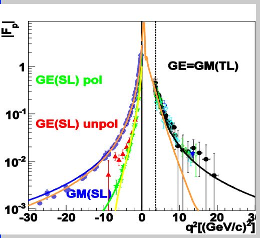
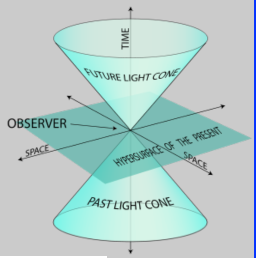
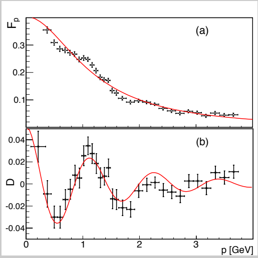
On the Physical Meaning of Time-like Electromagnetic Form Factors: The 4th Dimension of the Nucleon







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668. WE-Heraeus-Seminar on Baryon Form Factors: Where do we stand?

23-27 April 2018

Physikzentrum Bad Honnef, Germany

Europe/Berlin timezone

Supported by the WE-Heraeus Foundation

Wilhelm und Else Heraeus Stiftung

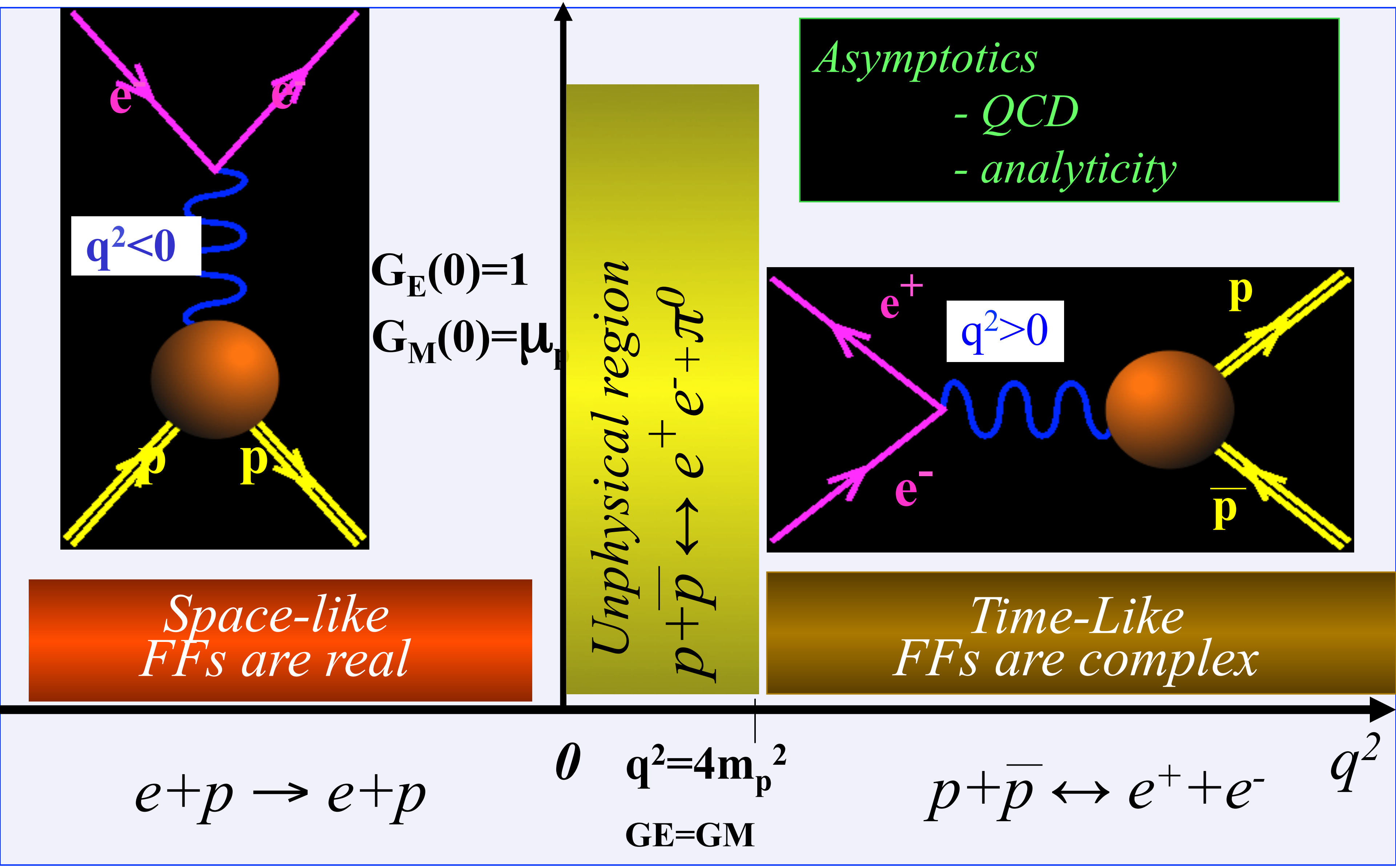
Plan

- Introduction
  - Space- and Time-like Form Factors
  - The dipole approximation
- Periodic oscillation of BaBar data
  - Data
  - Interpretation
    - Fourier transform
    - Optical model
- Modelization
  - Generalization of FF
  - The charge pair creation
  - A new picture for the nucleon structure
- Future prospects and Conclusions

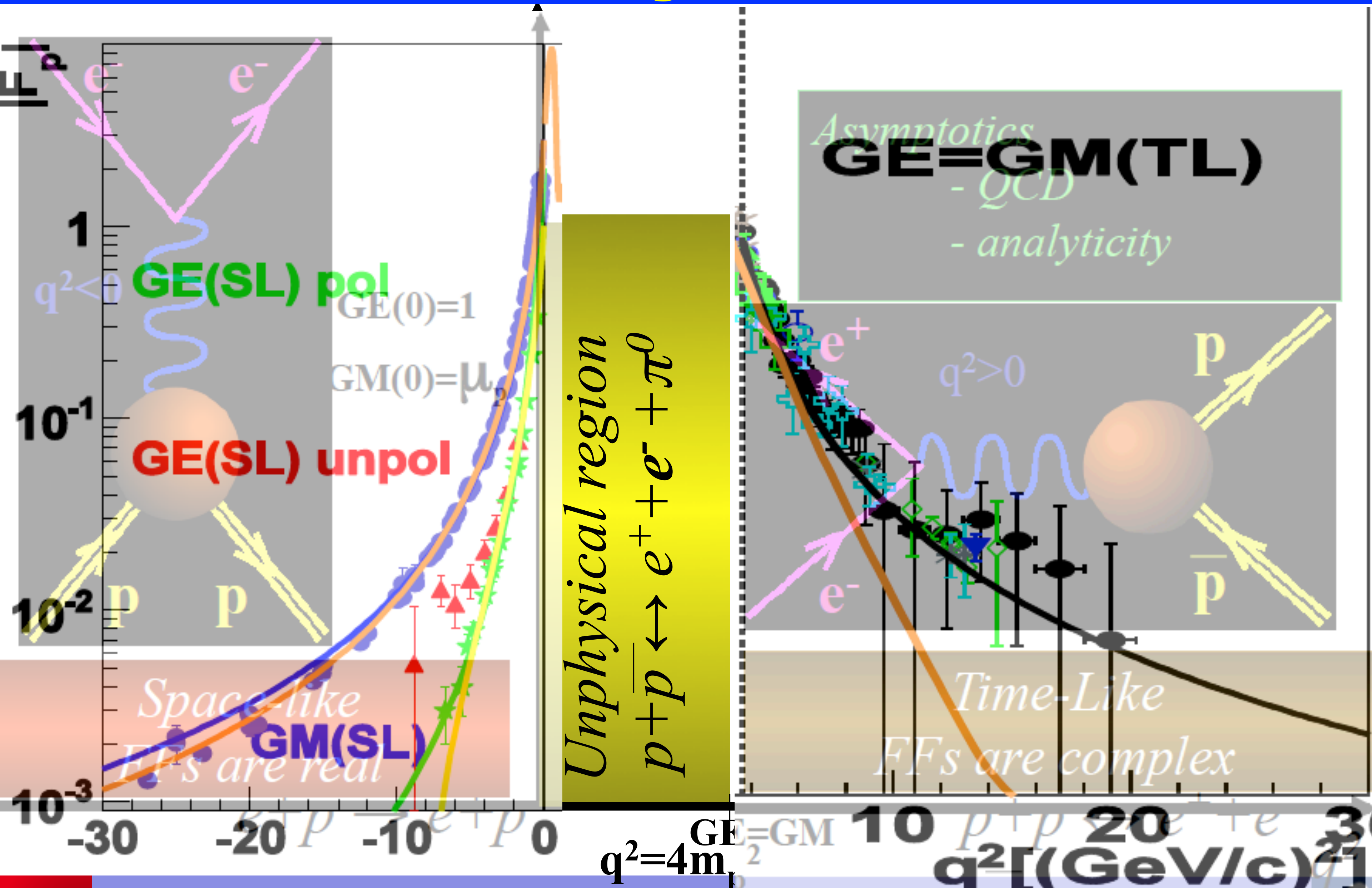
Plan

- Introduction
  - Space- and Time-like Form Factors
  - The dipole approximation

Proton Charge and Magnetic Distributions



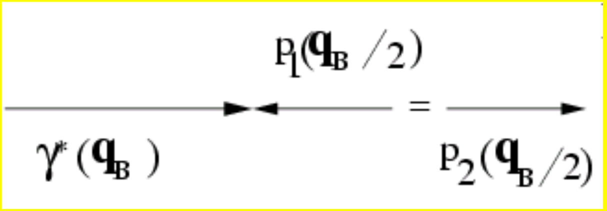
Hadron Electromagnetic Form factors



Dipole Approximation

$$G_D=(1+Q^2/\,0.71\ \mathrm{GeV}^2)^{-2}$$

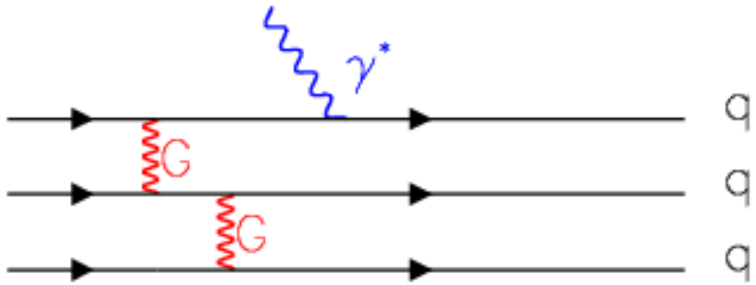
- Classical approach
  - Nucleon FF (in non relativistic approximation or in the Breit system) are Fourier transform of the charge or magnetic distribution.



Breit system

- The dipole approximation corresponds to **exponential density distribution.**
  - $\rho = \rho_0 \exp(-r/r_0)$,
  - $r_0^2 = (0.24\ fm)^2$, $<r^2> \sim (0.81\ fm)^2 \leftrightarrow m_D^2 = 0.71\ GeV^2$

Dipole Approximation and pQCD



Dimensional scaling

– $F_n(Q^2) = C_n\,[1/(1+Q^2/m_n)^{n-1}]$,
  - $m_n = n\beta^2$, <quark momentum squared>
  - n is the number of constituent quarks

– Setting $\beta^2 = (0.471 \pm .010)\ \mathrm{GeV}^2$ *(fitting pion data)*
  - pion: $F_\pi(Q^2) = C_\pi\,[1/(1+Q^2/0.471\ \mathrm{GeV}^2)^1]$,
  - nucleon: $F_N(Q^2) = C_N\,[1/(1+Q^2/0.71\ \mathrm{GeV}^2)^2]$,
  - deuteron: $F_d(Q^2) = C_d\,[1/(1+Q^2/1.41\,\mathrm{GeV}^2)^5]$

V. A. Matveev, R. M. Muradian, and A. N. Tavkhelidze (1973), Brodsky and Farrar (1973), Politzer (1974), Chernyak & Zhitnisky (1984), Efremov & Radyuskin (1980)…

Fourier Transform of the spatial density

$$F(q) = \frac{\int_\Omega d^3\vec{x} e^{i\vec{q}\cdot\vec{x}} \rho(\vec{x})}{\int_\Omega d^3\vec{x} \rho(\vec{x})}.$$

density $\rho(r)$	Form factor $F(q^2)$	r.m.s. $< r_c^2 >$	comments
δ	1	0	pointlike
e^{-ar}	$\frac{a^4}{(q^2+a^2)^2}$	$\frac{12}{a^2}$	dipole
$\frac{e^{-ar}}{r}$	$\frac{a^2}{q^2+a^2}$	$\frac{6}{a^2}$	monopole
$\frac{e^{-ar^2}}{r^2}$	$e^{-q^2/(4a^2)}$	$\frac{1}{2a}$	gaussian
ρ_0 for $x \leq R$ 0 for $r \geq R$	$\frac{3(\sin X - X\cos X)}{X^3}$ $X = qR$	$\frac{3}{5}R^2$	square well

Root mean square radius

$$< r_c^2 >= \frac{\int_0^\infty x^4 \rho(x)dx}{\int_0^\infty x^2 \rho(x)dx}.$$

$$F(q) \sim 1 - \frac{1}{6}q^2 < r_c^2 > +O(q^2),$$

IL NUOVO CIMENTO Vol. XXIV, N. 1 1° Aprile 1962

Proton-Antiproton Annihilation into Electrons, Muons and Vector Bosons.

A. Zichichi and S. M. Berman (*)

CERN - Geneva

N. Cabibbo and R. Gatto

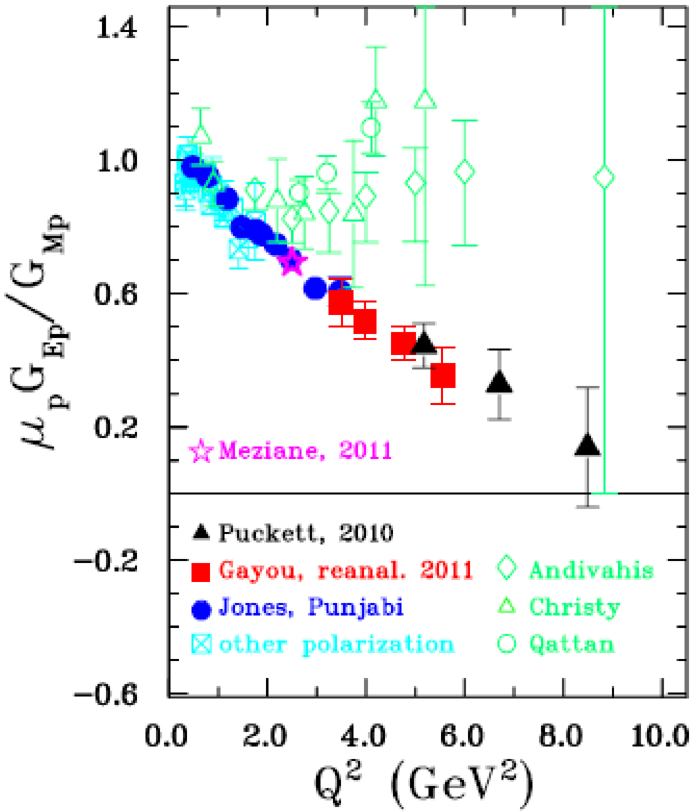
Università degli Studi - Roma e Cagliari
Laboratori Nazionali di Frascati del CNEN - Roma

Whereas in the spacelike experiments the form factors are given the physical interpretation *of the Fourier transforms of the spatial charge and magnetic structure* of the proton, *the timelike momentum transfers yield information about the frequency structure of the protons*.

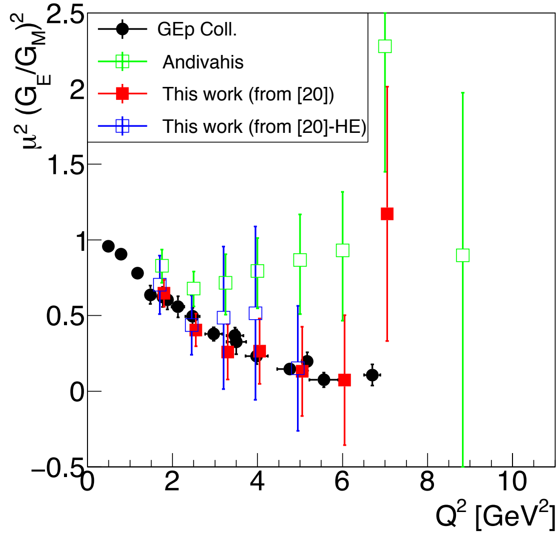
Dipole Approximation

Does not hold in SL region:

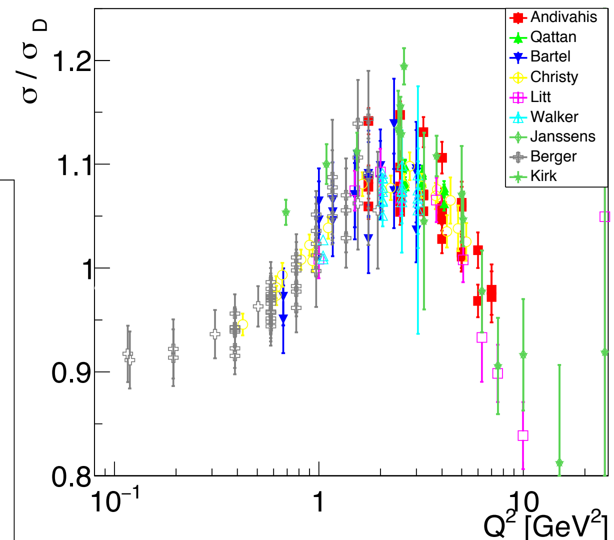
neither for GE ...nor for GM



The GEP collaboration,
A.J.R. Puckett et al, PRC96(2017)055203



R. Taylor, SLAC, 1967
S. Pacetti, E.T-G, PRC94, 055202 (2016)

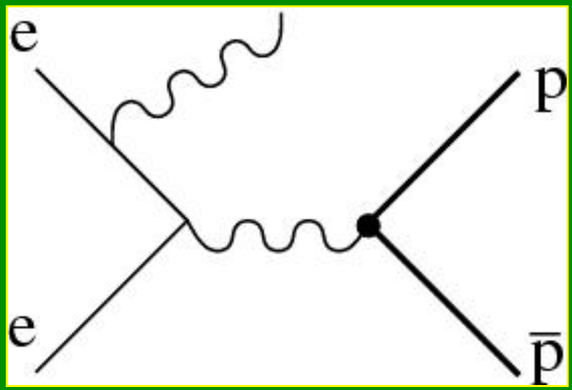


...and in TL ?

Plan

- Periodic oscillation of BaBar data
  - Data
  - Interpretation
    - Fourier transform
    - Optical model

Radiative Return (ISR)



$$e^+ + e^- \rightarrow p + \bar{p} + \gamma$$

$$\frac{d\sigma(e^+e^- \rightarrow p\bar{p}\gamma)}{dm\, d\cos\theta} = \frac{2m}{s} W(s,x,\theta)\, \boxed{\sigma(e^+e^- \rightarrow p\bar{p})(m),} \quad x = \frac{2E_\gamma}{\sqrt{s}} = 1 - \frac{m^2}{s},$$

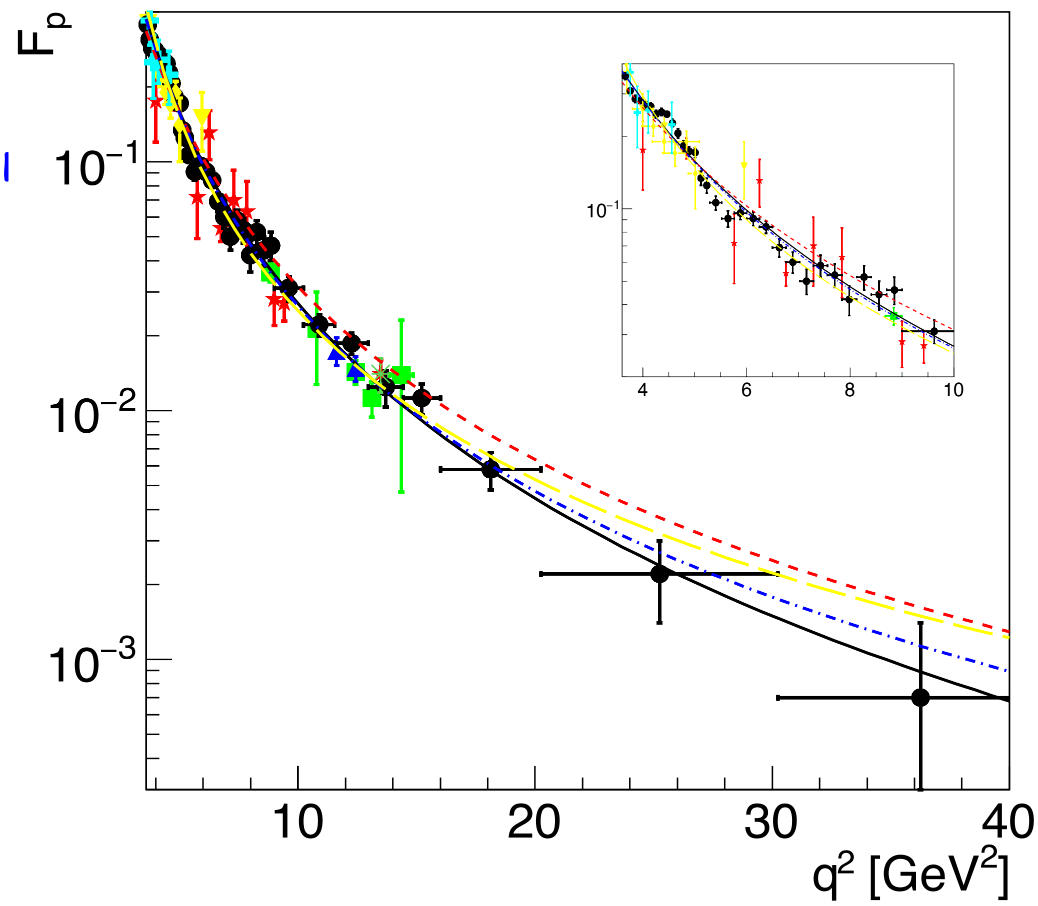
$$W(s,x,\theta) = \frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta} - \frac{x^2}{2}\right), \quad \theta \gg \frac{m_e}{\sqrt{s}}.$$

B. Aubert (BABAR Collaboration) Phys Rev. ***D73****, 012005 (2006)*

The Time-like Region

- The Experimental Status
  - No individual determination of GE and GM
  - TL proton FFs twice larger than in SL at the same Q^2
  - Steep behaviour at threshold
  - Babar: Structures? Resonances?

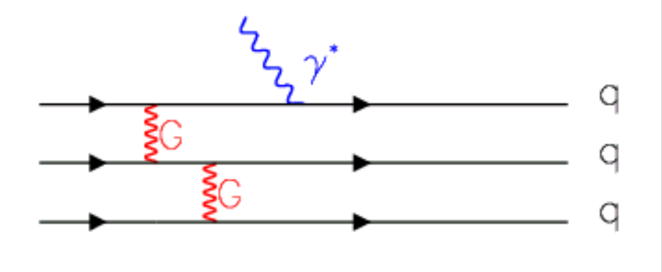
GE=GM



S. Pacetti, R. Baldini-Ferroli, E.T-G , Physics Reports, 514 (2014) 1

Panda contribution: *M.P. Rekalo, E.T-G , DAPNIA-04-01, ArXiv:0810.4245.*

The Time-like Region



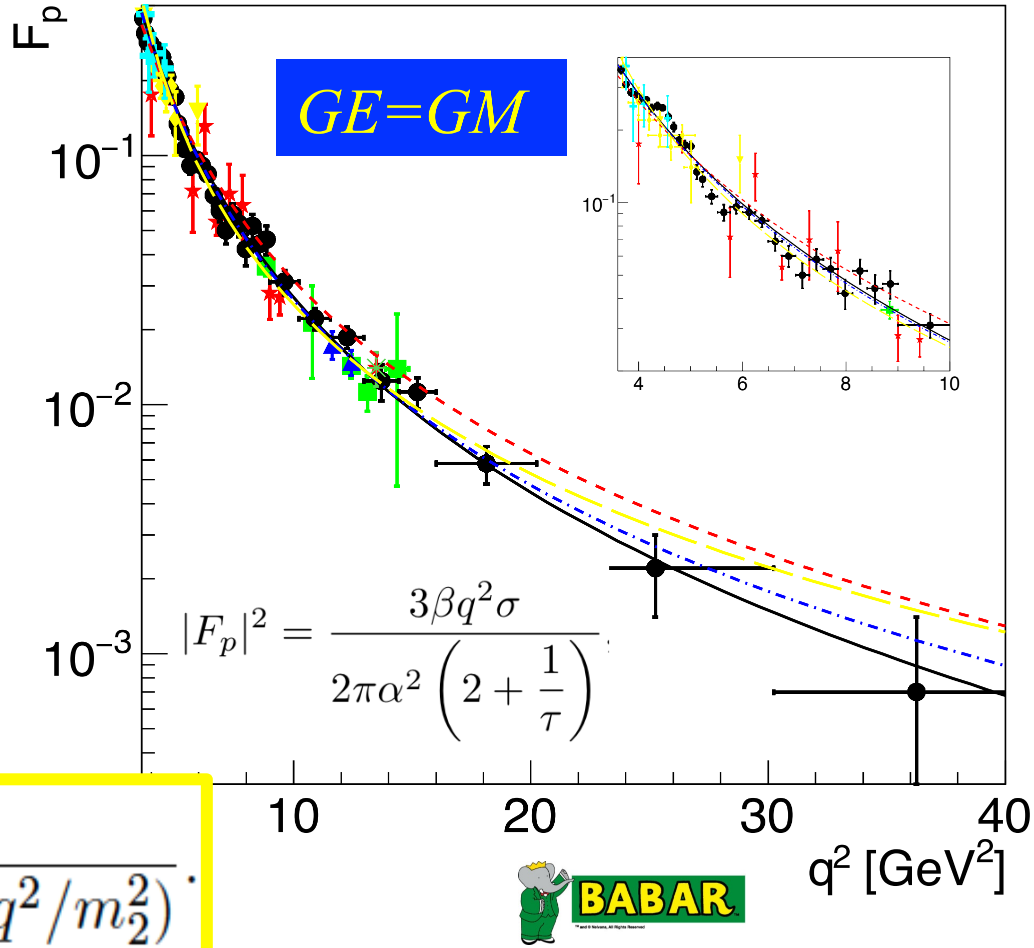
Expected QCD scaling $(q^2)^2$

$$|F_{scaling}(q^2)| = \frac{\mathcal{A}}{(q^2)^2 \log^2(q^2/\Lambda^2)}$$

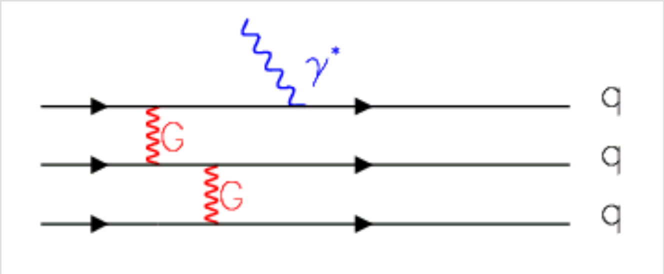
$$\frac{\mathcal{A}}{(1+q^2/m_a^2)\,[1-q^2/0.71]^2},$$

$$|F_{T3}(q^2)| = \frac{\mathcal{A}}{(1-q^2/m_1^2)(2-q^2/m_2^2)}.$$

$$e^+ + e^- \to \bar{p} + p\,,$$
$$\bar{p} + p \to e^+ + e^-$$



The Time-like Region



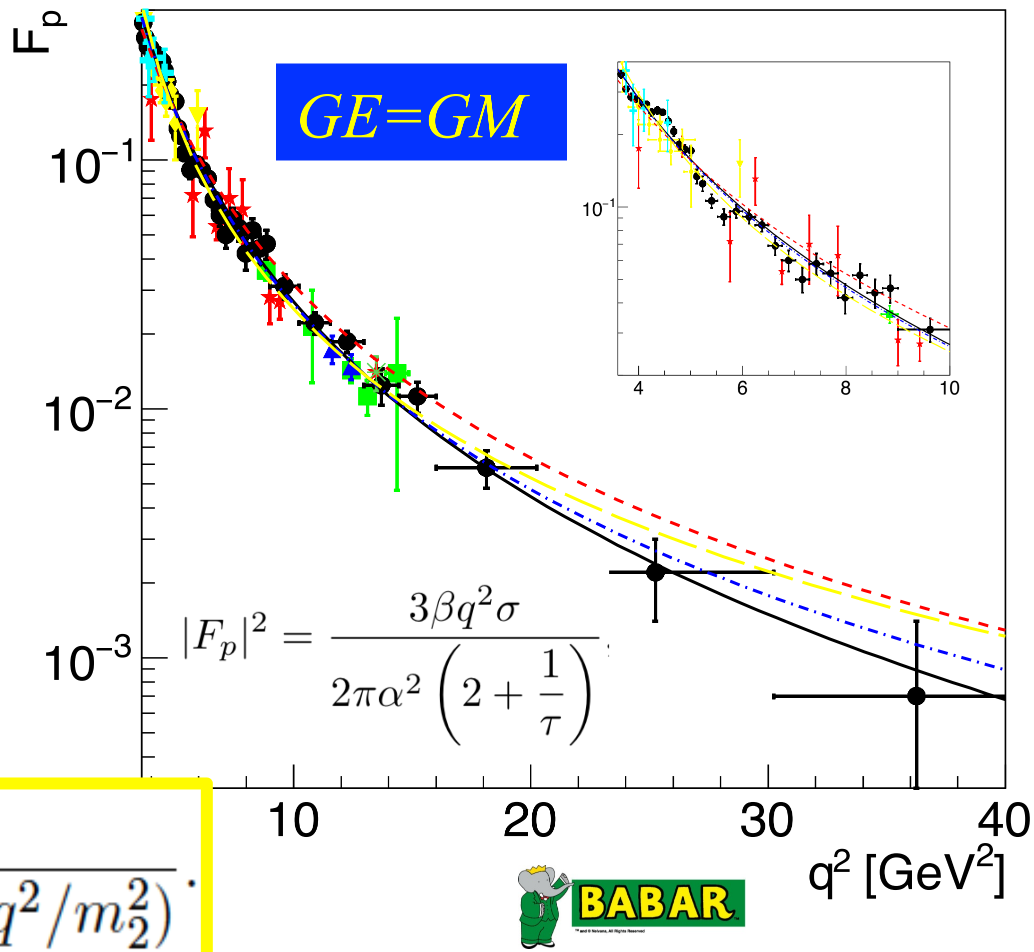
Expected QCD scaling $(q^2)^2$

$$e^+ + e^- \to \bar{p} + p\,,$$
$$\bar{p} + p \to e^+ + e^-$$

$$\frac{\mathcal{A}}{(q^2)^2\left[\log^2(q^2/\Lambda^2)+\pi^2\right]}.$$

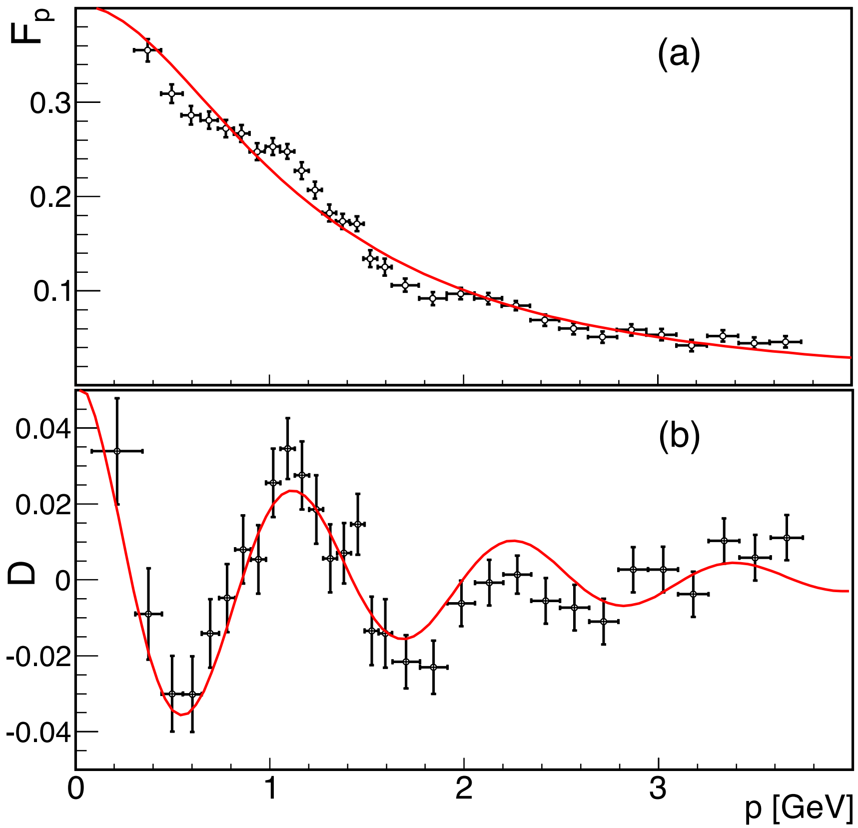
$$\frac{\mathcal{A}}{(1+q^2/m_a^2)\left[1-q^2/0.71\right]^2},$$

$$|F_{T3}(q^2)| = \frac{\mathcal{A}}{(1-q^2/m_1^2)(2-q^2/m_2^2)}.$$



Oscillations : regular pattern in P_{Lab}

The relevant variable is p_{Lab} associated to the relative motion of the final hadrons.



$$F_{osc}(p) \equiv A \exp(-Bp) \cos(Cp + D).$$

$A \pm \Delta A$	$B \pm \Delta B$ $[GeV]^{-1}$	$C \pm \Delta C$ $[GeV]^{-1}$	$D \pm \Delta D$	$\chi^2/n.d.f$
0.05 ± 0.01	0.7 ± 0.2	5.5 ± 0.2	0.03 ± 0.3	1.2

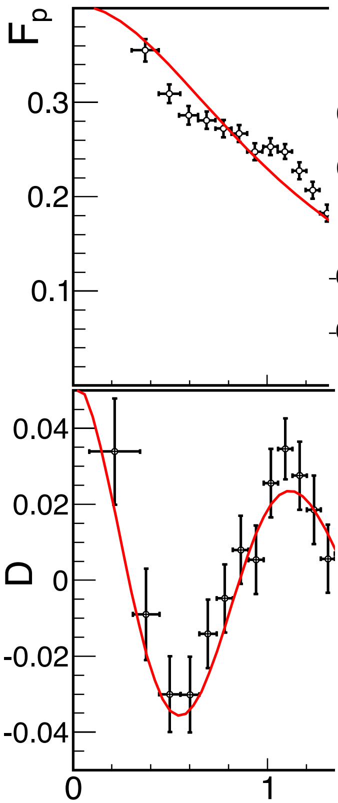
A: Small perturbation B: damping
C: r < 1fm D=0: maximum at p=0

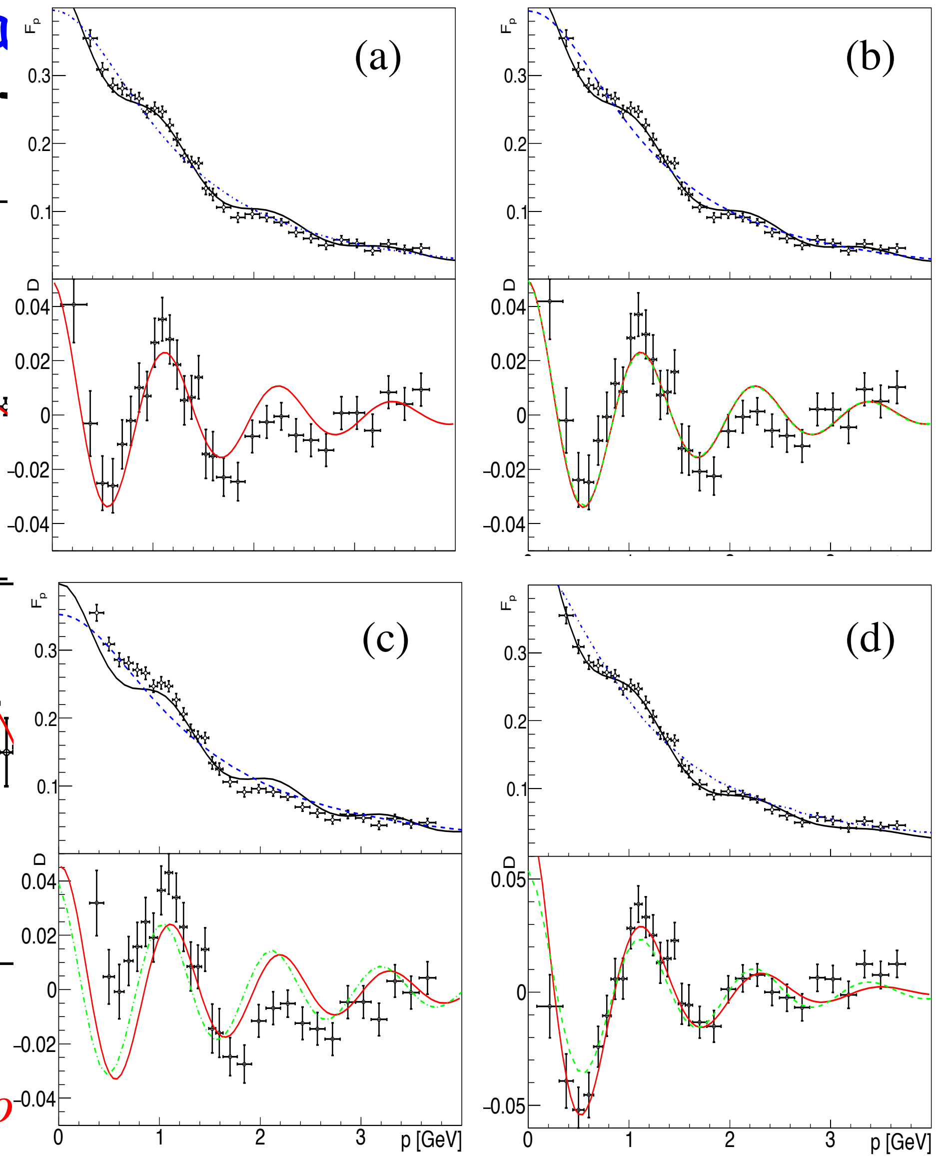
Simple oscillatory behaviour
Small number of coherent sources

A. Bianconi, E. T-G. Phys. Rev. Lett. 114,232301 (2015)

Oscillations : regular pattern in P_{Lab}

The releva… the relative motion of …





…p$(-Bp)\ \cos(Cp\ +\ D)$.

$C \pm \Delta C$ $[GeV]^{-1}$	$D \pm \Delta D$	$\chi^2/n.d.f$
5.5 ± 0.2	0.03 ± 0.3	1.2

…on B: damping
D=0: maximum at p=0

…ry behaviour
…f coherent sources

A. Bianco…)

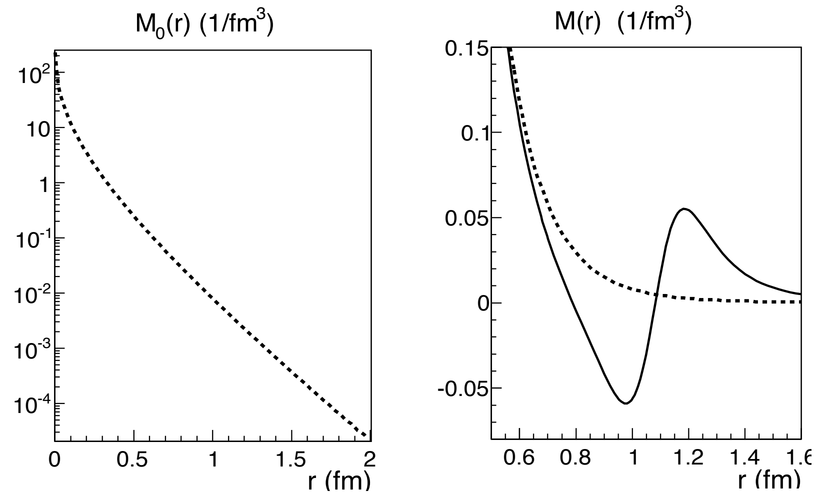
Fourier Transform

$$F_0(p) \equiv \int d^3\vec{r}\; exp(i\vec{p}\cdot\vec{r})\; M_0(r)$$

$$F(p) = F_0(p) + F_{osc}(p) \equiv \int d^3\vec{r}\; exp(i\vec{p}\cdot\vec{r}) M(r).$$

$$F_0 = \frac{\mathcal{A}}{(1 + q^2/m_a^2)\,[1 - q^2/0.71]^2},$$

$$F_{osc}(p) \equiv A\;\exp(-Bp)\;\cos(Cp + D).$$



- *Rescattering processes*
- *Large imaginary part*
- *Related to the time evolution of the charge density?*

  (E.A. Kuraev, E. T.-G., A. Dbeyssi, PLB712 (2012) 240)
- *Consequences for the SL region?*
- *Data expected at BESIII, PANDA*

Annihilation process

Fourier Transform: $$F_0(p) \equiv \int d^3\vec{r} \exp(i\vec{p}\cdot\vec{r}) M_0(r),$$

Plane wave IA: $\psi_f(\vec{r}) = \exp(i\vec{p}\cdot\vec{r})$ $(PWIA)$.

The matrix element:

$$F_0(p) = < \psi_f(x_1, ..., x_n)\psi_f(\vec{r})|T(r, x_1, .., x_n, x_{e^+e^-})|\psi_i(x_{e^+e^-}) >$$
$$\equiv \int d^3\vec{r}\ \psi_f(\vec{r})\ M_0(r),$$

Rescattering - Distorted wave IA:

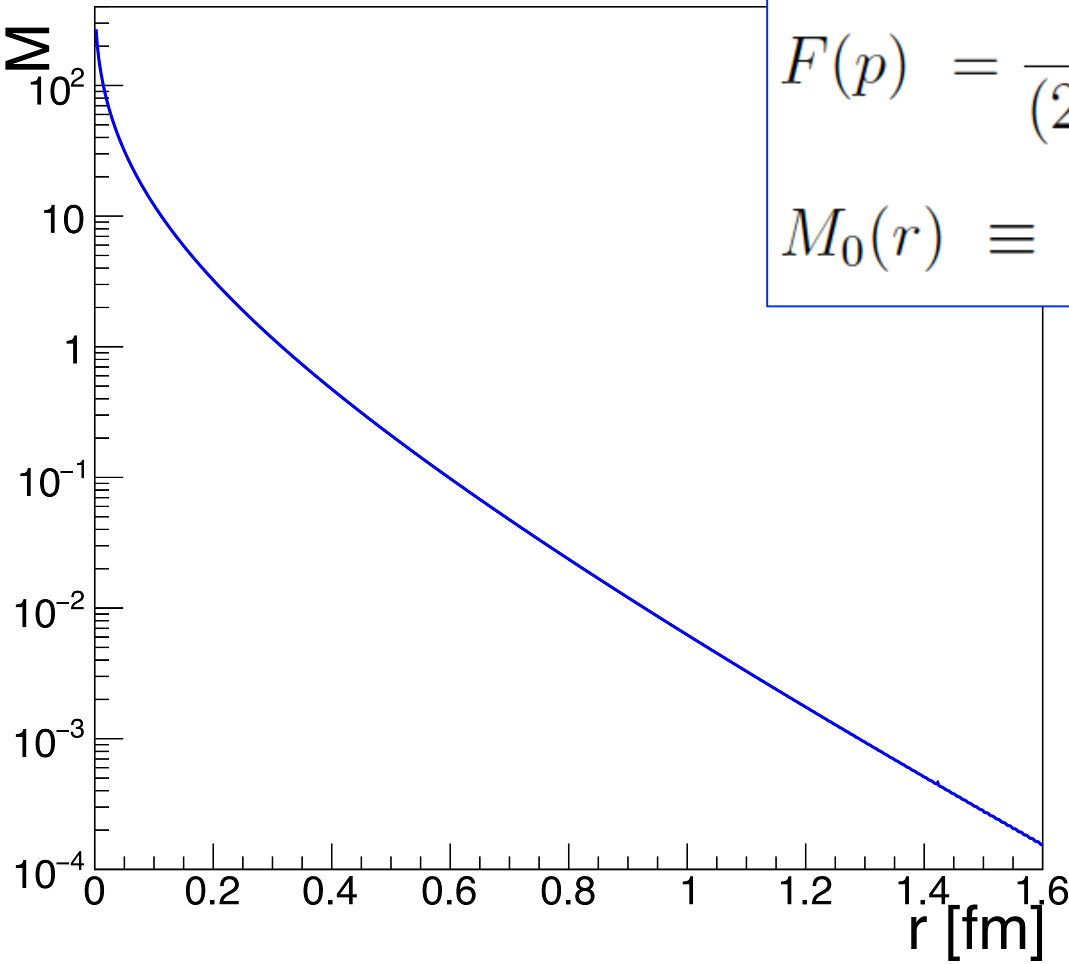
$$\psi_f(\vec{r}) = D(\vec{r}) \exp(i\vec{p}\cdot\vec{r})\ (DWIA)$$

Glauber distortion factor:

$$D(x,y,z) = \exp\Big(-ib \int_z^\infty \rho(x,y,z')dz' \Big)$$

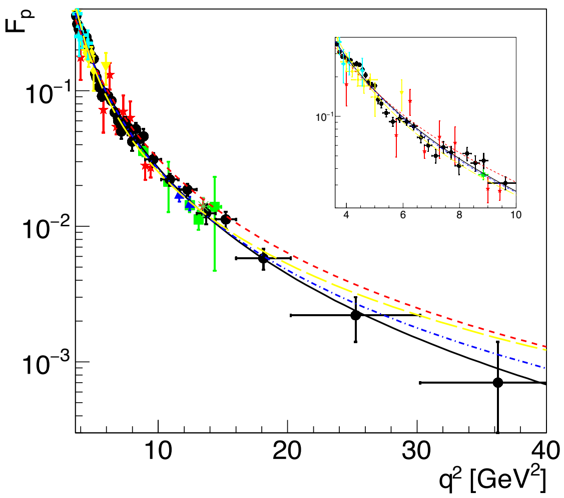
b : complex number ~ potential

Fourier Transform



$$F(p) = \frac{1}{(2\pi)^3} \int d^3\vec{r}\, e^{i\vec{p}\cdot\vec{r}} D(\vec{r}) M_0(r),$$

$$M_0(r) \equiv \int d^3\vec{p}\, e^{-i\vec{p}\cdot\vec{r}} F_0(p)$$



Potentials

Compact rescattering densities : Woods-Saxon, spherical, gaussian…:

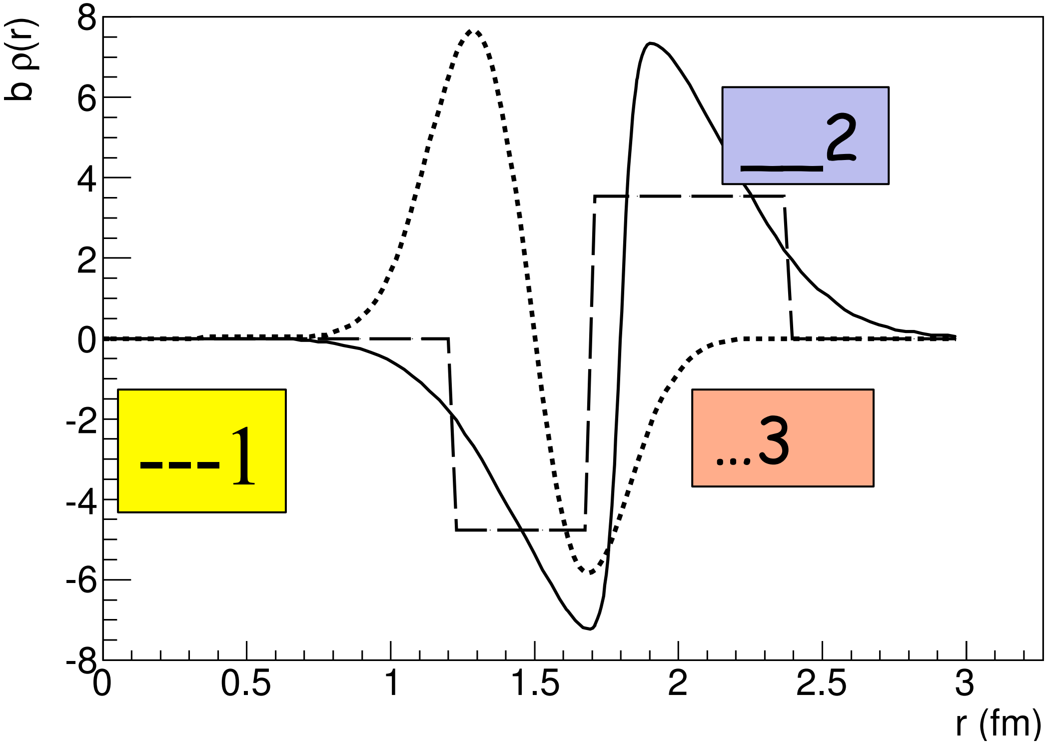
- Imaginary potentials are typical for low energy pbar-p(A) interactions
- no oscillations here -> t-channel momentum for (re)scattering versus relative momentum (s-channel)

Hollow rescattering densities: large for 0.2-2 fm, vanishing at small and large r, not changing sign.

- Real potential: peak >2 fm but M_0 <<3-4 orders of magnitude
- Imaginary potentials: need strong absorption which reduces M_0

Double layer potentials

Double layer rescattering densities : combination of two hollow potentials: one absorbing and one generating (imaginary potentials).



1) Multiple step function

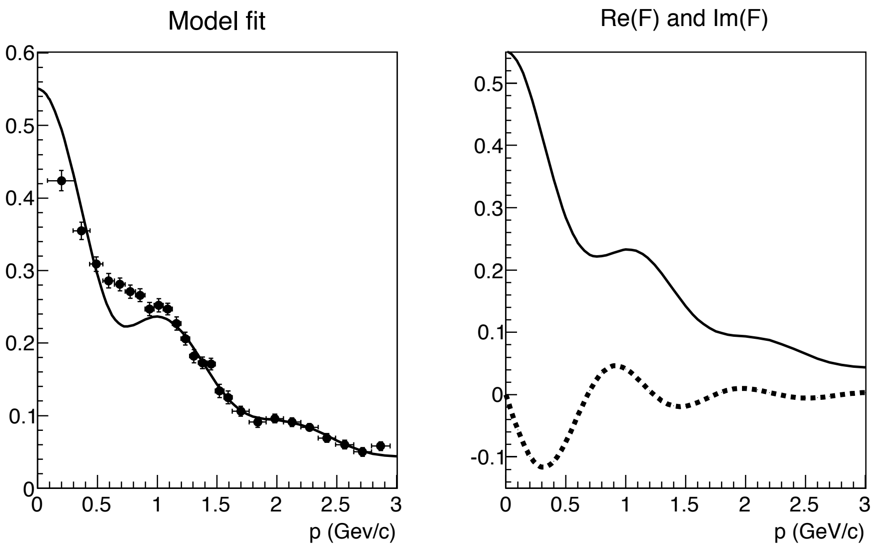
2) Soft multistep

3) Two-gaussian opposite sign potential

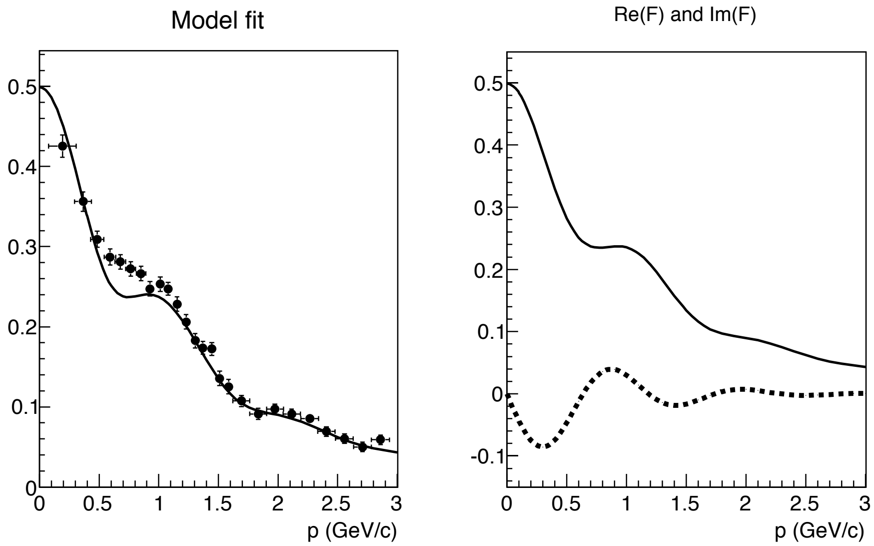
A. Bianconi, E. T-G., PRC 93, 035201 (2016)

Optical model analysis

1) Multiple step function



2) Soft multistep



- At large r: purely absorptive
- At small r: the product D(r)M(r)"resonates" with the FT factor
- Importance of the steep behavior (oscillation period)
- Related to threshold enhancement

Optical model analysis

The excited vacuum created by e+e- annihilation decays in multi-quark states: pbar-p is one of them

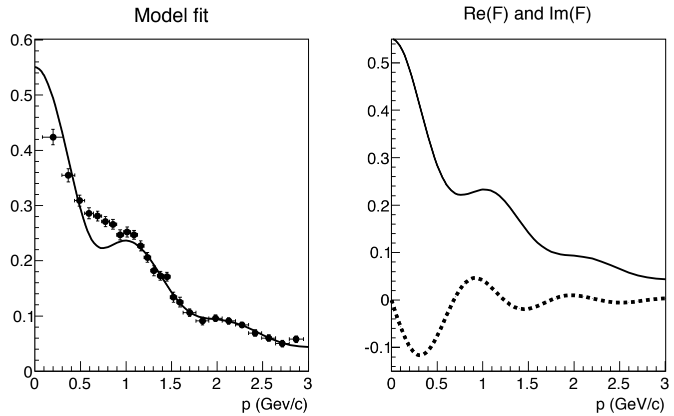
- feeding at small r by decay of higher mass states in pbar-p
- depletion at large r from pbar-p annihilation into mesons

From the pbar-p point of view, the coupling with the other channels transforms into an imaginary potential that

- destroys flux (absorption – negative potential)
- generates flux (creation – positive potential)

Optical model :

2 component imaginary potential:
absorbing outside,
regenerating inside,
with steep change of sign.



Regeneration-Absorbtion

Rescattering occurs in the spatial regions where:

- highly relativistic degrees of freedom (partons, internal properties of hadron)

and

- non- relativistic (relative motion of the two hadrons)

are both important

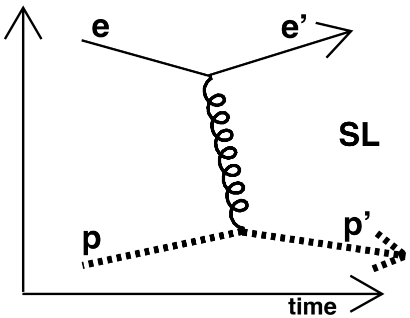
Plan

- Modelization
  - Generalization of FF
  - The charge pair creation
  - A new picture for the nucleon structure

TL-SL Generalization of Form Factors

E.A. Kuraev, E. T-G, A. Dbeyssi, Phys.Lett. B712 (2012) 240
A. Bianconi, E. T-G., PRC 95, 015204 (2017)

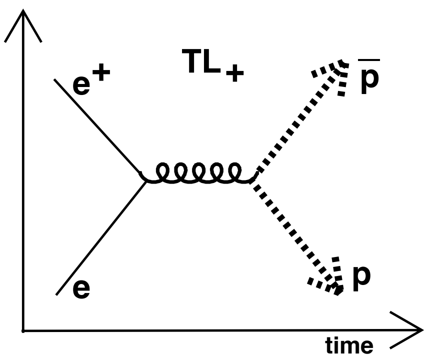
$$F(q^2) = \int_{\mathcal{D}} d^4x e^{iq_\mu x^\mu} F(x) \quad q_\mu x^\mu = q_0 t - \vec{q}\cdot\vec{x}$$

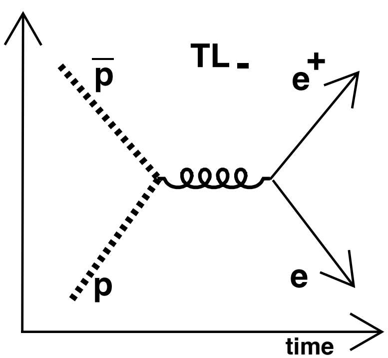


$F(x)=F(\vec{x},t)$ space-time distribution of the electric charge in the space-time volume $\mathcal{D}$.

$$\rho(x) = \rho(\vec{x}, t)$$

SL photon 'sees' a charge density





TL photon can NOT test a space distribution.

How to connect and understand the amplitudes?

Definition of TL-SL Form Factors

$$F(q) = \int d^4x e^{iqx} F(x).$$

$$F_{SL,Breit}(q) = \int d^3\vec{x}\, e^{-i\vec{q}\cdot\vec{x}} \int dt F(t,\vec{x}) \equiv \int d^3\vec{x}\, e^{-i\vec{q}\cdot\vec{x}} \rho(|\vec{x}|),$$
$$\rho(|\vec{x}|) = \int dt F(t,\vec{x}).$$

$$F_{TL,CM}(q) = \int dt\, e^{iqt} \int d^3\vec{x} F(t,\vec{x}) \equiv \int dt\, e^{iqt} R(t),$$
$$R(t) = \int d^3\vec{x} F(t,\vec{x}).$$

$\rho(\vec{x})$ and $R(t)$, represent projections of the same distribution in orthogonal subspaces

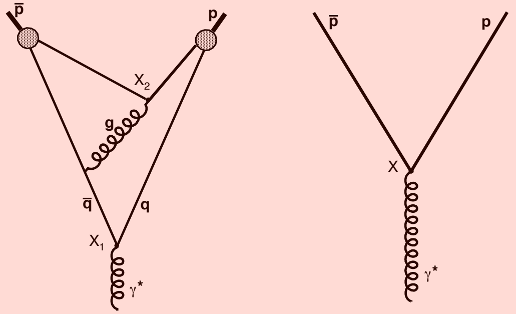
Photon-Charge coupling

$$\rho(x) = \rho(\vec{x}, t)$$

$\rho(\vec{x})$ Fourier transform of a stationary charge and current distribution

$R(t)$ Amplitude for creating charge-anticharge pairs at time t. Charge distribution => distribution in time of $\gamma^* \rightarrow charge - anticharge$ vertexes

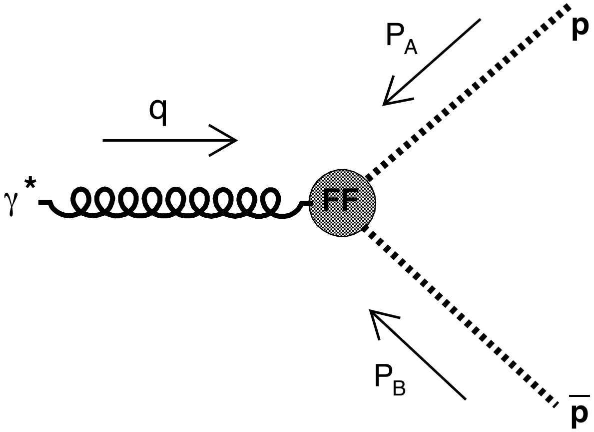
The photon



Resolved or Unresolved

- Both enter in the definition of a form factor
- Simplest configuration: qqbar+compact diquark

Photon-Charge coupling



$$SL : \gamma^*(q_\mu) + p(p_\mu) \to p(p'_\mu)$$
$$TL_+ : \gamma^*(q_\mu) \to p(p'_\mu) + \bar{p}(\bar{p_\mu}')$$
$$TL_- : p(p_\mu) + \bar{p}(\bar{p}_\mu) \to \gamma^*(q'_\mu)$$

q, P_A, P_B : formal arguments: scattering, hadron annihilation and production, by changing signs of the coordinates

$$\gamma^* + p \to p' \quad (SL : |q_0| < |\vec{q}\,|), \quad P_A = p, P_B = -p',$$
$$\gamma^* \to \bar{p} + p \quad (TL_+ : |q_0| > |\vec{q}\,|, q_0 > 0), \quad P_A = -p', P_B = -\bar{p}',$$
$$\bar{p} + p \to \gamma^* \quad (TL_- : |q_0| > |\vec{q}\,|, q_0 < 0), \quad P_A = p, P_B = \bar{p}, q = -q',$$

FFs as q-dependent quantities proportional to the amplitude

Examples

Homogeneous distribution for positive times:
Assume equal probability in time to form a complet p-pbar system, inside the future light cone of the first event.
(space probability: we integrate and set to one)

$$R(t) = \theta(-t),$$

$$F(q) = \int e^{iqt}\theta(-t) = \frac{\pi}{\epsilon - iq},$$

Exponential damping (a is finite): either the spectator pair and the ppbar system are created soon or the system evolves differently (jet fragmentation...)

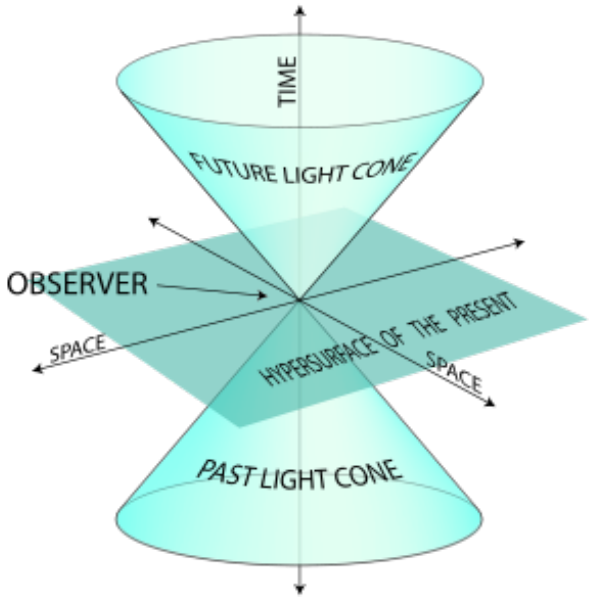
$$R(t) = \theta(-t)e^{-a|t|},$$

$$F(q) = \frac{\pi}{a - iq} = \frac{a\pi}{a^2+q^2} + i\frac{q\pi}{a^2+q^2},$$

Examples: Monopole-like shape

F(x) ≠ 0 in past and future LC.

- Annihilation and creation processes:
  - -time symmetric.
  - -differ by a phase.
- Summing two terms with the same phase: when t>> 1/a



$$R(t) = \theta(t)e^{-at} + \theta(-t)e^{at} = e^{-a|t|},$$

$$F(q)/\pi = \frac{1}{a-iq} + \frac{1}{a+iq} = \frac{2a}{a^2+q^2}.$$

1/a: formation time

=> zero mass resonance of width a

Examples: Lorentzian resonance

Replacing q->q-M : one obtains poles

$$F(q)/\pi = i\left(\frac{1}{q-M+ia} - \frac{1}{q-M-ia}\right) \propto \frac{1}{(q-M)^2+a^2}.$$

By Fourier transform:

$$R(t) \propto e^{iMt}\, e^{-a|t|}.$$

Response of a classical damped oscillator to an instantaneous external force $\delta(t)$

Negative energy states are allowed by particle-antiparticle symmetry.
To each pole q_0 =M+ia corresponds a pole q_0 =-(M +ia)
Positive poles => creation process
Negative poles=> annihilation process

Examples: Breit-Wigner

A Breit-Wigner probability contains all four poles:

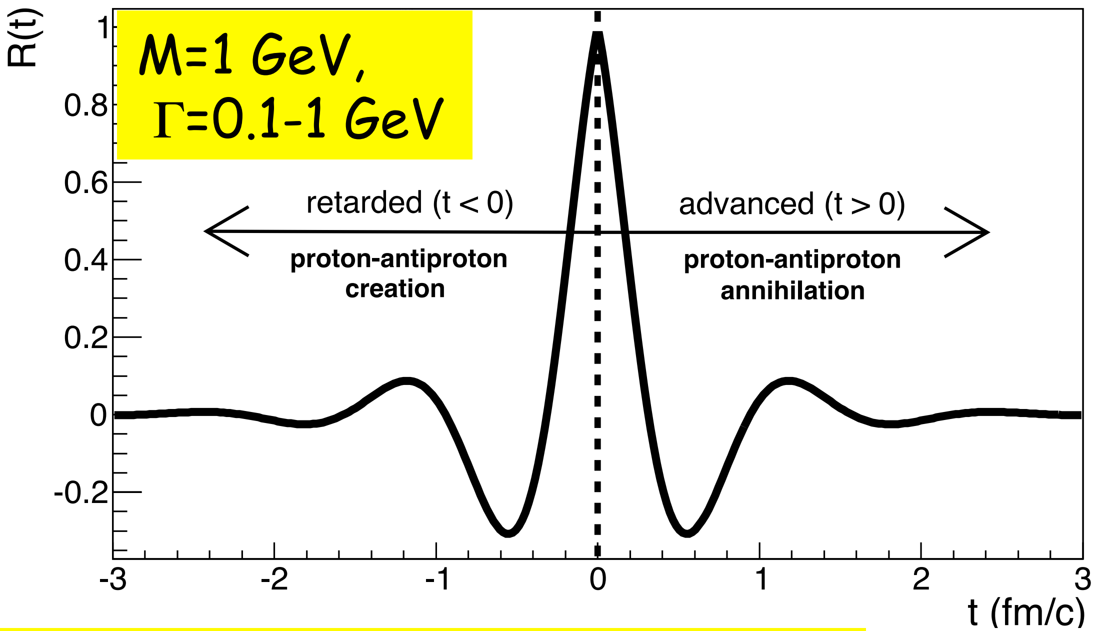
$$F_{\pm}(q) \;\propto\; \frac{1}{(q^2 - M^2) \pm iMa},$$

The combination:

$$F(q) \;\propto\; F_{+}(q) + F_{-}(q)$$

corresponds to

$$R(t) \;\propto\; cos(Mt)\, e^{-a|t|},$$



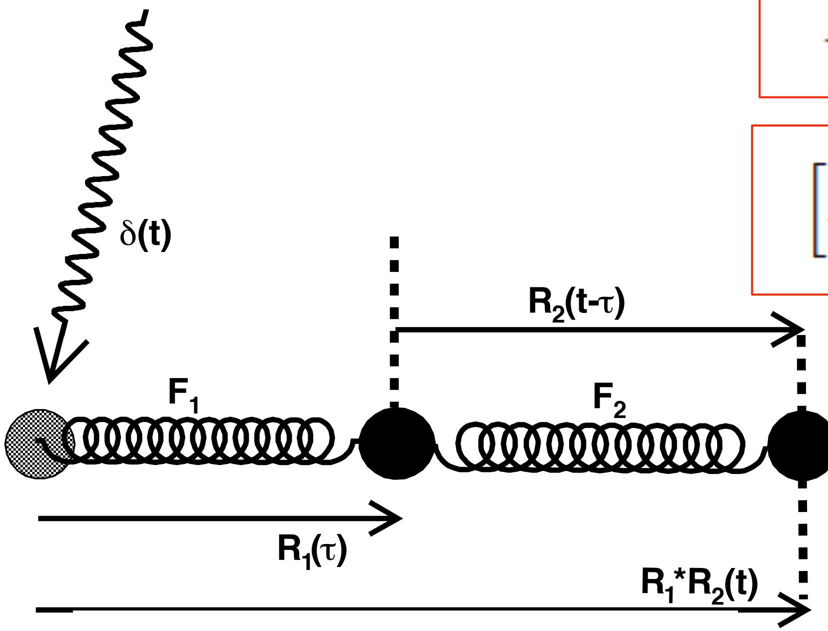
Retarded response of a classical bound and damped oscillator to a $\delta(t)$ external perturbation

$$R(t) \;\equiv\; R_{creation}(t)\theta(-t) \;+\; R_{ann}(t)\theta(t).$$

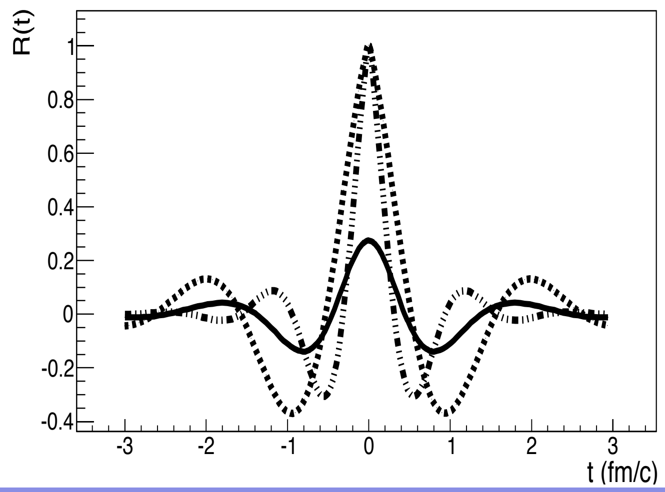
Several spectators: dipole and asymptotics

$$F_1(q)F_2(q) = F.T.\Big[R_1(t) * R_2(t)\Big],$$

$$\Big[R_1(t) * R_2(t)\Big] \equiv \int d\tau R_1(\tau) R_2(t-\tau).$$



Use FT properties of convolutions
Chain of two oscillators,
one directly connected to the photon
The second is a decaying correlation
between active quark and spectator



$$R(t) = \int d\tau \; e^{-a|t-\tau|} \; e^{-b|\tau|}.$$

$$F(q) \propto \frac{1}{(a^2+q^2)(b^2+q^2)},$$

More complicated examples

$$e^+e^- \to \bar{p}n\pi^+ \to \bar{p}p$$

Three quark-antiquarks pair in the intermediate state.

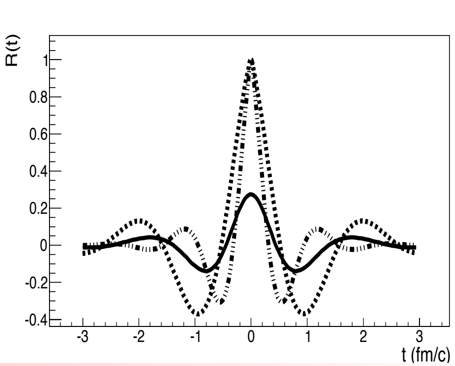
$$R(t) = \Big[[R_1(t) * R_2(t)] * R_3(t)\Big],$$

$$F(q) \propto \frac{1}{(q^2 \pm a^2)(q^2 \pm b^2)(q^2 \pm c^2)},$$

Sum of two contributions of equal shape:

$$\begin{aligned} R(t) &= R_0(t) + aR_0(t-b),\ a \ll 1, \\ F(q) &= F_0(q)[1 + ae^{ibq}], \end{aligned}$$

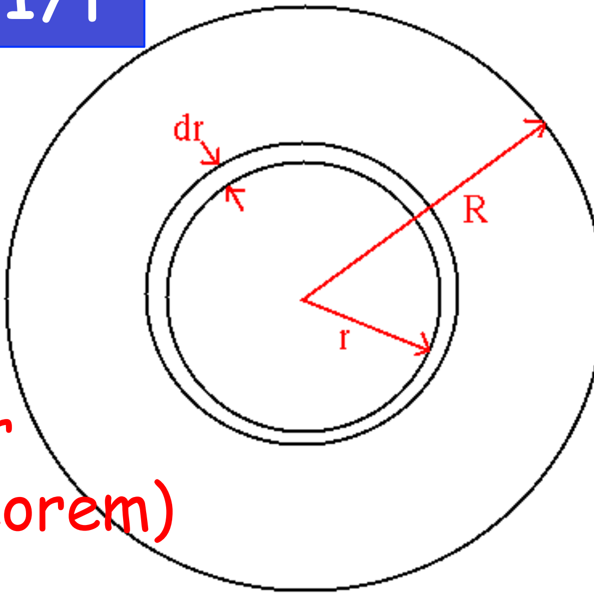
periodic modulation



The nucleon: homogenous, symmetric sphere?

Analogy with Gravitation
Coulomb Potential ~Gravitational Potential
Mass~charge

1/r



- Spherical symmetric distributed mass density
- A point at a distance r<R from the center feels only the matter inside (Newton theorem)

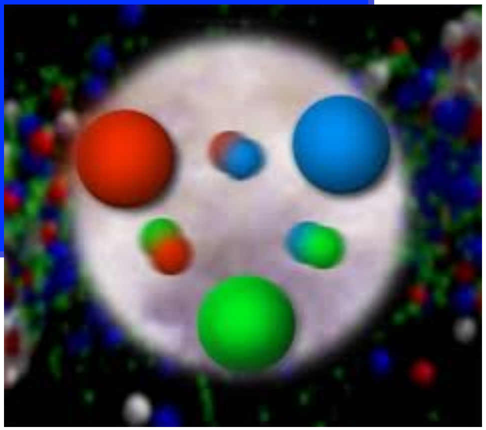
works for the SCALAR part, NOT for the VECTOR part of $A=(\Phi, \vec{A})$

$$\boldsymbol{R}=\mu G_E/G_M \simeq \left(\frac{1}{rQ}\right)^3, \; Q > \frac{1}{r}$$

$r \sim 0.7$ fm $\rightarrow Q = 0.29$ GeV

$\boldsymbol{R \sim 1/Q^3}$ *is NOT the observed experimental behavior !*

The nucleon



3 valence quarks and
a neutral sea of $\overline{q}q$ pairs

antisymmetric state of colored quarks

$$|p> \sim \epsilon_{ijk}|u^i u^j d^k>$$
$$|n> \sim \epsilon_{ijk}|u^i d^j d^k>$$

Main assumption

Does not hold in the spatial center of the nucleon: the center of the nucleon *is electrically neutral,* due to strong gluonic field

E.A. Kuraev, E. T-G, A. Dbeyssi, Phys.Lett. B712 (2012) 240

The nucleon

Inner region: gluonic condensate of clusters with randomly oriented chromo-magnetic field (Vainshtein, 1982):

Intensity of the gluon field in vacuum:

$$< 0|\alpha_s/\pi(G^a_{\mu\nu})^2|0 >\sim E^2 - B^2 \sim E^2 = 0.012 \text{ GeV}^4.$$

$$G^2 \simeq 0.012\,\pi/\alpha_s GeV^4, \text{ i.e., } E \simeq 0.245\, GeV^2. \quad \alpha_s/\pi \sim 0.1$$

In the internal region of strong chromo-magnetic field, the color quantum number of quarks does not play any role, due to stochastic averaging

$< G\|u^i u^j\|G >\sim \delta_{ij}$	proton
$d^i d^j$	neutron

Colorless quarks: Pauli principle

Model: SL and TL regions

Antisymmetric state of colored quarks

Colorless quarks: Pauli principle acts

1) uu (dd) quarks are repulsed from the inner region
2) The 3rd quark is attracted by one of the identical quarks, forming a compact di-quark
3) The color state is restored

Formation of di-quark: competition between attraction force and stochastic force of the gluon field

$$\frac{Q_q^2 e^2}{r_0^2} > e|Q_q|\,E.$$

proton: (u) Qq=-1/3

neutron: (d) Qq=2/3

attraction force > stochastic force of the gluon field

Model

Additional suppression for the scalar part due to colorless internal region: "charge screening in a plasma": the scalar part of the EM field obeys to

$$\Delta\phi = -4\pi e \sum Z_i n_i,\ n_i = n_{i0} exp\left[-\frac{Z_i e\phi}{kT}\right]$$

k:Boltzmann constant
T: temperature of the hot plasma

Neutrality condition: $\sum Z_i n_{i0} = 0$

$$\Delta\phi - \chi^2\phi = 0,\ \phi = \frac{e^{-\chi r}}{r},\ \chi^2 = \frac{4\pi e^2 Z_i^2 n_{i0}}{kT}$$

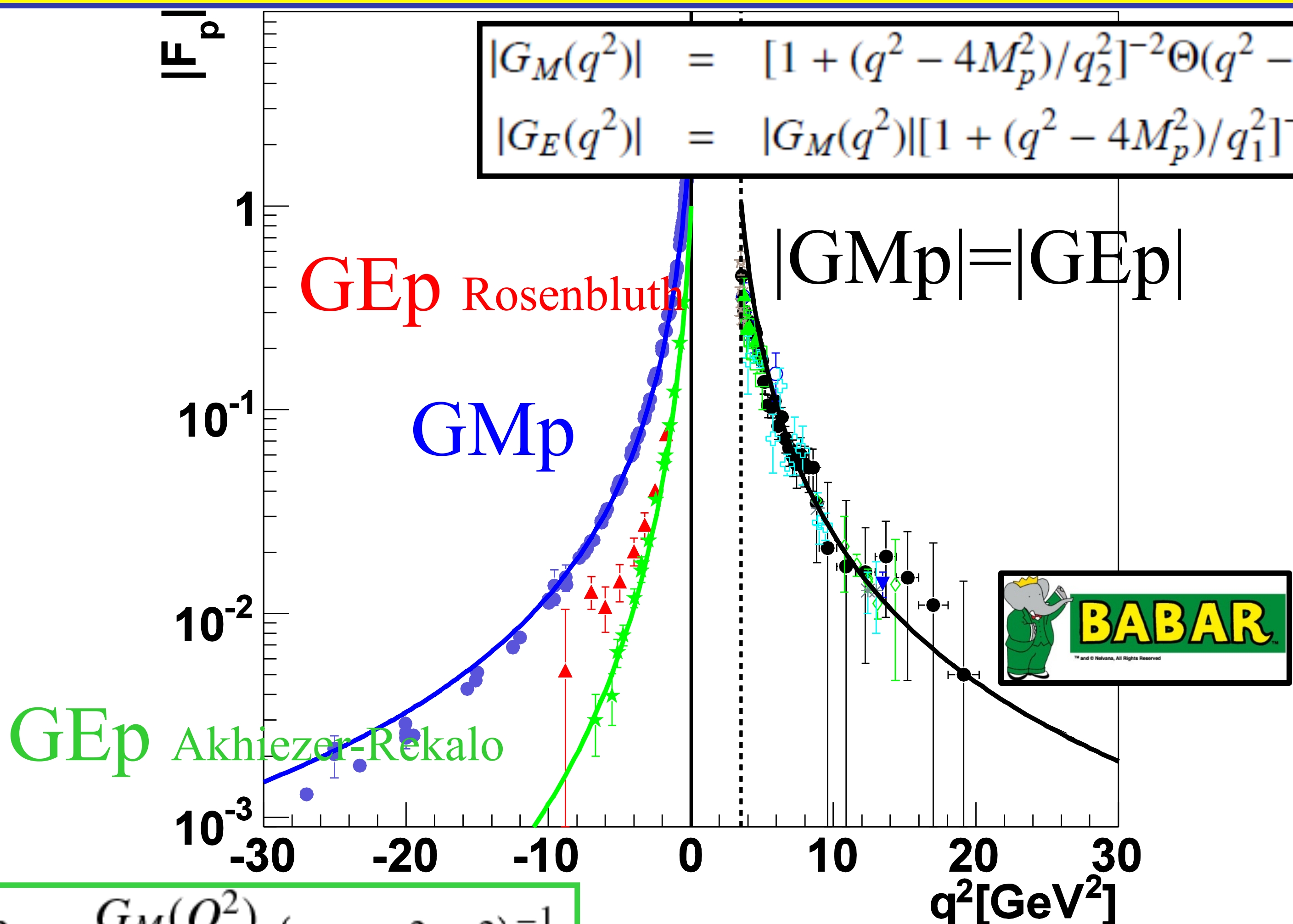
Additional suppression (Fourier transform)

$$G_E(Q^2) = \frac{G_M(Q^2)}{\mu}\left(1 + Q^2/q_1^2\right)^{-1}$$

$q_1 (\equiv \chi)$ fitting parameter

Proton Form Factors

$$|G_M(q^2)| = [1 + (q^2 - 4M_p^2)/q_2^2]^{-2}\Theta(q^2 - 4M_p^2),$$
$$|G_E(q^2)| = |G_M(q^2)|[1 + (q^2 - 4M_p^2)/q_1^2]^{-1}\Theta(q^2 - 4M_p^2),$$



$$G_E(Q^2) = \frac{G_M(Q^2)}{\mu}\left(1 + Q^2/q_1^2\right)^{-1}$$

Conclusions

- Theory: unified models in SL and TL regions:
  - *describe* proton and neutron, electric and magnetic FFs in SL and TL regions
  - *non-trivial* charge distribution at the hadron formation
  - pointlike behavior at threshold?

- New understanding of Form Factors in the Time-like region: time distribution of quark-antiquark pair creation vertices

- Experiment: measure
  - zero crossing of GE/GM in SL? 2γ? Proton radius?
  - GE and GM separately in TL
  - complex FFs in TL region: polarization!
  - new structures in TL

Model

Quark counting rules apply
to the vector part of the potential

$$G_M^{(p,n)}(Q^2) = \mu G_E(Q^2);$$
$$G_E^{(p,n)}(Q^2) = G_D(Q^2) = \left[1 + Q^2/(0.71\ \mathrm{GeV}^2)\right]^{-2}$$

$$G_E^{(p,n)}(0) = 1, 0,\ G_M^{(p,n)}(0) = \mu_{p,n}$$

Additional suppression for the scalar part

The annihilation channel: $e^+ + e^- \to \gamma^*(q) \to p + \bar{p}.$

The neutral plasma acts on the distribution of the electric charge (not magnetic).

<u>Prediction:</u> additional suppression due to the neutral plasma ⟶ similar behavior in SL and TL regions

$$|G_M(q^2)| = [1 + (q^2 - 4M_p^2)/q_2^2]^{-2}\Theta(q^2 - 4M_p^2),$$
$$|G_E(q^2)| = |G_M(q^2)|[1 + (q^2 - 4M_p^2)/q_1^2]^{-1}\Theta(q^2 - 4M_p^2),$$

- *Implicit normalization at* $q^2=4M_p^2$: $|GE|=|GM|$ =1
- *No poles in unphysical region*

The annihilation channel: $e^+ + e^- \to \gamma^*(q) \to p + \bar{p}.$

The repulsion of p and $\bar{p}$ with kinetic energy

$$T = \sqrt{q^2} - 2M_p c^2$$

is balanced by the confinement potential

$$q_0 - 2M_p c^2 = (k/2)R^2$$

- The long range color forces create a stable colorless state of proton and antiproton
- The initial energy is dissipated from current to constituent quarks originating on shell $\bar{p}p$ separated by R.

The annihilation channel: $e^+ + e^- \to \gamma^*(q) \to p + \bar{p}$.

1) Creation of a $p\bar{p}$ state through $^3S_1 =< 0|J^\mu|p\bar{p} >$ intermediate state with $q = (\sqrt{q^2}, 0, 0, 0)$.

2) The vacuum state transfers all the released energy to a state of matter consisting of:

- 6 massless valence quarks
- Set of gluons
- Sea of current $q\bar{q}$ pairs of quarks with energy $q_0 > 2M_p$, J=1, dimensions $\hbar/(2M_p) \sim 0.1$ fm

3) Pair of p and $\bar{p}$ formed by three bare quarks:

•Structureless

•Colorless → pointlike FFs !!!

The annihilation channel: $e^+ + e^- \rightarrow \gamma^*(q) \rightarrow p + \bar{p}$

- The point-like hadron pair expands and cools down: the current quarks and antiquarks absorb gluon and transform into constituent quarks

- The residual energy turns into kinetic energy of the motion with relative velocity $2\beta = 2\sqrt{1 - 4M_p^2/q_0^2}$

- The strong chromo-EM field leads to an effective loss of color. Fermi statistics: identical quarks are repulsed. The remaining quark of different flavor is attracted to one of the identical quarks, creating a compact diquark (*du*-state)

The annihilation channel: $e^+ + e^- \to \gamma^*(q) \to p + \bar{p}$.

The repulsion of p and $\bar{p}$ with kinetic energy

$$T = \sqrt{q^2} - 2M_p c^2$$

is balanced by the confinement potential

$$q_0 - 2M_p c^2 = (k/2)R^2$$

- The long range color forces create a stable colorless state of proton and antiproton
- The initial energy is dissipated from current to constituent quarks originating on-mass shell $\bar{p}p$ separated by R.

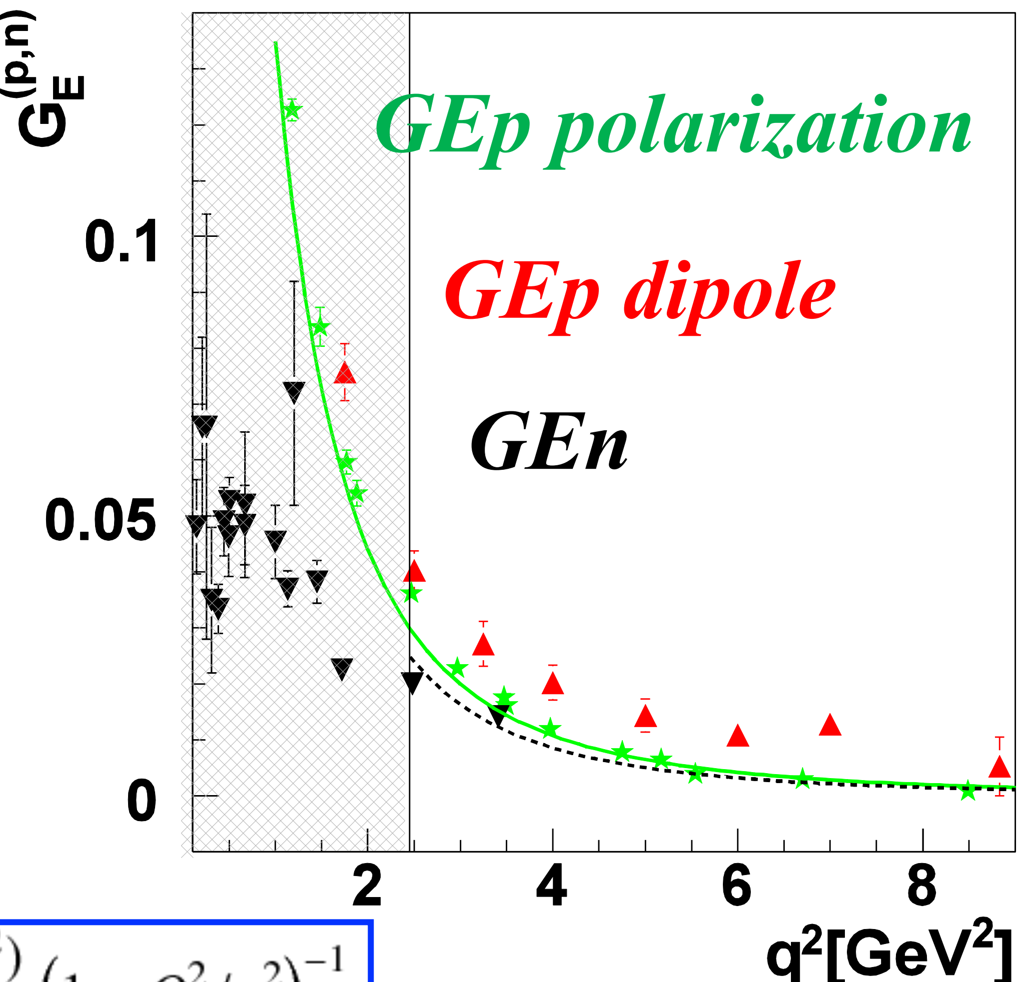
The annihilation channel: $e^+ + e^- \to \gamma^*(q) \to p + \bar{p}$.

At larger distances, the inertial force exceeds the confinement force: p and $\bar{p}$ start to move apart with relative velocity β

p and p_ leave the interaction region: at large distances the integral of Q(t) must vanish.

For very small values of the velocity $\alpha\pi/\beta \simeq 1$ *FSI lead to the creation of a bound $\bar{N}N$ system .*

Space-Like region

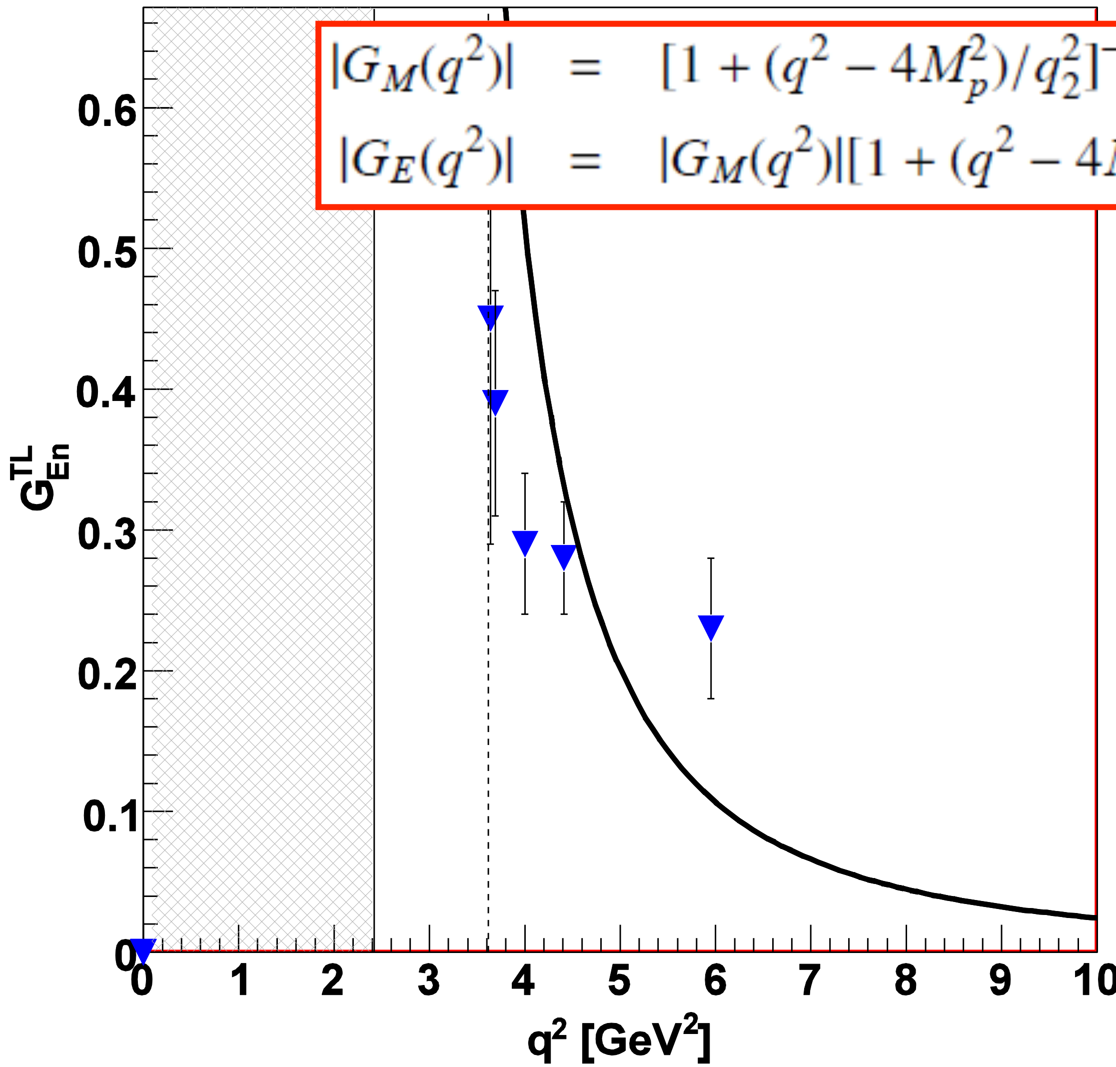


$$G_E(Q^2) = \frac{G_M(Q^2)}{\mu}\left(1 + Q^2/q_1^2\right)^{-1}$$

Neutron TL region

$$|G_M(q^2)| = [1+(q^2-4M_p^2)/q_2^2]^{-2}\Theta(q^2-4M_p^2),$$

$$|G_E(q^2)| = |G_M(q^2)|[1+(q^2-4M_p^2)/q_1^2]^{-1}\Theta(q^2-4M_p^2),$$



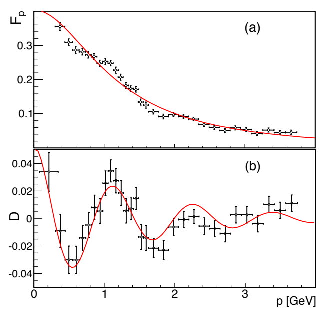
Time-like FFs

- New understanding of Form Factors in the Time-like region: time distribution of quark-antiquark pair creation vertices

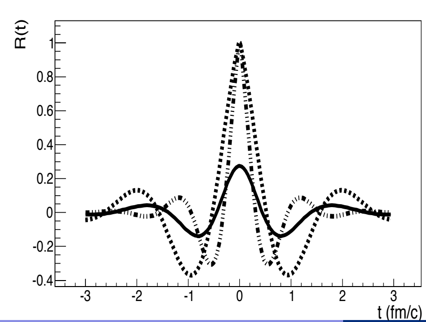
$$F(q^2) = \int_{\mathcal{D}} d^4x e^{iq_\mu x^\mu} \rho(x), \; q_\mu x^\mu = q_0 t - \vec{q} \cdot \vec{x}$$

- The distributions tested by the virtual photon are projections in orthogonal 1 and 3-dim spaces of the function

$\rho(x) = \rho(\vec{x}, t)$ $R(t)$ and $\rho(\vec{x})$

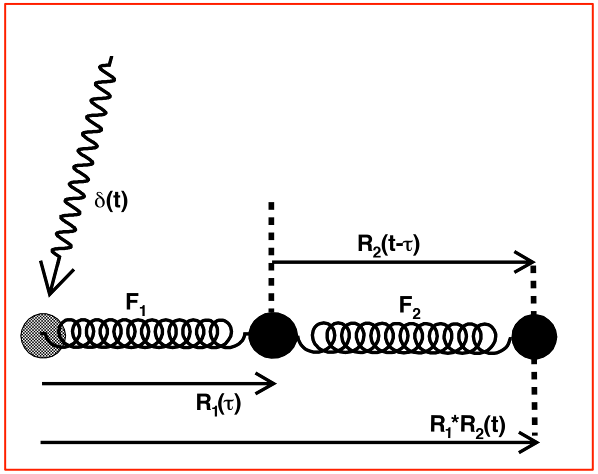


- Simple functions R(t) can explain the origin of oscillatory phenomena



A. Bianconi, E. T-G., PRC 95, 015204 (2017)

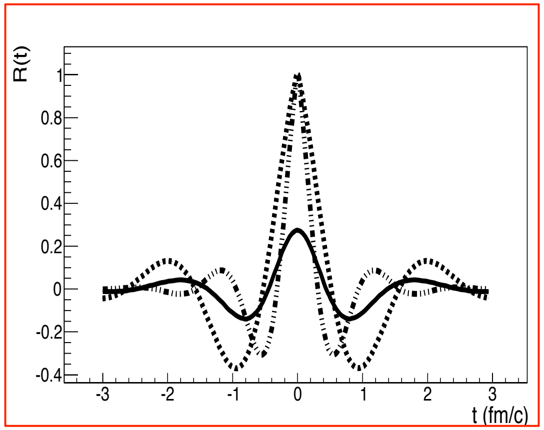
How to get quark counting ?



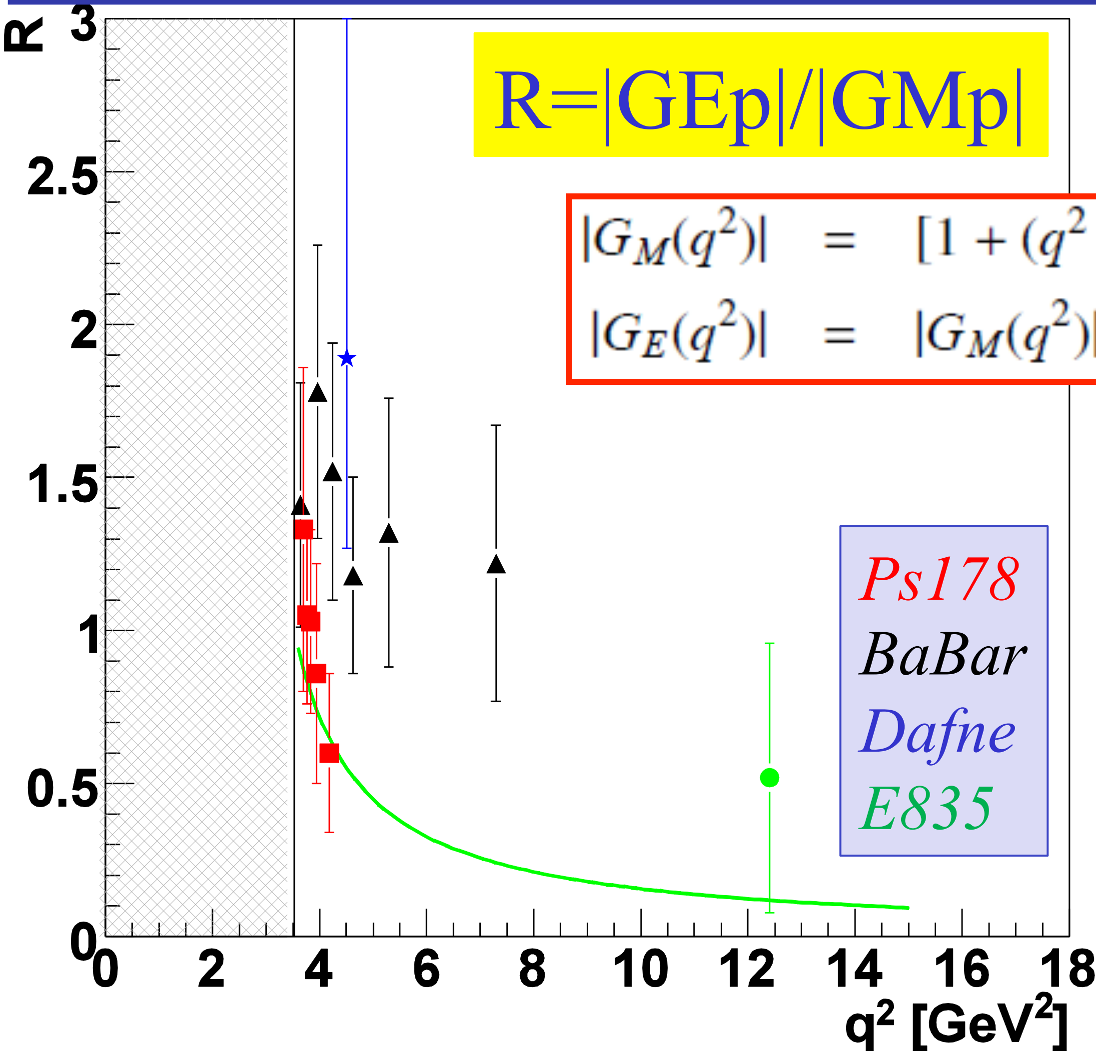
Use FT properties of convolutions :
A 3-constituent Fock state has 2 internal 4-dim degrees of freedom.
1) vertex hit by the virtual photon.
2) decaying correlation between the active and the spectator degree of freedom.

Each valence quark plays the role of active quark
Chain of two oscillators, one directly connected to the photon

If the decay times are different, at large t R1*R2 concides with the longest
R1*R2 may decay
- Because acquire opposite phase $t=\pi/(M1-M2)$
- Because $t>1/a_l$, a_l being the longer life-time pole.



Time-Like region



$$|G_M(q^2)| = [1 + (q^2 - 4M_p^2)/q_2^2]^{-2}\Theta(q^2 - 4M_p^2),$$

$$|G_E(q^2)| = |G_M(q^2)|[1 + (q^2 - 4M_p^2)/q_1^2]^{-1}\Theta(q^2 - 4M_p^2),$$

Not a fit

Point-like form factors?

Sommerfeld Enhancement and Resummation Factors

S. Pacetti

Coulomb Factor $\mathcal{C}$ for S-wave only:

- Partial wave FF: $G_S = \dfrac{2G_M\sqrt{q^2/4M^2} + G_E}{3}$ $\quad G_D = \dfrac{G_M\sqrt{q^2/4M^2} - G_E}{3}$
- Cross section: $\sigma(q^2) = 2\pi\alpha^2\beta\,\dfrac{4M^2}{(q^2)^2}\left[\mathcal{C}\,|G_S(q^2)|^2 + 2|G_D(q^2)|^2\right]$

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

- Enhancement factor: $\mathcal{E} = \pi\alpha/\beta$
- Step at threshold: $\sigma_{p\overline{p}}(4M_p^2) = \dfrac{\pi^2\alpha^3}{2M^2}\dfrac{\beta}{\beta}|G_S^p(4M_p^2)|^2 = 0.85\,|G_S^p(4M_p^2)|^2$ nb

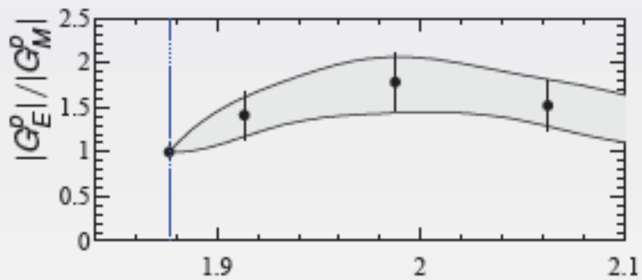
- Resummation factor: $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$
- Few MeV above threshold: $\mathcal{C} \simeq 1 \;\Rightarrow\; \sigma_{p\overline{p}}(q^2) \propto \beta\,|G_S^p(q^2)|^2$

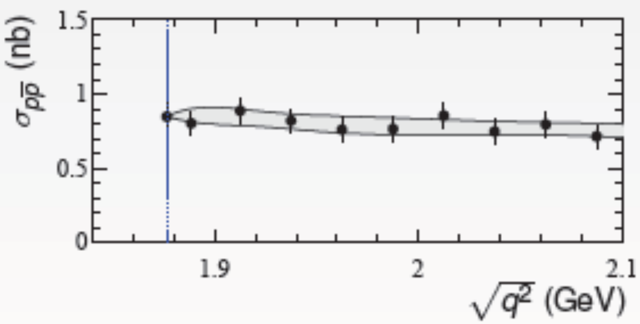
Point-like form factors?

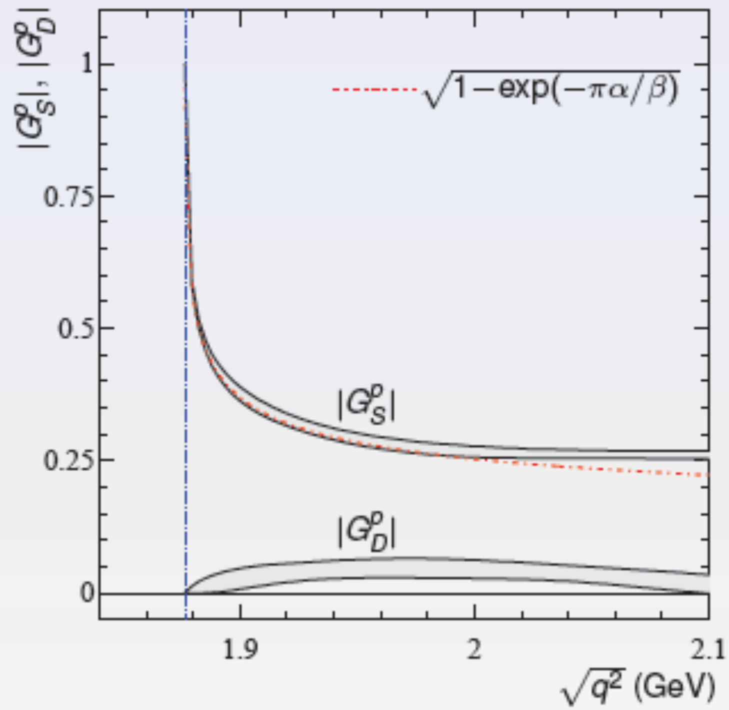
BABAR: $|G_E^p|/|G_M^p|$ and $\sigma(e^+e^- \to p\bar{p})$ [PRD73, 012005]

S. Pacetti

Extracting $|G_S^p|$ and $|G_D^p|$ using
- data on $\sigma_{p\bar{p}}$
- data on $|G_E^p|/|G_M^p|$
- G_E^p/G_M^p phase $\phi \simeq 0$

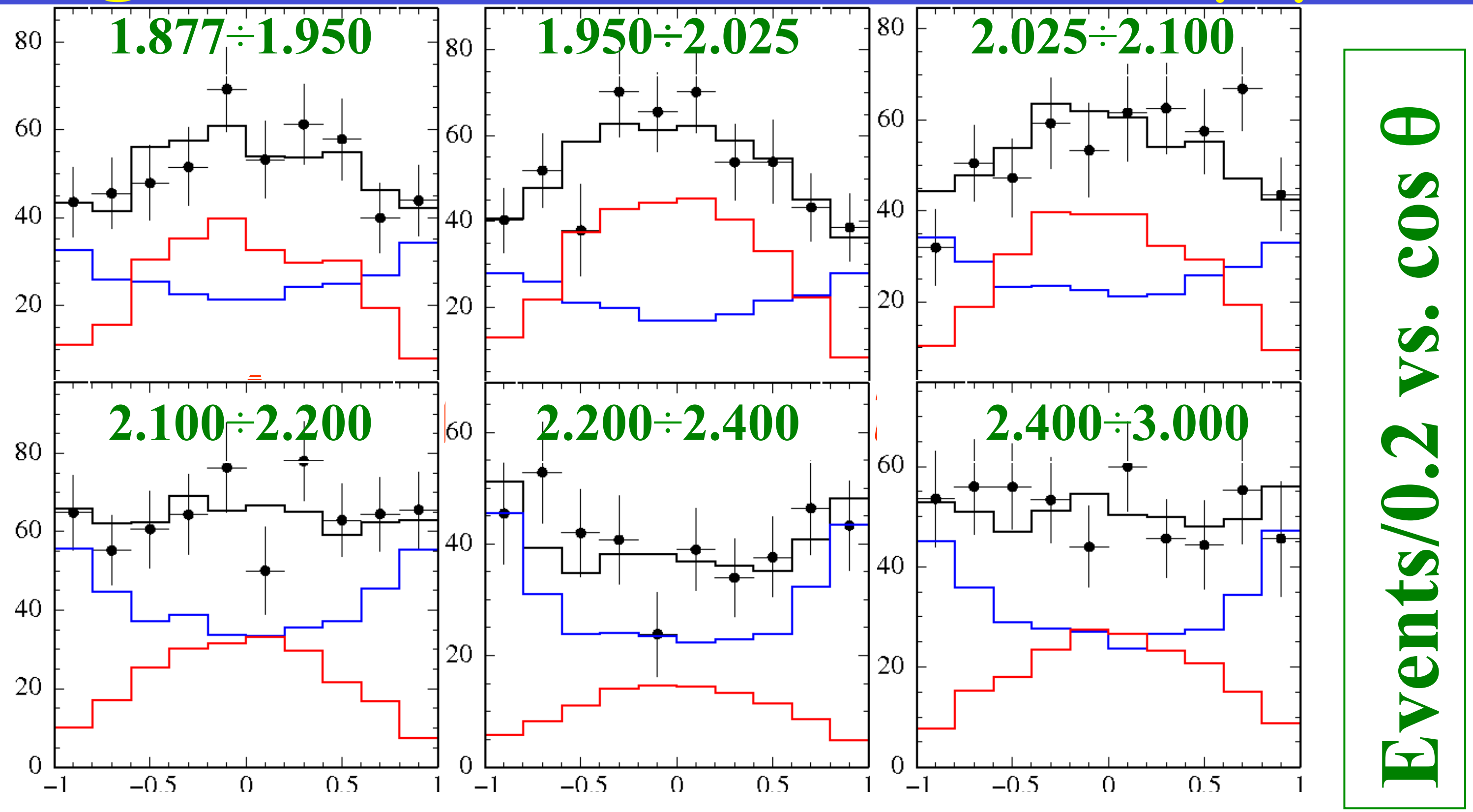






- $|G_S^p| \simeq \sqrt{1-\exp(-\pi\alpha/\beta)}$
- **No need of resummation factor?**

Angular Distributions $e^+e^- \to p\,\bar{p}$



$$\frac{dN}{d\cos\theta_p} = A\left[H_M(\cos\theta, M_{p\bar{p}}) + \left|\frac{G_E}{G_M}\right|^2 H_E(\cos\theta, M_{p\bar{p}})\right]$$

BABAR

2γ−exchange?

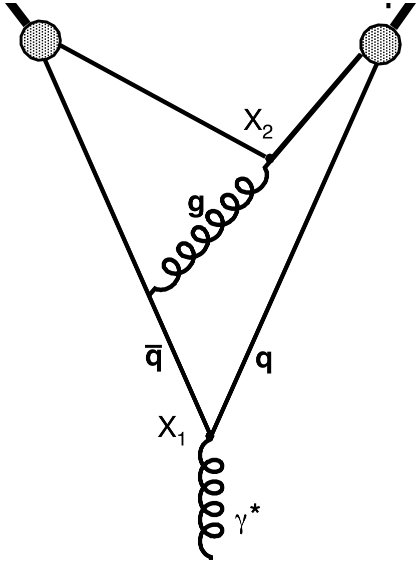
B. Aubert (BABAR Collaboration) Phys Rev. ***D73****, 012005 (2006)*

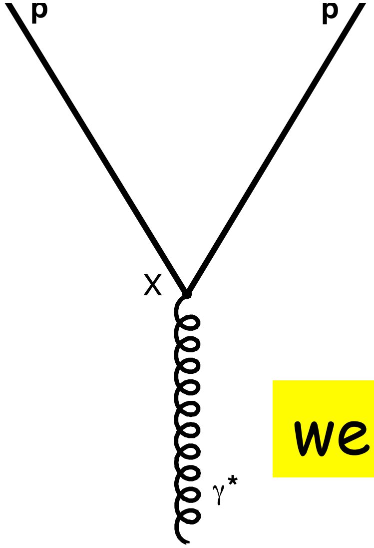
Implementing causality

X (TL) => q(TL)

$$\psi(X_1, X_2, ...X_N) \equiv e^{ipX}\Phi(x_1, x_2, ...x_N),$$

Fock state of N constituents





$$X = \sum w_i X_i,\ i = 1, ...N,$$
$$x_i \equiv X_i - X,$$
$$\sum w_i x_i = 0,$$

weights related to the masses

$$A_{TL,charge} = R_{point,charge}(q, p, \bar{p})\ e_1 \int dX_1 dX_2 ....e^{iiqX_1}\psi^+(X_1, X_2, ...)\ \psi'(X_1, X_2, ...) =$$

$$= R_{point,charge}(q, p, \bar{p})\ e_1 \int dX e^{[i(q-p-\bar{p})X]} \int dx_1 e^{iqx_1} \int dx_2 ...\delta^4(\sum w_i x_i)\Phi^+(x_1, x_2, ...)\ \Phi'(x_1, x_2, ...) \equiv$$

$$\equiv R_{point,charge}(q, p, \bar{p})\ \delta^4(q - p - \bar{p}) \int d^4x_1 e^{iqx_1}F(x_1), \quad x \equiv x_1.$$

Examples: Breit-Wigner

The Breit-Wigner example contains basic points

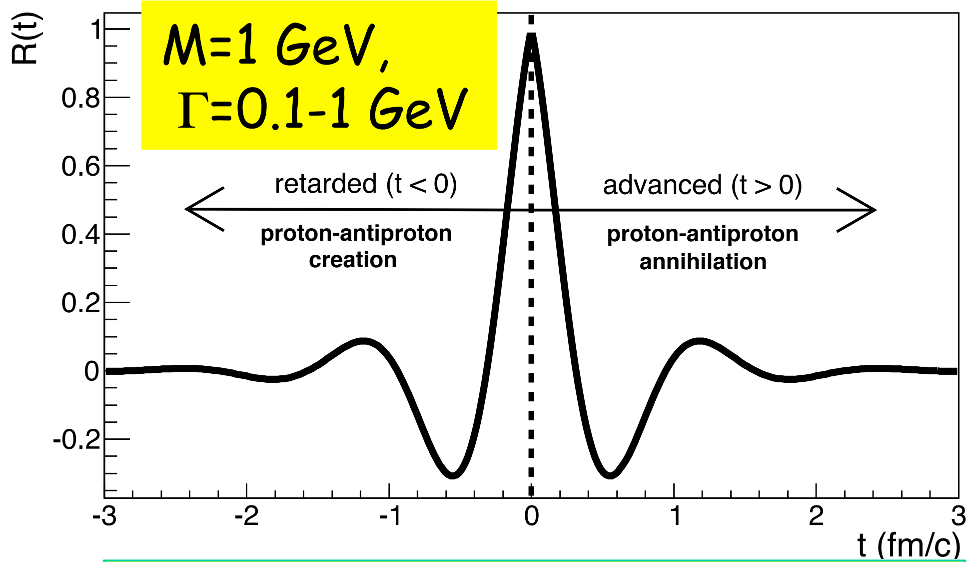
$$F_{\pm}(q) \propto \frac{1}{(q^2 - M^2) \pm iMa},$$

Two dimensional scaling violating parameters, corresponding to the pole mass and width

In SL: $q^2 \to -q^2$ how a charge distribution decreases with the distance to the pole mass

In TL the mass is associated to the frequency of oscillations in time of the gamma*q qbar coupling.

The pole width tells us how fast decreases the probability of formation of the ppbar pair.



Small number of visible oscillations.

W=0 => oscillations continue forever.

In SL: finite charge radius 1/M and monopole shape

Model

$$\frac{Q_q^2 e^2}{r_0^2} > e|Q_q|\, E.$$

attraction force > stochastic force of the gluon field

$$p_0 = \sqrt{\frac{E}{e|Q_q|}} = 1.1\ \text{GeV}.$$

Proton: $r_0 = 0.22$ fm, $p_0^2 = 1.21\ \text{GeV}^2$

Neutron: $r_0 = 0.31$ fm, $p_0^2 = 2.43\ \text{GeV}^2$

Applies to the scalar part of the potential

Properties of F(x)

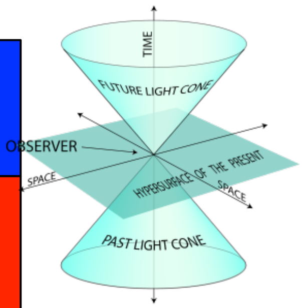
SL: t->-t is forbidden : the symmetry properties of a scalar amplitude do not depend on frame (Lorentz boost can make t negative)

TL: t->-t not forbidden: Lorentz boost does not mix past and present light cones

$$\vec{x}^2 > t^2 : F_{outLC}(x_\mu) = f(x^\mu x_\mu),\ t \to -t \ \ forbidden$$

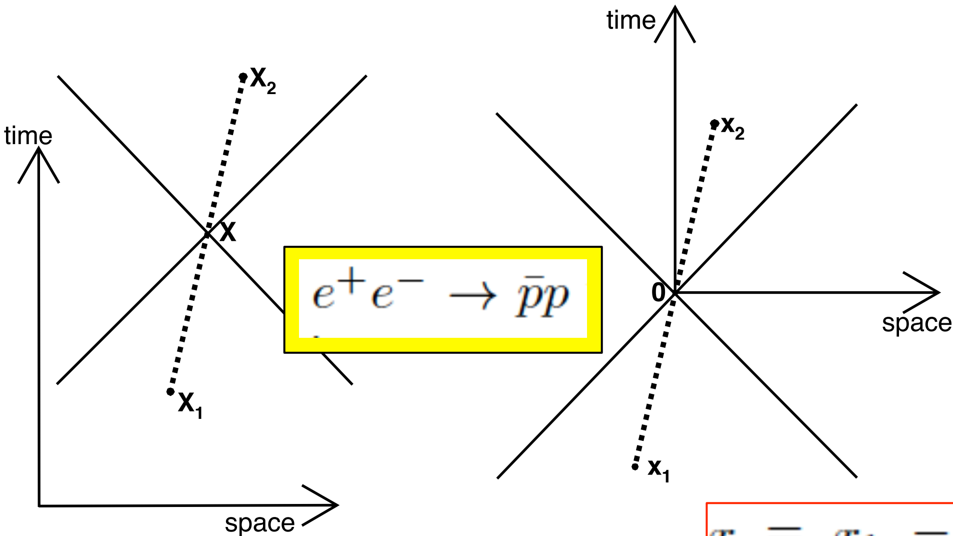
$$t^2 > \vec{x}^2 : F_{inLC}(x_\mu) = f_+(x^\mu x_\mu)\theta(t) + f_-(x^\mu x_\mu)\theta(-t),$$

$$F_{inLC}(x_\mu) = 1/2\,[f_+ + f_-] + 1/2\,[f_+ - f_-][\theta(t) - \theta(-t)].$$



leads to imaginary part of F(q) even if F(x) is real- affects only phases (T-conservation)

Implementing causality



$e^+e^- \to \bar{p}p$

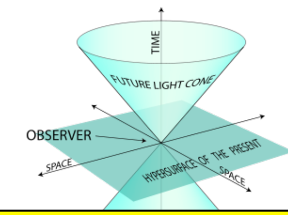
Fock state of N constituents

$$X = \sum w_i X_i, \; i = 1, ...N,$$
$$x_i \equiv X_i - X,$$
$$\sum w_i x_i = 0,$$

weights related to the masses

$$x \equiv x_1 = (x_1 - x_2)w, \quad w > 0.$$

Causality *implies* $t_1 < t_2$



Assume equal probability in time to form a complet p-pbar system, inside the future light cone of the first event. *(space probability: we integrate and set to one).*

$$R(t) = \theta(-t),$$

$$F(q) = \int e^{iqt}\theta(-t) = \frac{\pi}{\epsilon - iq},$$

Photon-charged pair coupling

$$
\begin{aligned}
A_{\text{point SL}}(q,p,p') &= \langle \mu' | A_\nu(X)\, J^\nu(X) | \mu \rangle \\
&= e \int d^4X \; e^{iqX} e^{-ip'X} e^{ipX} e_\nu \bar{u}(p')\, \gamma^\nu u(p) \\
&= e \int d^4X e^{iqX} e^{-ip'X} e^{ipX} \left[e_0 u^+(p')\, u(p) - \vec{e}\, \bar{u}(p')\, \vec{\gamma} u(p) \right] \\
&= \; \delta^4(q+p-p') \left[T_{\text{point charge}}(q,p,p') - T_{\text{point current}}(q,p,p') \right].
\end{aligned}
$$

$$
A_{\text{point}}(q,P_A,P_B) \equiv \delta^4(q+P_A+P_B) \left[T_{\text{point charge}}(q,P_A,P_B) - T_{\text{point current}}(q,P_A,P_B) \right],
$$

Introduce FFs as q-dependent quantities proportional to the amplitude, F(q), G(q)

$$
\begin{aligned}
A(q,P_A,P_B) &\equiv A_{\text{charge}}(q,P_A,P_B) - A_{\text{current}}(q,P_A,P_B) \qquad (21) \\
&\equiv \delta^4(q+P_A+P_B) \left[T_{\text{point charge}}(q,P_A,P_B) F(q) - T_{\text{point current}}(q,P_A,P_B) G(q) \right],
\end{aligned}
$$

Model: generalized form factors

Definition:

$$F(q^2) = \int_{\mathcal{D}} d^4x e^{iq_\mu x^\mu} \rho(x),\ q_\mu x^\mu = q_0 t - \vec{q}\cdot\vec{x}$$

$\rho(x) = \rho(\vec{x}, t)$ *space-time distribution of the electric charge in the space-time volume* $\mathcal{D}$.

In SL- Breit frame (zero energy transfer):

$$F(q^2) = \delta(q_0) F(Q^2),\ Q^2 = -(q_0^2 - \vec{q}^2) > 0.$$

In TL-(CMS): $$F(q^2) = \int_{\mathcal{D}} dt e^{i\sqrt{q^2}t} \int d^3\vec{r} \rho(\vec{r}, t) = \int_{\mathcal{D}} dt e^{i\sqrt{q^2}t} Q(t),$$

$Q(t)$: *time evolution of the charge distribution in the domain* $\mathcal{D}$.