Numerical simulations of star formation in the interstellar medium

Patrick Hennebelle

(with many others…)

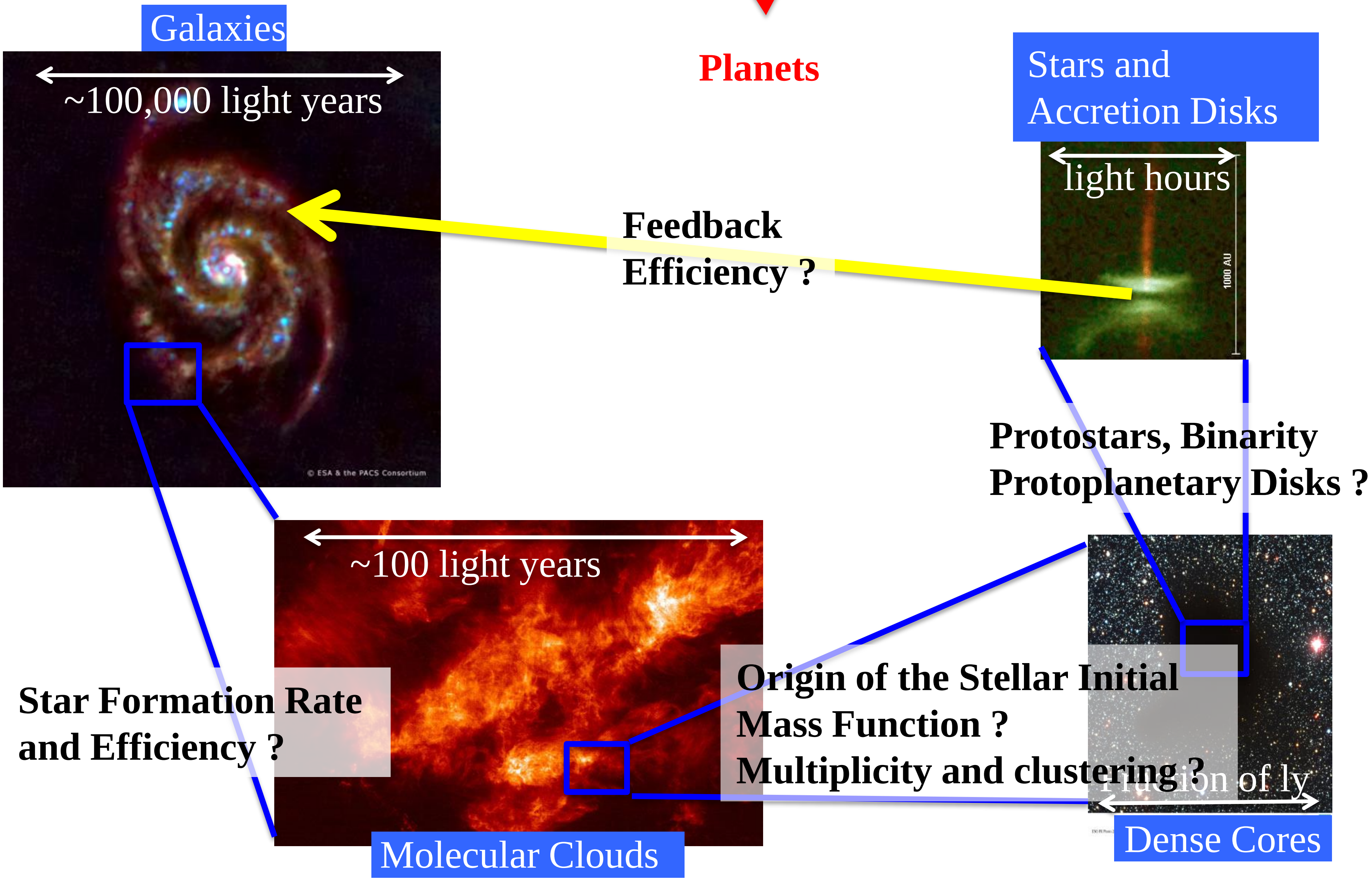
Large Scale Structures

↓

Interstellar Cycle and Star Formation

↓

Planets



Galaxies

Stars and Accretion Disks

Feedback Efficiency ?

Protostars, Binarity Protoplanetary Disks ?

Star Formation Rate and Efficiency ?

Origin of the Stellar Initial Mass Function ?
Multiplicity and clustering ?

Molecular Clouds

Dense Cores

Interstellar Medium: 2 main difficulties

Huge dynamical range:

~10-15 orders of magnitude in space and time
~20-30 orders of magnitude in density
~6 orders of magnitude in temperature

Profusion of physical processes:

Radiation ≈ Thermal ≈ Kinetic ≈ Magnetic ≈ Cosmic Rays
+Gravity

≈ 1 eV cm^{-3}

=> Energy equipartition
=> Strong coupling between several physical processes
=> Difficult to simplify and isolate the problems

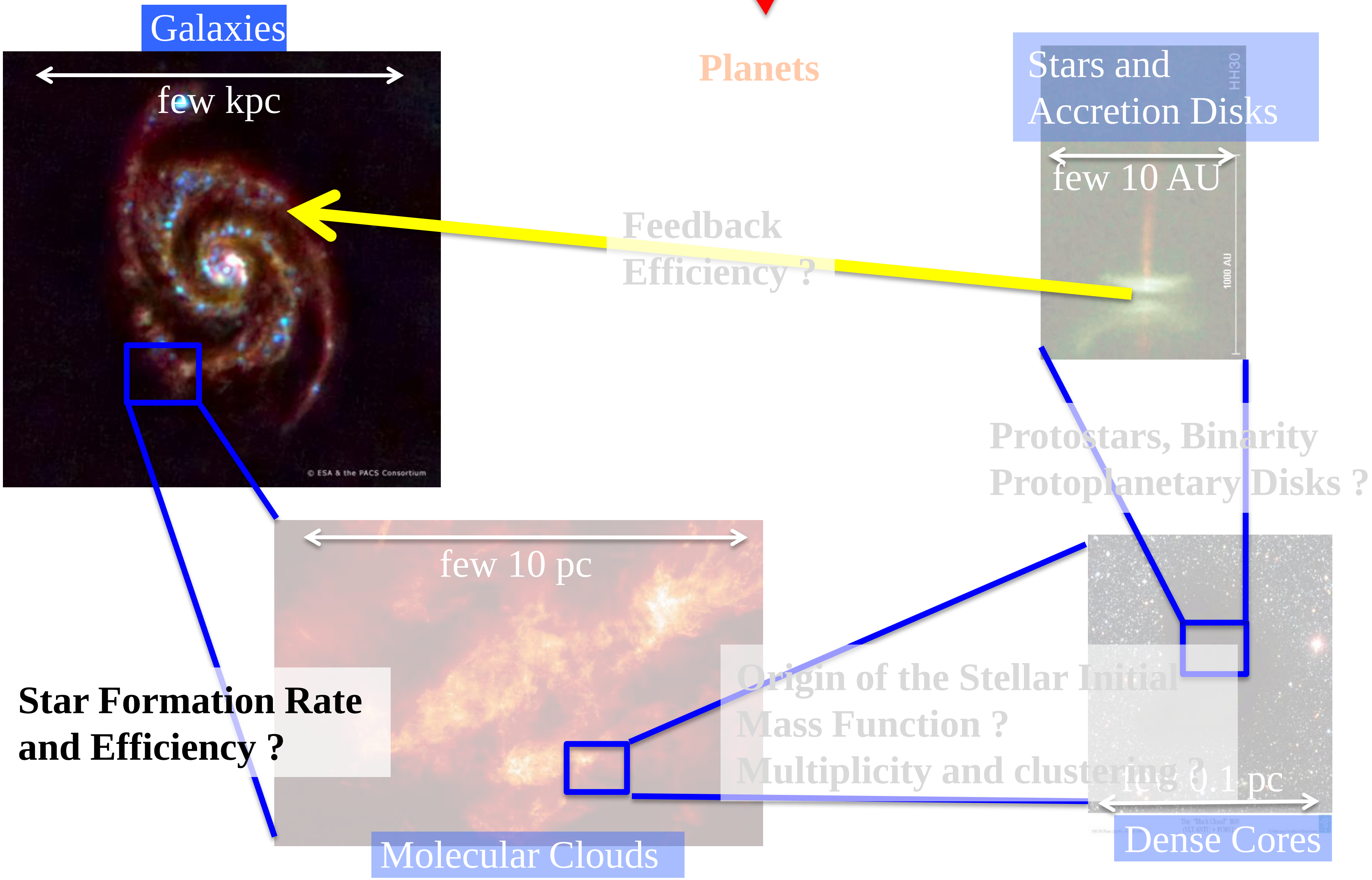
Modern Astrop

$$\omega^4 - (\Omega^2 + 2k^2 v_A^2)\omega^2 + k^2 v_A^2 (k^2 v_A^2 - 3\Omega^2) = 0.$$

$$\frac{dN}{dM} \propto \frac{1}{R^6} \left(\frac{M}{R^3}\right)^{\frac{3}{2} - \frac{1}{2\sigma^2}\ln\left(M/R^3\right)}, M = R\left(1 + \mathcal{M}_*^2 R^{2\eta}\right), \mathcal{M}_* \propto \frac{V_0}{C_s}\frac{\lambda_J}{1pc}$$

Large Scale Structures

Interstellar Cycle and Star Formation



Galaxies

few kpc

Planets

Stars and Accretion Disks

few 10 AU

Feedback Efficiency ?

Protostars, Binarity
Protoplanetary Disks ?

few 10 pc

Origin of the Stellar Initial Mass Function ?
Multiplicity and clustering ?

few 0.1 pc

Dense Cores

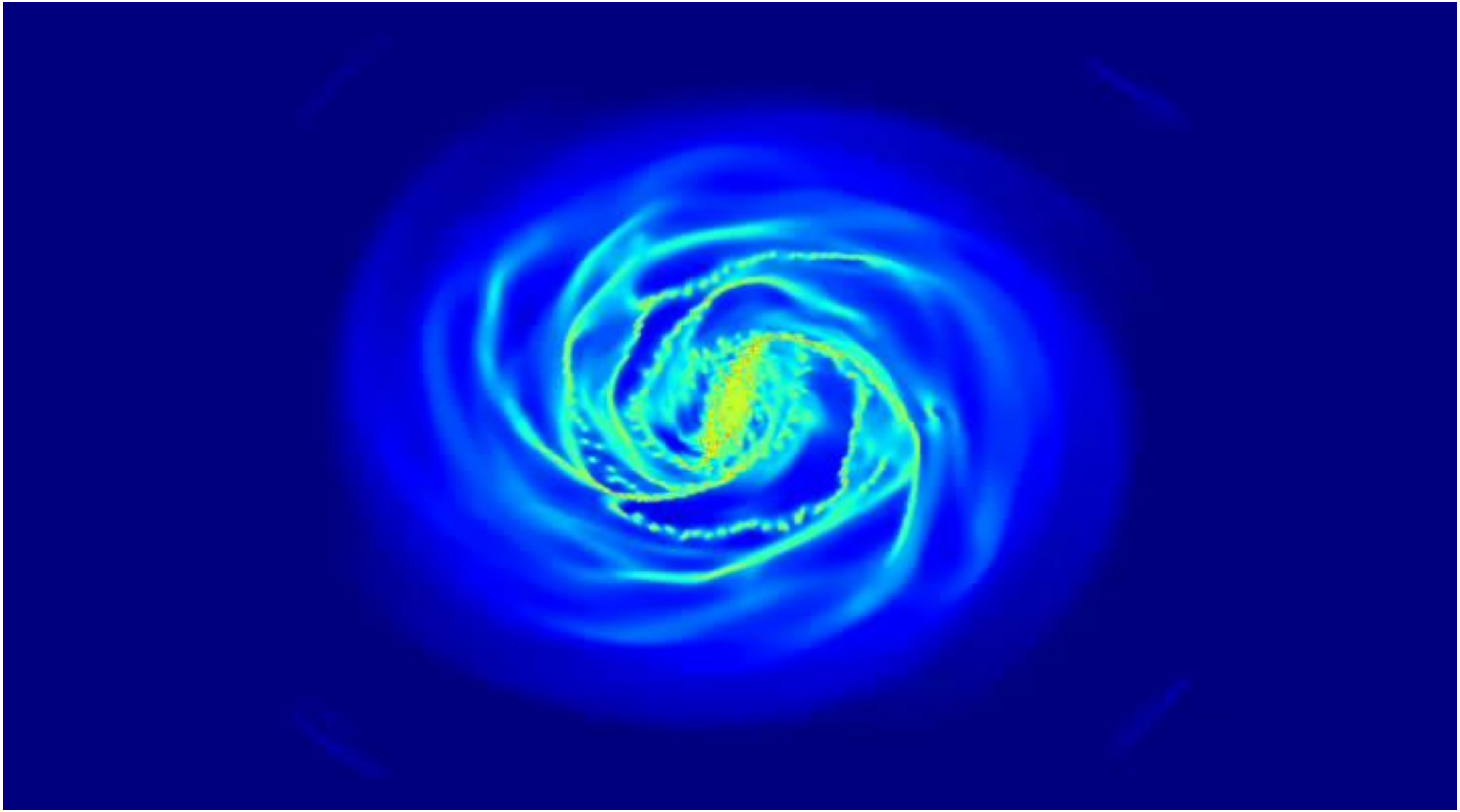
Star Formation Rate and Efficiency ?

Molecular Clouds

Numerical simulation of a whole galaxy: a global approach

Gas dynamics, dark matter halo, star formation

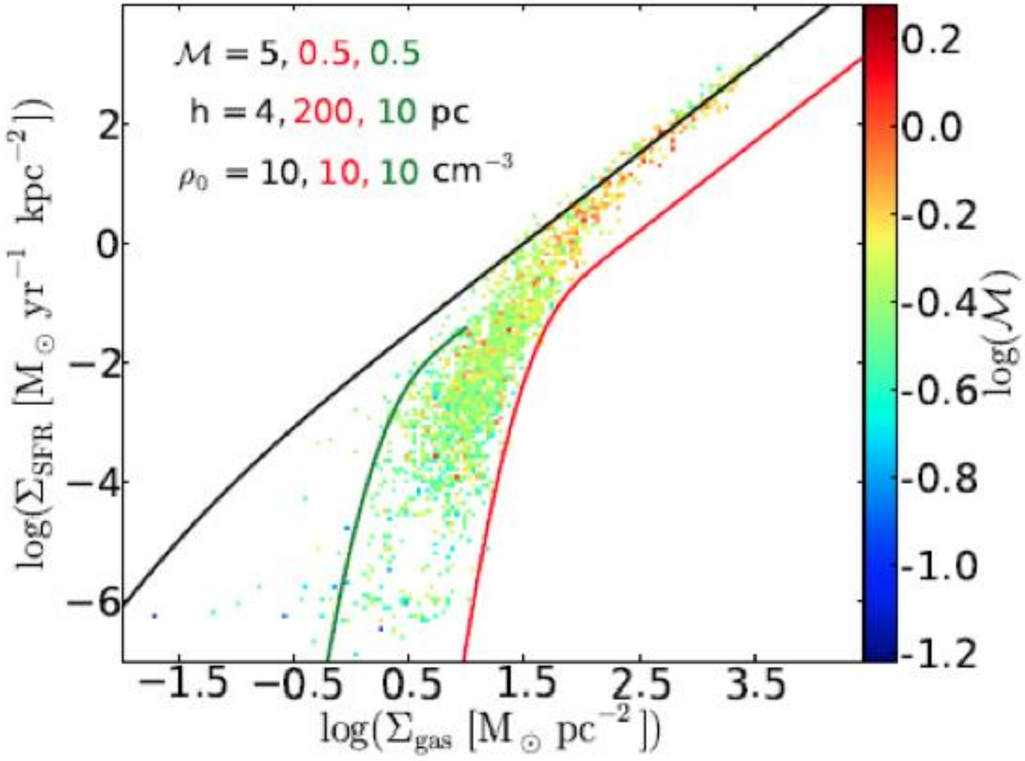
~100,000 light years



Bournaud et al. 2010, Renaud et al. 2013
Several millions of CPU hours (PRACE project)

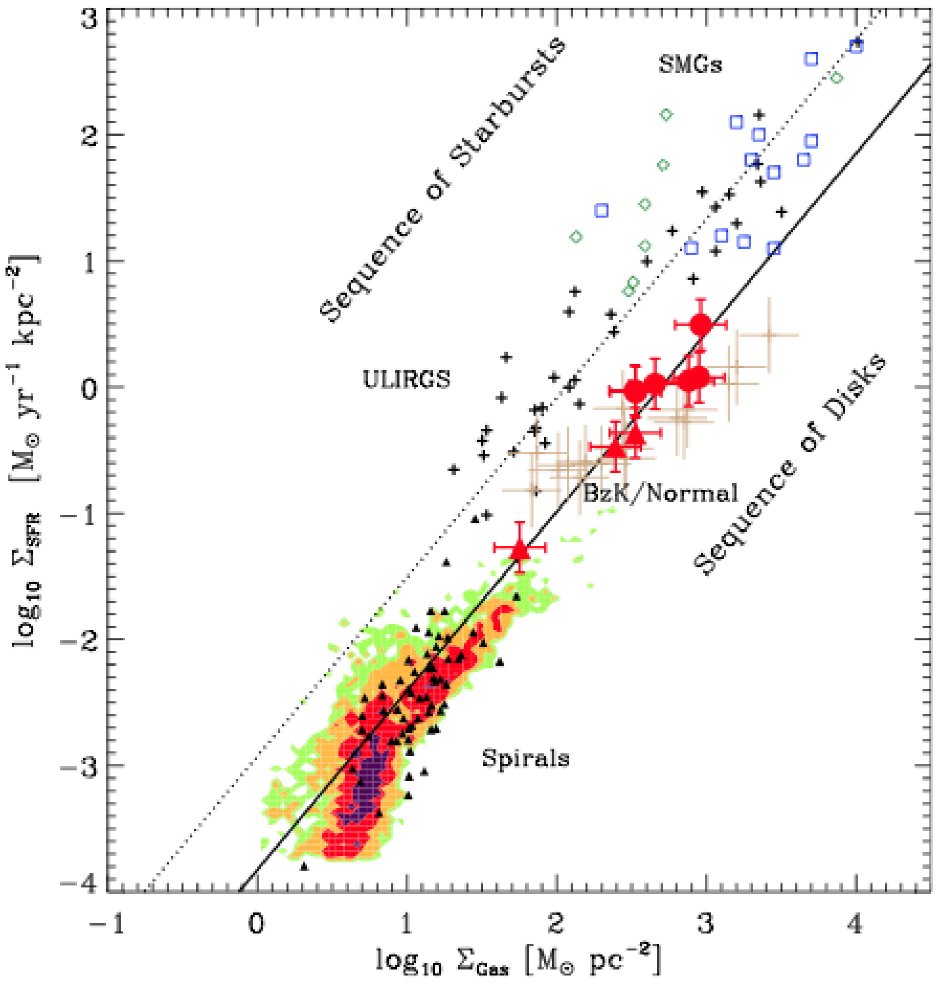
Star formation rate:
Confrontation between simulation results and observations

Simulations and models: isolated galaxies



Kraljic et al. 2014

Observations



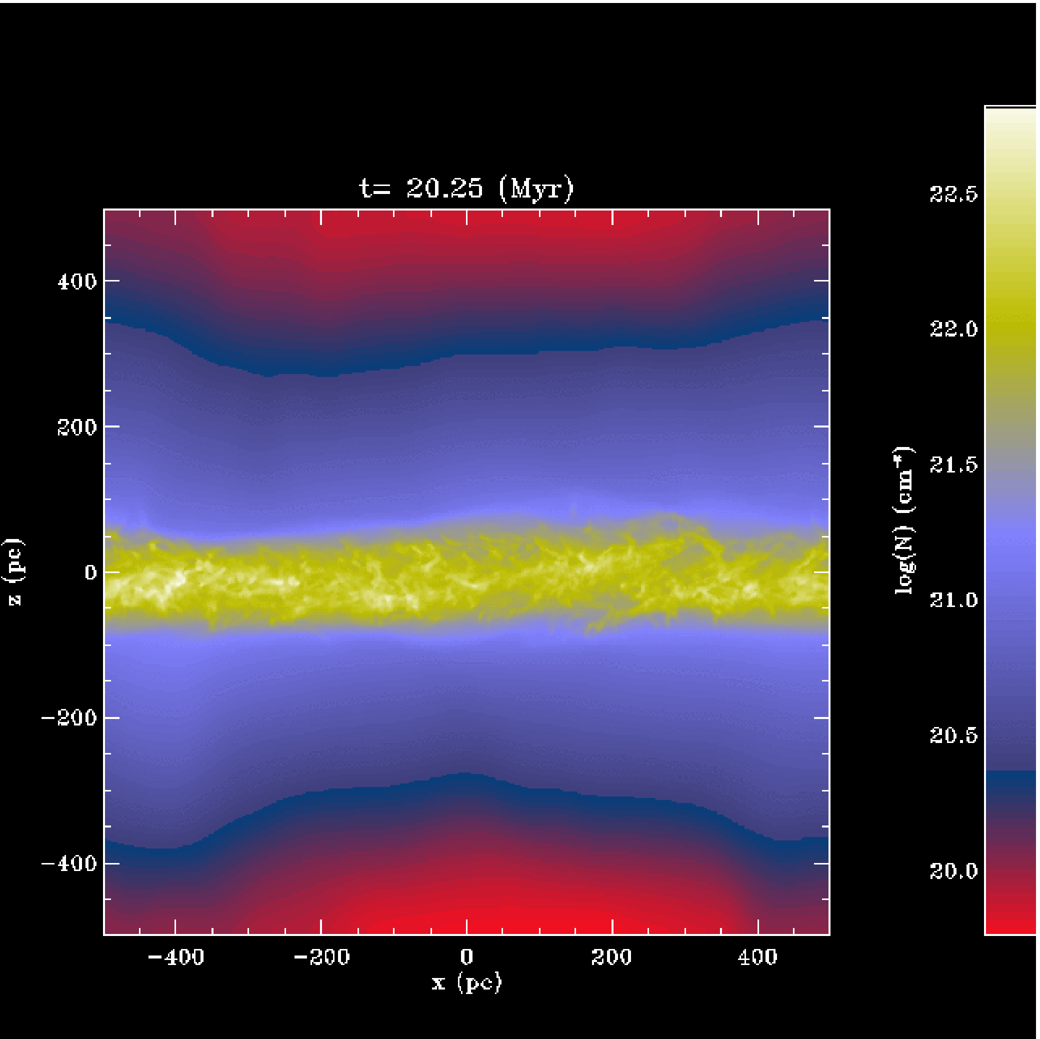
Daddi et al. 2010

Supernovae regulated ISM (from few 100 pc to 1kpc)

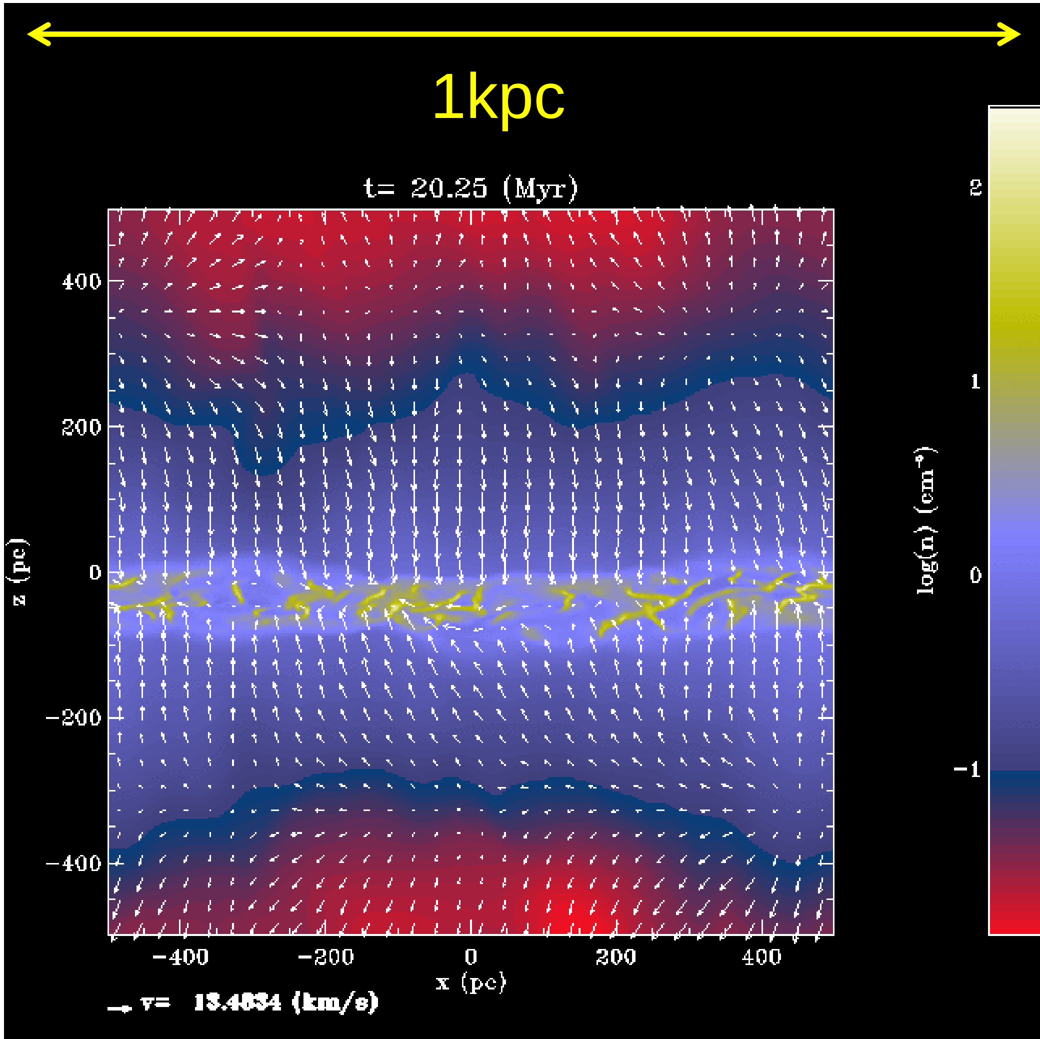
(H & Iffrig 2014)

External gravitational field (due to stars and DM), multi-phase ISM, self-gravity, magnetic field
Supernovae explosions (different schemes)

Column density

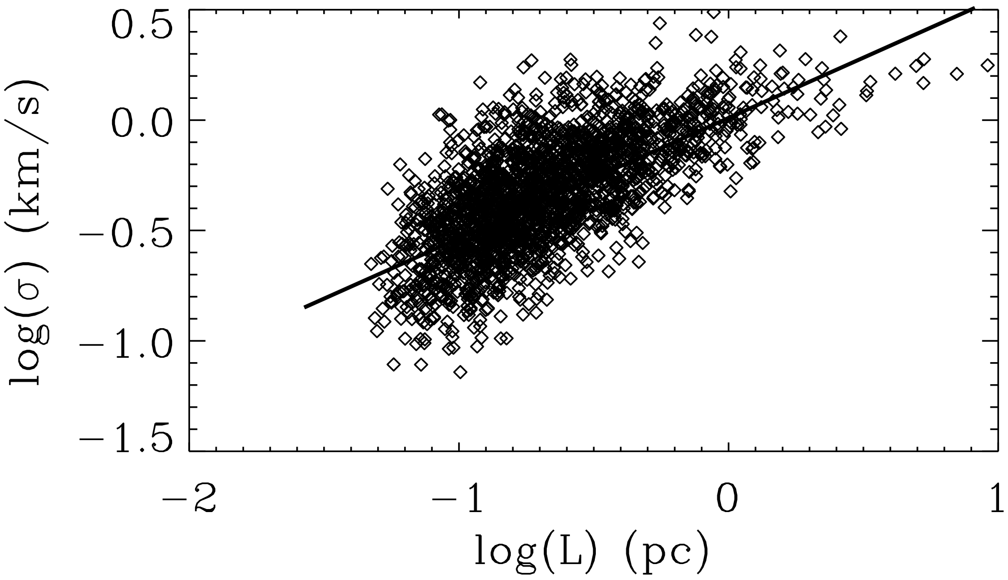


density



Internal clump velocity dispersion vs size

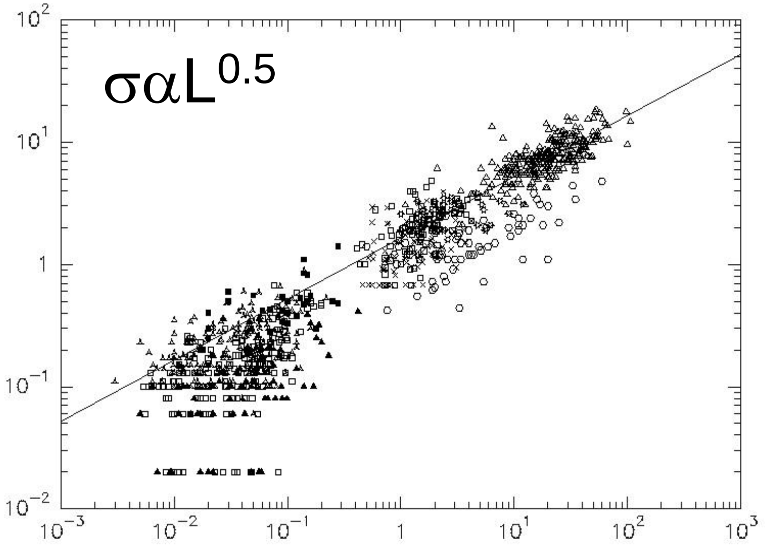
Numerical simulations



$$\sigma(L) \approx 1\,\mathrm{km\,s^{-1}}\,(L/1pc)^{0.5}$$

Compatible with observation
=>is turbulence globally injected by supernovae ?
=>is turbulence *within dense clouds* driven from outside ?
=>is it driven by continuous accretion ?

Observations



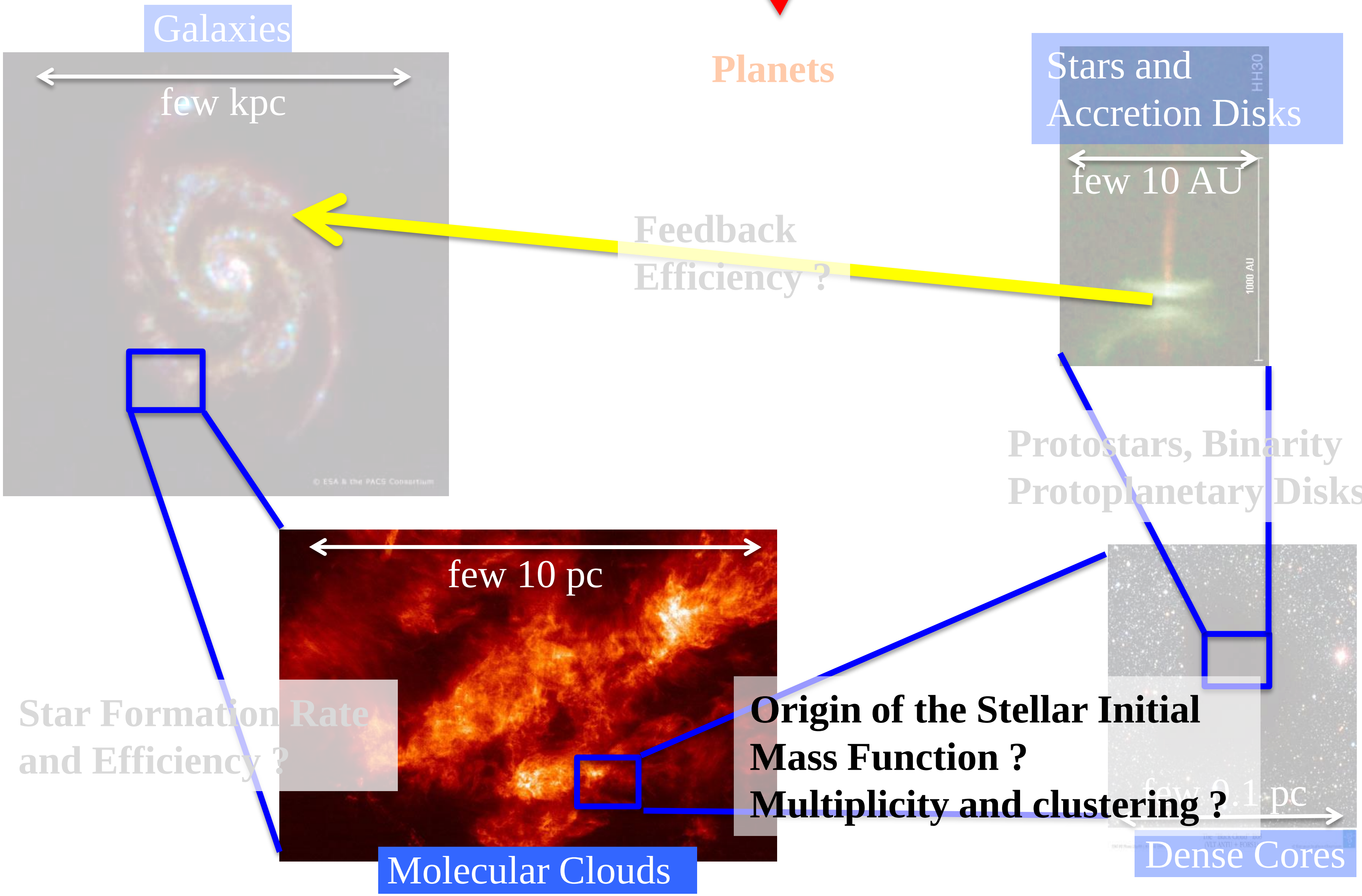
$\sigma \alpha L^{0.5}$

Falgarone 2000

Large Scale Structures

Interstellar Cycle and Star Formation

Planets



Galaxies

few kpc

Stars and Accretion Disks

few 10 AU

Feedback Efficiency ?

Protostars, Binarity Protoplanetary Disks ?

few 10 pc

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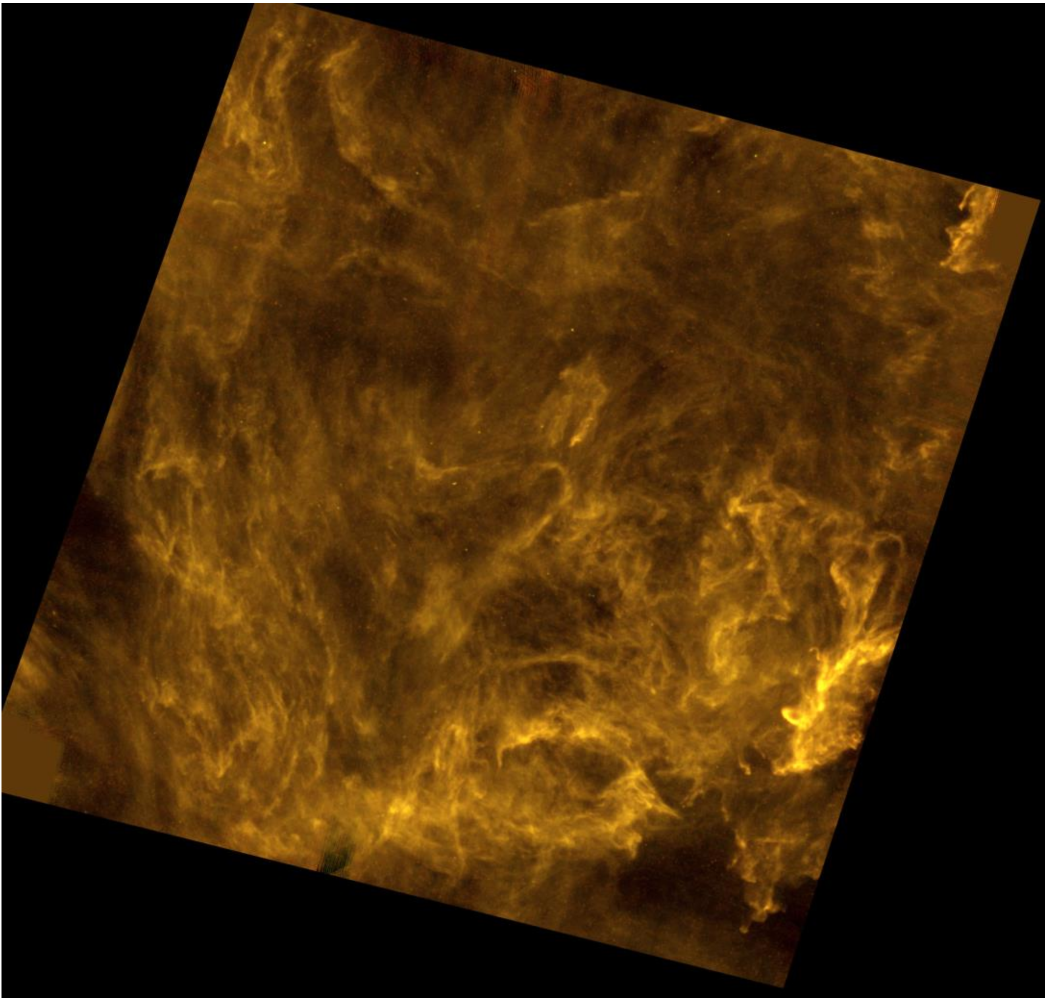
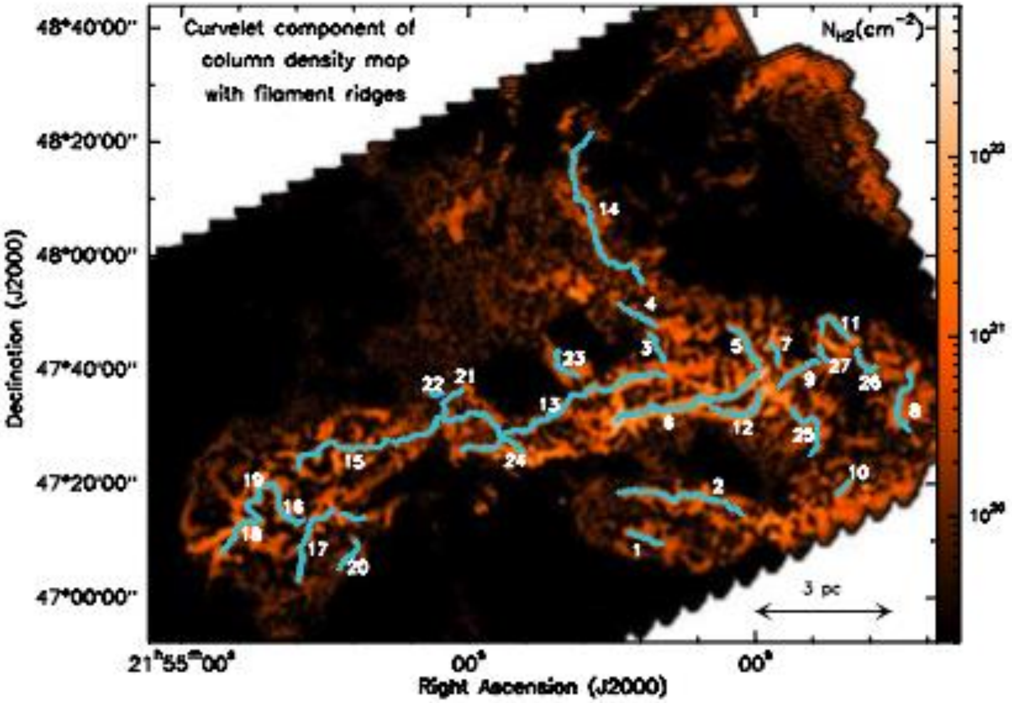
few 0.1 pc

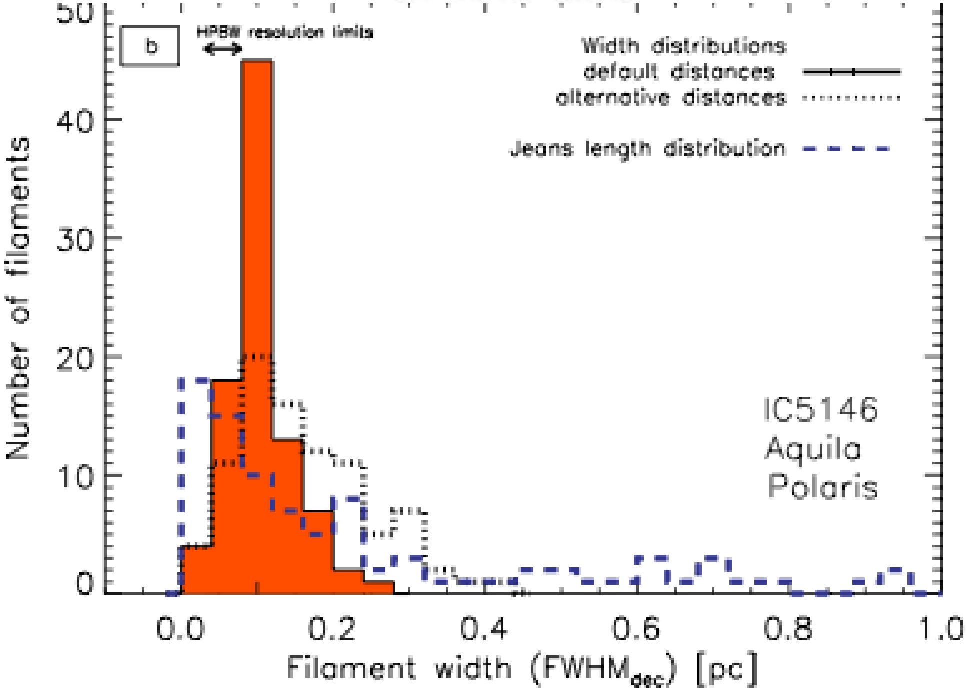
Dense Cores

Molecular Clouds

Herschel data: Recent analysis from the Gould belt survey







A characteristic width ?

Arzoumanian et al. 2011

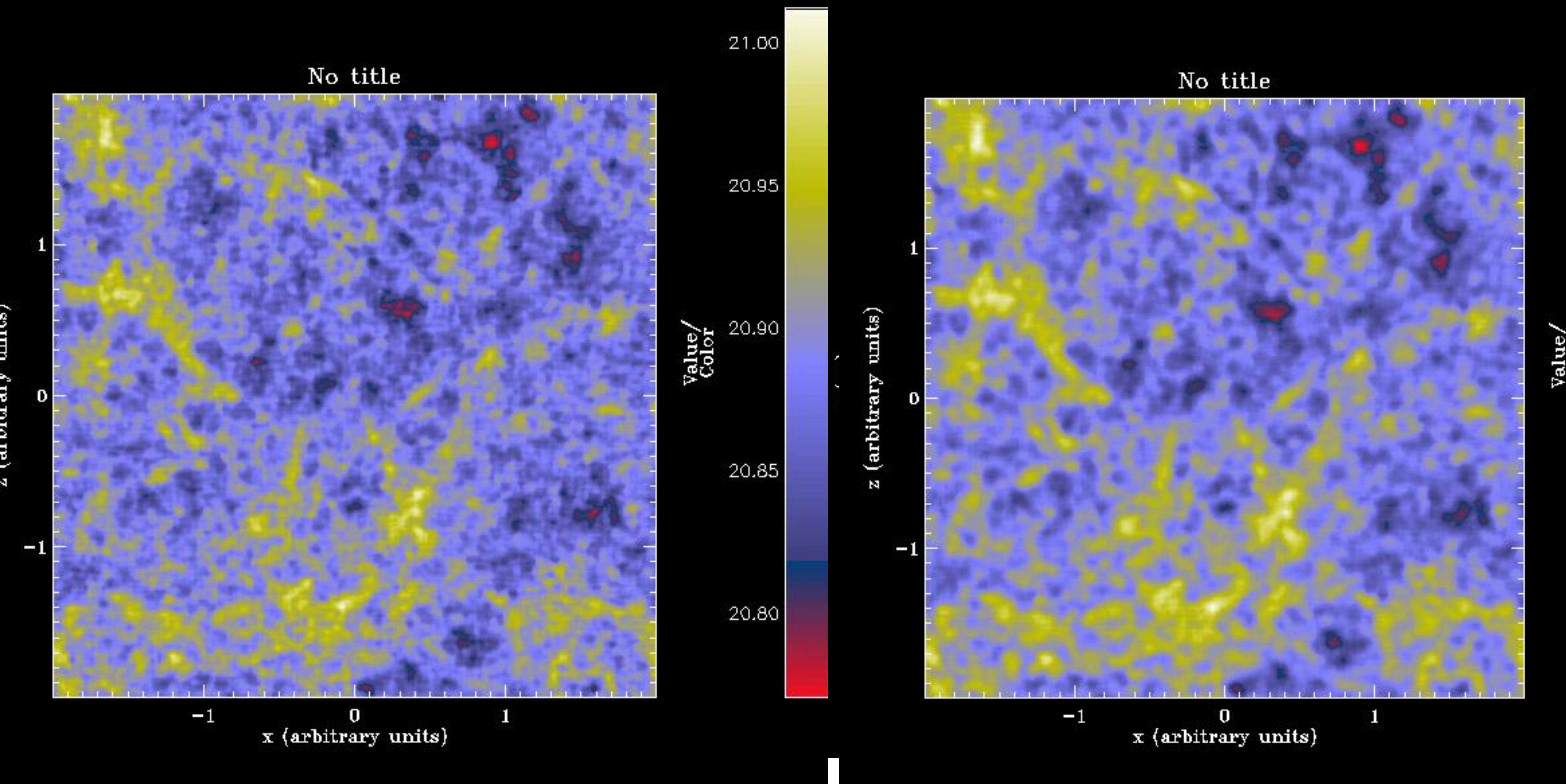
Comparison between hydro and MHD simulations

Decaying turbulence, 2 phase-medium, no gravity, 5 cm^{-3}

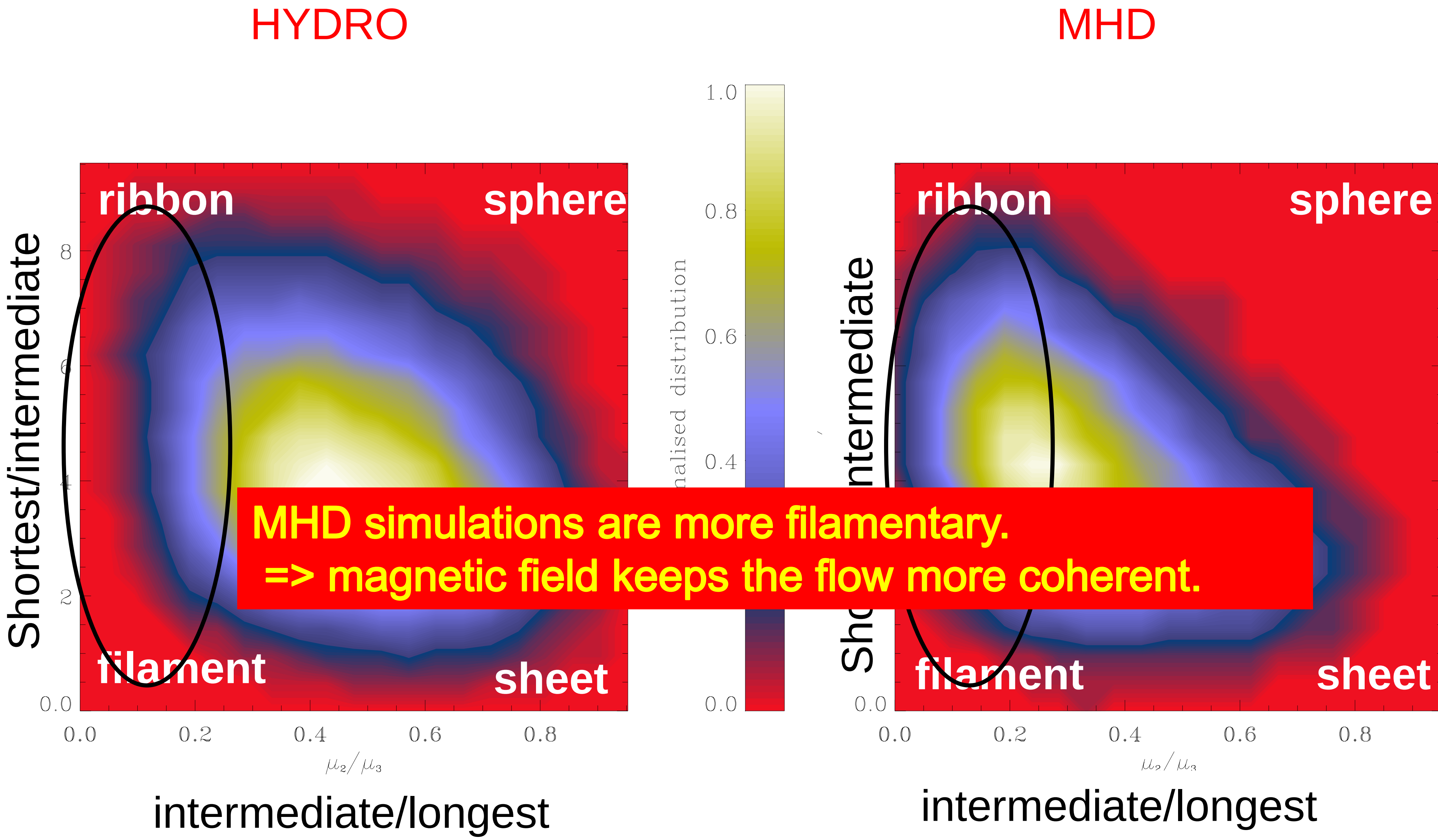
Initial Mach (wrt cold gas) : 10, B=0 or 5 μG

HYDRO

MHD



Distribution of clump aspect ratio



MHD simulations are more filamentary.
=> magnetic field keeps the flow more coherent.

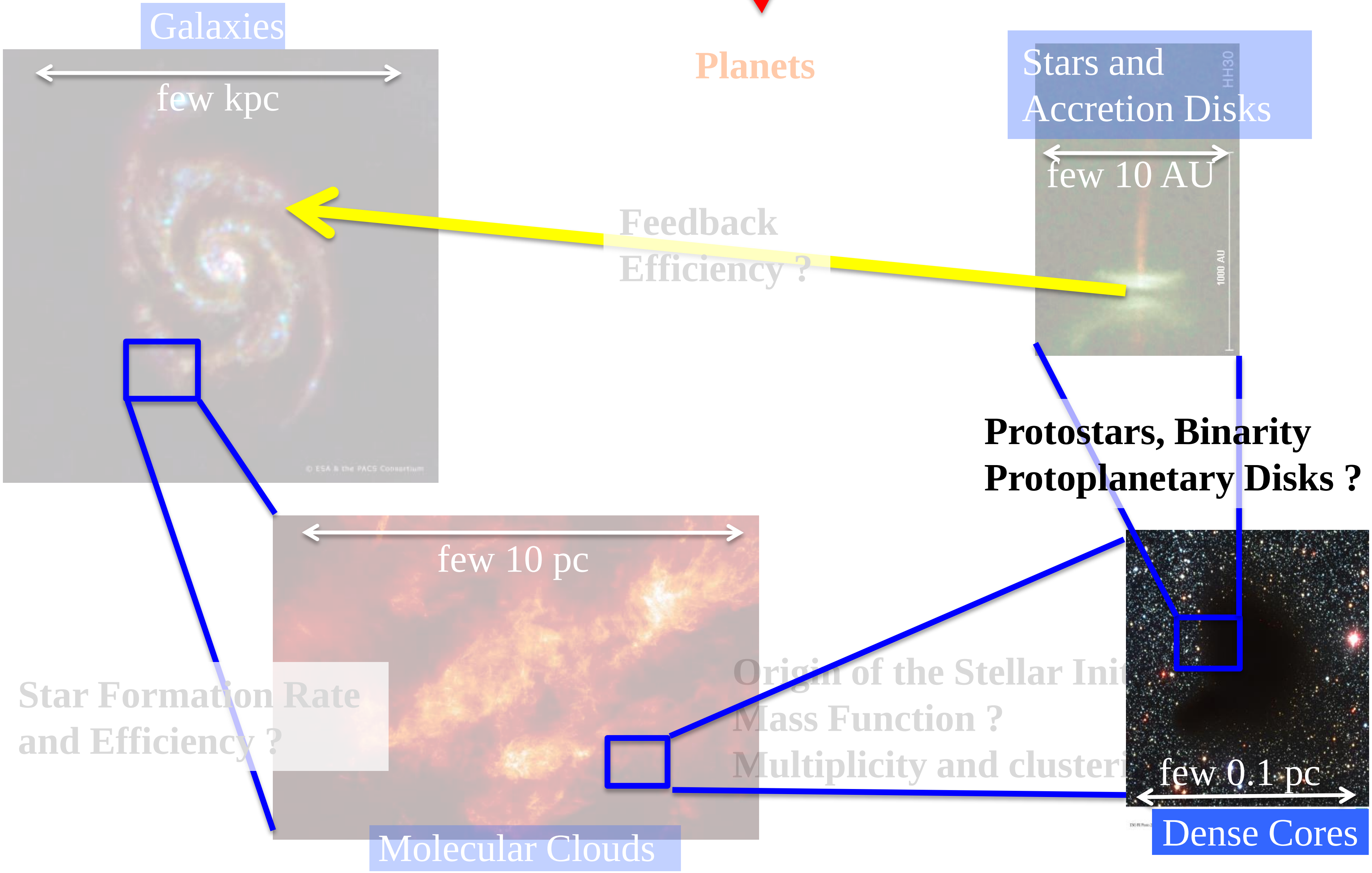
H2013

Large Scale Structures

↓

Interstellar Cycle and Star Formation

↓



Protostars, Binarity
Protoplanetary Disks ?

Thermal Support

Consider a cloud of initial radius R

If γ<4/3, when R decreases, Etherm/Egrav decreases:

$$\frac{E_{therm}}{E_{grav}} = \frac{PV}{GM^2/R} \mu\, r^{g} R^{4} \mu\, R^{4-3g}$$

Centrifugal Support and Angular Momentum Conservation

When R decreases, Erot/Egrav increases:

$$j = R^2 w(t) = R_0^{\,2} w_0$$

$$\frac{E_{rot}}{E_{grav}} = \frac{MR^2 w^2}{GM^2/R} \mu \frac{1}{R}$$

Magnetic Support and Flux Conservation

$$f \mu BR^2$$

$$\frac{E_{mag}}{E_{grav}} = \frac{B^2 R^3}{M^2/R} \mu (f/M)^2$$

When R decreases, Emag/Egrav is constant:

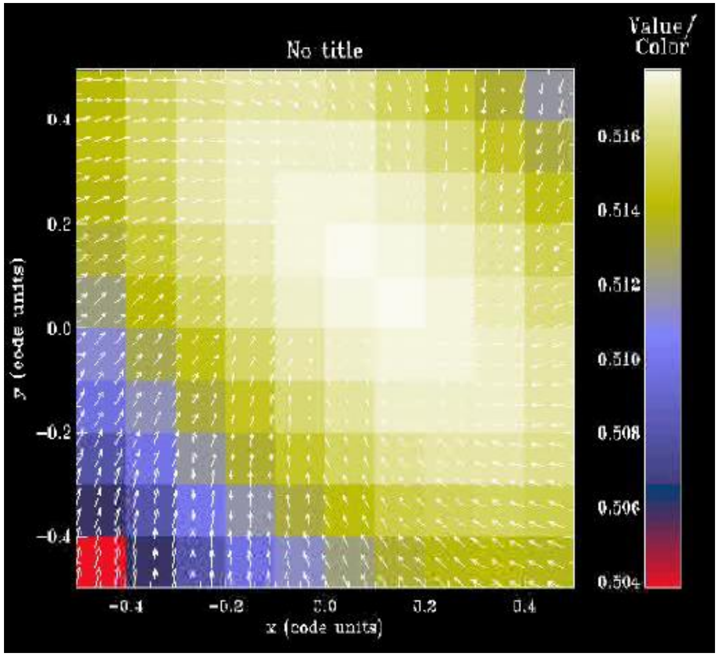
Typically one infers $\mu=(M/\phi)/(M/\phi)_c=1\text{-}4$

(Crutcher et al. 1999, 2004)

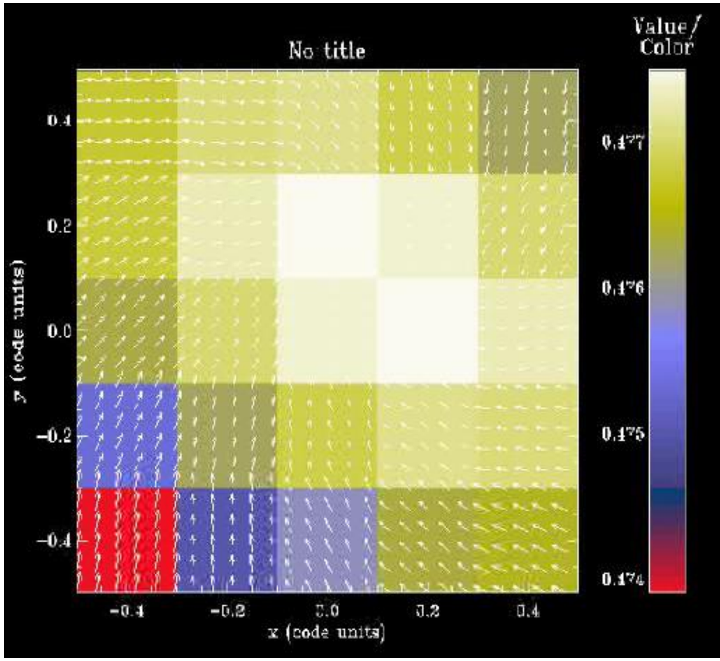
Zoom into the central part of a collapse calculation

~30 light hours

XY
hydro

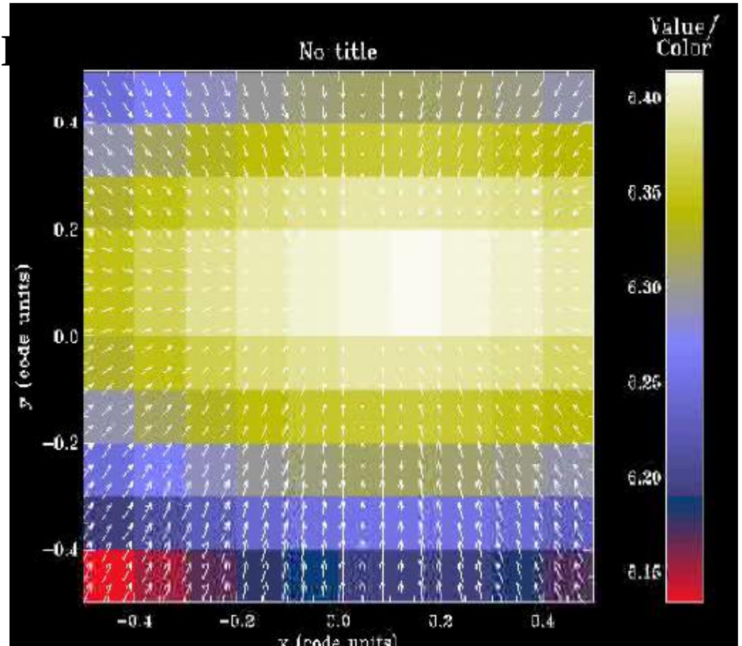


XY
MHD
μ=2



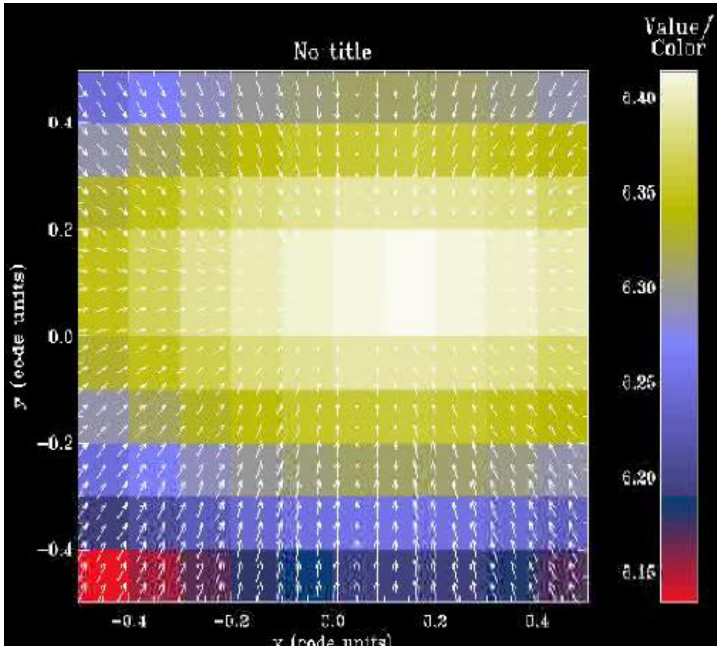
⊙ B, ω

XZhydr

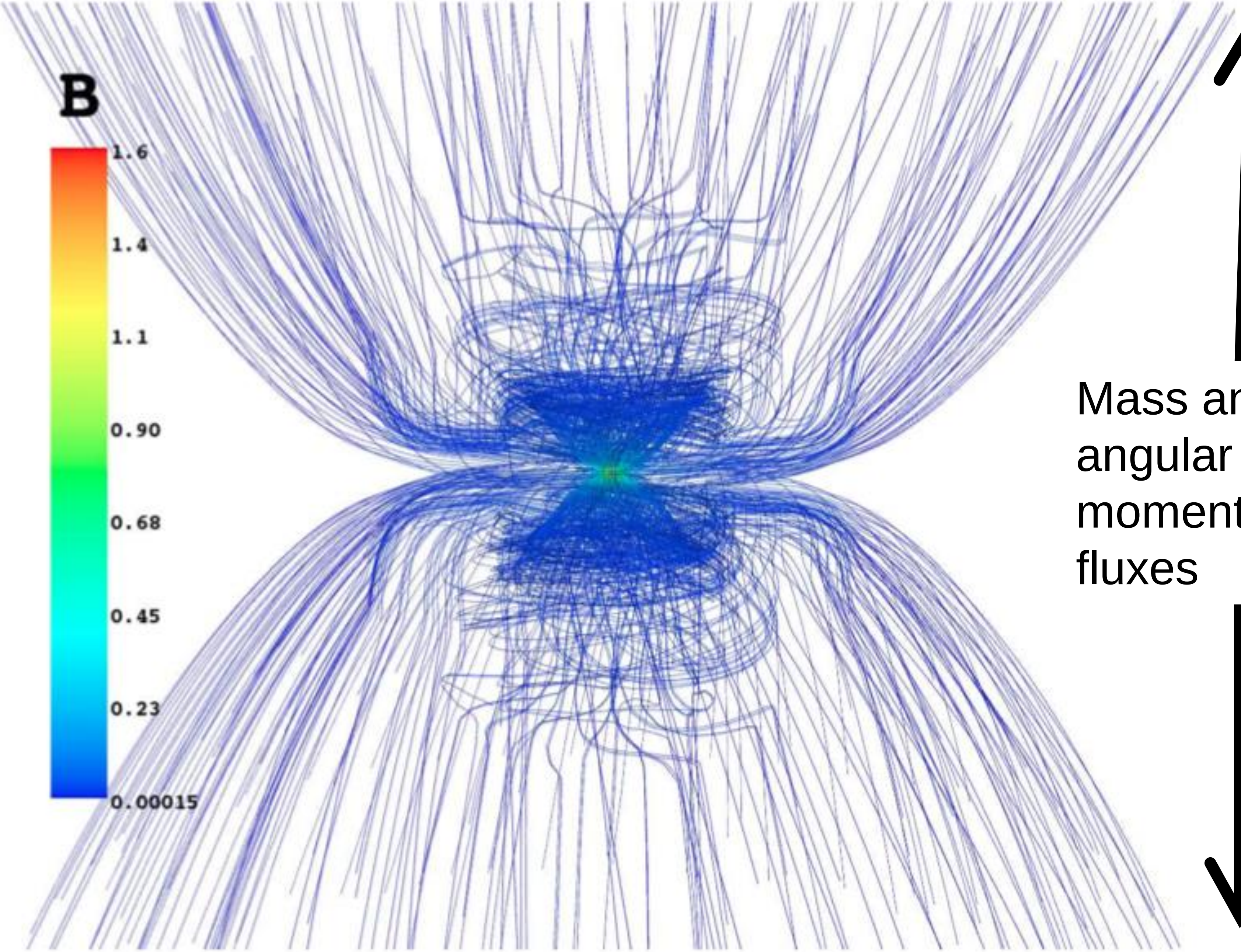


↑ B, ω

XZ
MHD
μ=2



Magnetic field lines have been pinched and wrapped.
Strong braking occurs. Angular momentum is transported outwards.

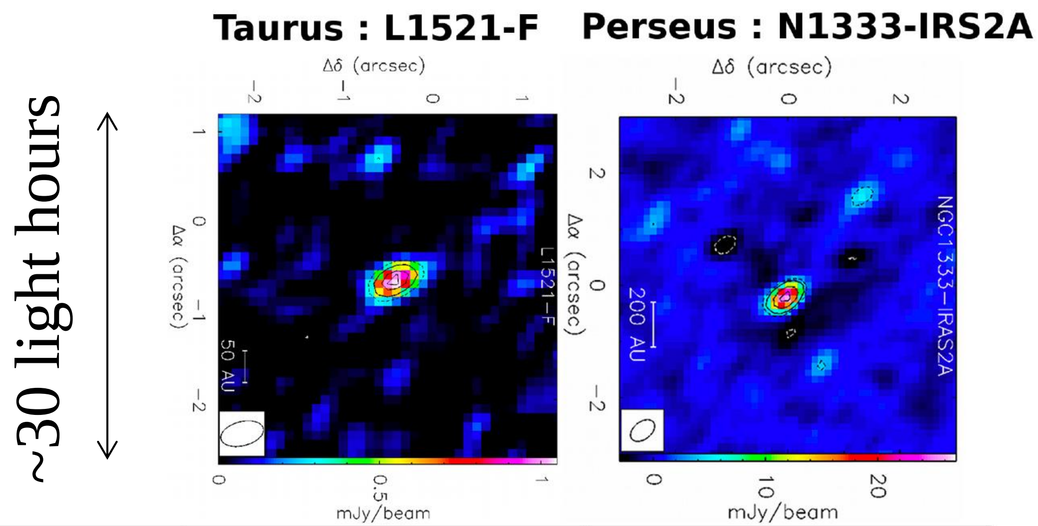


Mass and angular momentum fluxes

Comparison of observations with MHD simulations

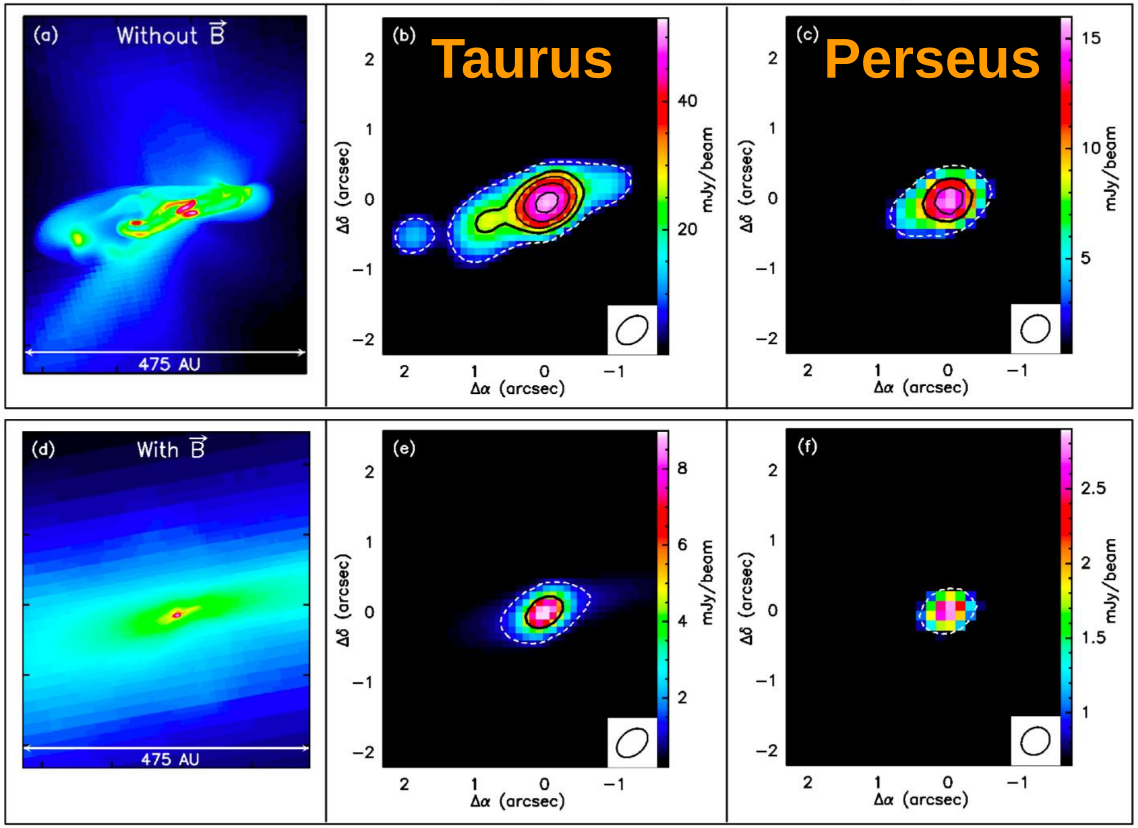
Hydrodynamical simulations produce too much extended (+ multiple) structures if compared to Maury et al. 2010 Observations.

→ MHD simulations ?



Synthetic observations from hydro simulations

Synthetic observations from mhd simulations



Maury et al. 2010

Conclusion

Star formation is one of the key process in our universe

Combinations: observations – simulations – theory
is necessary to address the multi-scale / multi-physics interstellar medium

Great challenge tightly linked to cosmology and planet topic

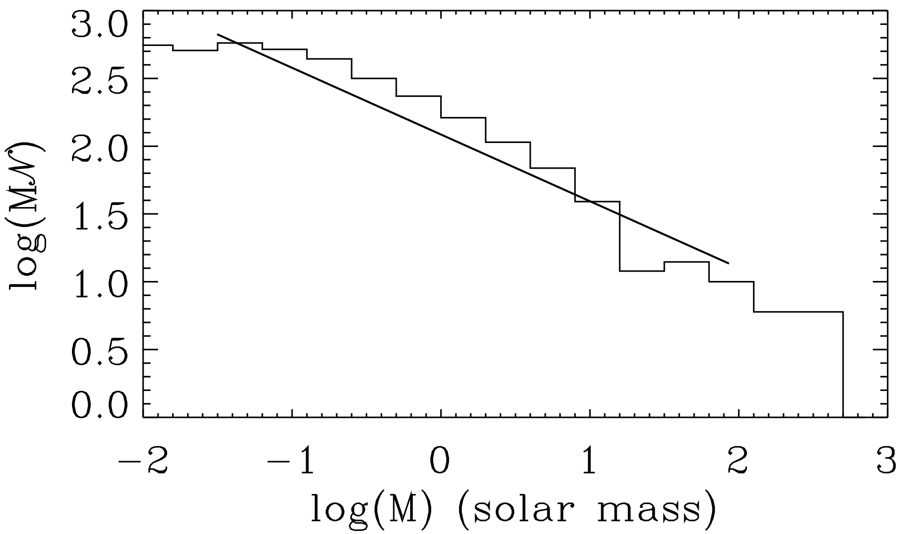
New physical processes and new physical regimes to be discovered

Mass spectrum and mass size relation of molecular clouds

Numerical simulations

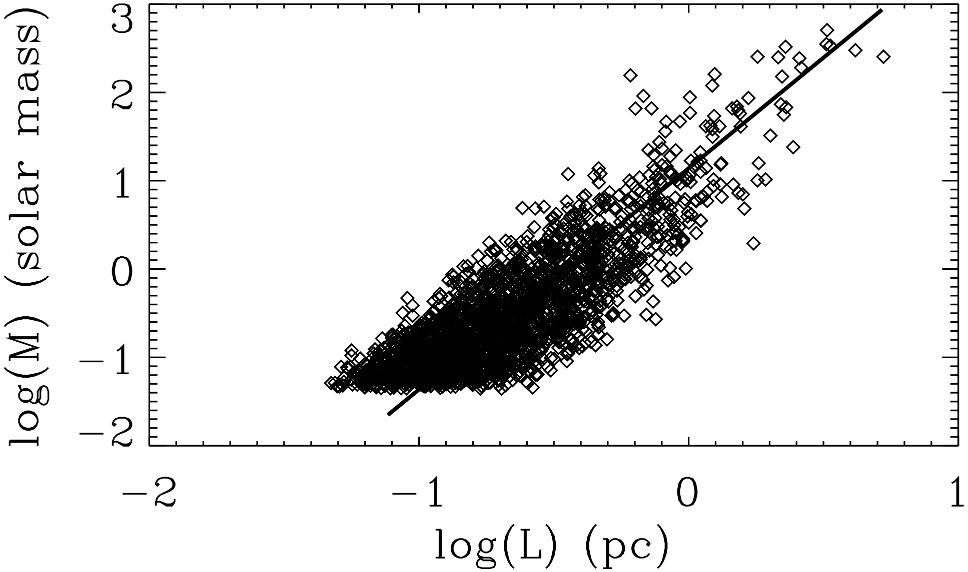
Mass spectrum of clumps

$dN/dM \alpha M^{-1.7}$



Mass versus size of clumps

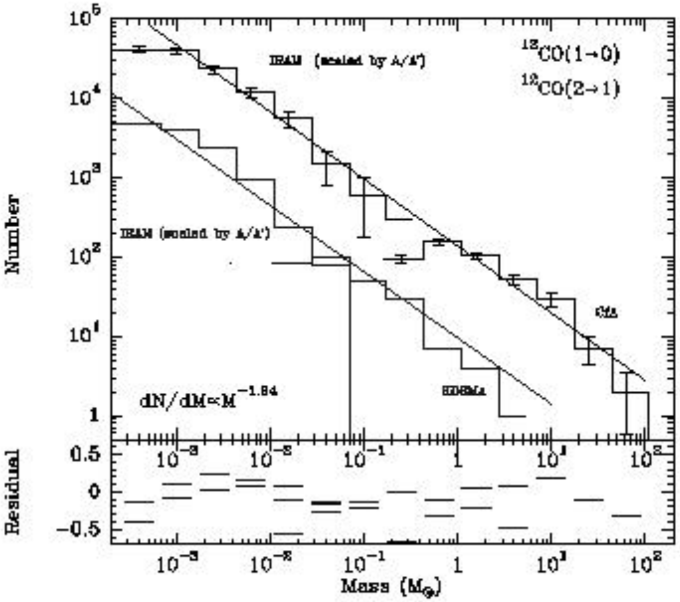
$M \alpha R^{2.3-2.5}$



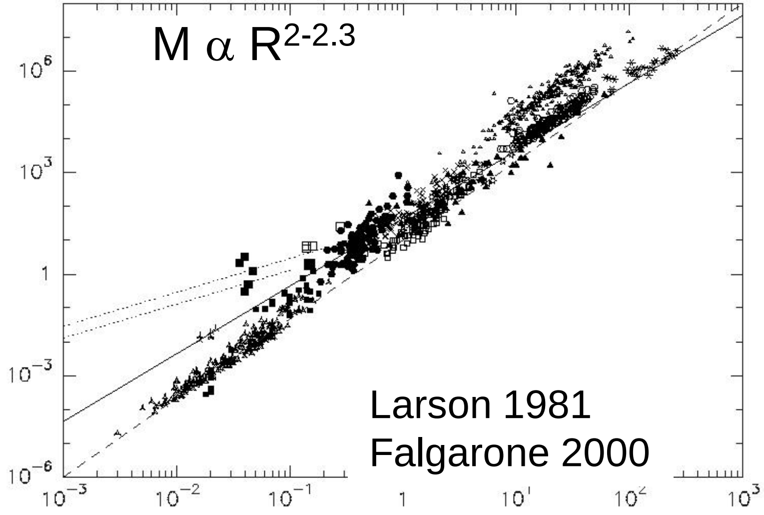
Observations

Universal Mass Spectrum

$dN/dM \ \alpha \ M^{-1.6-1.8}$ (Heithausen et al .98)



Mass versus size of CO clumps



$M \ \alpha \ R^{2-2.3}$

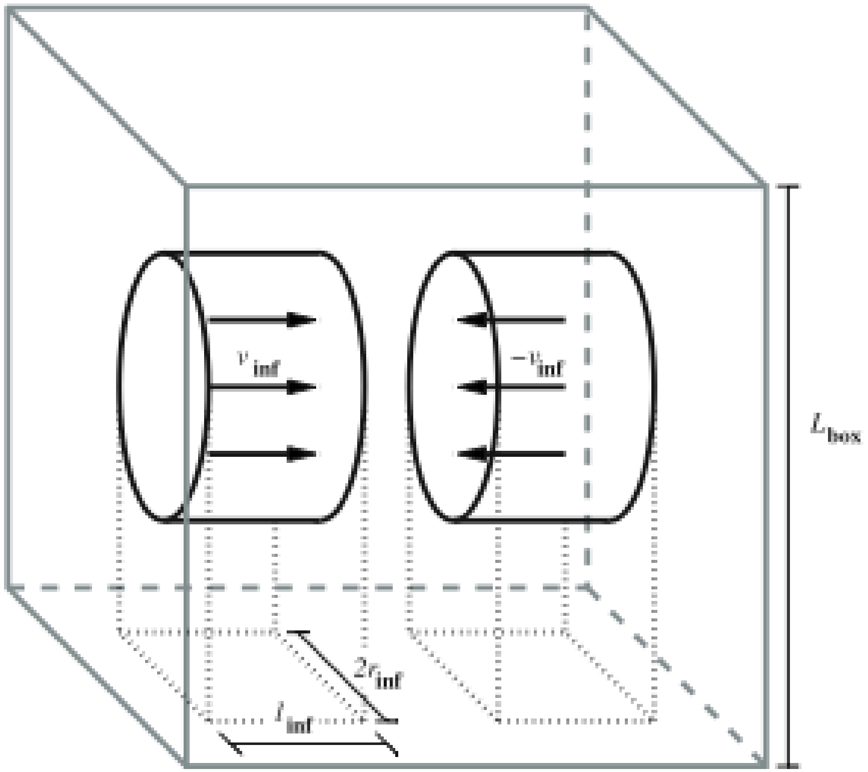
Larson 1981
Falgarone 2000

Formation of a *molecular clouds* from diffuse atomic hydrogen

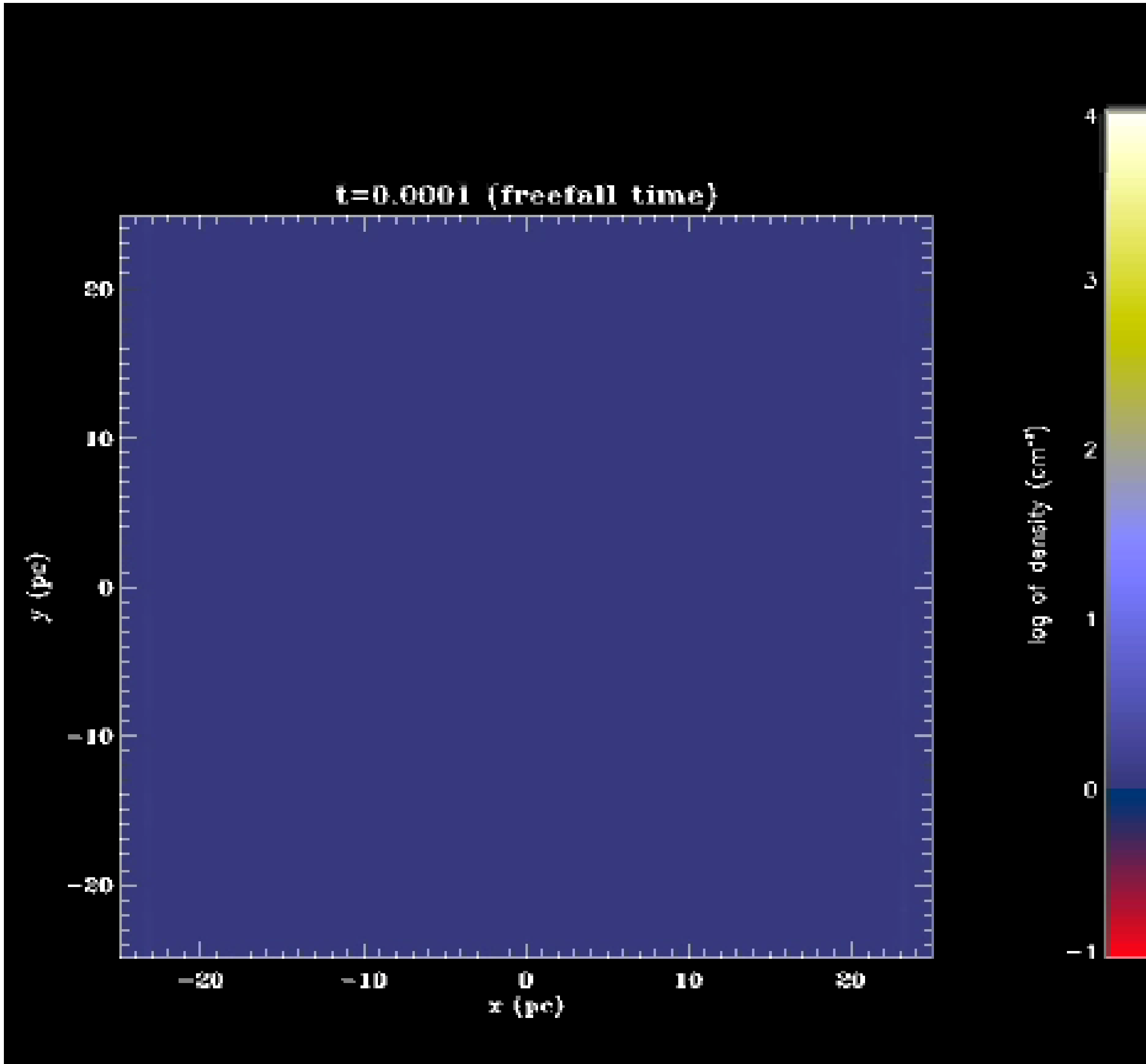
Flow of WNM (density 1cc), velocity 20km/s each side, initial magnetic field 5μG

Include gravity and cooling

Colliding flows



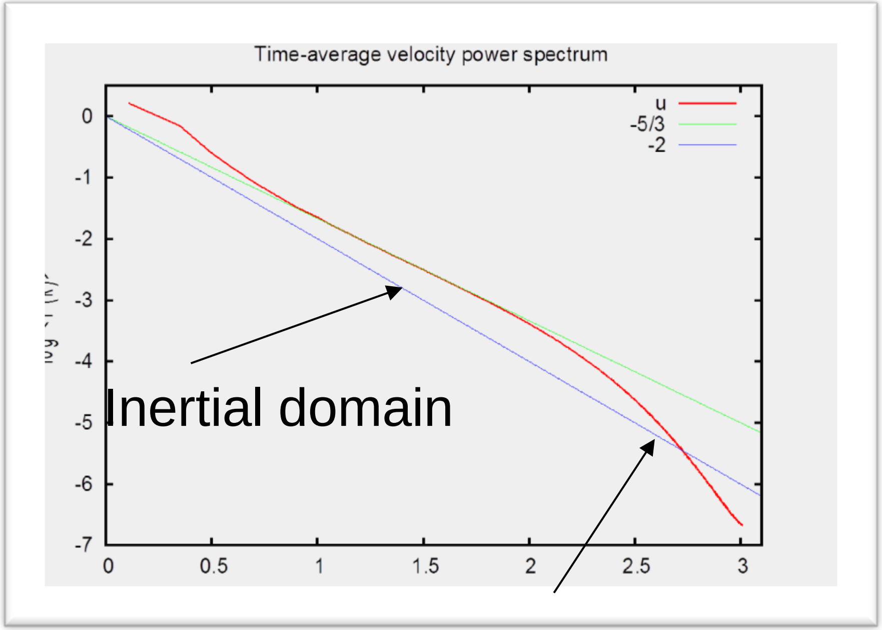
~100 light years



Hennebelle et al. 2008,
Heitsch et al. 2008
Vazquez-Semadeni et al. 2007, 2011

Power spectrum of supersonic turbulence

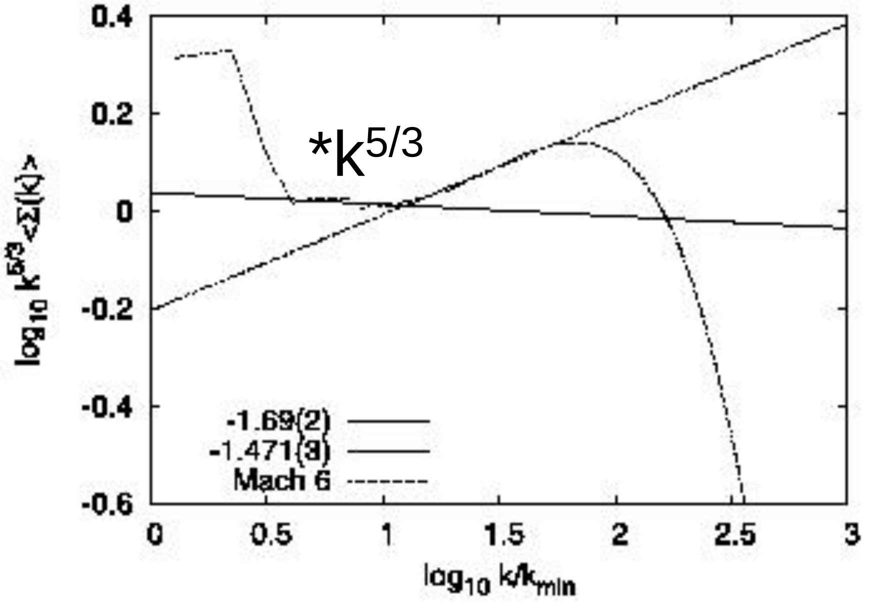
Power spectra of velocity



Inertial domain

Bottle neck effect

Compensated Power spectrum of corrected velocity



$*k^{5/3}$

Exponent of PS between around 1.9 between K41 and Burgers

Value 1.69 i.e. closer to K41

$$e=\frac{rv^3}{l} \text{Þ} \quad r^{1/3}v \text{»} l^{1/3}$$

Kritsuk et al. 2007

The equations

(Spitzer 1978, Shu 1992)

Equation of state:

$$P = k_b / m_p r T$$

Ionisation Equilibrium:

$$r >> r_i \, , r_i = c\sqrt{r} \quad (r > 10^3 cm^{-3})$$

Energy Equation:

$$\partial_t e + \vec{v}.\vec{\nabla} e + (\gamma - 1) e \vec{\nabla}\vec{v} = -L$$

Continuity Equation:

$$\partial_t \rho + \nabla(\rho\vec{v}) = 0$$

Momentum Conservation:

$$\rho(\partial_t \vec{v} + \vec{v}\nabla\vec{v}) = -\vec{\nabla}P + \rho\vec{\nabla}\phi + \nu_{in}\rho\rho_i(\vec{v}_i - \vec{v})$$

Mom. Cons. for ions:

$$\rho_i(\partial_t \vec{v}_i + \vec{v}_i\nabla\vec{v}_i) = \nu_{in}\rho\rho_i(\vec{v} - \vec{v}_i) + \frac{1}{4\pi}\vec{\nabla}\vec{B}\times\vec{B}$$

Induction Equation:

$$\partial_t \vec{B} + \vec{\nabla}(\vec{B}\times\vec{v}) = 0$$

Poisson Equation:

$$\Delta\phi = -4\pi G\rho$$

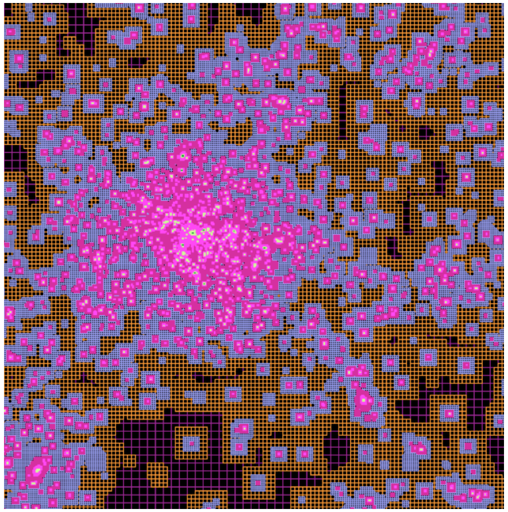
Numerical approach

Code RAMSES (Teyssier 2002, Fromang et al. 2006)

Godunov method:

$$\boxed{\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}=0 \Rightarrow \frac{U^{n+1}-U^{n}}{\Delta t}+\frac{F^{n+1/2}_{i+1/2}-F^{n+1/2}_{i-1/2}}{\Delta x}=0}$$

Exact conservation: mass, momentum, energy

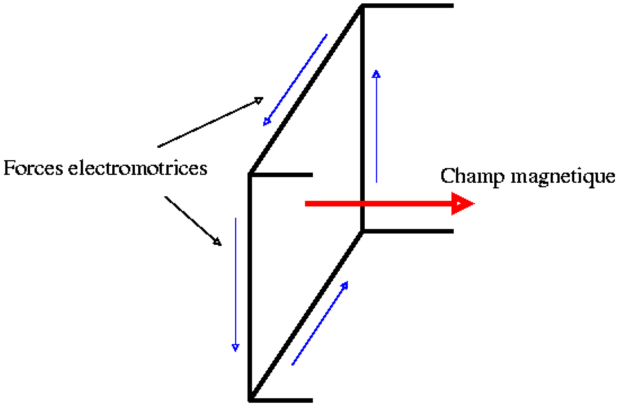


AMR technique (adaptive mesh refinement):

Increase locally the resolution

Constrained transport: insure nullity of divB

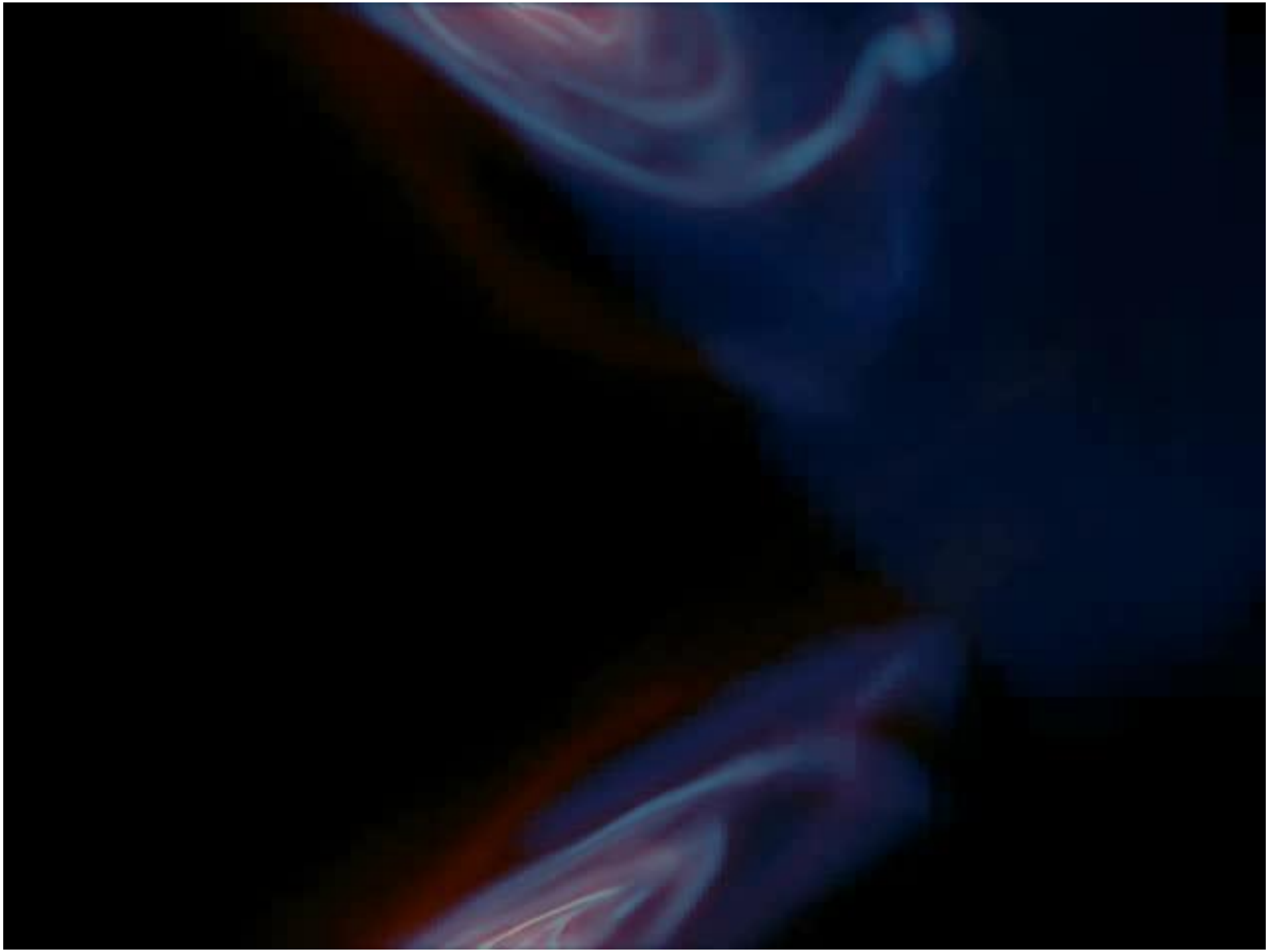
$$\frac{\partial B}{\partial t}=\nabla\times(v\times B)\Rightarrow \iint_{S} B.dS=\oint(v\times B).dl$$



Numerical simulation of interacting galaxies

Gas dynamics, dark matter halo, star formation

~1000,000 light years



Renaud et al. 2014

Can a galactic disk self-regulate ?
=>energy injection by supernovae remnants

Energy Dissipation:

turbulent energy is dissipated in about a crossing time $\dot{\epsilon} = \frac{1}{2}\frac{\rho V_{rms}^{2}}{t_{diss}} = \frac{1}{2}\frac{\rho V_{rms}^{3}}{L}$

For typical numbers, we get $\dot{\epsilon} = 3 \times 10^{-27}\, erg\, cm^{-3}\, s^{-1} \frac{n}{1\, cm^{-3}} \left(\frac{V_{rms}}{10\, km/s} \right)^{3} \frac{100\, pc}{L}$

Energy injection by supernovae:

$$\dot{\epsilon} = 3 \times 10^{-26}\, erg\, cm^{-3}\, s^{-1} \frac{\eta_{SN}}{0.1} \frac{E_{SN}}{10^{51}\, erg} \frac{\sigma_{SN}}{SNu} \left(\frac{15\, kpc}{R_{sf}} \right)^{-2} \left(\frac{150\, pc}{H} \right)^{-1}$$

Star Formation Efficiency a fundamental parameter

The case of the Milky-way

Star formation efficiency varies enormously from place to place
(from about 0%, e.g. Magdalena's Cloud to 50%, e.g. Orion)

The star formation rate in the Galaxy is: **3 solar mass per year**

However, a simple estimate fails to reproduce it.

Mass of gas in the Galaxy denser than 10^3 cm^{-3}: 10^9 Ms

Free fall gravitational time of gas denser than 10^3 cm^{-3} is about:

From these two numbers, we can infer a Star Formation Rate of: **500 solar mass per year**

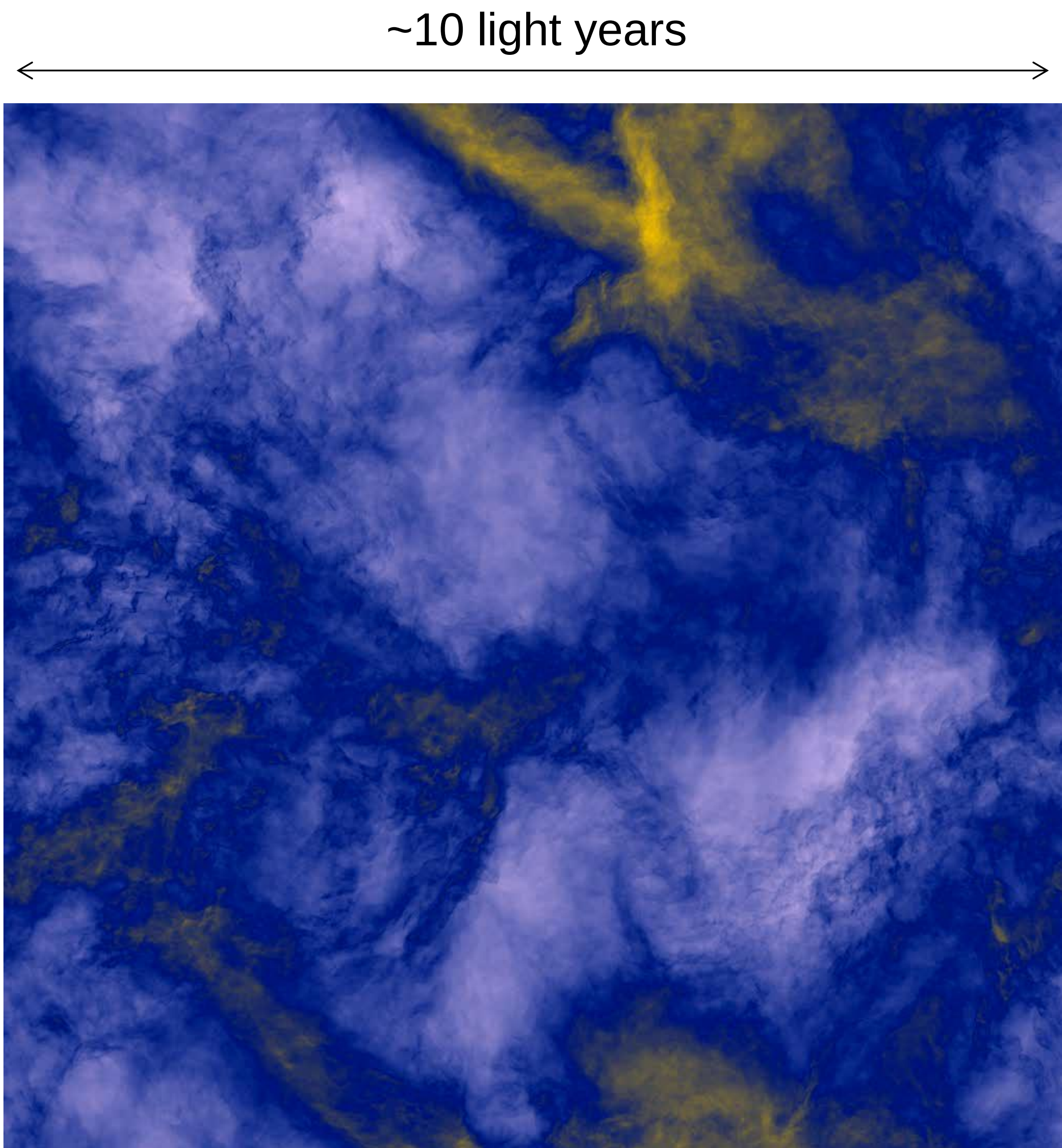
=> 100 times larger than the observed value

$$t_{dyn} = \sqrt{3\pi/32G\rho} \approx 2\,10^6\, years$$

=> Gas is not in freefall and is supported by some agent:

Turbulence, magnetic field, stellar feedback ?

Supersonic hydrodynamical isothermal turbulence: an idealised approach



3D simulation of supersonic
isothermal turbulence with AMR
2048 equivalent resolution
Kritsuk et al. 2007

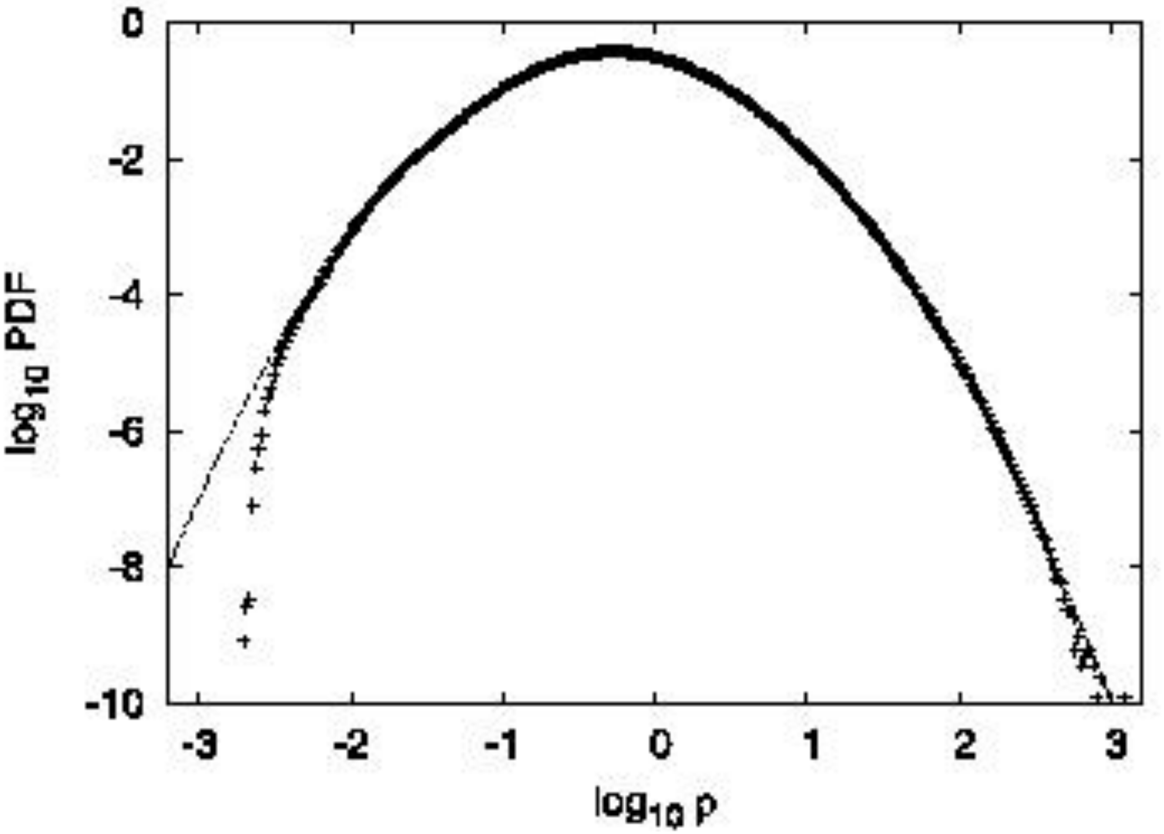
Periodic boxes
Random solenoidal forcing is applied at large scales ensuring constant rms Velocity.

Typically Mach=6-10

Kritsuk et al. 2007

PDF of density field

(Vazquez-Semadeni 1994, Padoan et al. 1997, Kritsuk et al. 2007)



A lognormal distribution:

$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(\delta + \sigma^2/2)^2}{2\sigma^2} \right)$$

$$\delta = \ln(\rho/\bar{\rho}), \sigma^2 \approx \ln\left(1 + 0.25 \times M^2\right)$$

$$\frac{\sigma_\rho}{\bar{\rho}} \approx bM^2$$

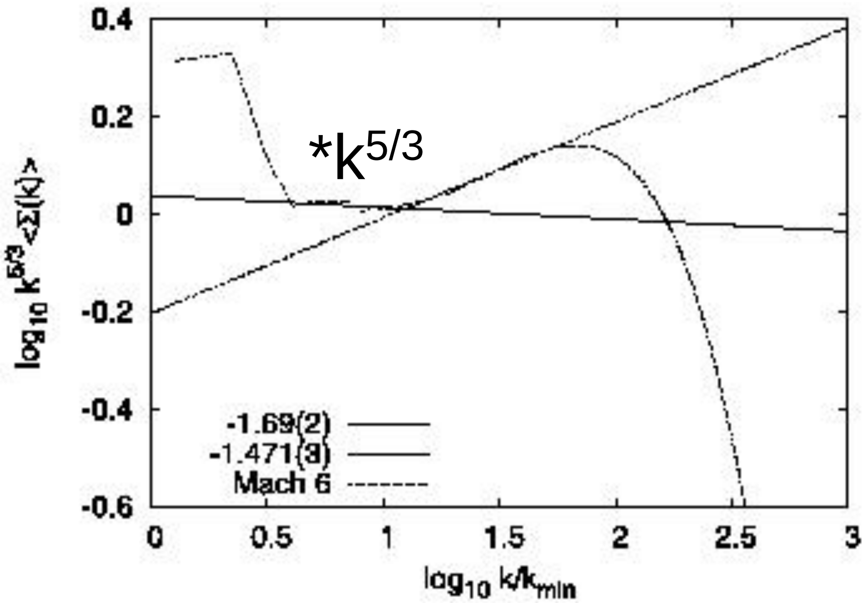
$$P(\rho)d\rho = P(\delta)d\delta$$

$$\sigma_\rho^2 = \int (\rho - \bar{\rho})^2 P(\rho)d\rho$$

$$= \bar{\rho}^2 \int (\exp(\delta) - 1)^2 P(\delta)d$$

Energy cascade is similar with the incompressible cascade

Compensated power spectrum of corrected velocity / energy



Kritsuk et al. 2007

Value 1.69 i.e. close to Kolmogorov theory of turbulence

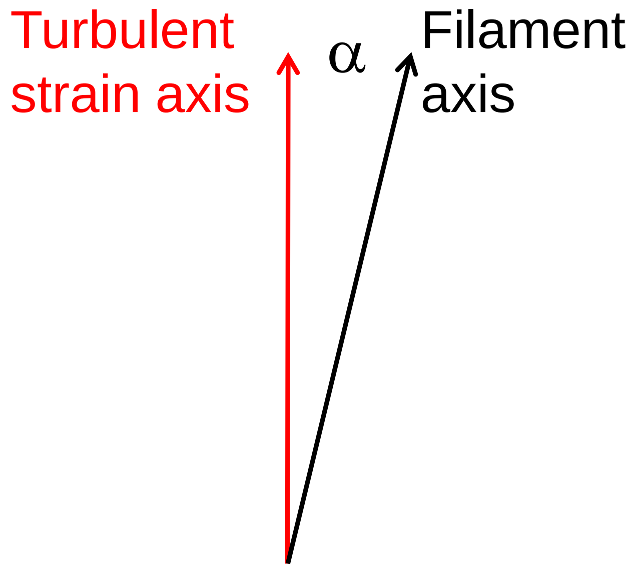
A new equation to describe the energy flux through the turbulent cascade.

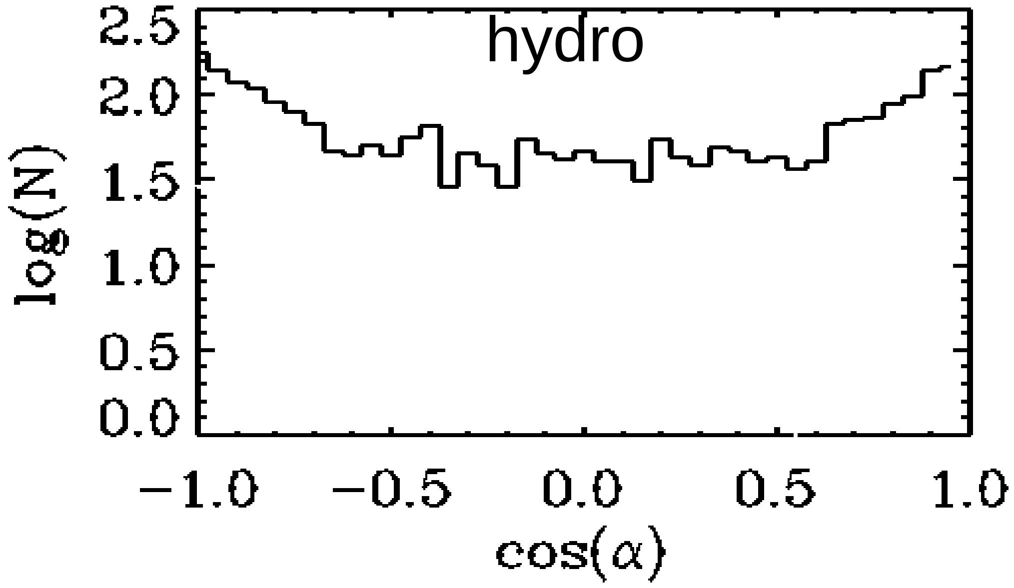
$$-2\varepsilon = \mathcal{S}(r) + \frac{1}{r^2}\partial_r(r^2 \mathcal{F}_r) \, ,$$

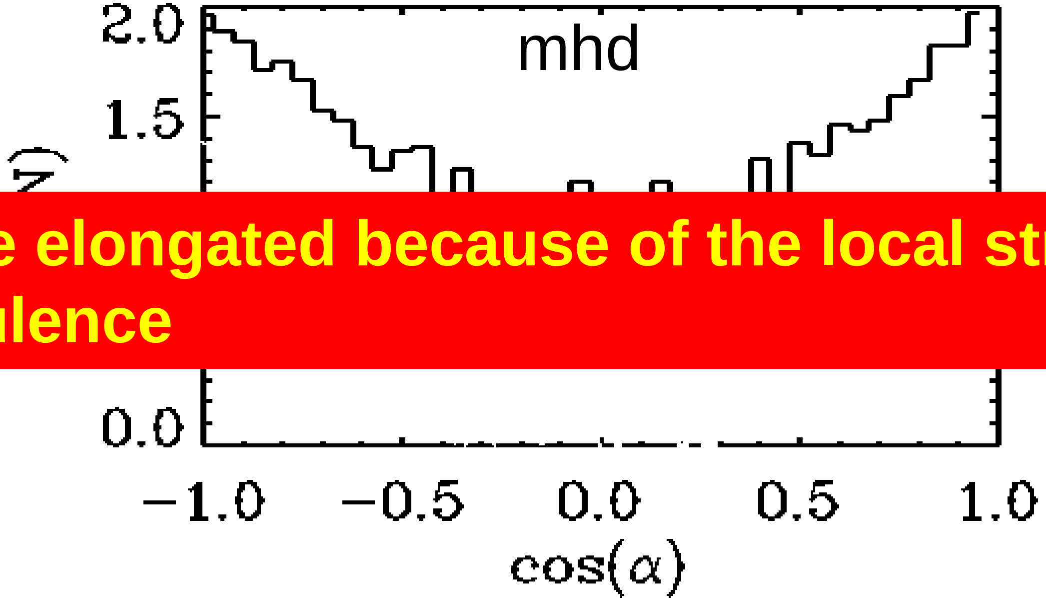
Galtier & Banerjee 2011

Compressibility appears as a source term.

Correlation between the principal axis of the clumps and the principal axis of the strain tensor







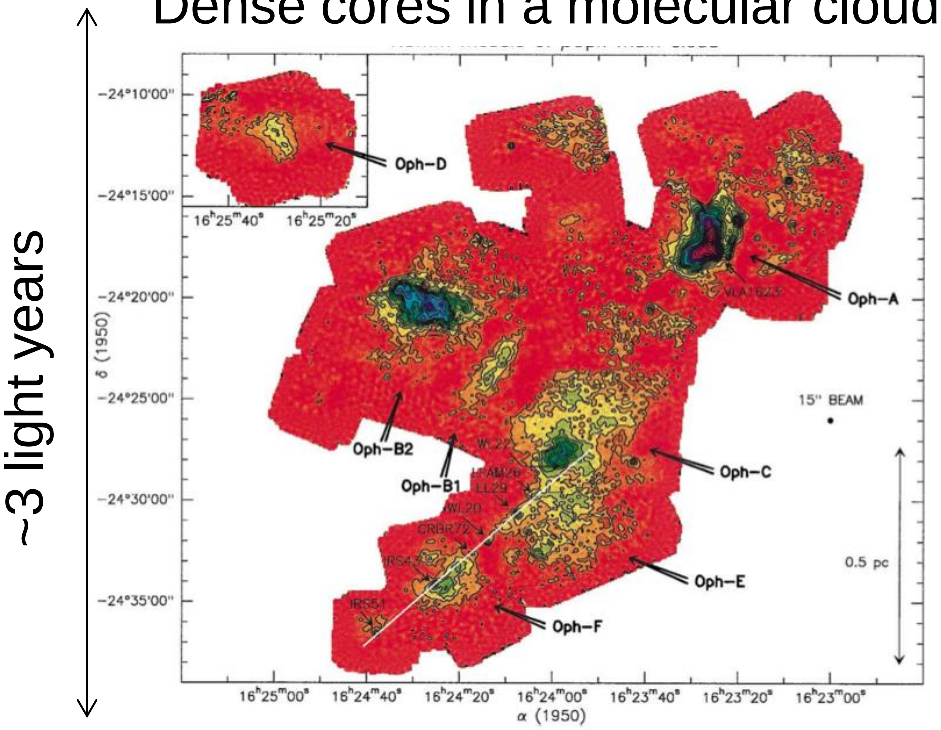
The filaments are elongated because of the local strain induced by turbulence

H2013

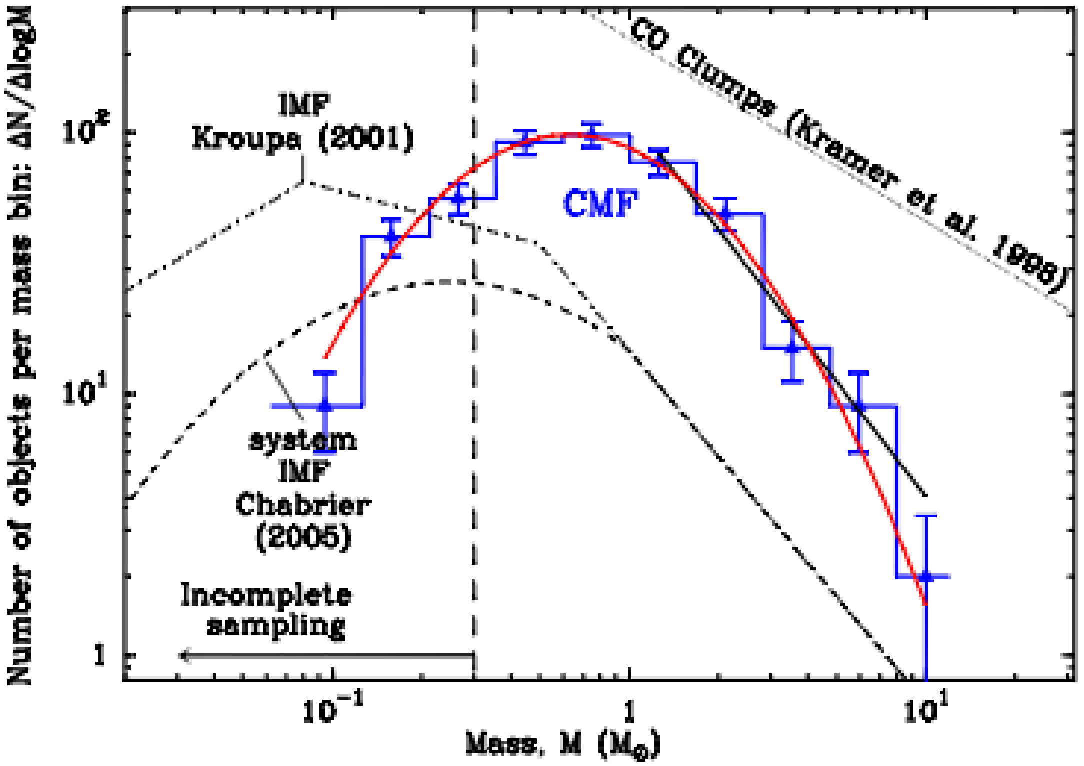
The Initial mass function and the Core Mass Function

(Motte et al. 1998, Testi & Sargent 1998, Alves et al. 2007, Johnstone et al. 2002, Enoch et al. 2008, Simpson et al. 2008)

Dense cores in a molecular cloud



Motte et al. 98

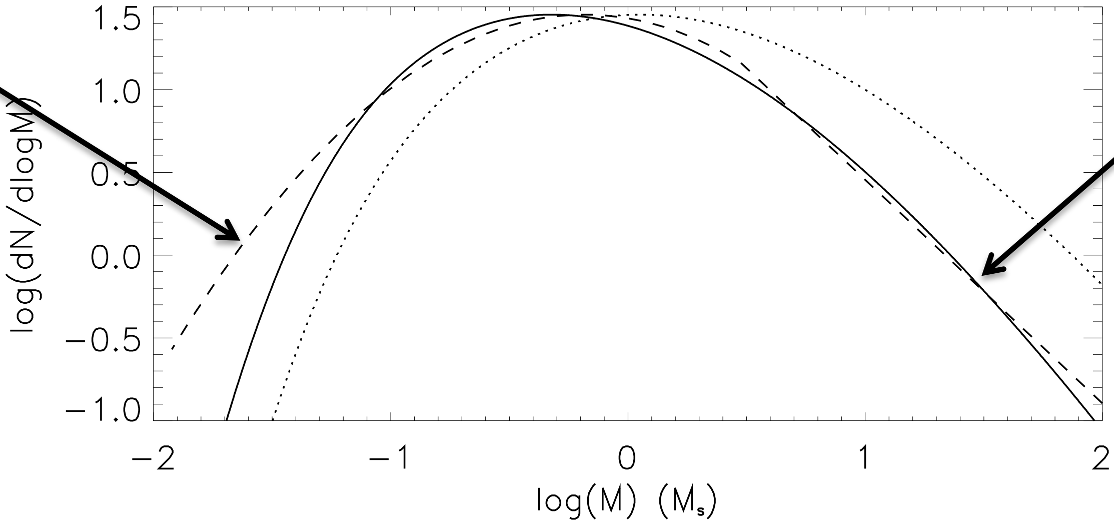


Konyves, André et al. 2010

The core mass function has a shape which is very similar to the IMF. This suggests that cores could constitute the mass reservoir of stars.

Comparison between CMF and Chabrier IMF

Chabrier's IMF

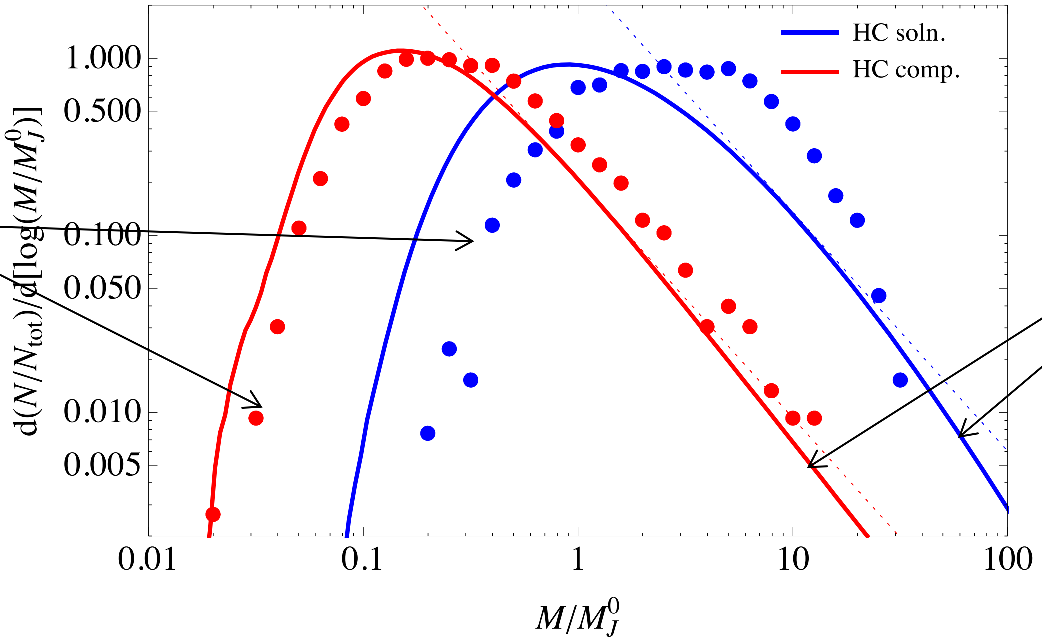


Analytical prediction

Hennebelle & Chabrier 2008, 2013

Comparison with high resolution numerical simulations

Simulation result



Analytical prediction

Schmidt et al. 2010