# **Optimization of the transmission in an alternating gradient muon collection channel**

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**Abstract.** This paper reports on recent results regarding the optimization of the transmission in a muon production channel based on quadrupole focusing technique.

#### Introduction

The alternating gradient pion decay channel ("AG" in the following) is comprised of a 10 m long upstream section with very large aperture that funnels pion beams from four horns into a single 20 m long FODO decay section with 80 cm aperture (Fig. 1). Descriptions of the system and first stages of its optimization have been subject to earlier publications one can refer to for more details [1,2].





The main goal in the optimization is to maximize the muon transmission. For that purpose ray-tracing methods are used, that on the one hand permit the use of realistic models of magnetic fields in the large aperture magnets traversed by the beam and on the other hand allow accounting for  $\pi \to \mu$  decay based on Monte Carlo technique.

Compared to most recent results [2] the transmission efficiency in the AG channel has been sensibly increased, thanks mostly to the optimization of magnetic fields along the channel, element by element. For the sake of comparison, optimization of muon transmission through a 30 m long, 80 cm aperture solenoidal channel is also reported.

### Working hypothesis

Fig. 2 shows the MARS pion beam conditions at horn exit [3] as used in the present transmission optimizations. Note that time at origin is set to zero for all pions (analysis of proton bunch length effects is addressed in an analytical way in [4], detailed tracking studies are postponed to further investigations, numerical estimates in the solenoid case can be found in [5]).



**FIGURE 2.** MARS distributions at horn exit - channel entrance. Left : kinetic energy, middle and right : xx' and zz' phase spaces.

# **Preliminary tracking results**

Both the AG and the solenoid channels feature acceptance much smaller than the beam initial emittance shown in Fig. 2, this is illustrated in Fig. 3 that displays the subset of the initial coordinates of the pions yielding muons that make it to the end of



**FIGURE 3.** Initial vertical phase space of pions yielding muons that make it to the end of the 30 m channel (the horizontal phase space is similar).

the channel. It can be observed that the solenoid transverse acceptance is wider than the AG one, this is a property clearly established by the transmission simulations, however these need to take into account the longitudinal acceptance (of the RF bucket) which has a strong effect on transmission as made clear in Fig. 4 : the time extent of the muon beam at the exit of the solenoid (about  $[0.1, > 0.4] \ \mu s$ ) is much wider than in the AG case (about  $[0.1, 0.11] \ \mu s$ ).

A strong cause of transverse loss in the AG channel is the momentum spread in the beam (Fig. 2-left), by far larger than the momentum acceptance of the AG optics. A consequence is that the energy spread of the transmitted muons is strongly reduced compared to the initial pions' one as can be observed by comparison of the projected time spectra in Fig. 4-left with the initial one in Fig. 2-left.

#### Transmission calculation and optimization

The transmission efficiency through the channel is defined as

Number of muons in a given 6-D acceptance at channel exit Number of pions at channel entrance

Comments on this formula are in order :

- the pion beam at channel entrance is that of Fig. 2, and the "Number of pions at channel entrance" is 5  $10^4$ , this number is associated with  $10^6$  p.o.t.

- the "Number of muons in a given 6-D acceptance at channel exit" is maximized using an automatic procedure that optimizes, for each one of the 3 phase spaces, the shape (i.e., the parameters  $\alpha$ ,  $\beta$ ) of the acceptance ellipse

$$\gamma(y-y_c)^2 + 2\alpha(y-y_c)(y'-y'_c) + \beta(y'-y'_c)^2 = \varepsilon_y/\pi$$

 $(\gamma = (1 + \alpha^2)/\beta$ ; y stands for x, z, l for respectively the (x, x'), (z, z') and (time, energy) motions), and the positioning  $(y_c, y'_c)$  of that ellipse ; the only parameter being fixed in that procedure is the acceptance  $\varepsilon_y/\pi$ .

This is illustrated in Fig. 4-right that shows the various so-optimized longitudinal phase space ellipses with surfaces  $\varepsilon_l = 0.1, 0.5, 1$  and  $2 \pi$  eV.s.



**FIGURE 4.** Time-energy phase space of the muon beam at AG and solenoid channel ends, and *rms* ellipses (D). Optimum longitudinal ellipses at downstream end are shown for the solenoid case.

Maximization of the transmission in the AG channel has been obtained by adjusting the value of the magnetic fields in all the magnets, one by one. This has the advantage of accounting for the variation of the average rigidity of the  $(\pi + \mu)$  beam along the channel due to the decay [2,4]. The same adjustment has been performed on the solenoid field. It comes out that for both types of channels, the optimum value of the fields, yielding maximum transmission, sensibly depends on the 6-D acceptance at channel end. It is found for instance that in the case of the solenoid, if the longitudinal acceptance is taken infinite the optimum field is 4.8 T about whereas it is in the range 1.5 - 2 T with limited longitudinal acceptance. In the case of the AG channel the optimum field is rather high, of the order of 3 T at pole tip in the 40 cm radius quadrupoles of the FODO cells. Relaxing on transmission efficiency by about 30% has to be conceded if one wants to utilize warm magnets with field in the range 1.5 T at pole tip.

#### **Optimization results**



**FIGURE 5.** Transmissions versus longitudinal acceptance  $\varepsilon_l/\pi$ , in the AG and in the solenoid channels, for  $\varepsilon_{x,z}/\pi = 1$  cm and  $\varepsilon_{x,z}/\pi = 4$  cm transverse acceptances at exit.

Fig. 5 summarizes transmission results. In both non-cooling case (1 cm transverse acceptance) and cooling case (4 cm transverse acceptance), it can be seen that the AG has slightly larger transmission than the solenoid channel. One key reason for that is the wide initial momentum byte of the beam, which causes large beam lengthening and hence larger surface in longitudinal phase space at solenoid exit. Low field AG (1.5 T about at pole tip in the FODO quadrupoles) yields transmissions (not shown here) comparable to that of the solenoid.

## Conclusion

For the case of 40 cm inner radius, the 6-D transmission efficiencies of the AG channel and of the solenoid channel are comparable. Starting with a pion distribution generated with the MARS program for a proton energy of 2.2 GeV, about 8% of the muons produced arrive in emittances of  $0.01 \pi$  m.rad transverse and  $0.5 \pi$  eV.s longitudinal. For  $10^{16}$  p.o.t./s this yields 4  $10^{13}$  muons/s, i.e. 40% of the nominal value.

These emittances can probably be transmitted into the RLA and the decay ring ; how the beam will be accelerated to its final energy is still to be studied. This opens the possibility of proposing a NuFact design without cooling, the latter being considered as an upgrade of the project.

With transverse and longitudinal coolings, emittances of  $0.04 \pi$  m.rad transverse and  $1 \pi$  eV.s longitudinal can be considered, yielding more than twice as much muons/s. This has to be weighted against losses in the cooling channel.

#### REFERENCES

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