Θ^+-pentaquark parity from associative baryon production in πD collisions, $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$

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Abstract

We derive, in model independent way, the spin structure of the matrix element for the reaction of associative Θ^+ pentaquark production, $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$, in the threshold region and in collinear kinematics. The expressions for the polarization observables in this reaction are found assuming spin 1/2 and different parities for Θ^+. We have proved that such reaction can be used for a model independent determination of the P-parity of Θ^+ only by measuring the Θ^+ polarization. Other polarization observables, such as the dependence of the $\Sigma^\pm$ polarization on the vector and tensor deuteron polarizations, are insensitive to the Θ^+ parity in the considered kinematical conditions. All linear and quadratic relations between polarization observables in $\pi^\pm + \vec{D} \to \Theta^+ + \vec{\Sigma}^\pm$($\Theta^+$ is unpolarized) do not depend on the parity of the Θ^+ pentaquark.

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I. INTRODUCTION

The possible existence of exotic hadrons was suggested in connection with the KN scattering data [1] (before the advent of QCD). In QCD the multiquark states were considered as natural extension of the ordinary hadrons. A state with mass about 1.5 GeV and a strong decay width smaller than 15 MeV, now called Θ^+ pentaquark, was predicted in Ref. [2]. This work motivated and oriented experimental searches recently leading to the observation of a narrow resonance at about the predicted mass [3]. So far this observation has been confirmed or infirmed by other experimental groups using various projectiles and targets (for the details see the review [4]).

The narrow exotic (B=1, S=1) baryon resonance $\Theta^+(1540)$ was called pentaquark since the simplest quark content of Θ^+ is $(uudd\bar{s})$. The quantum numbers of the pentaquarks are very important for the determination of its quark structure and in particular its multiplet assignment [5].

At present the existence of a narrow pentaquarks is not fully confirmed: several laboratories have reported the evidence of such states, while others have negative results. The question of whether pentaquarks exist may be solved by a second generation of high statistics experiments (for example, at JLab [6]).

The parity of the Θ^+ pentaquark, $\pi(\Theta)$, being a very important characteristics of this exotic state, is especially challenging for a model independent determination. The study of polarization phenomena for different processes of Θ^+ production seems unavoidable. Unfortunately, in many cases, the necessary set of observables includes the Θ^+ polarization which measurement, through the decay $\Theta^+ \to NK$, is not an easy task.

Up to now, the most perspective method for the determination of $\pi(\Theta)$, in model independent way, has been suggested for associative hyperon production in nucleon-nucleon collisions, $N + N \to Y + \Theta^+$, $Y = \Lambda$ or Σ, and $p + p \to \Theta^+ + \Lambda + \pi^+$ ([7] and refs therein). It was shown, that in the threshold region, (with s-wave production of the final hyperons) double spin polarization observables, such as the spin correlation coefficients C_{xx} or C_{yy} (in the collision of polarized nucleons), and the spin transfer coefficient D_{xx} (from one initial nucleon, beam or target, to the produced hyperon Y) is sensitive to $\pi(\Theta)$.

We discuss in the present work another possible, simple reaction, the two body associative hyperon production, with baryon number equal 2, similar to $N + N \to \Theta^+ + Y$, but with a deuteron in the initial state:

$$\pi^{\pm} + D \to \Theta^{+} + \Sigma^{\pm} \tag{1}$$

in the threshold region [$E_{\pi,th}$= 1.04 GeV, with $M(\Theta^{+})$=1.54 GeV] or in collinear kinematics, with the aim of finding some polarization observables which are sensitive to $\pi(\Theta)$. Note that a similar reaction, $\pi^{-} + D \to n + n$, has been suggested many years ago [8] for the model independent determination of the charged pion parity.

The study of the reaction (1) in collinear kinematics looks more preferable in comparison with the threshold region, due to the larger cross section in forward direction, and to the fact that the formalism applies to any pion energy.

II. THE REACTION $\pi^{\pm} + D \to \Theta^{+} + \Sigma^{\pm}$

The simplest mechanism for the reaction (1) is shown in Fig. 1. It is similar to the impulse approximation, where the new deuteron vertex, $D \to \Theta^{+} + \Lambda$, generates a deuteron component made from eight quarks: $D \to 3u + 3d + s\bar{s}$. This component is not new. For example, the presence of an intrinsic $s\bar{s}$ component in the nucleon has been advocated to explain the strong violation of the OZI rule, which has been observed in different processes of ϕ production, in particular in $p\bar{p}$ annihilation at rest (for a recent review see Ref. [9]). It is possible to rearrange the np system, composed by $(uud + s\bar{s}) + (udd)$, as superposition of Θ^{+} and Λ hyperons and to derive a connection between the deuteron structure at short distances and the Θ^{+} physics. However, our interest here is focused on the dependence of the polarization effects in the reaction (1) on the parity of the Θ^{+} hyperon, $\pi(\Theta)$. To solve this problem we use only general symmetry properties of the strong interaction, such as the P-invariance and the isotropy of space.

In the general case, the matrix element for $\pi^{\pm} + D \to \Theta^{+} + \Sigma^{\pm}$ contains six independent spin structures, assuming spin 1/2 for Θ^{+}, which have different parametrizations according to $\pi(\Theta)$. From the isotopic invariance we have $\pi(p) = \pi(n)$. The capture process, $\pi^{-} + D \to n + n$ [8], proved the pseudoscalar nature of the pion. In this case no polarization observable is necessary, as the identity of the produced neutrons generates only one spin structure. Unfortunately, one can not use such procedure for $\pi^{\pm} + D \to \Theta^{+} + \Sigma^{\pm}$, due to different final hyperons.

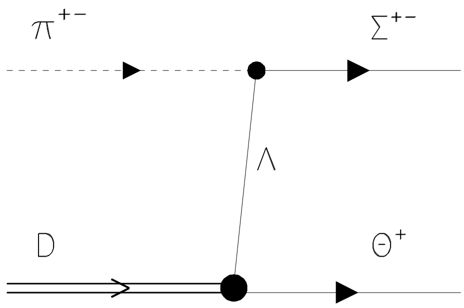

FIG. 1: Possible mechanism for $\pi^{\pm} + D \to \Theta^{+} + \Sigma^{\pm}$.

To avoid the complexity of the calculation of the polarization phenomena in reaction (1), we restrict our analysis to two different kinematical regimes: threshold production and collinear regime. Although the dynamics is different in these two cases, the analysis of polarization effects is similar, due to the axial symmetry of the two regimes: only one physical direction is defined: the three-momentum of the colliding particles. This essentially simplifies the calculations. Let us start from considerations which hold in threshold regime.

III. MATRIX ELEMENT AND POLARIZATION EFFECTS

The production of the final hyperons in s-state is characterized by two different partial transitions:

$$\begin{aligned}\underline{\pi(\Theta) = +1:} \quad & \ell = 1 \;\rightarrow\; \mathcal{J}^P = 0^+ \rightarrow S_f = 0, \\ & \ell = 1 \;\rightarrow\; \mathcal{J}^P = 1^+ \rightarrow S_f = 1,\end{aligned} \tag{2}$$

$$\begin{aligned}\underline{\pi(\Theta) = -1:} \quad & \ell = 0 \;\rightarrow\; \mathcal{J}^P = 1^- \rightarrow S_f = 1, \\ & \ell = 2 \;\rightarrow\; \mathcal{J}^P = 1^- \rightarrow S_f = 1,\end{aligned} \tag{3}$$

where ℓ is the orbital angular momentum of the initial pion (in the reaction CMS), S_f is the total spin of the the $\Theta\Sigma$-system, $\mathcal{J}^P$ is the total angular momentum and parity of colliding (and final) particles.

The spin structure of the corresponding matrix element can be written as:

$$\mathcal{M}^{(+)} = \chi_2^\dagger \left[f_1^{(+)} \vec{\sigma} \cdot \vec{U} \times \hat{\vec{q}} + i f_2^{(+)} \hat{\vec{q}} \cdot \vec{U} \right] \sigma_y \tilde{\chi}_1^\dagger, \tag{4}$$

$$\mathcal{M}^{(-)} = \chi_2^\dagger \left[f_1^{(-)} \vec{\sigma} \cdot \vec{U} + f_2^{(-)} \vec{\sigma} \cdot \hat{\vec{q}} \hat{\vec{q}} \cdot \vec{U} \right] \sigma_y \tilde{\chi}_1^\dagger, \tag{5}$$

where χ_1 and χ_2 are the two component spinors of the Θ^+ and Σ hyperon, $\hat{\vec{q}}$ is the unit vector along the pion three momentum, $\vec{U}$ is the deuteron polarization three-vector, $f_i^{(\pm)} = f_i^{(\pm)}(w)$, $i = 1, 2$ are the partial amplitudes of the transitions (2) and (3), which are, in general case, complex functions of the excitation energy $w = \sqrt{s} - M_1 - M_2$, $M_{1,2}$ are the masses of Θ and Σ, $\sqrt{s}$ is the invariant total energy of the colliding particles.

Note that the spin structure of the matrix elements $\mathcal{M}^{(\pm)}$ in collinear regime (i.e., for any value of $\sqrt{s}$ and production angle equal 0^0 or 180^0) is described by the same formulas, Eqs. (4) and (5), but the amplitudes $f_i^{(\pm)} \rightarrow f_{i,col}^{(\pm)}$ describe two different allowed helicity transitions, which work in collinear kinematics. Therefore, so different kinematical conditions are described by different amplitudes, but, due to the axial symmetry inherent to the considered problem, the same analysis of polarization phenomena can be applied.

The axial symmetry and the P-invariance of the strong interaction allow only single spin polarization observables in $\pi + \vec{D} \rightarrow \Theta^+ + \Sigma$, namely, the analyzing power induced by the quadrupole polarization of the deuteron target:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_0 \left[1 + \mathcal{A} Q_{ab} \hat{q}_a \hat{q}_b \right], \tag{6}$$

where $\mathcal{A}$ is the analyzing power, and Q_{ab} is the tensor which describes the deuteron quadrupole polarization. The spin–density matrix of the deuteron is described by (in the reaction CMS):

$$\rho_{ab} = \frac{1}{3}\left(\delta_{ab} + \frac{q_a q_b}{M^2}\right) - \frac{i}{2}\epsilon_{abc}\tilde{S}_c + Q_{ab}, \tag{7}$$

$$Q_{aa} = \frac{q_a q_b}{E^2}Q_{ab},\ Q_{ab} = Q_{ba},\quad \tilde{S}_i = \frac{E}{M}S_i - \frac{\vec{q}\cdot\vec{S}}{ME}q_i,$$

where $\vec{S}$ and $E(M)$ are the vector polarization and energy (mass) of the deuteron target in the reaction CMS.

The dependence of the Σ-hyperon polarization $\vec{P}_\Sigma$, on the polarization characteristics of the deuteron target, can be written in the following general form:

$$\vec{P}_\Sigma = \vec{S}\mathcal{P}_1 + \hat{\vec{q}}\vec{S}\cdot\hat{\vec{q}}\mathcal{P}_2 + \vec{Q}\times\hat{\vec{q}}\mathcal{P}_3, \tag{8}$$

$$Q_a = Q_{ab}\hat{q}_b,$$

where $\mathcal{P}_i$, $i = 1-3$, are real functions which can be expressed in terms of bilinear combinations of the reaction amplitudes. These functions are related to the following coefficients of the polarization transfer from the initial deuteron to the produced Σ-hyperon:

$$D_{xx} = D_{yy} = \mathcal{P}_1,\ D_{zz} = \mathcal{P}_1 + \mathcal{P}_2,\ D_{x,yz} = -D_{y,xz} = \mathcal{P}_3,$$

where the z-axis is chosen along the $\hat{\vec{q}}$ direction, but the x and y directions can not be uniquely fixed, due to the axial symmetry.

Summarizing, it is possible to measure four T-even observables: $(d\sigma/d\Omega)_0$, $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{A}$ and one T-odd observable, the coefficient $\mathcal{P}_3$.

Therefore, we can state that there must be a definite linear relations between T-even observables, which is model independent and works in collinear and threshold regime, as well.

To find this relation, let us express the coefficients $\mathcal{A}$, $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ as quadratic combinations of the corresponding amplitudes (for both cases of the Θ^+ parity):

$\underline{\pi(\Theta) = +1:}$

$$\left(\frac{d\sigma_+}{d\Omega}\right)_0 = \frac{N}{3}\left(2|f_1^{(+)}|^2 + r^2|f_2^{(+)}|^2\right),\ \mathcal{A}^{(+)}\left(\frac{d\sigma_+}{d\Omega}\right)_0 = N\left(-r^{-2}|f_1^{(+)}|^2 + |f_2^{(+)}|^2\right),$$

$$\mathcal{P}_1^{(+)}\left(\frac{d\sigma_+}{d\Omega}\right)_0 = -NrRef_1^{(+)}f_2^{(+)*},\ \mathcal{P}_2^{(+)}\left(\frac{d\sigma_+}{d\Omega}\right)_0 = Nr\left(r^{-2}|f_1^{(+)}|^2 + Ref_1^{(+)}f_2^{(+)*}\right),$$

$$\mathcal{P}_3^{(+)}\left(\frac{d\sigma_+}{d\Omega}\right)_0 = 2NImf_1^{(+)}f_2^{(+)*},$$

<u>$\pi(\Theta) = -1:$</u>

$$\left(\frac{d\sigma_-}{d\Omega}\right)_0 = \frac{N}{3}\left(2|f_1^{(-)}|^2 + r^2|f_1^{(-)} + f_2^{(-)}|^2\right),\ \mathcal{A}^{(-)}\left(\frac{d\sigma_-}{d\Omega}\right)_0 = N\left(-r^{-2}|f_1^{(-)}|^2 + |f_1^{(-)} + f_2^{(-)}|^2\right),$$

$$\mathcal{P}_1^{(-)}\left(\frac{d\sigma_-}{d\Omega}\right)_0 = Nr\left(|f_1^{(-)}|^2 + Ref_1^{(-)}f_2^{(-)*}\right),\ \mathcal{P}_2^{(-)}\left(\frac{d\sigma}{d\Omega_-}\right)_0 = -Nr\left(Ref_1^{(-)}f_2^{(-)*} + \frac{\vec{q}^2}{E^2}|f_1^{(-)}|^2\right),$$

$$\mathcal{P}_3^{(-)}\left(\frac{d\sigma_-}{d\Omega}\right)_0 = -2NImf_1^{(-)}f_2^{(-)*},$$

where $r = E/M$ and $N = 1/(64\Pi^2 MW)(p/q_\pi)$ is a kinematical factor.

From these formulas, one can find a linear relation between T-even polarization observables:

$$\mathcal{P}_1^{(\pm)} + \mathcal{P}_2^{(\pm)} + \frac{r}{3}\mathcal{A}^{(\pm)} = r^{-1} \ \text{ or } D_{zz}^{(\pm)} + \frac{r}{3}\mathcal{A}^{(\pm)} = r^{-1}. \tag{9}$$

This relation is the same for the two possible values of the Θ^+ parity. Therefore, it can not be used for the determination of this parity. Being model independent, this relation can be useful to determine the extension of the threshold region, i.e., of the s-wave dominance.

The measurement of the analyzing power $\mathcal{A}$ and cross section $(d\sigma/d\Omega)_0$ allows to realize the first step of the complete experiment for $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$, i.e., the determination of the moduli of the two complex amplitudes:

$$N|f_1^{(+)}|^2 = (1 - \frac{r^2}{3}\mathcal{A}^{(+)})\left(\frac{d\sigma_+}{d\Omega}\right)_0, Nr^2\ |f_2^{(+)}|^2 = (1 + \frac{2r^2}{3}\mathcal{A}^{(+)})\left(\frac{d\sigma}{d\Omega_+}\right)_0,$$

$$N|f_1^{(-)}|^2 = (1 - \frac{r^2}{3}\mathcal{A}^{(-)})\left(\frac{d\sigma_-}{d\Omega}\right)_0,\ Nr^2|f_1^{(-)} + f_2^{(-)}|^2 = (1 + \frac{2r^2}{3}\mathcal{A}^{(-)})\left(\frac{d\sigma_-}{d\Omega}\right)_0. \tag{10}$$

Evidently, the T-odd observable $\mathcal{P}_3$, being proportional to $\sin\delta$ (δ is the relative phase of the complex amplitudes $f_1^{(\pm)}$ and $f_2^{(\pm)}$), is the most sensitive to this phase. This is the last step of the complete experiment. It is possible also to find definite quadratic relations between T-odd and T-even observables:

$$1 + \frac{r^2}{3}\mathcal{A}^{(\pm)} - \frac{2r^4}{9}(\mathcal{A}^{(\pm)})^2 - \frac{r^2}{4}(\mathcal{P}_3^{(\pm)})^2 = (\mathcal{P}_1^{(\pm)})^2, \ \text{ if } \pi(\Theta) = \pm 1. \tag{11}$$

It is another result, which shows also that the specific quadratic combination of different observables of T-odd and T-even nature does not depend on $\pi(\Theta)$. In order to test these relations, it necessary to have a deuteron target, vector and tensor polarized, and to measure the Σ-hyperon polarization. These linear and quadratic relations apply equally well for threshold and for collinear kinematics of the considered process.

But the situation is different for the Θ^+ polarization. The dependence of the Θ^+ pentaquark polarization $\vec{P}_\Theta$, on the polarization characteristics of the deuteron target, can be written in the following general form (similar to Eq. (8)):

$$\vec{P}_\Theta = \vec{S}\bar{\mathcal{P}}_1 + \hat{\vec{q}}\vec{S}\cdot\hat{\vec{q}}\bar{\mathcal{P}}_2 + \vec{Q}\times\hat{\vec{q}}\bar{\mathcal{P}}_3, \tag{12}$$

where $\bar{\mathcal{P}}_i$, $i = 1-3$, are real functions which can be expressed in terms of bilinear combinations of the reaction amplitudes. Using Eqs. (4) and (5) for the matrix elements we can obtain the following expressions for $\bar{P}_i$ in terms of quadratic combinations of the corresponding amplitudes (for both cases of the Θ^+ parity)

$\underline{\pi(\Theta) = +1:}$

$$\mathcal{P}_1^{(+)} = -\bar{\mathcal{P}}_1^{(+)},\ \ \bar{\mathcal{P}}_2^{(+)}\left(\frac{d\sigma_+}{d\Omega}\right)_0 = Nr\left(r^{-2}|f_1^{(+)}|^2 - Ref_1^{(+)}f_2^{(+)*}\right),\ \ \mathcal{P}_3^{(+)} = -\bar{\mathcal{P}}_3^{(+)},$$

$\underline{\pi(\Theta) = -1:}$

$$\mathcal{P}_i^{(-)} = \bar{\mathcal{P}}_i^{(-)},\ i = 1,2,3.$$

From these formulas one can see that only the transversal components of the Θ^+ polarization vector are sensitive to the pentaquark parity:

$$\bar{\mathcal{P}}_{1,3} = -\pi(\Theta^+)\mathcal{P}_{1,3},\ \bar{\mathcal{P}}_1 + \bar{\mathcal{P}}_2 = \mathcal{P}_1 + \mathcal{P}_2.$$

In terms of the coefficients of polarization transfer $D_{ab}(\Theta)$ $[D_{ab}(\Sigma)]$ from vector–polarized deuteron target to the Θ^+ (Σ) hyperons, these relations can be written as follows:

$$D_{aa}(\Theta) = -\pi(\Theta)D_{aa}(\Sigma),\ aa = xx; yy; x, yz \ \text{ or } y, xz,\ D_{zz}(\Theta) = D_{zz}(\Sigma).$$

Evidently, it is a very difficult experiment.

IV. CONCLUSIONS

We performed a model independent analysis of the spin structure of the matrix element for associative hyperon production in $\pi^\pm D$ collisions, $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$, in collinear or threshold regime. We calculated different polarization observables and showed that they are related by very specific linear and quadratic equations. Unfortunately, these model independent relations are insensitive to the parity of the Θ-pentaquark. Therefore, difficult measurements of the Θ^+ polarization are necessary in order to determine the parity of the Θ^+.

What is the physical reason of such insensitivity of the $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$ process to $\pi(\Theta)$? What is the main difference between $N + N \to Y + \Theta$ and $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$, which look very similar processes? The problem is that the initial two-nucleon system in $N + N \to Y + \Theta$ is more flexible, as it can be in triplet and singlet state, as well, whereas in $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$ the initial two-nucleon system, being a deuteron, is always in triplet state.

However, if the parity of Θ^+ will be determined, the study of the process $\pi^\pm + D \to \Theta^+ + \Sigma^\pm$ will be useful for the study of other questions, related to the Θ^+ physics, as, for example, the exotic baryon content of the deuteron at small distances.

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