#### DE LA RECHERCHE À L'INDUSTRIE





## Representations of Generalized Parton Distributions in the Dyson-Schwinger Formalism



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Few-Body Problems 2015 | Hervé MOUTARDE

May 19<sup>th</sup>, 2015

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### Motivations. 3D imaging of nucleon's partonic content but also...



Representations of Generalized Parton Distributions

### Introduction

### Theoretical framework

- Definition
- Polynomiality
- Double Distributions
- Overlap representation

### GPD modeling

Diagrams Algebraic model

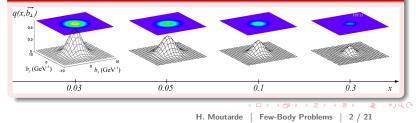
### Results

Checks Form factor Pion PDF

### Conclusions

- Correlation of the longitudinal momentum and the transverse position of a parton in the nucleon.
- Insights on:
  - **Spin** structure,
  - **Energy-momentum** structure.
- Probabilistic interpretation of Fourier transform of GPD(x, ξ = 0, t) in transverse plane.

### Transverse plane density (Goloskokov and Kroll model)





### Overview.

Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.



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- Important topic for several past, existing and future experiments: H1, ZEUS, HERMES, CLAS, CLAS12, JLab Hall A, COMPASS, EIC, ...
- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- Recent applications of the Dyson-Schwinger and Bethe-Salpeter framework to hadron structure studies.



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- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- Recent applications of the Dyson-Schwinger and Bethe-Salpeter framework to hadron structure studies.
- Here develop pion GPD model for simplicity.
- No planned experiment on pion GPDs but existing proposal of DVCS on a virtual pion.

Amrath et al., Eur. Phys. J. C58, 179 (2008)

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### Overview.

Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.



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- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- Recent applications of the Dyson-Schwinger and Bethe-Salpeter framework to hadron structure studies.

Steps towards a **pion GPD** model:

- GPDs: Theoretical Framework
- GPDs in the Dyson-Schwinger and Bethe-Salpeter Approach



Results: Theoretical Constraints and Phenomenology

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### **GPDs:** Theoretical Framework

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### Pion Generalized Parton Distribution. Definition and symmetry relations.

Representations of Generalized Parton Distributions

Introduction Theoretical framework Definition Polynomiality

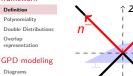
$$H_{\pi}^{q}(x,\xi,t) = \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left( -\frac{z}{2} \right) \gamma^{+}q \left( \frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^{+}=0\\z_{\perp}=0}}$$
with  $t = \Delta^{2}$  and  $\xi = -\Delta^{+}/(2P^{+})$ .

 $z^3$ 

Müller et al., Fortschr. Phys. 42, 101 (1994) Ji, Phys. Rev. Lett. 78, 610 (1997) Radyushkin, Phys. Lett. B380, 417 (1996)

From isospin symmetry, all the information about pion GPD is encoded in  $H^{u}_{\pi^{+}}$  and  $H^{d}_{\pi^{+}}$ . Further constraint from charge conjugation:  $H^{u}_{\pi^{+}}(x,\xi,t) = -H^{d}_{\pi^{+}}(-x,\xi,t).$ 

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### Diagrams Algebraic model Results

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### Conclusions





### Properties. Generalization of form factors and Parton Distribution Functions.



Representations of Generalized Parton Distributions

### PDF forward limit

$$H^q(x,0,0) = q(x)$$

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Generalization of form factors and Parton Distribution Functions.



### PDF forward limit

Form factor sum rule

$$\int_{-1}^{+1} dx \, H^q(x,\xi,t) = F_1^q(t)$$

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Generalization of form factors and Parton Distribution Functions.



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- Polynomiality

$$\int_{-1}^{+1} dx x^n H^q(x,\xi,t) = \text{polynomial in } \xi$$



Generalization of form factors and Parton Distribution Functions.



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- Form factor sum rule
- Polynomiality
- Positivity

$$H^{q}(x,\xi,t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right)q\left(\frac{x-\xi}{1-\xi}\right)}$$



Generalization of form factors and Parton Distribution Functions.



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- Form factor sum rule
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- $H^q$  is an even function of  $\xi$  from time-reversal invariance.



Generalization of form factors and Parton Distribution Functions.



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- Positivity
- $H^q$  is an **even function** of  $\xi$  from time-reversal invariance.
- *H*<sup>q</sup> is **real** from hermiticity and time-reversal invariance.



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- $H^q$  is an **even function** of  $\xi$  from time-reversal invariance.
- *H<sup>q</sup>* is **real** from hermiticity and time-reversal invariance.

•  $H^q$  has support  $x \in [-1, +1]$ .



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- *H<sup>q</sup>* is **real** from hermiticity and time-reversal invariance.
  - $H^q$  has support  $x \in [-1, +1]$ .
- **Soft pion theorem** (pion target)

$$H^{q}(x,\xi=1,t=0) = \frac{1}{2}\phi_{\pi}^{q}\left(\frac{1+x}{2}\right)$$

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Generalization of form factors and Parton Distribution Functions.



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• 
$$H^q$$
 has support  $x \in [-1, +1]$ .

**Soft pion theorem** (pion target)

### Numerous theoretical constraints on GPDs.

- There is no known GPD parameterization relying only on first principles.
- Modeling becomes a key issue.



### Polynomiality. Mixed constraint from Lorentz invariance and discrete symmetries.



Introduce isovector and isoscalar GPDs: Representations of Generalized Parton  $H^{l=0}(x,\xi,t) = H^{u}_{\pi^{+}}(x,\xi,t) + H^{d}_{\pi^{+}}(x,\xi,t)$ Distributions  $H^{l=1}(x,\xi,t) = H^{u}_{\pi^{+}}(x,\xi,t) - H^{d}_{\pi^{+}}(x,\xi,t)$ Introduction Compute Mellin moments of GPDs: Theoretical framework Definition  $\int dx x^m H^{l=0}(x,\xi) = 0 \ (m \text{ even})$ Polynomiality Double Distributions Overlan representation  $\int_{-1}^{1} \mathrm{d}x \, x^m H^{l=0}(x,\xi) = \sum_{m=1}^{m} (2\xi)^i C_{mi}^{l=0} + (2\xi)^{m+1} C_{m\,m+1}^{l=0} \ (m \text{ odd})$ GPD modeling Diagrams Algebraic model Results Checks  $\int_{-1}^{1} \mathrm{d}x \, x^m H^{l=1}(x,\xi) = \sum_{i=0}^{m} (2\xi)^i C_{mi}^{l=1} \ (m \text{ even})$ Form factor Pion PDF Conclusions even  $\int^{1} \mathrm{d}x \, x^{m} H^{l=1}(x,\xi) = 0 \ (m \text{ odd})$ 

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### Double Distributions. A convenient tool to encode GPD properties.



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Define Double Distributions F<sup>q</sup> and G<sup>q</sup> as matrix elements of twist-2 quark operators:

$$P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^{\{\mu_i \overleftrightarrow{D}^{\mu_1} \dots i \overleftrightarrow{D}^{\mu_m\}}} q(0) \left| P - \frac{\Delta}{2} \right\rangle = \sum_{k=0}^m \binom{m}{k}$$

$$\int_{\text{Batributions}}^{\text{for}} \left[ F_{mk}^{q}(t) 2 P^{\left\{\mu\right\}} - G_{mk}^{q}(t) \Delta^{\left\{\mu\right\}} \right] P^{\mu_{1}} \dots P^{\mu_{m-k}} \left( -\frac{\Delta}{2} \right)^{\mu_{m-k+1}} \dots \left( -\frac{\Delta}{2} \right)^{\mu_{m}}$$

with

$$F^{q}_{mk} = \int_{\Omega} d\beta d\alpha \, \alpha^{k} \beta^{m-k} F^{q}(\beta, \alpha)$$
$$G^{q}_{mk} = \int_{\Omega} d\beta d\alpha \, \alpha^{k} \beta^{m-k} G^{q}(\beta, \alpha)$$

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994) Radyushkin, Phys. Rev. **D59**, 014030 (1999) Radvsuhkin, Phys. Lett. **B449**. 81 (1999) H. Moutarde | Few-Body Problems | 8/21



### Double Distributions. Relation to Generalized Parton Distributions.



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Representation of GPD:

$$H^{q}(x,\xi,t) = \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \,\delta(x-\beta-\alpha\xi) \big(F^{q}(\beta,\alpha,t) + \xi G^{q}(\beta,\alpha,t)\big)$$

Support property: 
$$x \in [-1, +1]$$
.

Discrete symmetries:  $F^q$  is  $\alpha$ -even and  $G^q$  is  $\alpha$ -odd.



### Overlap representation. A first-principle connection with Light Front Wave Functions.



#### Representations of Generalized Parton Distributions

$$H; P, \lambda \rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_{\perp}]_{N} \psi_{N}^{(\beta,\lambda)}(x_{1}, \mathbf{k}_{\perp 1}, \dots, x_{N}, \mathbf{k}_{\perp N}) |\beta, k_{1}, \dots, k_{N} \rangle$$

Decompose an hadronic state  $|H; P, \lambda\rangle$  in a Fock basis:

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• Derive an expression for the pion GPD in the DGLAP region  $\xi \le x \le 1$ :

$${}^{q}(x,\xi,t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{x}\mathrm{d}\bar{\mathbf{k}}_{\perp}]_{N} \delta_{j,q} \delta(x-\bar{x}_{j}) \left(\psi_{N}^{(\beta,\lambda)}\right)^{*} (\hat{x}',\hat{\mathbf{k}}_{\perp}') \psi_{N}^{(\beta,\lambda)}(\tilde{x},\tilde{\mathbf{k}}_{\perp})$$

with  $\tilde{x}, \tilde{k}_{\perp}$  (resp.  $\hat{x}', \hat{k}'_{\perp}$ ) generically denoting incoming (resp. outgoing) parton kinematics.

### Diehl et al., Nucl. Phys. B596, 33 (2001)

■ Similar expression in the ERBL region -ξ ≤ x ≤ ξ, but with overlap of N- and (N+2)-body LFWF.

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### Overlap representation. Advantages and drawbacks.



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- Physical picture.
- Positivity relations are fulfilled **by construction**.
- Implementation of symmetries of N-body problems.

### What is not obvious anymore

What is *not* obvious to see from the wave function representation is however the **continuity of GPDs at**  $x = \pm \xi$ and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies **nontrivial relations between the wave functions** for the different Fock states relevant in the two regions. An *ad hoc* Ansatz for the wave functions would **almost certainly lead** to GPDs that **violate the above requirements**.

### Diehl, Phys. Rept. 388, 41 (2003)

# GPDs in the Dyson-Schwinger and Bethe-Salpeter Approach

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### GPDs in the rainbow ladder approximation. Evaluation of triangle diagrams.



#### Representations of Generalized Parton Distributions

$$\langle x^{m} \rangle^{q} = \frac{1}{2(P^{+})^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^{+} (i\overleftrightarrow{D}^{+})^{m} q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

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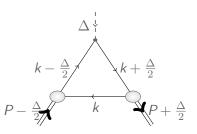
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• Compute **Mellin moments** of the pion GPD *H*.

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- $k \frac{\Delta}{2}$   $k + \frac{\Delta}{2}$   $P \frac{\Delta}{2}$   $k + \frac{\Delta}{2}$   $P + \frac{\Delta}{2}$
- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.

### GPDs in the rainbow ladder approximation. Evaluation of triangle diagrams.

Representations of Generalized Parton Distributions

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

Compute Mellin moments

Triangle diagram approx.

Resum **infinitely many** 

of the pion GPD H.

contributions.

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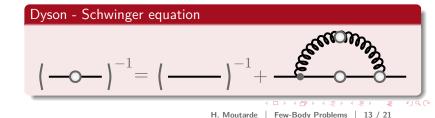
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### GPDs in the rainbow ladder approximation. Evaluation of triangle diagrams.

Representations of Generalized Parton Distributions

$$\langle x^{m} \rangle^{q} = \frac{1}{2(P^{+})^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^{+} (i\overleftrightarrow{D}^{+})^{m} q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

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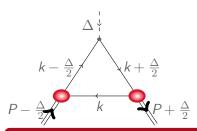
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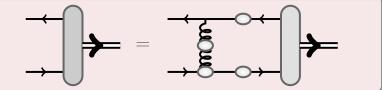
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- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.
- Resum infinitely many contributions.

### Bethe - Salpeter equation



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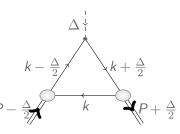
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- Resum infinitely many contributions.
- **Nonperturbative** modeling.

Most GPD properties satisfied by construction.

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- Most GPD properties satisfied by construction.
- Also compute crossed triangle diagram.

### Mezrag *et al.*, arXiv:1406.7425 [hep-ph] and Phys. Lett. **B741**, 190 (2015)

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# Cea

### Algebraic model.

Intermediate step before using numerical solutions of Dyson-Schwinger and Bethe-Salpeter equations.



Representations of Generalized Parton Distributions

 $2(P^{+})^{m+1} \langle x^{m} \rangle^{u} = \operatorname{tr}_{CFD} \int \frac{\mathrm{d}^{4}k}{(2\pi)^{4}} (k^{+})^{m} i\Gamma_{\pi} \left(k - \frac{\Delta}{2}, P - \frac{\Delta}{2}\right) \\ \times S(k - \frac{\Delta}{2}) i\gamma^{+} S(k + \frac{\Delta}{2}) i\overline{\Gamma}_{\pi} \left(k + \frac{\Delta}{2}, P + \frac{\Delta}{2}\right) S(k - P)$ 

Expressions for vertices and propagators:

Expression for GPD Mellin moments:

$$S(p) = \left[ -i\gamma \cdot p + M \right] \Delta_M(p^2)$$
  

$$\Delta_M(s) = \frac{1}{s + M^2}$$
  

$$\Gamma_\pi(k, p) = i\gamma_5 \frac{M}{f_\pi} M^{2\nu} \int_{-1}^{+1} dz \, \rho_\nu(z) \, \left[ \Delta_M(k_{+z}^2) \right]^\nu$$
  

$$\rho_\nu(z) = R_\nu (1 - z^2)^\nu$$

with  $R_{\nu}$  a normalization factor and  $k_{+z} = k - p(1-z)/2$ . Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

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Expressions for vertices and propagators:

$$\begin{split} S(p) &= \begin{bmatrix} -i\gamma \cdot p + M \end{bmatrix} \Delta_M(p^2) \\ \Delta_M(s) &= \frac{1}{s + M^2} \\ \Gamma_\pi(k, p) &= i\gamma_5 \frac{M}{f_\pi} M^{2\nu} \int_{-1}^{+1} \mathrm{d}z \, \rho_\nu(z) \, \left[ \Delta_M(k_{+z}^2) \right]^\nu \\ \rho_\nu(z) &= R_\nu (1 - z^2)^\nu \end{split}$$

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Intermediate step before using numerical solutions of Dyson-Schwinger and Bethe-Salpeter equations.

Expressions for vertices and propagators:



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# $S(p) = \left[ -i\gamma \cdot p + \mathbf{M} \right] \Delta_{\mathbf{M}}(p^{2})$ $\Delta_{\mathbf{M}}(s) = \frac{1}{s + \mathbf{M}^{2}}$ $\Gamma_{\pi}(k, p) = i\gamma_{5} \frac{\mathbf{M}}{f_{\pi}} \mathbf{M}^{2\nu} \int_{-1}^{+1} \mathrm{d}z \,\rho_{\nu}(z) \left[ \Delta_{\mathbf{M}}(k_{+z}^{2}) \right]^{\nu}$ $\rho_{\nu}(z) = R_{\nu}(1 - z^{2})^{\nu}$

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Only two parameters:

Dimensionful parameter M.

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- Dimensionless parameter  $\nu$

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- Dimensionful parameter *M*.
- Dimensionless parameter v. Fixed to 1 to recover asymptotic pion DA.

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# **Results: Theoretical Constraints and Phenomenology**

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Analytic expression in DGLAP and ERBL regions. Representations of Generalized  $\begin{array}{lll} \text{Parton} \\ \text{Distributions} \ H^{\nu}_{x \geq \xi}(x,\xi,0) & = & \displaystyle \frac{48}{5} \ \left\{ \frac{3 \left( -2(x-1)^4 \left( 2x^2 - 5\xi^2 + 3 \right) \log(1-x) \right)}{20 \left( \xi^2 - 1 \right)^3} \right. \end{array} \right.$  $3\left(+4\xi\left(15x^{2}(x+3)+(19x+29)\xi^{4}+5(x(x(x+11)+21)+3)\xi^{2}\right)\tanh^{-1}\left(\frac{(x-1)}{x-\xi^{2}}\right)$ Introduction Theoretical  $20 (\xi^2 - 1)^3$ framework  $+\frac{3 \left(x^3 (x (2 (x - 4) x + 15) - 30) - 15 (2 x (x + 5) + 5) \xi^4\right) \log \left(x^2 - \xi^2\right)}{20 (\xi^2 - 1)^3}$ Definition Polynomiality Double Distributions  $+\frac{3 \left(-5 x (x (x + 2) + 3 6) + 18) \xi ^2-15 \xi ^6\right) \log \left(x^2-\xi ^2\right)}{20 \left(\xi ^2-1\right) ^3}$ Overlan representation GPD modeling Diagrams  $+\frac{3\left(2(x-1)\left((23x+58)\xi^{4}+(x(x(x+67)+112)+6)\xi^{2}+x(x((5-2x)x+15)+3)\xi^{2}+(x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+6)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x(x+67)+112)+2)\xi^{2}+x(x+67)+112)+2)\xi^{2}+x(x+67)$ Algebraic model  $20(\varepsilon^2-1)^3$ Results  $+\frac{3\left(\left(15(2x(x+5)+5)\xi^{4}+10x(3x(x+5)+11)\xi^{2}\right)\log\left(1-\xi^{2}\right)\right)}{20\left(\xi^{2}-1\right)^{3}}$ Checks Form factor Pion PDF  $+ \left. \frac{3 \left(2 x (5 x (x+2)-6)+15 \xi ^6-5 \xi ^2+3\right) \log \left(1-\xi ^2\right)}{20 \left(\xi ^2-1\right)^3}\right\}$ Conclusions

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Representations of Generalized Parton Distributions

- Analytic expression in DGLAP and ERBL regions.
- **Explicit check** of **support property** and **polynomiality** with correct powers of *ξ*.

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- Also direct verification using Mellin moments of *H*.





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- Soft pion theorem obtained by construction in a symmetry-preserving treatment.

Mezrag et al., Phys. Lett. B741, 190 (2015)





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Mezrag et al., Phys. Lett. B741, 190 (2015)

Valence  $H^{u}(x, \xi, t)$  as a function of x and  $\xi$  at vanishing t.

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-1.0

-0.5





### Pion form factor.

Determination of the model dimensionful parameter M.



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Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{l=1}(x,\xi,t) = 2F_{\pi}(t)$$

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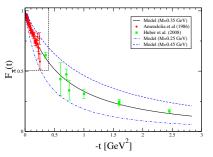
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Single dimensionful parameter  $M \simeq 350$  MeV.



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

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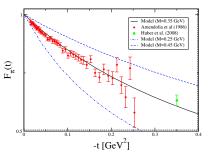
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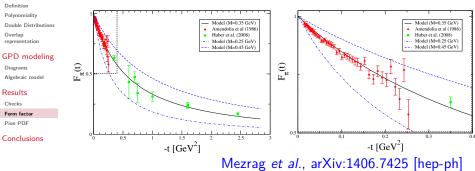
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# Pion Parton Distribution Function.

Algebraic model in the overlap representation.



Representations of Generalized Parton Distributions

Evaluate LFWF in algebraic model: 
$$\psi(\textbf{x},\textbf{k}_{\perp}) \propto \frac{\textbf{x}(1-\textbf{x})}{[(\textbf{k}_{\perp}-\textbf{x}\textbf{P}_{\perp})^2+\textit{M}^2]^2}$$

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• Expression for the GPD at 
$$t = 0$$
:

$$H(x,\xi,0) \propto \frac{(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2}$$

Expression for the PDF:

$$q(x) = 30x^2(1-x)^2$$

Compare to "naive" triangle diagram computation:

$$q(x) = \frac{72}{25} \Big( (30 - 15x + 8x^2 - 2x^3)x^3 \log x \Big)$$

 $+(3+2x^{2})(1-x)^{4}\log(1-x)+(3+15x+5x^{2}-2x^{3})x(1-x)\Big)$ 

### Chang et al., Phys. Lett. B737, 23 (2014)

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### Conclusions and prospects. First steps in a GPD modeling program.



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Checks Form factor

Conclusions

- Computation of GPDs, DDs, PDFs, LFWFs and form factors in the nonperturbative framework of Dyson-Schwinger and Bethe-Salpeter equations.
- **Explicit check** of several theoretical constraints, including polynomiality, support property and soft pion theorem.
- Simple algebraic model exhibits most features of the numerical solutions of the Dyson-Schwinger and Bethe-Salpeter equations.
- Very good agreement with existing pion form factor and PDF data.
- Clear limits of impulse approximation in the evaluation of quark twist-2 matrix elements.

### Commissariat à l'énergie atomique et aux énergies alternatives DSM Centre de Saclay | 91191 Gif-sur-Yvette Cedex Irfu T, +330(19 60 67 38 | F, +330(1) 60 68 78 84 SPINI

Etablissement public à caractère industriel et commercial R.C.S. Paris B 775 685 01

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