Institute of Research into the
Fundamental Laws of the Universe
Astrophysics Division

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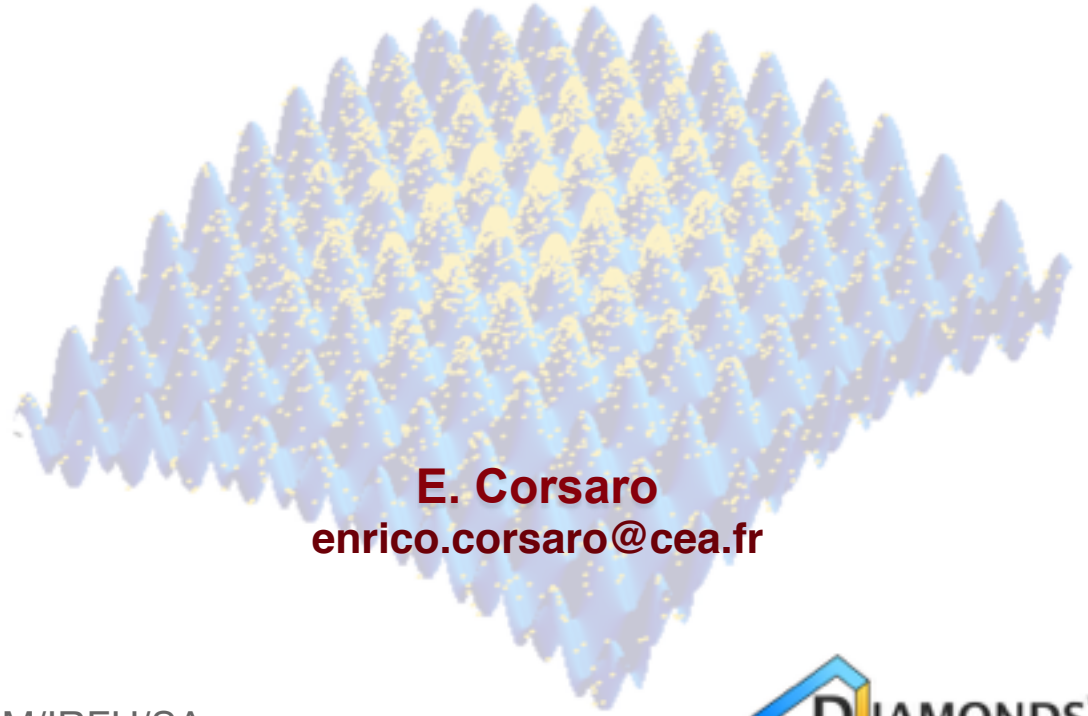
Space-Inn School on Astero/Helioseismology
and Stellar/Solar Physics

Bayesian vs. frequentist statistics and asteroseismic data analysis



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DSM/IRFU/SAp
27 OCTOBER 2015

1. - BAYESIAN STATISTICS

2. - NESTED SAMPLING MONTE CARLO

3. - THE DIAMONDS CODE

4. - FITTING A PSD: BACKGROUND COMPONENTS

5. - FITTING A PSD: THE OSCILLATION MODES

The background model equation for a *Kepler* star

$$H_{\rm osc} \exp\left[-\frac{(\nu-\nu_{\rm max})^2}{2\sigma_{\rm env}^2}\right] \qquad \sum_{i=1}^{3} \frac{\xi a_i^2/b_i}{1+(\nu/b_i)^4}$$

$G(\nu)$ Gaussian envelope oscillations

$B(\nu)$ super-Lorentzian profiles

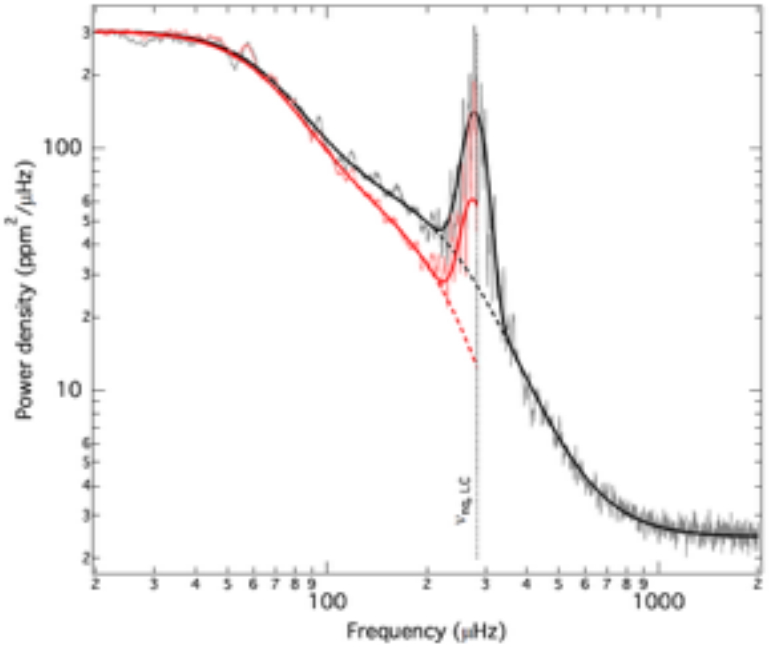
W Photon noise (flat)

The background model equation

$$P_{\rm bkg}(\nu) = [G(\nu) + B(\nu)]R(\nu) + W$$

$$R(\nu) = \sin^2\left(\frac{\pi\nu}{2\nu_{\rm Nyq}}\right)$$

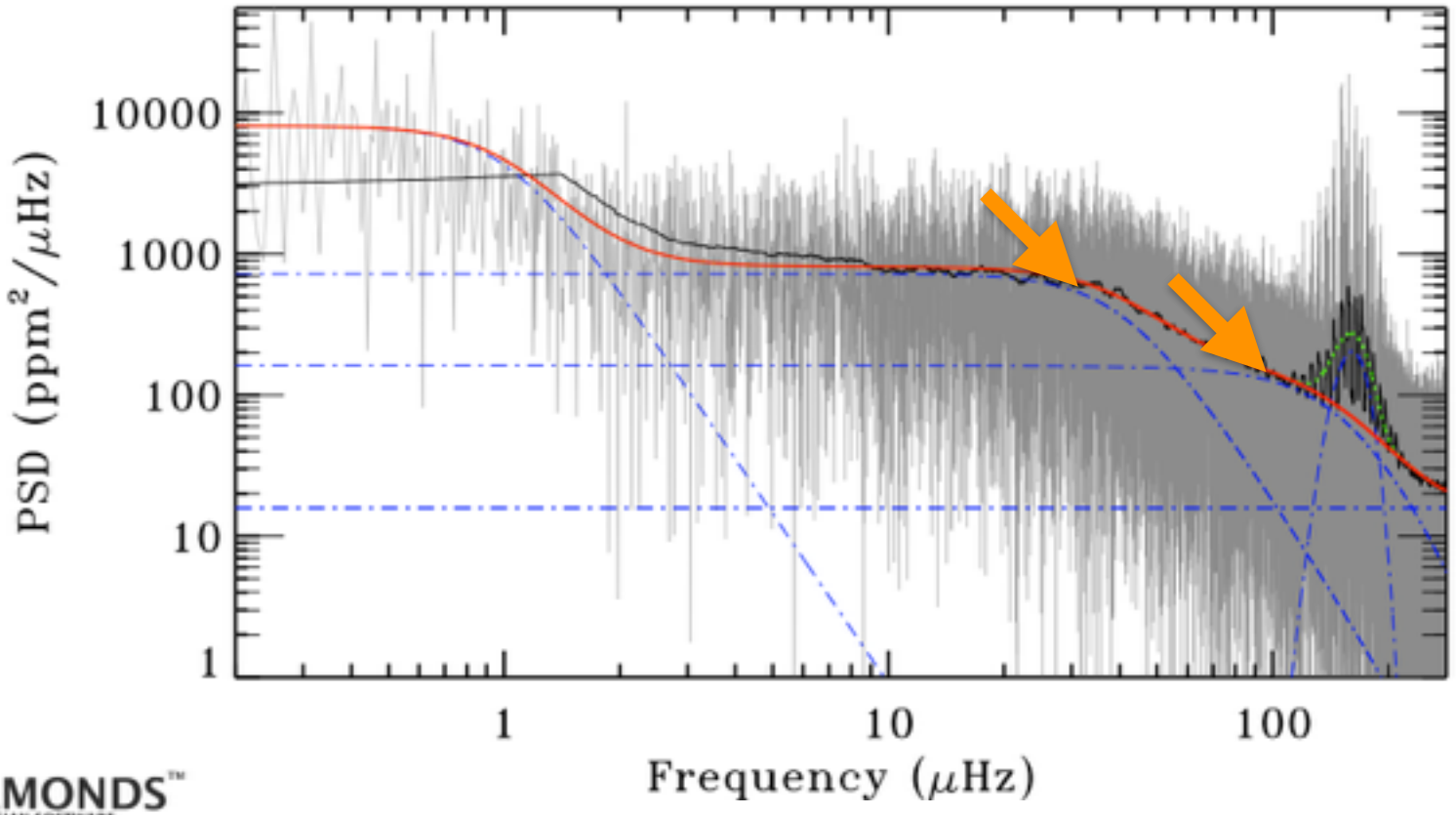
Apodization - Decrease of the signal for discrete time observations



Kallinger et al. 2014

The background model fit for a *Kepler* star

Corsaro, De Ridder, García 2015 A&A, 579, 83



DIAMONDS™ BAYESIAN SOFTWARE

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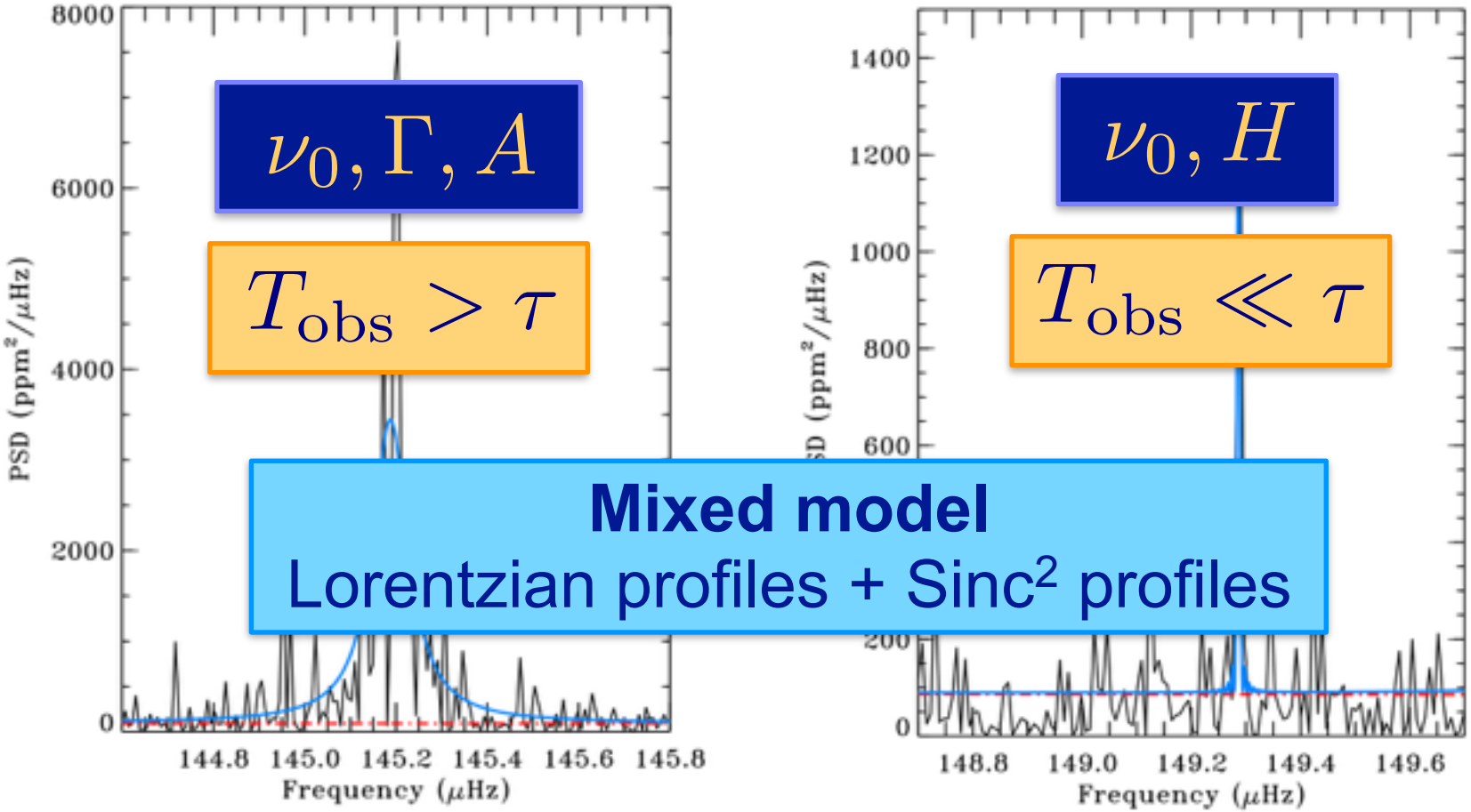
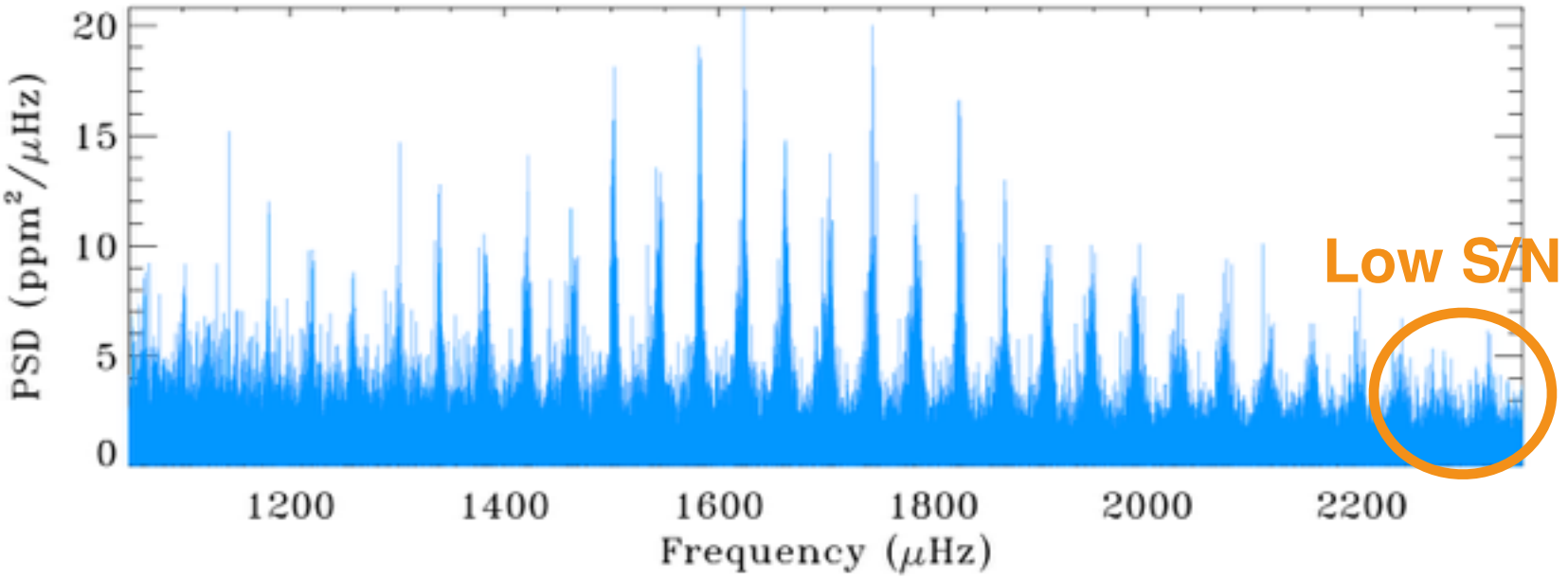
The Lorentzian and sinc^2 profiles



Mixed model
Lorentzian profiles + Sinc^2 profiles

Problems in the fitting of the oscillations



- **Problem 1**: big dataset + fitting numerous oscillation modes (peaks) per star (can be more than 100)

- **Problem 2**: testing if a peak is real or not (noise)

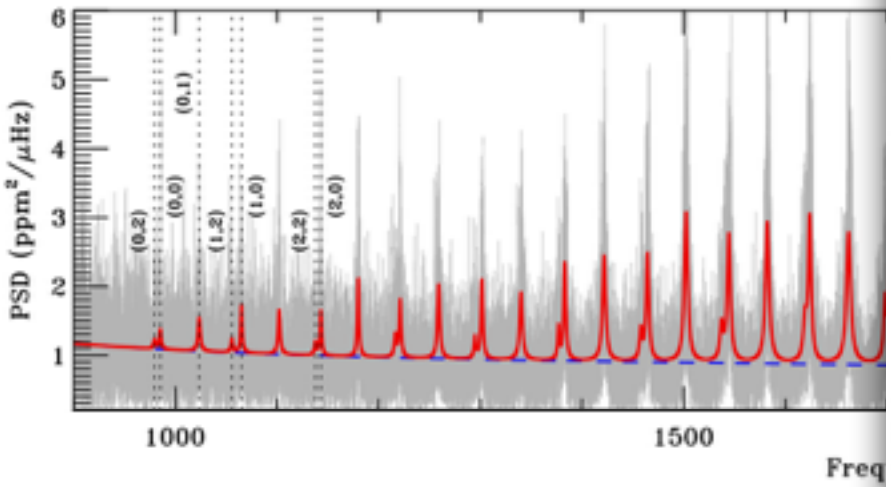
Problem 1

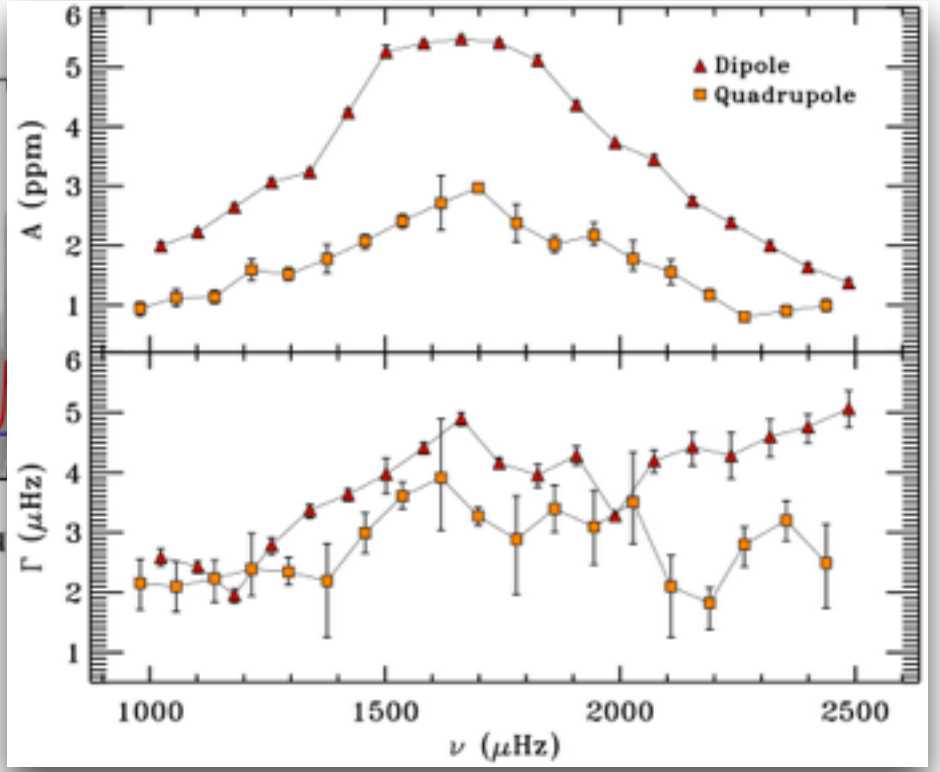
Solving a high-dimensional fitting problem

High-dimensional Model

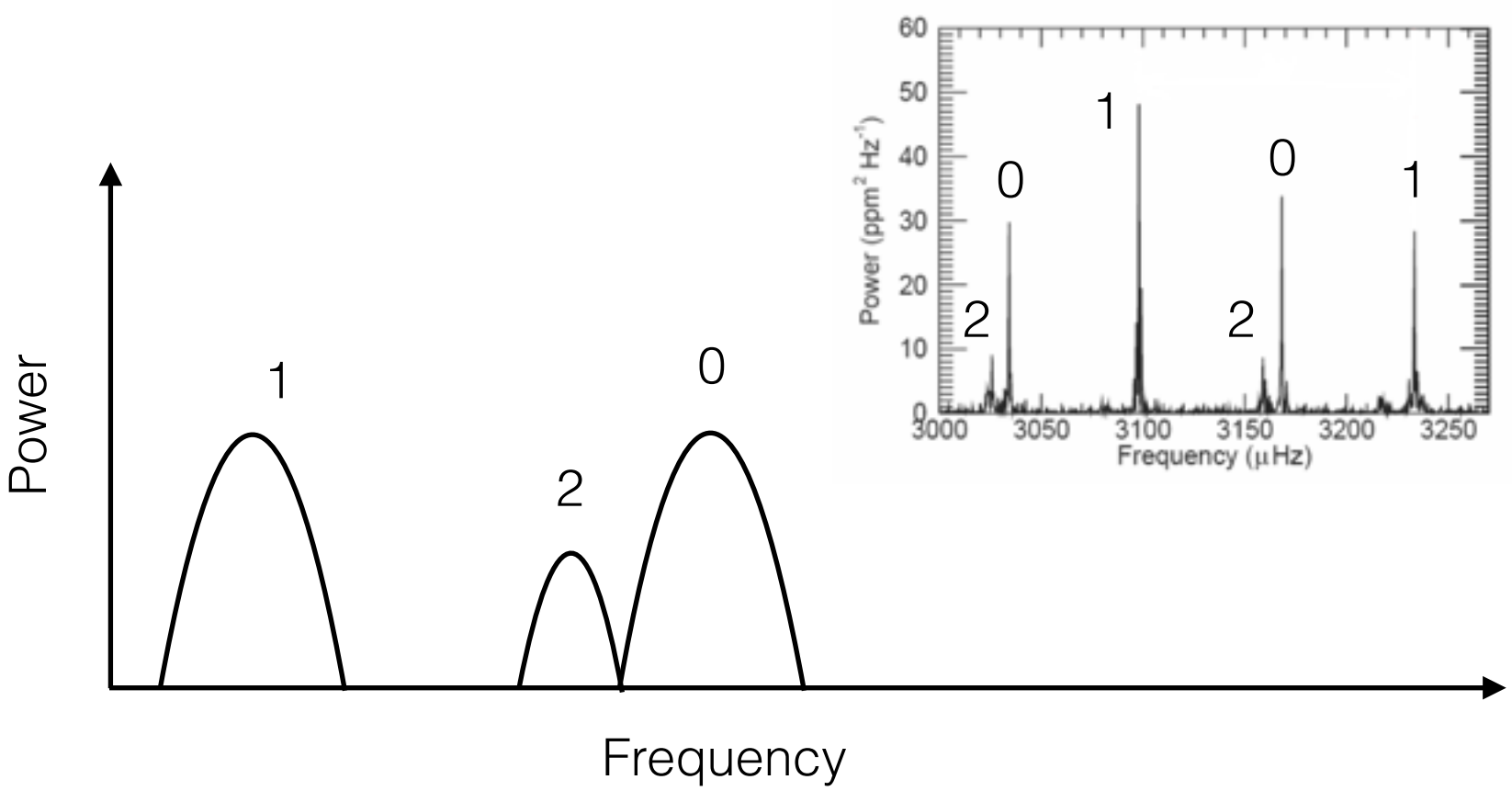
About 180 free parameters!
Computational time increases a lot

KIC 9139163

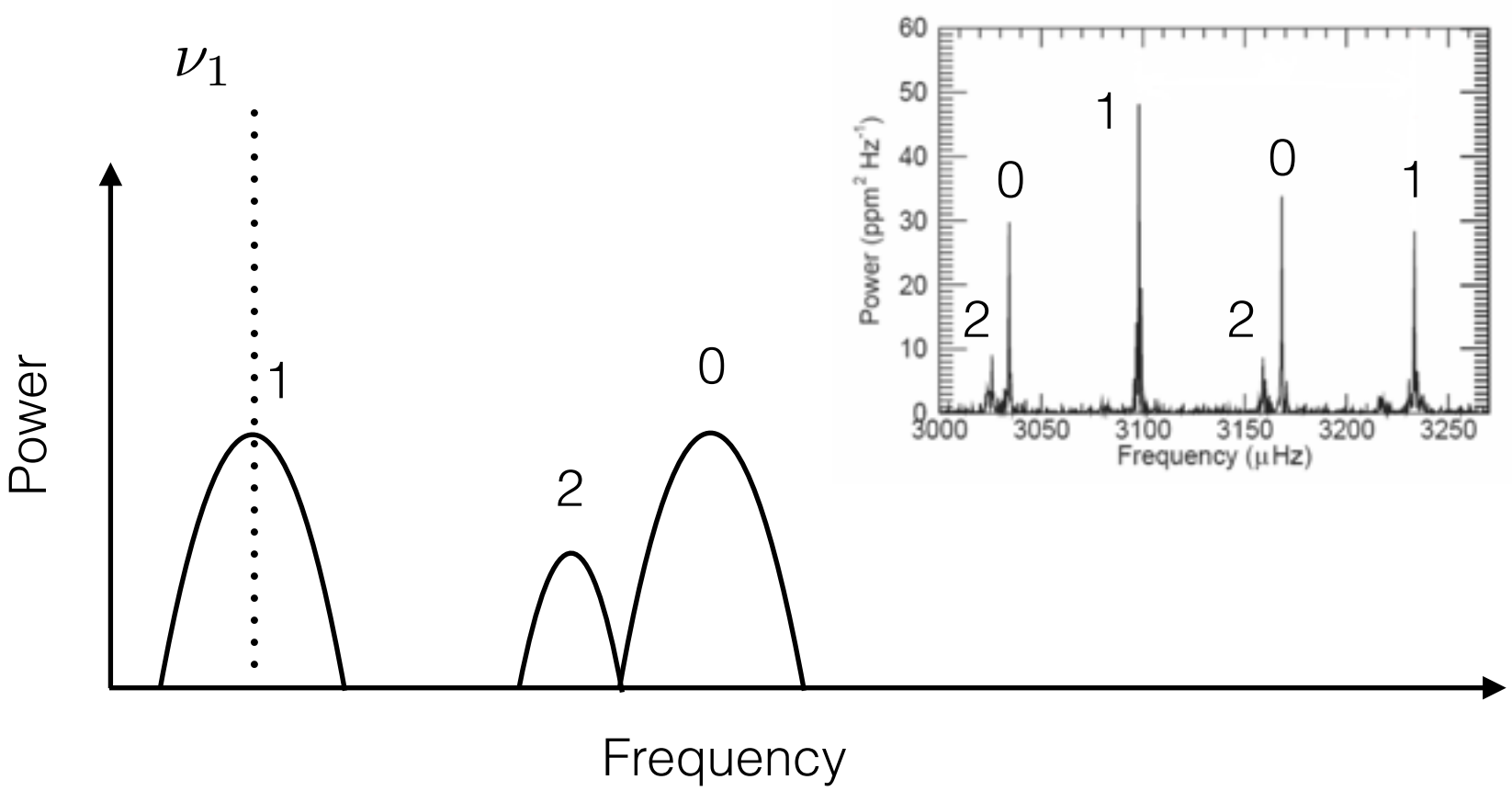




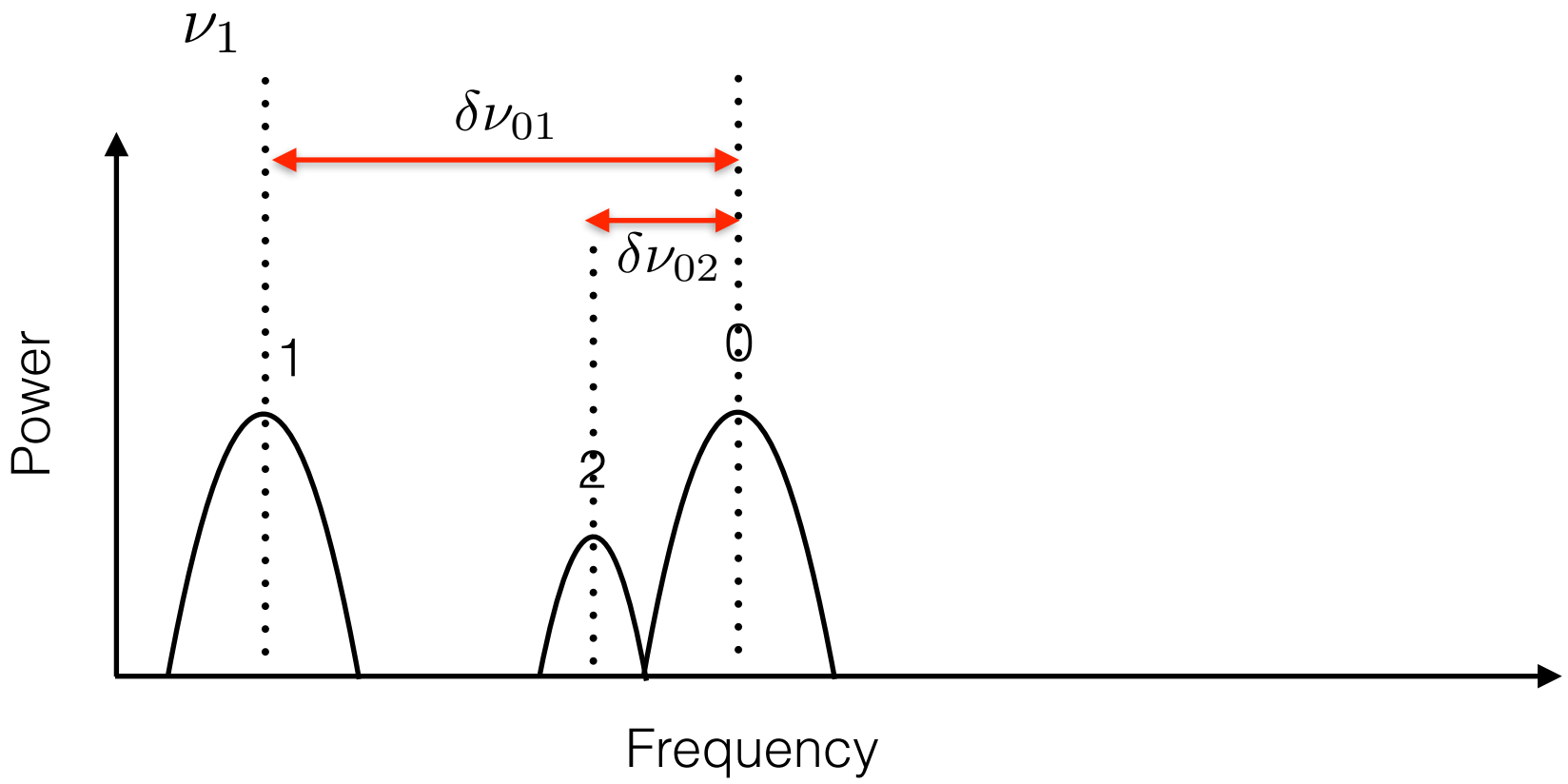
Multi-modal Model



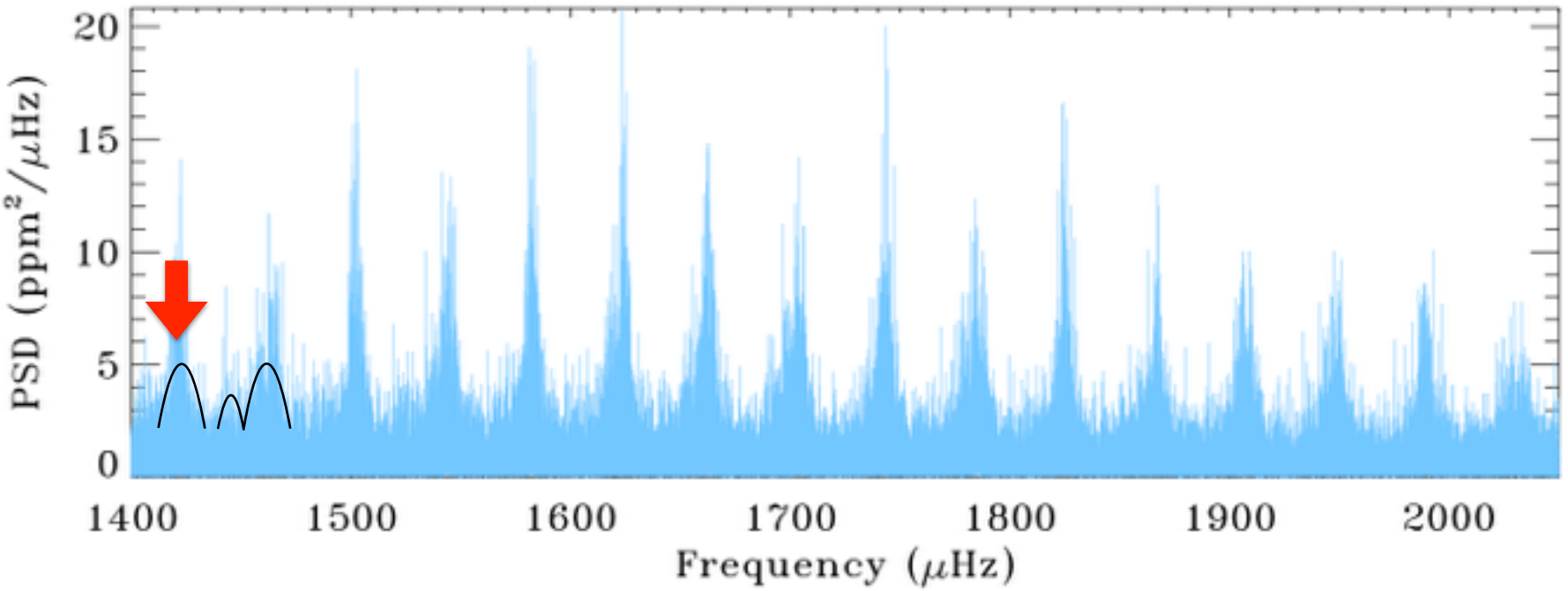
Multi-modal Model



Multi-modal Model



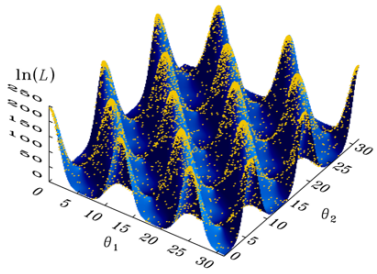
Multi-modal Model



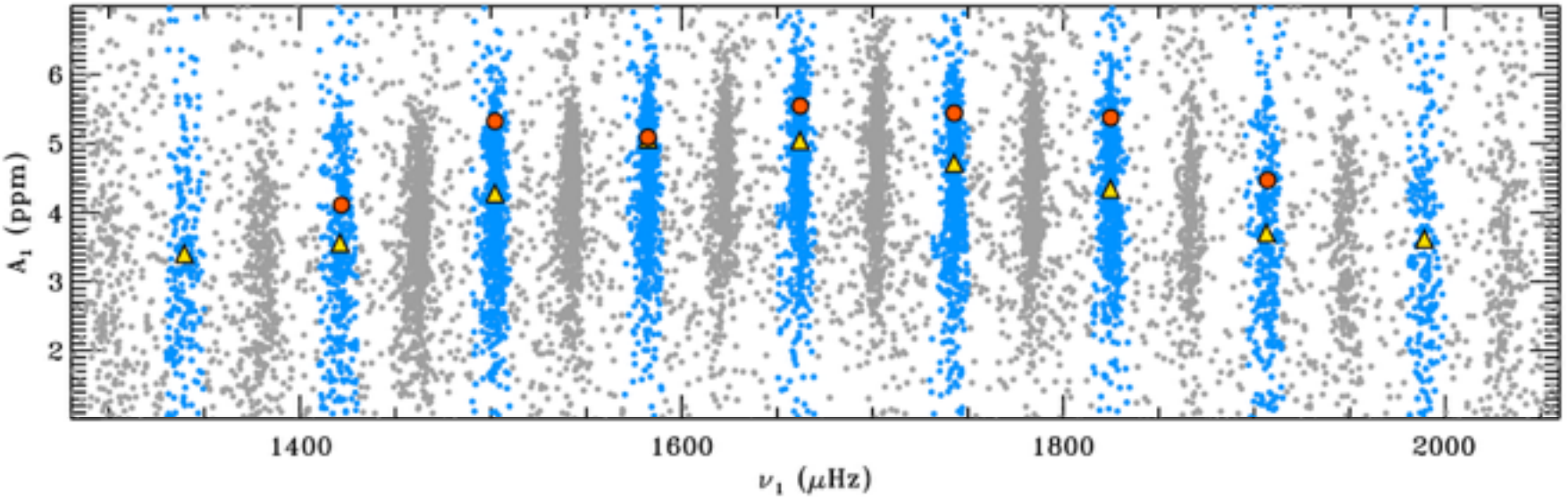
Corsaro & De Ridder 2014 A&A, 571, 71

Results

Multi-modal inference problem on 9 consecutive radial orders (27 peaks)



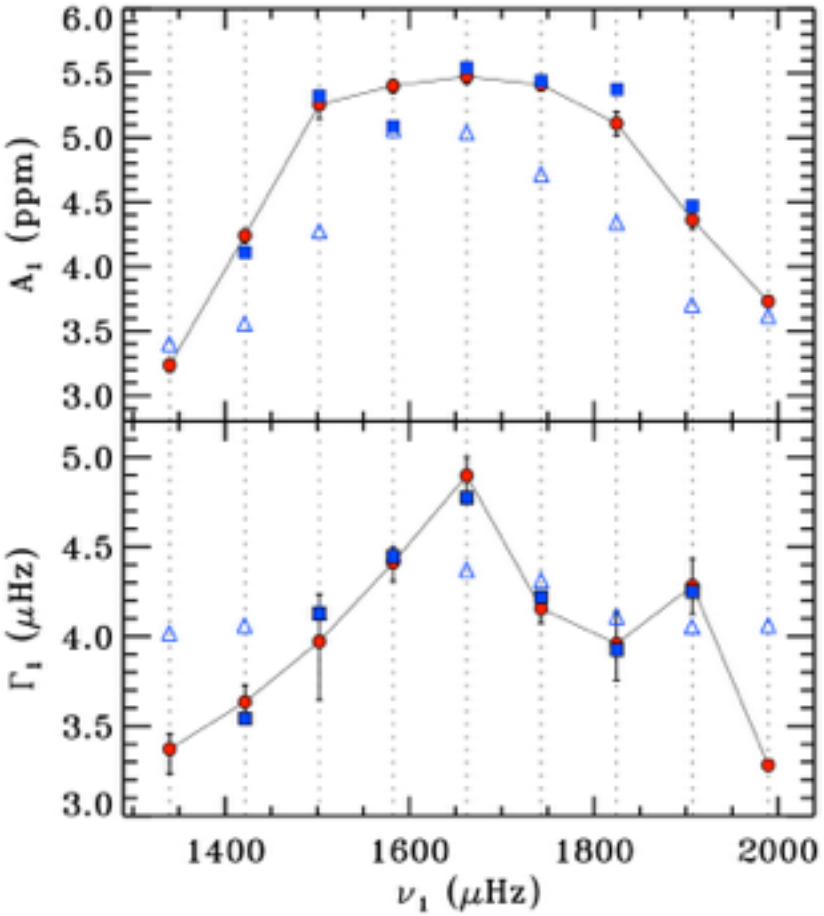
Only 9 free parameters!



Corsaro & De Ridder 2014 A&A, 571, 71

Comparison

Red: uni-modal fit
Blue: multi-modal fit



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Comparison

Red: uni-modal fit
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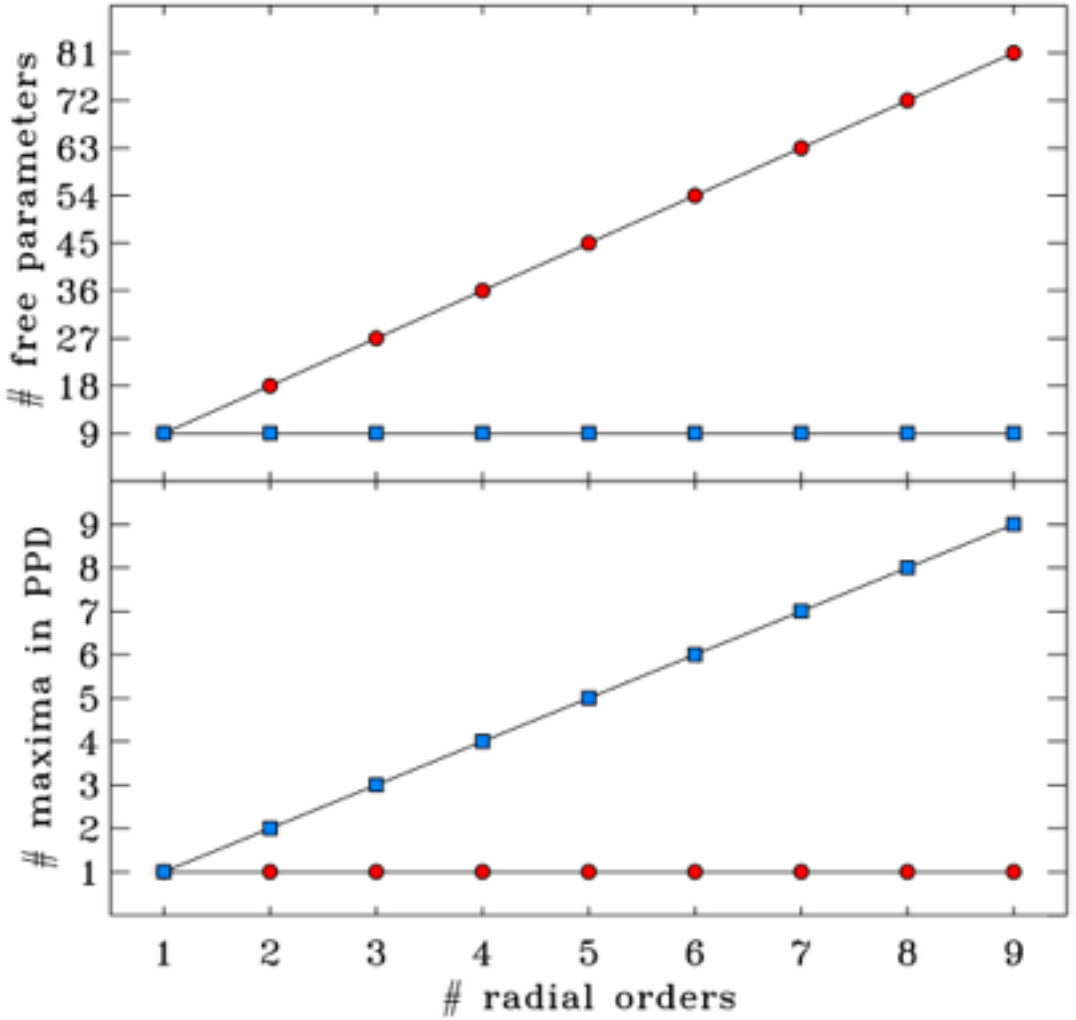


Corsaro & De Ridder 2014 A&A, 571, 71

Problem 2

Test the significance of an oscillation peak

Bayesian Model Comparison

Bayesian Evidence

$\mathcal{E}$ ↑ Fit quality

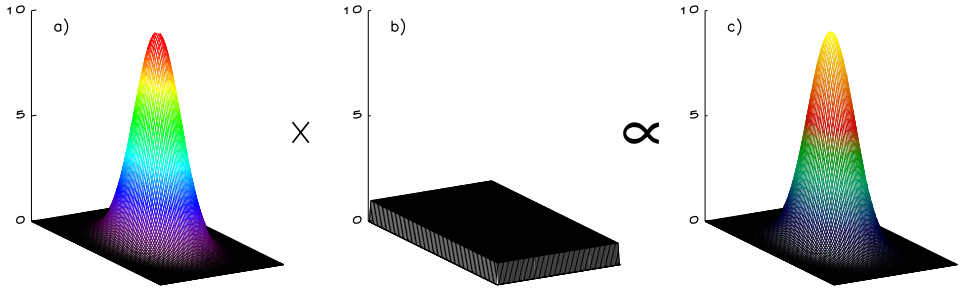
$\mathcal{E}$ ↓ # free parameters

WEIGHT: simple models are preferred (Occam's razor)

Bayes' Theorem

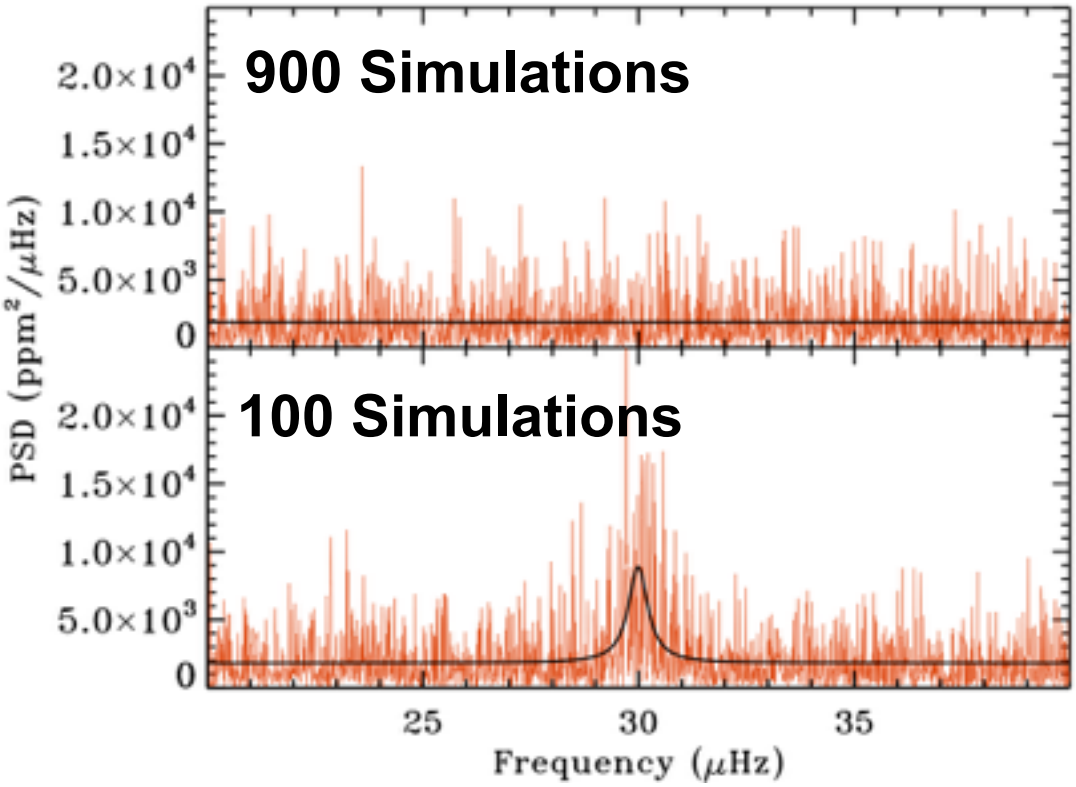
$$p(\boldsymbol{\theta}) = \frac{\mathcal{L}(\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})}{\mathcal{E}}$$

Likelihood **Prior** **Posterior**



Peak Significance Criterion

- Simulations test
- **1000** artificial chunks of PSD
- Blind search for those with a peak



Bayes' factor

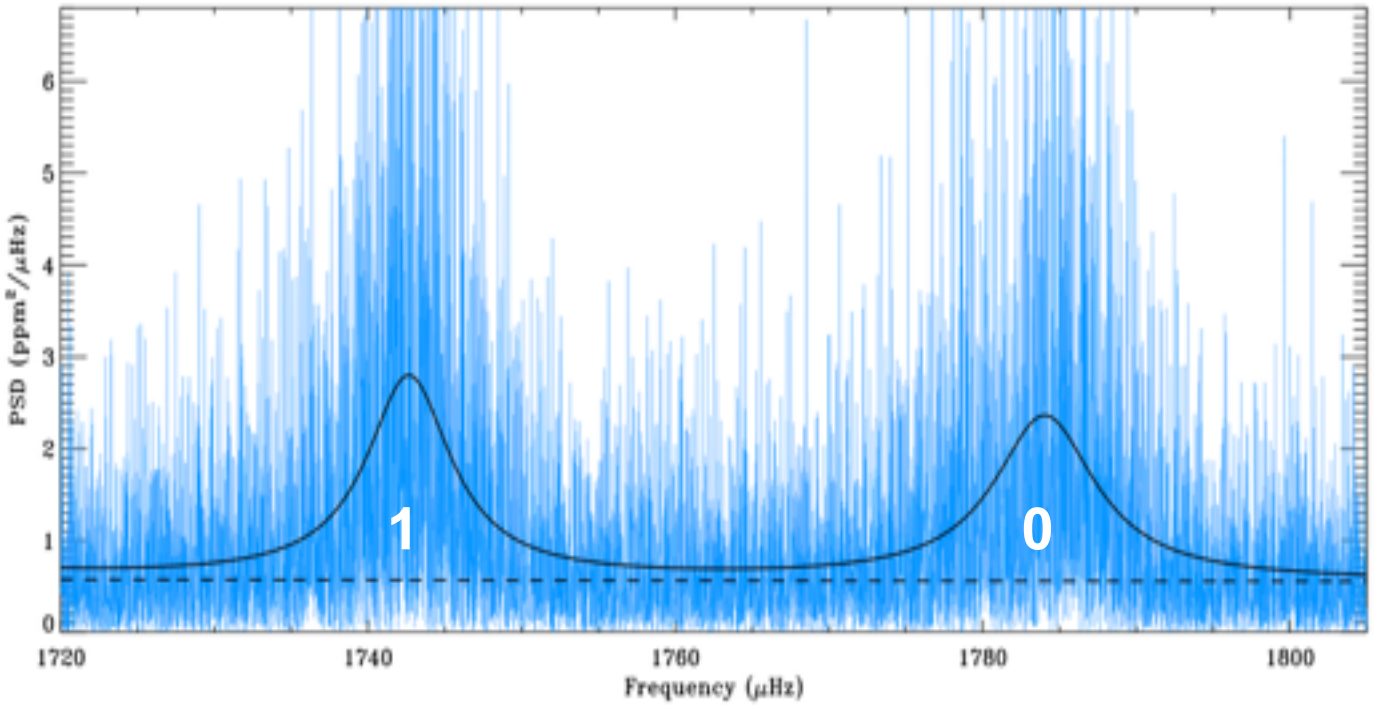
$$B_{yes,no} = \frac{\mathcal{E}_{yes}}{\mathcal{E}_{no}}$$

$$B_{yes,no} \sim 150$$

Strong Evidence (Jeffreys' scale)

All peaks found!

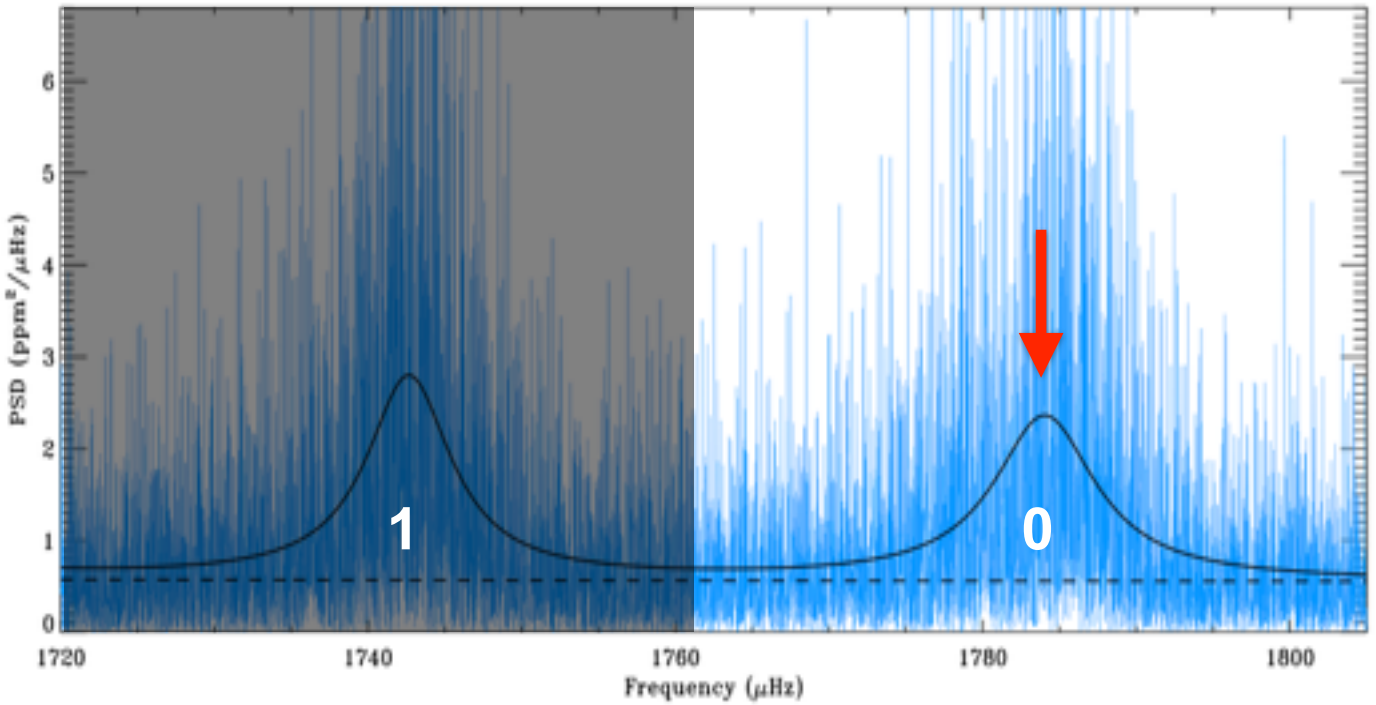
Peak significance



Peak significance

$\mathcal{M}_{\ell=0}$ **Only ℓ = 0**

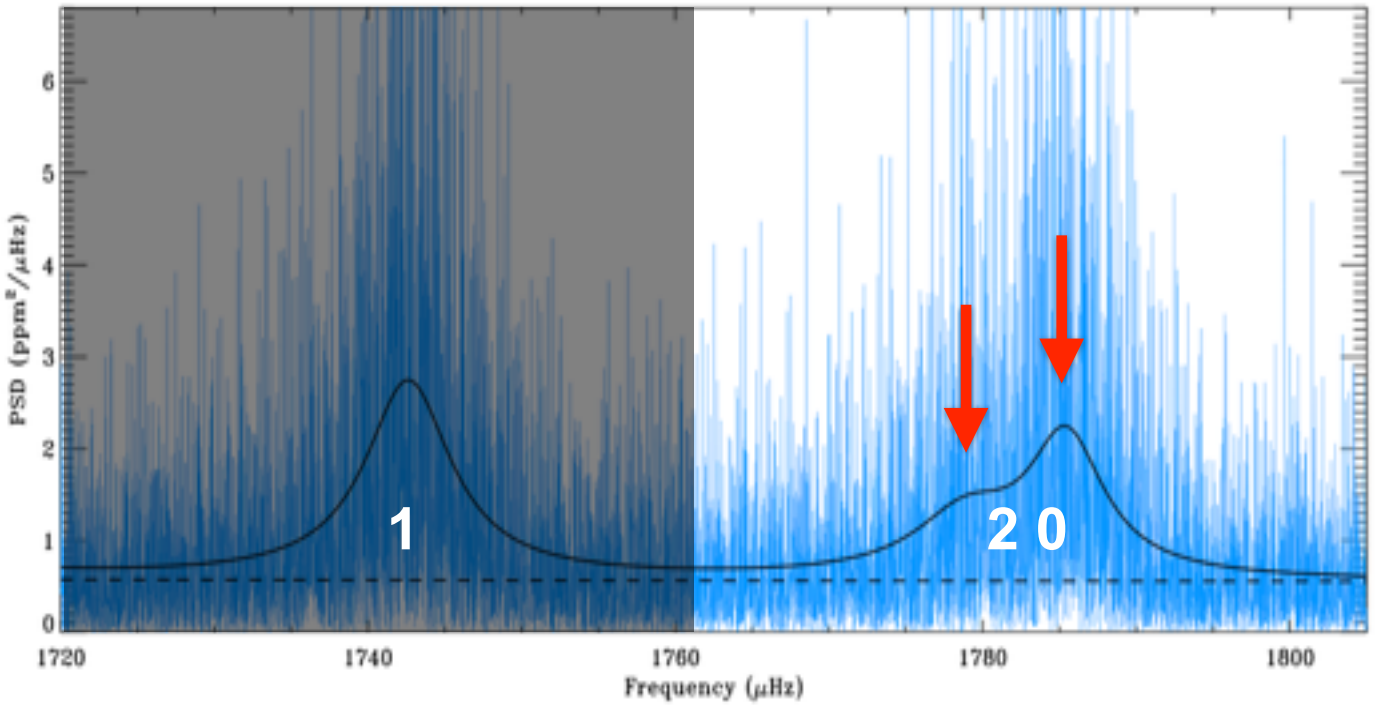
$\mathcal{E}_{\ell=0}$ **Bayesian Evidence**



Peak significance

$\mathcal{M}_{\ell=2}$ **Both ℓ = 2 and ℓ = 0**

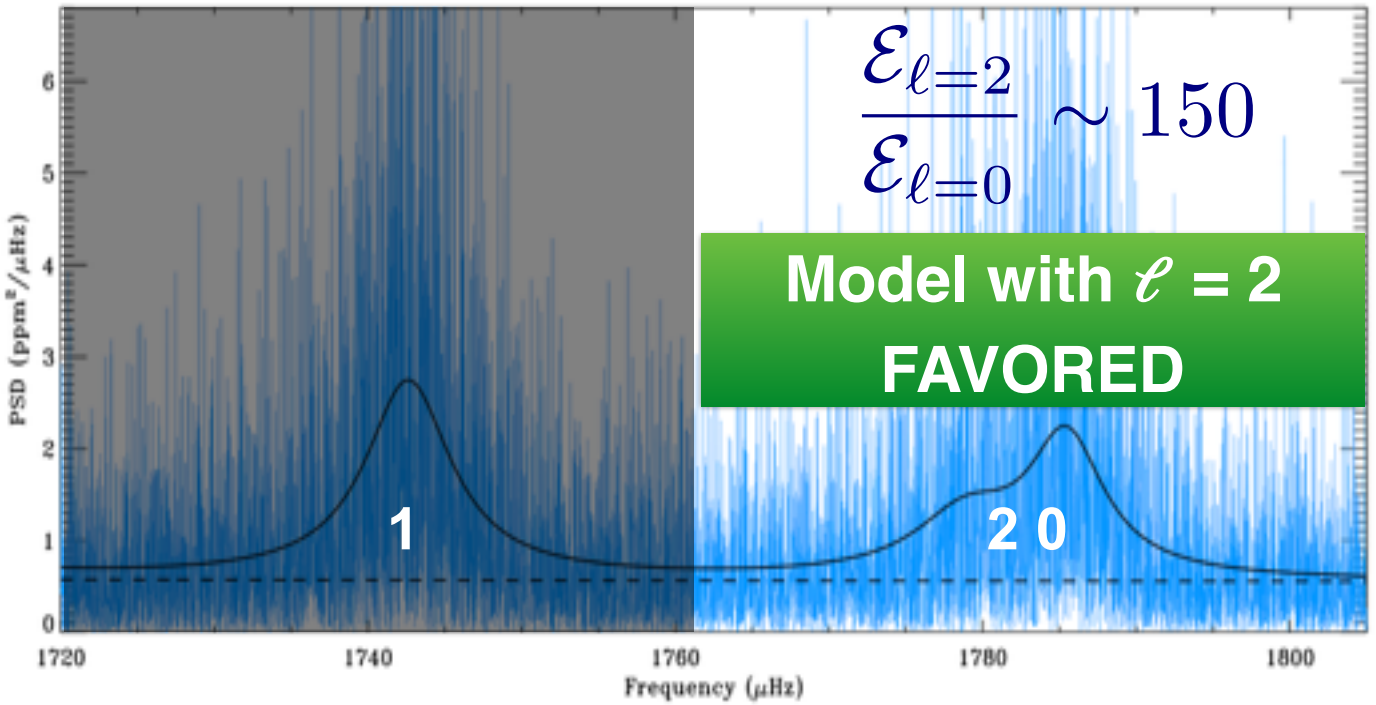
$\mathcal{E}_{\ell=2}$ **Bayesian Evidence**



Peak significance

$\mathcal{M}_{\ell=2}$ **Both ℓ = 2 and ℓ = 0**

$\mathcal{E}_{\ell=2}$ **Bayesian Evidence**



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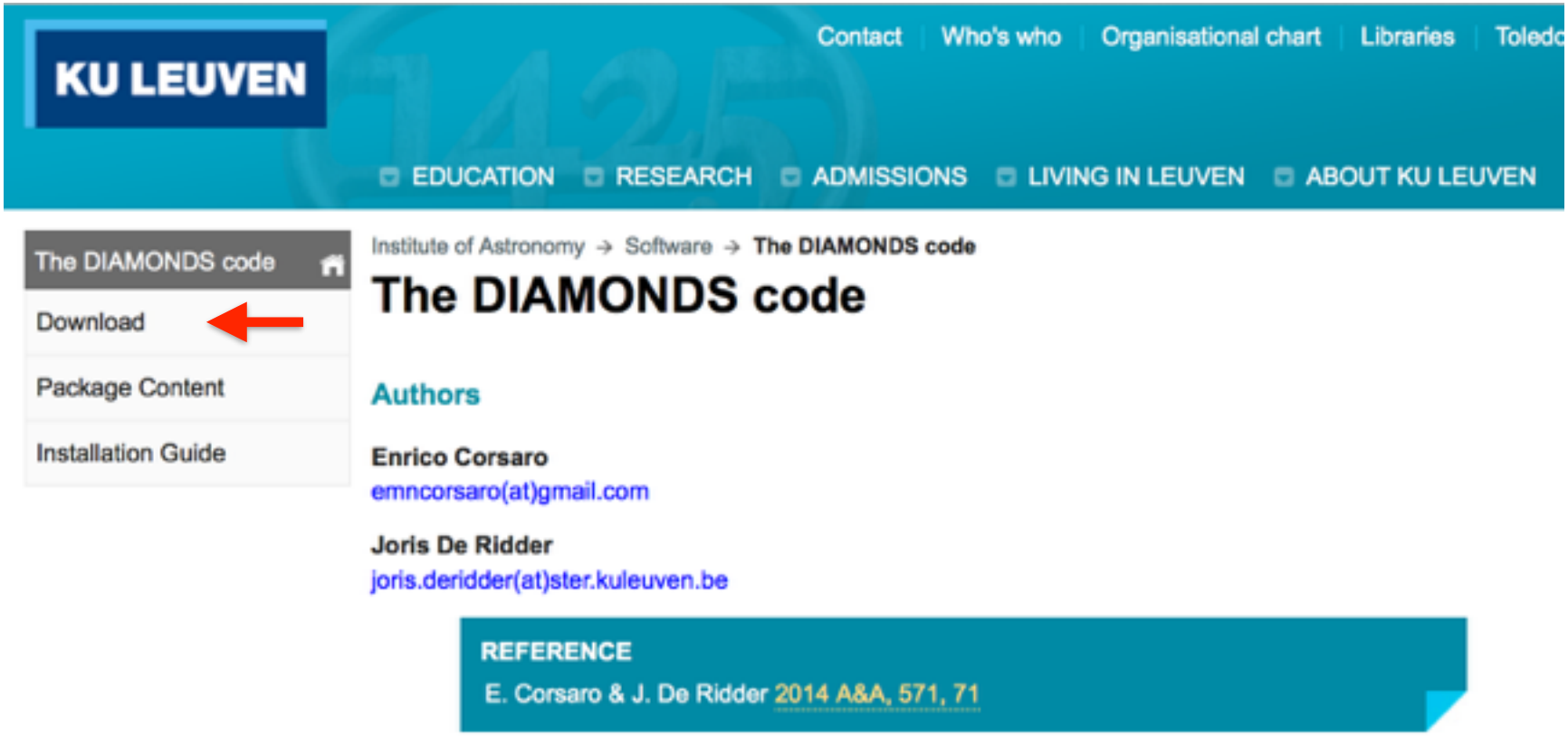

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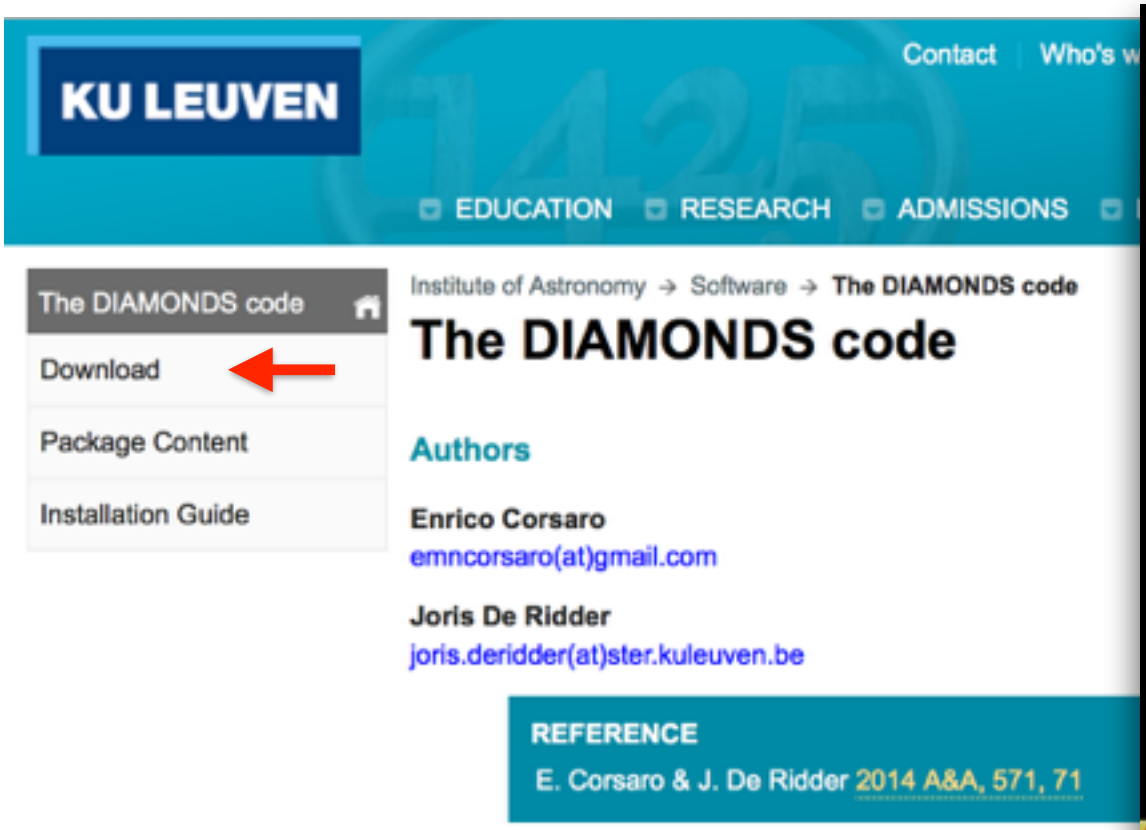
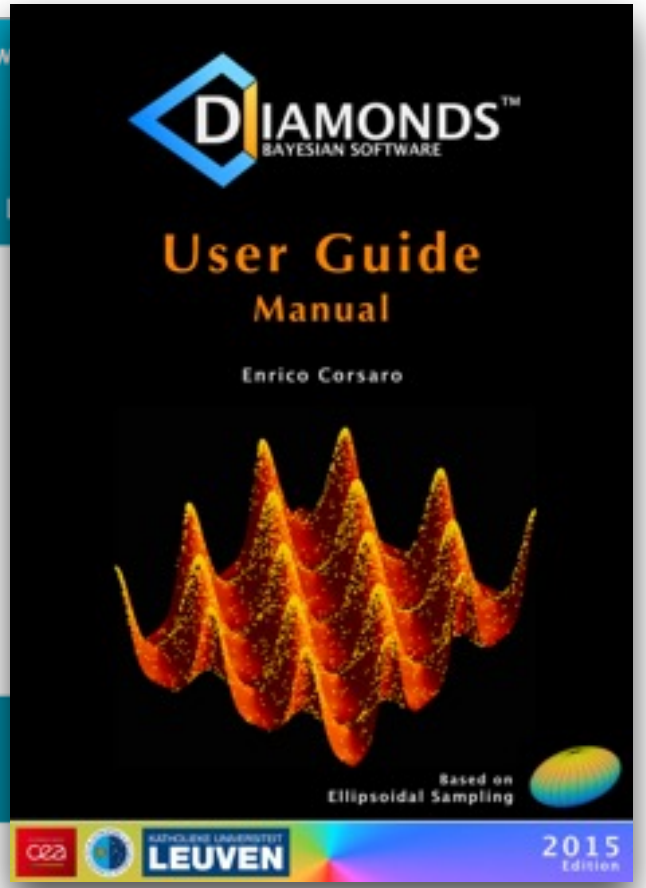


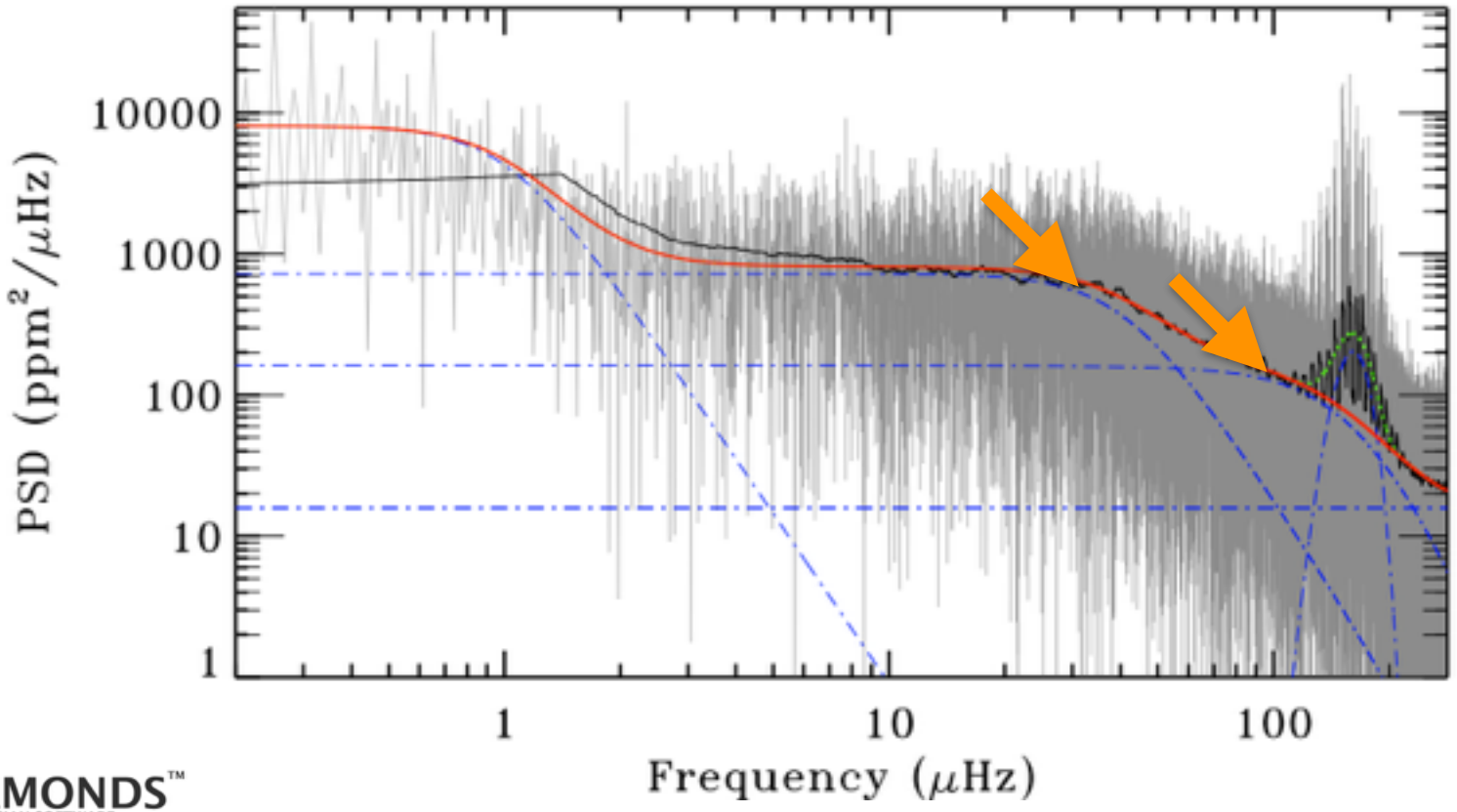


4. - FITTING A PSD: BACKGROUND COMPONENTS

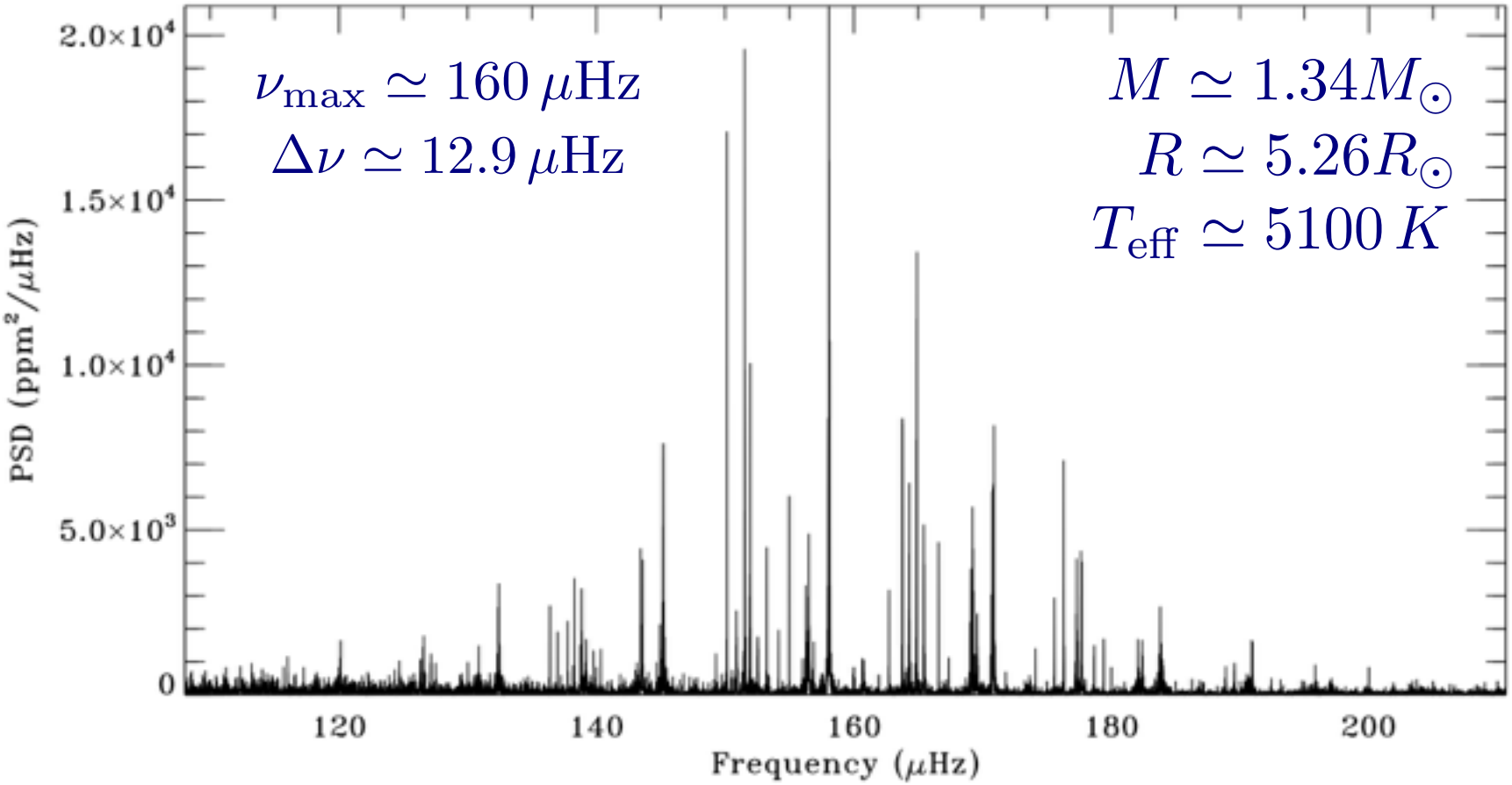
The background model fit for a *Kepler* star

KIC 12008916

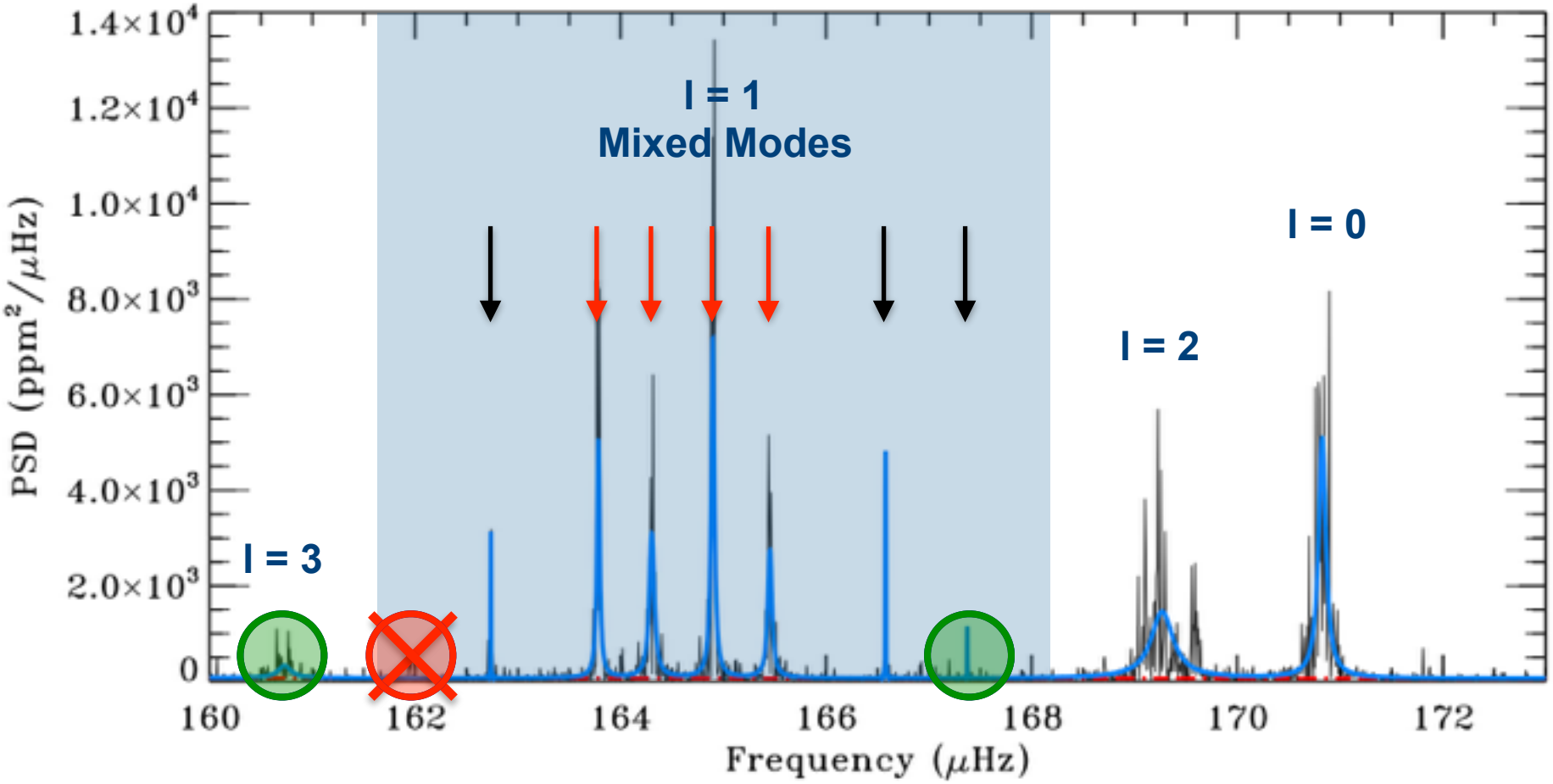
Corsaro, De Ridder, García 2015 A&A, 579, 83



DIAMONDS™ BAYESIAN SOFTWARE


$\nu_{\max} \simeq 160\,\mu\text{Hz}$
$\Delta\nu \simeq 12.9\,\mu\text{Hz}$
$M \simeq 1.34 M_\odot$
$R \simeq 5.26 R_\odot$
$T_{\text{eff}} \simeq 5100\,K$
PSD (ppm²/μHz)
2.0×10⁴
1.5×10⁴
1.0×10⁴
5.0×10³
0
120
140
160
180
200
Frequency (μHz)

KIC 12008916



Detection Probability

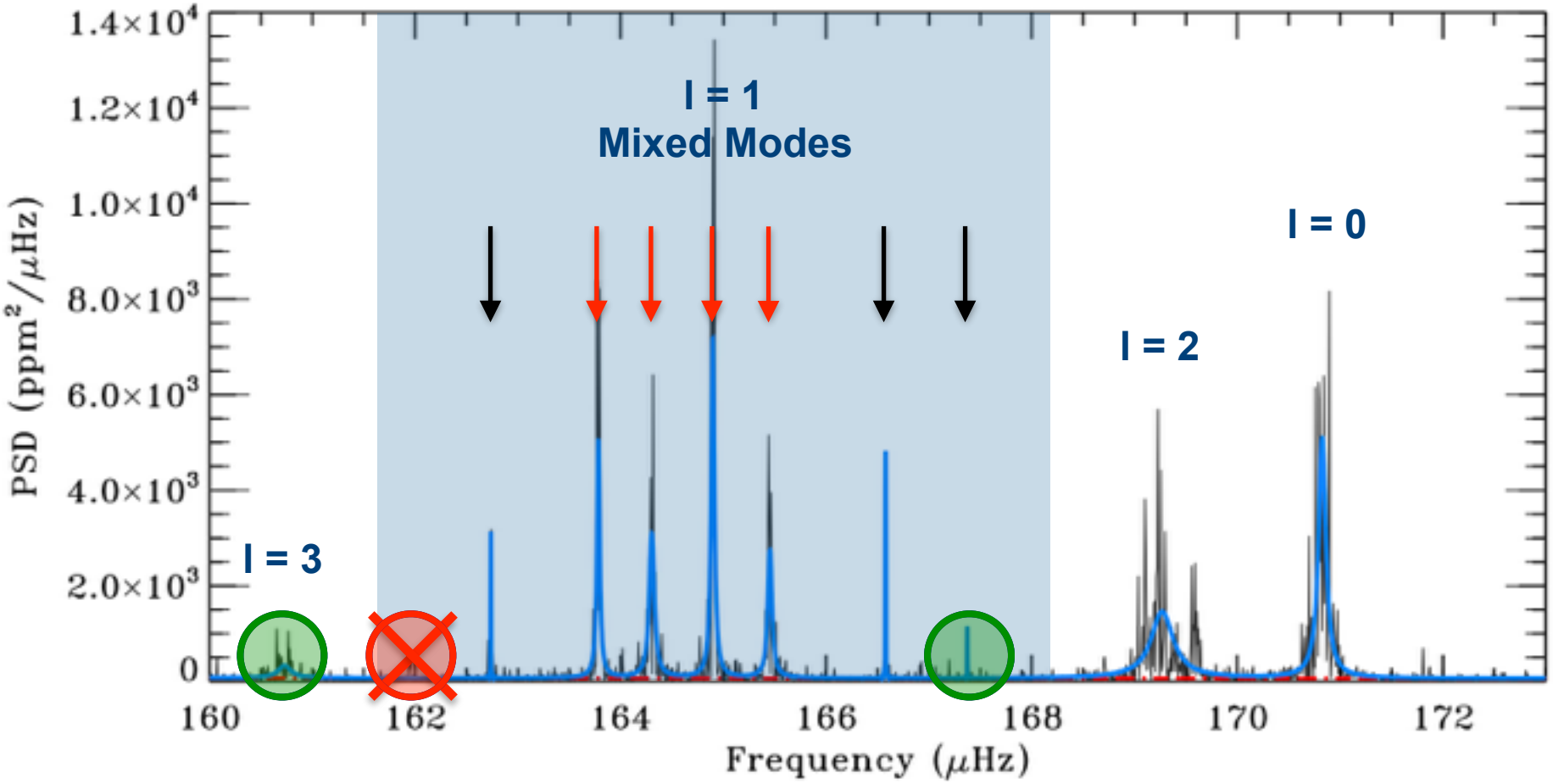
$$p_{\mathrm{peak}} \equiv \frac{\mathcal{E}_{\mathrm{peak}}}{\mathcal{E}_{\mathrm{no\,peak}} + \mathcal{E}_{\mathrm{peak}}}$$

$$p_{\mathrm{peak}} \gtrsim 99\,\%$$

Peak detected!

$$B_{1,2} = \frac{\mathcal{E}_1}{\mathcal{E}_2}$$

KIC 12008916



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