Numerical methods for 6-D dynamics simulations in FFAG rings

F. Lemuet *, F. Méot † A. Verdier *

A geometrical method for 3-D modeling of the magnetic field in FFAG magnets, installed in a stepwise ray-tracing code, is presented. It is applied for illustration to 6-D multiturn tracking in a 150 MeV radial sector proton FFAG.

1. Introduction

The method finds applications in machine design and multiturn tracking. It is derived from a former *DIPOLE* procedure implemented in the 70's in the ray-tracing code Zgoubi [1] for the design of large acceptance spectrometers (*e.g.*, SATURNE's SPES2, SPES3, and resorts to the sole geometrical parameters which describe the magnet. It yields an efficient ray-tracing tool, high precision tracking, and allows the use of automatic fitting procedures for magnet geometry and machine parameter optimizations.

2. Simulation of FFAG magnetic field

The two procedures so obtained, called "*DIPOLES*" and "*FFAG*", account for overlapping fields in the case of neighboring dipoles in an N-uplet (Fig. 1). Dipoles are defined by their parameters as wedge angles, pole curvatures, fringe fields extents, etc., [1, b], and are positioned within a sector region with angle AT, by means of angles ACN_i. Field dependence has the form $B_{z\,i}(r,\theta) = B_{z\,0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)$ (the index i stands for the dipole of concern) . "*DIPOLES*" and "*FFAG*" differ mostly by the radial behavior, respectively

$$\mathcal{R}_i(r) = \quad b_{0_i} + b_{1_i}\frac{r-R_{0,i}}{R_{0,i}} + b_{2_i}\left(\frac{r-R_{0,i}}{R_{0,i}}\right)^2 + ...$$
$$\mathcal{R}_i(r) = \quad (r/R_{0,i})^{K_i} \qquad (1)$$

The factor $\mathcal{F}_i(r,\theta)$ models the azimuthal dependence of the field. The first form of $\mathcal{R}_i(r)$ is proper to simulate fields for instance in muon chicane magnets [2], magnets for isochronous rings [3], SC magnets [4], etc., by *ad hoc* values of the b_{j_i} coefficients. The field fall-offs at *EFBs* is modeled using $\mathcal{F}_{EFB}(d) = (1+\exp[P(d)])^{-1}$ with $P(d) = C_0 + C_1 d/g + C_2(d/g)^2 + \,...\, + C_5(d/g)^5$ wherein d is the distance to the *EFB* and depends on r and θ ; the normalizing coefficient g is in general of the form $g(r) = g_0(R_0/r)^\kappa$ ($\kappa \geq 0$) with g_0 the dipole gap. A dipole having two *EFBs* (entrance and exit) with each one its own fringe

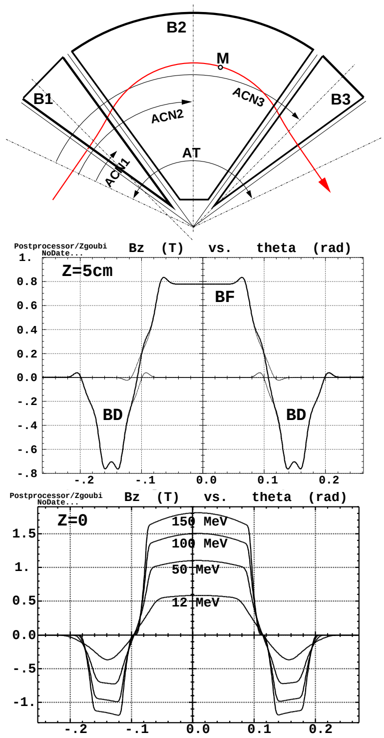

Figure 1. Top : FFAG sector triplet. Middle : overlapping fields at constant radius (non-zero z) ; the three field contributions are represented separately, as well as their merging. Bottom : mid-plane field at traversal of the triplet, on 12, 50, 100, 150 MeV closed orbits.

*CERN AB, Geneva
†CEA DAPNIA, Saclay

field factor, the resulting form factor at (r, θ) due to dipole (i) of the N-uplet is thus taken to be $\mathcal{F}_i(r, \theta) = \mathcal{F}_{\text{Entrance}}(r, \theta) \times \mathcal{F}_{\text{Exit}}(r, \theta)$.

The total mid-plane field and field derivatives at (r, θ) in an N-uplet are obtained by addition of the contributions of the N dipoles taken separately (*e.g.*, $N = 3$ in Fig. 1), namely

$$\begin{aligned} B_z(r, \theta) &= \sum_{i=1,N} B_{z0,i}\, \mathcal{F}_i(r, \theta)\, \mathcal{R}_i(r)\ , \\ \frac{\partial^{k+l} \vec{B}_z(r, \theta)}{\partial \theta^k \partial r^l} &= \sum_{i=1,N} \frac{\partial^{k+l} \vec{B}_{z\,i}(r, \theta)}{\partial \theta^k \partial r^l} \end{aligned} \qquad (2)$$

with $\mathcal{R}_i(r)$ defined in Eq. 1. Eventually, the 6-D modeling $\vec{B}(r, \theta, z)$ is deduced by z-extrapolation. Sample $B_z(r, \theta)$ patterns are given in Fig. 1, a simulation of the field in an FFAG triplet with characteristics drawn from the KEK 150 MeV proton machine [5]. Two different methods have been implemented to calculate the field derivatives in the median plane (Eq. 2), namely, either numerical interpolation using field values in the vicinity of the particle position, or analytical expressions drawn straightforwardly from the geometrical description of the magnet [6]. The first method has the merit of making it easy to change the source code so as to modify the mid-plane magnetic field model $B_z(r, \theta)$, for instance if simulation of shims, defects, or other special r, θ dependence need be introduced. The second method insures better symplecticity in principle and faster tracking.

3. 6-D tracking in a 150 MeV proton FFAG

To conclude, we show that these methods provide correct results. A 12-cell FFAG ring is considered, representative of the KEK 150 MeV proton FFAG [5], the cell is a 30 degree sector encompassing a DFD triplet with field on closed orbits as schemed in Fig. 1. First order results, as drawn from multiturn tracking, are displayed in the Table below for the 12, 50, 100 and 150 MeV motions in the vicinity of the closed orbit, and show satisfying consistency with published data [5].

E (MeV)	$\frac{d\mathcal{L}/\mathcal{L}}{dp/p}$	β_r/β_z at drift center (m)	tunes (r/z)	f_{rev} (MHz)
150	0.1138	0.718 / 2.80	4.060 / 2.207	4.463
100	0.1140	0.692 / 2.71	4.079 / 2.219	3.865
50	0.1136	0.649 / 2.60	4.111 / 2.226	2.953
12	0.1145	0.564 / 2.58	4.194 / 2.099	1.620

Closed orbits in a cell and one-turn tunes are displayed in Fig. 2, they have the expected behavior, in particular the vertical chromaticity is not exactly zero due to the fringe fields (zero vertical chromaticity is obtained if a geometrical model with hard edge is used).

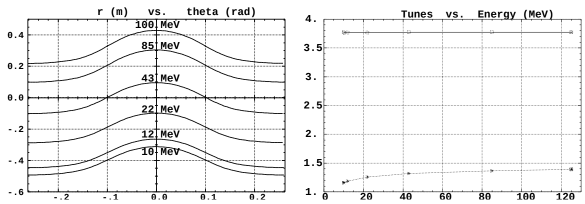

Figure 2. Left : closed orbits in a cell. Right : machine tunes (12 cells).

Fig. 3 shows sample phase space motion at 50 MeV. The horizontal symplecticity is good. The vertical motion shows confined emittance spread, attributed to non-linear coupling to horizontal motion.

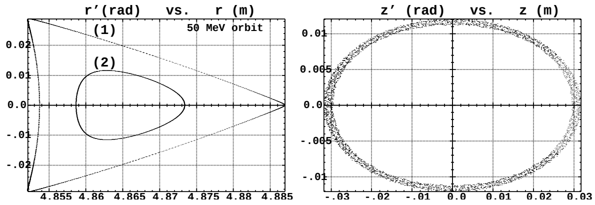

Figure 3. Left : pure radial motion, particles launched with (1) : $r_0 = r_{c.o.} + 1.9$ cm $= 4.887$ m (the stability limit), (2) : $r_0 = r_{c.o.} + 0.5$ cm. Right : vertical motion, given $r \approx r_{c.o.}$.

Next, stationary buckets in Fig. 4 have been obtained assuming a single cavity located in a straight section, with peak voltage 19 kV. The agreement with theory (bucket height, synchrotron tunes, etc.) is excellent. A full accel-

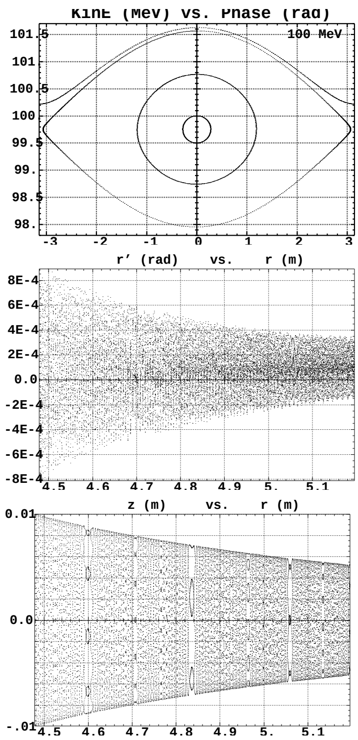

Figure 4. Top : stationary bucket in the 100 MeV orbit region. Middle and bottom : evolution of respectively the (r, r') and (r, z) motions during acceleration cycle, for a particle launched near the 12 MeV horizontal closed orbit with $z_0 = 1$ cm.

eration cycle, $2\,10^4$ turns from 12 to 150 MeV, using 20 deg. synchronous phase, is displayed in Fig. 4. The vertical motion undergoes expected $\sqrt{B\rho}$ damping.

4. Comments

Computation of field derivatives by numerical differentiation from the mid-plane geometrical field model (Fig. 1) yields good tracking symplecticity, in particular transverse motion (Fig. 3) can be explored up to stability limits. However, using analytical expressions instead for computing the derivatives insures best precision, and allows faster tracking, by a factor of more than 2.

Acceleration, and in a general manner 6-D motion, are very well handled (Fig. 4). These developments yield an efficient ray-tracing tool for beam, or long-term, tracking based studies, and, accounting for the built-in fitting procedure, for FFAG magnetic field and machine design studies.

It is planned to compare the magnetic fields (Fig. 1) as obtained with the geometrical method described in this paper and the ensuing tracking results, with 3-D magnet calculations and tracking in field maps. Works have already been tackled on that topic and will be pursued [7].

CPU time

Computing speed tests were performed upon acceleration in the 12 cell FFAG ring (conditions as in Fig. 4), using two different processors, Pentium III 1 GHz or Xeon 2.8 GHz, under Linux system. Results are as follows.

An integration step size $\Delta s = 0.5$ cm is considered, derivatives are computed with either the analytical or the numerical method, up to either second or fourth order as indicated in the Table.

CPU time (seconds per turn) :

	Pentium III 1 GHz		Xeon 2.8 GHz	
	Analyt.	Num.	Analyt.	Num.
2nd order	0.17 s	0.40 s	0.10 s	0.25 s
4th order	0.44 s	1.00 s	0.17 s	0.64 s

Such computing speed means that one can envisage overnight runs on computer network systems, aiming at such goals as long-term DA tracking, 6-D multi-turn beam transmission, resonance crossing studies with thousands-particle beam.

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