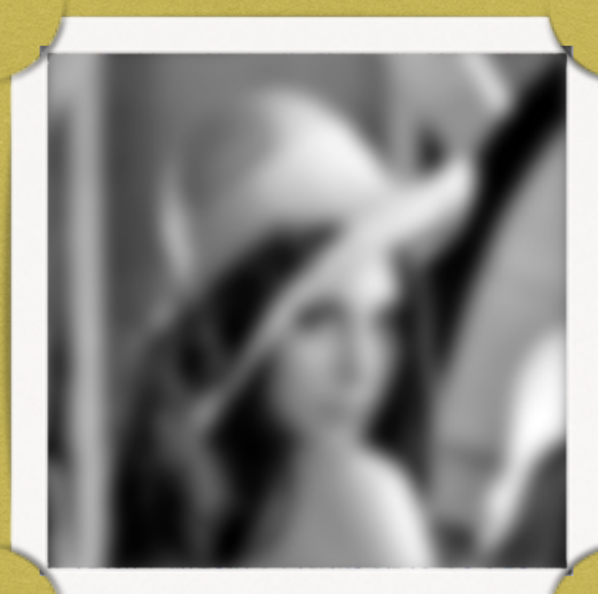
INVERSE PROBLEM

X

M



MX



$M^{-1}MX$

Inverse Problem

Data contains noise component.

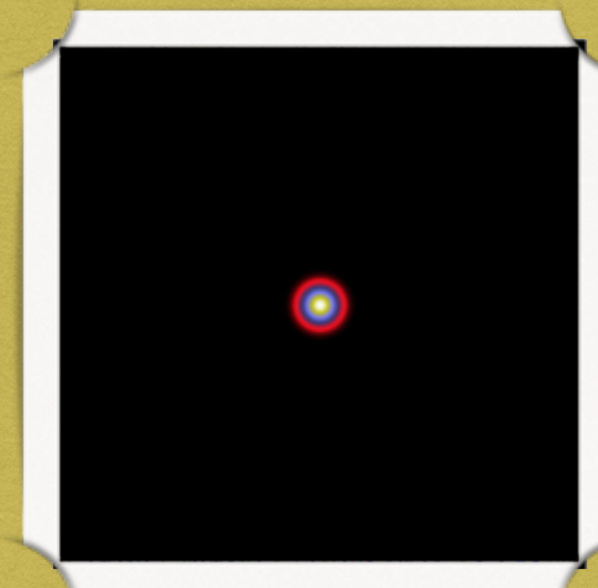
$$Y = MX + n$$

$$X \neq M^{-1}Y$$

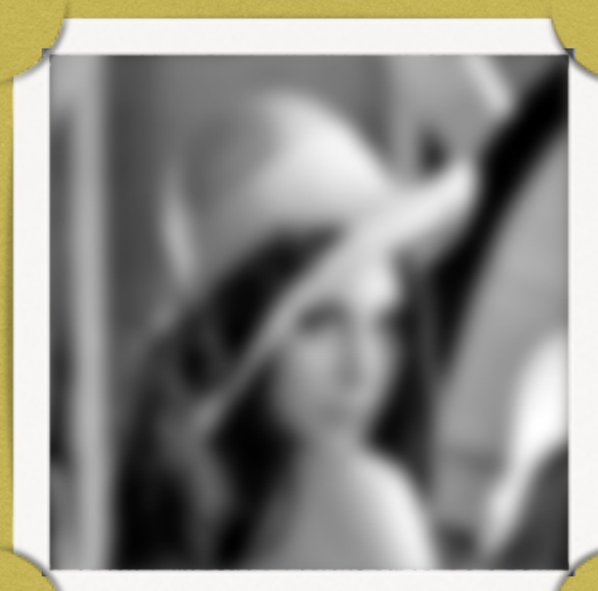
INVERSE PROBLEM

X

M



$MX + n$



$M^{-1}(MX + n)$

GRADIENT DESCENT

Find the gradient of the minimising function.

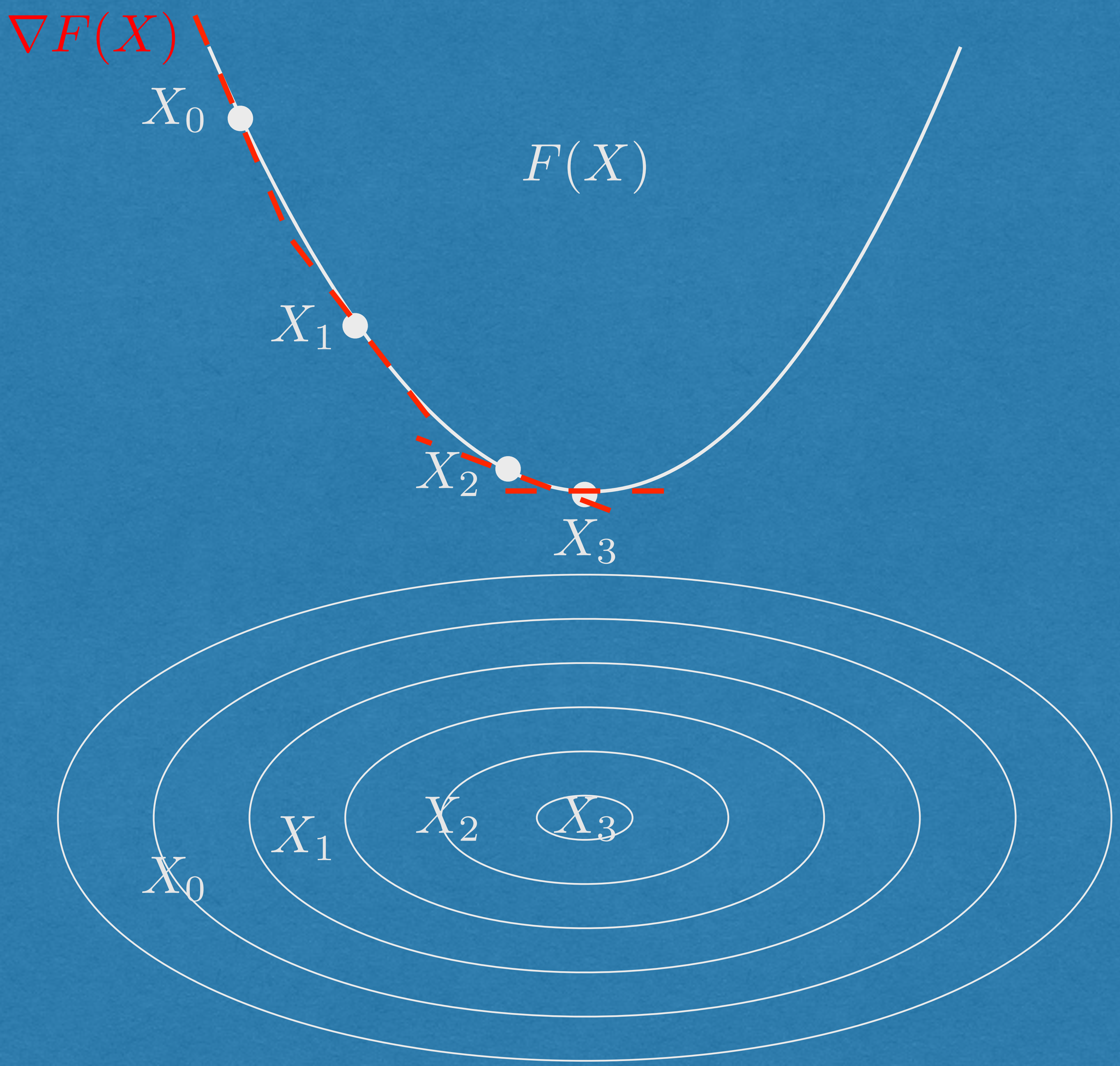
$$F(X) = \frac{1}{2}\|Y - MX\|_2^2$$

$$\nabla F(X) = M^T(Y - MX)$$

Iteratively update X until it converges.

$$X_{n+1} = X_n - \gamma_n \nabla F(X_n)$$

GRADIENT DESCENT



REGULARISATION

Use prior information to constrain *X*

$$F(X) = \frac{1}{2}\|Y - MX\|_2^2 + \lambda R(X)$$

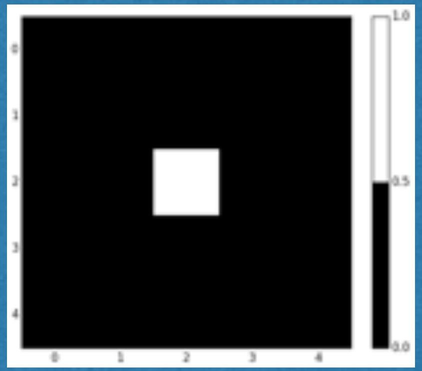
Require that the reconstruction be positive

$$X \geq 0$$

Sparsity

A sparse signal is one comprised mostly of zeros.

$$X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



A signal maybe sparse in a different domain.

$$X = \begin{bmatrix} 0.04 & -0.032 & 0.012 & 0.012 & -0.032 \\ -0.032 & 0.012 & 0.012 & -0.032 & 0.04 \\ 0.012 & 0.012 & -0.032 & 0.04 & -0.032 \\ 0.012 & -0.032 & 0.04 & -0.032 & 0.012 \\ -0.032 & 0.04 & -0.032 & 0.012 & 0.012 \end{bmatrix} \rightarrow \mathcal{F}(X) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Sparsity

The sparsity of a signal can be measured with the ℓ_0 pseudo-norm.

$$\|X\|_0 = \text{Card}(\text{Supp}(X)) = 1 \qquad X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Need to relax ℓ_0 to ℓ_1 norm to maintain convexity.

$$\|X\|_1 = \max_{j=1,\ldots,n} \sum_{i=1}^{m} |X_{ij}| = 1 \qquad X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Sparsity

Preserve sparse solutions with thresholding.

$$\mathrm{HT}_\lambda(x) = \begin{cases} x & \text{if } |x| \geq \lambda \\ 0 & \text{if otherwise} \end{cases}$$

e.g. $\lambda = 0.5$

$$X = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.2 \\ 0.1 & 0.7 & 0.1 & 0.3 & 0.1 \\ 0.1 & 0.1 & 1.0 & 0.1 & 0.3 \\ 0.3 & 0.8 & 0.1 & 0.6 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.4 \end{bmatrix} \quad \mathrm{HT}_{0.5}(X) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0.8 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

LOW-RANK

The rank is a measure of the number of linearly independent columns (or rows) in a matrix.

$$X = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix} \qquad \text{rank}(X) = 2$$

$$X = \begin{bmatrix} -9 & 8 & 0 \\ 5 & 3 & 7 \\ 2 & -5 & 3 \end{bmatrix} \qquad \text{rank}(X) = 3$$

LOW-RANK

The rank of a signal can be approximated with the nuclear norm.

$$\|X\|_* = \sum_i \sigma_i$$

Singular value decomposition (SVD).

$$X = U\Sigma V^*$$

$$\Sigma = \begin{bmatrix} \sigma_0 & 0 & \dots & 0 \\ 0 & \sigma_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_i \end{bmatrix}$$

LOW-RANK

Preserve low-rank solutions with thresholding.

$$\mathrm{HT}_\lambda(x) = \begin{cases} x & \text{if } |x| \geq \lambda \\ 0 & \text{if otherwise} \end{cases}$$

e.g. $\lambda = 12$

$$\Sigma = \begin{bmatrix} 15 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{HT}_{12}(\Sigma) = \begin{bmatrix} 15 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

MINIMISATION

Positive and sparse reconstruction

$$\underset{X}{\operatorname{argmin}} \quad \frac{1}{2}\|Y - MX\|_2^2 + \|W \odot \phi X\|_1 \quad \text{s.t.} \quad X \geq 0$$

Positive and low rank reconstruction

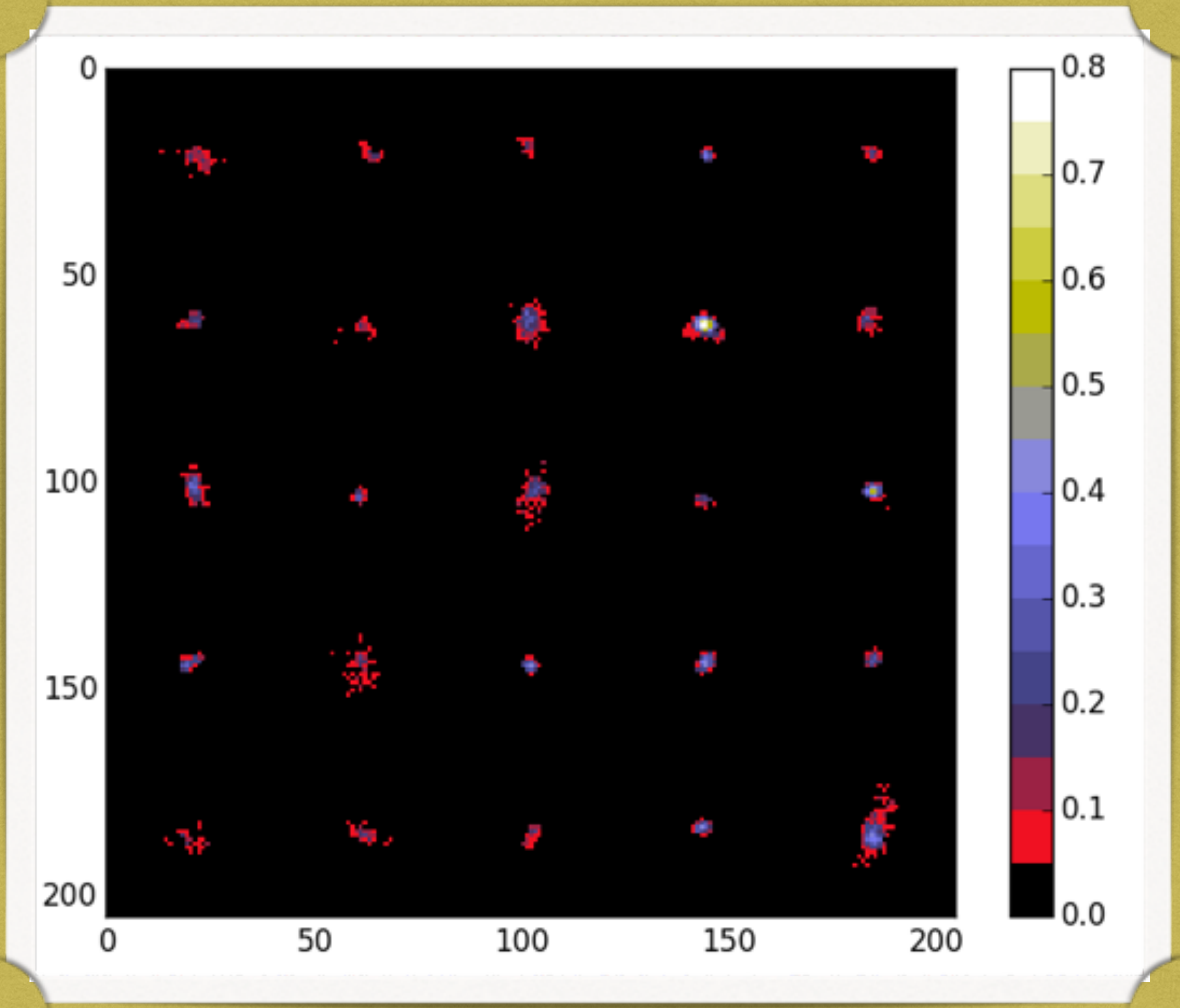
$$\underset{X}{\operatorname{argmin}} \quad \frac{1}{2}\|Y - MX\|_2^2 + \lambda\|X'\|_* \quad \text{s.t.} \quad X \geq 0$$

Positive, sparse and low rank reconstruction

$$\underset{X}{\operatorname{argmin}} \quad \frac{1}{2}\|Y - MX\|_2^2 + \|W \odot \phi X\|_1 + \lambda\|X'\|_* \quad \text{s.t.} \quad X \geq 0$$

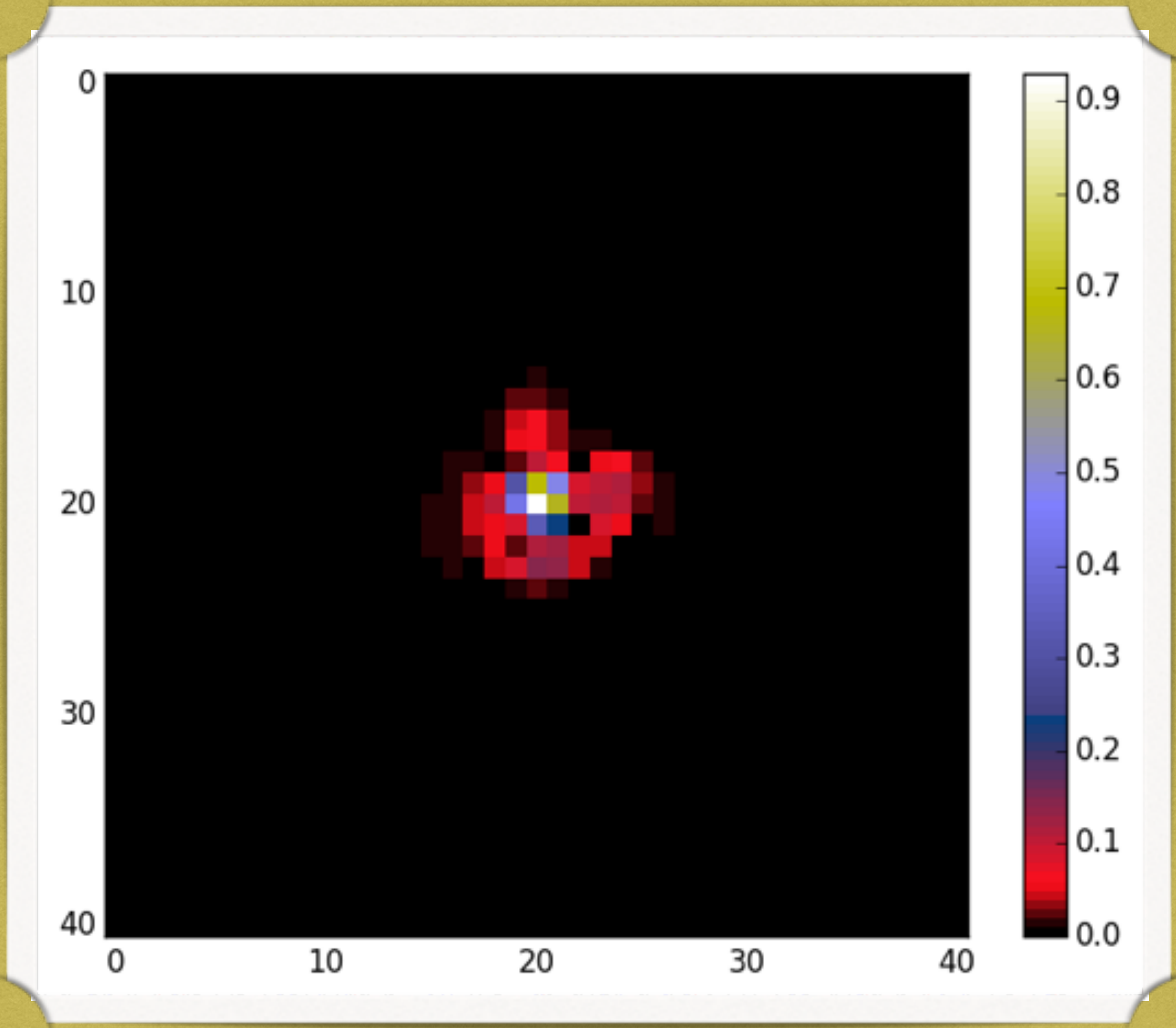
DATA

10 random samples of 25 Hubble Deep Field images



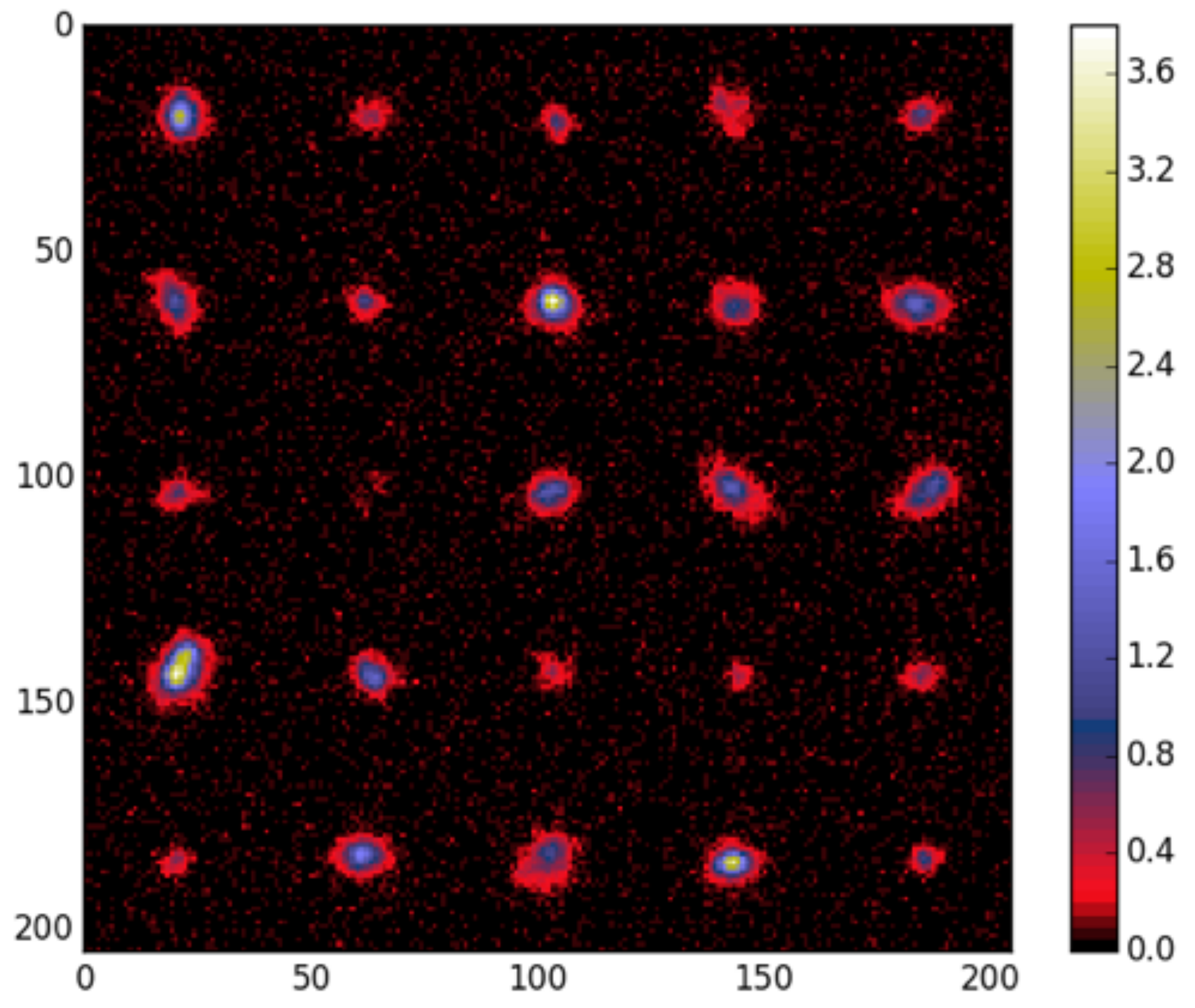
DATA

Euclid-like PSFs generated by K. Okumura, J. Amiaux, and P. Hudelot



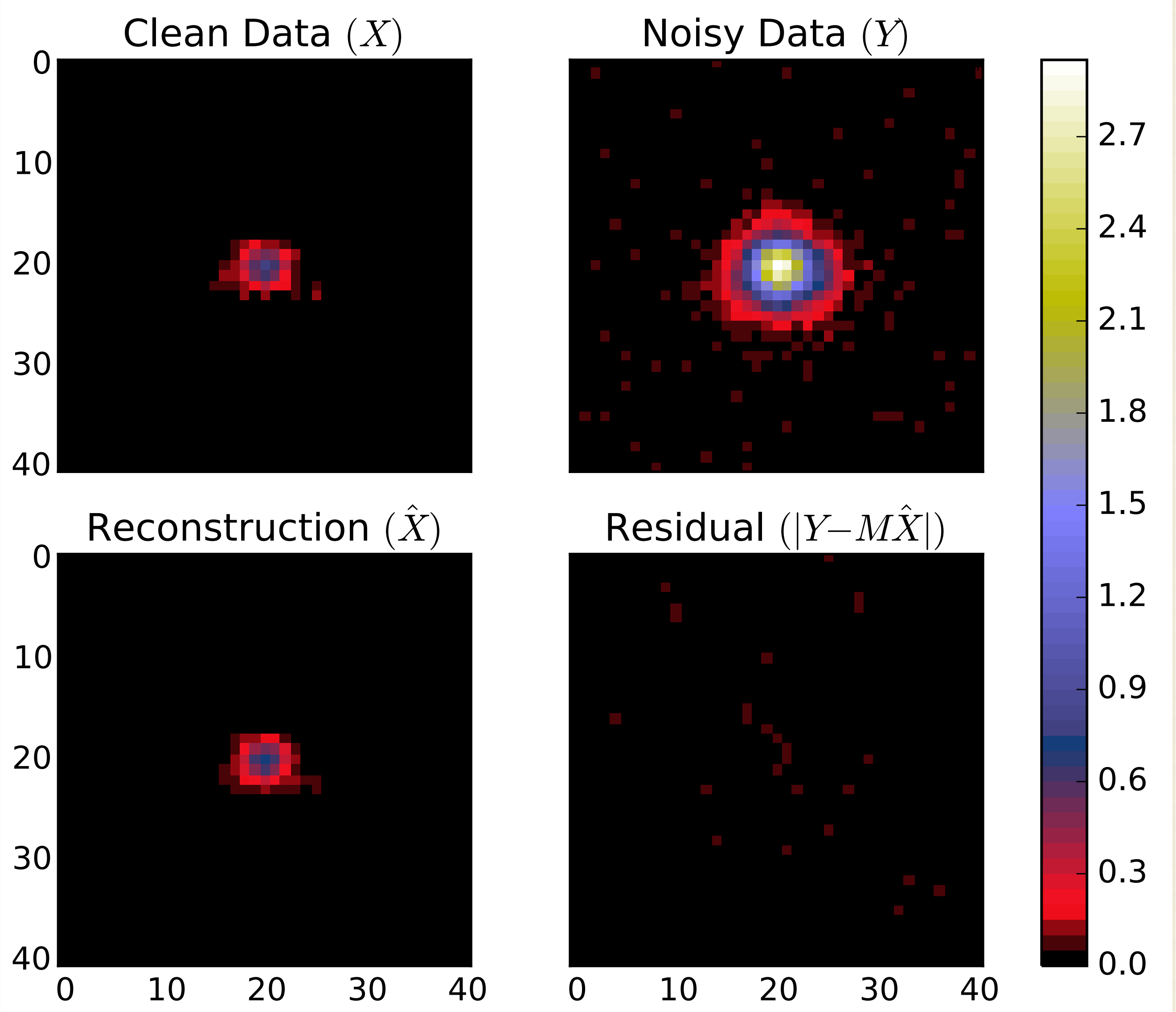
DATA

Galaxy images are convolved with Euclid-like PSFs and random noise is added.



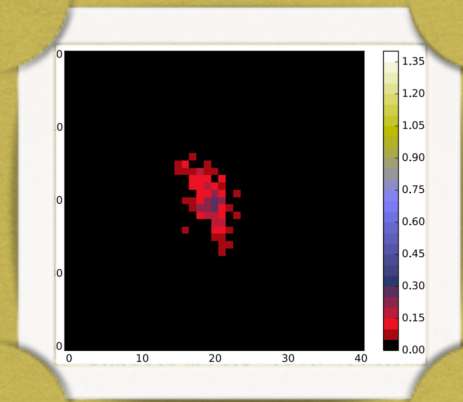
RESULTS

Example of combined reconstruction



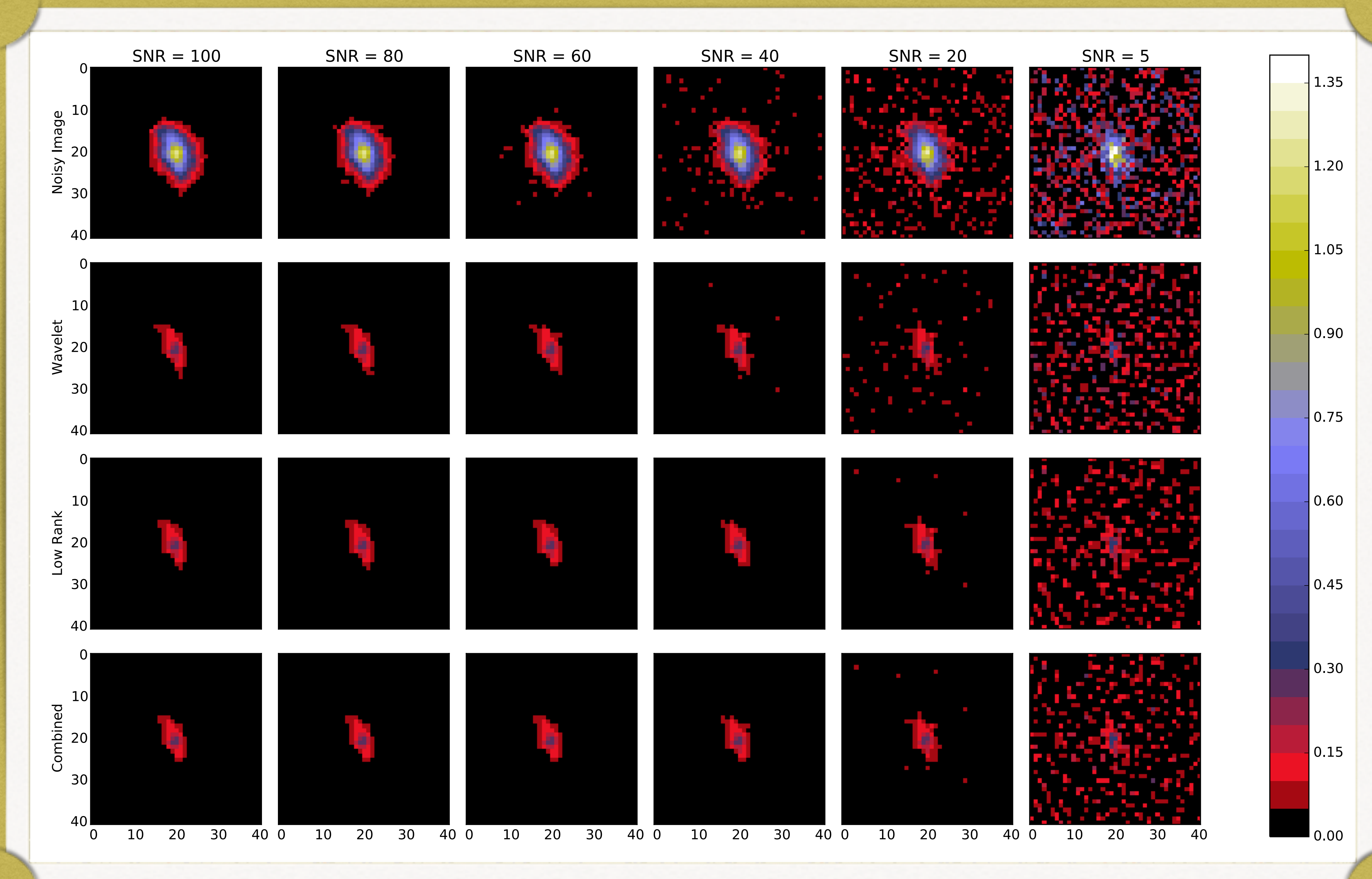
Clean Image

X



Y

$\hat{X}$



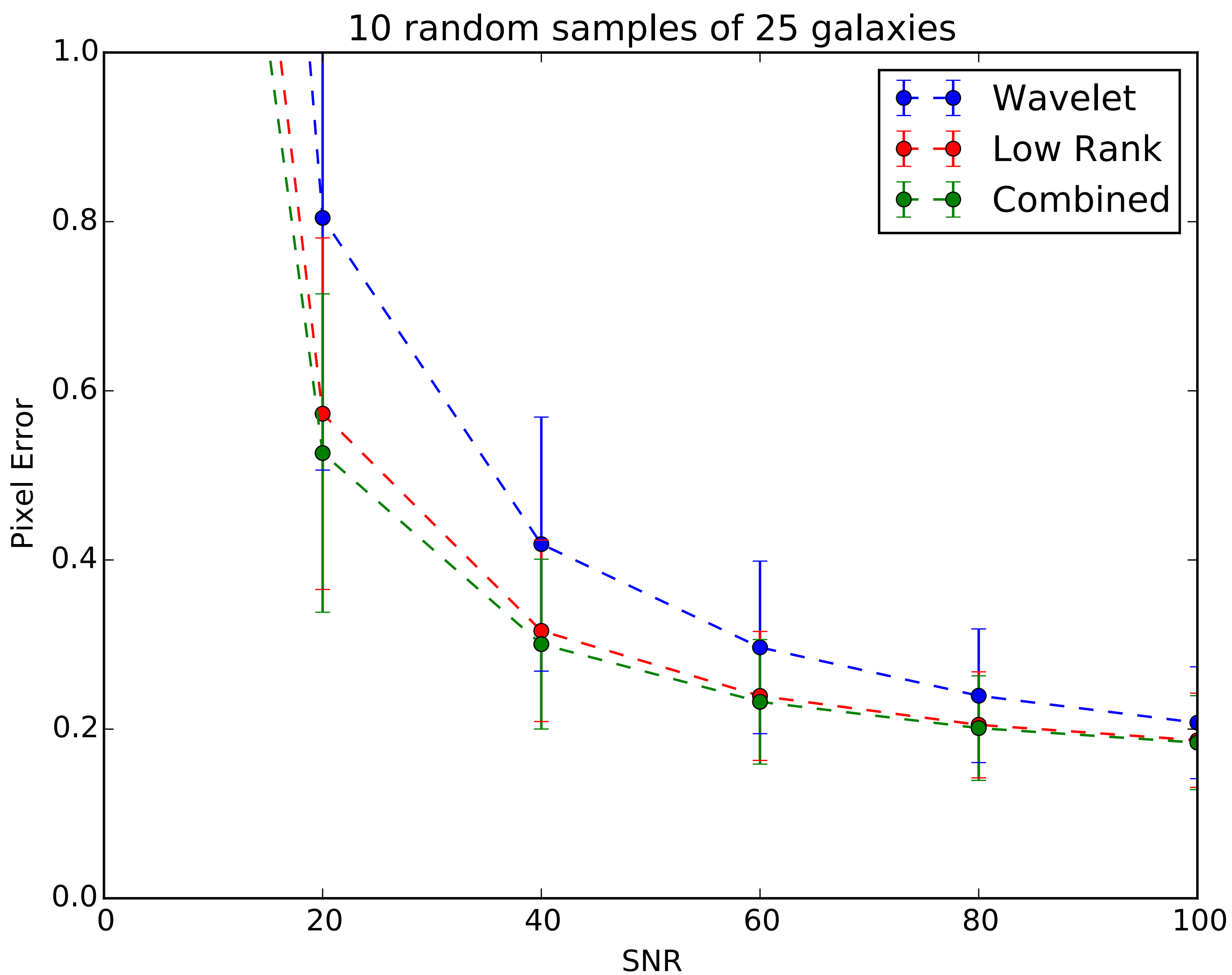
RESULTS

Pixel error of whole sample.

$$P_{\mathrm{err}} = \frac{\|X - \hat{X}\|_F}{\|X\|_F}$$

RESULTS

Comparison of reconstruction methods.



THE NEAR FUTURE

- Investigate the potential improvements in the low-rank results when a larger data set is used.
- Compare shape measurement errors against Euclid requirements.
- Test the impact of a pixel-variant PSF on the results.
- Attempt reconstruction when the PSF is only partially known.

Merci