Excitation energies of superdeformed states in ^{196}Pb: towards a systematic study of the second well in Pb isotopes

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Two transitions of energy 3.698 and 4.062 MeV have been observed in the depopulation of the lowest-energy superdeformed band in ^{196}Pb. Correlations across isomeric states in the normal minimum have been used to elucidate the decay paths. The lowest observed SD level has excitation energy $E_x = 5.859$ MeV and spin/parity $I^{\pi} = 6^{+}$. Together with previous excitation energy measurements on ^{192}Pb and ^{194}Pb, this result allows a systematic comparison of different theoretical calculations with experimental data. The trend to increasing excitation energy with increasing neutron number predicted by both a macroscopic Strutinsky method approach and microscopic mean field calculations is confirmed, but none of the calculations reproduces the results in detail. The data also allow the extraction of two-proton and two-neutron separation energies in the second well, providing an additional test for the calculations.

The existence of excited states in atomic nuclei characterised by very elongated ellipsoidal shapes with an axis ratio approaching 2 : 1 is one of the most exciting discoveries of nuclear structure studies. Over the past two decades, rotational bands associated with these *superdeformed* (SD) shapes have been observed in several regions of the nuclear chart [1], with 85 SD bands observed in nuclei with $79 \leq Z \leq 84$ (the $A \approx 190$ region) alone. Unfortunately, measurement of the fundamental properties of these states – excitation energy, spin and parity – has only rarely been possible.

The difficulty lies with observing the very weak discrete transitions which link SD states with levels of normal deformation. To date, linking transitions have been identified in the decay of only five SD bands in the $A \approx 190$ region: two bands in ^{194}Hg [2, 3], and one band in each of ^{194}Pb [4, 5], ^{192}Pb [6] and ^{191}Hg [7]. This is despite many attempts using the high-efficiency multi-detector arrays Gammasphere and Euroball. Less precise measurements of SD state excitation energies have been achieved in the nuclei ^{192}Hg [8] and ^{195}Pb [9] following analyses of the quasicontinuum component of the decay from the SD minimum.

The present work reports on the measurement of the excitation energy of the yrast (lowest energy for a given spin) SD band in ^{196}Pb. Together with the earlier measurements of the excitation energies of SD states in ^{194}Pb [4, 5] and ^{192}Pb [6], the results allow a systematic study of the energy of the SD well in a single isotope chain.

The data were obtained in an experiment carried out using the Euroball IV array [10] at the Institut de Recherches Subatomiques, Strasbourg. High-spin states were populated in the reaction ^{170}Er(^{30}Si,4n)^{196}Pb. The target consisted of a 1.65 mg/cm^2 layer of ^{170}Er on a Au backing of thickness 6.6 mg/cm^2. The beam was provided by the Vivitron Tandem accelerator at an energy of 144 MeV. An "inner ball" of 210 BGO detectors was used to measure the sum energy and multiplicity of the γ rays emitted in each reaction. A master trigger signal was generated and an event recorded when signals were obtained from at least five of the inner ball elements and at least four escape-suppressed Ge detectors. In addition to the energy of each γ ray, the time of its detection relative to the master trigger was also recorded. The experimental set-up allowed γ rays to be detected and correlated within a window of ≈ 500 ns width. After a preliminary treatment of the data,1.1×10^9 triple- or higher-fold γ-ray coincidence events were obtained.

Recently, it was demonstrated that isomeric states in the normal minimum can sometimes simplify the study of the decay out of a SD minimum [6]. In that work, all γ rays in a particular event were timed with respect to

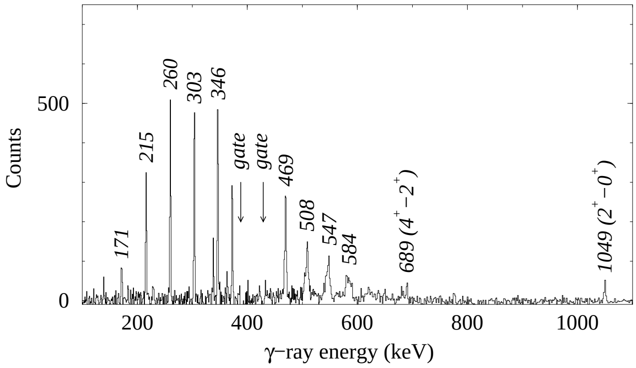

FIG. 1: Background-subtracted spectrum of γ rays in coincidence with both the 388 and 429 keV SD transitions. Members of the SD band are marked with the transition energies.

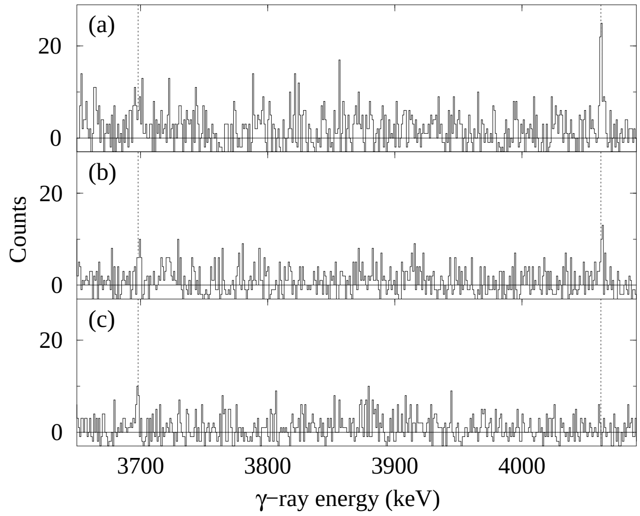

FIG. 2: (a) Gamma rays in prompt coincidence with at least three SD transitions. (b) and (c): Gamma rays in prompt coincidence with at least two SD transitions and (b) either the 689 keV $4^+ \rightarrow 2^+$ or the 1049 keV $2^+ \rightarrow 0^+$ transitions in delayed coincidence; (c) either the 372 keV $7^- \rightarrow 5^-$, the 164 keV $8^- \rightarrow 7^-$ or the 421 keV $8^+ \rightarrow 7^-$ transitions in prompt coincidence.

a beam pulse, which provided a reference time that was fixed relative to the fusion-evaporatin reaction time. The present data, however, were obtained with a continuous beam. Under such conditions, the reference time t_0 is usually taken as the time of the master trigger, which in this case is generated when the last of four escape-suppressed signals from the Euroball Ge array and five signals from the inner ball elements is detected. However, several strongly populated isomers can be fed during the decay from the highest spin states in ^{196}Pb to the ground state, and thus a t_0 derived in this way is not fixed relative to the reaction time.

To circumvent this problem, we have exploited the known energies of the SD band in order to generate a reference time on an event-by-event basis. When creating a spectrum gated on the SD band, the average time of the gating transitions (rather than the master trigger) was used to define t_0. Energy-dependent time gates were then applied to determine whether a particular γ ray was prompt or delayed with respect to the SD band.

The yrast levels of spin and parity $I^\pi = 5^-, 9^-$, 11^- and 12^+ in ^{196}Pb have been previously been identified as isomeric [11]. These states have excitation energies 1.797, 2.307, 2.695 and 3.193 MeV respectively, and their lifetimes (between 50 and 250 ns) are such that, if any are fed in the decay from a SD state towards the ground state, the techniques of time-correlated γ-ray spectroscopy should be applicable.

Figure 1 shows a spectrum obtained by double-gating on the 388 and 429 keV transitions in the yrast SD band in ^{196}Pb and requiring that all γ rays are in prompt coincidence. The in-band transitions are marked with their energies. The peak at 1049 keV corresponds to the $2^+ \rightarrow 0^+$ ground-state transition in ^{196}Pb. A comparison of the intensity of this transition with the intensity of the 260 keV transition (which carries 100% of the in-band SD intensity) gives an indication of the fraction of the SD intensity decaying through isomeric states. By studying this and similar spectra, after efficiency and internal conversion correction, we find that only 23(3)% of the SD band intensity is in prompt coincidence with the ground-state transition, indicating that 77(3)% of the intensity of the decay occurs via paths which feed one or more of the isomeric states discussed above. Further analysis indicates that 14(4)% of the intensity feeds states which decay via the 9^- isomer (and hence the 5^- isomer below it), while 63(3)% feeds states which decay via the 5^- isomer only. No evidence for decay via the higher-lying 11^- and 12^+ states was observed. This is consistent with the spins assigned to the SD band in previous work [12], where the lowest observed level was assigned as having $I^\pi = 6^+$ and the decay out of the band occurs from the levels of $I^\pi = 10^+$ and below.

Figure 2a shows the higher-energy region of a background-subtracted spectrum of γ rays detected in prompt coincidence with at least three of the in-band SD transitions. The energy range covers the region in which transitions which directly link the SD band to yrast and near-yrast states in the normal well might be expected. A peak is clearly visible at 4062 keV; a second, weaker peak is tentatively observed at 3698 keV (both marked with dotted lines).

Due to their low intensity, the statistics in spectra gated by one or two SD transitions and these candidate linking transitions are very low. While the data show that the 4062 keV γ ray is in prompt coincidence with the lowest energy in-band SD transition, indicating that it de-excites the SD level with $I^\pi = 6^+$, it is difficult to pinpoint the ND level it feeds using prompt coincidence requirements alone. Detailed investigation of the

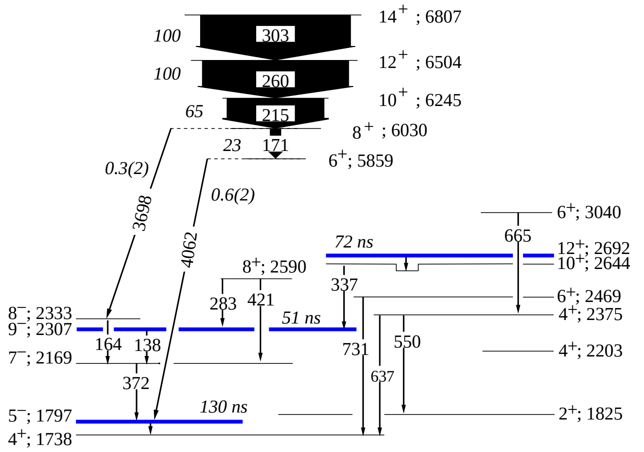

FIG. 3: Partial level scheme indicating the decay from the lowest observed SD states to the yrast normal-deformed levels.

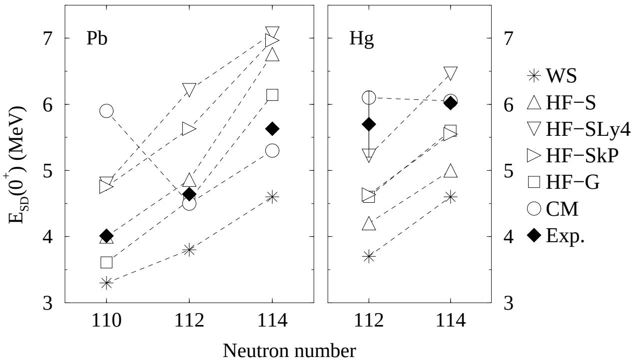

FIG. 4: Comparison with various model predictions: WS: Strutinsky method using Woods-Saxon [13]; HF-S: HFB using density-dependent Skyrme interaction [14]; HF-SLy4: HFB using SLy4 interaction [15]; HF-SkP: HFB using SkP interaction [15]; HF-G: HFB using Gogny [16]; CM: cluster model [17]. Experimental energies are from [2, 4, 6, 8] and this work.

coincidence relations shows that the 4062 keV γ ray is *not* in coincidence with the 1049 keV $2^+ \rightarrow 0^+$ and 689 keV $4^+ \rightarrow 2^+$ transitions. This suggests that it may feed an isomeric level or a level which itself feeds an isomer. In fact, the presence of the four known isomers can be exploited to determine which levels are fed by both the 3698 keV and 4062 keV γ rays.

Figure 2b shows the γ rays in prompt coincidence with at least two SD transitions, with the additional requirement that either the $2^+ \rightarrow 0^+$ or the $4^+ \rightarrow 2^+$ transition (which de-excite the 5^- isomer) was detected in the delayed period $\approx 30 - 300$ ns after the SD gating transitions. The 4062 and 3698 keV peaks are still present, demonstrating that both transitions feed an isomer or level which depopulates via an isomer.

Figure 2c shows the γ rays in prompt coincidence with at least two SD transitions and one of the transitions which feed the 5^- isomer. The 4062 keV transition is not present in this spectrum. Further investigation shows that it is not present in spectra of γ rays detected in coincidence with the delayed transitions between the 9^- and 5^- isomers. It is therefore most likely that it feeds the 5^- level directly. The peak at 3698 keV, on the other hand, is present in the spectrum created requiring prompt coincidence with one of the transitions feeding the 5^- isomer (Fig. 2c). It is not present in the spectrum of γ rays in coincidence with the delayed decay of the 9^- isomer (not shown). The time correlation evidence and the energy differences suggest that this peak is the transition from the 8^+ SD level to the yrast 8^- level in the normal-deformed minimum.

The suggested decay scheme is shown in Fig. 3. The two paths to the normal-deformed states fix the excitation energy of the lowest observed level as $E_{SD}(6^+) = 5.859$ MeV. Assuming that the band is based on a $K^\pi = 0^+$ configuration (as would be expected for a band built on a quasi-vacuum state in an even-even nucleus), the in-band levels will have even spin and positive parity. The spin assignments indicated in the figure are then the most likely. An assignment of $2\hbar$ higher would result in the 4062 and 3698 keV transitions having $E3$ and $M2$ multipolarity respectively, while an assignment $2\hbar$ lower would result in an $M2$ assignment for the 3698 keV transition, any of which would be extremely unlikely to compete with the highly collective in-band $E2$ transitions. On the basis of the current assignment, both observed single-step linking transitions are of $E1$ multipolarity.

A rotational model extrapolation of excitation energy as a function of spin has been used to obtain an estimate of the bandhead excitation energy, resulting in $E_{SD}(0^+) = 5.630(4)$ MeV. This is significantly higher than the values measured for ^{194}Pb (4.643 MeV) [4] and ^{192}Pb (4.011 MeV) [6].

Several predictions of SD bandhead energies in $A \approx 190$ nuclei have been made using both macroscopic and microscopic approaches. In the following we consider only those calculations which have predicted energies for all three even-even Pb isotopes. Early calculations using the Strutinsky method and a Woods-Saxon potential were performed by Satula *et al.* [13]; the Hartree-Fock-Bogoliubov (HFB) method has been employed by several authors, including the Skyrme force calculations of Krieger *et al.* [14] and Heenen *et al.* [15] and the Gogny force calculations of Libert *et al.* [16]. Most recently, a cluster model approach has been applied by Adamian *et al.* [17]. The results of each of these calculations for the Pb isotopes are compared to the extrapolations from experimental data in Fig. 4.

While most of the calculations predict the gross trend of increasing energy with increasing neutron number, it appears that none reproduces the SD excitation energies of all three Pb isotopes. The Strutinsky calculations of Satula *et al.* [13] predict the differences between the Pb isotopes to within 250 keV, but con-

	Expt.	SkM*	SLy4	SkP
S_{2n}				
^{196}Pb	18.287	17.191	16.587	16.063
^{194}Pb	18.432	17.946	16.381	16.934
S_{2p}				
^{196}Pb	7.412	6.971	8.267	7.536

TABLE I: Comparison of experimental and theoretical [15] two-neutron and two-proton separation energies. All value are in MeV.

sistently underestimate the absolute values. In addition, the differences between isotones of Pb and Hg are not reproduced. The HFB calculations of Krieger *et al.* (Skyrme) [14] and Libert *et al.* (Gogny) [16] overestimate $E_{SD}(0^+,{}^{196}\text{Pb})$ and the difference between $E_{SD}(0^+,{}^{196}\text{Pb})$ and $E_{SD}(0^+,{}^{194}\text{Pb})$. Both the SLy4 and SkP calculations of Heenen *et al.* [15] overestimate the energies by around 1 MeV for each of the three Pb isotopes and fail to reproduce the higher excitation energies in the Hg isotopes. In fact, the SkM* calculations also presented in that work provide a much closer match to the data - however, the authors state that this interaction has the wrong isospin behaviour and should not be used in binding energy extrapolations. Finally, the cluster model approach of Adiaman *et al.* [17] does reasonably well for all cases except ^{192}Pb.

Heenen *et al.* [15] caution that the absolute excitation energies are not necessarily a good test of the HFB approach, since small changes in the model (such as treatment of the pairing channel) can result in changes of the order of 1 MeV in the results. More useful probes of the models are the two-proton and two-neutron separation energies, S_{2p} and S_{2n}, which can be calculated if the ground-state binding energies and SD excitation energies are known. The present data allow the extraction of both S_{2p} and S_{2n} for ^{196}Pb (using the tabulated binding energies [18]). These are listed in table I, together with the predictions of ref [15]. The value of S_{2n} obtained for ^{194}Pb is also included.

The two-neutron separation energies are not well-reproduced by any of the three Skyrme parameterizations. The calculations using the SkM* interaction come closest to the experimental values, but as mentioned above there are problems with this parameterization which indicate it should not be used here. In contrast, the two-proton separation for ^{196}Pb is best reproduced by the SkP calculations.

It has recently been suggested that correlations beyond the mean field can play an important role in determing the properties of the ground state wavefunctions [19–21] of nuclei which exhibit potential energy surfaces which are soft as a function of shape or which contain several minima. Although the SD well in the Pb isotopes is well defined at high spins, it is expected to become shallower at low spins. At the spins at which the decay occurs out of the SD band, the potential surfaces exhibit minima associated with spherical and oblate shapes as well as the SD minimum. Thus it may be necessary to adopt this "beyond mean field" approach, which includes extra binding energies due to rotational and vibrational correlations (it should be noted that octupole vibrational states are also thought to play an important role in the SD bands in this mass region [12, 22–24]).

In principle, the binding energies of the SD bandheads should be among the simpler quantities to be calculated. Thus this step towards a systematic study of the "ground state" state associated with the SD minimum provides a real test for the new calculations. The theoretical focus to date has been on the lighter Pb isotopes with $A \leq 192$, and thus it is not possible to compare these more sophisticated approaches with the experimental results at present.

In conclusion, we have observed two high energy transitions in the decay of the yrast SD band of ^{196}Pb which directly feed yrast states in the normal well. The data fix the excitation energy of the SD states, and a rotational model extrapolation indicates a bandhead excitation energy of 5.630(4) MeV. Comparison with the two other experimentally established excitation energies of SD states in even-even Pb isotopes confirm the predicted trend of increasing excitation energy with increasing neutron number. However, a more detailed comparison of the data with the available model predictions indicate that none correctly reproduces the excitation energies (or two particle separation energies). These data provide an excellent challenge for the new "beyond mean field" calculations.

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