Polarization effects in the reaction $\bar{p} + p \to e^+ + e^-$ in presence of two–photon exchange

G. I. Gakh *

National Science Centre "Kharkov Institute of Physics and Technology", 61108 Akademicheskaya 1, Kharkov, Ukraine

E. Tomasi–Gustafsson

DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France

Abstract

Polarization observables for the reaction $\bar{p}+p \to e^+ + e^-$ are given in terms of three independent complex amplitudes, in presence of two photon exchange. General expressions for the differential cross section and the polarization observables are given and model independent properties are derived. Polarization effects depending on the polarization of the antiproton beam, the target and of the electron in the final state, have been calculated.

PACS numbers: 12.20.-m, 13.40.-f, 13.60.-Hb, 13.88.+e

* e-mail: *gakh@kipt.kharkov.ua*

I. INTRODUCTION

The problem of the presence of the two–photon–exchange (TPE) contribution for elastic electron–proton scattering at relatively large momentum transfer is very actual. Intensive theoretical and experimental activity is under way, related, in particular, to the discrepancy between the experimental results on the proton electromagnetic form factors (FFs), extracted by different procedures, through the Rosenbluth fit [1] or from the polarization transfer method [2] (for a recent discussion see Ref. [3]).

Estimations of the TPE contribution to the elastic electron–deuteron scattering were firstly discussed in Refs. [4, 5] in the framework of the Glauber theory. It was shown [4] that this contribution decreases very slowly with momentum transfer squared q^2 and may dominate the cross section at high q^2 values. Since the TPE amplitude is essentially imaginary, the difference between positron and electron scattering cross sections depends upon the small real part of the TPE amplitude [4]. Recoil polarization effects may be substantial, in the region where the one– and two–photon–exchange contributions are comparable. If the TPE mechanism becomes sizeable, the straightforward extraction of FFs from the experimental data is no longer possible [4].

It is known that double scattering dominates in collisions of high–energy hadrons with deuterons at high q^2 values, and in Ref. [5] it was predicted that the TPE effect in elastic electron–deuteron scattering should represent 10% effect at $q^2 \cong 1.3$ GeV^2. At the same time the importance of the TPE mechanism was considered in Ref. [6]. The fact that the TPE mechanism, where the momentum transfer is shared between the two virtual photons, can become important with increasing q^2 value was already indicated more than thirty years ago [4–6].

This mechanism was never directly observed in an experiment, but recent measurements of the asymmetry in the scattering of transversely polarized electrons on unpolarized protons, give values different from zero, contrary to what is expected in the Born approximation [7, 8]. This observable is related to the imaginary part of the interference between one and two photon exchange and can be related only indirectly to the real part of the interference, which plays a role in the elastic ep cross section.

Measurements of the ratio of the electric to the magnetic proton FFs, G_E/G_M, have been performed at JLab in polarized ep elastic scattering, $\vec{e}+p \to e+\vec{p}$ [2]. The transverse,

P_t, and the longitudinal, P_l, components of the recoil proton polarization in the electron scattering plane are directly related to the ratio of the electromagnetic proton FFs. This method, firstly suggested in Ref. [9], could be applied only recently, due to the availability of high intensity, high polarized electron beams, hadron polarimeters in the GeV range and large acceptance spectrometers.

The data [2] have been obtained in the region 0.3 GeV$^2 \le Q^2 \le$ 5.6 GeV^2, and reveal a remarkable fall of the ratio G_E/G_M when Q^2 increases, in disagreement with the data obtained by the Rosenbluth technique, which show that this ratio is constant.

In Ref. [10] it has been shown, on the basis of a VMD inspired model taking into account ten resonances, that the polarization data [2] may be consistent with all known FF properties, including also QCD asymptotics and that G_E will vanish around q^2=-15 GeV^2. A zero, and eventually negative values of G_E, if confirmed by the planned experiment [11], will seriously constrain the nucleon models.

From the theoretical point of view, it seems unavoidable to consider the problem of the TPE contribution in the $\bar{p}+p \to e^+ + e^-$ reaction. The process $\bar{p}+p \to e^+ + e^-$ and its crossing channel, $e+p \to e+p$, must have common mechanisms. The process $\bar{p}+p \to e^+ + e^-$ is very convenient to study the polarization effects induced by collisions of polarized protons and antiprotons, but the measurement of the final–lepton polarization cannot be considered as a realizable experiment. However, for completeness, we will also calculate observables related to the final electron polarization.

The TPE contribution in the $\bar{p}+p \to e^+ + e^-$ reaction results, first of all, in a nonlocal spin structure of the matrix element. This makes the analysis of polarization effects more complicated with respect to the case of the one–photon–exchange mechanism. Such analysis can be done similarly to the case of hadronic reactions among spin 1/2 particles, such as, for example, $n+p \to n+p$ scattering [12].

At our knowledge, the annihilation reaction $\bar{p}+p \to \ell^+ + \ell^-$, $\ell = e$ or μ was firstly considered in Ref. [13] in the case of unpolarized particles, where the differential cross section was calculated both in the center of mass (CMS) and in the laboratory (Lab) systems. As already mentioned, if nucleon FFs decrease rapidly in time–like region, then, just as in space–like region, it is possible that the TPE mechanism becomes important.

The general case of polarized initial particles (antiproton beam or/and proton target) in $\bar{p}+p \to e^+ + e^-$ has been firstly investigated in Ref. [14], with particular attention to the determination of the phases of FFs, and more recently in Ref. [15]. The relations between the measurable asymmetries in terms of the electromagnetic FFs, G_M and G_E, in the time–like region were derived, assuming one photon exchange.

In this paper we consider the reaction

$$\bar{p} + p \to e^+ + e^-. \tag{1}$$

We derive here the expressions for the differential cross section and various polarization observables for the case when the matrix element contains the TPE contribution. The parametrization of the TPE term is done following the analytic continuation to the time–like region of the approach used (in the space–like region) in Refs. [16–20]. Using some approximations or in framework of a model, it was shown TPE could account, at least partially, for the apparent discrepancy between the Rosenbluth and the polarization transfer methods.

Another approach is taken in Refs. [17–19], where the purpose is to derive general expressions for the polarization observables in the elastic electron–nucleon scattering and to suggest model independent methods to extract nucleon electromagnetic FFs even in presence of the TPE contribution (parametrized in the tensor [17] or axial [18] forms), without underlying assumptions. We use the tensor form of the TPE contribution parametrization, and follow the approach of Ref. [18].

II. DIFFERENTIAL CROSS SECTION

Let us consider the process (1) in the general case of polarized beam and target and measuring the polarization of the outgoing electron. The starting point of our analysis of the reaction (1) is the following general parametrization of the spin structure of the matrix element for this reaction, according to the approach used in Refs. [16–20]

$$M = -\frac{e^2}{q^2} j_\mu J_\mu, \text{ with } j_\mu = \bar{u}(k_2)\gamma_\mu u(-k_1), \tag{2}$$

and

$$J_\mu = \bar{u}(-p_2)[\tilde{G}_M(q^2,t)\gamma_\mu + \frac{P_\mu}{m}\tilde{F}_2(q^2,t) + \frac{P_\mu}{m^2}\hat{K}F_3(q^2,t)]u(p_1),$$

where $K = (k_1 - k_2)/2$, $P = (p_2 - p_1)/2$, p_1 (p_2) and k_1 (k_2) are the four–momenta of proton (antiproton) and positron (electron), respectively; $q^2 = (p_1 + p_2)^2$, $t = K \cdot P$, m is the nucleon mass, $\tilde{G}_M$, $\tilde{F}_2$ and F_3 are complex functions of two independent variables q^2 and t.

The spin structure of the matrix element of the electron–nucleon scattering (2) can be derived in analogy with the elastic neutron–proton scattering [12] assuming general properties of the electron–hadron interaction, such as P–invariance and relativistic invariance. Taking into account the identity of the initial and final states and the T–invariance of the electromagnetic interaction, the process of the electron– nucleon scattering is characterized by three invariant complex amplitudes (in the limit of zero electron mass). The spin structure of the matrix element for the $\bar{p}+p\to e^{+}+e^{-}$ reaction is obtained from the matrix element of the elastic electron–nucleon scattering by analytic continuation.

In the Born (one–photon–exchange) approximation we have

$$\tilde{G}_M^{Born}(q^2,t)=G_M(q^2),\ \tilde{F}_2^{Born}(q^2,t)=F_2(q^2),\ F_3^{Born}(q^2,t)=0, \tag{3}$$

where $G_M(q^2)$ and $F_2(q^2)$ are the magnetic and Pauli proton electromagnetic FFs, respectively, which are complex functions of the variable q^2. The complex nature of FFs in time-like region is due to the strong interaction between proton and antiproton in the initial state.

In the following we use the Sachs magnetic $G_M(q^2)$ and charge $G_E(q^2)$ proton FFs which are related to the Dirac proton FF $F_1(q^2)$ and to $F_2(q^2)$ as follows

$$G_M=F_1+F_2,\ G_E=F_1+\tau F_2,\ \tau=\frac{q^2}{4m^2}. \tag{4}$$

To disentangle the effects of the Born and TPE contributions, let us single out the dominant contribution and define the following decompositions of the amplitudes:

$$\tilde{G}_M(q^2,t)=G_M(q^2)+\Delta G_M(q^2,t),\ \tilde{F}_2(q^2,t)=F_2(q^2)+\Delta F_2(q^2,t). \tag{5}$$

Instead of the amplitude $\tilde{F}_2$ we use the linear combination

$$\tilde{G}_E(q^2,t)=G_E(q^2)+\Delta G_E(q^2,t). \tag{6}$$

We neglect below the bilinear combinations of the terms ΔG_M, ΔG_E and F_3 since they are smaller (at least of the order of α), in comparison with the dominant ones.

Then the differential cross section of the reaction (1) can be written in CMS as follows:

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4q^6}\frac{E}{p}L_{\mu\nu}H_{\mu\nu},\ L_{\mu\nu}=j_\mu j_\nu^*,\ H_{\mu\nu}=J_\mu J_\nu^*, \tag{7}$$

where $E(p)$ is the energy (momentum) of the antiproton. In the case of longitudinally polarized electrons the leptonic tensor has the form

$$L_{\mu\nu}=-q^2g_{\mu\nu}+2(k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu})+2i<\mu\nu qk_2>, \tag{8}$$

where $< \mu\nu ab >= \varepsilon_{\mu\nu\rho\sigma}a_\rho b_\sigma$. Other components of the electron polarization lead to a suppression by a factor m_e/m.

Taking into account the polarization states of the beam and target, the hadronic tensor can be written as the sum of three tensors as follows:

$$H_{\mu\nu} = H^{(0)}_{\mu\nu} + H^{(1)}_{\mu\nu} + H^{(2)}_{\mu\nu}, \qquad (9)$$

where the tensor $H^{(0)}_{\mu\nu}$ corresponds to the unpolarized beam and target, the tensor $H^{(1)}_{\mu\nu}$ describes the production of e^+e^- by polarized beam or target and the tensor $H^{(2)}_{\mu\nu}$ corresponds to polarized beam and polarized target.

Since the presence of the TPE contribution leads to the term of the hadronic current which contains the momenta from the leptonic vertex, the general structure of the $H^{(0)}_{\mu\nu}$ tensor becomes more complicated: instead of the two standard structure functions we have five ones (as in the case of $\gamma^* d \to np$ or $\gamma^* N \to \pi N$). So, the general structure of this tensor can be written as

$$H^{(0)}_{\mu\nu} = H_1\tilde{g}_{\mu\nu} + H_2P_\mu P_\nu + H_3K_\mu K_\nu + H_4(K_\mu P_\nu + K_\nu P_\mu) + iH_5(K_\mu P_\nu - K_\nu P_\mu), \qquad (10)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2$. One gets the following expressions for these structure functions for the case of the hadronic current given by Eq. (2):

$$\begin{aligned}
H_1 &= -2q^2(|G_M|^2 + 2ReG_M\Delta G_M^*),\\
H_2 &= \frac{8}{\tau-1}\Big[|G_E|^2 - \tau|G_M|^2 + 2ReG_E\Delta G_E^* - 2\tau ReG_M\Delta G_M^* + \\
&\quad 2\sqrt{\tau(\tau-1)}\cos\theta Re(G_E-\tau G_M)F_3^*\Big],\\
H_3 &= 0,\ H_4 = -8\tau ReG_MF_3^*,\ H_5 = -8\tau ImG_MF_3^*,
\end{aligned} \qquad (11)$$

where θ is the angle between the electron and the antiproton momenta in the $\bar{p}+p \to e^+ + e^-$ reaction CMS. One can see that the structure functions H_4 and H_5 are completely determined by the TPE terms: in the absence of these terms we have the standard tensor structure for $H^{(0)}_{\mu\nu}$.

The differential cross section of the reaction (1) for the case of unpolarized particles has the form:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4q^2}\sqrt{\frac{\tau}{\tau-1}}D, \qquad (12)$$

$$
\begin{aligned}
D &= (1+\cos^2\theta)(|G_M|^2 + 2ReG_M\Delta G_M^*) + \frac{1}{\tau}\sin^2\theta(|G_E|^2 + 2ReG_E\Delta G_E^*) + \\
& 2\sqrt{\tau(\tau-1)}\cos\theta\sin^2\theta Re(\frac{1}{\tau}G_E - G_M)F_3^*.
\end{aligned}
$$

One can see that in the Born approximation the expression (12) reduces to the result obtained in Refs. [14, 15]. The contribution of the one–photon–exchange diagram leads to an even function of $\cos\theta$, whereas the TPE contribution leads to four new terms of the order of α compared to the dominant contribution.

At the reaction threshold where $q^2 = 4m^2$, one gets $G_M = G_E$ and the differential cross section becomes $\theta-$independent in the Born approximation. This is not anymore true in presence of TPE terms.

As it was pointed out in Ref. [21], for the processes of the type $e^+ + e^- \to h^+ + h^-$, except in particular cases, the term of the cross section due to TPE is an odd function of the variable $\cos\theta$. Therefore, it does not contribute to the differential cross section for $\theta = 90^0$.

III. SINGLE SPIN POLARIZATION OBSERVABLES

Let us consider the case when the antiproton beam is polarized. Then, if the hadronic current is given by Eq. (2), the hadronic tensor $H_{\mu\nu}^{(1)}$ can be written as:

$$
\begin{aligned}
H_{\mu\nu}^{(1)} &= -\frac{2i}{m}\Big[m^2|\tilde{G}_M|^2 < \mu\nu q s_2 > +(\tau-1)^{-1}Re\tilde{G}_M(\tilde{G}_E - \tilde{G}_M)^*(< \mu p_1 p_2 s_2 > P_\nu - \\
& < \nu p_1 p_2 s_2 > P_\mu) + ReG_M F_3^*(< \mu k q s_2 > P_\nu - < \nu k q s_2 > P_\mu)\Big] + \\
& \frac{2}{m(\tau-1)}\Big[Im\tilde{G}_M\tilde{G}_E^*(< \mu p_1 p_2 s_2 > P_\nu + < \nu p_1 p_2 s_2 > P_\mu) + \\
& (\tau-1)ImG_M F_3^*(< \mu k q s_2 > P_\nu + < \nu k q s_2 > P_\mu) - \\
& \frac{2}{m^2} < s_2 p_2 p_1 k > Im(G_E - G_M)F_3^* P_\mu P_\nu\Big],
\end{aligned}
\tag{13}
$$

where $s_{2\mu}$ is the antiproton polarization four–vector ($p_2 \cdot s_2 = 0$).

Note that, unlike the elastic electron–nucleon scattering in the Born approximation, the hadronic tensor in the time–like region contains a symmetric part even in the Born approximation due to the fact that nucleon FFs are complex. Taking into account the TPE contribution leads to additional terms in the symmetric part of this tensor.

The polarization four–vector of a relativistic particle, s_μ, in a reference system where its momentum, $\vec{p}$, is connected with the polarization vector, $\vec{\chi}$, in its rest frame by a Lorentz boost is:

$$\vec{s} = \vec{\chi} + \frac{\vec{p}\cdot\vec{\chi}\vec{p}}{m(E+p)},\ s^0 = \frac{1}{m}\vec{p}\cdot\vec{s}.$$

Let us define a coordinate frame in CMS of the reaction (1), where the z axis is directed along the antiproton momentum $\vec{p}$, the y axis is directed along the vector $\vec{p}\times\vec{k}$, ($\vec{k}$ is the electron momentum), and the x axis forms a left–handed coordinate system. In this frame the components of the unit vectors are: $\hat{\vec{p}} = (0,0,1)$ and $\hat{\vec{k}} = (\sin\theta, 0, \cos\theta)$ with $\hat{\vec{p}}\cdot\hat{\vec{k}} = \cos\theta$.

The presence of a symmetrical part in the hadronic tensor (13) leads to a non–zero single–spin asymmetry which can be written as

$$A_y(\theta) = \frac{2\sin\theta}{\sqrt{\tau}D}\Big[\cos\theta\ Im(G_M G_E^* + G_M\Delta G_E^* - G_E\Delta G_M^*) + \tag{14}$$

$$+\sqrt{\tau(\tau-1)}Im(\cos^2\theta\ G_M + \sin^2\theta\ G_E)F_3^*\Big].$$

Again, in the Born approximation this expression reduces to the result of Ref. [14]. One can see that:

- $A_y(\theta)$ is determined by the spin vector component which is perpendicular to the reaction plane;

- $A_y(\theta)$, being a T–odd quantity, does not vanish even in the one–photon–exchange approximation due to the complex nature of the nucleon FFs in the time–like region. This is the principal difference with the elastic electron–nucleon scattering.

Let us consider two particular kinematical cases:

- when the electron is scattered at $\theta = 90^0$.

- the reaction threshold.

For $\theta = 90^0$, in the Born approximation $A_y(\theta)$ vanishes. The presence of the TPE contributions leads to a non–zero value of $A_y(\theta)$ at $\theta = 90^0$ and this value is given by a simple expression

$$A_y(90^0) = 2\frac{\sqrt{\tau-1}}{\overline{D}} Im G_E F_3^*,\quad \overline{D} = D(\theta = 90^0).$$

This quantity is expected to be small due to the fact that it is determined by the interference of the one–photon and two–photon exchange amplitudes and is of the order of α. One can see that this asymmetry is an increasing function of the variable q^2: this is due to the presence of the kinematical factor containing τ and to the steep decreasing of the nucleon FFs with q^2 while the TPE mechanism becomes more important when q^2 increases. So, the measurement of this asymmetry at $\theta = 90^0$ can give information about the TPE contribution and its behaviour as a function of q^2.

At threshold, in the Born approximation, $A_y^{th}(\theta)$ has to vanish, due to the relation $G_E = G_M$. Including the TPE contributions, the asymmetry becomes:

$$A_y^{th}(\theta) = \frac{\sin 2\theta}{D_{th}} Im G_M (\Delta G_E - \Delta G_M)^*.$$

Note that, at threshold, this asymmetry can be equal to zero, if $\Delta G_E = \Delta G_M$. In this case the differential cross section does not contain any explicit dependence on the angular variable θ, but only through the amplitudes $\Delta G_{E,M}$ which, in the general case, depend on the variable θ.

The importance of the TPE contributions in $A_y^{th}(\theta)$ at an arbitrary scattering angle will increase as q^2 increases. This is due to the presence of the kinematical factor containing τ and it is expected that the TPE amplitudes decrease more slowly with q^2 compared with the nucleon FFs.

The antisymmetrical part of the hadronic tensor $H_{\mu\nu}^{(0)}$ leads to another single–spin observable: the final electron gets a transverse polarization (orthogonal to the reaction plane) in the annihilation of unpolarized proton and antiproton. The expression for this polarization is:

$$P_y^{(e)}(\theta) = 2\frac{m_e}{m}\frac{\sqrt{\tau - 1}}{D} \sin\theta Im G_M F_3^*,$$

where m_e is the electron mass. One can see that

- $P_y^{(e)}$ has a T–odd nature, since it is determined by the imaginary part of the product of G_M and of the amplitude F_3.

- $P_y^{(e)}$ is entirely due to the TPE mechanism and it vanishes in the Born approximation.

- Since it is a transverse polarization, it is suppressed by a factor (m_e/m). The polarization for the case of production of $\mu^+\mu^-$-pair is essentially larger (m_μ/m_e=200) and for $\tau^+\tau^-$ production one finds no additional suppression. Another advantage of detecting heavy leptons is that the polarization of unstable particles (μ and τ) can be measured through the angular distribution of their decay products.

- $P_y^{(e)}$ vanishes at threshold, also in presence of TPE contribution.

- $P_y^{(e)}$ increases when q^2 becomes larger. The reasons are the same as for the asymmetry A_y (see the discussion above).

Let us consider now the polarization transfer when the antiproton beam is polarized and the polarization of the produced electron is measured. We consider only the longitudinal polarization of the final electron because in this case the suppression factor m_e/m is absent. The corresponding observables are:

$$
\begin{aligned}
A_x &= \frac{2\sin\theta}{\sqrt{\tau}D}\Big[ReG_M G_E^* + Re(G_M\Delta G_E^* + G_E\Delta G_M^*) + \sqrt{\tau(\tau-1)}\cos\theta ReG_M F_3^*\Big], \\
A_z &= \frac{2}{D}\Big[\cos\theta(|G_M|^2 + 2ReG_M\Delta G_M^*) - \sqrt{\tau(\tau-1)}\sin^2\theta ReG_M F_3^*\Big]. \qquad (15)
\end{aligned}
$$

These coefficients are T–even observables and they are nonzero in the Born approximation, and also for elastic electron–nucleon scattering. The coefficient A_z vanishes at $\theta = 90^0$ in the Born approximation. But the presence of the TPE term F_3 in the electromagnetic hadron current leads to a nonzero value of this quantity, driven by the term $ReG_M F_3^*$.

The expressions (15), in the one–photon–exchange approximation, coincide with the results for the polarization vector components of the nucleon in the $e^+ + e^- \to N + \bar{N}$ reaction, when the electron beam is longitudinally polarized [22, 23].

IV. DOUBLE SPIN POLARIZATION OBSERVABLES

Let us consider the case when the polarized antiproton beam annihilates with a polarized proton target. The corresponding hadronic tensor $H_{\mu\nu}^{(2)}$ can be written as:

$$
\begin{aligned}
H_{\mu\nu}^{(2)} = {} & C_1 g_{\mu\nu} + C_2 P_\mu P_\nu + C_3(P_\mu s_{1\nu} + P_\nu s_{1\mu}) + C_4(P_\mu s_{2\nu} + P_\nu s_{2\mu}) + \\
& C_5(s_{1\mu}s_{2\nu} + s_{1\nu}s_{2\mu}) + C_6(P_\mu K_\nu + P_\nu K_\mu) + iC_7(P_\mu s_{1\nu} - P_\nu s_{1\mu}) + \\
& iC_8(P_\mu s_{2\nu} - P_\nu s_{2\mu}) + iC_9(P_\mu K_\nu - P_\nu K_\mu), \qquad (16)
\end{aligned}
$$

where $s_{1\mu}$ is the proton polarization four-vector $(p_1 \cdot s_1 = 0)$ and the terms proportional to q_μ or q_ν were omitted, since they do not contribute to the cross section and to the polarization observables (due to the conservation of the leptonic current). The structure functions have the following form

$$
\begin{aligned}
C_1 &= \frac{1}{2}(q^2 s_1\cdot s_2 - 2q\cdot s_1 q\cdot s_2)|\tilde{G}_M|^2, \\
C_2 &= \frac{2}{\tau-1}\Big[\tau|\tilde{G}_M|^2 - |\tilde{G}_E|^2 + 2\frac{K\cdot P}{m^2}Re(\tau G_M - G_E)F_3^*\Big]s_1\cdot s_2 + \\
&\quad \frac{q\cdot s_1 q\cdot s_2}{m^2(\tau-1)^2}|\tilde{G}_E - \tilde{G}_M|^2 +
\end{aligned}
$$

$$
\begin{aligned}
& \frac{2}{m^2(\tau-1)}(q\cdot s_1 K\cdot s_2 - q\cdot s_2 K\cdot s_1) Re(G_E-\tau G_M)F_3^*,\\
C_3 &= ReE_1,\ C_4=ReE_2,\ C_6=ReE_3,\ C_5=-\frac{q^2}{2}|\tilde{G}_M|^2,\\
E_1 &= \frac{q\cdot s_2}{\tau-1}(\tau|\tilde{G}_M|^2-\tilde{G}_E\tilde{G}_M^*)+\frac{1}{2m^2}(2K\cdot Pq\cdot s_2-q^2K\cdot s_2)F_3G_M^*,\\
E_2 &= -\frac{q\cdot s_1}{\tau-1}(\tau|\tilde{G}_M|^2-\tilde{G}_E\tilde{G}_M^*)-\frac{1}{2m^2}(2K\cdot Pq\cdot s_1+q^2K\cdot s_1)F_3G_M^*,\\
E_3 &= \frac{1}{2m^2}(q^2s_1\cdot s_2-2q\cdot s_1 q\cdot s_2)F_3G_M^*,\\
C_7 &= ImE_1,\ C_8=ImE_2,\ C_9=ImE_3. \qquad (17)
\end{aligned}
$$

The non–zero spin correlation coefficients between the polarizations of beam and target (when the final leptons are unpolarized) can be written as:

$$
\begin{aligned}
D_{xx} &= \frac{\sin^2\theta}{D}\Big[|G_M|^2+2ReG_M\Delta G_M^*+\frac{1}{\tau}(|G_E|^2+2ReG_E\Delta G_E^*)+\\
&\quad 2\sqrt{\tau(\tau-1)}\cos\theta Re(G_M+\frac{1}{\tau}G_E)F_3^*\Big],\\
D_{yy} &= \frac{\sin^2\theta}{D}\Big[\frac{1}{\tau}(|G_E|^2+2ReG_E\Delta G_E^*)-|G_M|^2-2ReG_M\Delta G_M^*-\\
&\quad 2\sqrt{\tau(\tau-1)}\cos\theta Re(G_M-\frac{1}{\tau}G_E)F_3^*\Big],\\
D_{zz} &= \frac{1}{D}\Big[(1+\cos^2\theta)(|G_M|^2+2ReG_M\Delta G_M^*)-\frac{1}{\tau}\sin^2\theta(|G_E|^2+2ReG_E\Delta G_E^*)-\\
&\quad 2\sqrt{\tau(\tau-1)}\cos\theta\sin^2\theta Re(G_M+\frac{1}{\tau}G_E)F_3^*\Big],\\
D_{xz} &= D_{zx}=\frac{\sin 2\theta}{\sqrt{\tau}D}\Big[Re(G_MG_E^*+G_M\Delta G_E^*+G_E\Delta G_M^*)+\\
&\quad \sqrt{\tau(\tau-1)}\cos\theta Re(G_M-tan^2\theta G_E)F_3^*\Big]. \qquad (18)
\end{aligned}
$$

For completeness, we give here the nonzero coefficients for the case of a longitudinally polarized electron:

$$
\begin{aligned}
D_{xy}=D_{yx} &= \frac{1}{D}\sqrt{\tau(\tau-1)}\sin^2\theta ImG_MF_3^*,\\
D_{zy}=D_{yz} &= \frac{\sin\theta}{\sqrt{\tau}D}Im\Big(G_{Mp}G_{Ep}^*+G_{Mp}\Delta G_{Ep}^*-G_{Ep}\Delta G_{Mp}^*+\\
&\quad \sqrt{\tau(\tau-1)}\cos\theta G_{Mp}F_{3p}^*\Big). \qquad (19)
\end{aligned}
$$

One can see that:

- The coefficients D_{xx}, D_{yy}, D_{zz}, D_{xz}, and D_{zx} are T–even observables, whereas the coefficients D_{xy}, D_{yx}, D_{yz}, and D_{zy} are T–odd observables.

- In the Born approximation the expressions for the T–even correlation coefficients coincide with the results of Ref. [14]. The expressions for the T–odd ones coincide with the corresponding components of the polarization correlation tensor of the baryon B and the antibaryon $\bar{B}$ created through the one–photon–exchange mechanism in the $e^+e^- \to B\bar{B}$ process [22].

- The relative contribution of the interference terms (between one- and two–photon–exchange mechanisms) increases as q^2 becomes larger (see the discussion above).

At the reaction threshold the correlation coefficients have some specific properties:

- All correlation coefficients do not depend on the function F_3.

- In the Born approximation the $(D_{xx} + D_{yy} + D_{zz})$ observable does not depend on the θ variable, but the TPE contribution induces such dependence.

- In the Born approximation the D_{yy} observable is zero, but the inclusion of the TPE term leads to a nonzero value, determined by the quantity $ReG_M(\Delta G_E - \Delta G_M)^*$.

- The relation $D_{yy} + D_{zz} = 0$ holds for $\theta = 90^0$.

- All T–odd double-spin observables vanish.

Taking into account the P–invariance of the hadron electromagnetic interaction, we can write the following general formula for the differential cross section as a function of the polarizations of the proton, $(\vec{\xi}_1)$, of the antiproton $(\vec{\xi}_2)$ and of the longitudinal polarization of the produced lepton, (λ_e):

$$\begin{aligned}\frac{d\sigma}{d\Omega}(\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}) = & \left(\frac{d\sigma}{d\Omega}\right)_0 \Big\{1 + A_n \vec{n}\cdot\vec{\xi}_1 + \bar{A}_n \vec{n}\cdot\vec{\xi}_2 + P_n^{(e)} \vec{n}\cdot\vec{\xi} + D_{mm} \vec{m}\cdot\vec{\xi}_1 \vec{m}\cdot\vec{\xi}_2 + \\ & D_{nn} \vec{n}\cdot\vec{\xi}_1 \vec{n}\cdot\vec{\xi}_2 + D_{\ell\ell} \vec{\ell}\cdot\vec{\xi}_1 \vec{\ell}\cdot\vec{\xi}_2 + D_{m\ell} \vec{m}\cdot\vec{\xi}_1 \vec{\ell}\cdot\vec{\xi}_2 + D_{\ell m} \vec{\ell}\cdot\vec{\xi}_1 \vec{m}\cdot\vec{\xi}_2 + \\ & \lambda_e \Big[A_m \vec{m}\cdot\vec{\xi}_1 + \bar{A}_m \vec{m}\cdot\vec{\xi}_2 + A_\ell \vec{\ell}\cdot\vec{\xi}_1 + \bar{A}_\ell \vec{\ell}\cdot\vec{\xi}_2 + D_{mn} \vec{m}\cdot\vec{\xi}_1 \vec{n}\cdot\vec{\xi}_2 + \\ & D_{nm} \vec{n}\cdot\vec{\xi}_1 \vec{m}\cdot\vec{\xi}_2 + D_{\ell n} \vec{\ell}\cdot\vec{\xi}_1 \vec{n}\cdot\vec{\xi}_2 + D_{n\ell} \vec{n}\cdot\vec{\xi}_1 \vec{\ell}\cdot\vec{\xi}_2\Big]\Big\}, \qquad (20)\end{aligned}$$

where

$$\vec{\ell} = \frac{\vec{p}}{|\vec{p}|}, \quad \vec{n} = \frac{\vec{p}\times\vec{k}}{|\vec{p}\times\vec{k}|}, \quad \vec{m} = \vec{n}\times\vec{\ell},$$

$\vec{\xi}$ is the electron polarization four–vector and all polarization observables are functions of two independent variables q^2 and $\cos\theta$. The function A_n $(\bar{A}_n)$ is the asymmetry in the $\bar{p}+\vec{p}$ $(\vec{\bar{p}}+p)$ collision induced by the component of the polarization $\vec{\xi}_1(\vec{\xi}_2)$ in the direction $\vec{n}$; A_m and A_ℓ ($\bar{A}_m$ and $\bar{A}_\ell$) are the polarization transfer coefficients when the target (beam) and the electron are polarized due to the component of polarization vector $\vec{\xi}_1(\vec{\xi}_2)$ in the directions

$\vec{m}$ and $\vec{\ell}$, correspondingly; D_{ij} $(ij = mm, \ell\ell, nn, m\ell, \ell m)$ and D_{ij} $(ij = mn, nm, n\ell, \ell n)$ are the spin correlation coefficients induced by the collision of both polarized initial particles for the case of unpolarized and longitudinally polarized final electron, respectively; $P_n^{(e)}$ is the electron polarization in the case of unpolarized target and beam.

The following polarization observables

$$A_n, \ \bar{A}_n, \ P_n^{(e)}, \ D_{mn}, \ D_{nm}, \ D_{\ell n}, \ D_{n\ell}$$

are T–odd observables, whereas the other ones are T–even observables.

In the general case, all these polarization observables are nonzero, and their q^2- and $\cos\theta-$dependence depends on the dynamics of the process. On the basis of C–invariance it is not possible to predict any definite behavior of these observables.

V. CONCLUSIONS

We have studied the properties of the annihilation process $\bar{p} + p \to e^+ + e^-$ in presence of two photon exchange. We have derived the expressions of the cross section and of all polarization observables in terms of the nucleon electromagnetic FFs and of the amplitudes describing the TPE mechanism. We have analyzed the properties of these observables in different kinematical conditions.

The reasons of the possible contribution of the two photon contribution at large q^2 have been discussed long ago for $e + p \to e + p$ elastic scattering and apply equally well to the crossing channels. The importance of the experimental evidence and of the quantitative determination of TPE is related to the extraction of the electromagnetic FFs from the differential cross section. The simple formalism based on the one-photon mechanism, becomes much more complicated in presence of TPE.

Note that if the charge of the electron and positron is not detected (the detection is symmetric under interchange of the positron and electron), then the interference term between the one- and two–photon–exchange channels will not contribute to the differential cross section [13, 21, 24]. This symmetry between the positron and the electron can then be used either to eliminate or to make evident the influence of the TPE mechanism on the nucleon electromagnetic structure.

This analysis is especially useful in view of the future experiments planned at the FAIR facility, at GSI [25], where the first measurement of the relative phase of the proton magnetic and electric FFs in the time–like region is planned [26]. This information can discriminate strongly between the existing models for the nucleon FFs. This phase can be most simply measured via single–spin asymmetry in the annihilation reaction (1) with a transversely polarized target or beam.

VI. ACKNOWLEDGMENTS

We acknowledge Prof. M. P. Rekalo for fundamental contributions and ideas on the two-photon exchange problem and for enlightning discussions.

---

[1] L. Andivahis *et al.*, Phys. Rev. D **50**, 5491 (1994).

[2] M. K. Jones, et al., Phys. Rev. Lett. **84**, 1398 (2000); O. Gayou, et al., Phys. Rev. Lett. **88**, 092301 (2002).

[3] J. Arrington, Phys. Rev. C **68**, 034325 (2003).

[4] J. Gunion, L. Stodolsky, Phys. Rev. Lett. **30**, 345 (1973).

[5] V. Franco, Phys. Rev. **D8**, 826 (1973).

[6] V. N. Boitsov, L.A. Kondratyuk, and V. B. Kopeliovich, Sov. J. Nucl. Phys. **16**, 238 (1973); F. M. Lev, Sov. J. Nucl. Phys. **21**, 45 (1973).

[7] S. P. Wells *et al.* [SAMPLE collaboration], Phys. Rev. C **63**, 064001 (2001).

[8] F. E. Maas *et al.*, Phys. Rev. Lett. **94**, 082001 (2005).

[9] A. Akhiezer and M. P. Rekalo, Dokl. Akad. Nauk USSR, **180**, 1081 (1968); Sov. J. Part. Nucl. **4**, 277 (1974).

[10] S. Dubnicka and A. Z. Dubnickova, FizikaB **13**, 287 (2004).

[11] JLab Proposal 04-108: 'Measurement of G_E/G_M to Q^2=9 GeV^2 via Recoil Polarization', (Spokepersons: C.F. Perdrisat, V. Punjabi, M.K. Jones and E. Brash), JLab, July 2004.

[12] M. L. Goldberger, Y. Nambu and R. Oehme, Annals Phys. **2**, 226 (1957).

[13] A. Zichichi, S. M. Berman, N. Cabibbo, R Gatto, Nuovo Cim. **24**, 170 (1962).

[14] S. M. Bilenky, C. Giunti, V. Wataghin, Z. Phys. **C59**, 475 (1993).

[15] E. Tomasi-Gustafsson, F. Lacroix, C. Duterte and G. I. Gakh, arXiv:nucl-th/0503001.

[16] P.A.M. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003).

[17] M. P. Rekalo and E. Tomasi-Gustafsson, Eur. Phys. J. A. **22**, 331 (2004).

[18] M. P. Rekalo, E. Tomasi-Gustafsson, Nucl. Phys. **A740**, 271 (2004).

[19] M. P. Rekalo, E. Tomasi-Gustafsson, Nucl. Phys. **A742**, 322 (2004).

[20] Y.-C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaeghen, Phys. Rev. Lett. **93**, 122301 (2004).

[21] G. Putzolu, Nuovo Cim. **20**, 542 (1961).

[22] A. Z. Dubnickova, S. Dubnicka and M. P. Rekalo, Nuovo Cim. **A109**, 241 (1996).

[23] S. J. Brodsky, C. E. Carlson, J. R. Hiller and D. S. Hwang, Phys. Rev. **D69**, 054022 (2004).

[24] S. D. Drell, Ann. Phys. **4**, 75 (1958); J. D. Bjorken, S. D. Drell and S. C. Frautschi, Phys. Rev. **112**, 1409 (1958).

[25] An International Accelerator Facility for Beams of Ions and Antiprotons, *Conceptual Design Report*, http://www.gsi.de

[26] F. Rathmann and P. Lenisa, arXiv:hep-ex/0412078.