# Study of the $\eta \mathbf{N}$ scattering amplitude through the associated photoproduction of $\phi$ - and $\eta$-mesons in the region of the $\mathrm{N}^{*}(1535)$ resonance 

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#### Abstract

The $\gamma p \rightarrow \phi \eta p$ reaction is studied in the kinematic region where the $\eta p$ final state originates dominantly from the decay of the $\mathrm{N}^{*}(1535)$ resonance. The threshold laboratory photon energy for this reaction (at the peak of the S11 resonance) is $E_{\gamma}^{L a b}=3 \mathrm{GeV}$. We will discuss it somewhat above threshold, at $E_{\gamma}^{L a b} \simeq 4-5 \mathrm{GeV}$, in order to reach lower (absolute) values of the squared 4-momentum transfer from the initial photon to the final $\phi$-meson. In these conditions, we expect the t-channel $\pi^{0}$ - and $\eta$-meson exchanges to drive the dynamics underlying the $\gamma p \rightarrow \phi \eta p$ process. The initial photon dissociates into the final $\phi$-meson and a virtual pseudoscalar meson ( $\pi^{0}$ or $\eta$ ). The virtual pseudoscalar meson scatters from the proton target to produce the final $\eta p$ state. The $\pi^{0} p \rightarrow \eta p$ and $\eta p \rightarrow \eta p$ amplitudes are derived in the framework of a coupled-channel effective field theory of meson-baryon scattering. We found the $\eta$-meson exchange to be largely dominant. The $\eta-\pi^{0}$ interference is of the order of $20-30 \%$. The sign of this term is not known and has a significant influence on the results. The $\pi N \rightarrow \eta N$ amplitude being largely determined by data on the $\pi^{-} p \rightarrow \eta n$ reaction, we found that the $\gamma p \rightarrow \phi \eta p$ reaction cross section is rather directly related to the $\eta$-nucleon scattering amplitude in the $\mathrm{N}^{*}(1535)$ resonance region. Accurate data on the $\gamma p \rightarrow \phi \eta p$ process would therefore put additional constraints on this still poorly known amplitude.


Key words: Meson photoproduction; eta-nucleon scattering; N* (1535)
PACS: 13.30.Eg;13.60.Le;13.60.Rj;14.20.Gk

## 1 Introduction

The structure of the $\eta$-nucleon scattering amplitude close to threshold ( $\sqrt{s}=$ 1.486 GeV ) is of much interest because it appears dominated by the presence of a baryon resonance slightly above threshold, the $\mathrm{N}^{*}{ }_{1 / 2^{-}}(1535)$. The width of the $\mathrm{N}^{*}(1535)$ is of the order of 150 MeV . Its two main decay channels are the $\mathrm{N} \pi(35-55 \%)$ and the $\mathrm{N} \eta(30-55 \%)$ final states [1].

The $\eta$-nucleon scattering length, $a_{\eta N}$, characterizes the behaviour of the $\eta$ nucleon interaction at threshold. It is a complex quantity whose real part is still poorly determined. We refer to the recent work of Green and Wycech [2] for a discussion of the range of values found in the literature (from $\sim 0.3$ to $\sim 1 \mathrm{fm})$. The main reason for the spread in values is that $a_{\eta N}$ is obtained indirectly and through model-dependent analyses of pion- and photon-induced $\eta$ production reactions. The $\eta$-nucleon scattering length is a key quantity to assess the possibility of forming $\eta$-nuclear quasi-bound states [3].

In this work, we propose to study the threshold behaviour of the $\eta$-nucleon scattering amplitude through the $\gamma p \rightarrow \phi \eta p$ reaction in the particular kinematics where the invariant mass of the $\eta p$ pair is close to the $\mathrm{N}^{*}(1535)$ mass. We tune the incident laboratory photon energy in order to reach low momentum transfers, i.e. a sufficiently small $\left|t_{\min }\right|$. At the $\phi$ threshold, $E_{\gamma}^{L a b}=3 \mathrm{GeV}$ and $\left|t_{m i n}\right|$ is $1.2 \mathrm{GeV}^{2}$. We will consider values of $E_{\gamma}^{\text {Lab }}$ ranging from 4 to 5 GeV . At $E_{\gamma}^{L a b}=4 \mathrm{GeV}$, $\left|t_{\text {min }}\right|$ is $0.38 \mathrm{GeV}^{2}$. At 5 GeV , it is $0.26 \mathrm{GeV}^{2}$. Our main argument is that the $\gamma p \rightarrow \phi \eta p$ process in these kinematics $(|t|<1$ $\mathrm{GeV}^{2}$ ) is dominated by the $\eta$ t-channel exchange and offers the possibility to test the $\eta$-nucleon scattering amplitude close to threshold. Both $\pi$ - and $\eta$ exchanges can contribute. The dominance of the $\eta$-exchange in the $t$-channel comes mainly from the property that the $\phi$-meson radiative decay probability to the $\eta \gamma$ channel is an order of magnitude larger than to the $\pi \gamma$ channel (despite the larger phase space available for the latter decay). This is ultimately related to the large $s \bar{s}$ content of the $\eta$-meson. The s-wave $\pi p \rightarrow \eta p$ amplitude is also significantly smaller than the $\eta p \rightarrow \eta p$ amplitude.

Our results on the $\gamma p \rightarrow \phi \eta p$ reaction are based on $\pi p \rightarrow \eta p$ and $\eta p \rightarrow \eta p$ scattering amplitudes obtained in the unitary coupled-channel model of Ref. [4]. These amplitudes reproduce a large set of pion-nucleon and photon-nucleon scattering data in the energy range $1.4<\sqrt{s}<1.8 \mathrm{GeV}$, in particular the pionand photon-induced $\eta$-meson production cross sections. The value obtained for $a_{\eta N}$ is $(1.03+\mathrm{i} 0.49) \mathrm{fm}$ [4], in close agreement with the findings of Ref. [2].

Our t-channel calculation of the $\gamma p \rightarrow \phi \eta p$ reaction cross section in the $\mathrm{N}^{*}(1535)$ region is described in Section 2. We discuss the $\eta$ - and $\pi^{0}$-exchanges and their interference. The latter is constructive or destructive depending on
the relative sign of the couplings constants $g_{\phi \pi \gamma}$ and $g_{\phi \eta \gamma}$ of the corresponding anomalous interaction Lagrangians. We display numerical results for the $\gamma p \rightarrow$ $\phi \eta p$ reaction cross section in Section 3. We show the expected t-distributions at $E_{\gamma}^{L a b}=4 \mathrm{GeV}$ and 5 GeV and emphasize the role of the double $\eta$-pole term and of the $\eta-\pi^{0}$ interference in these quantities. A few concluding remarks are given in Section 4.

## 2 The $\gamma p \rightarrow \phi \eta p$ reaction cross section in the $\mathbf{N}^{*}(1535)$ region

The t-channel $\pi$ - and $\eta$-exchange amplitudes contributing to the $\gamma p \rightarrow \phi \eta p$ process in the $\mathrm{N}^{*}(1535)$ region are shown in Figs. 1 and 2.


Fig. 1. Pion-exchange contribution to the $\gamma p \rightarrow \phi \eta p$ process.


Fig. 2. $\eta$-exchange contribution to the $\gamma p \rightarrow \phi \eta p$ process.
We calculate the cross section for the $\gamma p \rightarrow \phi \eta p$ reaction, assuming it is driven by the mechanisms displayed in Figs. 1 and 2. We have three contributions, associated with the $\pi$-exchange, the $\eta$-exchange and their interference
respectively. The t-channel dominance is clearly an hypothesis which can but be confirmed by measurements of differential cross sections. We note that recent data on the $\gamma p \rightarrow \phi p$ reaction at $E_{\gamma}^{L a b}=2.4 \mathrm{GeV}[5,6]$ show that this process is dominated by t-channel exchanges from threshold onwards. The exact nature of these exchanges is however not clear [5,6]. It is therefore of interest to study a $\phi$-photoproduction process in which the excitation of the target to the $\mathrm{N}^{*}{ }_{1 / 2^{-}}(1535)$ is expected to favour specifically unnatural parity exchanges, i.e. the $\pi$ and $\eta$ t-channel contributions.

The 4 -momenta of the photon, the proton, the $\phi$, the $\eta$ and the final proton are denoted by $q, p, \bar{q}_{\phi}, \bar{q}_{\eta}$ and $\bar{p}$. The photon, initial proton and final proton polarizations are indicated by the symbols $\lambda_{\gamma}, \lambda$ and $\bar{\lambda}$. The total cross section reads

$$
\begin{align*}
\sigma_{\gamma p \rightarrow \phi \eta p}= & \frac{1}{\left|\vec{v}_{\gamma}-\vec{v}_{p}\right|} \frac{1}{2 q^{0}} \frac{m_{p}}{p^{0}} \int \frac{d^{3} \overrightarrow{\bar{q}}_{\phi}}{(2 \pi)^{3}} \frac{1}{2 \bar{q}_{\phi}^{0}} \int \frac{d^{3} \overrightarrow{\bar{q}}_{\eta}}{(2 \pi)^{3}} \frac{1}{2 \bar{q}_{\eta}^{0}} \int \frac{d^{3} \overrightarrow{\vec{p}}}{(2 \pi)^{3}} \frac{m_{p}}{\bar{p}^{0}} \\
& \times(2 \pi)^{4} \delta^{4}\left(q+p-\bar{q}_{\phi}-\bar{q}_{\eta}-\bar{p}\right) \sum_{\lambda_{\gamma}, \lambda, \bar{\lambda}_{\phi}, \bar{\lambda}} \frac{1}{4}\left|M_{\gamma p \rightarrow \phi \eta p}\right|^{2} . \tag{1}
\end{align*}
$$

The photon- $\phi$-pseudoscalar meson vertices are described by the anomalous interaction Lagrangian

$$
\begin{equation*}
\mathcal{L}_{\phi \chi \gamma}^{i n t}=e \frac{g_{\phi \chi \gamma}}{2 m_{\phi}} \varepsilon^{\mu \nu \alpha \beta} \phi_{\mu}\left(\partial_{\nu} \chi\right) F_{\alpha \beta}, \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
F_{\alpha \beta}=\partial_{\alpha} A_{\beta}-\partial_{\beta} A_{\alpha} \tag{3}
\end{equation*}
$$

and $\chi$ denotes the chiral pseudoscalar mesons of interest, the pion or the $\eta$-meson. Using this interaction Lagrangian to calculate the $\phi \rightarrow \pi^{0} \gamma$ and $\phi \rightarrow \eta \gamma$ partial widths, we have

$$
\begin{equation*}
\Gamma_{\phi \rightarrow \chi \gamma}=g_{\phi \chi \gamma}^{2} \frac{\alpha}{24} m_{\phi}\left(1-\frac{m_{\chi}^{2}}{m_{\phi}^{2}}\right)^{3} . \tag{4}
\end{equation*}
$$

Identifying this expression with the experimental decay widths [1]

$$
\begin{align*}
& \Gamma_{\phi \pi^{0} \gamma}=(5.24 \pm 0.49) \mathrm{keV},  \tag{5}\\
& \Gamma_{\phi \eta \gamma}=(55.17 \pm 1.71) \mathrm{keV}, \tag{6}
\end{align*}
$$

we obtain

$$
\begin{equation*}
\left|g_{\phi \pi \gamma}\right| \simeq 0.13, \quad\left|g_{\phi \eta \gamma}\right| \simeq 0.70 \tag{7}
\end{equation*}
$$

If we want to compute the interference between the $\pi$ - and $\eta$-exchanges, we should know the relative sign of the coupling constants $g_{\phi \pi \gamma}$ and $g_{\phi \eta \gamma}$. It is generally assumed that the $\phi \rightarrow \pi \gamma$ transition is dominated by the $\phi-\omega$ mixing. The $\phi-\omega$ mixing angle is not easy to determine because it depends on the overlap of two narrow resonances far apart from each other. It is therefore model-dependent and sensitive to the assumptions used to calculate the $\omega$ meson self-energy far from the $\omega$ pole. The $\phi \rightarrow \eta \gamma$ transition depends on the $\eta-\eta^{\prime}$ mixing, a phenomenon closely connected with the $\mathrm{U}(1)$ anomaly of QCD and still under analysis in different schemes [7]. Based on recent analyses of the available data, it is most often found that both $g_{\phi \pi \gamma}$ and $g_{\phi \eta \gamma}$ have the same sign as $g_{\omega \pi \gamma}$, and hence a positive relative sign [8,9]. The opposite conclusion has also been reached [10]. In view of the uncertainties in the $\mathrm{SU}(3)$ symmetry breaking mechanisms responsible for the mixing angles, we consider this relative sign as yet undetermined.

We will now establish the dominance of the $\eta$-exchange process in the $\gamma p \rightarrow$ $\phi \eta p$ reaction in the kinematics of interest ( $E_{\gamma}^{L a b} \simeq 4-5 \mathrm{GeV}$ and $\left|t_{\text {min }}\right|<$ $|t| \lesssim 1 \mathrm{GeV}^{2}$ ). Using the interaction Lagrangian (2), the squared amplitudes corresponding to the $\pi$-exchange, the $\eta$-exchange and their interference are

$$
\begin{gather*}
\sum_{\lambda_{\gamma}, \lambda, \overline{\lambda_{\phi}}, \bar{\lambda}} \frac{1}{4}\left|M_{\gamma p \rightarrow \phi \eta p}^{\pi-\text { exchange }}\right|^{2}=\frac{e^{2} g_{\phi \pi \gamma}^{2}}{4 m_{\phi}^{2}} \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\pi}^{2}\right)^{2}} \frac{1}{2} \sum_{\lambda, \bar{\lambda}}\left|M_{\pi p \rightarrow \eta p}\right|^{2},  \tag{8}\\
\sum_{\lambda_{\gamma}, \lambda, \bar{\lambda}_{\phi}, \bar{\lambda}} \frac{1}{4}\left|M_{\gamma p \rightarrow \phi \eta p}^{\eta-\text { exchange }}\right|^{2}=\frac{e^{2} g_{\phi \eta \gamma}^{2}}{4 m_{\phi}^{2}} \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\eta}^{2}\right)^{2}} \frac{1}{2} \sum_{\lambda, \bar{\lambda}}\left|M_{\eta p \rightarrow \eta p}\right|^{2},  \tag{9}\\
\sum_{\lambda_{\gamma}, \lambda, \bar{\lambda}_{\phi}, \bar{\lambda}} \frac{1}{4}\left|M_{\gamma p \rightarrow \phi \eta p}^{\text {interference }}\right|^{2}=\frac{e^{2} g_{\phi \pi \gamma} g_{\phi \eta \gamma}}{4 m_{\phi}^{2}} \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\pi}^{2}\right)\left(t-m_{\eta}^{2}\right)} \\
\frac{1}{2} \sum_{\lambda, \bar{\lambda}}\left(M_{\pi p \rightarrow \eta p}^{+} M_{\eta p \rightarrow \eta p}+M_{\eta p \rightarrow \eta p}^{+} M_{\pi p \rightarrow \eta p}\right) . \tag{10}
\end{gather*}
$$

We work in the photon-proton center of mass reference frame where the total energy of the reaction is denoted by $\sqrt{s}$. In that reference frame, the photon 3 -momentum is $-\vec{p}$ and the 3 -momentum of the $\eta p$ pair is $-\vec{q}_{\phi}$. We define the invariant mass $\sqrt{\bar{w}^{2}}$ of the final $\eta p$ pair by

$$
\begin{equation*}
\bar{w}^{2}=\left(p+q-\bar{q}_{\phi}\right)^{2}=s+m_{\phi}^{2}-2 \sqrt{s} \sqrt{m_{\phi}^{2}+\vec{q}_{\phi}^{2}} \tag{11}
\end{equation*}
$$

and express the 4-momentum transfer $t=\left(q-\bar{q}_{\phi}\right)^{2}$ as function of that variable,

$$
\begin{align*}
t\left(s, \bar{w}^{2}, \cos \theta\right)=m_{\phi}^{2}-\frac{1}{2 s}(s- & \left.m_{p}^{2}\right)\left(s+m_{\phi}^{2}-\bar{w}^{2}\right) \\
& \times\left(1-\sqrt{1-\frac{4 m_{\phi}^{2} s}{\left(s+m_{\phi}^{2}-\bar{w}^{2}\right)^{2}}} \cos \theta\right) \tag{12}
\end{align*}
$$

where $\theta$ is the angle between the initial photon and the produced $\phi$-meson. We define the notation $t_{-}\left(s, \bar{w}^{2}\right) \equiv t_{\min }\left(s, \bar{w}^{2}\right)=t\left(s, \bar{w}^{2}, \cos \theta=+1\right)$ and $t_{+}\left(s, \bar{w}^{2}\right)=t\left(s, \bar{w}^{2}, \cos \theta=-1\right)$.

Using the above variables, the differential cross section for the $\gamma p \rightarrow \phi \eta p$ reaction with respect to $t$ and $\bar{w}^{2}$ is given by

$$
\begin{align*}
\frac{d \sigma_{\gamma p \rightarrow \phi \eta p}}{d t d \bar{w}^{2}}= & \frac{\alpha m_{p}}{16 m_{\phi}^{2} s|\vec{p}|^{2}} \int \frac{d^{3} \overrightarrow{\underline{q}}_{\eta}}{(2 \pi)^{3}} \frac{1}{2 \bar{q}_{\eta}^{0}} \int \frac{d^{3} \vec{p}}{(2 \pi)^{3}} \frac{m_{p}}{\bar{p}^{0}} \\
& \times(2 \pi)^{4} \delta^{4}\left(q+p-\bar{q}_{\phi}-\bar{q}_{\eta}-\bar{p}\right) \\
& \times\left\{\frac{g_{\phi \pi \gamma}^{2}}{4 \pi} \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\pi}^{2}\right)^{2}} \sum_{\lambda, \bar{\lambda}} \frac{1}{2}\left|M_{\pi p \rightarrow \eta p}\right|^{2}\right. \\
& +\frac{g_{\phi \eta \gamma}^{2}}{4 \pi} \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\eta}^{2}\right)^{2}} \sum_{\lambda, \bar{\lambda}} \frac{1}{2}\left|M_{\eta p \rightarrow \eta p}\right|^{2} \\
& +\frac{g_{\phi \pi \gamma} g_{\phi \eta \gamma}}{4 \pi} \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\pi}^{2}\right)\left(t-m_{\eta}^{2}\right)} \\
& \left.\times \frac{1}{2} \sum_{\lambda, \bar{\lambda}}\left(M_{\pi p \rightarrow \eta p}^{+} M_{\eta p \rightarrow \eta p}+M_{\eta p \rightarrow \eta p}^{+} M_{\pi p \rightarrow \eta p}\right)\right\} \tag{13}
\end{align*}
$$

The total cross section is obtained by integrating over $t$ from $t_{+}$to $t_{-}$and over $\bar{w}^{2}$ from $\left(m_{p}+m_{\eta}\right)^{2}$ to $\left(\sqrt{s}-m_{\phi}\right)^{2}$. In all the results presented in the next section, we will restrict the interval of values of $t$ from $t=-1 \mathrm{GeV}^{2}$ to $t_{\text {min }}$, the expected range of validity of our model. Because $t_{+}$extends to much lower values (for $\bar{w}=1.54 \mathrm{GeV}, t_{+}=-5.8 \mathrm{GeV}^{2}$ at $E_{\gamma}^{L a b}=5 \mathrm{GeV}$ for example), we will refrain from displaying integrated cross sections.

We note that Eq. (13) is gauge-invariant because of the specific form of the interaction Lagrangian (2).

It is of interest to study the pole structure of the three contributions to the cross section ( $\pi^{0}$-exchange, $\eta$-exchange, interference) as it determines the shape of the t-distributions. We have the following decompositions:

$$
\begin{align*}
& \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\pi}^{2}\right)^{2}}=\frac{\left(m_{\phi}^{2}-m_{\pi}^{2}\right)^{2}}{\left(t-m_{\pi}^{2}\right)^{2}}-2 \frac{m_{\phi}^{2}-m_{\pi}^{2}}{t-m_{\pi}^{2}}+1  \tag{14}\\
& \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\eta}^{2}\right)^{2}}=\frac{\left(m_{\phi}^{2}-m_{\eta}^{2}\right)^{2}}{\left(t-m_{\eta}^{2}\right)^{2}}-2 \frac{m_{\phi}^{2}-m_{\eta}^{2}}{t-m_{\eta}^{2}}+1  \tag{15}\\
& \frac{\left(m_{\phi}^{2}-t\right)^{2}}{\left(t-m_{\eta}^{2}\right)\left(t-m_{\pi}^{2}\right)}=\frac{\left(m_{\phi}^{2}-m_{\eta}^{2}\right)^{2}}{t-m_{\eta}^{2}} \frac{1}{m_{\eta}^{2}-m_{\pi}^{2}} \\
& \quad-\frac{\left(m_{\phi}^{2}-m_{\pi}^{2}\right)^{2}}{t-m_{\pi}^{2}} \frac{1}{m_{\eta}^{2}-m_{\pi}^{2}}+1 \tag{16}
\end{align*}
$$

Simple effects can be understood from these expressions irrespectively of the dynamical aspects of the scattering amplitudes and of the strength of the $\gamma \chi \phi$ vertex. It is easy to see from Eq. (14) that the double pion pole term (proportional to $\frac{1}{\left(t-m_{\pi}^{2}\right)^{2}}$ ) dominates the pion-exchange contribution close to $t_{\text {min }}$, becomes comparable to the single pion pole term (proportional to $\frac{1}{\left(t-m_{\pi}^{2}\right)}$ ) around $t=-0.5 \mathrm{GeV}^{2}$ and smaller than the latter for larger values of $|t|$. In the case of the $\eta$-exchange displayed in Eq. (15), the single $\eta$ pole term is always dominant in the kinematic range under consideration. Close to $t_{\text {min }}$, the double $\eta$ pole term represents typically $25-30 \%$ of the total $\eta$-exchange contribution. It is nevertheless instrumental in producing a rather sharp drop in the differential cross section. In the interference term (16), the t-dependence is largely given by the single pion pole term at low $|t|$.

To derive Eq. (13), we have assumed that the $\gamma \eta \phi$ transition form factor is one. We have no information on that quantity but it influences significantly the outcome of our calculation. If the form factor is hard, its effect will be rather small. If it is soft, it could affect substantially the t-dependence of the $\gamma p \rightarrow \phi \eta p$ reaction. The only argument we see in favour of a hard form factor is that there is no obvious intermediate state to build a form factor in the $\eta$ direction. The $\eta$-meson couples dominantly to two photons and to three pions. To construct a form factor in the $\eta$-channel, one would need a significant decay of the $\phi$-meson into a photon and two vector particles or into a photon and three pions. The only available information is an upper limit of $510^{-4}$ on the branching ratio of the $\phi$-meson to the $\rho \gamma \gamma$ channel [1]. In the absence of more significant data, it is not possible to gain a reasonable understanding of the $\gamma \eta \phi$ form factor for space-like $\eta$-mesons. We will therefore set it to one, keeping this assumption in mind.

Finally, if our model takes into account the $\eta$-nucleon final state interaction to all orders, it does not treat $\phi$-nucleon rescattering in the outgoing channel. We do not expect this rescattering to be very important in the kinematics under consideration, i.e. with a large relative momentum between the $\phi$-meson emitted at small angles and the recoiling target products. In particular, we are not in the threshold regime where cryptoexotic $B_{\phi}$ baryons or $\phi N$ resonances
could enlarge final state interactions [11].

## 3 Numerical results for the $\gamma p \rightarrow \phi \eta p$ reaction

We proceed to the calculation of the $\gamma p \rightarrow \phi \eta p$ cross section as outlined in the previous section using the $\pi p \rightarrow \eta p$ and $\eta p \rightarrow \eta p$ scattering amplitudes obtained in the model of Ref. [4]. These amplitudes are displayed in Fig. 3 and 4.


Fig. 3. Real and imaginary parts of the s-wave $\pi \mathrm{N} \rightarrow \eta \mathrm{N}$ scattering amplitude (from Ref. [4]).

We notice that the $\pi \mathrm{N} \rightarrow \eta \mathrm{N}$ amplitude is about two to three times smaller than the $\eta \mathrm{N}$ scattering amplitude in the region of the $\mathrm{N}^{*}(1535)$ resonance. Combined with the very unfavourable ratio $\left|g_{\phi \pi \gamma}\right|^{2} /\left|g_{\phi \eta \gamma}\right|^{2} \approx 1 / 29$ of the anomalous coupling constants, this effect suggests that the pion contribution to the $\gamma p \rightarrow \phi \eta p$ cross section will be much smaller, typically by two orders of magnitude, than the $\eta$ contribution.

It is interesting to remark that the amplitude displayed in Fig. 4 is very similar to the corresponding quantity obtained in Ref. [2], suggesting that our calculation does not depend too much on the specific model used to derive the amplitudes. We emphasize also that the real part of the $\eta \mathrm{N}$ scattering


Fig. 4. Real and imaginary parts of the s-wave $\eta \mathrm{N}$ scattering amplitude (from Ref. [4]).
amplitude is mostly visible in a narrow band of total center of mass energies, i.e. from threshold until 1.5 GeV . Beyond $\sqrt{s}=1.52$, the cross section will be dominated by the imaginary part of the amplitude.

Consequently we will show results for two values of the invariant mass of the $\eta \mathrm{N}$ pair, $\bar{w}=1.49 \mathrm{GeV}$ (very close to threshold) and $\bar{w}=1.54 \mathrm{GeV}$ (at the resonance peak).

We consider first $\bar{w}=1.49 \mathrm{GeV}$ and display $d \sigma_{\gamma p \rightarrow \phi \eta_{p}} / d t d \bar{w}$ at $E_{\gamma}^{L a b}=4 \mathrm{GeV}$ and $E_{\gamma}^{\text {Lab }}=5 \mathrm{GeV}$. This is shown in Figs. 5 and 6 respectively.

As discussed earlier, we do not know the relative sign of the $g_{\phi \pi \gamma}$ and $g_{\phi \eta \gamma}$ coupling constants. We have therefore considered both signs. If the coupling constants are of the same sign, the $\eta-\pi^{0}$ interference is destructive because of the opposite sign of the amplitudes (solid curve). If $g_{\phi \pi \gamma}$ and $g_{\phi \phi \gamma}$ are of opposite sign, the $\eta-\pi^{0}$ interference is constructive (dashed curve). In order to evaluate the importance of this interference, we show in addition the $\eta$-exchange contribution (dot-dashed line) and the $\pi^{0}$-exchange contribution (dotted line). Close to $t_{\text {min }}$, the $\eta-\pi^{0}$ interference suppresses or enhances the differential cross section by about $30 \%$. As anticipated, the $\pi^{0}$-exchange contribution is completely negligible. We plot also the double pole contribution to the $\eta$-exchange (dot-double-dashed line). It represents typically a quarter of the $\eta$-exchange term but contributes very significantly to the t-dependence of the differential cross section.

If the $\eta-\pi^{0}$ interference is constructive, the pole structure of the $\eta$-exchange and of the interference leads to a rather sharp t -dependence close to $t_{\min }$. This


Fig. 5. Differential cross section $d \sigma_{\gamma p \rightarrow \phi \eta p} / d t d \bar{w}$ at $E_{\gamma}^{L a b}=4 \mathrm{GeV}$ for a total center of mass energy of the $\eta \mathrm{N}$ pair of 1.49 GeV . The full and dashed lines are the total differential cross sections assuming a destructive and a constructive $\pi^{0}-\eta$ interference respectively. The dot-dashed line is the full contribution of the $\eta$-meson exchange while the dot-double-dashed line shows the double $\eta$-pole contribution. The dotted line is the $\pi^{0}$-exchange contribution.


Fig. 6. Same as Fig. 5 at 5 GeV .
effect increases with increasing laboratory photon energy as a lower $\left|t_{\text {min }}\right|$ can be reached. If the $\eta-\pi^{0}$ interference is destructive, the terms driving the
increase of the differential cross section at low $|t|$ cancel significantly, leading to a rather flat behaviour.

We show in Figs. 7 and 8 the t-distributions for the total $\eta \mathrm{N}$ center of mass energy $\bar{w}=1.54 \mathrm{GeV}$, taken to be close to the $\mathrm{N}^{*}(1535)$ mass. The comparison to the results obtained at $\bar{w}=1.49 \mathrm{GeV}$ for the same value of the incident photon laboratory energy shows that the cross section increases. This is a consequence of the opening of the $\eta \mathrm{N}$ phase space and of the presence of the $\mathrm{N}^{*}(1535)$ resonance (implying a large imaginary part in the amplitude). Otherwise the features of the differential cross section $d \sigma_{\gamma p \rightarrow \phi \eta p} / d t d \bar{w}$ are very similar for both values of $\bar{w}$.


Fig. 7. Differential cross section $d \sigma_{\gamma p \rightarrow \phi \eta p} / d t d \bar{w}$ at $E_{\gamma}^{L a b}=4 \mathrm{GeV}$ for a total center of mass energy of the $\eta \mathrm{N}$ pair of 1.54 GeV . The full and dashed lines are the total differential cross sections assuming a destructive and a constructive $\pi^{0}-\eta$ interference respectively. The dot-dashed line is the full contribution of the $\eta$-meson exchange while the dot-double-dashed line shows the double $\eta$-pole contribution. The dotted line is the $\pi^{0}$-exchange contribution.

We emphazise however that the differential cross section $d \sigma_{\gamma p \rightarrow \phi \eta p} / d t d \bar{w}$ at $\bar{w}=1.49 \mathrm{GeV}$ and at $\bar{w}=1.54 \mathrm{GeV}$ tests different parts of the $\eta \mathrm{N}$ scattering amplitudes, the real and imaginary parts at threshold on the one hand and the imaginary part at the resonance peak on the other hand. We remark also that the $\pi N \rightarrow \eta N$ scattering amplitude is very much constrained by data on the $\pi^{-} p \rightarrow \eta n$ cross section close to threshold [2,4]. If our model is correct, accurate data on the $d \sigma_{\gamma p \rightarrow \phi \eta p} / d t d \bar{w}$ reaction at low $|t|$ could therefore be interpreted in terms of the $\eta N$ scattering amplitude. We have underlined two uncertainties in such an analysis, the relative sign of the $g_{\phi \pi \gamma}$ and $g_{\phi \eta \gamma}$ coupling constants and a possible effect due to a $\gamma \eta \phi$ transition form factor for space-

$$
\gamma \mathrm{p} \rightarrow \phi \eta \mathrm{p} \quad \mathrm{E}_{\gamma}^{\mathrm{Lab}}=5 \mathrm{GeV} \quad \overline{\mathrm{w}}=1.54 \mathrm{GeV}
$$



Fig. 8. Same as Fig. 7 at $E_{\gamma}^{L a b}=5 \mathrm{GeV}$.
like $\eta$ 's.

## 4 Conclusion

We have studied the $\gamma p \rightarrow \phi \eta p$ reaction with the idea of using future accurate data on this process to gain understanding of the threshold behaviour of the $\eta$-nucleon scattering amplitude. We chose kinematic conditions where a t-channel meson-exchange description is expected to be valid (i.e. low momentum transfers). We showed that the process in which the initial photon dissociates into a $\phi$-meson and a virtual $\eta$-meson scattering from the proton target to produce an $\eta$ proton final state is dominant for ( $\eta \mathrm{p}$ ) pairs with invariant masses close to the $\mathrm{N}^{*}(1535)$ mass. It can however interfere with the analogous process where a $\pi^{0}$-meson is exchanged. The $\pi^{0}$-exchange term is negligible. The sign of the $\eta-\pi^{0}$ interference is at present not known. We show how it changes both the t-dependence and the absolute value of the differential cross section.

In view of the fact that the $\pi N \rightarrow \eta N$ scattering amplitude in the $\mathrm{N}^{*}(1535)$ resonance region is rather well-known, accurate data on the $\gamma p \rightarrow \phi \eta p$ reaction at $E_{\gamma}^{L a b}=4-5 \mathrm{GeV}$ and low t would put additional constraints on the still poorly controlled $\eta$-proton scattering amplitude close to threshold. The expected cross sections are small but do not appear out of reach of present experimental facilities.

## 5 Acknowledgement

One of us (M.S) is indebted to Slawomir Wycech for a very useful discussion and to the hospitality of the GSI Theory Group, where part of this work was done. We are thankful to our referees for comments which led us to significantly improve the contents of this paper. We acknowledge the support of the European Community-Research Infrastructure Activity under the FP6 "Structuring the European Research Area" programme (Hadron Physics, contract number RII3-CT-2004-506078).

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