How to reconcile the Rosenbluth and the polarization transfer method in the measurement of the proton form factors

P.A.M. Guichon[1] and M. Vanderhaeghen[2]

[1]SPhN/DAPNIA, CEA Saclay, F91191 Gif sur Yvette, France

[2]Institut für Kernphysik, Johannes Gutenberg Universität, D-55099 Mainz, Germany

The apparent discrepancy between the Rosenbluth and the polarization transfer method for the ratio of the electric to magnetic proton form factors can be explained by a two-photon exchange correction which does not destroy the linearity of the Rosenbluth plot. Though intrinsically small, of the order of a few percent of the cross section, this correction is kinematically enhanced in the Rosenbluth method while it small for the polarization transfer method, at least in the range of Q^2 where it has been used until now.

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The electro-magnetic form factors are essential pieces of our knowlegde of the nucleon structure and this justifies the efforts devoted to their experimental determination. They are defined as the matrix elements of the electro-magnetic current $J^\mu(x)$ according to :

$$
\begin{aligned}
&< N(p')|J^\mu(0)|N(p) > \\
&= e\,\bar{u}(p')\left[G_M(Q^2)\gamma^\mu - F_2(Q^2)\frac{p+p'}{2M}\right]u(p), \quad (1)
\end{aligned}
$$

where $e \simeq \sqrt{4\pi/137}$ is the proton charge, M the nucleon mass, and Q^2 the squared momentum transfer. The magnetic form factor G_M is related to the Dirac (F_1) and Pauli (F_2) form factors by $G_M = F_1 + F_2$, and the electric form factor is given by $G_E = F_1 - \tau F_2$, with $\tau = Q^2/4M^2$. For the proton, $F_1(0) = 1$, and $F_2(0) = \mu_p - 1 = 1.79$. In the one-photon exchange or Born approximation, elastic lepton-nucleon scattering :

$$
l(k) + N(p) \to l(k') + N(p'), \quad (2)
$$

gives direct access to the form factors in the spacelike region ($Q^2 > 0$), through its cross section :

$$
d\sigma_B = C_B(Q^2,\varepsilon)\left[G_M^2(Q^2) + \frac{\varepsilon}{\tau}G_E^2(Q^2)\right], \quad (3)
$$

where ε is the photon polarization parameter, and where $C_B(Q^2,\varepsilon)$ is a phase space factor which is irrelevant in what follows. For a given value of Q^2, Eq. (3) shows that it is sufficient to measure the cross section for two values of ε to determine the form factors G_M and G_E. This is referred to as the Rosenbluth method [1]. The fact that $d\sigma/C_B(Q^2,\varepsilon)$ is a linear function of ε (Rosenbluth plot criterion) is generally considered as a test of the validity of the Born approximation.

Polarized lepton beams give another way to access the form factors [2]. In the Born approximation, the polarization of the recoiling proton along its motion (P_l) is proportional to G_M^2 while the component perpendicular to the motion (P_t) is proportional to $G_E G_M$. We call this the polarization method for short. Because it is much easier to measure ratios of polarizations, it has been used mainly to determine the ratio G_E/G_M through a measurement of P_t/P_l using [2, 3] :

$$
\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\frac{G_E}{G_M}. \quad (4)
$$

Thus, in the framework of the Born approximation, one has two independent measurements of $R = G_E/G_M$. On Fig. 1 we show the corresponding results, which we call $R^{exp}_{Rosenbluth}$ and $R^{exp}_{Polarisation}$, for the range of Q^2 which is common to both methods. The data are taken from Refs. [4, 5, 6]. It is seen that the deviation between the two methods starts around $Q^2 = 2$ GeV^2 and increases with Q^2, reaching a factor 4 at $Q^2 = 6$ GeV^2. Recently, a global re-analysis of the SLAC cross section data was performed [7], where it was found that the individual cross section data are self-consistent, but still yield results inconsistent with the polarization measurements. An extensive set of Rosenbluth measurements in the range $0.5 < Q^2 < 5.5$ GeV^2 was also obtained recently at JLab (Hall C) [8] as a byproduct of the experiment E94-110 [9]. These results are also in excellent agreement with the global re-analysis of Ref. [7] confirming the discrepancy between the Rosenbluth and polarization extractions of the ratio G_E/G_M. This discrepancy is a serious problem as it generates confusion and doubt about the whole methodology of lepton scattering experiments.

In this letter we take a first step to unravel this problem by interpreting the discrepancy as a failure of the Born approximation which nevertheless does not destroy the linearity of the Rosenbluth plot. This means that we give up the beloved one-photon exchange concept and enter the not well paved path of multi-photon physics. By this we do not mean the effect of soft (real or virtual) photons, that is the radiative corrections. The effect of the latter is well under control because their dominant (infra-red) part can be factorized in the observables and therefore does not affect the ratio G_E/G_M. Here we consider genuine exchange of hard photons between the lep-

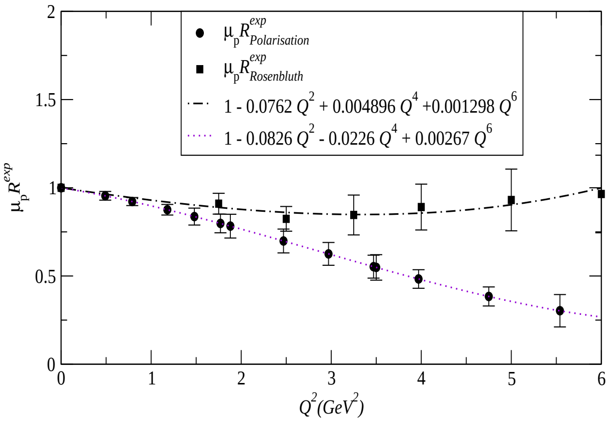

FIG. 1: Experimental values of $R^{exp}_{Rosenbluth}$ and $R^{exp}_{Polarisation}$ and their polynomial fits.

ton and the hadron.

Even if we restrict to the two-photon exchange case, the evaluation of the box diagram (Fig. 2) involves the *full* reponse of the nucleon to doubly virtual Compton scattering and we do not know how to perform this calculation in a model independent way. Therefore we adopt a modest strategy based on the phenomenological consequences of using the full eN scattering amplitude rather than its Born approximation. Though it cannot lead to a full answer it produces the following interesting results:

- the two-photon exchange amplitude needed to explain the discrepancy is actually of the expected order of magnitude, that is a few percent of the Born amplitude.
- there may be a simple explanation to the fact that the Rosenbluth plot looks linear eventhough it is strongly affected by the two-photon exchange.
- the polarization method result is little affected by the two-photon exchange, *at least in the range of* Q^2 *which has been studied until now.*

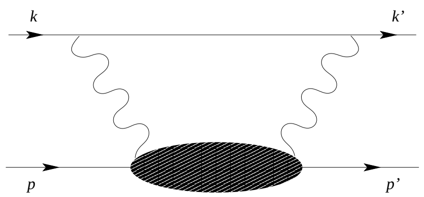

FIG. 2: The box diagram. The filled blob represents the response of the nucleon to the scattering of the virtual photon.

To proceed with the general analysis of elastic electron-nucleon scattering (2), we adopt the usual definitions :

$$P = \frac{p+p'}{2},\ K = \frac{k+k'}{2},\ q = k-k' = p'-p, \qquad (5)$$

and choose

$$Q^2 = -q^2,\ \nu = K.P, \qquad (6)$$

as the independent invariants of the scattering. The polarization parameter ε of the virtual photon is related to the invariant ν as (neglecting the electron mass) :

$$\varepsilon = \frac{\nu^2 - M^4\tau(1+\tau)}{\nu^2 + M^4\tau(1+\tau)}. \qquad (7)$$

For a theory which respects Lorentz, parity and charge conjugation invariance, the general amplitude for elastic scattering of two spin 1/2 particles depends on six invariant amplitudes [10]. To simplify, we neglect the amplitudes which flip the helicity of the electron since this amounts to neglect terms of the order of the electron mass. One is then left with three amplitudes [10], and the T-matrix can be written in the form :

$$\begin{aligned} T &= \frac{e^2}{Q^2}\,\bar{u}(k')\gamma_\mu u(k) \\ &\times\ \bar{u}(p')\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2\frac{P^\mu}{M} + \tilde{F}_3\frac{\gamma.KP^\mu}{M^2}\right)u(p), \quad (8) \end{aligned}$$

where $\tilde{G}_M$, $\tilde{F}_2$, $\tilde{F}_3$ are complex functions of ν and Q^2, and where the factor e^2/Q^2 has been introduced for convenience. In the Born approximation, one obtains :

$$\begin{aligned} \tilde{G}_M^{Born}(\nu,Q^2) &= G_M(Q^2), \\ \tilde{F}_2^{Born}(\nu,Q^2) &= F_2(Q^2), \\ \tilde{F}_3^{Born}(\nu,Q^2) &= 0. \quad (9) \end{aligned}$$

Since $\tilde{F}_3$ and the phases of $\tilde{G}_M$ and $\tilde{F}_2$ vanish in the Born approximation, they must originate from processes involving at least the exchange of two photons. Relative to the factor e^2 introduced in Eq. (8), we see that they are at least of order e^2. This, of course, assumes that the phases of $\tilde{G}_M$ and $\tilde{F}_2$ are defined, which amounts to suppose that, in the kinematical region of interest, the modulus of $\tilde{G}_M$ and $\tilde{F}_2$ do not vanish, which we take for granted in the following. Defining :

$$\tilde{G}_M = e^{i\phi_M}|\tilde{G}_M|,\ \tilde{F}_2 = e^{i\phi_2}|\tilde{F}_2|,\ \tilde{F}_3 = e^{i\phi_3}|\tilde{F}_3|, \qquad (10)$$

and using standard techniques, we get the following expressions for the observables of interest :

$$\begin{aligned} d\sigma &= C_B(\nu,Q^2)\,\frac{\varepsilon(1+\tau)}{\tau} \\ &\times \Big\{|\tilde{G}_M|^2\,\frac{\rho^2-\tau+\tau^2}{\rho^2-\tau-\tau^2} + |\tilde{F}_2|^2(1+\tau) \\ &\quad -2\,|\tilde{G}_M|\,\Big(\cos\phi_{2M}|\tilde{F}_2| - \cos\phi_{3M}|\tilde{F}_3|\rho\Big) \\ &\quad -2\,\cos\phi_{23}\,|\tilde{F}_2\tilde{F}_3|\rho + |\tilde{F}_3|^2(\rho^2-\tau^2)\Big\}, \quad (11) \end{aligned}$$

$$\begin{aligned} \frac{P_t}{P_l} &= -\sqrt{\frac{\rho^2-\tau-\tau^2}{\tau}} \\ &\times\ \frac{|\tilde{G}_M| - \cos\phi_{2M}|\tilde{F}_2|(1+\tau) + \cos\phi_{3M}|\tilde{F}_3|\rho}{|\tilde{G}_M|\rho + \cos\phi_{3M}|\tilde{F}_3|(\rho^2-\tau-\tau^2)}, \quad (12) \end{aligned}$$

with $\phi_{2M} = \phi_2 - \phi_M$, $\phi_{3M} = \phi_3 - \phi_M$, $\phi_{23} = \phi_2 - \phi_3$, and $\rho = \nu/M^2$. If one substitutes the Born approximation values of the amplitudes (9) then Eqs. (11,12) give back the familiar expressions of Eqs. (3,4).

To simplify the above general expressions, we make the very reasonable assumption that only the two-photon exchange needs to be considered. This amounts to keep only the terms of order e^2 with respect to the leading one in Eqs. (11,12). Using the fact that ϕ_M, ϕ_2 and $\tilde{F}_3$ are of order e^2 we get the approximate expressions :

$$\frac{d\sigma}{C_B(\varepsilon,Q^2)} = \frac{|\tilde{G}_M|^2}{\tau}\left\{\tau + \varepsilon\frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2\varepsilon\left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right)\mathcal{R}\left(\frac{\nu\tilde{F}_3}{M^2|\tilde{G}_M|}\right)\right\}, \quad (13)$$

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\left\{\frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right)\mathcal{R}\left(\frac{\nu\tilde{F}_3}{M^2|\tilde{G}_M|}\right)\right\}, \quad (14)$$

where, by analogy, we have defined :

$$\tilde{G}_E = \tilde{G}_M - (1+\tau)\tilde{F}_2, \;\; \tilde{G}_E^{Born}(\nu,Q^2) = G_E(Q^2), \quad (15)$$

and $\mathcal{R}$ denotes the real part. As for $\tilde{G}_M$ and $\tilde{F}_2$, we assume that the modulus of $\tilde{G}_E$ does not vanish in the kinematical domain of interest. To set the scale for the size of the pure two-photon exchange term ($\sim \tilde{F}_3$) we introduce the dimensionless ratio :

$$Y_{2\gamma}(\nu,Q^2) = \mathcal{R}\left(\frac{\nu\tilde{F}_3}{M^2|\tilde{G}_M|}\right), \quad (16)$$

which should be a good measure of the effect since, if we neglect $|\tilde{G}_E|/|\tilde{G}_M|$ with respect to τ in Eq. (13), we see that the cross section would be of the form $|\tilde{G}_M|^2(1+\varepsilon\, Y_{2\gamma})^2$. Therefore we expect $Y_{2\gamma}$ to be of the order of $\alpha \simeq 1/137$.

Eqs. (13,14) already exhibit the solution to our problem. In the expression for the cross section, the coefficient of ε has a two-photon correction $2(\tau + |\tilde{G}_E|/|\tilde{G}_M|)Y_{2\gamma}$ which is essentially $2\tau Y_{2\gamma} = Q^2 Y_{2\gamma}/2M^2$ at large Q^2. Moreover it competes with a leading term $|\tilde{G}_E|^2/|\tilde{G}_M|^2$ which is small. This produces an amplification of the two-photon effect which is not present in P_t/P_l.

From Eqs. (13,14) we see that the experimental couple ($d\sigma$, P_t/P_l) depends on $|\tilde{G}_M|$, $|\tilde{G}_E|$, and $\mathcal{R}(\tilde{F}_3)$. In first approximation, we know that $|\tilde{G}_M(\nu\, Q^2)| \simeq G_M(Q^2)$, $|\tilde{G}_E(\nu\, Q^2)| \simeq G_E(Q^2)$, and only $\mathcal{R}(\tilde{F}_3)$ is really a new unknown parameter. Thus allowing for two-photon exchange somewhat complicates the interpretation of the lepton scattering experiments but not in a dramatic way. In principle one could determine the three unknown parameters $|\tilde{G}_M|$, $|\tilde{G}_E|$ and $\mathcal{R}(\tilde{F}_3)$ if one had one more observable to complete the set of Eqs. (13,14). This could be provided, for instance, by a separate measurement of P_l and P_t rather than their ratio. However this is not really necessary because the dependence on ν of $|\tilde{G}_M|$ and $|\tilde{G}_E|$, being of order e^2, is certainely weak enough that it can be described by only a few parameters. Therefore if, as it is already the case for $d\sigma$, one had a set of values P_t/P_l at several values of ε, it is likely that one could solve Eqs. (13,14) by a global fit method.

As the present data are not enough to realize the program of extracting directly $|\tilde{G}_M(\nu\, Q^2)|$, $|\tilde{G}_E(\nu\, Q^2)|$ and $\mathcal{R}(\tilde{F}_3(\nu\, Q^2))$ we need to further simplify the problem. If we look at the data of Ref. [6] for $\sigma/C_B(\varepsilon,Q^2)$ as a function of ε we observe that for each value of Q^2 the set of points are pretty well aligned. We see on Eq. (13) that this can be understood if, at least in first approximation, the product $\nu\,\tilde{F}_3$ is independent of ε. We do not have a first principle explanation for this but we feel allowed to take it as an experimental evidence. To explain the linearity of the plot one must also suppose that $|\tilde{G}_M|$ and $|\tilde{G}_E|$ are independent of ε (that is ν) but since the dominant term of these amplitudes depends only on Q^2 this is a very mild assumption. We then see from Eq. (13) that what is measured using the Rosenbluth method is :

$$(R^{exp}_{Rosenbluth})^2 = \frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2\left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right)Y_{2\gamma}, \quad (17)$$

with $|\tilde{G}_E|/|\tilde{G}_M|$ and $Y_{2\gamma}$ essentially independent of ε , rather than $(R^{exp}_{Rosenbluth})^2 = (G_E/G_M)^2$, as implied by the one-photon exchange approximation. On the other hand the experimental results of the polarization method have been obtained for a rather narrow range of ε, typically from $\varepsilon = 0.6$ to 0.9. So, in pratice, we can neglect the ε dependence of $R^{exp}_{Polarisation}$ and from Eq. (14) we see that this experimental ratio must be interpreted as :

$$R^{exp}_{Polarisation} = \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right)Y_{2\gamma}, \quad (18)$$

rather than $R^{exp}_{Polarisation} = G_E/G_M$. In order that Eq. (18) be consistent with our hypothesis we should find that $Y_{2\gamma}$ is small enough that the factor $2\varepsilon/(1+\varepsilon)$ introduces no noticeable ε dependence in $R^{exp}_{Polarisation}$.

We can now solve Eqs. (17,18) for $|\tilde{G}_E|/|\tilde{G}_M|$ and $Y_{2\gamma}$ for each Q^2 . Due to the kinematical enhancement of the two-photon effect in the cross section we cannot treat it as a perturbation when solving the system of equations. Since the latter is equivalent to a quadratic equation it is more efficient to solve it numerically. For this we have fitted the data by a polynomial in Q^2 as shown on Fig. 1, and we shall consider this fit as the experimental values. In particular we do not attempt to represent the effect of the error bars which can be postponed to a more complete re-analysis of the data. The solution of Eqs. (17,18)

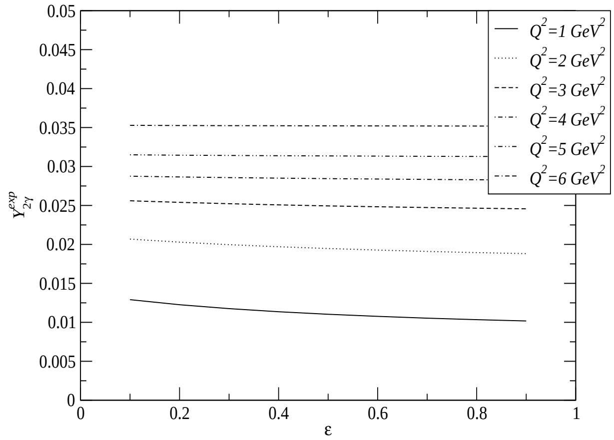

FIG. 3: The ratio $Y_{2\gamma}^{exp}$ versus ε for several values of Q^2.

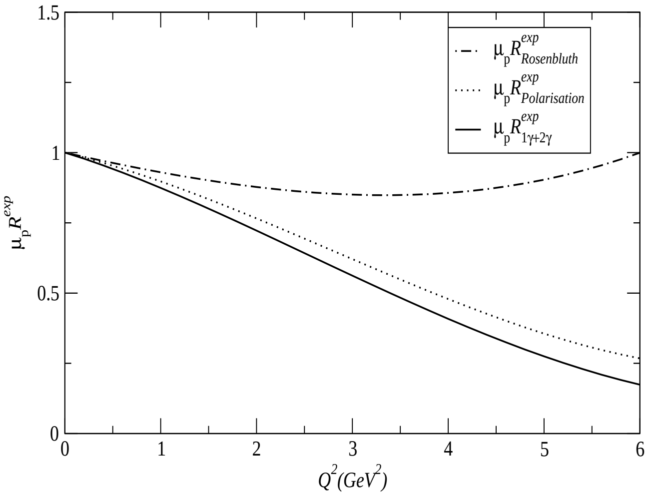

FIG. 4: Comparison of the experimental ratios $\mu_p R_{Rosenbluth}^{exp}$ and $\mu_p R_{Polarisation}^{exp}$ with the value of $\mu_p R_{1\gamma+2\gamma}^{exp}$ deduced from our analysis.

for the ratio $Y_{2\gamma}^{exp}$ is shown on Fig. 3 where we can see that it is actually small, of the order of a few percents. The largest value is obtained at the highest Q^2 but here the error bars [6] are rather large. We also observe that $Y_{2\gamma}^{exp}$ is essentially flat as a function of ε which is consistent with our hypothesis. The result for $Y_{2\gamma}^{exp}$ indicates that the corrections to the Born approximation are actually small in absolute value. In the Rosenbluth method their effect is accidentally amplified but there is no reason to think that this kind of accident will also occur in $\tilde{G}_E - G_E$ or $\tilde{G}_M - G_M$. So it makes sense to compare the value we get for $R_{1\gamma+2\gamma}^{exp} = |\tilde{G}_E|/|\tilde{G}_M|$ with the starting experimental ratios $R_{Rosenbluth}^{exp}$ and $R_{Polarisation}^{exp}$. This is shown on Fig. 4, from which we see that $R_{1\gamma+2\gamma}^{exp}$ is close to $R_{Polarisation}^{exp}$.

In summary, the discrepancy between the Rosenbluth and the polarization method for G_E/G_M can be attributed to a failure of the one-photon approximation which is dramatically amplified at large Q^2 in the case of the Rosenbluth method. No such amplification occurs in the polarization method, and this result does not depend on a specific evaluation of the two-photon correction. The expression for the cross section also suggests that the two-photon effect does not destroy the linearity of the Rosenbluth plot because the product $\mathcal{R}(\nu\tilde{F}_3)$ is independent of ν. It remains to be investigated if there is a fundamental reason for this behavior or if it is fortuitous. Using the existing data we have extracted the essential piece of the puzzle, that is the ratio $Y_{2\gamma}^{exp}$ which measures the relative size of the two-photon amplitude $\tilde{F}_3$. Within our approximation scheme, we find that $Y_{2\gamma}^{exp}$ is of the order of a few percent. This is a very reassuring result since this is the order of magnitude expected for two-photon corrections. What is needed as a next step is a realistic evaluation of this particular amplitude. From our analysis we extract the ratio $|\tilde{G}_E|/|\tilde{G}_M|$ which in first approximation can be assimilated to G_E/G_M. We find that it is close to the value obtained by the polarization method when one assumes the one-photon exchange approximation. Of course it will be necessary to relate the general amplitudes $\tilde{G}_E$, $\tilde{G}_M$ to G_E, G_M before drawing definitive conclusions but the small value that we get for the ratio $Y_{2\gamma}^{exp}$ suggests that the difference should be modest. We recall that our analysis always assumes that the modulus of the amplitudes $\tilde{G}_E$, $\tilde{G}_M$ do not vanish. When this happens the analysis has to be modified.

Finally, we point out that " à quelque chose malheur est bon". The two-photon corrections are generally small in absolute value but are in some cases amplified by kinematical factors. This causes some trouble to extract G_E/G_M but we can also see a positive aspect. Thanks to this amplification we have a way to measure two-photon amplitudes and this may open a new path to investigate nucleon structure.

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