

QUANTUM FIELD THEORY: AN INTRODUCTION

J. ZINN-JUSTIN

IRFU/CEA, Paris-Saclay University, France

and

上海大学 | (*Shanghai University*)

*Email : jean.zinn-justin@cea.fr

Course delivered at IRFU/CEA, Paris-Saclay University, academic year
2015-2016

Contents

1 Random walk, continuum limit and renormalization group	1
1.1 Translation invariant local random walk	3
1.2 Universality: the renormalization group strategy	17
1.3 Brownian motion and path integral	31
Exercises	38
2 The essential role of functional integrals in theoretical physics	41
2.1 Classical equations: The mysterious variational principle	44
2.2 Relativistic quantum field theory: unitarity and covariance	54
2.3 The non-linear σ -model	57
2.4 Quantum field theory and lattice regularization	59
2.3 Lattice gauge theories and numerical simulations	63
2.4 Instantons, barrier penetration and vacuum instability	64

2.5 Instantons, large order behaviour and the problem of Borel summability	65
2.3 Relation between classical and quantum statistical physics . .	68
2.4 Finite temperature QFT, finite size effects in statistical field theory and dimensional reduction	70
2.5 Quantization of non-Abelian gauge theories	74
3 Path integrals in quantum mechanics	81
3.1 Position, momentum operators and matrix elements	85
3.2 Markov process	88
3.3 Locality of short-time evolution	92
3.4 Quantum partition function	107
3.5 Explicit calculation: Gaussian path integrals	110
3.6 Correlation functions: generating functional	119
3.7 Correlation functions and time-ordered products	127
3.8 General Gaussian path integral and correlation functions . .	133

3.9 Quantum harmonic oscillator: the partition function	144
3.10 Perturbed harmonic oscillator	146
3.11 Quantum evolution and Scattering matrix	153
3.12 Path integral representation: Evolution and S -matrix . . .	158
Exercises	169
4 Bosons in the holomorphic representation	174
4.1 Formal complex integration and Wick's theorem	177
4.2 Perturbative expansion	188
4.3 Quantum mechanics in the holomorphic representation . . .	193
4.4 Bosons in the second quantization formalism	198
4.5 Operators: kernel representation	201
4.6 Path integral: the harmonic oscillator	211
4.7 Path integral: general Hamiltonians	225
4.8 Several complex variables	229
4.9 Boson states and Hamiltonian in second quantization	232

4.10 Quantum statistical physics: the partition function	241
4.11 S -matrix and holomorphic formalism	246
Exercises	255
5 Bosons: from non-relativistic to relativistic quantum field theory	272
5.1 Bose–Einstein condensation	275
5.2 Generalized path integrals: the quantum Bose gas	285
5.3 Partition function: the field integral representation	293
5.4 The Helium superfluid transition	301
5.5 The Bose gas: time evolution	308
5.6 The relativistic neutral scalar field	312
5.7 The free field	318
5.8 Free field theory and the holomorphic formalism	324
5.9 The S -matrix: generalities	333
5.10 S -matrix and field asymptotic conditions	342
5.11 Two-point function: spectral representation and field renormaliza-	

tion	357
5.12 The non-relativistic limit	368
6 The scalar field: formal perturbation theory	373
6.1 Correlation functions and generating functionals	375
6.2 Gaussian or free field theory. Wick's theorem	382
6.3 Perturbative expansion	389
6.4 Feynman diagrams: loops	394
6.5 Example: the ϕ^4 interaction	402
6.6 Loop expansion	411
6.7 Divergences in perturbation theory	422
6.8 Divergences: general analysis and power counting	434
6.9 Classification of renormalizable field theories	449
6.10 S -matrix and correlation functions	458
7 From QED to the general renormalization group	467
7.1 A brief history	470

7.2 QED a local quantum field theory	474
7.3 QED: the problem of infinities	478
7.4 Renormalization procedure	483
7.5 The nature of divergences and the meaning of renormalization	487
7.6 QFT and renormalization group	489
7.7 The triumph of renormalizable QFT: the Standard Model . .	492
7.8 Critical phenomena: other infinities	495
7.9 Scale decoupling in physics	498
7.10 Effective quantum field theories	510
8 The scalar field: renormalization and renormalization group . .	516
8.1 The notion of effective field theory	523
8.2 Rescaling, Gaussian renormalization and power counting . .	531
8.3 Renormalization group and Gaussian fixed point	540
8.4 Renormalization theorem and renormalization group	548
8.5 Renormalization group equations for the massless theory . .	556

8.6	Renormalized correlation functions and RG	562
8.7	Solution of RG equations	565
8.8	RG functions at one-loop order and triviality of ϕ_4^4	570
8.9	The massive ϕ^4 theory in the symmetric phase	579
8.10	An early RG upper-bound on the Higgs particle mass	588
9	Dimensional continuation: application to critical phenomena	598
9.1	Dimensional continuation	600
9.2	Dimensional regularization	608
9.3	The minimal subtraction scheme	624
9.4	The ϕ^4 field theory near and below dimension four	630
9.5	Calculations to one-loop order: fixed-point and scaling	643
10	Fermions in the Grassmann formalism	655
10.1	Grassmann algebras	657
10.2	Differentiation in Grassmann algebras	662
10.3	Integration in Grassmann algebras	666

10.4	Gaussian integrals. Expectation values. Wick's theorem	674
10.5	Perturbative expansion: a quartic perturbation	685
10.6	Generating functions	691
10.7	Fermion vector space and operators: one state	698
10.8	General Grassmann analytic functions	707
10.9	Operators and kernels	711
10.10	The one-state example	718
10.11	Many-fermion states. Hamiltonians	720
10.12	Second quantization representation	723
10.13	Grassmann path integral: one-state problem	730
10.14	Grassmann path integrals: generalization	742
	Exercises	752
11	From non-relativistic to relativistic fermions	766
11.1	Fermion states: Hilbert space	768
11.2	Partition function: the field integral	774

11.3 Fermi gas: evolution operator	781
11.4 Relativistic fermions	783
11.5 Chiral symmetry and massless fermions	798
11.6 Interacting theory and S -matrix	801
11.7 Massive fermions: the non-relativistic limit	803
11.8 Free euclidean relativistic fermions	809
11.9 Partition function. Correlations	819
11.10 Generating functionals	828
11.11 Connection between spin and statistics	830
11.12 The Gross–Neveu–Yukawa model: a Higgs-top toy model .	836
11.13 An effective field theory: the Gross–Neveu model	850
A11 Euclidean Dirac fermions, spin group and γ matrices	869
A11.1 Orthogonal and spin groups	870
A11.2 The γ matrices: a hermitian representation	882
A11.3 The Fierz transformation	899

Exercises	902
12 Symmetries and symmetry breaking	910
12.1 Algebraic preliminaries	914
12.2 Linear global symmetries	922
12.3 Linear symmetry breaking	933
12.4 Spontaneous symmetry breaking	944
12.5 Approximate chiral symmetry in hadron physics	956
12.6 An example: the linear σ -model	963
12.7 Ward–Takahashi identities	974
12.8 $(\phi^2)^2$ field theory and non-linear σ -model	984
12.9 Non-linear σ -model: perturbation theory, power counting	992
12.10 RG analysis at and above two dimensions	1005
A12 Currents and Noether’s theorem	1010
A12.1 Currents in classical field theory	1011
A12.2 Euclidean quantum field theory	1015

A12.3	The energy–momentum tensor	1018
A12.4	Energy–momentum tensor and euclidean field theory . .	1024
A12.5	Dilatation and conformal invariance	1026
13	Abelian gauge theories: the framework of QED	1032
13.1	The free massive vector field	1035
13.2	Interaction with fermion matter	1049
13.3	Massless vector field: Abelian gauge symmetry	1054
13.4	Canonical quantization: non-covariant gauges	1062
13.5	Equivalence with covariant gauges	1073
13.6	Perturbation theory: regularization	1081
13.7	WT identities	1093
13.8	Renormalization	1097
13.9	Gauge dependence: the fermion two-point function . . .	1100
13.10	Renormalization group equations	1110
13.11	The one-loop β -function: charged fermions	1113

13.12	The Abelian Higgs model	1118
13.13	Quantization of the Abelian Higgs model	1123
13.14	Physical observables. Unitarity of the S -Matrix	1133
14	Non-Abelian gauge theories: the basis of the Standard Model	1135
14.1	Parallel transport and gauge invariance	1137
14.2	Gauge invariant action	1147
14.3	Hamiltonian formalism. Quantization	1150
14.4	Perturbation theory, regularization	1170
14.5	The non-Abelian Higgs mechanism	1178
A14	BRST symmetry	1193
A14.1	BRST symmetry: the origin	1193
15	The Standard Model of fundamental interactions	1205
15.1	Weak–electromagnetic interactions: gauge and Higgs fields	1210
15.2	Leptons: minimal SM extension with Dirac neutrinos	1219
15.3	Quarks and weak–electromagnetic interactions	1229

15.4 Quantum ChromoDynamics: renormalization group . . .	1241
15.5 The Abelian anomaly	1258
15.6 Non-Abelian anomaly	1287
15.7 Physics implications	1293
16 Lattice gauge theories: an introduction	1301
16.1 Gauge invariance on the lattice	1304
16.2 The pure gauge theory	1310
16.3 Wilson's loop and confinement	1318
16.4 Mean field approximation	1338