Radiative Corrections in Reactions involving Electrons & Protons

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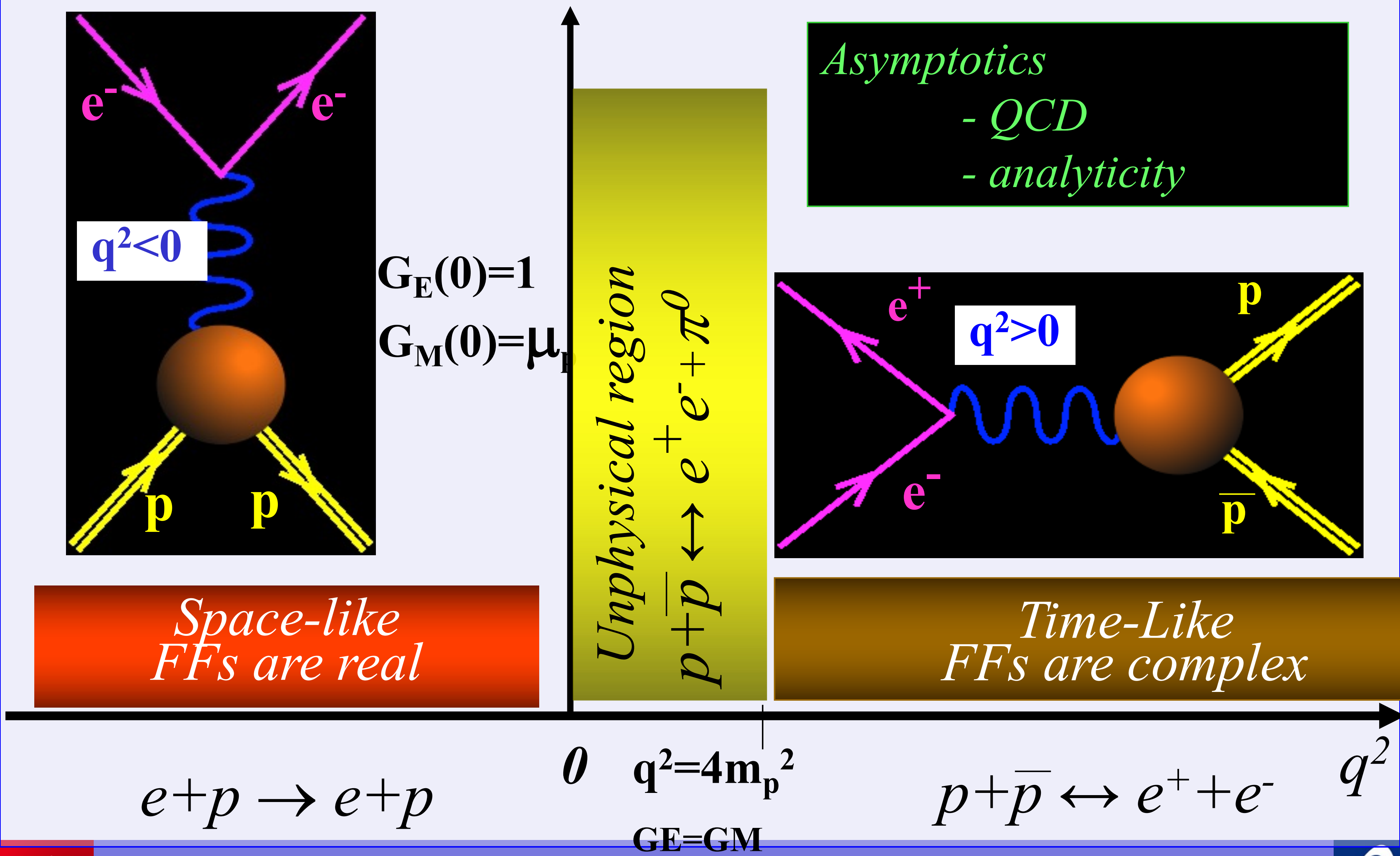
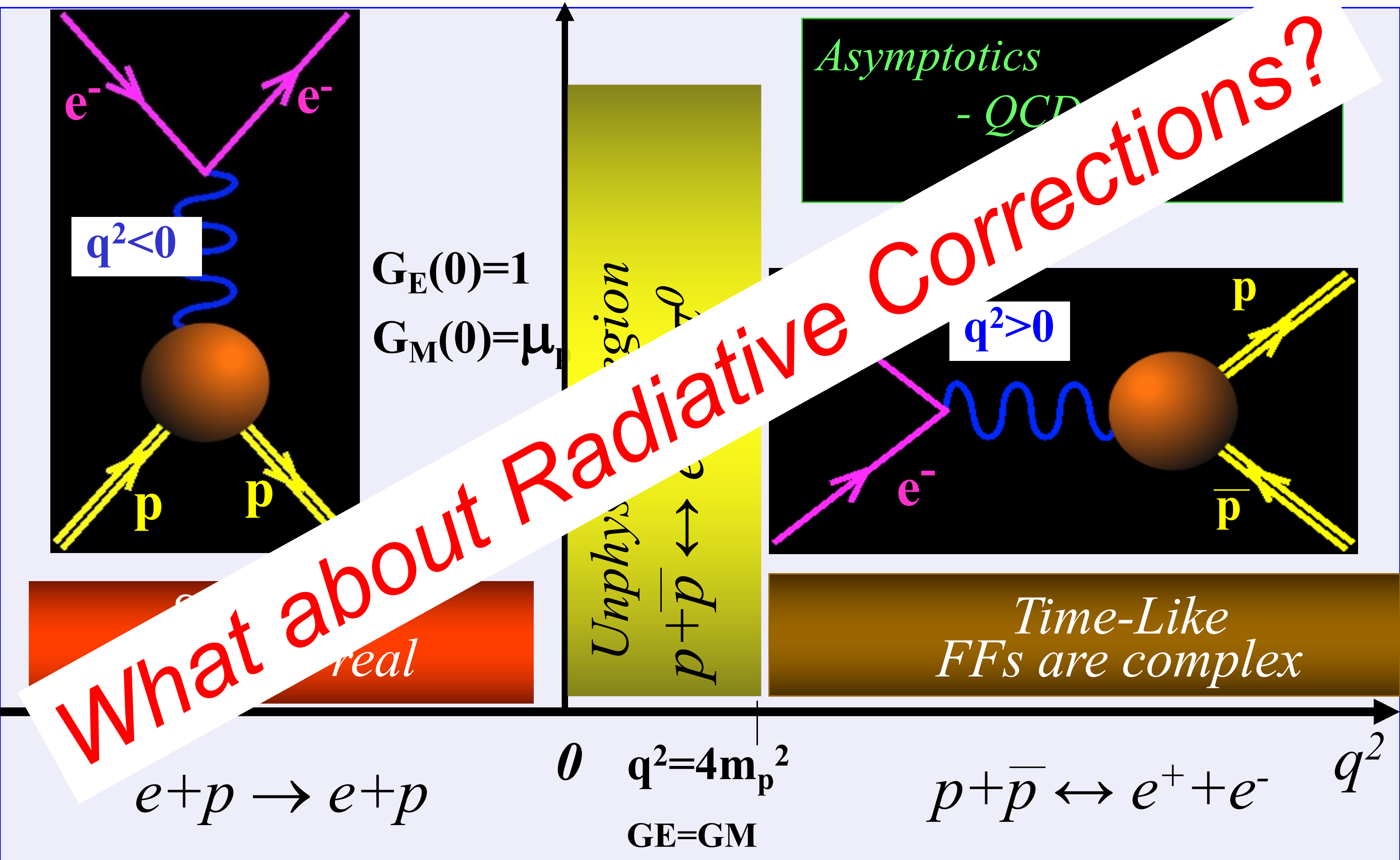
ECT* Workshop on Radiative Corrections from Medium to High Energy Experiments
July 18-22, 2022

Nucleon Charge and Magnetic Distributions



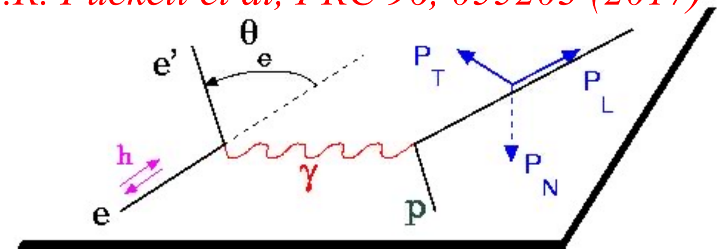
Nucleon Charge and Magnetic Distributions

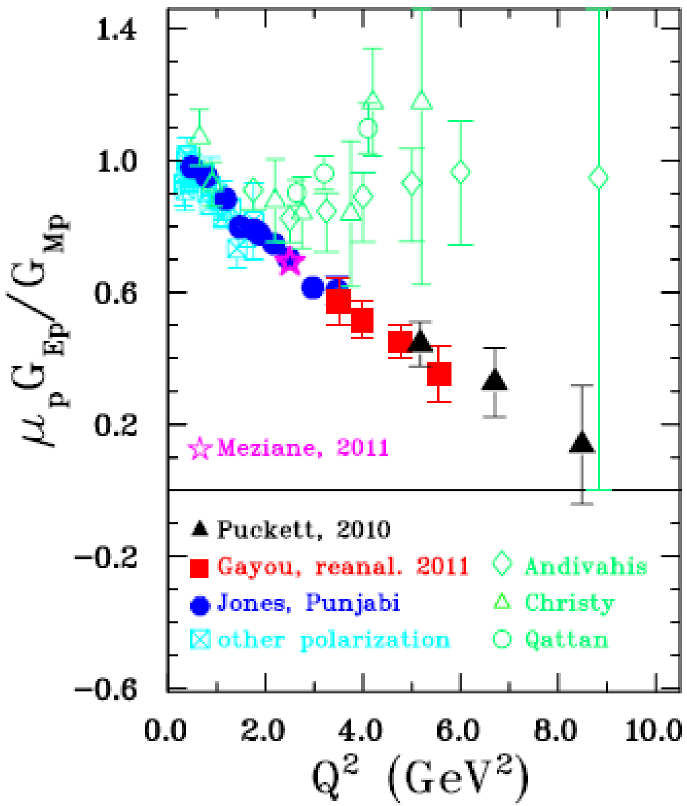


Experimental fact: *ep → ep*

- Precise data on the proton space-like form factors by the Akhiezer-Rekalo recoil proton polarization method show that the electric and magnetic distributions in the proton are different, suggesting a steaper Q^2-monopole-like decrease of the ratio and eventually a zero-crossing of G_E.
- It is well accepted today that the polarization method gives THE reliable measurement of the EM FF ratio at large Q^2 (compared to the Rosenbluth method).
- The difference has been attributed to radiative corrections (including 2γ ?)
- Applying radiative corrections at first order in α brings a % uncertainty in cross section measurements.

JLab-GEp Collaboration
J.R. Puckett et al, PRC 96, 055203 (2017)





Ch. Perdrisat, V. Punjabi

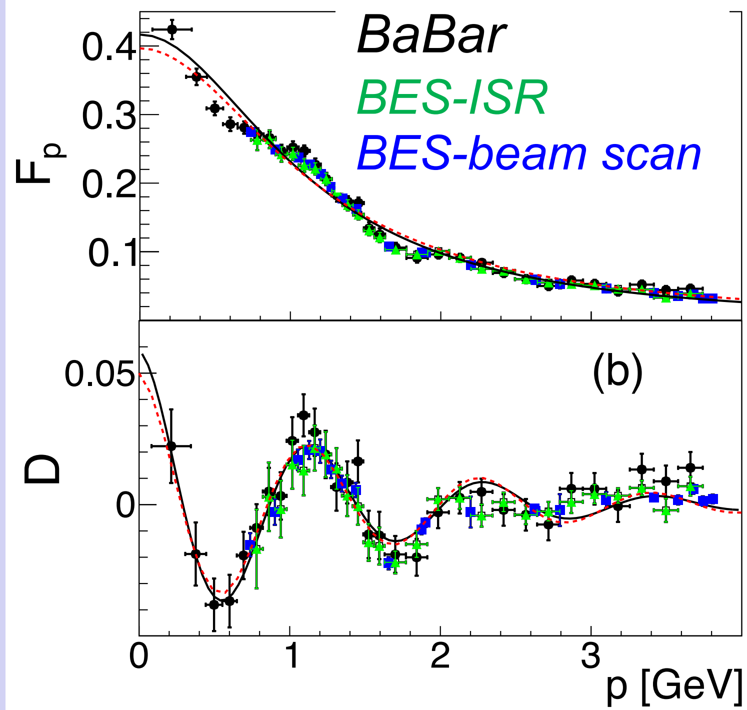
Experimental fact: $e^+e^- \to \bar{p}p$

- BaBar and BESIII data on the proton time-like effective form factor show a systematic sinusoidal modulation in terms od the p-$\bar{\text{p}}$ relative 3-momentum in the near-threshold region.

- ~ 10% size oscillations on the top of a regular background (dipole x monopole)

- The periodicity and the simple shape of the oscillations point to an interference of mechanisms of scale 0.2 and ~1 fm.

- The hadronic matter is distributed in non-trivial way.
- High order radiative corrections are applied (structure functions method)



$$F_p^{\rm fit}(s) = F_{\rm 3p}(s) \ + \ F_{\rm osc}(p(s))$$

$$F_{\rm 3p}(s) \ = \ \frac{F_0}{\left(1+\frac{s}{m_a^2}\right)\left(1-\frac{s}{m_0^2}\right)^2},$$

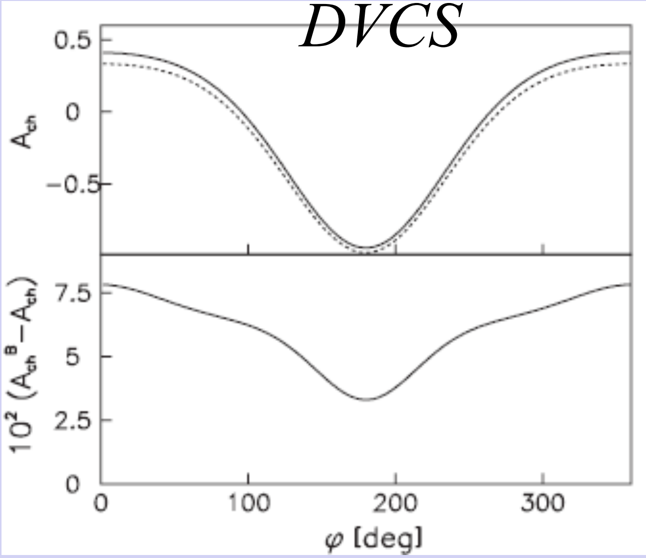
$$F_{\rm osc}(p(s)) \ = \ Ae^{-Bp}\cos(Cp+D)\,.$$

A.Bianconi, E. T-G. Phys. Rev. Lett. 114,232301 (2015)

Plan

- Radiative corrections modify not only the absolute values but also the dependence of the observables on the relevant kinematical variables

V.V. Bytev, E.A. Kuraev, E.T.-G. PRC77, 055205 (2008)



- Higher orders:
necessity and importance

- Reaction mechanism: 1γ-2γ interference

- Other issues:
  - *Normalizations*
  - *Correlations*

What can we learn from time-like processes?

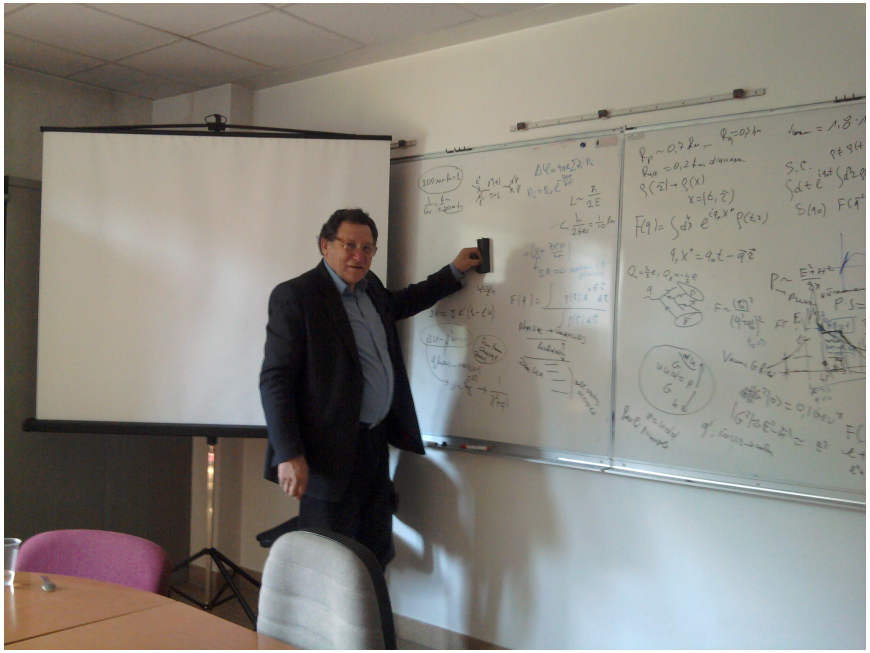
Cross section of (quasi)elastic *ep*-scattering

Classify radiative corrections…

Elastic $e^-(p_1)+p(p)\to e^-(p_1')+p(p')$

Inelastic $\to e^-(p_1')+p(p')+\gamma(k)$

E.A. Kuraev (1940-2014)



Higher order inelastic

double brehmstrahlung,
pair production..

$$\to e^-(p_1')+p(p')+\gamma(k_1)+\gamma(k_2)$$
$$\to e^-(p_1')+p(p')+e^-(q_-)+e^+(q_-)$$
$$\to \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

….in terms of leading logarithm

Short history (I)

- Schwinger (1949) : corrections to the cross section for electron scattering in external field

$$\sigma=\sigma_0(1+\delta) \quad (1)^*$$

- Yennie, Frauchi, Suura (1961) : the cross section of any pure process (without real photon emission) is zero.

- Kessler, Ericsson, Baier, Fadin, Khoze, Y. Tsai (1968) : quasi real electron method. The emission of a hard photon is described in terms of a convolution of a radiative function with the Born cross section.

$\rightarrow$ * (1) Not sufficient

Short history (II)

1977: Altarelli, Parisi, Gribov, Lipatov, Dokshitzer: (DGLAP)

- Asymptotic freedom, evolution equation
- Collins factorization theorem
- Drell-Yan picture of hard processes in QED (application of QCD ideas to QED): radiative corrections in form of structure functions and Drell-Yan picture

Leading $\sim \left(\frac{\alpha}{\pi} L\right)^n$ and non leading $\sim \frac{\alpha}{\pi} \left(\frac{\alpha}{\pi} L\right)^n$ terms are explicitely taken into account in DGLAP evolution equations.

In QED they are known as *Lipatov equations* (1975).

Structure Function Method

E. A. Kuraev and V.S. FADIN, Sov. J. of Nucl. Phys. 41, 466 (1985)

Distinguish:

-leading contributions of higher order $\sim \left(\frac{\alpha}{\pi}L\right)^n$

-non leading ones $\sim \frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}L\right)^n$

Large log

$$L = \ln \frac{Q^2}{m_e^2}$$

The SF method is based on:

- *Renormalization group - evolution equation*
- *Drell-Yan parton picture of the cross section in QCD*

$$d\sigma(y) = \int_{x_0}^{1} \frac{dx}{x}\rho_x \frac{d\sigma_0(Ex)}{|1-\Pi(Q^2x)|^2}\mathcal{D}(x,L)\,\mathcal{D}\left(\frac{y\rho_x}{x},L\right)\left(1+\frac{\alpha}{\pi}K\right)$$

$$\mathcal{D}(x,L)$$

Electron SF: probability to 'find' an electron in the initial electron, with energy fraction x and virtuality up to Q^2

Structure Function Method

E. A. Kuraev and V.S. Fadin, Sov. J. of Nucl. Phys. 41, 466 (1985)

- SF method applied to QED processes: calculation of radiative corrections with precision ~ 0.1%.

- Takes into account the dynamics of the process

- Is formulated in terms of parton densities (leptons, antileptons, photons)

- Many applications to different processes

Structure Function Method *(some applications)*

- $e^+e^- \to$ hadrons(J/Ψ width)
  E. A. KURAEV and V.S. FADIN, Sov. J. of Nucl. Phys. 41, 466 (1985)
- ep $\to$ e'X (elastic and inelastic scattering)
  E. A. KURAEV ;N.P. MERENKOV and V.S. FADIN, Sov. J. of Nucl. Phys. 47,1009 (1988)
- Decay width of mesons (FSI)
  E. A. KURAEV, JETP Lett.65, 127 (1997)
- Radiative corrections for LEP beam (small angle BHABHA scattering)
  A.B.Arbuzov, E.A.Kuraev et al, Phys. Lett.B 399, 312 (1997)
- Compton and double Compton scattering
  A.N.Ilyichev, E.A. Kuraev, V.Bytev and Y. P. Peresun'ko, J. Exp. Theor. Phys.100 31 (2005)
- Structure function method applied to polarized and unpolarized electron-proton scattering: A solution of the GE(p)/GM(p) discrepancy.
  Y. Bystricky, E.A.Kuraev, E. Tomasi-Gustafsson, Phys. Rev. C75, 015207 (2007).
- Radiative corrections to DVCS electron tensor
  V.Bytev, E.A.Kuraev, E. Tomasi-Gustafsson, Phys. Rev. C78, 015205 (2008)
- Radiative proton-antiproton annihilation to a lepton pair
  A.I. Ahmadov, V.V. Bytev, E.A.Kuraev, E. T-G, Phys. Rev. D82, 094016 (2010)

PHYSICAL REVIEW C **75**, 015207 (2007)

Structure function method applied to polarized and unpolarized electron-proton scattering: A solution of the $G_E(p)/G_M(p)$ discrepancy

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(Received 23 March 2006; revised manuscript received 7 August 2006; published 25 January 2007)

PHYSICAL REVIEW C **89**, 065207 (2014)

Radiative corrections for electron-proton elastic scattering taking into account high orders and hard-photon emission

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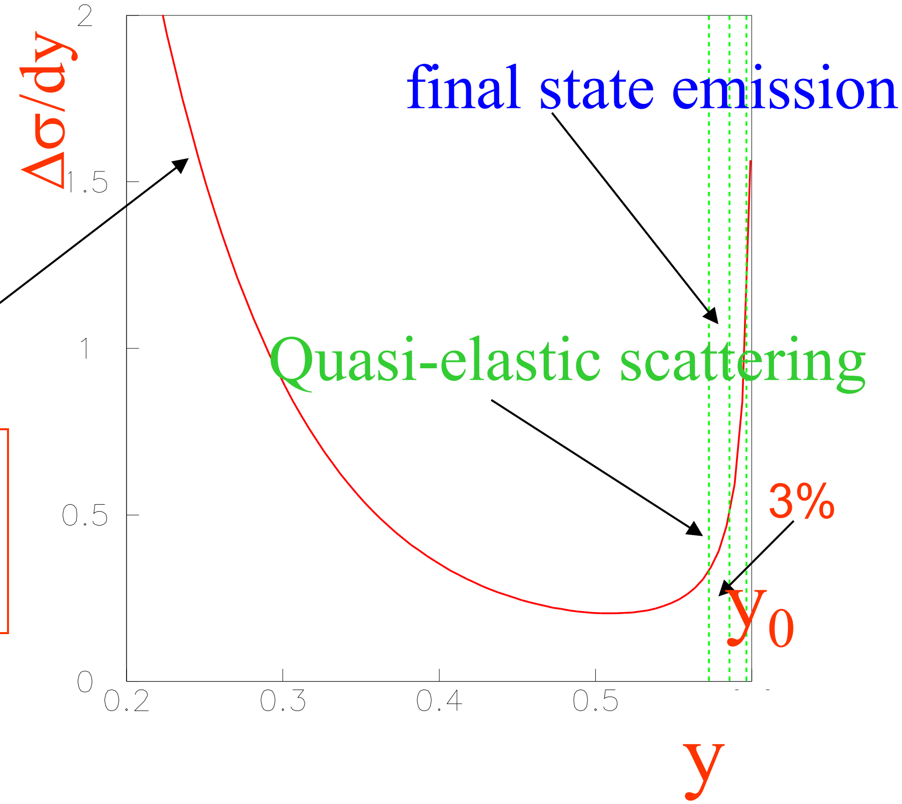
(Received 31 October 2013; revised manuscript received 10 January 2014; published 18 June 2014)

Scattered electron energy

$$E'/E = y\,;\quad y_0 = \frac{1}{\rho}$$

$$\rho = 1 + \frac{2E}{M}\sin^2\theta/2 .$$

Initial state emission



$$\Delta\frac{d\sigma}{d\Omega} \sim \frac{d\sigma_0}{d\Omega}\cdot\frac{\alpha}{\pi}\ln\frac{E}{\Delta E}\ln\frac{2EM}{m_e^2} .$$

Not so small!

ISR: Shift to LOWER Q^2

All orders of PT needed →

beyond Mo & Tsai approximation

LSF: ‰ precision

E. A. K. and V.S. FADIN, Sov. J. of Nucl. Phys. 41, 466 (1985)

LLA (Leading Logarithm Approximation)

$$\frac{\alpha}{\pi} L \approx 1, L = \ln \frac{Q^2}{m_e^2}$$

Precision of LLA

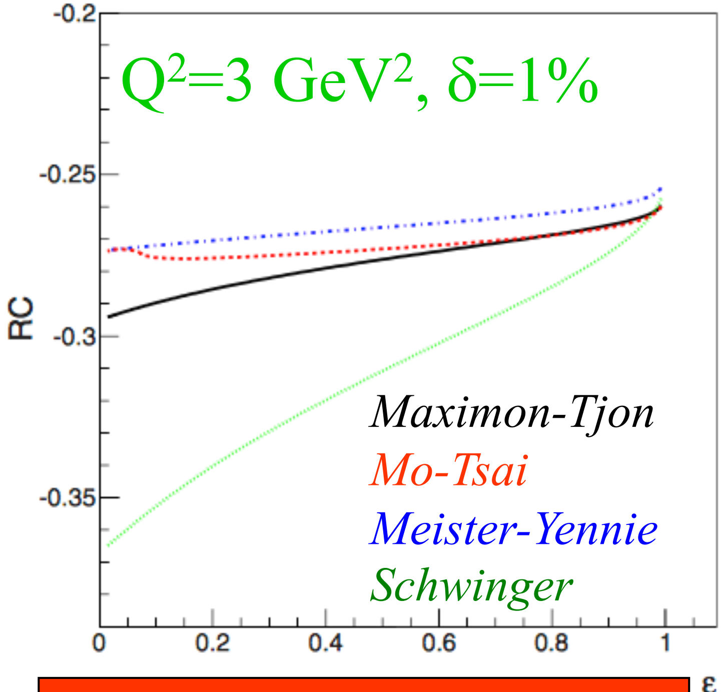
$$\left(\frac{\alpha}{\pi}\right)\left(\frac{\alpha}{\pi} L\right) \approx \frac{1}{400} \approx 0.2\%$$

Including K-factor

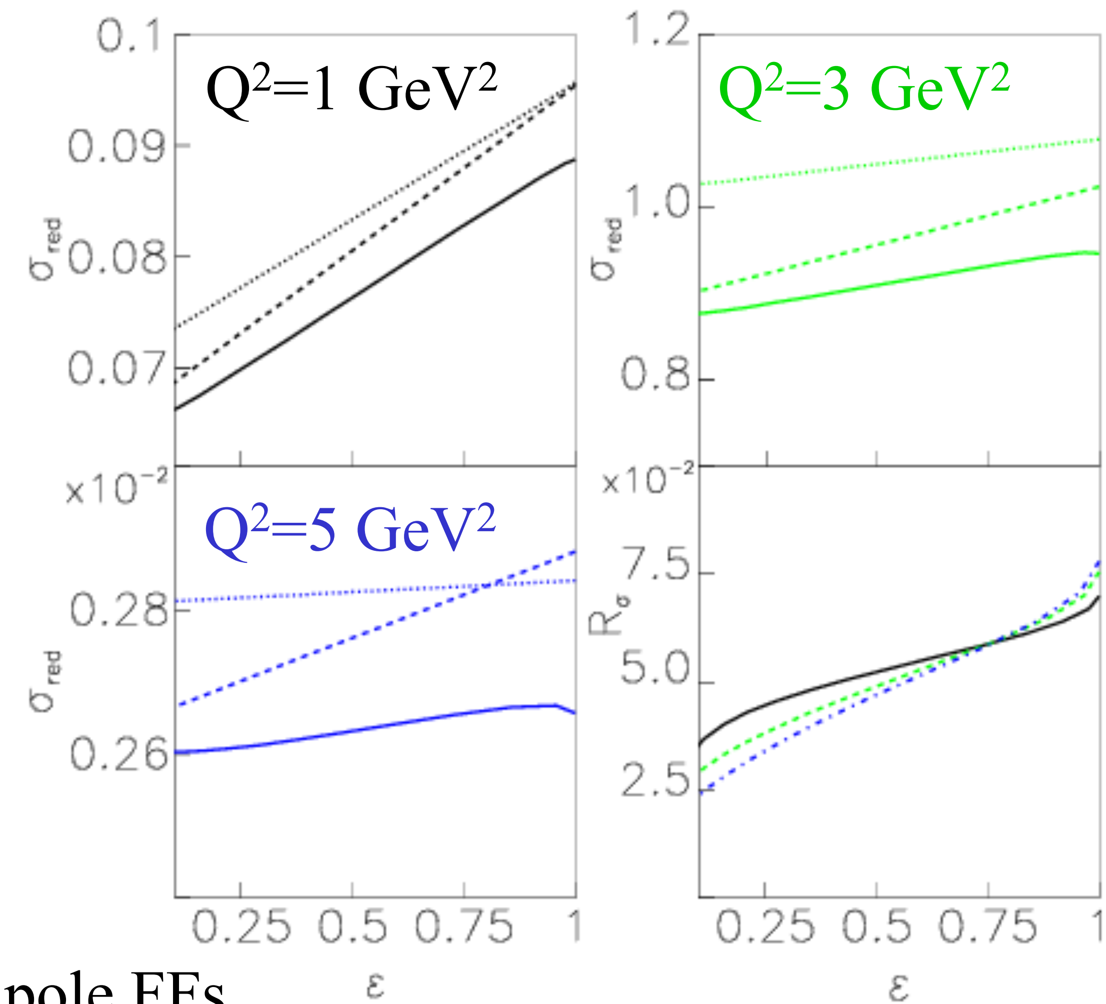
$$\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{\alpha}{\pi} L\right) \leq 0.01\%$$

Even when corrections in first order PT are $\delta \sim 100\%$, the accuracy of higher order RC (LSF) is $\alpha/\pi\ \delta \leq 1\%$!

RC & LSF



$$\sigma_{red} = \tau G_{Mp}^2 + \epsilon G_{Ep}^2$$



Both calculations assume dipole FFs

The slope changes due to different RC !

GEp exp ·········

Born cs ---------

SF cs ———

*E.T-G, Phys. Part. Nucl. Lett. **4**, 281-288 (2007)*

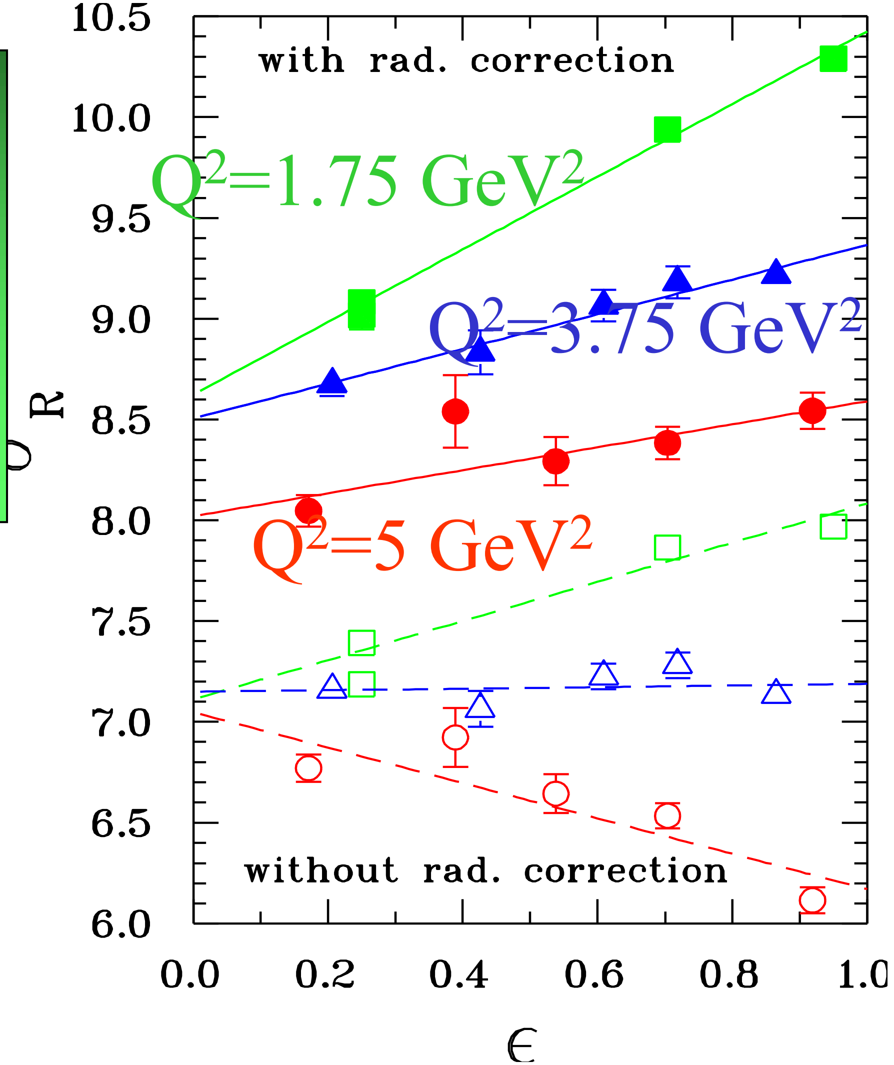
Radiative Corrections

Data from L. Andivahis et al., PRD50, 5491 (1994)

RC
to the Rosenbluth cross section:
– large (may reach 40%)
– ε and Q^2 dependent
– calculated at first order

May change
the slope of σ_R
(and even the sign !!!)

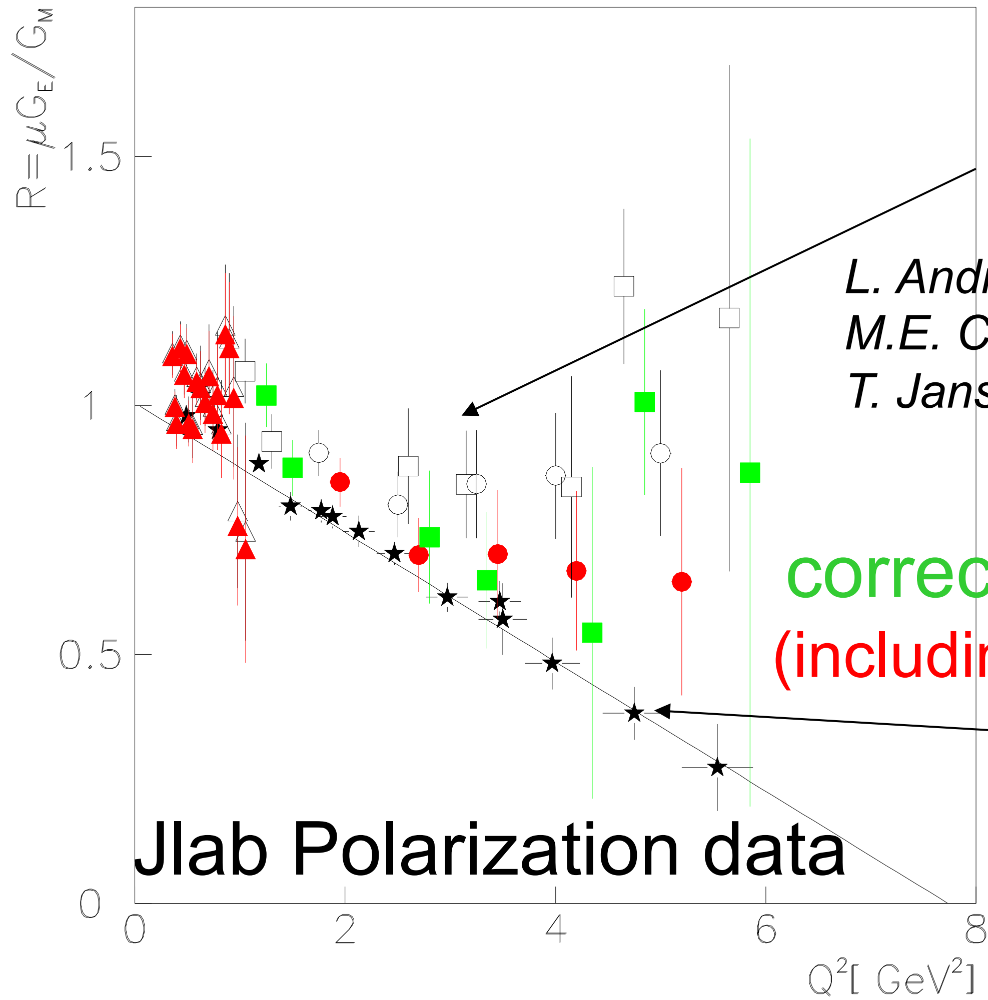
$$\sigma=\sigma_0(1+\delta)$$



E. T.-G., G. Gakh, PRC 72, 015209 (2005)

C. Perdrisat, V. Punjabi, M. Vanderhaeghen, Progr. in Part. and Nucl. Physics (2007)

Radiative Corrections (SF method)



SLAC data

L. Andivahis et al., PRD50, 5491 (1994)
M.E. Christy et al., PRC70, 015206 (2004)
T. Janssens, et al. PR142, 922 (1966)

corrected by SF
(including 2γ and Δ contribution)

Yu. Bystricky, E.A.Kuraev, E. T.-G, Phys. Rev. C 75, 015207 (2007)

PHYSICAL REVIEW C 93, 055201 (2016)

Reanalysis of Rosenbluth measurements of the proton form factors

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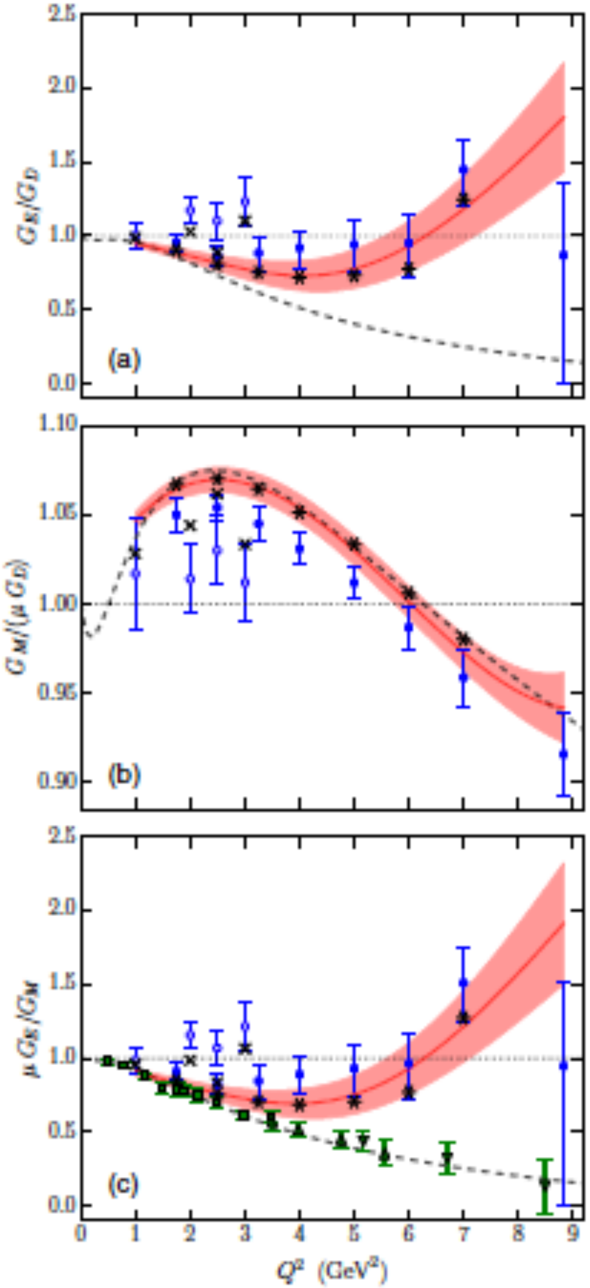
(Received 28 March 2016; published 10 May 2016)



V. Fadin ,R.E.Gerasimov

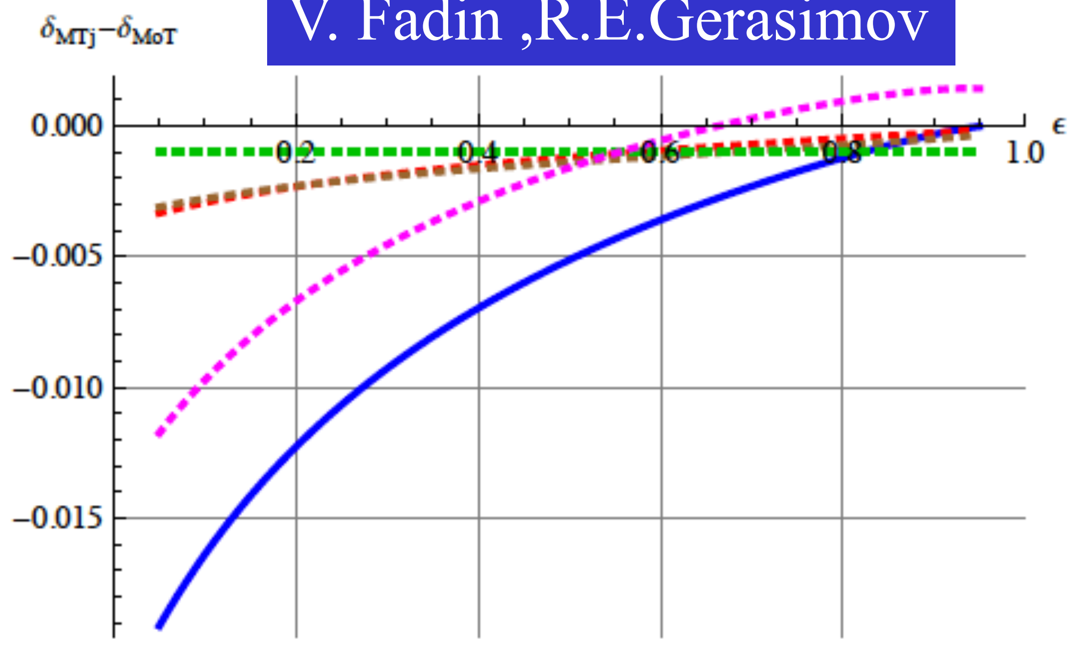
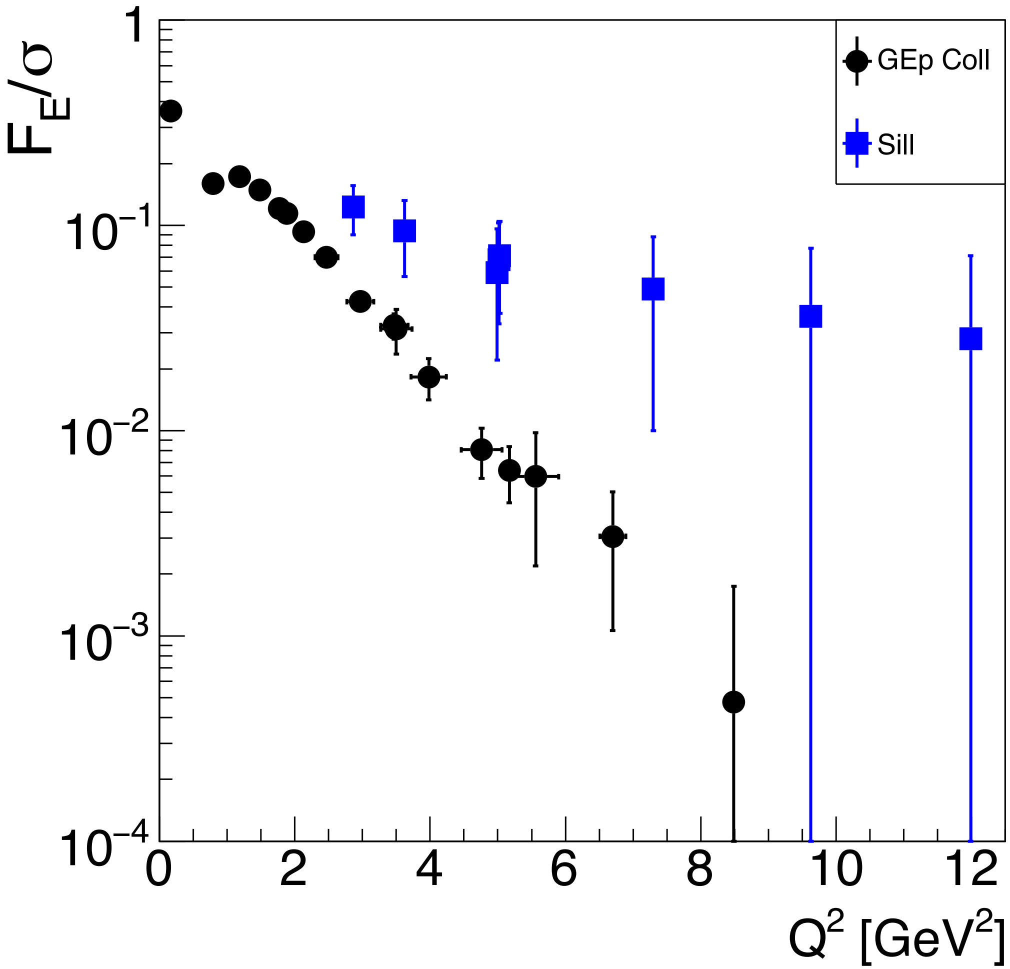


Figure 3: Difference at $Q^2 = 5\ GeV^2$.

Electric contribution to ep cross section

$$F_E = \frac{\epsilon G_E^2}{1+\tau/(\epsilon R^2)}.$$

$$\sigma_R = \varepsilon G_E^2 + \tau G_M^2$$



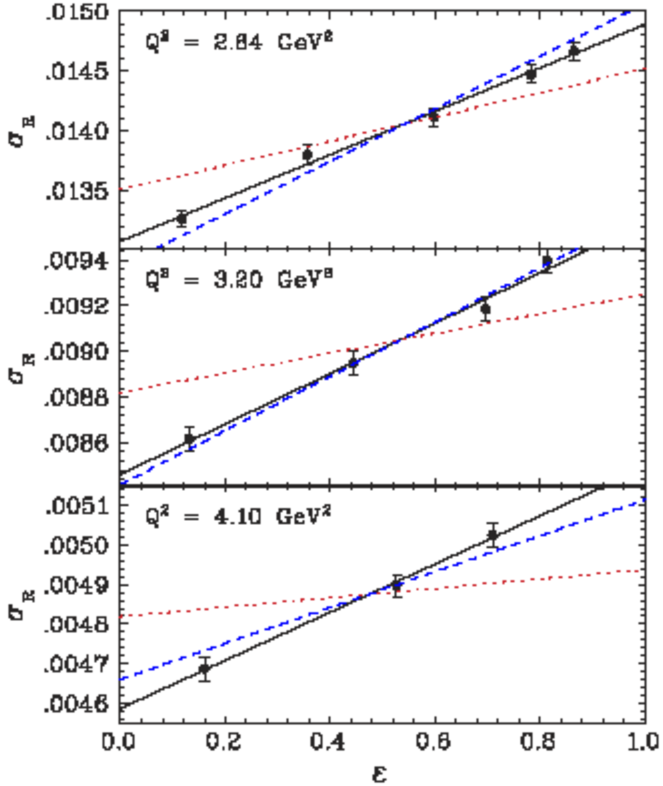
$G_E \approx G_D$

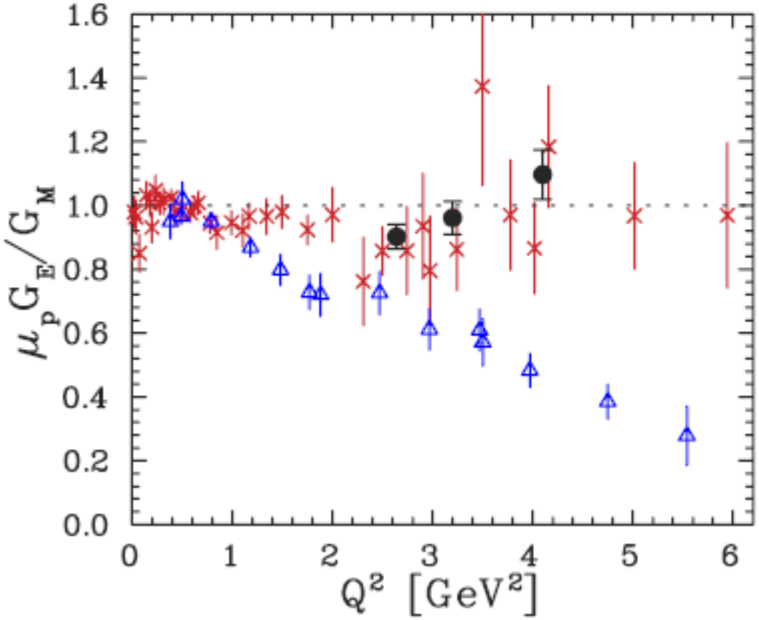
$G_E < G_D$

PRL 94, 142301 (2005) PHYSICAL REVIEW LETTERS week ending 15 APRIL 2005

Precision Rosenbluth Measurement of the Proton Elastic Form Factors

I. A. Qattan,[1,2] J. Arrington,[2] R. E. Segel,[1] X. Zheng,[2] K. Aniol,[3] O. K. Baker,[4] R. Beams,[2] E. J. Brash,[5] J. Calarco,[6] A. Camsonne,[7] J.-P. Chen,[8] M. E. Christy,[4] D. Dutta,[9] R. Ent,[8] S. Frullani,[10] D. Gaskell,[11] O. Gayou,[12] R. Gilman,[13,8] C. Glashausser,[13] K. Hafidi,[2] J.-O. Hansen,[8] D. W. Higinbotham,[8] W. Hinton,[14] R. J. Holt,[2] G. M. Huber,[5] H. Ibrahim,[14] L. Jisonna,[1] M. K. Jones,[8] C. E. Keppel,[4] E. Kinney,[11] G. J. Kumbartzki,[13] A. Lung,[8] D. J. Margaziotis,[3] K. McCormick,[13] D. Meekins,[8] R. Michaels,[8] P. Monaghan,[9] P. Moussiegt,[15] L. Pentchev,[12] C. Perdrisat,[12] V. Punjabi,[16] R. Ransome,[13] J. Reinhold,[17] B. Reitz,[8] A. Saha,[8] A. Sarty,[18] E. C. Schulte,[2] K. Slifer,[19] P. Solvignon,[19] V. Sulkosky,[12] K. Wijesooriya,[2] and B. Zeidman[2]

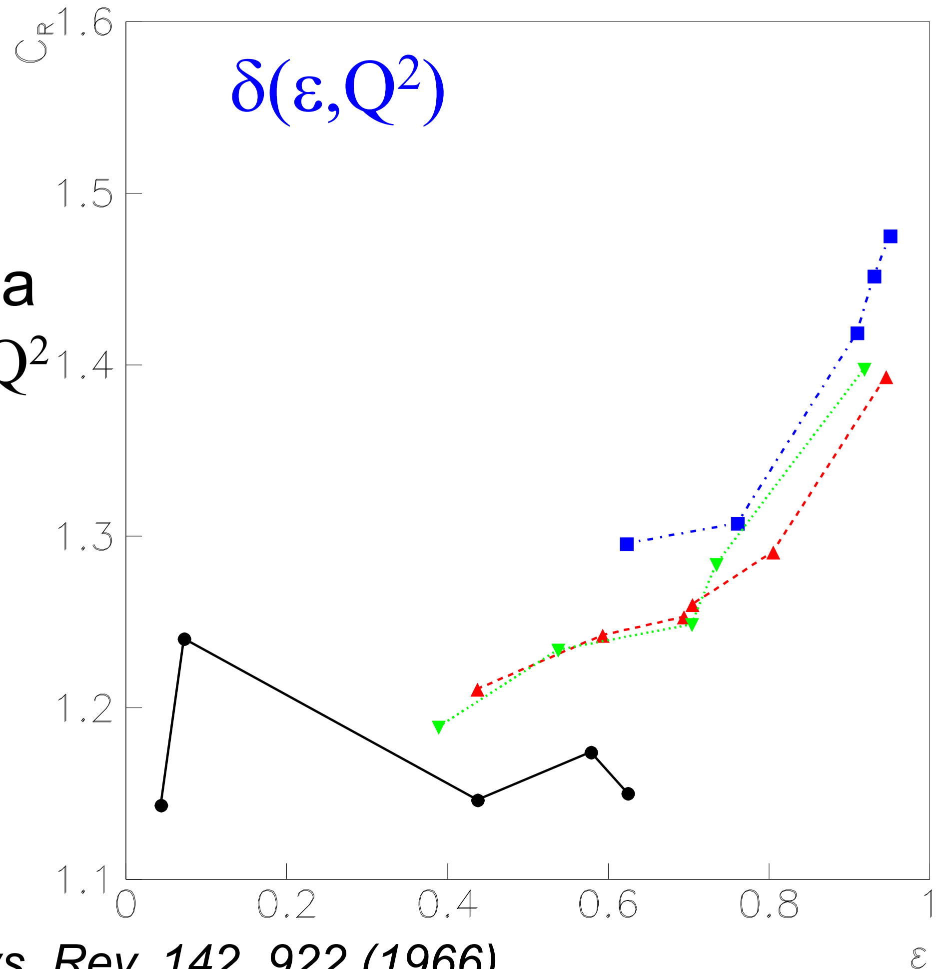




Radiative Corrections

$$\sigma=\sigma_0(1+\delta(\varepsilon,Q^2))$$

Radiative Corrections as a function of ε for different Q^2



0.32 GeV²: T. Janssens et al., Phys. Rev. 142, 922 (1966).

3 GeV²: R.C. Walker *et al.*, PRD49, 5671 (1971)

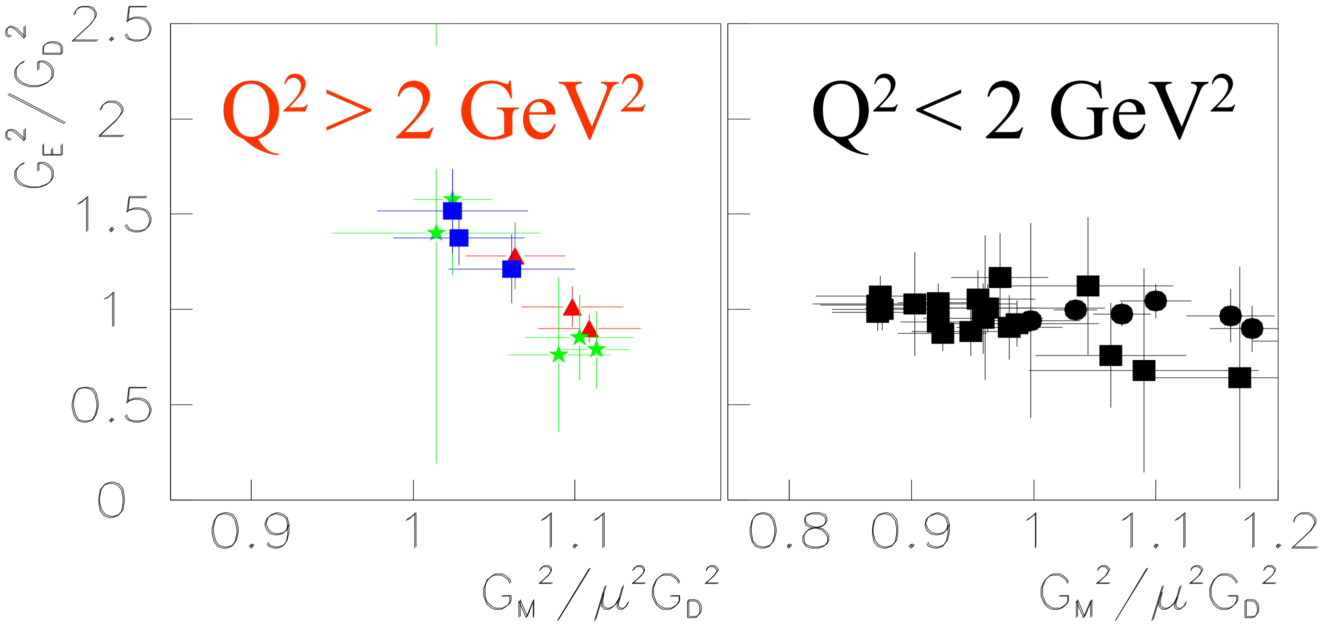
4 GeV² & 5 GeV² : L. Andivahis et al., PRD50, 5491 (1994).

E.T-G, Phys. Part. Nucl. Lett. ***4,*** *281 (2007)*

Correlations

$$\sigma=\sigma_0(1+\delta(\varepsilon,Q^2));\ \ \sigma_{red}=\varepsilon\, G_E^2+\tau\, G_M^2\ \ ;\ \tau=Q^2/4M_p^2$$

G_E^2 versus G_M^2



E.T-G, Phys. Part. Nucl. Lett. ***4,*** *281 (2007)*

Correlations

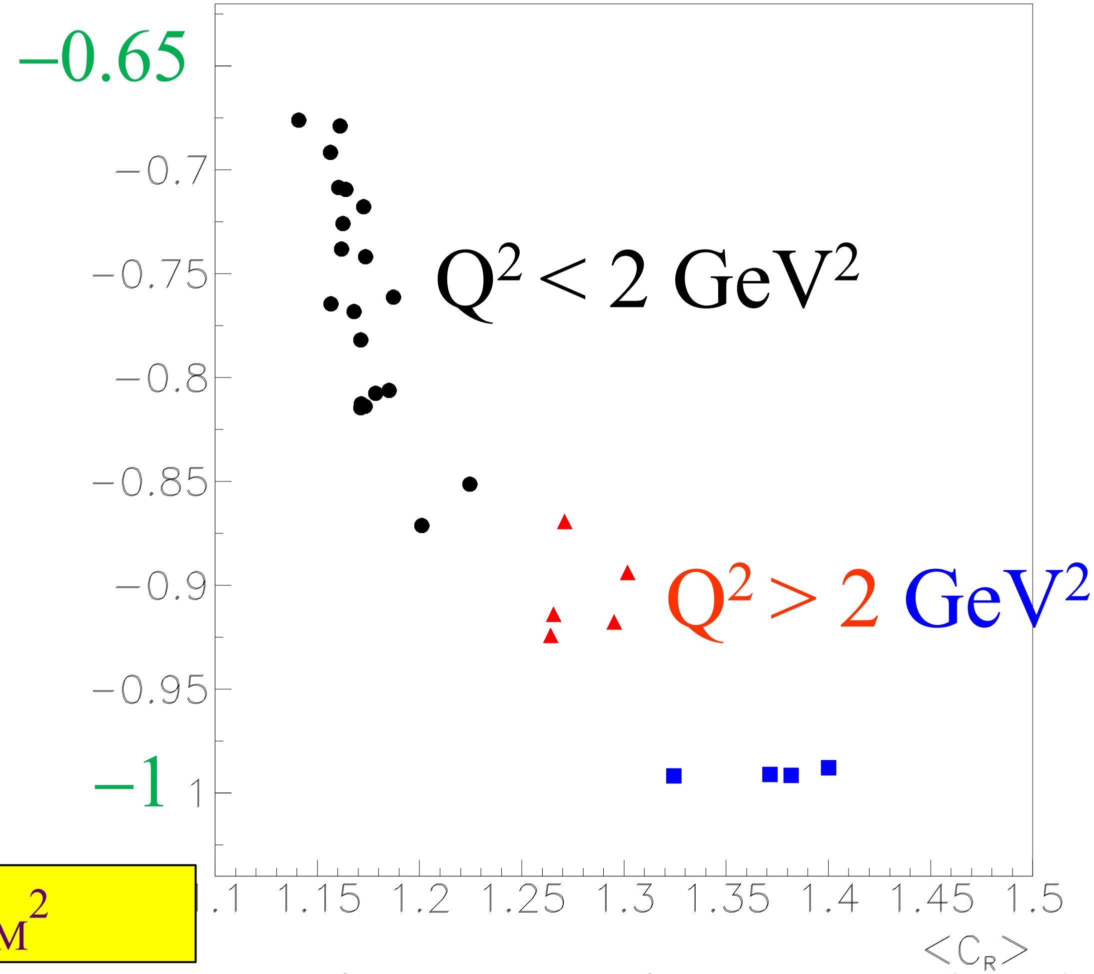
$$\sigma=\sigma_0(1+\delta(\varepsilon,Q^2))$$

$$\sigma=\varepsilon\, G_E^2+\tau\, G_M^2$$

Correlation coefficient $\xi = \mathrm{cov}(a, b)/\sigma_a\sigma_b$ for different sets of data

Q^2, GeV	ξ	Ref.	Q^2, GeV	ξ	Ref.
2.6400	−0.8823	[5]	0.2717	−0.7258	[24]
3.2000	−0.8973		0.2911	−0.7818	
4.1000	−0.9060		0.3105	−0.7085	
1.7500	−0.8693	[4]	0.3493	−0.7683	
2.5000	−0.9141		0.3881	−0.7417	
3.2500	−0.9242		0.4269	−0.7093	
4.0000	−0.9178		0.4657	−0.7381	
5.0000	−0.8940		0.5045	−0.8126	
1.0000	−0.9918	[22]	0.5433	−0.7646	
2.0030	−0.9915		0.5821	−0.8076	
2.4970	−0.9910		0.6209	−0.8061	
3.0070	−0.9878		0.6598	−0.8137	
0.1552	−0.6761	[24]	0.6986	−0.8713	
0.1785	−0.6788		0.7374	−0.8145	
0.1940	−0.6915		0.7762	−0.8512	
0.2329	−0.7177		0.8538	−0.7612	

Correlation coefficient as a function of $\langle C_R\rangle_\varepsilon$



Q^2 behavior of G_E^2 driven by G_M^2

*E.T-G, Phys. Part. Nucl. Lett. **4**, 281 (2007)*

Correlations

$$\sigma=\sigma_0(1+\delta(\varepsilon,Q^2))$$

$$\sigma=\varepsilon\, G_E^2+\tau\, G_M^2$$

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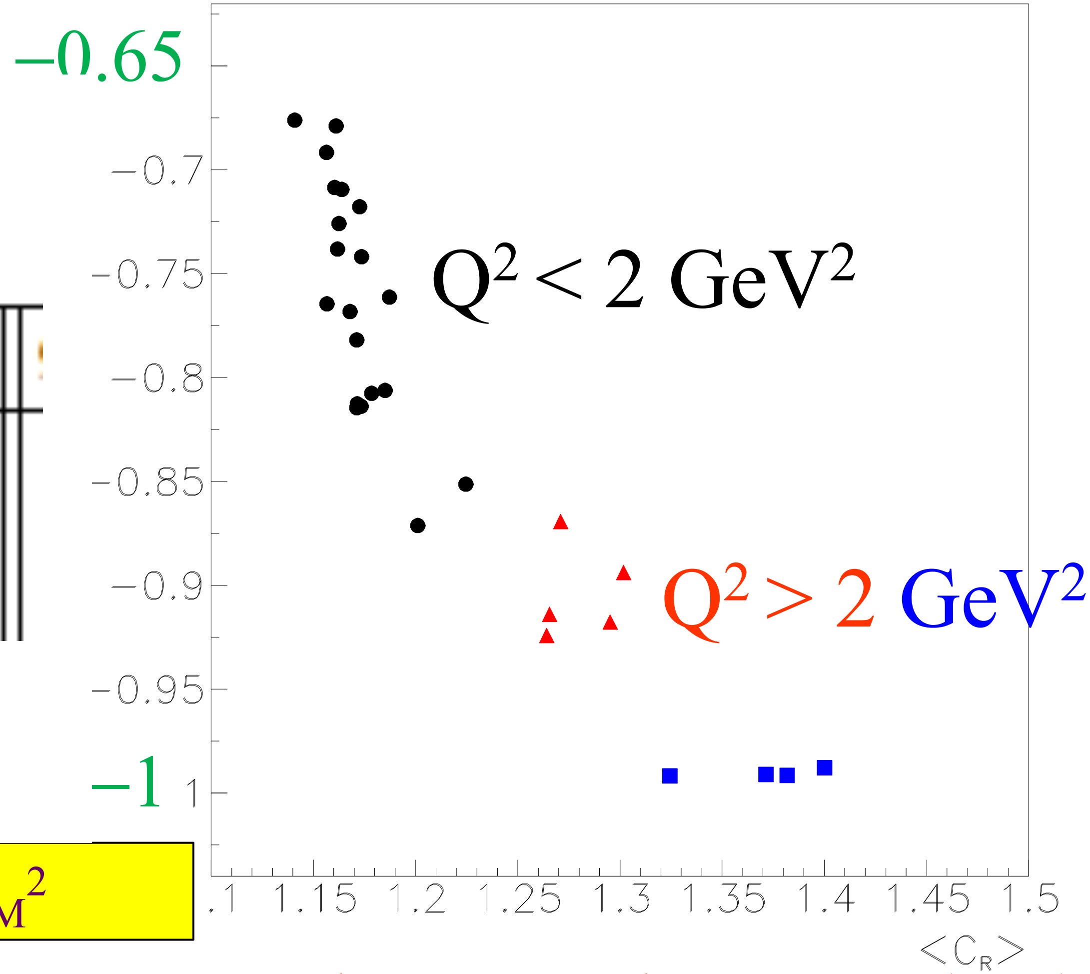
0.1940	−0.6915		0.7762	−0.8512	
0.2329	−0.7177		0.8538	−0.7612	

Correlation coefficient as a function of average <RC>

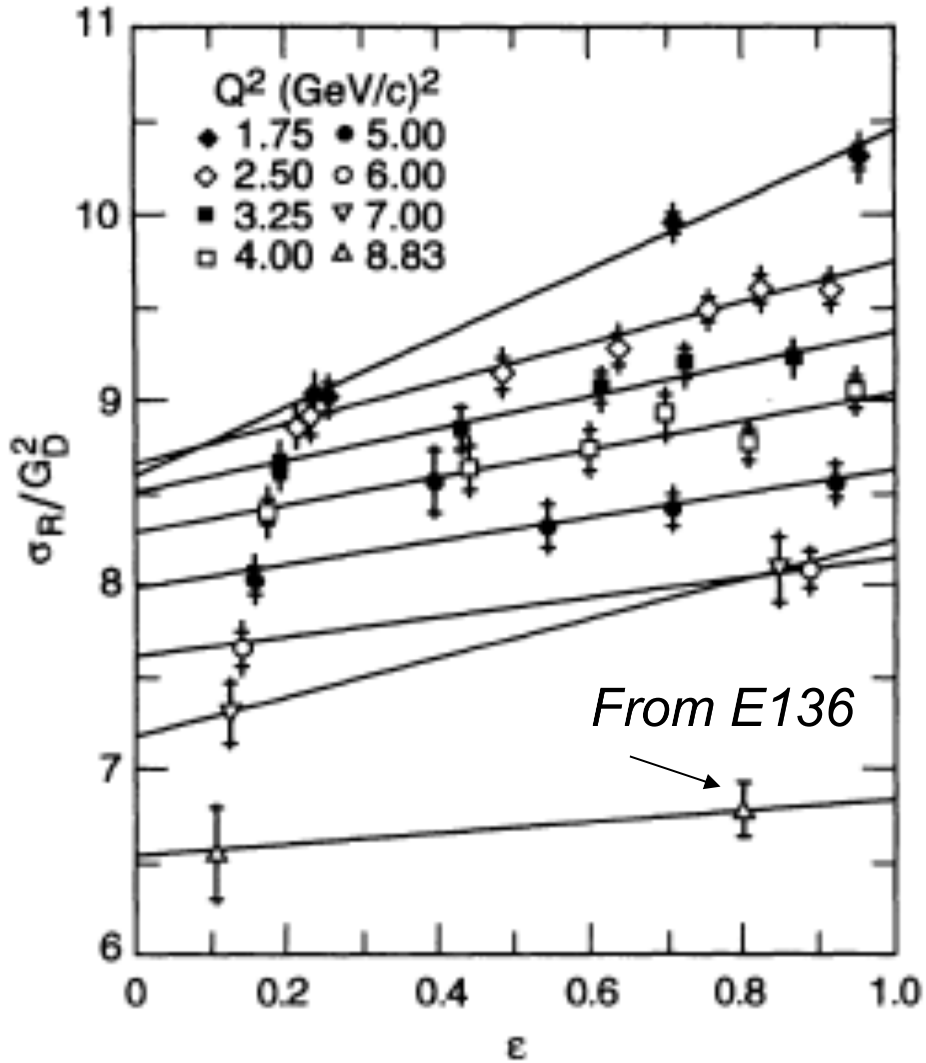


Q^2 behavior of G_E^2 driven by G_M^2

E.T-G, Phys. Part. Nucl. Lett. ***4,*** *281 (2007)*

Normalization

Andivahis et al., PRD50, 5491 (1994)



Two spectrometers
(8 and 1.6 GeV)

2 points at low ε

Fixed renormalization
for the lowest ε point
c=0.956
(acceptance correction)

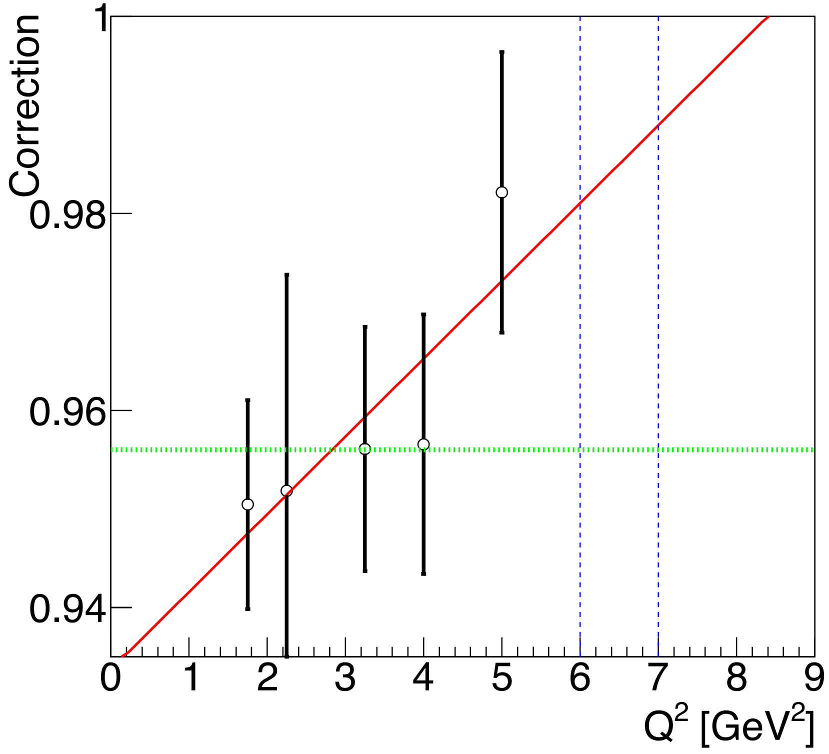
Increases the slope!

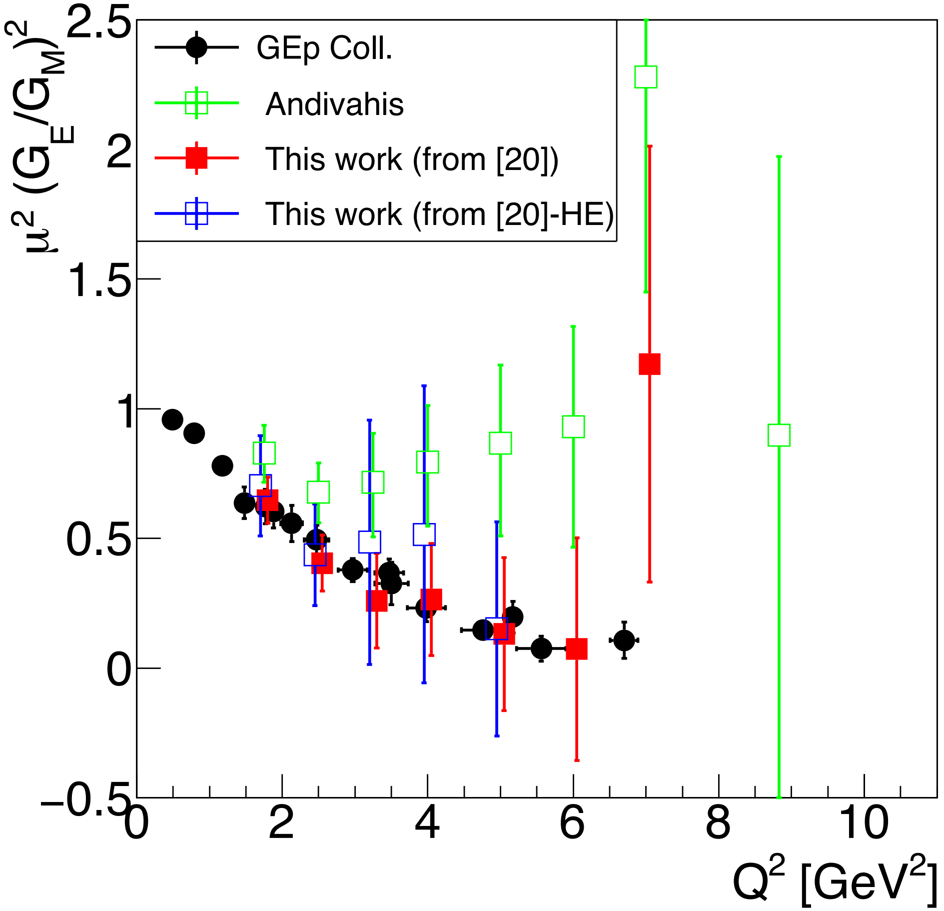
$G_E \approx G_D$

Direct extraction of the Ratio

Andivahis et al., PRD50, 5491 (1994)

$$\sigma_{\rm red} = G_M^2(R^2\epsilon + \tau),$$



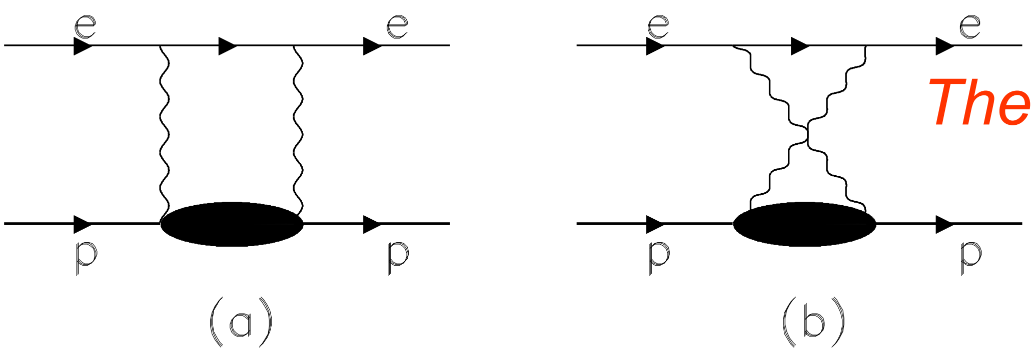


Simone Pacetti and Egle Tomasi-Gustafsson
Phys. Rev. C **94**, 055202, 2016

Two photon exchange

- *1γ-2γ interference is of the order of $\alpha=e^2/4\pi=1/137$*

- In the 70's it was shown [J. Gunion and L. Stodolsky, V. Franco, F.M. Lev, V.N. Boitsov, L. Kondratyuk and V.B. Kopeliovich, R. Blankenbecker…] that, at large momentum transfer, the sharp decrease of the FFs, if the momentum is shared between the two photons, may compensate α
- The calculation of the box amplitude requires the description of intermediate nucleon excitation and of their FFs at any Q^2
- Different calculations give quantitatively different results



Theory not enough constrained!

Interaction of 4 spin ½ fermions

16 amplitudes in the general case.
- P- and T-invariance of EM interaction,
- helicity conservation

- **One-photon exchange**

  - Two form factors (real in SL, complex in TL)
  - Functions of one variable

- **Two-photon exchange**

  - Three (complex) amplitudes
  - Functions of two variables

Is it still possible to extract the « real » FFs in presence of 2γ exchange in <u>Model Independent</u> way?

In Space-like region ->

Possible but difficult !

Space-like region :

-electron and positron beams

- longitudinally polarized ,
- in identical kinematical conditions

Generalization of the polarization method
(A. Akhiezer and M.P. Rekalo)

- Three T-odd polarization observables
(Ay, Py(le), Dab(le))

or

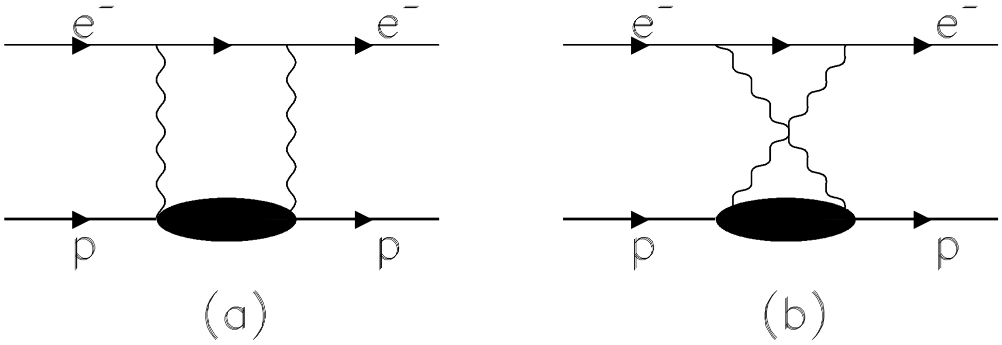
- five T-even polarization observables….
($d\sigma/d\Omega$, Px(λe), Pz(λe), Dxx, Dyy,or Dzz, Dxz)

M. P. Rekalo and E. T-G Nucl. Phys. A740 (2004) 271,
M. P. Rekalo and E. T-G Nucl. Phys. A742 (2004) 322
M. P. Rekalo, E. T.-G. , EPJA (2004), Nucl. Phys. A (2003)

Interference of $1\gamma \otimes 2\gamma$ exchange

– One reason that it might be large: enhancement due to the fast decreasing of form factors (transferred momentum equally shared between the two photons).

Interference of $1\gamma \otimes 2\gamma$ exchange



- Explicit calculation for structureless proton
  - The contribution is small, for unpolarized and polarized ep scattering
  - Does not contain the enhancement factor *L*
  - The relevant contribution to *K* is ~ 1

E.A.Kuraev, V. Bytev, Yu. Bystricky, E.T-G, Phys. Rev. D74, 013003 (2006)

Interference of 1$\gamma \otimes 2\gamma$ exchange

- One reason that it might be large: enhancement due to the fast decreasing of form factors (transferred momentum equally shared between the two photons).
- *In this case it should be larger for deuteron & at large Q^2*

BUT NO EVIDENCE

from Hall A and Hall C data on *ed* elastic scattering

M.P. Rekalo, E.T-G, D. Prout, Phys. Rev. C 60, 042202

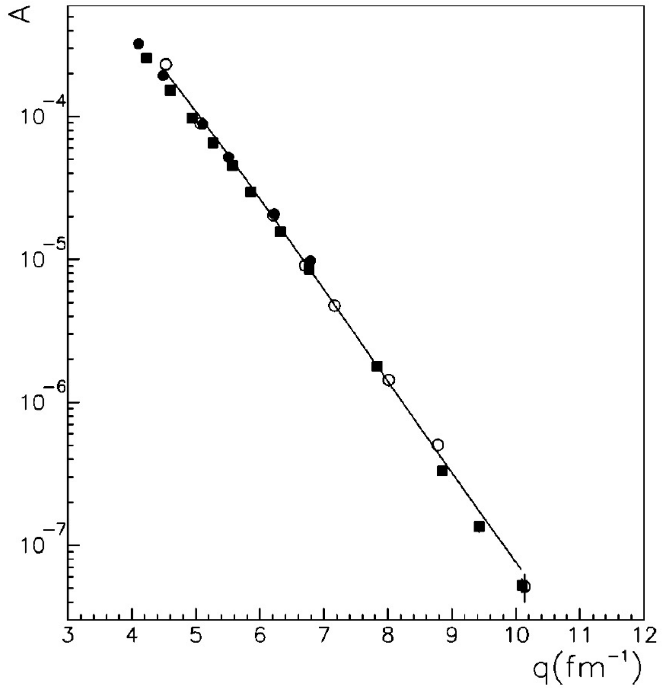
Interference of 1γ ⊗2γ exchange

M.P. Rekalo, E.T-G, D. Prout, Phys. Rev. C 60, 042202 (1999)

for deuteron at large Q²

$$\frac{d\sigma}{d\Omega_e} = \left(\sigma_0 \cot^2 \frac{\theta_e}{2} \right) \Bigg|_{\theta_e = 8^\circ} \frac{p_1}{(1+q^2/p_2)^{p_3}},$$

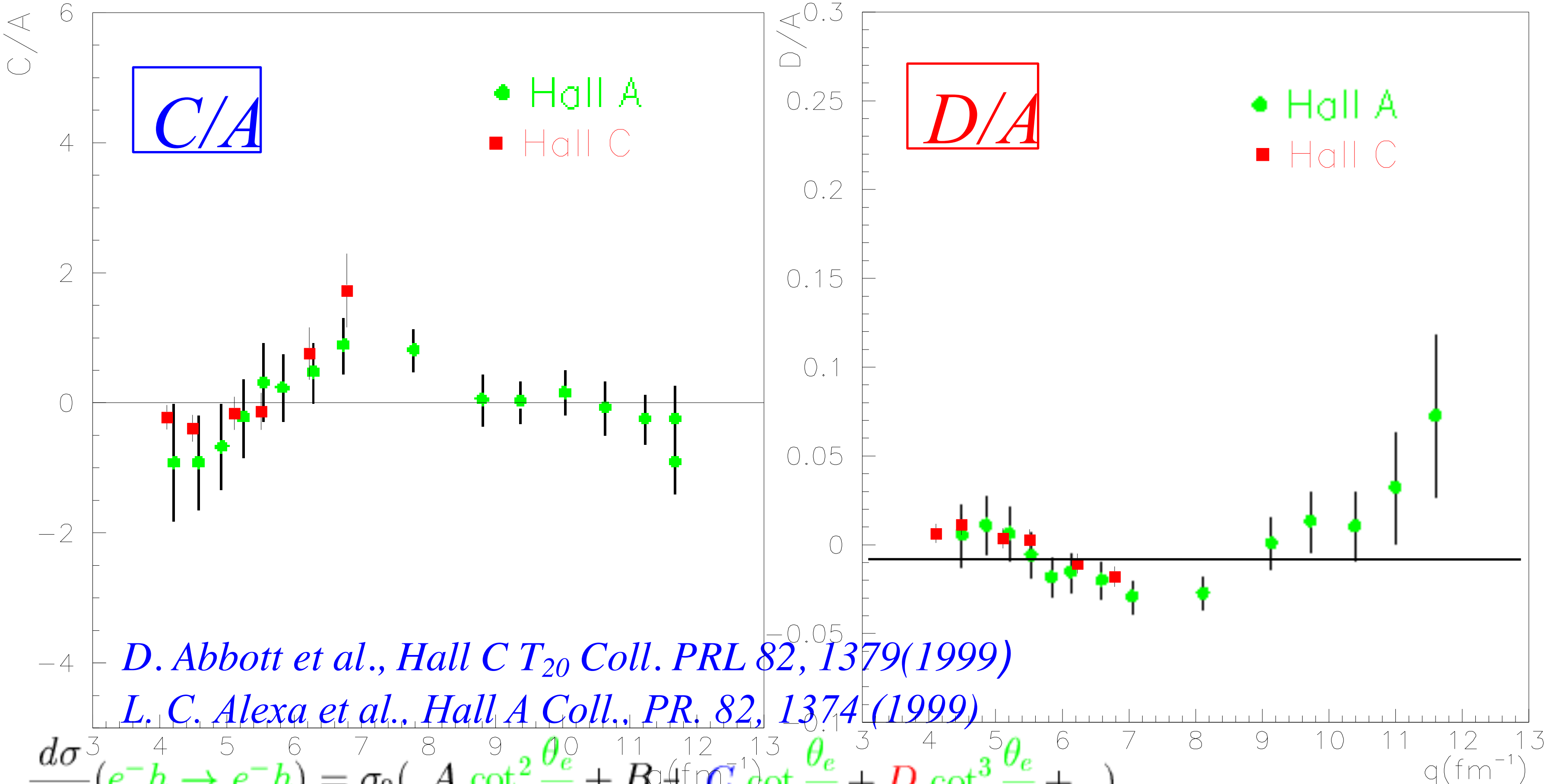
$$\frac{\left[\frac{d\sigma}{d\Omega_e}(e^- d) - \frac{d\sigma}{d\Omega_e}(e^+ d) \right]}{\left[\frac{d\sigma}{d\Omega_e}(e^- d) + \frac{d\sigma}{d\Omega_e}(e^+ d) \right]} = \frac{C \cot \frac{\theta_e}{2} + D \cot^3 \frac{\theta_e}{2}}{A \cot^2 \frac{\theta_e}{2} + B}.$$



R. G. Arnold et al., Phys. Rev. Lett. 35, 776 (1975)

2γ exchange in *ed* elastic scattering

M.P. Rekalo, E.T-G, D. Prout, Phys. Rev. C 60, 042202 (1999)



D. Abbott et al., Hall C T_{20} Coll. PRL 82, 1379(1999)

L. C. Alexa et al., Hall A Coll., PR. 82, 1374 (1999)

$$\frac{d\sigma}{d\Omega_e}(e^- h \to e^- h) = \sigma_0\left(A\cot^2\frac{\theta_e}{2} + B + C\cot\frac{\theta_e}{2} + D\cot^3\frac{\theta_e}{2} + ..\right)$$

Interference of $1\gamma \otimes 2\gamma$ exchange

Several reasons for which it should keep small :

- No evidence from Hall A and Hall C data on *ed* elastic scattering *M.P. Rekalo, E.T-G, D. Prout, Phys. Rev. C 60, 042202*
- Cancellation between elastic and inelastic channels. Sum rules *Yu. Bystricky, E.A.Kuraev, E. T.-G, Phys. Rev. C 75, 015207 (2007)*
- $e\mu$ elastic scattering can be calculated exactly and it is an upper limit of *ep* elastic scattering.

E.A.Kuraev, E. T.-G, Physics of Particles and Nuclei Letters, 7, (2010) 67
A.de Rujula, J. M. Kaplan, and E. De Rafael NPB35, 365 (1971); NPB 53, 545 (1973)

- No evidence from the ε-dependence on PL/PT ratio
- No evidence from time-like region at large Q^2

Is it still possible to extract the « real » FFs in presence of 2γ exchange in Model Independent way?

Time-like region -> much easier!

- Large Q^2
- *Large efforts put in Radiative Corrections calculations and MonteCarlo at e^+e^- colliders!*

Time-like observables: $|G_E|^2$ and $|G_M|^2$

-The cross section for $\overline{p} + p \to e^+ + e^-$ (1 γ-exchange):

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{8m^2\sqrt{\tau-1}}\left[\tau|G_M|^2(1+\cos^2\theta) + |G_E|^2\sin^2\theta\right]$$

θ: angle between e^- and $\overline{p}$ in cms.

A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto, Il Nuovo Cimento XXIV, 170 (1962)
B. Bilenkii, C. Giunti, V. Wataghin, Z. Phys. C 59, 475 (1993).
G. Gakh, E.T-G., Nucl. Phys. A761,120 (2005).

As in SL region:
- Dependence on q^2 contained in FFs
- Even dependence on $\cos^2\theta$ (1γ exchange)
- No dependence on sign of FFs
- Enhancement of magnetic term

but TL form factors are complex!

Unpolarized cross section

-The cross section for $\overline{p} + p \to e^+ + e^-$ (1 γ-exchange):

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{8m^2\sqrt{\tau-1}}\left[\tau|G_M|^2(1+\cos^2\theta) + |G_E|^2\sin^2\theta\right]$$

θ: angle between e^- and $\overline{p}$ in cms.

Two Photon Exchange:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4q^2}\sqrt{\frac{\tau}{\tau-1}}D,$$

- Induces four new terms
- Odd function of θ:
- Does not contribute at θ=90°

$$D = (1+\cos^2\theta)(|G_M|^2 + 2ReG_M\Delta G_M^*) + \frac{1}{\tau}\sin^2\theta(|G_E|^2 + 2ReG_E\Delta G_E^*) +$$
$$2\sqrt{\tau(\tau-1)}\cos\theta\sin^2\theta Re(\frac{1}{\tau}G_E - G_M)F_3^*.$$

M.P. Rekalo and E. T.-G., EPJA 22, 331 (2004)
G.I. Gakh and E. T.-G., NPA761, 120 (2005)

Symmetry Relations(annihilation)

- Differential cross section at complementary angles:

The SUM cancels the 2γ contribution:

$$\frac{d\sigma_+}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi-\theta) = 2\frac{d\sigma^{Born}}{d\Omega}(\theta)$$

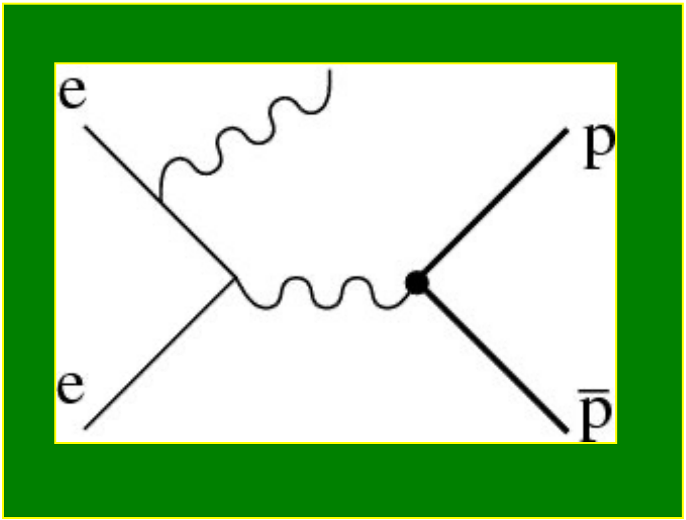
The DIFFERENCE enhances the 2γ contribution:

$$\frac{d\sigma_-}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi-\theta) = 4N\Big[(1+x^2)ReG_M\Delta G_M^* + \\ +\frac{1-x^2}{\tau}ReG_E\Delta G_E^* + \sqrt{\tau(\tau-1)}x(1-x^2)Re(\frac{1}{\tau}G_E - G_M)F_3^*\Big]$$

$$\tau = \frac{q^2}{4m^2}, \quad x = cos\theta$$

What about data?

Radiative Return (ISR)



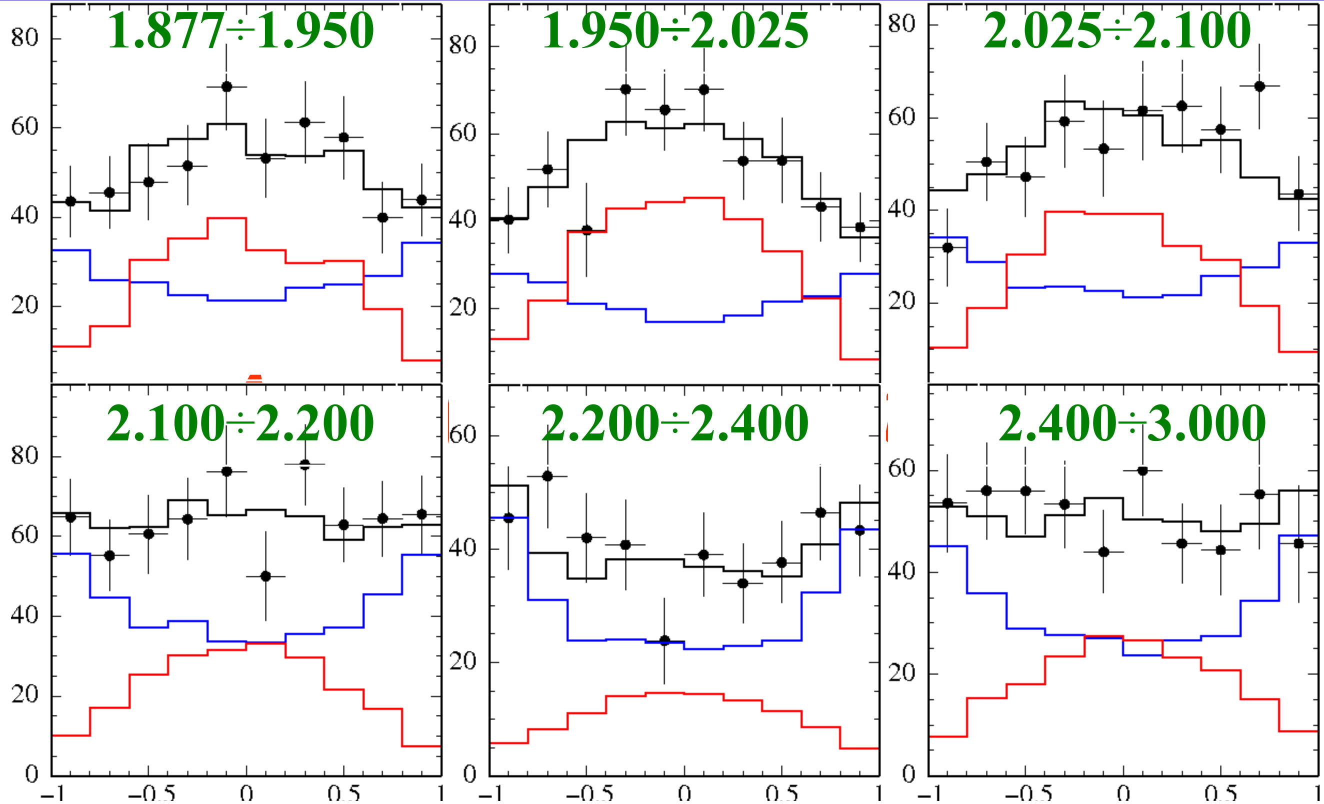
$$e^+ + e^- \rightarrow p + \bar{p} + \gamma$$

$$\frac{d\sigma(e^+e^- \rightarrow p\bar{p}\gamma)}{dm\, d\cos\theta} = \frac{2m}{s} W(s,x,\theta)\,\boxed{\sigma(e^+e^- \rightarrow p\bar{p})(m),} \quad x = \frac{2E_\gamma}{\sqrt{s}} = 1 - \frac{m^2}{s},$$

$$W(s,x,\theta) = \frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta} - \frac{x^2}{2}\right), \quad \theta \gg \frac{m_e}{\sqrt{s}}.$$

B. Aubert (BABAR Collaboration) Phys Rev. ***D73****, 012005 (2006)*

Angular Distributions



Events/0.2 vs. cos θ

$$\frac{dN}{d\cos\theta_p} = A\left[H_M(\cos\theta, M_{p\bar{p}}) + \left|\frac{G_E}{G_M}\right|^2 H_E(\cos\theta, M_{p\bar{p}})\right]$$

BABAR

2γ-exchange?

Angular Asymmetry

BABAR

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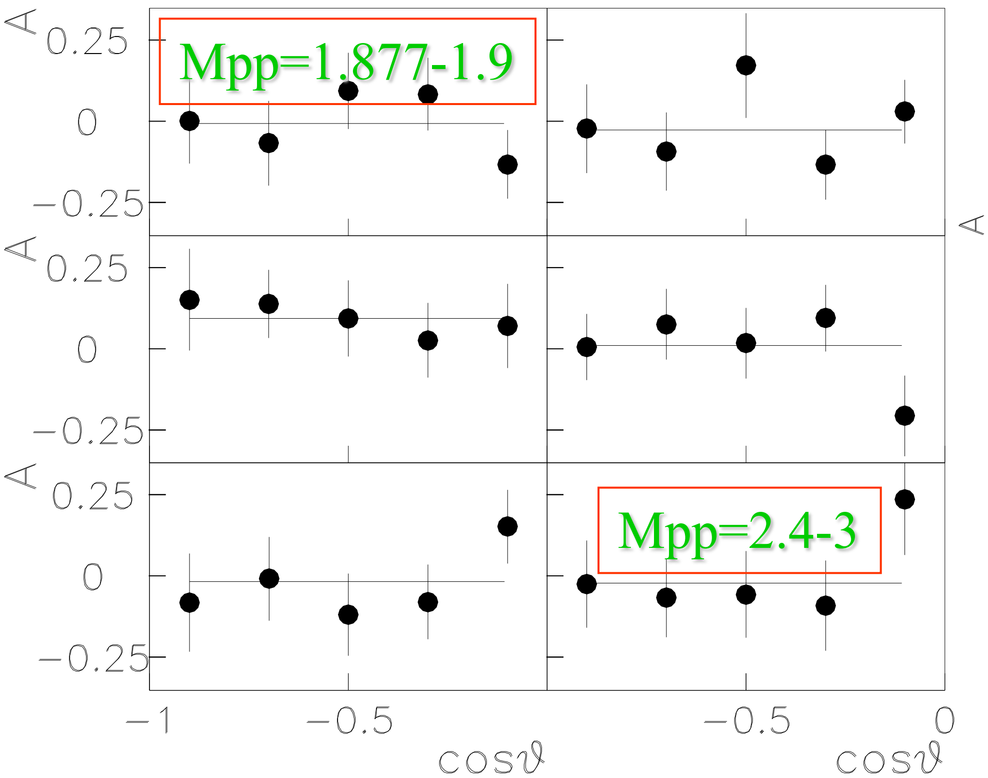
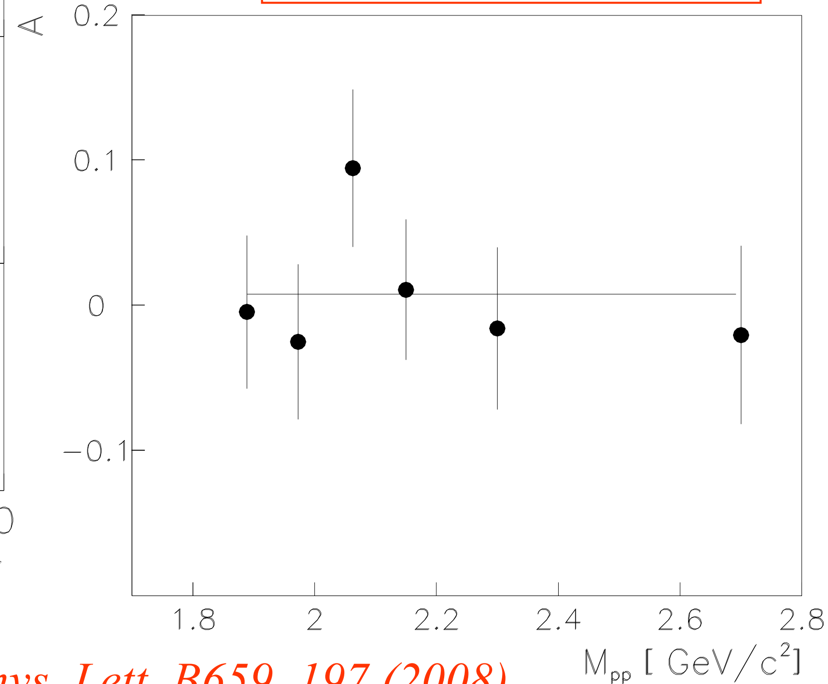
$$\mathcal{A}(c) = \frac{\frac{d\sigma}{d\Omega}(c) - \frac{d\sigma}{d\Omega}(-c)}{\frac{d\sigma}{d\Omega}(c) + \frac{d\sigma}{d\Omega}(-c)}$$



A=0.01±0.02

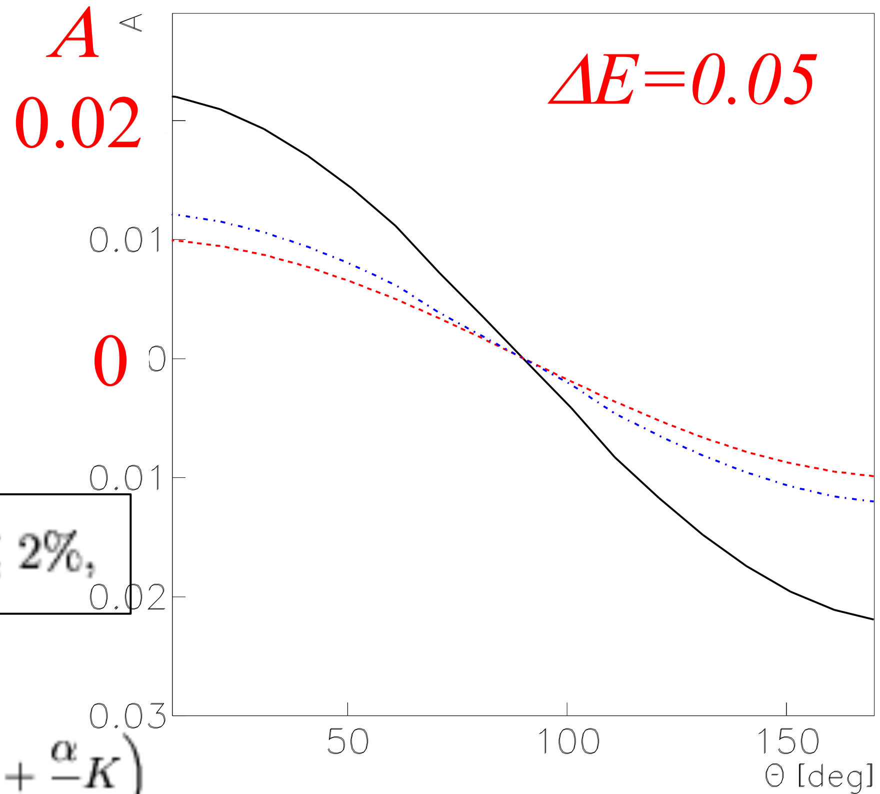


E. T.-G., E.A. Kuraev, S. Bakmaev, S. Pacetti, Phys. Lett. B659, 197 (2008)

Structure Function Method

E.A. Kuraev, V. Meledin, Nucl. Phys. B122, 485 (1977)

$$e^+ + e^- \to p + \bar{p} + \gamma$$



$$A^{soft}(E) \simeq \frac{2\alpha}{\pi}\left(\ln\frac{1+\beta c}{1-\beta c}\ln\frac{\Delta E}{E}\right)$$

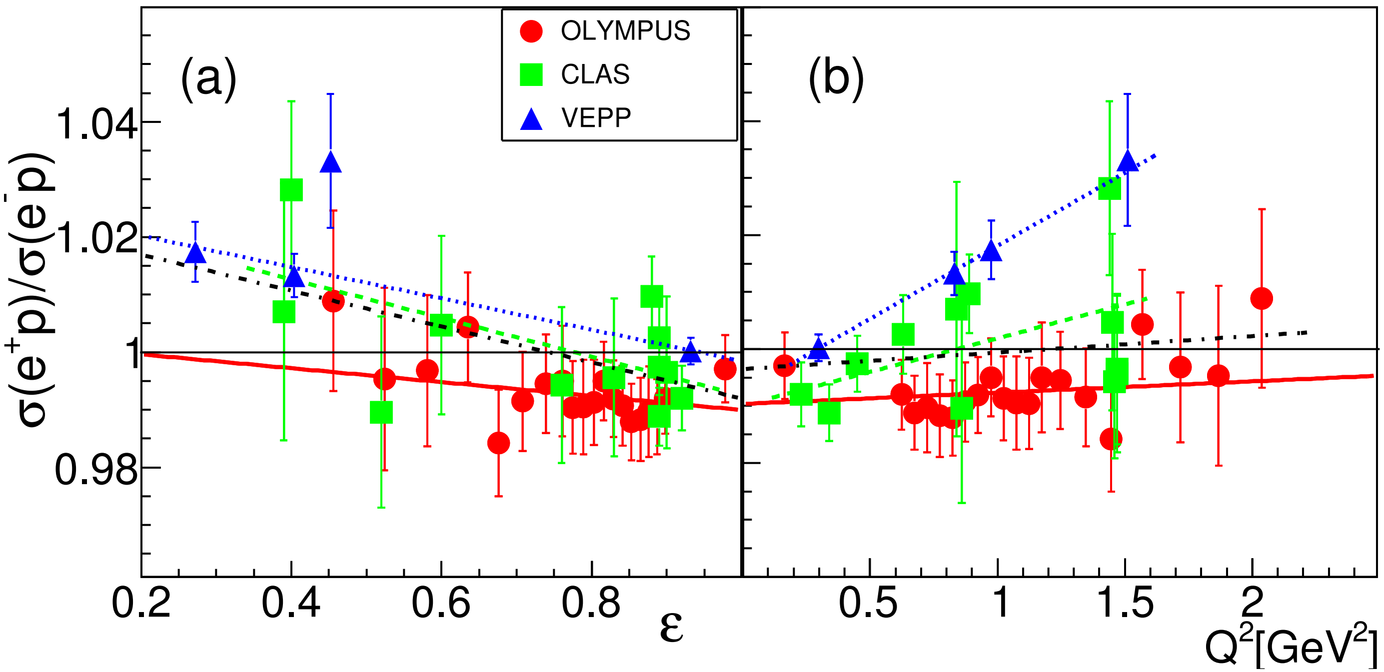
$$A^{tot} = A^{soft} + A^{hard} = \frac{2\alpha}{\pi}\psi(c,\beta), \ |A^{tot}| \leq 2\%,$$

$$\frac{d\sigma}{d\Omega}(c) \pm \frac{d\sigma}{d\Omega}(-c) \sim \int dx_1 \mathcal{D}(x_1,L)\mathcal{D}(x_2,L)dx_2\left(1+\frac{\alpha}{\pi}K\right)$$
$$[d\sigma_B(p_- x_1, p_+ x_2, c) \pm d\sigma_B(p_- x_1, p_+ x_2, -c)]\,,$$

$$\frac{d\sigma}{d\Omega}(c) + \frac{d\sigma}{d\Omega}(-c) = 2\frac{d\sigma_0}{d\Omega}\left[1+\frac{\alpha}{\pi}\left(\frac{3}{2}L - 2(L-1)\ln\frac{\Delta E}{E} + \frac{\pi^2}{3} - 2\right)\right],\ L = \ln\frac{t}{m^2},$$

E. T.-G., E.A. Kuraev, S. Bakmaev, S. Pacetti, Phys. Lett. B659, 197 (2008)

Electron & positron beams



CLAS, VEPP, OLYMPUS….

- $Q^2<2$ GeV^2
- Effect < 2%
- No evident increase with Q^2

Radiative Corrections in α^3

The Born $e^{\pm}$ cross section σ_{el} is the measured cross section after applying radiative corrections

$$d\sigma^{\pm}_{\text{meas}} = d\sigma_{el}(1+\delta^{\pm}), \; d\sigma_{el} = \frac{d\sigma^{\pm}_{\text{meas}}}{(1+\delta^{\pm})},$$

C-Even

C-Odd

$$\delta_{\text{odd}} = \delta_{2\gamma} + \delta_s.$$

Brehmstrahlung

Virtual 2γ exchange

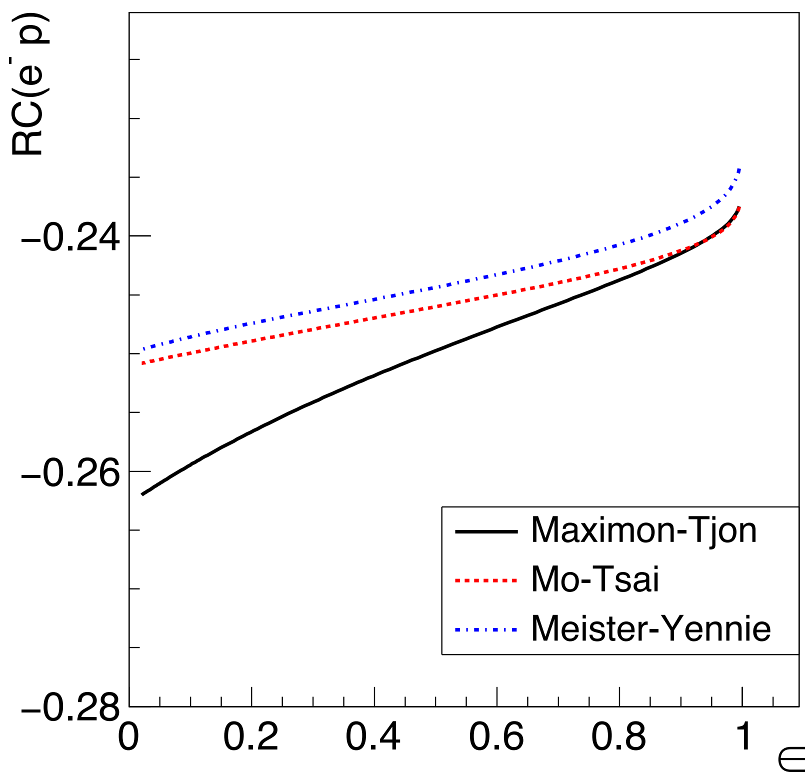
Soft part: *infrared divergencies that cancel with the real soft photon contribution* - *QED exact*

Hard part:
both virtual photons are hard
3(6) FFs, off-shell proton…
Model dependent

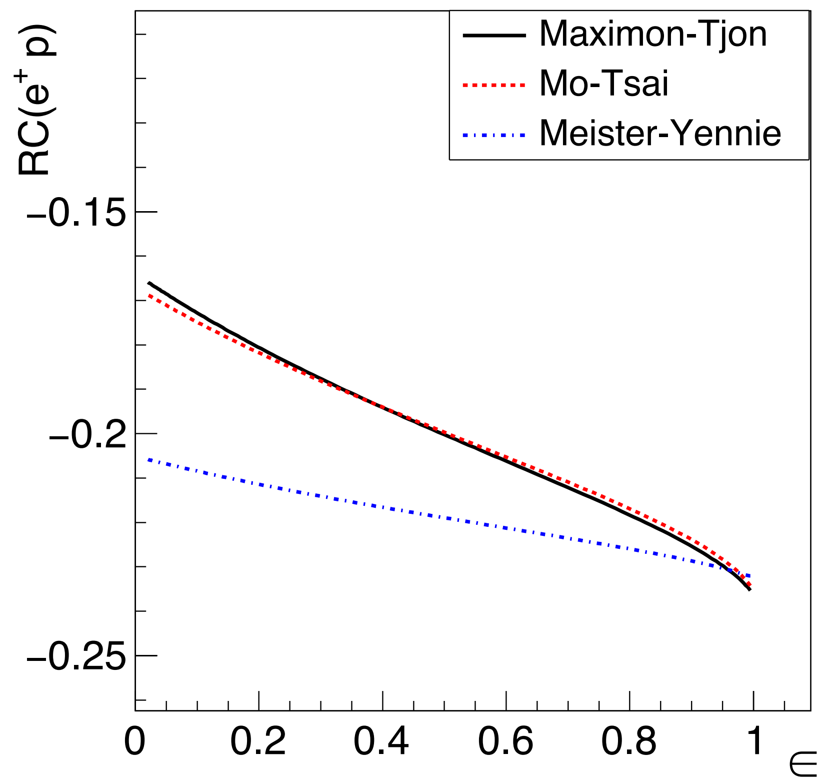
The splitting in different terms may differ in calculations

Radiative Corrections (α^3)

electrons



positrons



$Q^2 = 1\ \mathrm{GeV}^2 \quad \Delta E = 0.01\ E'$

Electron & positron beams

A deviation from unity of the ratio:

$$R^{\text{meas}} = \frac{d\sigma^{\text{meas}}(e^+p \to e^+p)}{d\sigma^{\text{meas}}(e^-p \to e^-p)} = \frac{1 + \delta_{\text{even}} - \delta_{2\gamma} - \delta_s}{1 + \delta_{\text{even}} + \delta_{2\gamma} + \delta_s}$$

Is a clear signature of C-odd contributions (soft or hard)

A C-odd contribution to the cross section is enhanced in the Ratio

$$A^{\text{odd}} = \frac{d\sigma(e^+p \to e^+p) - d\sigma(e^-p \to e^-p)}{d\sigma(e^+p \to e^+p) + d\sigma(e^-p \to e^-p)} = \frac{\delta_{\text{odd}}}{1 + \delta_{\text{even}}} = \frac{R-1}{R+1}, \quad R = \frac{1 + A_{\text{odd}}}{1 - A_{\text{odd}}}.$$

By correcting R^{meas} by δ_{even} and δ_s

$$R_{2\gamma} \simeq \frac{1 - \delta_{2\gamma}}{1 + \delta_{2\gamma}},$$

Charge Asymmetry

The charge asymmetry including soft γ and hard 2γ

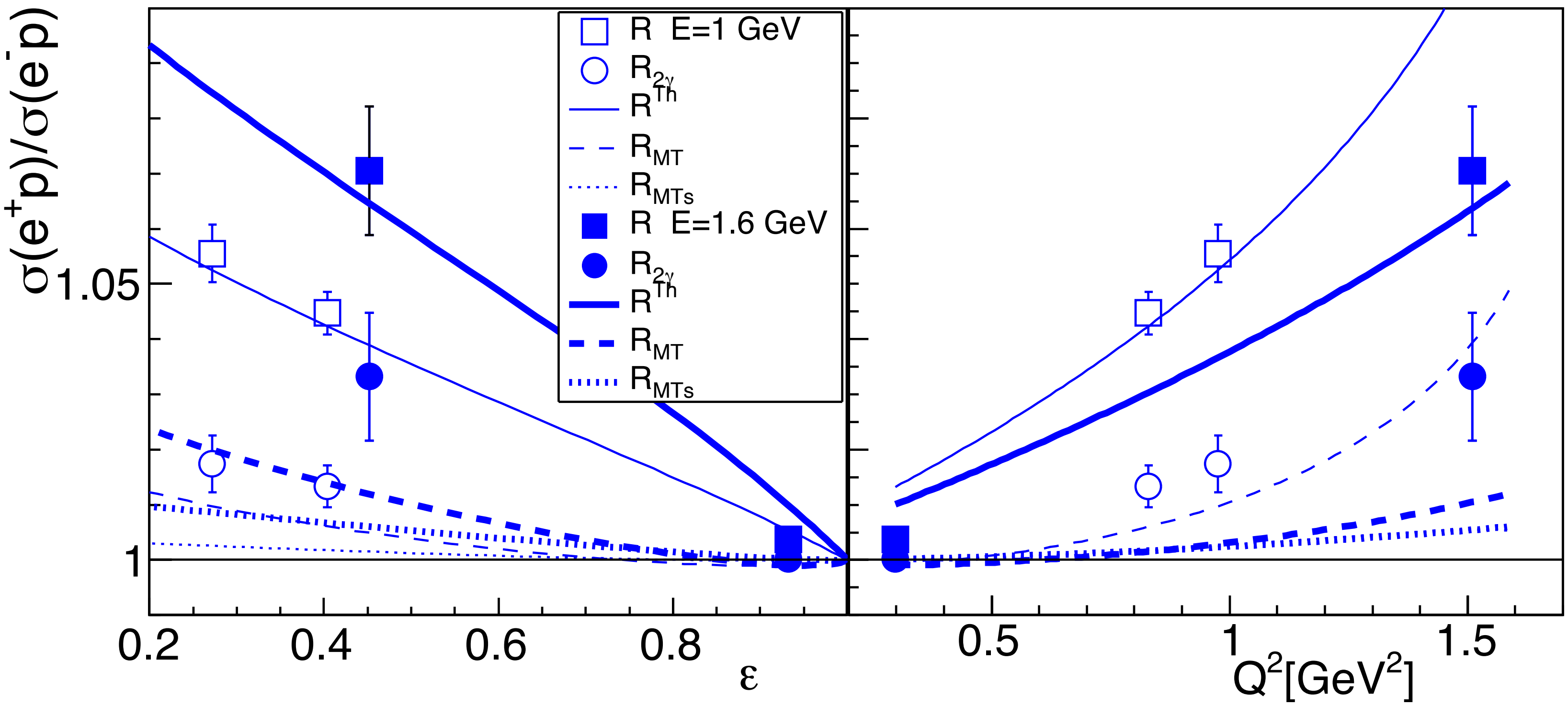
$$A_{\rm odd}^{K} = \frac{d\sigma^{e^+p} - d\sigma^{e^-p}}{d\sigma^{e^+p} + d\sigma^{e^-p}}$$

$$= \frac{2\alpha}{\pi(1+\delta_{\rm even})}\left[\ln\frac{1}{\rho}\ln\frac{(2\Delta E)^2}{ME} - \frac{5}{2}\ln^2\rho + \ln x \ln\rho + \mathrm{Li}_2\left(1-\frac{1}{\rho x}\right) - \mathrm{Li}_2\left(1-\frac{\rho}{x}\right)\right],$$

$$\rho = \left(1-\frac{Q^2}{s}\right)^{-1} = 1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}, \quad x = \frac{\sqrt{1+\tau}+\sqrt{\tau}}{\sqrt{1+\tau}-\sqrt{\tau}}.$$

E. A. Kuraev, V. V. Bytev, S. Bakmaev, and E. T.-G., PRC 78, 015205 (2008)

VEPP

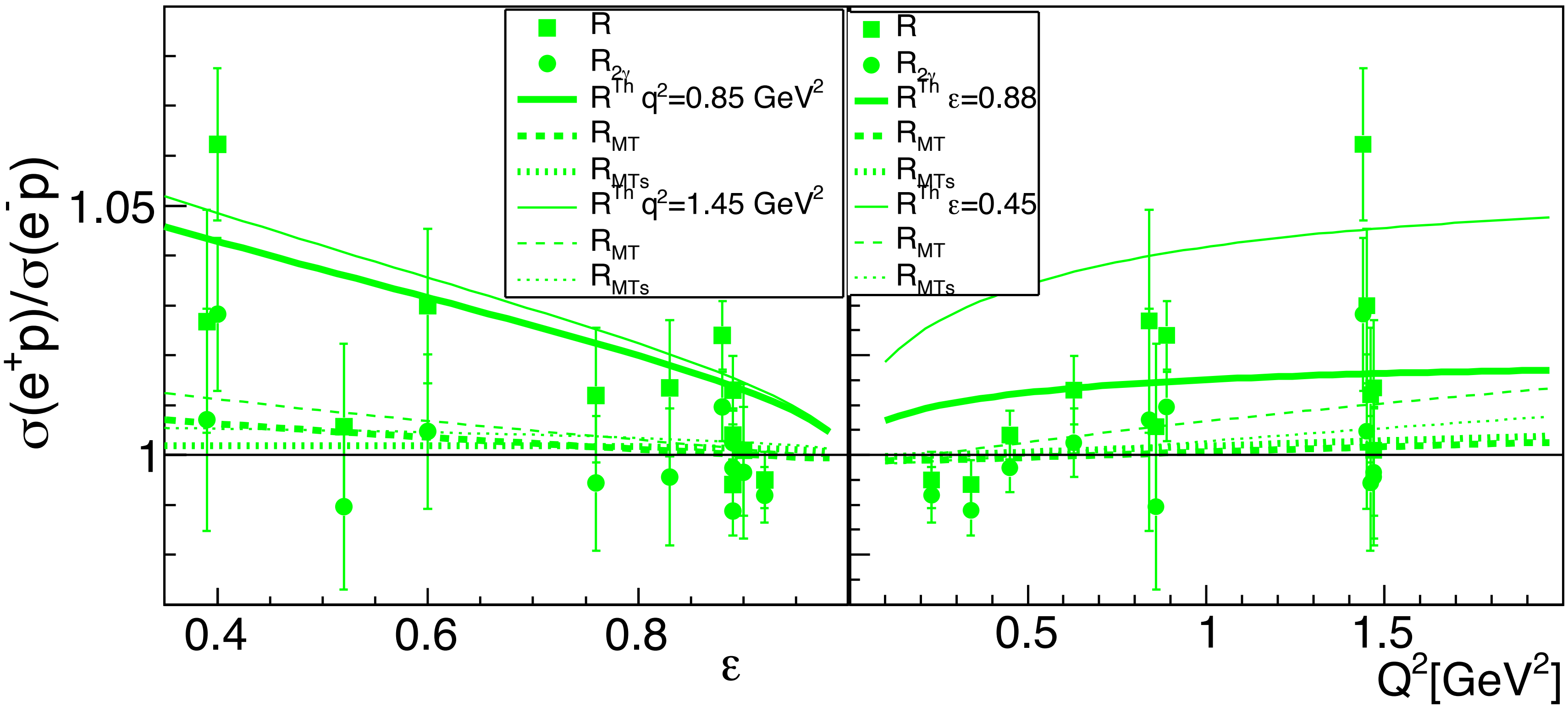


From R^{meas} to $R_{2\gamma}{}^{K}$

remove the odd contribution included in the data

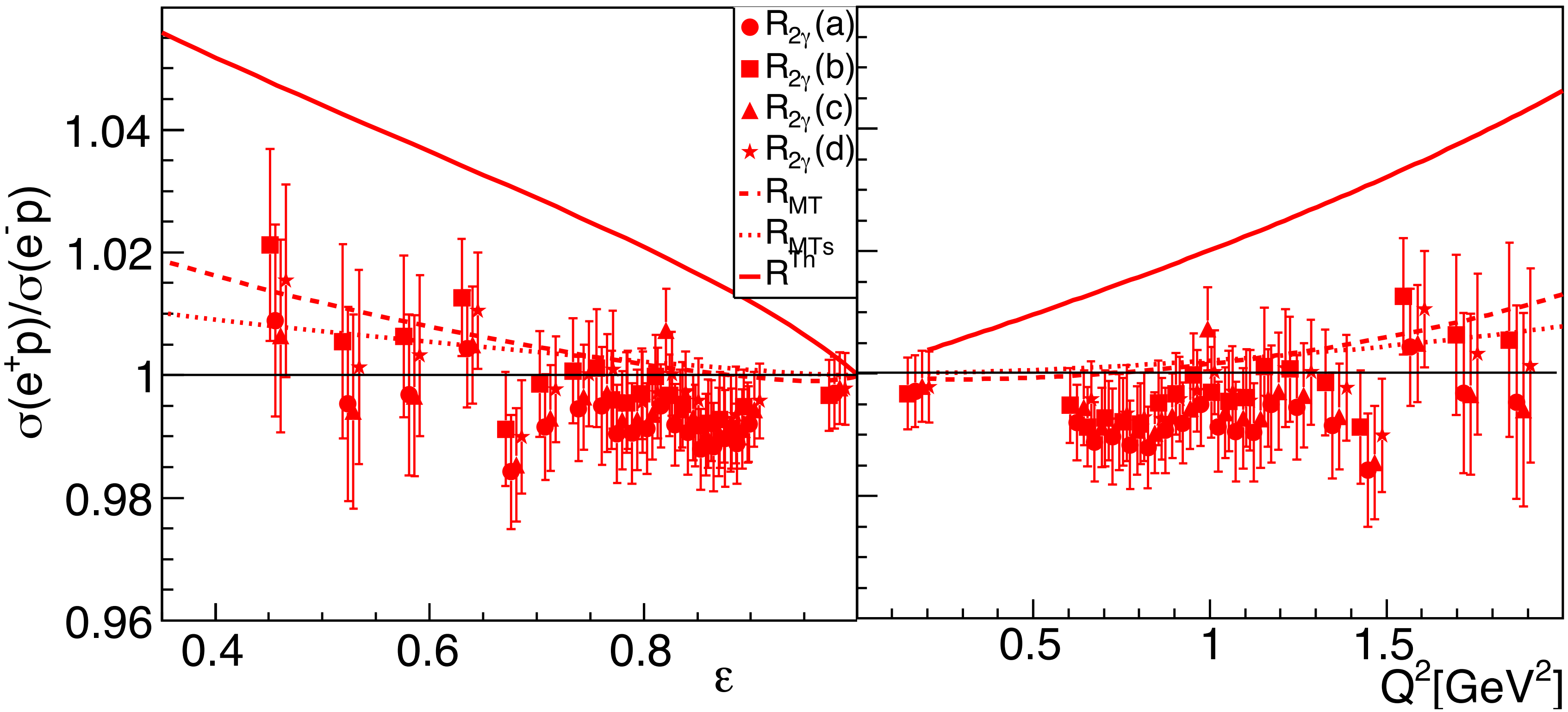
$$R_{2\gamma}^{K} = \frac{1 - A_{\rm odd}^{K}(1+\delta_{\rm even}) + \delta_M}{1 + A_{\rm odd}^{K}(1+\delta_{\rm even}) - \delta_M},$$

CLAS



R
$R_{2\gamma}$
R^{Th} q^2=0.85 GeV^2
R_{MT}
R_{MTs}
R^{Th} q^2=1.45 GeV^2
R_{MT}
R_{MTs}
R
$R_{2\gamma}$
R^{Th} ε=0.88
R_{MT}
R_{MTs}
R^{Th} ε=0.45
R_{MT}
R_{MTs}
$\sigma(e^+p)/\sigma(e^-p)$
1.05
1
0.4
0.6
0.8
ε
0.5
1
1.5
Q^2[GeV^2]

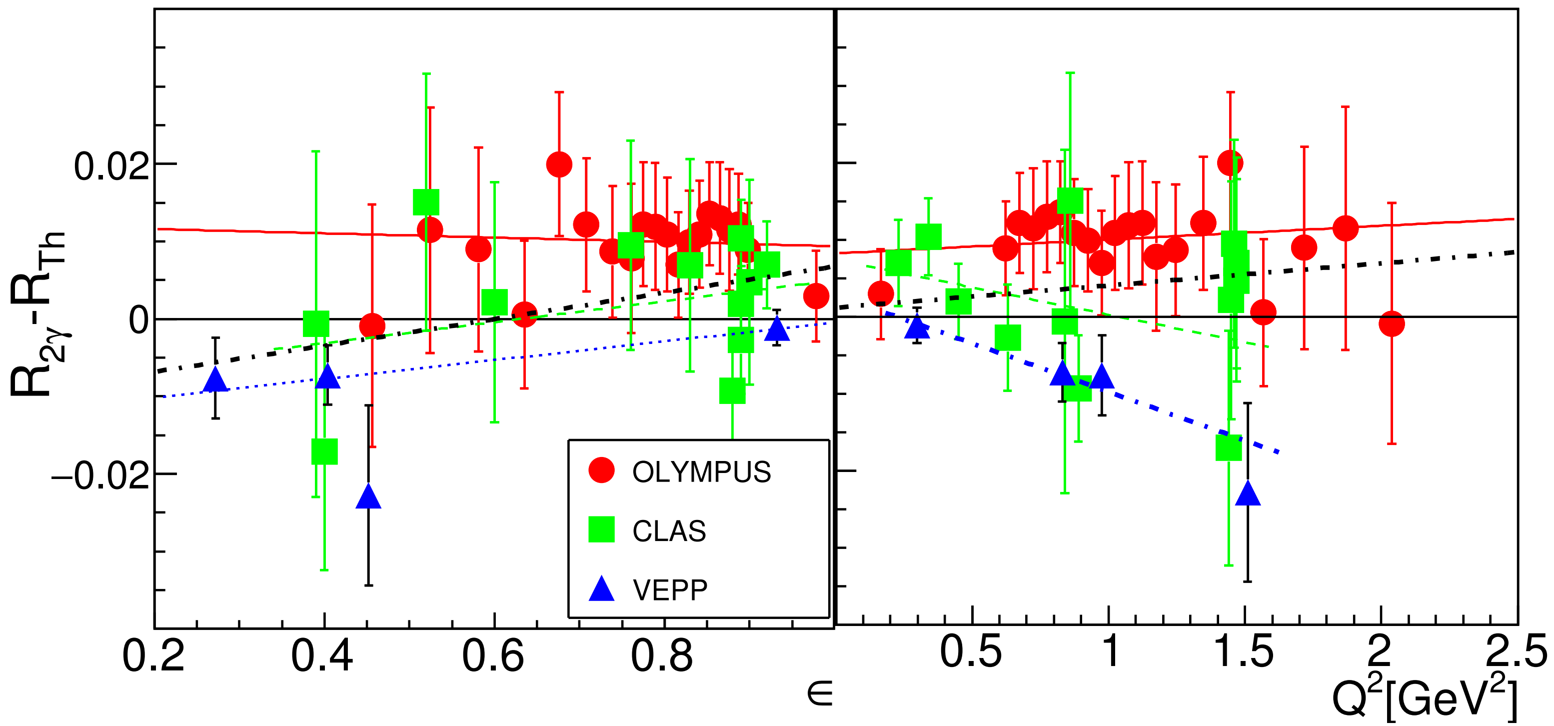
OLYMPUS



$\sigma(e^+p)/\sigma(e^-p)$
1.04
1.02
1
0.98
0.96
0.4
0.6
0.8
ε
$R_{2\gamma}$(a)
$R_{2\gamma}$(b)
$R_{2\gamma}$(c)
$R_{2\gamma}$(d)
R_{MT}
R_{MTs}
R^{Th}
0.5
1
1.5
Q^2[GeV^2]

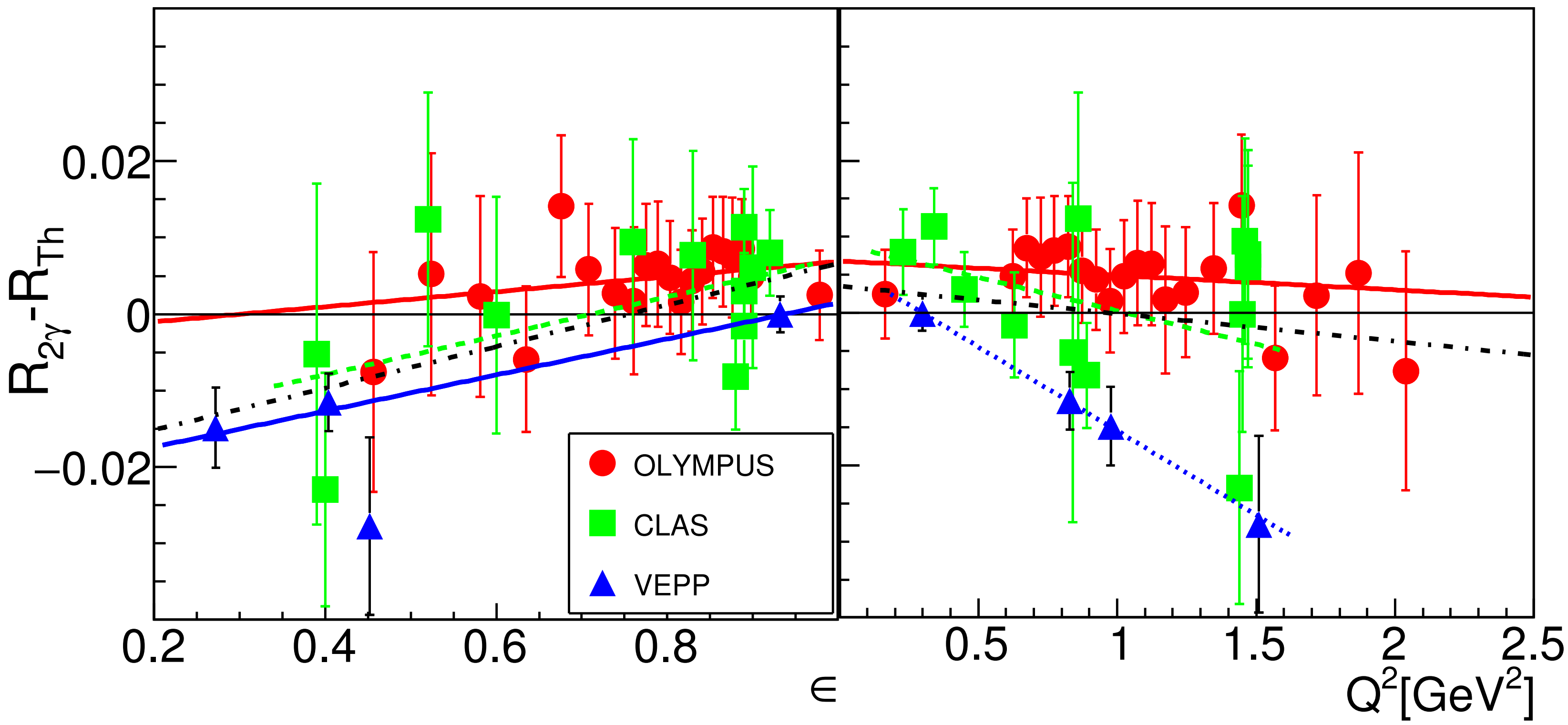
$R_{2\gamma} - R_{Th}$ (Mo & Tsai)

$$R_{2\gamma}^{K} = \frac{1 - A_{\rm odd}^{K}(1+\delta_{\rm even}) + \delta_M}{1 + A_{\rm odd}^{K}(1+\delta_{\rm even}) - \delta_M}$$



$R_{2\gamma} - R_{Th}$ (Maximon & Tjon)

$$R_{2\gamma}^{K} = \frac{1 - A_{\text{odd}}^{K}(1+\delta_{\text{even}}) + \delta_M}{1 + A_{\text{odd}}^{K}(1+\delta_{\text{even}}) - \delta_M}$$



Conclusions

- *High order radiative corrections* are mandatory to claim a percent precision on the observables

- Effects as correlations and normalizations should be carefully scrutinized

  see GEp as a parameter

  ε-derivative of the reduced elastic cross section

- Two photon exchange as *K*-factor

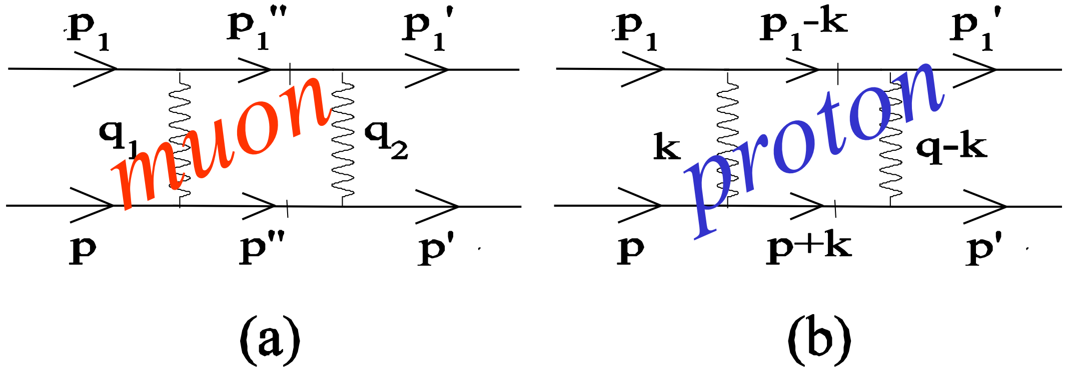
Radiative corrections modify not only the absolute values but also the dependence of the observables on the relevant kinematical variables

Thank you for attention

Results obtained in collaboration with M.P. Rekalo, G.I.Gakh, E.A. Kuraev, V.V. Bytev, Yu. Bystriskiy, S. Pacetti

QED versus QCD

Imaginary part of the 2γ amplitude



$$\mathcal{M}_{1a} = \frac{1}{\sqrt{8s}} \int \frac{dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}$$

$$\mathcal{M}_{1b} = \frac{1}{\sqrt{8s}} \int \frac{dQ_1^2 dQ_2^2 F(Q_1^2)F(Q_2^2)}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}$$

$$dO_1'' = \frac{2dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1 Q_0^2}}, \quad \mathcal{D}_1 = 2(Q_1^2+Q_2^2)Q^2Q_0^2 - 2Q^2Q_1^2Q_2^2 - (Q_1^2-Q_2^2)Q_0^2 - (Q^2)^2Q_0^2$$

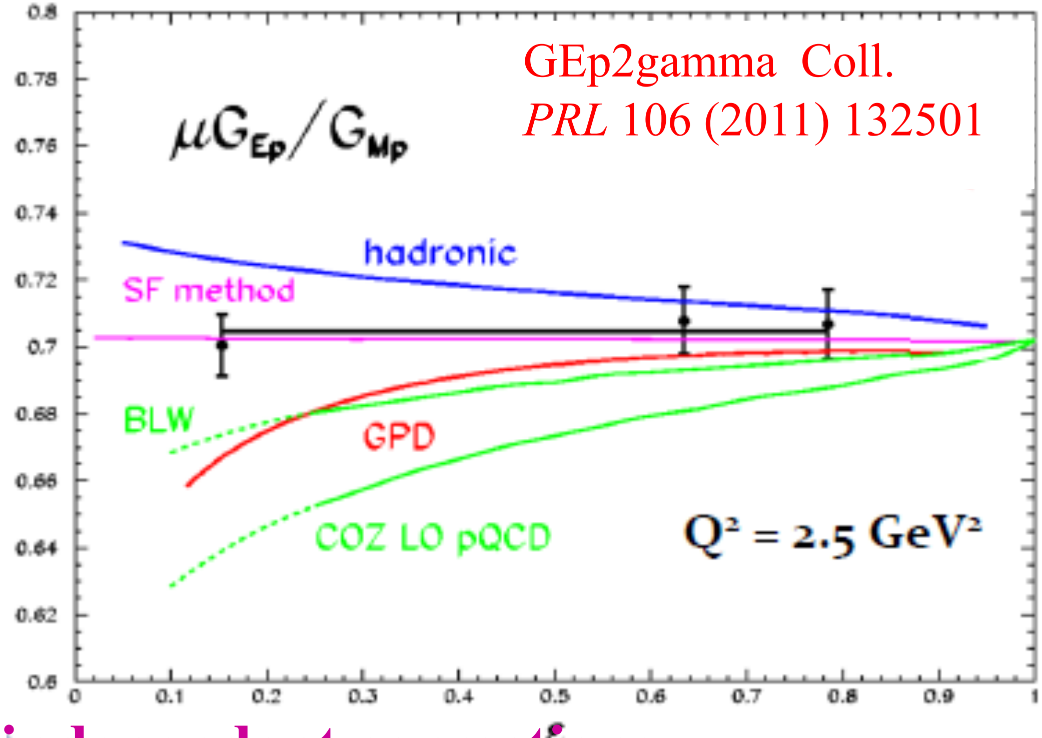
e-μ scattering constitutes an upper limit of e-p!

E.A.Kuraev, E. T.-G, Physics of Particles and Nuclei Letters, 7, (2010) 67

Polarization ratio (ε-dependence)

- DATA: No evidence of ε-dependence at 1% level

- *MODELS: large correction (opposite sign) at small ε*



•SF method: ε-(almost)independent corrections

- *Theory: corrections to the Born approximation at Q^2= 2.5 GeV2*
  - Y. Bystritskiy, E.A. Kuraev and E.T.-G, Phys.Rev.C75: 015207 (2007)
  - P. Blunden et al., Phys. Rev. C72:034612 (2005) (mainly G_M)
  - A. Afanasev et al., Phys. Rev. D72:013008 (2005) (mainly G_E)
  - N.Kivel and M.Vanderhaeghen, Phys. Rev. Lett.103:092004 (2009). (high Q^2)

PHYSICAL REVIEW C 77, 055205 (2008)

Radiative corrections to the deeply virtual Compton scattering electron tensor

V. V. Bytev and E. A. Kuraev
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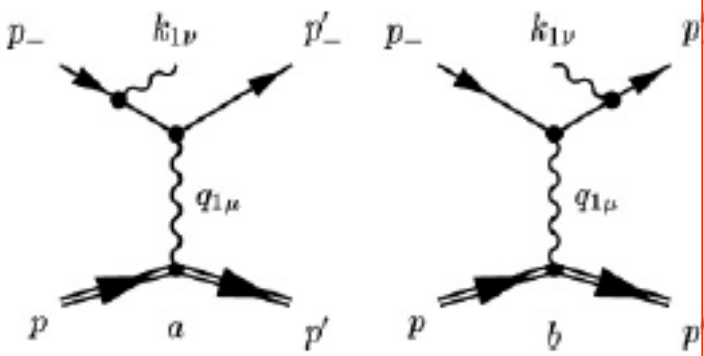
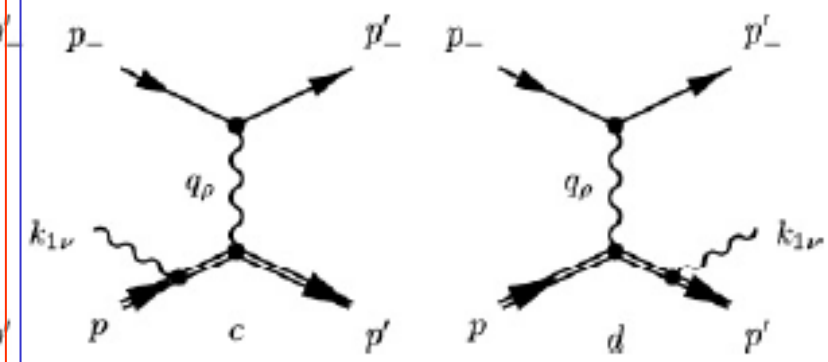
$$e^-(p_-) + \mu(p) \to e^-(p'_-) + \mu(p') + \gamma(k_1).$$

Bethe-Heitler



DVCS



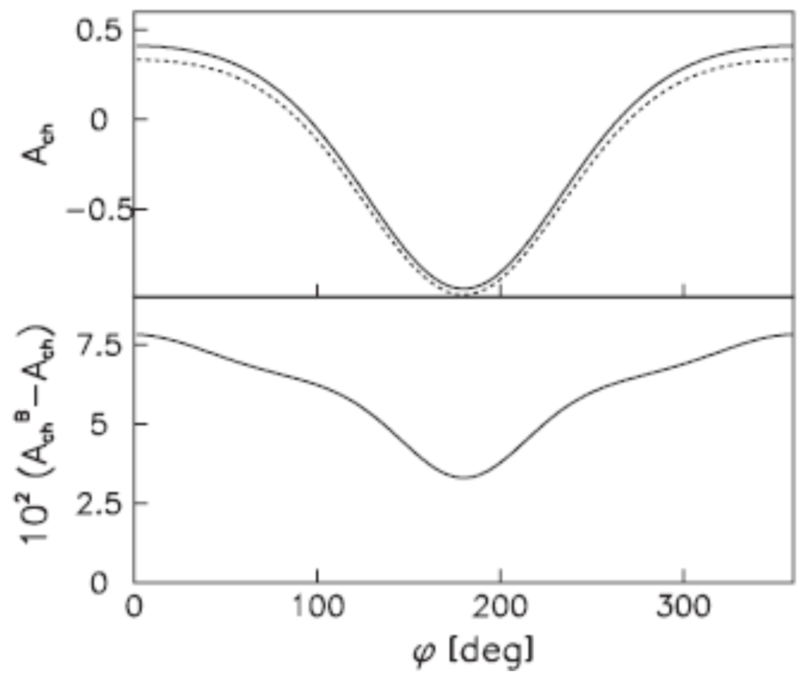
$$d\sigma^{\text{tot}}(e^- p \to e^- p\gamma) = d\sigma^{\text{BH}} + d\sigma^{\text{DVCS}} + d\sigma^{\text{odd}},$$

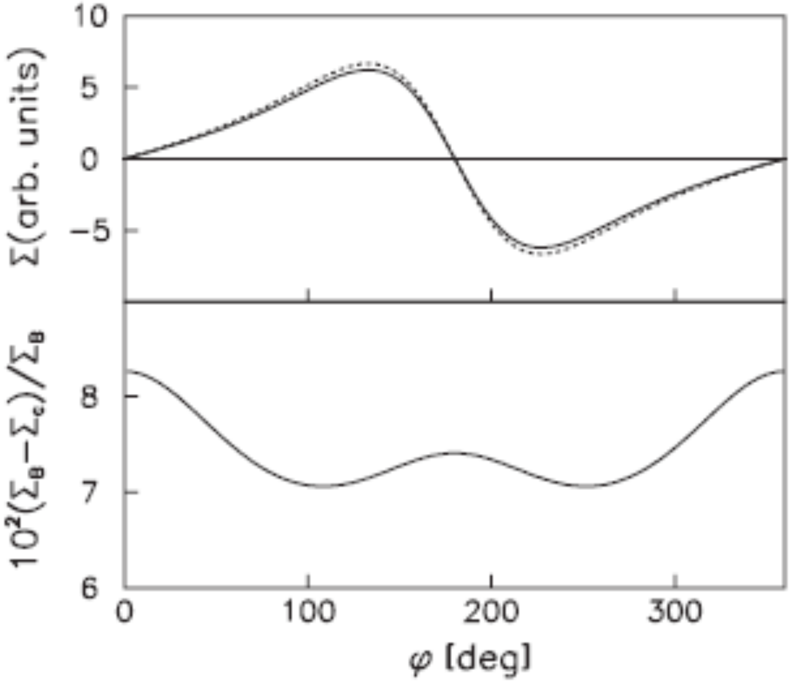
Interference

DVCS CHARGE and HELICITY Asymmetry

$$A_{ch} = \frac{d\sigma^{e^-\mu\to e^-\mu\gamma} - d\sigma^{e^+\mu\to e^+\mu\gamma}}{d\sigma^{e^-\mu\to e^-\mu\gamma} + d\sigma^{e^+\mu\to e^+\mu\gamma}}.$$

$$\frac{d^4\Sigma}{d\phi} = \frac{1}{2}\left(\frac{d\sigma^{\rightarrow}}{d\phi} - \frac{d\sigma^{\leftarrow}}{d\phi}\right)$$

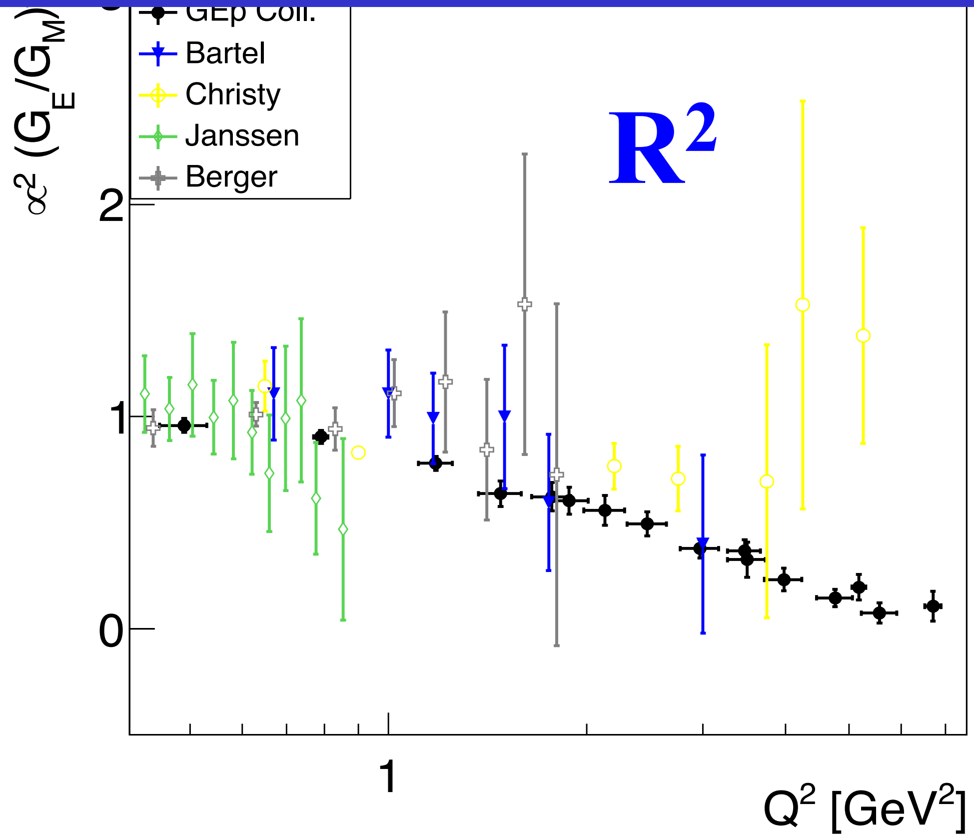


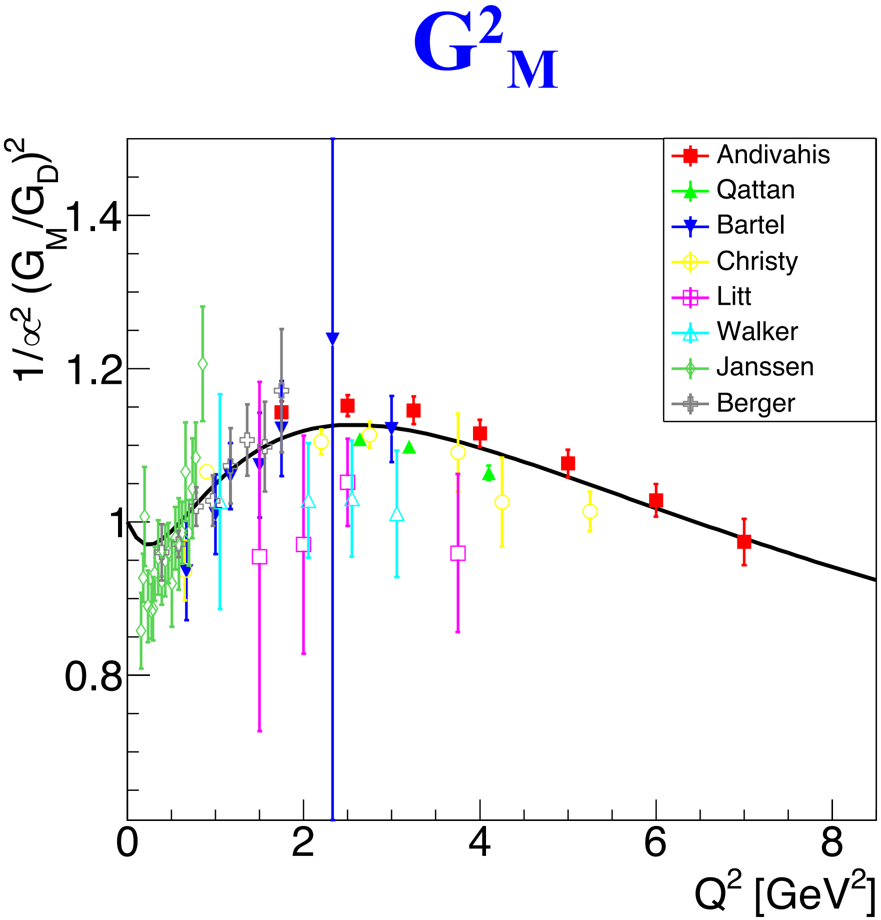


RC (LSF)

RC(1st order)

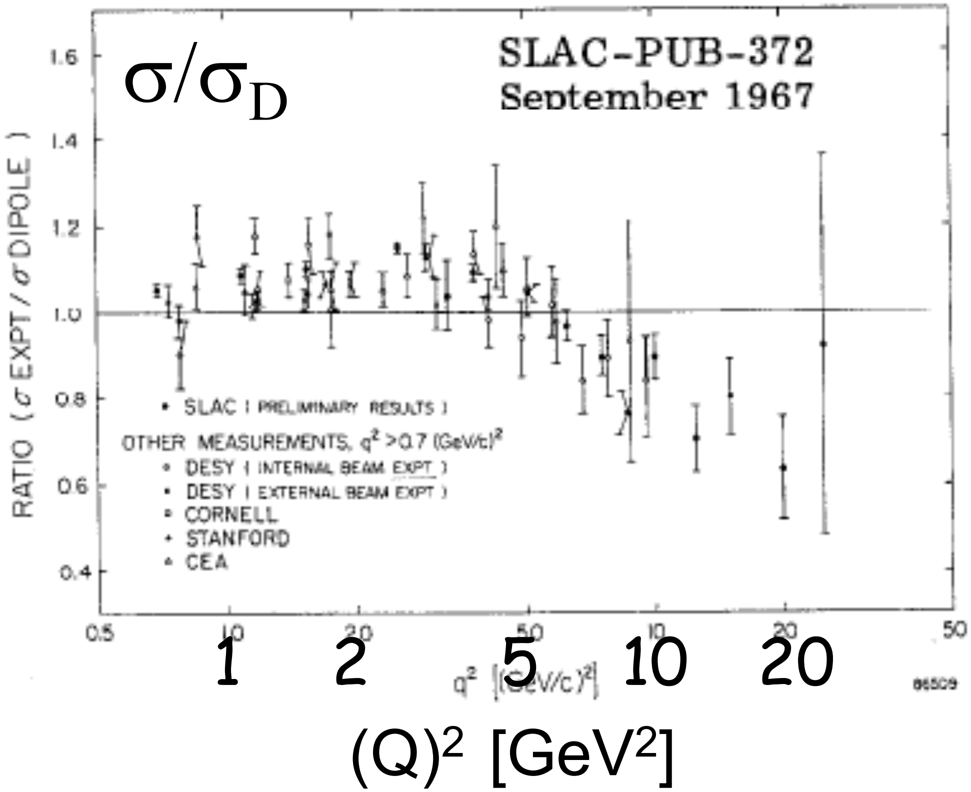
Different Data Sets





Simone Pacetti and Egle Tomasi-Gustafsson
Phys. Rev. C **94**, 055202, 2016

Nucleon FFs above 6 GeV

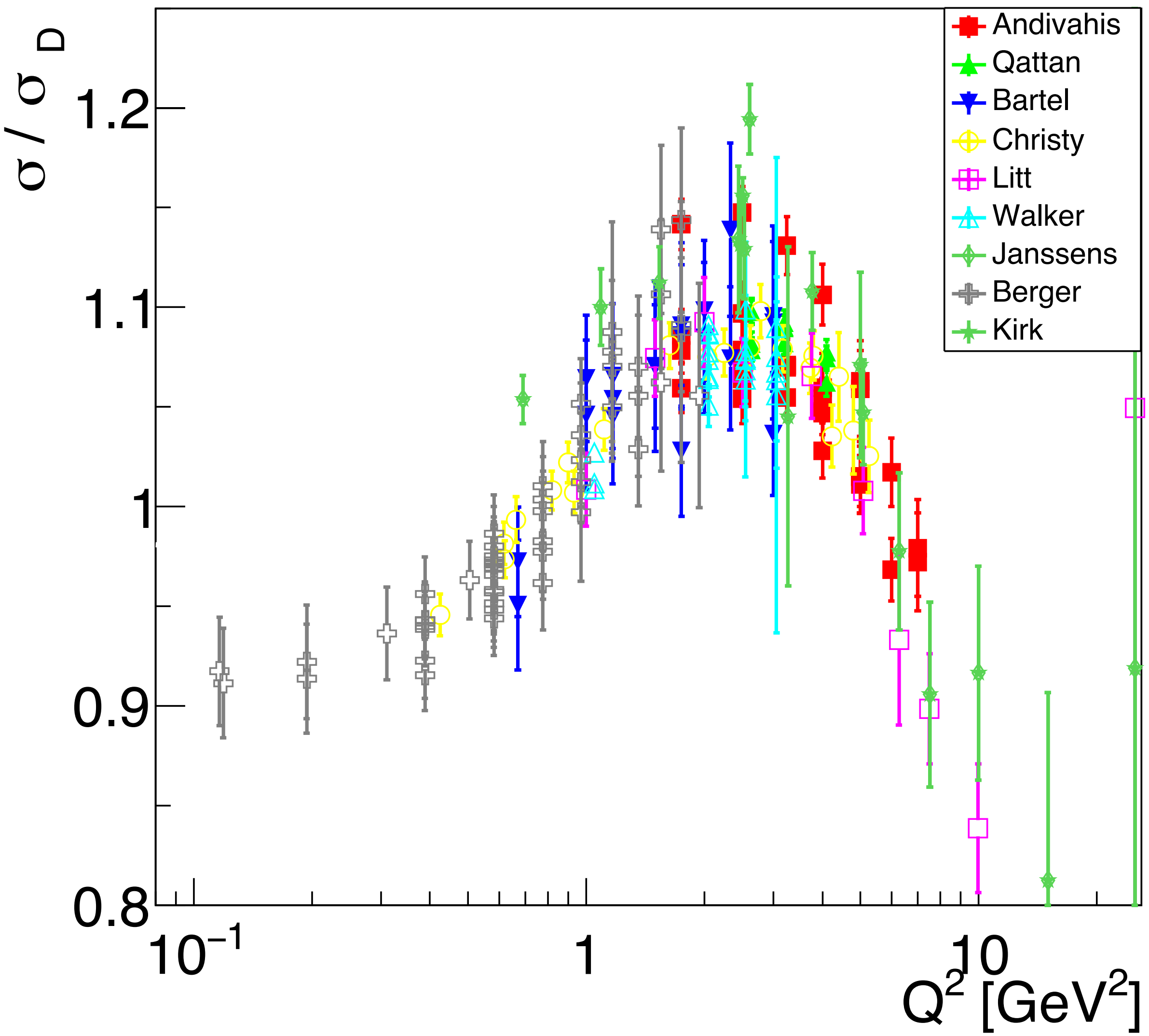


R. Taylor
Proc. of the Int. Symp. On Electron and Photon Interactions at High Energies, SLAC, 1967 p. 78-101.

...which makes evident any disagreement with the dipole prediction

Simone Pacetti and Egle Tomasi-Gustafsson
Phys. Rev. C **94**, 055202, 2016

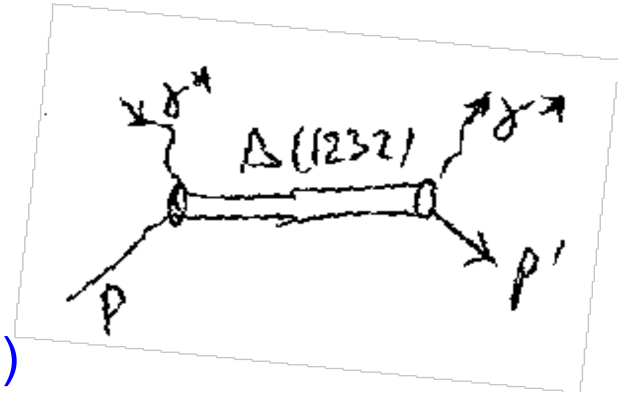
Nucleon FFs above 6 GeV



Example of inelastic channels: Δ–resonance

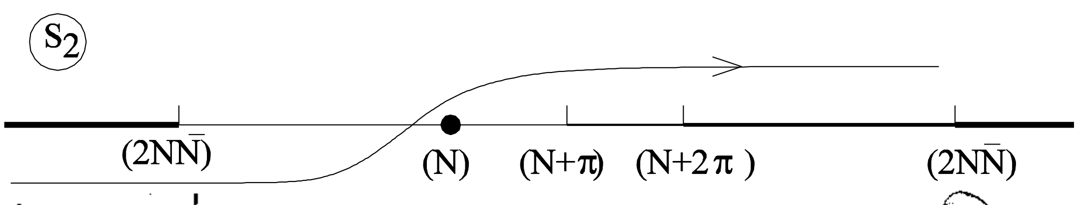
Yu. Bystricky, E.A.Kuraev, E. T.-G, Phys. Rev. C 75, 015207 (2007)

- Small contribution ~0.5%
- Opposite sign with respect to proton intermediate state
- Confirmed by other independent calculations)

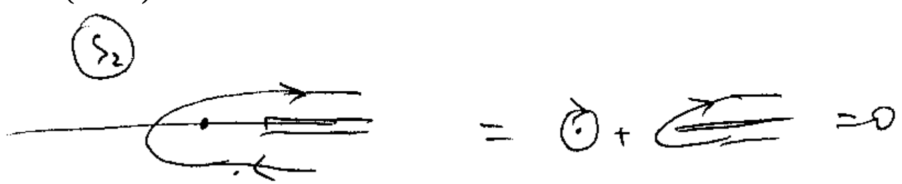


Cancellation of contributions in elastic and inelastic channels (sum rules, analytical properties of the Compton amplitude)

Neglect left contribution and close contour on the right side (*10% accuracy*)



(Pole + Right Cut) contribution to box amplitude ≡ 0

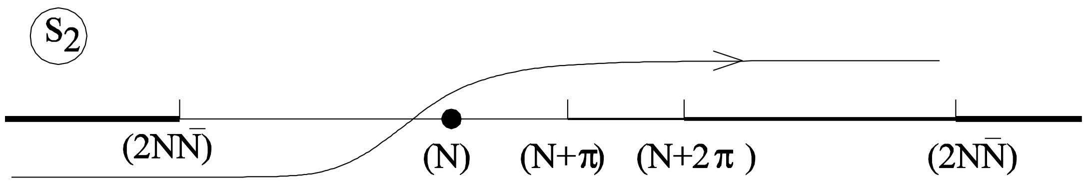


Cancellation proved exactly in QED:

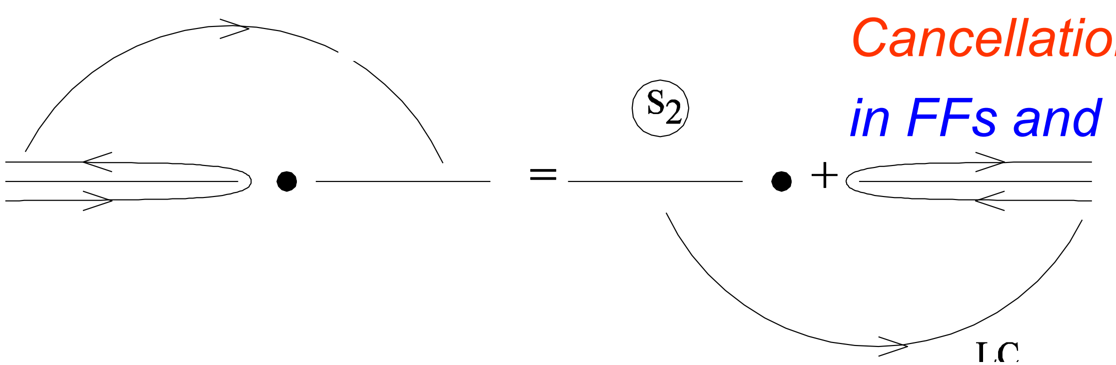
the L^2 γ^* -contribution to FFs is cancelled by soft photon emission

Analytical properties of Compton amplitude

- *Elastic form factors and inelastic channels are not independent : SUM RULES*



- Neglect left contribution and close contour on the right side (*10% accuracy*)

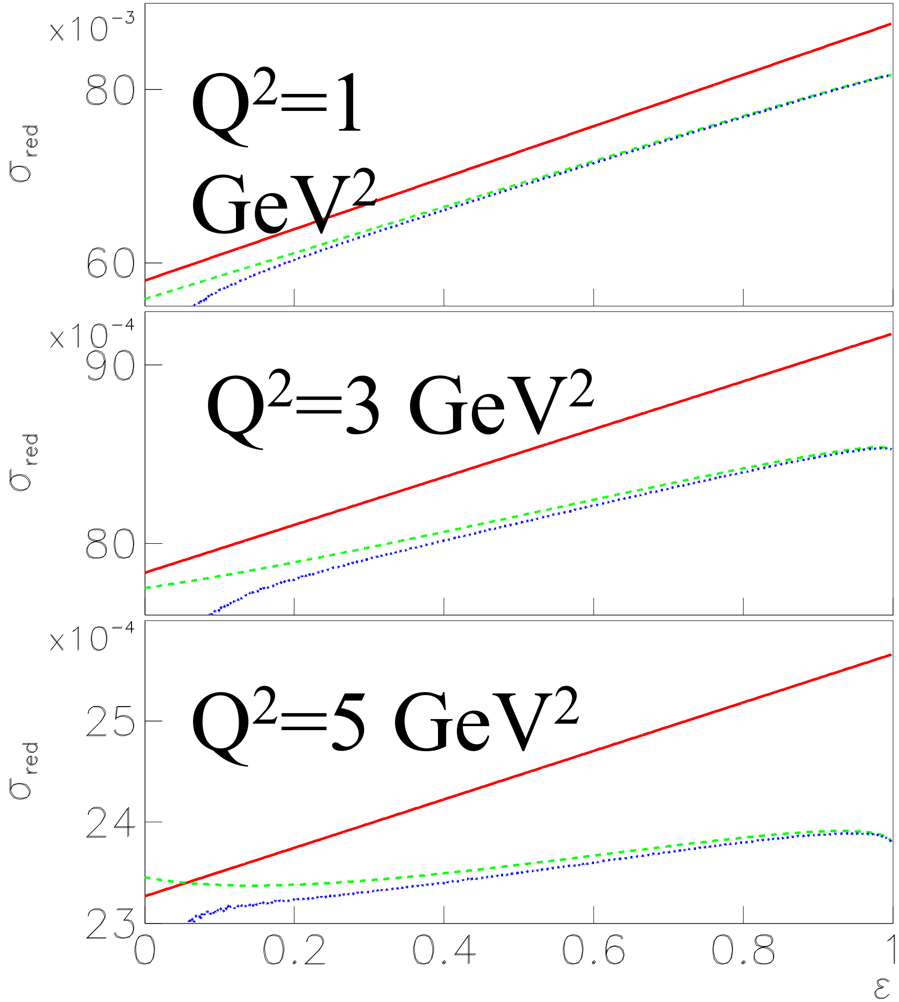


Cancellation of strong interaction effects in FFs and inelastic channels!

Cancellation proved exactly in QED:
the L^2 γ^* -contribution to FFs is cancelled by soft photon emission

Unpolarized Cross section

Yu. Bystricky, E.A.Kuraev, E. T.-G, Phys. Rev. C 75, 015207 (2007)



$$\sigma_{red} = \tau G_{Mp}^2 + \epsilon G_{Ep}^2$$

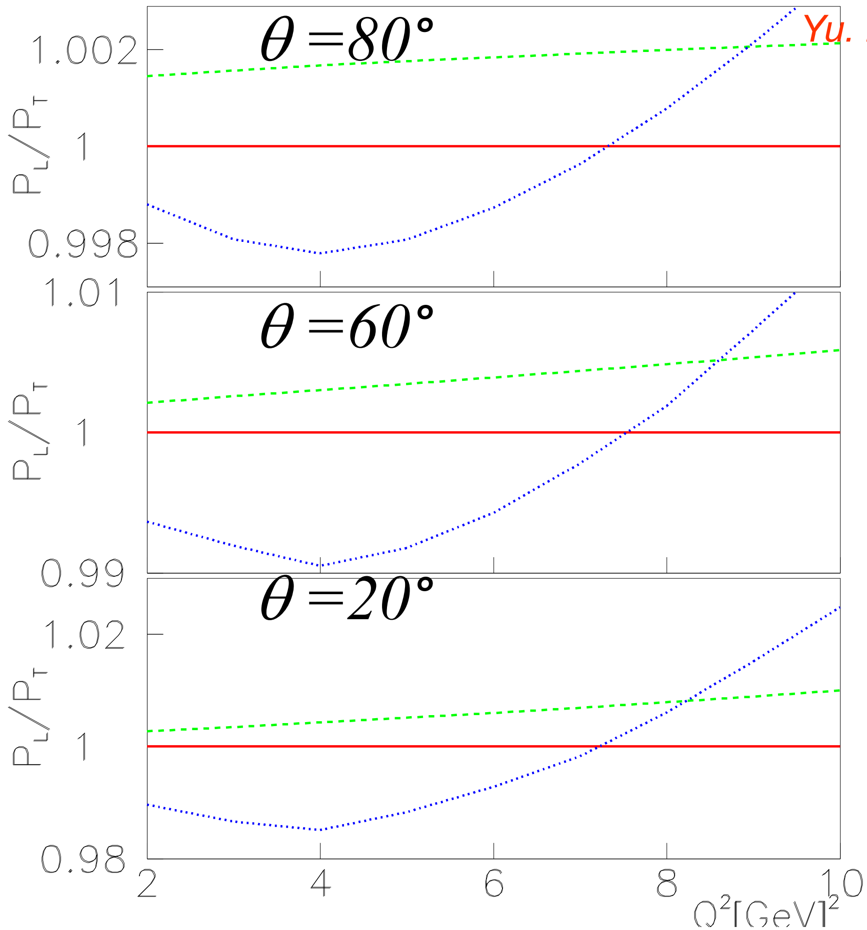
Born +dipole FFs
(=unpolarized experiment+Mo&Tsai)
SF (with dipole FFs)
SF+2γ exchange

SF: change the slope!

2γ exchange very small!

Yu. Bystricky, E.A.Kuraev, E. T.-G, Phys. Rev. C 75, 015207 (2007)

Polarization ratio

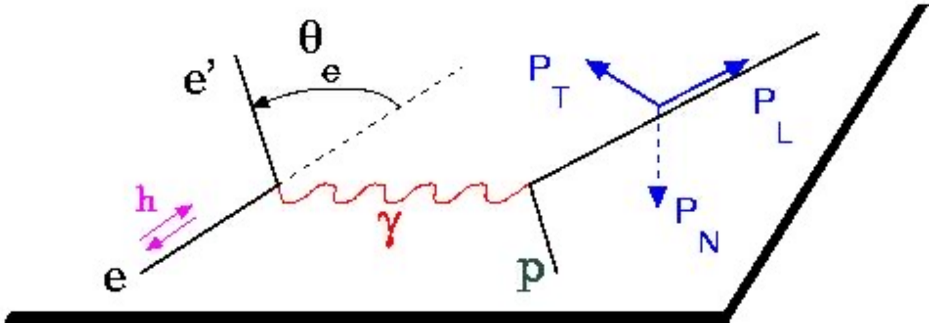


Yu. Bystricky, E.A.Kuraev, E. T.-G, Phys. Rev. C 75, 015207 (2007)

Born

SF

SF+2γ exchange



2γ exchange very small!

2γ destroys linearity!

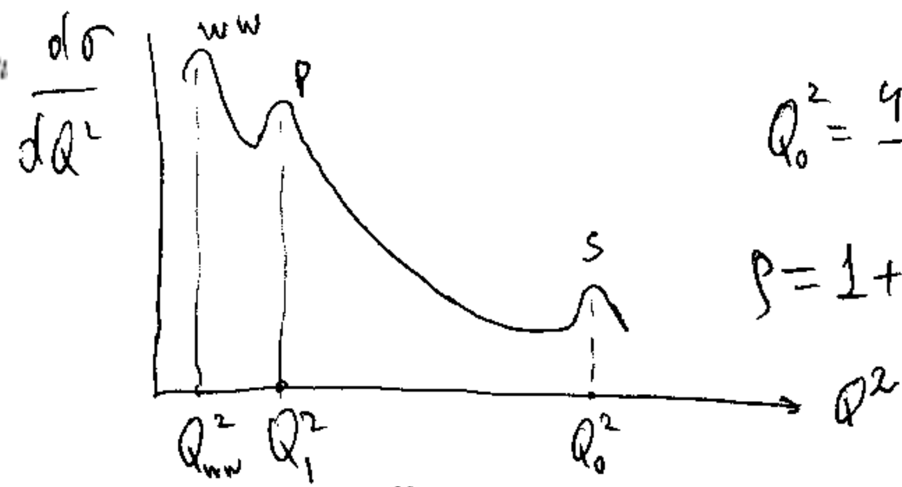
$$\left(\mathcal{P}_t\frac{d\sigma}{d\Omega}\right)_{corr} = -\lambda\int_{z_1}^{1} dz\frac{D(z,\beta)}{[1-\Pi(Q_z^2)]^2}\frac{\alpha^2}{Q_z^2}\left(\frac{1}{\rho_z}\right)^2\sqrt{\frac{\tau_z}{\tan^2(\theta/2)(1+\tau_z)}}G_E(Q_z^2)G_M(Q_z^2)\left(1+\frac{\alpha}{\pi}K_t\right);$$

$$\left(\mathcal{P}_\ell\frac{d\sigma}{d\Omega}\right)_{corr} = -\lambda\int_{z_1}^{1} dz\frac{D(z,\beta)}{[1-\Pi(Q_z^2)]^2}\frac{\alpha^2}{2M^2}\left(\frac{1}{\rho_z}\right)^2\sqrt{1+\frac{1}{\tan^2(\theta/2)(1+\tau_z)}}G_M^2(Q_z^2)\left(1+\frac{\alpha}{\pi}K_\ell\right).$$

REVIEWS OF MODERN PHYSICS VOLUME 41, NUMBER 1 JANUARY 1969

Radiative Corrections to Elastic and Inelastic *ep* and *μp* Scattering*

L. W. MO, Y. S. TSAI
Stanford Linear Accelerator Center, Stanford University, St



$$Q_0^2 = \frac{4E^2 \sin^2\theta/2}{\rho}$$

$$\rho = 1 + \frac{2E}{m}\sin^2\theta/2 .$$

Q_0^2 – elastic scattering – final state emission

$Q_1^2 = Q_0^2(EX)$, $X = 1 - \frac{W}{E}$ initial state emission

$Q_{WW}^2 \sim M^2\left(\frac{S_q}{S}\right)^2$ – Weizsaecker-Williams kinematics: virtual photon almost real.