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Nucleon Tomography: Imaging the origin of mass



 $\begin{array}{c} a_{3} \\ a_{2} \\ a_{1} \\ a_{2} \\ a_{3} \\$

 $H^{+}(x, t; \xi=0.2, Q^{1}-4)$

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FAMN seminar | Hervé MOUTARDE

Oct. 26th, 2016

www.cea.fr



Quantum Chromodynamics as a paradigm. The **theory** (and not an *effective theory*) of the strong interaction.



Facts Nucleon Tomography

Restricted number of parameters.

- Mass without mass Nucleon structure Content of GPDs

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- Mathematically consistent.
- Large scope.
- Validated up to large energy $\lesssim 13$ TeV.
- Accurate algorithmic answer.

- Confinement.
- Chiral symmetry breaking.
- Existence of a mass gap.



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No observed free color charges (PDG 2009) FREE QUARK SEARCHES

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The basis for much of the theory of particle scattering and hadron spectroscopy is the construction of the hadrons from a set of fractionally charged constituents (quarks). A central but unproven hypothesis of this theory, Quantum Chromodynamics, is that quarks cannot be observed as free particles but are confined to mesons and baryons.

Experiments show that it is at best difficult to "unglue" quarks. Accelerator searches at increasing energies have produced no evidence for free quarks, while only a few cosmic-ray and matter searches have produced uncorroborated events.



- Confinement.
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From quarks to hadrons

- What are the relevant degrees of freedom?
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What are the effective forces between them?

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Clay Millenimum Prize (Jaffe and Witten)

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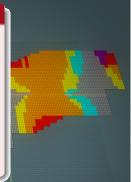
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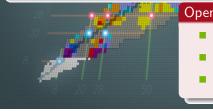
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Finally, QFT is the jumping-off point for a quest that may prove central in 21st century physics—the effort to unify gravity and quantum mechanics, perhaps in string theory. For mathematicians to participate in this quest, or even to understand the possible results, QFT must be developed further as a branch of mathematics. It is important not only to understand the solution of specific problems arising from physics, but also to set stude results within a new mathematical framework. One hopes that this framework will provide a unified development of several fields of mathematics and physics, and that it will also provide an arean for the development of new mathematics and physics.

For these reasons the Scientific Advisory Board of CMI has chosen a Millennium problem about quantum gauge theories. Solution of the problem requires both understanding one of the deep unsolved physics mysteries, the existence of a mass gap, and also producing a mathematically complete example of quantum gauge field theory in four-dimensional space-time.





- Confinement.
- **Chiral symmetry** breaking.
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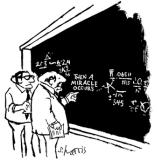
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"I think you should be more explicit here in step two." he solution of specific problems hin a new mathematical frameunified development of several also provide an arena for the

f CMI has chosen a Millennium of the problem requires both steries, the existence of a mass ate example of quantum gauge

- Confinement.
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Motivation.

QCD large distance dynamics from the hadron structure viewpoint.



Nucleon Tomography

 Lattice QCD clearly shows that the mass of hadrons is generated by the interaction, not by the guark masses.

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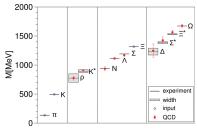
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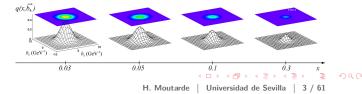
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Durr et al., Science 322, 1224 (2008)

Can we map the location of mass inside a hadron?







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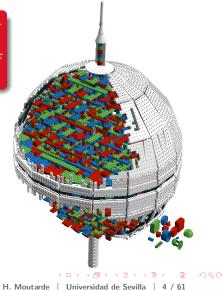
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Mass?







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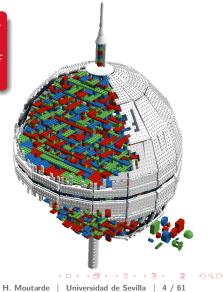
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> Mass? Spin?







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> Mass? Spin? Charge?







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> Mass? Spin? Charge?

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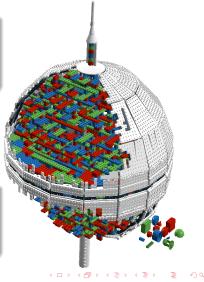
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How can we recover the wellknown characterics of the nucleon from the properties of its **colored building blocks**?

> Mass? Spin? Charge?

What are the relevant **effective degrees of freedom** and **effective interaction** at large distance?







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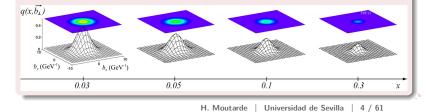
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Structuring questions for the hadron physics community

- QCD mechanisms behind the origin of mass in the visible universe?
- **Cartography** of interactions giving its mass to the nucleon?
- **Pressure** and **density** profiles of the nucleon as a continuous medium?
- Localization of quarks and gluons inside the nucleon?

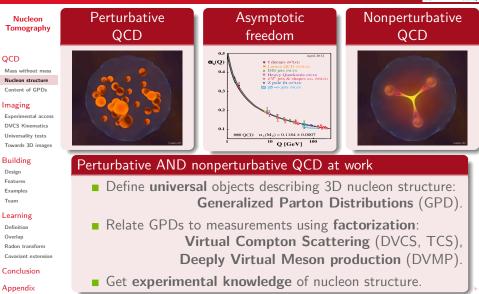


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Motivation.

Study nucleon structure to shed new light on nonperturbative QCD.









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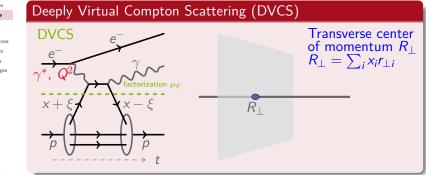
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- Correlation of the longitudinal momentum and the transverse position of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.







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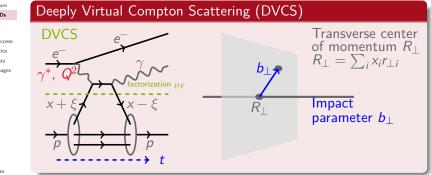
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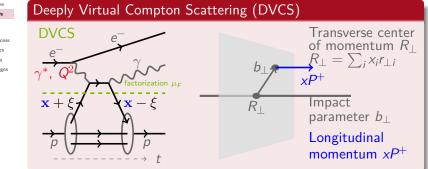
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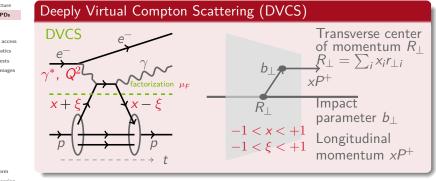
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• 24 GPDs $F^i(\mathbf{x}, \boldsymbol{\xi}, t, \mu_F)$ for each parton type i = g, u, d, ...for leading and sub-leading twists. $(\Box \rightarrow \langle \boldsymbol{g} \rangle \langle \boldsymbol{\xi} \rangle \langle$





Nucleon Tomography

Probabilistic interpretation of Fourier transform of $GPD(x, \xi = 0, t)$ in **transverse plane**.

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 $\rho(\mathbf{x}, b_{\perp}, \lambda, \lambda_{N}) = \frac{1}{2} \left[\mathbf{H}(\mathbf{x}, 0, b_{\perp}^{2}) + \frac{b_{\perp}^{j} \epsilon_{ji} S_{\perp}^{i}}{M} \frac{\partial \mathbf{E}}{\partial b_{\perp}^{2}} (\mathbf{x}, 0, b_{\perp}^{2}) + \lambda \lambda_{N} \tilde{\mathbf{H}}(\mathbf{x}, 0, b_{\perp}^{2}) \right]$

 Notations : quark helicity λ, nucleon longitudinal polarization λ_N and nucleon transverse spin S_⊥.

Burkardt, Phys. Rev. D62, 071503 (2000)





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Most general structure of matrix element of energy momentum tensor between nucleon states:

$$P + \frac{\Delta}{2} \left| T^{\mu\nu} \left| N, P - \frac{\Delta}{2} \right\rangle = \bar{u} \left(P + \frac{\Delta}{2} \right) \left[\mathbf{A}(t) \gamma^{(\mu} P^{\nu)} + \mathbf{B}(t) P^{(\mu} i \sigma^{\nu)\lambda} \frac{\Delta_{\lambda}}{2M} + \frac{\mathbf{C}(t)}{M} (\Delta^{\mu} \Delta^{\nu} - \Delta^{2} \eta^{\mu\nu}) \right] u \left(P - \frac{\Delta}{2} \right)$$

with $t = \Delta^2$.

Key observation: link between GPDs and gravitational form factors

$$\int d\mathbf{x} \mathbf{x} \mathbf{H}^{q}(\mathbf{x}, \xi, t) = \mathbf{A}^{q}(t) + 4\xi^{2} \mathbf{C}^{q}(t)$$
$$\int d\mathbf{x} \mathbf{x} \mathbf{E}^{q}(\mathbf{x}, \xi, t) = \mathbf{B}^{q}(t) - 4\xi^{2} \mathbf{C}^{q}(t)$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

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Spin sum rule:

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$\int dx x (H^{q}(x,\xi,0) + E^{q}(x,\xi,0)) = A^{q}(0) + B^{q}(0) = 2J^{q}$

Ji, Phys. Rev. Lett. 78, 610 (1997)

• Shear and pressure of a hadron considered as a continuous medium:

$$\langle N | T^{ij}(\vec{r}) | N \rangle N = s(r) \left(\frac{r^i r^j}{\vec{r}^2} - \frac{1}{3} \delta^{ij} \right) + p(r) \delta^{ij}$$

Polyakov and Shuvaev, hep-ph/0207153

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Towards hadron tomography. GPDs as a scalpel-like probe of hadron structure.



Nucleon Tomography **Status of 3D imaging** We can apply the GPD formalism to existing data.

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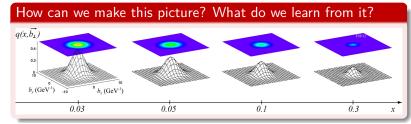
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2 Building the tools

We develop the tools to analyze near-future data.

Learning from GPDs

We build symmetry-preserving GPD models.



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Phenomenological status of nu-

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Exclusive processes of current interest (1/2). Factorization and universality.



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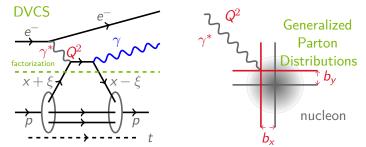
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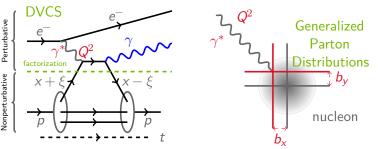
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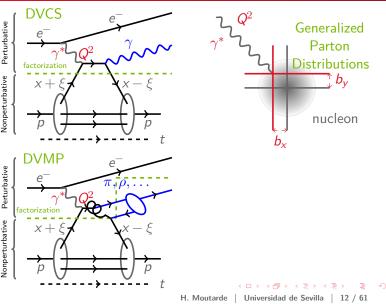
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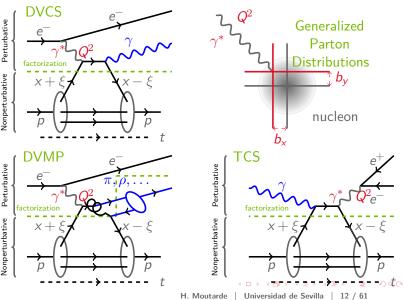
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$\begin{array}{c} & \quad \mathsf{Exclusive processes of current interest (1/2).} \\ & \quad \mathsf{Factorization and universality.} \end{array}$



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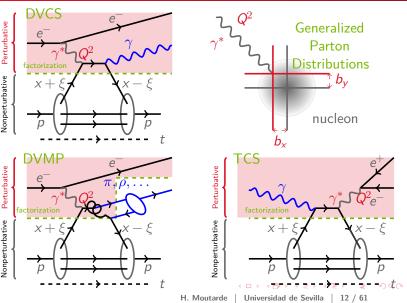
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Exclusive processes of current interest (1/2). Factorization and universality.



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Perturbative

Nonperturbative

Perturbative

Nonperturbative

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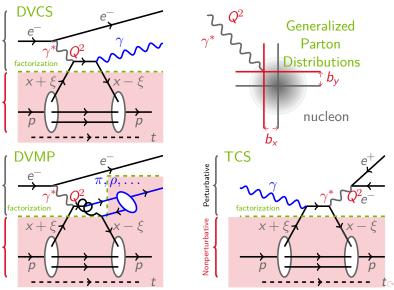
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Exclusive processes of present interest (2/2). Factorization and universality.



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Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on factorization theorems.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale μ_F .
- **Consistency** requires the study of **different channels**.

GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx C\left(x, \xi, \alpha_{\mathcal{S}}(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F.

• CFF \mathcal{F} is a **complex function**.

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Towards 3D images

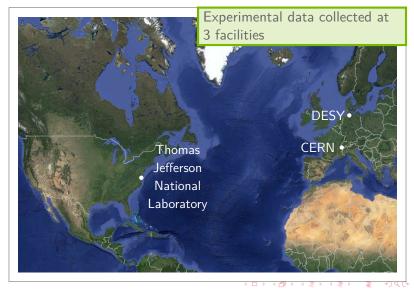
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Nucleon Tomography

QCD

Mass without mass Nucleon structure Content of GPDs

Imaging

Experimental access
DVCS Kinematics
Universality tests

Towards 3D images

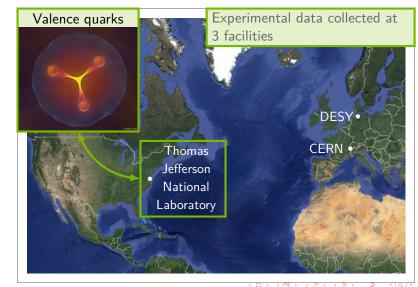
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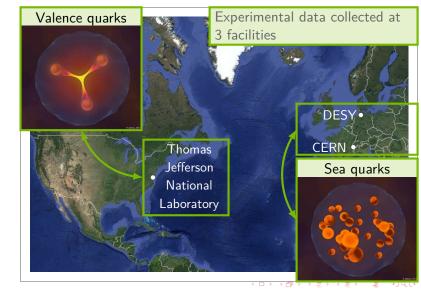
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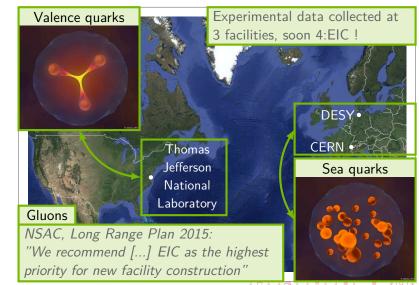
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Typical DVCS kinematics. Probing gluons, sea and valence guarks through DVCS.



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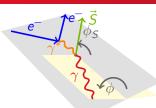
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Study the harmonic structure of $ep \rightarrow ep\gamma$ amplitude.

Diehl *et al.*, Phys. Lett. **B411**, 193 (1997)

| - | Kinematics | | | |
|------------|------------|-------------------------|--------------------|--|
| Experiment | ХB | $Q^2 \; [\text{GeV}^2]$ | $t [\text{GeV}^2]$ | |
| HERA | 0.001 | 8.00 | -0.30 | |
| COMPASS | 0.05 | 2.00 | -0.20 | |
| HERMES | 0.09 | 2.50 | -0.12 | |
| CLAS | 0.19 | 1.25 | -0.19 | |
| HALL A | 0.36 | 2.30 | -0.23 | |

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Goloskokov-Kroll (GK) model on DVCS. No parameter of the GK model was tuned to analyse DVCS.



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Beam Charge Asymmetry, HERMES

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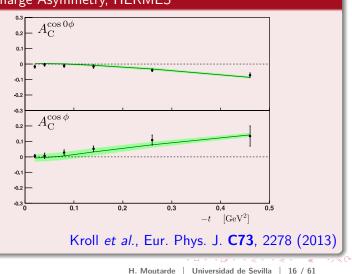
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Goloskokov-Kroll (GK) model on DVCS. No parameter of the GK model was tuned to analyse DVCS.



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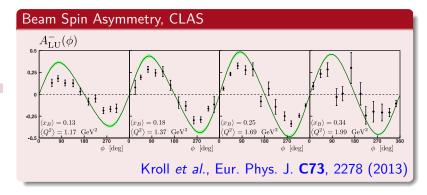
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Summary of first extractions. Feasibility of twist-2 analysis of existing data.



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- **Dominance** of twist-2 and **validity** of a GPD analysis of DVCS data.
- *ImH* **best determined**. Large uncertainties on *ReH*.
 - However sizable **higher twist contamination** for DVCS measurements.
- Already some indications about the invalidity of the H-dominance hypothesis with unpolarized data.

See more on fits.

Imaging the nucleon. How? Extracting GPDs is not enough...Need to extrapolate!



1. Experimental data fits 2. GPD extraction Nucleon Tomography $H^{+}(x, t; \Xi=0.2, O^{2}=4)$ $\Delta \sigma$ [pb.GeV⁻⁴] 15. QCD 0.1 Mass without mass Nucleon structure = 0.5-10 Content of GPDs $= 6.3 \text{ GeV}^2$ -1.08,05,0,4,02 0,02,0,4,08,08,1 0 0.735 GeV^2 Imaging Experimental access 0.2 ϕ [deg] DVCS Kinematics Universality tests 3. Nucleon imaging Towards 3D images Building Design Images from Guidal et al.. Rept. Prog. Phys. 76 (2013) 066202 The 2015 Long Range Plan for Nuclear Science Examples Team Sidebar 2.2: The First 3D Pictures of the Nucleon Learning 2 A computed tomography (CT) scan can help physicians Definition pinpoint minute cancer tumors, diagnose tiny broken bones, and spot the early signs of osteoporosis. Overlan 0,[fm] Now physicists are using the principles behind the Radon transform 0 procedure to peer at the inner workings of the proton. Covariant extension This breakthrough is made possible by a relatively new -1 concept in nuclear physics called generalized parton Conclusion distributions. -2 -1 0 1 Ó -2 -1 b, [fm] b_x [fm] An intense beam of high-energy electrons can be used Appendix H. Moutarde Universidad de Sevilla 18 / 61



Imaging the nucleon. How? Extracting GPDs is not enough...Need to extrapolate!



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1 Extract $H(x, \xi, t, \mu_F^{ref})$ from experimental data.

- **2 Extrapolate** to vanishing skewness $H(x, 0, t, \mu_F^{ref})$.
- **3 Extrapolate** $H(x, 0, t, \mu_F^{ref})$ up to infinite *t*.
- **4 Compute** 2D Fourier transform in transverse plane:

$$H(x, b_{\perp}) = \int_{0}^{+\infty} \frac{\mathrm{d}|\Delta_{\perp}|}{2\pi} |\Delta_{\perp}| J_0(|b_{\perp}||\Delta_{\perp}|) H(x, 0, -\Delta_{\perp}^2)$$

- 5 Propagate uncertainties.
- **6 Control** extrapolations with an accuracy matching that of experimental data with **sound** GPD models.

The challenge of the high precision era. Higher order and higher twist contributions, and GPD modeling.



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• Evaluation of the impact of higher order effects.

See more on NLO evaluations.

• Evaluation of the impact of target mass and finite-*t* corrections.

See more on DVCS kinematics.

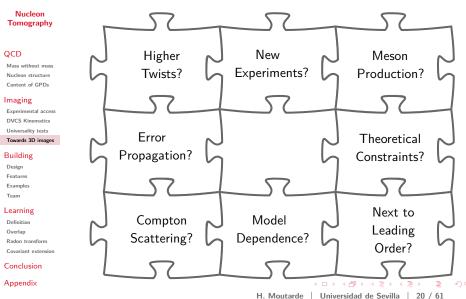
- Extrapolations with **GPD models**.
- See more on DVCS at LO
- Evaluation of the contribution of **higher twist** GPDs.
- DVMP: sensitivity to **DA models**.

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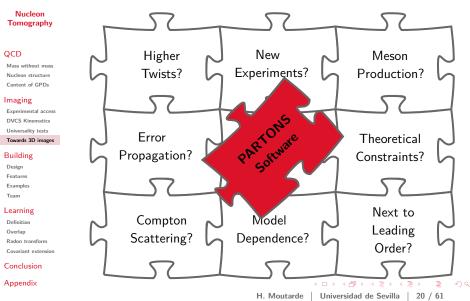
Software for the phenomenology of GPDs. Different questions to be answered with the same tools.





Software for the phenomenology of GPDs. Different questions to be answered with the same tools.





Building the tools for high precision: the PARTONS project



PARtonic Tomography Of Nucleon Software



Computing chain design. Differential studies: physical models and numerical methods.



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Experimental data and phenomenology

Computation of amplitudes

principles and

fundamental

parameters

First

Small distance contributions

Large distance contributions

Full processes

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Computing chain design. Differential studies: physical models and numerical methods.



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Nucleon Tomography Experimental data and Full processes phenomenology Mass without mass Nucleon structure Content of GPDs Imaging Experimental access DVCS Kinematics Small distance Computation Universality tests Towards 3D images contributions of amplitudes Building First Learning Large distance principles and Radon transform contributions fundamental Covariant extension Conclusion parameters Appendix A (1) > A (2) > A H. Moutarde Universidad de Sevilla



Differential studies: physical models and numerical methods.





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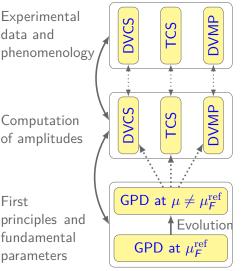
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Computation of amplitudes First principles and fundamental

Experimental

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Differential studies: physical models and numerical methods.





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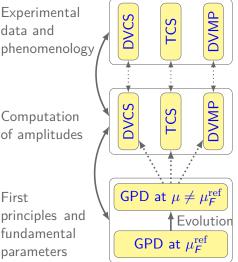
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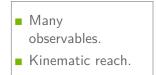
Computation of amplitudes

Experimental

data and

First principles and fundamental parameters





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Differential studies: physical models and numerical methods.



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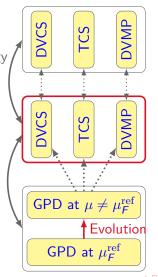
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data and phenomenology Need for modularity Computation of amplitudes First principles and fundamental parameters

Experimental



Many observables.

Kinematic reach.

Perturbative approximations.

Physical models.

Fits.

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- Numerical methods.
- Accuracy and speed.

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Differential studies: physical models and numerical methods.



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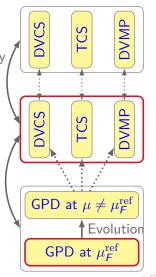
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Many observables.

- Kinematic reach.
- Perturbative approximations.
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Fits.

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- Numerical methods.
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Differential studies: physical models and numerical methods.



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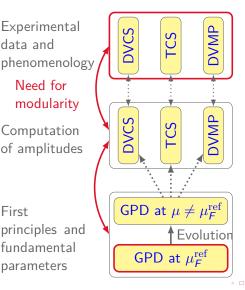
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data and



Many observables.

- Kinematic reach.
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Differential studies: physical models and numerical methods.



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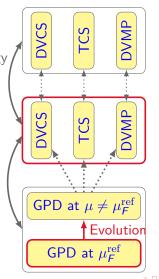
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- Numerical methods.
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Differential studies: physical models and numerical methods.



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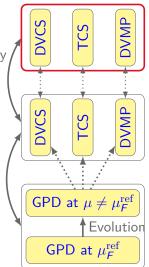
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- Perturbative approximations.
 - Physical models.

Fits.

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- Numerical methods.
- Accuracy and speed.

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Towards the first release. Currently: tests, benchmarking, documentation, tutorials.



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- 3 stages:
 - 1 Design.
 - 2 Integration and validation.
 - 3 Benchmarking and production.
- Flexible software architecture.
 - B. Berthou *et al.*, *PARTONS: a computing platform for the phenomenology of Generalized Parton Distributions* arXiv:1512.06174, *to appear in Eur. Phys. J. C.*

• See more on software architecture.

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- 1 new physical development = 1 new module.
- Aggregate knowledge and know-how:
 - Models
 - Measurements
 - Numerical techniques
 - Validation

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Systematic studies made easy. A faster and safer way to GPD phenomenology.



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Automation allows...:

- to run **numerous computations** with various physical assumptions,
- to run **nonregression** tests.
- to perform **fits** with various models.
- physicists to focus on physics!

Without PARTONS



With PARTONS



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GPD computations made fast.

Improved performances thanks to clever architecture design.



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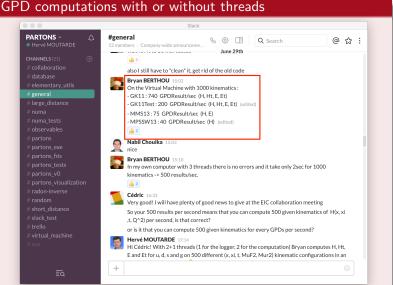
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Systematic studies made fast. What can be done from scratch in about 1 hour.



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Luca Colaneri. **DVCS** beam-spin asymmetries at EIC Nabil Chouika (PARTONS) – GK11 VGG 0.24 $Q^2 = 16 \text{ GeV}^2$ $x_B = 0.01$ 0.16 $E_e E_p = 500 \text{ GeV}^2$ 0.08 -0.08 -0 16 -0.24 $Q^2 = 8 \text{ GeV}^2 \ x_B = 0.005$ 0.24 SPELIMIMARY! 0.16 0.08 A_{LU} o -0.08 -0.16 0.24 0.24 $O^2 = 4 \text{ GeV}^2$ $x_B = 0.003$ 0.16 0.08 -0.08 See next talk on -0.16 PARTONS by C. Mezrag -0.24 40 80 120 160 200 160 200 240 280 320 360 0 200 240 280 320 $\phi(^{\circ})$ $\phi(^{\circ})$ $\phi(^{\circ})$ $-t = 0.1 \text{ GeV}^2$ $-t = 0.25 \text{ GeV}^2$ $-t = 1 \text{ GeV}^2$

From D. Sokhan's talk, EIC User Group Meeting, ANL, 2016

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GPD or CFF fits.

From GPDs to measurements and from measurements to GPDs.

First fit of pseudo DVCS data, Sep. 26th, 2016



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partons_fits ~ 7 🏯 PARTONS Mon Sep 26 2016 pawel 3:16 PM ECN=1.00128e-11 EROM MTGRAD STATUS=CONVERGED 44 CALLS 45 TOTAL EDM=2.00186e-11 STRATEGY= 1 ERROR MATRIX ACCURATE EXT PARAMETER FIRST NAME VALUE FRROR SIZE DERIVATIVE NO. fit_CFF_H_Re 6.67247e-02 1.34241e+00 2.92531e-05 -7.02262e-07 2 fit CFF H Im 1.24231e+01 1.07342e+00 1.80608e-05 1.71071e-04 fit CFF E Re -3.94789e+00 fixed 4 fit_CFF_E_Im -1.64116e-01 fixed 5 fit CFF Ht Re 1.54183e+00 fixed 6 fit_CFF_Ht_Im 2.59017e+00 fixed fit CFF Et Re 5.41102e+01 fixed 8 fit CFF Et Im 3.79052e+01 EXTERNAL ERROR MATRIX. NDIM= 25 NPAR= 2 ERR DEF=1 1.804e+00 7.961e-03 7.961e-03 1.153e+00 PARAMETER CORRELATION COEFFICIENTS NO. GLOBAL 1 0.00552 1.000 0.006 2 0 00552 0 006 1 000 ⊱ dbinosi The first reasonable fit with PARTONS Fits! 12 AUL and 12 ALU asymmetries fitted together. The true values of fit CFF H Re and fit CFF H Im are 0.06672466940113253 and 12 423114181138908 Write a message... Help H. Moutarde Universidad de Sevilla 27 / 61

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GPD computing made simple. Each line of code corresponds to a physical hypothesis.



| Nucleon | | gpdExample() | | |
|--------------------------------------|----|--|--|--|
| Tomography | 1 | // Lots of includes | | |
| | | <pre>#include <src partons.h=""></src></pre> | | |
| QCD | 3 | | | |
| Mass without mass | 4 | // Retrieve GPD service | | |
| Nucleon structure Content of GPDs | 5 | GPDService* pGPDService = Partons::getInstance()->getServiceObjectRegistry | | |
| Imaging Experimental access | 6 | ()->getGPDService(); // Load GPD module with the BaseModuleFactory | | |
| DVCS Kinematics | 7 | GPDModule* pGK11Model = Partons::getInstance()->getModuleObjectFactory | | |
| Universality tests | | ()->newGPDModule(GK11Model::classId); | | |
| Towards 3D images | 8 | // Create a GPDKinematic(x, xi, t, MuF, MuR) to compute | | |
| Building | | GPDKinematic gpdKinematic(0.1, 0.00050025, -0.3, 8., 8.); | | |
| Design | | // Compute data and store results | | |
| Features | 11 | | | |
| Examples | ** | computeGPDModelRestrictedByGPDType(gpdKinematic, pGK11Model, | | |
| Team Learning | | GPDType::ALL); | | |
| Definition | 12 | // Print results | | |
| Overlap | 13 | <pre>std::cout << gpdResult.toString() << std::endl;</pre> | | |
| Radon transform | 14 | | | |
| Covariant extension | 15 | delete pGK11Model; | | |
| Conclusion | | pGK11Model = 0; | | |
| Appendix | | (ロ) (日) (日) (日) (日) (日) (日) (日) (日) (日) (日 | | |
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OF LARGERENE À L'INDUSTR

GPD computing automated. Each line of code corresponds to a physical hypothesis.



| Nucleon | | computeOneGPD.xml |
|---------------------------------------|----|--|
| Tomography | 1 | xml version="1.0" encoding="UTF-8" standalone="yes" ? |
| | 2 | <pre><scenario date="" description="Example_:_computation_of_one_GPD</pre></th></tr><tr><th>QCD Mass without mass</th><th>3</th><th><pre>model_(GK11)_without_evolution" id="01"> <!-- Select type of computation--></scenario></pre> |
| Nucleon structure | 4 | <task method="computeGPDModel" service="GPDService"></task> |
| Content of GPDs | 5 | </math Specify kinematic $>$ |
| Imaging | 6 | <pre><kinematics type="GPDKinematic"></kinematics></pre> |
| Experimental access | 7 | <pre><param name="x" value="0.1"/></pre> |
| DVCS Kinematics Universality tests | 8 | <pre><param name="xi" value="0.00050025"/></pre> |
| Towards 3D images | 9 | <param name="t" value="-0.3"/> |
| Building | 10 | <param name="MuF2" value="8"/> |
| Design | 11 | <param name="MuR2" value="8"/> |
| Features | 12 | |
| Examples | 13 | </math Select GPD model and set parameters $>$ |
| Team | 14 | <computation configuration=""></computation> |
| Learning | 15 | <module type="GPDModule"></module> |
| Definition Overlap | 16 | <pre><pre>className" value="GK11Model" /></pre></pre> |
| Radon transform | 17 | |
| Covariant extension | 18 | |
| Conclusion | 19 | |
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GPD computing automated. Each line of code corresponds to a physical hypothesis.



| Nucleon | | computeOneGPE | D.xml |
|--|----|--|--------------------------------------|
| Tomography | | xml version="1.0" encoding="UTF-8" stand</td <td>$H^{\mu} = 0.822557$</td> | $H^{\mu} = 0.822557$ |
| | 2 | <scenario date="" description="Exam</th><th>U U</th></tr><tr><th>QCD</th><th></th><th><math>_{ m u}</math>model<math>_{ m u}</math>(GK11)<math>_{ m u}</math>without<math>_{ m u}</math>evolution" id="01"></scenario> | $H^{u(+)} = 0.165636$ |
| Mass without mass | 3 | </math Select type of computation $>$ | $H^{u(-)} = 1.47948$ |
| Nucleon structure | 4 | <task gpdkinematic"="" method="con</th><th>11 * * = 1.47940</th></tr><tr><th>Content of GPDs</th><td>5</td><td><! Specify kinematic></td><td></td></tr><tr><th>Imaging</th><th>6</th><th><pre><kinematics type=" service="GPDService"></task> | $H^d = 0.421431$ |
| Experimental access DVCS Kinematics | 7 | <param]<="" name="x" td="" value="0.1"/> <td></td> | |
| Universality tests | 8 | <param <="" name="xi" t"="" td="" value="-0.3"/> <td>$H^{d(-)} = 0.762344$</td> | $H^{d(-)} = 0.762344$ |
| Building | 10 | <param ,<="" name="MuF2" th="" value="8"/> <th>11 - 0.102511</th> | 11 - 0.102511 |
| Design | 11 | <param <="" name="MuR2" th="" value="8"/> <th></th> | |
| Features | 12 | | $H^{s} = 0.00883408$ |
| Examples | 13 | Select GPD model and set parameter</th <th></th> | |
| Team | 14 | <computation_configuration></computation_configuration> | $H^{s(+)} = 0.0176682$ |
| Learning | 15 | <module type="GPDModule"></module> | $H^{s(-)} = 0$ |
| Definition | 16 | <pre><param name="className" pre="" va<=""/></pre> | |
| Overlap Radon transform | 17 | | |
| Covariant extension | 18 | | $H^{g} = 0.385611$ |
| Conclusion | 19 | | and $E, \tilde{H}, \tilde{E}, \dots$ |
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Observable computing automated. Each line of code corresponds to a physical hypothesis.



| | | computeManyKinematicsOneModel.xml |
|--|----|---|
| Nucleon Tomography | 1 | <pre><scenario date="2016-10-18" description="Use_kinematics_list"></scenario></pre> |
| romography | 2 | <task method="</td></tr><tr><td></td><td></td><td>computeManyKinematicOneModel" service="ObservableService" storeindb="1"></task> |
| QCD | 3 | <pre><kinematics type="ObservableKinematic"></kinematics></pre> |
| Mass without mass | 4 | <param name="file" value="observable_kinematics.dat"/> |
| Nucleon structure Content of GPDs | 5 | |
| Incontinue | 6 | <computation_configuration></computation_configuration> |
| Imaging Experimental access | 7 | <module type="Observable"></module> |
| DVCS Kinematics | 8 | <param name="className" value="Alu"/> |
| Universality tests | 9 | |
| Towards 3D images | 10 | <module type="DVCSModule"></module> |
| Building | 11 | <param name="className" value="BMJ2012Model" $/>$ |
| Design Features | 12 | $<$ param name="beam_energy" value="1066" $/>$ |
| Examples | 13 | |
| Team | 14 | <module type="DVCSConvolCoeffFunctionModule"> |
| Learning | 15 | <param name="className" value="DVCSCFFModel" $/>$ |
| Definition | 16 | $<$ param name="qcd_order_type" value="LO" $/>$ |
| Overlap | 17 | |
| Radon transform Covariant extension | 18 | <module type="GPDModule"></module> |
| Conclusion | 19 | <param name="className" value="GK11Model" $/>$ |
| | 20 | |
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DE LA RECHERCHE À L'INDUST

Observable plotting automated. Plot production is automated too!



| | | QueryDatabaseObservablePlotFile.xml |
|--|----|--|
| Nucleon Tomography | 1 | xml version="1.0" encoding="UTF-8" standalone="yes" ? |
| romography | 2 | <scenario date="2016-10-18" description=""></scenario> |
| | 3 | </math Generate plot file from database for GK model $>$ |
| QCD | 4 | <task method="generatePlotFile" service="ObservableService"></task> |
| Mass without mass Nucleon structure | 5 | <task_param type="output"></task_param> |
| Content of GPDs | 6 | <param <="" name="filePath" td="" value="observable_GK11_plot.csv"/> |
| Imaging | | > |
| Experimental access | 7 | |
| DVCS Kinematics | 8 | </math Variables of 2d plot $>$ |
| Universality tests | 9 | <task_param type="select"></task_param> |
| Towards 3D images | 10 | <param name="xPlot" value="phi" $/>$ |
| Building | 11 | $<$ param name="yPlot" value="observable_value" $/>$ |
| Design Features | 12 | $$ |
| Examples | 13 | </math Select results in database $>$ |
| Team | 14 | <task_param type="where"></task_param> |
| Learning | 15 | <param name="xB" value="0.1763" $/>$ |
| Definition | 16 | <param name="t" value="-0.1346" $/>$ |
| Overlap | 17 | <param name="Q2" value="1.3651" $/>$ |
| Radon transform Covariant extension | 18 | $<$ param name="computation_id" value="2" $>$ |
| Conclusion | 19 | $$ |
| Conclusion | 20 | |
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DE LA RECHERCHE À L'INDUST

Observable plotting automated. Plot production is automated too!



| | | Query[| DatabaseO | bservablePlotFile.xml | | | | |
|---------------------------------------|----------|--|--------------|-------------------------|--|--|--|--|
| Nucleon Tomography | 1 | xml version="1.0" encoding="UTF-8" standalone="yes" ? | | | | | | |
| | 2 | <scenario date="2016-10-18" description=""></scenario> | | | | | | |
| QCD | 3 | </math Generate plot file from database for GK model $>$ | | | | | | |
| QCD Mass without mass | 4 | <task method="generatePlotFile" service="ObservableService"></task> | | | | | | |
| Nucleon structure | 5 | <task_param type="output"></task_param> | | | | | | |
| Content of GPDs | 6 | <param <="" name="filePath" td="" value="observable_GK11_plot.csv"/> | | | | | | |
| Imaging | | > , . | | | | | | |
| Experimental access | 7 | | | | | | | |
| DVCS Kinematics Universality tests | 8 | </math Variables of 2d plot $>$ | | | | | | |
| Towards 3D images | 9 | <task_param td="" ty<=""><td>¢ [dog]</td><td>Λ</td></task_param> | ¢ [dog] | Λ | | | | |
| Building | 10 | <param nar<="" td=""/> <td>ϕ [deg]</td> <td>A_{LU}</td> | ϕ [deg] | A _{LU} | | | | |
| Design | 11 | <pre>param nar</pre> | 0. | 0. | | | | |
| Features | 12 13 | | 10. | 0.024736075012605108 | | | | |
| Examples | 14 | <task_param th="" ty<=""><th></th><th></th></task_param> | | | | | | |
| Learning | 15 | <pre>param nar</pre> | 20. | 0.048810639423911277 | | | | |
| Definition | 16 | <param nar<br=""/> <param nar<="" td=""/> <td>30.</td> <td>0.071572336121144678</td> | 30. | 0.071572336121144678 | | | | |
| Overlap | 17 | <param nar<br=""/> <param nar<="" td=""/> <td></td> <td></td> | | | | | | |
| Radon transform | 18 | <param nar<="" td=""/> <td></td> <td></td> | | | | | | |
| Covariant extension | 19 | | 350. | -0.024736075012605111 | | | | |
| Conclusion | 20 | | 360. | -9.0547874403168658e-17 | | | | |
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Pre-PARTONS times... First mention of the PARTONS project in a conference.



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PROPHET.

Platform for Representing the Organization of Partons inside Hadrons and Experimental Tomographies.

- Comprehensive database of experimental results.
- 2 Comprehensive database of theoretical predictions.
- Fitting engine.
- Opposition of statistic and systematic uncertainties.
- Solution Strate Stra model expectations.
- Onnection to experimental set-up descriptions to design new experiments.
- Interactive website providing free access to model and experimental values.

H. MOUTARDE (Irfu/SPhN, CEA-Saclay)

Hadron 2011 - 14 / 06 / 2011 イロト イヨト イモト イモト H. Moutarde Universidad de Sevilla 32 / 61

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Pivotal year for GPDs 2011 situation GPDs and DVCS

Selected data Status of GPD analysis

Universality Key results

orientations COMPASS-II JLab's 12 GeV upgrade Spin observables on an EIC The PROPHET

Conclusions

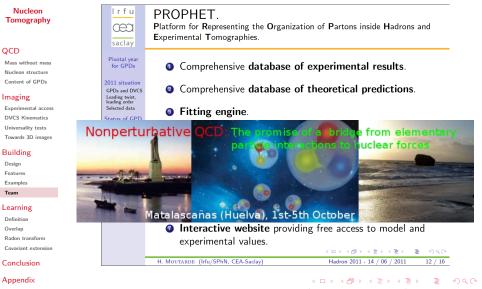
Leading twist. leading order

methods



Pre-PARTONS times... First mention of the PARTONS project in a conference.





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PARTONS times! An active community.



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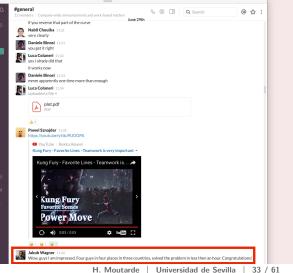
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PARTONS



F Invite people



Many (unfortunately not all!) problems can be solved fast

1 multidisciplinary team over 5 countries Theorists, experimentalists, 1 mathematician + 1 software engineer



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Learning on the strong interaction from GPD models

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Spin-0 Generalized Parton Distribution. Definition and simple properties.



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| $H^q_\pi(x,\xi,t) =$ |
|--|
| $\frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{i x P^{+} z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle \bar{q} \left(-\frac{z}{2} \right) \gamma^{+} q \left(\frac{z}{2} \right) \middle \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^{+} = 0\\z_{\perp} = 0}}$ |
| with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$. |
| $\rightarrow \tau^0$ |

References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994) Ji, Phys. Rev. Lett. **78**, 610 (1997) Radyushkin, Phys. Lett. **B380**, 417 (1996)

PDF forward limit

 z^3

$$H^q(x,0,0) = q(x)$$

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$$\begin{aligned} H^{q}_{\pi}(x,\xi,t) &= \\ \frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q} \left(-\frac{z}{2} \right) \gamma^{+} q \left(\frac{z}{2} \right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^{+}=0\\z_{\perp}=0}} \end{aligned}$$

with
$$t = \Delta^2$$
 and $\xi = -\Delta^+/(2P^+)$.

References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994) Ji, Phys. Rev. Lett. **78**, 610 (1997) Radyushkin, Phys. Lett. **B380**, 417 (1996)

PDF forward limit

 z^3

Form factor sum rule

$$\int_{-1}^{+1} dx H^q(x,\xi,t) = F_1^q(t)$$

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Spin-0 Generalized Parton Distribution. Definition and simple properties.



Nucleon Tomography $H^q_{\pi}($

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$$H_{\pi}^{q}(x,\xi,t) = \frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left(-\frac{z}{2} \right) \gamma^{+}q \left(\frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^{+}=0\\z_{\perp}=0}}$$
with $t = \Delta^{2}$ and $\xi = -\Delta^{+}/(2P^{+})$.

References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule
- H^q is an even function of ξ from time-reversal invariance.

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Spin-0 Generalized Parton Distribution. Definition and simple properties.



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$$H_{\pi}^{q}(x,\xi,t) = \frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left(-\frac{z}{2} \right) \gamma^{+}q \left(\frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^{+}=0\\z_{\perp}=0}}$$
with $t = \Delta^{2}$ and $\xi = -\Delta^{+}/(2P^{+})$.

References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)

PDF forward limit

- Form factor sum rule
- H^q is an even function of ξ from time-reversal invariance.
- H^q is real from hermiticity and time-reversal invariance.

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Radyushkin, Phys. Lett. **B380**, 417 (1996)





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 $\int_{-1}^{+1} dx x^n H^q(x,\xi,t) = \text{polynomial in } \xi$





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Lorentz covariance

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$H^{q}(x,\xi,t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right)q\left(\frac{x-\xi}{1-\xi}\right)}$





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Lorentz covariance

Positivity of Hilbert space norm

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Lorentz covariance

Positivity of Hilbert space norm

• H^q has support $x \in [-1, +1]$.

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Positivity

Positivity of Hilbert space norm

l orentz covariance

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• H^q has support x \in [-1, +1].
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Relativistic quantum mechanics





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QCD

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Lorentz covariance

Positivity of Hilbert space norm

•
$$H^q$$
 has support $x \in [-1, +1]$.

Relativistic quantum mechanics

Soft pion theorem (pion target)

$$H^{q}(x,\xi=1,t=0) = \frac{1}{2}\phi_{\pi}^{q}\left(\frac{1+x}{2}\right)$$

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Positivity

Positivity of Hilbert space norm

l orentz covariance

• H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

Soft pion theorem (pion target)

Dynamical chiral symmetry breaking





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Polynomiality

Lorentz covariance

Positivity of Hilbert space norm

• H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

Soft pion theorem (pion target)

Dynamical chiral symmetry breaking

How can we implement a priori these theoretical constraints?

 There is no known GPD parameterization relying only on first principles.

In the following, focus on **polynomiality** and **positivity**.



Double Distributions. Relation to Generalized Parton Distributions.

Representation of GPD:

Nucleon Tomography

$$H^{q}(x,\xi,t) = \int_{\Omega_{\rm DD}} \mathrm{d}\beta \mathrm{d}\alpha \,\delta(x-\beta-\alpha\xi) \big(F^{q}(\beta,\alpha,t) + \xi G^{q}(\beta,\alpha,t)\big)$$

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- Support property: $x \in [-1, +1]$.
- Discrete symmetries: F^q is α -even and G^q is α -odd.
- **Gauge**: any representation (F^q, G^q) can be recast in one representation with a single DD f^q :

$$H^{q}(x,\xi,t) = x \int_{\Omega_{\rm DD}} \mathrm{d}\beta \mathrm{d}\alpha \, f^{q}_{\rm BMKS}(\beta,\alpha,t) \delta(x-\beta-\alpha\xi)$$

Belitsky et al., Phys. Rev. D64, 116002 (2001) $H^{q}(x,\xi,t) = (1-x) \int_{\Omega_{\rm DD}} \mathrm{d}\beta \mathrm{d}\alpha \, f^{q}_{\rm P}(\beta,\alpha,t) \delta(x-\beta-\alpha\xi)$

> Pobylitsa, Phys. Rev. D67, 034009 (2003) Müller, Few Body Syst. 55, 317 (2014)

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Double Distributions. Lorentz covariance by example.



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• Choose $F^q(\beta, \alpha) = 3\beta\theta(\beta)$ ad $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$:

$$H^{q}(x,\xi) = 3x \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \,\delta(x - \beta - \alpha\xi)$$

Simple analytic expressions for the GPD:

$$\begin{aligned} \mathcal{H}(x,\xi) &= \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1, \\ \mathcal{H}(x,\xi) &= \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1. \end{aligned}$$

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Double Distributions. Lorentz covariance by example.



| Nucleon | Compute first Mellin moments. | | | | | | |
|---|--|---|---|--|--|--|--|
| Tomography | п | $\int_{-\xi}^{+\xi} \mathrm{d}x x^n H(x,\xi)$ | $\int_{+\xi}^{+1} \mathrm{d}x x^n H(x,\xi)$ | $\int_{-\xi}^{+1} \mathrm{d}x x^n H(x,\xi)$ | | | |
| QCD Mass without mass Nucleon structure Content of GPDs | 0 | $\frac{1+\xi-2\xi^2}{1+\xi}$ | $\frac{2\xi^2}{1+\xi}$ | 1 | | | |
| Imaging Experimental access DVCS Kinematics Universality tests | 1 | $\frac{1\!+\!\xi\!\!+\!\xi^2\!-\!3\xi^3}{2(1\!+\!\xi)}$ | $\frac{2\xi^3}{1+\xi}$ | $\frac{1+\xi^2}{2}$ | | | |
| Towards 3D images Building Design | 2 | $\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$ | $\frac{6\xi^4}{5(1+\xi)}$ | $\frac{3(1+\xi^2)}{10}$ | | | |
| Features Examples Team Learning | 3 | $\frac{1\!+\!\xi\!\!+\!\xi^2\!+\!\xi^3\!+\!\xi^4\!-\!5\xi^5}{5(1\!+\!\xi)}$ | $\frac{6\xi^5}{5(1+\xi)}$ | $\frac{1+\xi^2+\xi^4}{5}$ | | | |
| Definition Overlap Radon transform Covariant extension | 4 | $\frac{1 + \xi + \xi^2 + \xi^3 + \xi^4 + \xi^5 - 6\xi^6}{7(1 + \xi)}$ | $\frac{6\xi^6}{7(1+\xi)}$ | $\frac{1+\xi^2+\xi^4}{7}$ | | | |
| Conclusion | Expressions get more complicated as <i>n</i> increases But | | | | | | |
| Appendix | they always yield polynomials! | | | | | | |
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Overlap representation. A first-principle connection with Light Front Wave Functions.



Nucleon Tomography

• Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

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$$H; P, \lambda \rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_{\perp}]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \dots, x_N, \mathbf{k}_{\perp N}) |\beta, k_1, \dots, k_N \rangle$$

• Derive an expression for the pion GPD in the DGLAP region $\xi \le x \le 1$:

$$H^{q}(x,\xi,t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{x}\mathrm{d}\bar{\mathbf{k}}_{\perp}]_{N} \delta_{j,q} \delta(x-\bar{x}_{j}) \left(\psi_{N}^{(\beta,\lambda)}\right)^{*} (\hat{x}',\hat{\mathbf{k}}_{\perp}') \psi_{N}^{(\beta,\lambda)}(\tilde{x},\tilde{\mathbf{k}}_{\perp})$$

with $\tilde{x}, \tilde{\mathbf{k}}_{\perp}$ (resp. $\hat{x}', \hat{\mathbf{k}}'_{\perp}$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl et al., Nucl. Phys. B596, 33 (2001)

Similar expression in the ERBL region $-\xi \le x \le \xi$, but with overlap of *N*- and (N+2)-body LFWFs.

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Overlap representation. Advantages and drawbacks.



Nucleon Tomography

- Positivity relations are fulfilled **by construction**.
- Implementation of symmetries of *N*-body problems.

What is not obvious anymore

What is not obvious to see from the wave function representation is however the **continuity of GPDs at** $x = \pm \xi$ and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies nontrivial relations between the wave functions for the different Fock states relevant in the two regions. An ad hoc Ansatz for the wave functions would **almost certainly lead** to GPDs that **violate** the above requirements.

Diehl, Phys. Rept. 388, 41 (2003)

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Physical picture.

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The Radon transform. Definition and properties.





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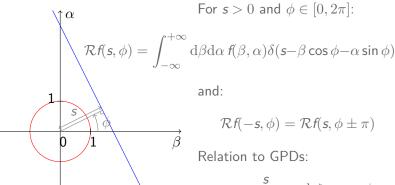
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$$x = \frac{s}{\cos \phi} \text{ and } \xi = \tan \phi$$

Relation between GPD and DD in Belistky et al. gauge

$$\frac{\sqrt{1+\xi^2}}{x}H(x,\xi) = \mathcal{R}f_{\rm BMKS}(s,\phi) ,$$

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The Radon transform. Definition and properties.





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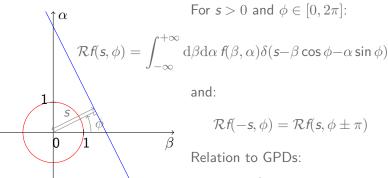
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$$x = \frac{s}{\cos \phi} \text{ and } \xi = \tan \phi$$

Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1+\xi^2}}{1-x}H(x,\xi) = \mathcal{R}f_{\mathrm{P}}(s,\phi) ,$$

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The range of the Radon transform. The polynomiality property a.k.a. the Ludwig-Helgason condition.



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 The Mellin moments of a Radon transform are homogeneous polynomials in ω = (sin φ, cos φ).

The converse is also true:

Theorem (Hertle, 1983)

Let $g(s, \omega)$ an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the **Ludwig-Helgason consistency condition** hold:

i) g is
$$C^{\infty}$$
 in ω ,

(ii) $\int ds s^m g(s, \omega)$ is a homogeneous polynomial of degree m for all integer $m \ge 0$.

 Double Distributions and the Radon transform are the natural solution of the polynomiality condition.

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Implementing Lorentz covariance. Extend an overlap in the DGLAP region to the whole GPD domain.



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DGLAP and ERBL regions

| $\begin{array}{rcl} (x,\xi) \in & \mathrm{DGLAP} & \Leftrightarrow & s \ge \sin \phi \\ (x,\xi) \in & \mathrm{ERBL} & \Leftrightarrow & s \le \sin \phi \end{array}$ | |
|--|---|
| $\alpha = \frac{1}{\xi}(x - \beta)$ | Each point $(\beta, \alpha) \neq 0$ contribute to both DGLAP ERBL r |
| $\Omega_{\rm DD} \left(\alpha + \beta \le 1 \right)$ | Express suppor theorem |

Each point (β, α) with $\beta \neq 0$ contributes to **both** DGLAP and ERBL regions.

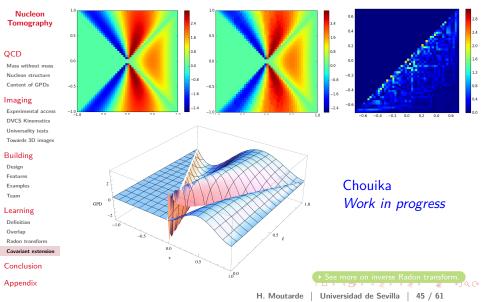
Expressed in support theorem.

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Covariant and positive GPD models. First systematic procedure to build models satisfying all constraints.





Conclusion

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Conclusions and prospects. Towards a unifying framework for GPD studies.



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- What makes hadron structure studies so interesting:
 - Deep **physical questions** waiting for answers!
 - Well-defined theoretical framework and observables.
 - Active experimental programs worldwide.
- Challenging constraints expected from:
 - Jefferson Lab in the valence sector,
 - CERN in the sea sector,
 - EIC (later) in the gluon sector.
- Success of physics program requires new GPD models with proper implementations of symmetries.
- Development of the PARTONS framework for phenomenology and theory purposes.
 - Fitting engine ready for local fits. Global fits in progress.
- First release of PARTONS by the end of 2016!

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Appendix

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Overview of current extraction methods. Problems: Model dependence? Uncertainties?



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Take each kinematic bin independantly of the others. Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ...as independent parameters.

Global fit

Local fits

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

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Neural networks

Exploratory stage for GPDs.

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Overview of current extraction methods. Problems: Model dependence? Uncertainties?



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Take each kinematic bin independantly of the others. Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ...as independent parameters.

M. Guidal, Eur. Phys. J. A39, 5 (2009)

- Almost model-independent: relies on twist-2 dominance assumption and assume bounds for the fitting domain.
- Interpretation of uncertainties on extracted quantities? Contributions from measurements uncertainties, correlations between CFFs and fitting domain boundaries.
- Interpretation of extracted quantities? e.g.mixing of quark and gluon GPDs due to NLO effects.
- **Oscillations** between different (x_B, t, Q^2) bins may happen.
- **Extrapolation** problem left open.

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Overview of current extraction methods. Problems: Model dependence? Uncertainties?



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Local fits: What can be achieved in principle?

Structure of BSA at twist 2 : $BSA(\phi) = \frac{a \sin \phi + b \sin 2\phi}{1 + c \cos \phi + d \cos 2\phi + e \cos 3\phi}$

where
$$a = \mathcal{O}(Q^{-1}), \quad b = \mathcal{O}(Q^{-4}), \quad c = \mathcal{O}(Q^{-1})$$

 $d = \mathcal{O}(Q^{-2}), \quad e = \mathcal{O}(Q^{-5}).$

Overview of current extraction methods. Problems: Model dependence? Uncertainties?



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Structure of BSA at twist 2 : $BSA(\phi) = \frac{a \sin \phi + b \sin 2\phi}{1 + c \cos \phi + d \cos 2\phi + e \cos 3\phi}$

Underconstrained problem (8 fit parameters : real and imaginary parts of 4 CFFs *H*, *E*, *H* and *E*).



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Local fits: What can be achieved in principle?

Structure of BSA at twist 2 : $BSA(\phi) = \frac{a\sin\phi + b\sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$

Underconstrained problem.

Need other asymmetries on same kinematic bin to allow extraction of all CFFs (or add \simeq 5-10 % systematic uncertainty).

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Overview of current extraction methods. Problems: Model dependence? Uncertainties?



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Local fits: What can be achieved in principle?

Structure of BSA at twist 2 : $BSA(\phi) = \frac{a \sin \phi + b \sin 2\phi}{1 + c \cos \phi + d \cos 2\phi + e \cos 3\phi}$

Underconstrained problem.

- Need other asymmetries on same kinematic bin to allow extraction of all CFFs.
- Add physical input? Dispersion relations, etc. Kumericki et al., arXiv:1301.1230
 Guidal et al., Rept. Prog. Phys. 76, 066202 (2013)



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Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Kumericki, Nucl. Phys. B841, 1 (2010)

- **Model-dependent** approach.
- Allows the implementation of theoretical constraints on GPDs or CFFs.
- Guideline for extrapolation outside the physical domain.
- Compromise between number of parameters and number of described GPDs (flavor dependence, higher-twists, ...)?
 - Impact on the choice of a fitting strategy?

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Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

Moutarde, Phys. Rev. D79, 094021 (2009)

■ Avoid unphysical oscillations between different (*x_B*, *t*, *Q*²) bins by comparing to a **global fit by a smooth function**:

$$H^{+} = 2\sum_{n=0}^{N}\sum_{l=0}^{n+1} B_{nl}(t)\theta(|x| < \xi) \left(1 - \frac{x^{2}}{\xi^{2}}\right) C_{2n+1}^{(3/2)}\left(\frac{x}{\xi}\right) P_{2l}\left(\frac{x}{\xi}\right)$$

- Number of fit parameters describing the B_{nl} coefficients increases with N²...Extension to other GPDs seems difficult.
- **Extrapolation** problem left open.



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Neural networks

Exploratory stage for GPDs.

Kumericki et al., JHEP 1107, 073 (2011)

- Already used for PDF fits.
- Almost model-independent: neural network description, twist-2, *H*-dominance?
- Good agreement between model fit and neural network fit in the fitting domain.
- More reliable uncertainties in extrapolations?
 - **Overtraining** as a generic feature of (too) flexible models.

Back to summary of first extractions.

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Developing the theoretical framework. Are subdominant contributions negligible?



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Moutarde et al., Phys. Rev. D87, 054029 (2013)





Experimental access

Systematic tests of perturbative QCD assumptions. Wide kinematic range (from JLab to EIC).

Accuracy set by JLab 12 GeV expected statistical accuracy.

Model dependent evaluations.

-0.3

-0.8

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Kinematics of existing DVCS measurements. Looking for the Bjorken regime.



What is large Q^2 ? Measurements before 2015... Nucleon Tomography Q^2 (GeV²) Mass without mass Nucleon structure 3.0 Content of GPDs Imaging Experimental access DVCS Kinematics 2.0 Universality tests · JLab Hall A Towards 3D images • II ab Hall B Building 1.0 HERMES Features Examples • HERA 0.10.20.30.40.5XB Learning Definition World data cover complementary kinematic regions. Overlan

Radon transform Covariant extension

Conclusion

QCD

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What is large Q^2 ? Measurements before 2015... Nucleon Tomography $1.0 |t|/Q^2$ • II ab Hall A Mass without mass · JLab Hall B Nucleon structure 0.8 Content of GPDs HERMES Imaging 0.6 Experimental access • HERA **DVCS** Kinematics Universality tests 0.4 Towards 3D images Building Design 0.2 Examples 0.20.30.40.5XB Learning Definition Overlan World data cover **complementary kinematic regions**.

 Q^2 is **not so large** for most of the data.

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- World data cover **complementary kinematic regions**. Q^2 is **not so large** for most of the data.
 - Higher twists? Finite-t and target mass corrections?

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- World data cover **complementary kinematic regions**. Q^2 is **not so large** for most of the data.
 - Higher twists? Finite-t and target mass corrections?

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Dispersion relations and the cross-over line. Existence of a relation between $Re\mathcal{H}(\xi)$ and $H(x, \xi = x)$.



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■ Write dispersion relation **at fixed** *t* **and** *Q*²:

$$Re\mathcal{H}(\xi, t) = \Delta(t) + \frac{2}{\pi} \mathcal{P} \int_0^1 \frac{\mathrm{d}x}{x} \frac{Im\mathcal{H}(x, t)}{\left(\frac{\xi^2}{x^2} - 1\right)}$$

• Use LO relation $Im\mathcal{H}(x,t) = \pi (H(x,x,t) - H(-x,x,t)).$

■ Up to the D-term form factor Δ(t), all the information accessible at LO and fixed Q² is contained on the cross-over line.

Teryaev, hep-ph/0510031 Anikin and Teryaev, Phys. Rev. **D76**, 056007 (2007) Diehl and Ivanov, Eur. Phys. J. **C52**, 919 (2007)

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Dispersion relations and actual data. Too few kinematic bins to provide model-independent constraints?



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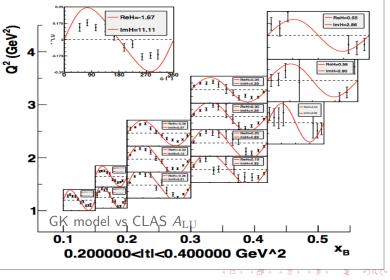
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Dispersion relations and actual data. Too few kinematic bins to provide model-independent constraints?

Nucleon Tomography ReH=-1.67 ReH=0.55 0.1 ImH=2.86 Q² (GeV²) ImH=11.11 3 4 Mass without mass Ηİ Ŧ Nucleon structure ReH=0.56 ImH=2.9 Content of GPDs I I Imaging Experimental access З DVCS Kinematics ζ. Universality tests Towards 3D images Building 2 Ξx ŦŦ-Examples 1 T Learning 3 to 4 (x_B , t, Q^2) bins Definition with $|t|/Q^2 \simeq 0.12$ GK model vs CLAS $A_{\rm LU}$ Radon transform 0.1 0.2 0.3 0.4 0.5 Covariant extension 0.200000<ltl<0.400000 GeV^2 Conclusion うくに < 🗆

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Dispersion relations and actual data. Too few kinematic bins to provide model-independent constraints?



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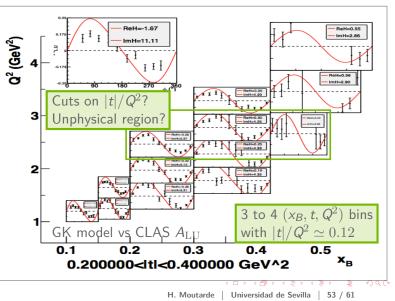
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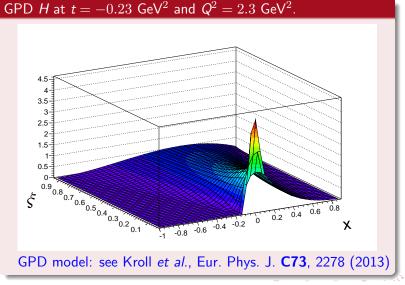
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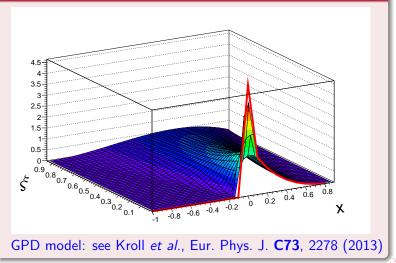
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Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.







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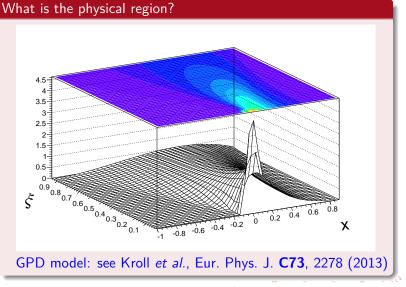
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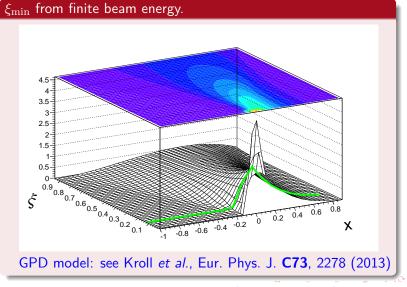
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 $\xi_{\rm max}$ from kinematic constraint on 4-momentum transfer.



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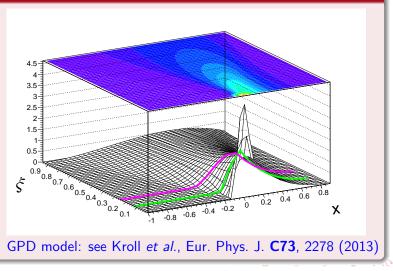
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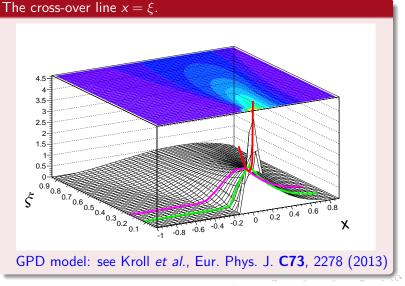
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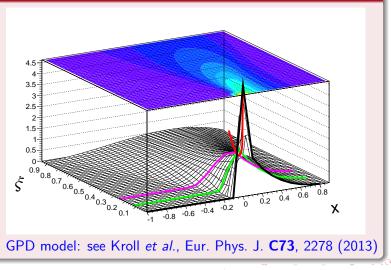
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The black curve is what is needed for transverse plane imaging!

Density plot of H at t = -0.23 GeV² and $Q^2 = 2.3$ GeV²



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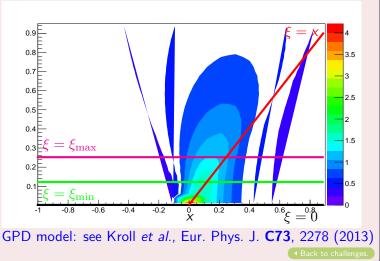
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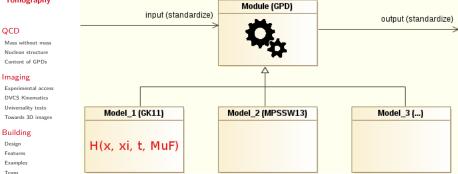
Modularity.

Inheritance, standardized inputs and outputs.



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Learning

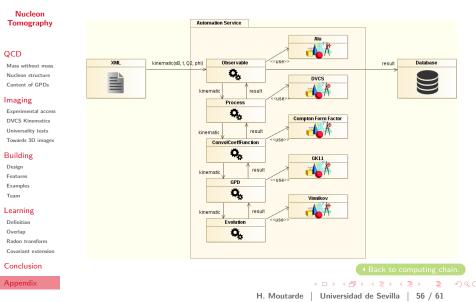
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- Steps of logic sequence in parent class.
- Model description and related mathematical methods in daughter class.

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Modularity and automation. Parse XML file, compute and store result in database.







Implementing Lorentz covariance. Extend an overlap in the DGLAP region to the whole GPD domain.



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For **any model of LFWF**, one has to address the following three questions:

1 Does the extension exist?

2 If it exists, is it unique?

3 How can we compute this extension?

Work in progress!

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Implementing Lorentz covariance. Uniqueness of the extension.



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Let f be a compactly-supported locally summable function defined on \mathbb{R}^2 and \mathcal{R} f its Radon transform. Let $(s_0, \omega_0) \in \mathbb{R} \times S^1$ and U_0 an open neighborhood of ω_0 such that:

for all $s > s_0$ and $\omega \in U_0$ $\mathcal{R}f(s, \omega) = 0$.

Then $f(\aleph) = 0$ on the half-plane $\langle \aleph | \omega_0 \rangle > s_0$ of \mathbb{R}^2 .

Consider a GPD H being zero on the DGLAP region.

- Take ϕ_0 and $s_0 \ s.t. \cos \phi_0 \neq 0$ and $|s_0| > |\sin \phi_0|$.
- Neighborhood U_0 of ϕ_0 s.t. $\forall \phi \in U_0 | \sin \phi | < |s_0|$.
- The underlying DD f has a zero Radon transform for all $\phi \in U_0$ and $s > s_0$ (DGLAP).
- Then $f(\beta, \alpha) = 0$ for all $(\beta, \alpha) \in \Omega_{DD}$ with $\beta \neq 0$.
 - Extension **unique** up to adding a **D-term**: $\delta(\beta)D(\alpha)$.

Cea

Computation of the extension. Numerical evaluation of the inverse Radon transform (1/2).



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Consider N + 1 Hilbert spaces H, H_1 , ..., H_N , and a family of continuous surjective operators $R_n : H \to H_n$ for $1 \le n \le N$. Being given $g_1 \in H_1$, ..., $g_n \in H_n$, we search f solving the following system of equations:

$$R_n f = g_n \quad \text{for } 1 \le n \le N$$

Fully discrete case

A discretized problem

Assume f piecewise-constant with values f_m for $1 \le m \le M$. For a collection of lines $(L_n)_{1 \le n \le N}$ crossing Ω_{DD} , the Radon transform writes:

$$g_n = \mathcal{R}f = \int_{L_n} f = \sum_{m=1}^M f_m \times \text{Measure}(L_n \cap C_m) \quad \text{ for } 1 \le n \le N$$

Computation of the extension. Numerical evaluation of the inverse Radon transform (2/2).



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And if the input data are inconsistent?

- Instead of solving $g = \mathcal{R}f$, find f such that $||g \mathcal{R}f||_2$ is minimum.
- The solution always exists.

• The input data are **inconsistent** if $||g - \mathcal{R}f||_2 > 0$.

Back to covariant extensions

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