Isotopic Scaling and the Symmetry Energy in Spectator Fragmentation

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Isotopic effects in the fragmentation of excited target residues following collisions of ^{12}C on ^{112,124}Sn at incident energies of 300 and 600 MeV per nucleon were studied with the INDRA 4π detector. The measured yield ratios for light particles and fragments with atomic number $Z \leq 5$ obey the exponential law of isotopic scaling. The deduced scaling parameters decrease with increasing centrality to values as low as $\alpha = 0.25 \pm 0.02$ for the central event group at 600 MeV per nucleon. Symmetry term coefficients, deduced from these data within the statistical description of isotopic scaling, are near $\gamma = 25$ MeV for peripheral and $\gamma < 10$ MeV for central collisions.

The growing interest in isospin effects in nuclear reactions is motivated by an increasing awareness of the importance of the symmetry term in the nuclear equation of state, in particular for astrophysical applications. Supernova simulations or neutron star models require inputs for the nuclear equation of state at extreme values of density and asymmetry [1, 2, 3]. The demonstration in the laboratory of the effects of the symmetry term at abnormal densities is, therefore, an essential first step within a program aiming at gaining such information experimentally [4, 5].

Multifragmentation is generally conceived as a low-density phenomenon. The short-range nature of the nuclear forces causes a clustering in nuclear systems that have expanded as a result of an initial compression or heating in the course of a violent nuclear collision. On the other hand, the scenario promoted by the rather successful class of statistical multifragmentation models [6, 7] considers normal-density fragments statistically distributed within an expanded volume. The density is only low on average, and standard liquid-drop parameters are used to describe the nascent fragments including their isotopic degrees of freedom. In the Copenhagen version of this model, the symmetry energy term $E_{\mathrm{sym}} = \gamma(A - 2Z)^2/A$ is used with coefficients $\gamma = 23$ to 25 MeV [7, 8].

An experimental value of nearly exactly this magnitude has recently been obtained within a statistical description of isotopic scaling in light-ion (p, d, α) induced reactions at relativistic energies of up to 15 GeV [8]. This result, while being fully consistent with the statistical scenario, may not be representative for multi-fragment decays because the used data samples were inclusive. The mean multiplicities of intermediate-mass fragments are known to be small in these reactions [9], and the internal degrees of freedom of heavy residues are expected to dominate.

In the present work, we extend the investigation of spectator decays at relativistic energies to the much higher excitation energies and correspondingly higher fragment multiplicities that can be reached with heavier projectiles. Isotopic scaling is studied exclusively in ^{12}C induced reactions on ^{112}Sn and ^{124}Sn targets at 300 and 600 MeV per nucleon incident energy. Here, according to the established systematics, maximum fragment pro-

duction is observed for central impact parameters [10]. We find that the symmetry energy coefficient indeed decreases toward the more central collisions. Its variation, however, is partly masked by the effects of secondary decay.

Isotopic scaling, also termed isoscaling, has been shown to be a phenomenon common to many different types of heavy ion reactions [8, 11, 12, 13]. It is observed by comparing product yields from otherwise identical reactions with isotopically different projectiles or targets, and it is constituted by an exponential dependence of the measured yield ratios $R_{21}(N, Z)$ on the neutron number N and proton number Z of the considered product. The scaling expression

$$R_{21}(N, Z) = Y_2(N, Z)/Y_1(N, Z) = C \cdot exp(\alpha N + \beta Z) \quad (1)$$

describes rather well the measured ratios over a wide range of complex particles and light fragments [14].

In the grand-canonical approximation, assuming that the temperature T is about the same, the scaling parameters α and β are proportional to the differences of the neutron and proton chemical potentials for the two systems, $\alpha = \Delta\mu_{\mathrm{n}}/T$ and $\beta = \Delta\mu_{\mathrm{p}}/T$. Of particular interest is their connection with the symmetry term coefficient. It has been obtained from the statistical interpretation of isoscaling within the SMM [8] and Expanding-Emitting-Source Model [14] and confirmed by an analysis of reaction dynamics [15]. The relation is

$$\alpha T = \Delta\mu_{\mathrm{n}} = \mu_{\mathrm{n},2} - \mu_{\mathrm{n},1} \approx 4\gamma\left(\frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2}\right) \quad (2)$$

where Z_i and A_i are the charges and mass numbers of the two systems (the indices 1 and 2 denote the neutron poor and neutron rich system, respectively). With the knowledge of the temperature and the isotopic compositions, the coefficient γ of the symmetry term [16] can be obtained from isoscaling.

The data were obtained with the INDRA multidetector [17] in experiments performed at the GSI. Beams of ^{12}C with 300 and 600 MeV per nucleon incident energy, delivered by the heavy-ion synchrotron SIS, were directed onto enriched targets of ^{112}Sn (98.9%) and ^{124}Sn (99.9%) with areal densities between 1.0 and 1.2 mg/cm^2. Light charged particles and fragments ($Z \leq 5$) were detected and isotopically identified with the calibration telescopes of rings 10 to 17 of the INDRA detector which cover the range of polar angles $45° \leq \theta_{\mathrm{lab}} \leq 176°$. These telescopes consist of pairs of an 80-μm Si detector and a 2-mm Si(Li) detector which are mounted between the ionization chamber and the CsI(Tl) crystal of one of the modules of a ring [17]. Further experimental details may be found in [18] and the references given therein. For impact-parameter selection, the charged-particle multiplicity M_{C} measured with the full detector was used, and four bins were chosen for the sorting of the data. For

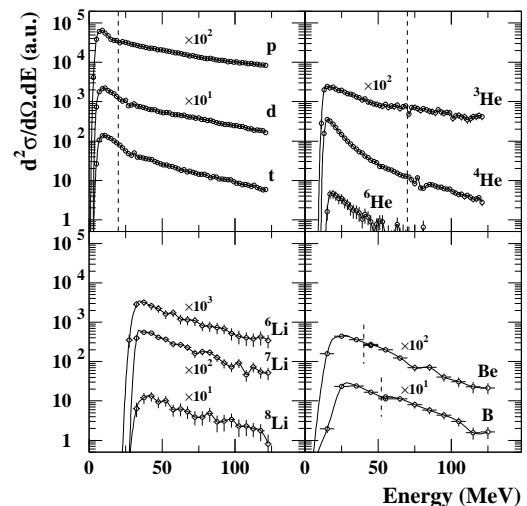
FIG. 1: Energy spectra of H, He, Li isotopes, and of Be and B elements, measured with the calibration telescopes for ^{12}C + ^{124}Sn at 300 MeV per nucleon. The dashed lines indicate the upper limits set for $Z = 1, 2$ in the analysis and the identification thresholds for $Z = 4, 5$.

TABLE I: Parameters obtained from fitting the measured isotopic yield ratios with the scaling function given in Eq. (1).

| | 300 MeV | | 600 MeV | |
$b/b_{\max}$	α	β	α	β
0.0 - 0.2	0.28 ± 0.01	-0.33 ± 0.03		
0.2 - 0.4	0.31 ± 0.01	-0.32 ± 0.01	0.25 ± 0.02	-0.28 ± 0.04
0.4 - 0.6	0.36 ± 0.01	-0.39 ± 0.02	0.32 ± 0.02	-0.34 ± 0.03
0.6 - 1.0	0.62 ± 0.01	-0.68 ± 0.02	0.52 ± 0.02	-0.59 ± 0.03

600 MeV per nucleon, the two most central bins were combined for reasons of counting statistics.

Kinetic energy spectra of light reaction products with $Z \leq 5$, integrated over the impact parameter and the angular range $\theta_{\mathrm{lab}} \geq 45°$, are shown in Fig. 1. To reduce preequilibrium contributions, upper limits of 20 MeV and 70 MeV were set for hydrogen and helium isotopes, respectively, which, however, are not crucial. The spectra of Li, Be, and B fragments were integrated above the energy thresholds for isotopic identification which amounted to 28, 40, and 52 MeV, respectively.

The ratios of the fragment yields measured for the two reactions and integrated over the chosen intervals of energy and angle ($\theta_{\mathrm{lab}} \geq 45°$) obey the law of isoscaling. This is illustrated in Fig. 2 which shows the scaled isotopic ratios $S(N) = R_{21}(N, Z)/exp(\beta Z)$. Their slope parameters change considerably with impact parameter, extending from $\alpha = 0.62$ to values as low as $\alpha = 0.25$ for the most central event group at 600 MeV per nucleon (Table I and Fig. 3, top).

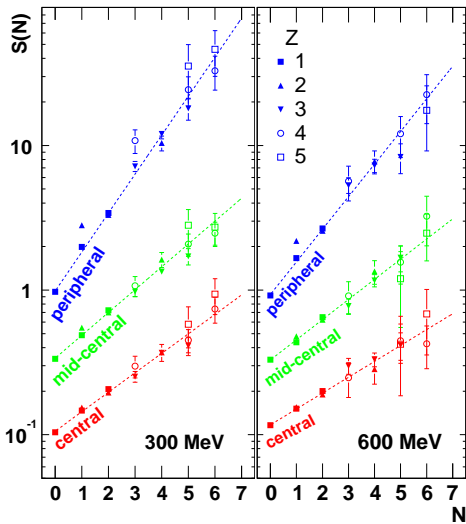Temperature estimates were obtained from the yields of ^{3,4}He and ^{6,7}Li isotopes, and the deduced T_{HeLi} con-

FIG. 2: Scaled isotopic ratios $S(N)$ for ^{12}C + ^{112,124}Sn at $E/A = 300$ MeV (left panel) and 600 MeV (right panel) for the intervals of reduced impact parameter specified in Table I, with "central" indicating $b/b_{\max} \leq 0.4$ and with offset factors of multiples of three. The dashed lines are the results of exponential fits according to Eq. (1). Only statistical errors are displayed.

FIG. 3: Isoscaling coefficient α (top), double-isotope temperatures T_{HeLi} (middle) and resulting γ_{app} (bottom) for $E/A = 300$ MeV (full symbols) and 600 MeV (open symbols), as a function of the centrality parameter $1 - b/b_{\max}$. The temperatures for the ^{112}Sn and ^{124}Sn targets are distinguished by full and dashed lines, respectively.

tains a correction factor 1.2 for the effects of sequential decay [19, 20]. The temperatures are quite similar for the two target cases and increase with centrality from about 6 MeV to 9 MeV (Fig 3, middle). This is consistent with the results obtained for ^{197}Au fragmentations [19, 20] and with the established dependence on the system mass [21]. The rise of T_{HeLi}, however, does not compensate for the decrease of α, as it did in the case of light-particle induced reactions [8], and $\Delta\mu_{\mathrm{n}}$, consequently, decreases toward the central collisions.

The analytical expression for $\Delta\mu_{\mathrm{n}}$ (Eq. 2) contains the isotopic composition of the sources, more precisely the difference of the squared Z/A values, $\Delta(Z^2/A^2) = (Z_1/A_1)^2 - (Z_2/A_2)^2$. For the target spectators, this quantity is not expected to deviate significantly from its original value [8], in contrast to mean-field dominated reaction systems at intermediate energies [15, 22]. Calculations performed with the Liège-cascade-percolation model [23] confirm this result: the individual Z/A values, evaluated for the residuals after the cascade and percolation stages (which in this model are used as input for subsequent statistical fragmentation calculations) change slightly but $\Delta(Z^2/A^2)$ remains nearly the same. It decreases by less than 4% and 7% for central collisions at 300 and 600 MeV per nucleon, respectively. Isospin effects of the nuclear mean field are explicitly considered in the Relativistic Mean Field Model of Ref. [24]. For central collisions at 600 MeV per nucleon, the calculations predict a reduction of $\Delta(Z^2/A^2)$ by 6% for the target-rapidity region after 90 fm/c collision time.
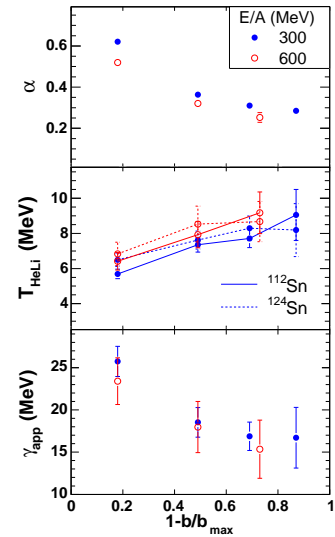
The suggested corrections are small and can be temporarily ignored. With the compositions of the original targets, $\Delta(Z^2/A^2) = 0.0367$, the expression $\gamma = \alpha T/0.147$ is obtained from Eq. 2. It was used to determine an apparent symmetry term γ_{app}, i.e. without sequential decay corrections for α, from the data shown in Fig. 3 (the mean values were used for T). The results are close to the normal-density coefficient for peripheral collisions but drop to lower values at the more central impact parameters (Fig. 3, bottom).

The effects of sequential decay were studied with the microcanonical Markov-chain version of the Statistical Multifragmentation Model [25]. The target nuclei ^{112,124}Sn with excitation energies of 4, 6, and 8 MeV per nucleon were chosen as inputs, and the symmetry term γ was varied between 4 and 25 MeV. The isoscaling coefficient α was determined from the calculated fragment yields before (hot fragments) and after (cold fragments) the sequential decay stage of the calculations for which standard values for the fragment masses were used. The energy balance at freeze-out and during the secondary deexcitation was taken into account as described in [7].

The hot fragments exhibit the linear relation of α with γ as expected (Fig. 4, top panel). With $\gamma = 25$ MeV, the sequential processes cause a slight broadening of the isotopic distributions and the resulting α is lowered by 10% to 20%, similar to what was reported in [8]. For smaller values of γ, however, the dominant effect is caused by the decay of the wings of the wider distributions of hot fragments which is directed toward the valley of stability. The resulting cold distributions are narrower and
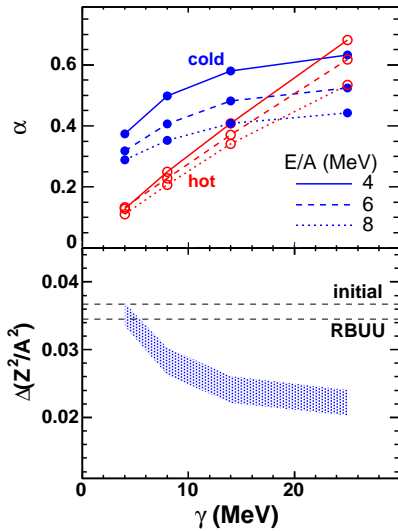
FIG. 4: Top panel: isoscaling coefficient α for hot (open circles) and cold fragments (dots) as a function of the symmetry term coefficient γ as predicted by the Markov-chain calculations for ^{112,124}Sn.
Bottom panel: region in the $\Delta(Z^2/A^2)$-versus-γ plane consistent with the measured value $\alpha = 0.29$ for central collisions and with the Markov-chain predictions for cold fragments (shaded). The dashed lines indicate the $\Delta(Z^2/A^2) = 0.0367$ of ^{112,124}Sn and the RBUU prediction.

the isoscaling coefficients correspondingly larger. Overall, the sequential decay has the effect that the variation of α with γ is strongly reduced. The decrease of γ with centrality should, thus, be much larger than that of γ_{app} displayed in Fig. 3. A low $\alpha < 0.3$, as measured for central collisions, is only reproduced with $\gamma = 4$ MeV and excitation energies of 6 to 8 MeV (Fig. 4, top).

The bottom panel of Fig. 4 shows the situation if the variation of the isotopic compositions of the two systems is included as a degree of freedom. The shaded band in the plane of $\Delta(Z^2/A^2)$ versus γ represents the region consistent with the weighted mean $\alpha = 0.29$ measured for the central bins ($b/b_{\mathrm{max}} \leq 0.4$) at the two energies. It was obtained from the predictions $\alpha(\gamma)$ for cold fragments at excitation energies between 6 and 8 MeV per nucleon (Fig. 4, top) according to $\Delta(Z^2/A^2) = 0.0367 \cdot 0.29/\alpha(\gamma)$, i.e. using that α is proportional to $\Delta(Z^2/A^2)$ (Eq. 2). If $\Delta(Z^2/A^2)$ remains near its original value or close to the predictions of the cascade and RBUU models the resulting symmetry term coefficient for the central collisions is very small, $\gamma \leq 10$ MeV. It is, in particular, quite insensitive to small variations of $\Delta(Z^2/A^2)$ around its initial value. To restore the consistency with $\gamma \approx 25$ MeV for the central reaction channels would require considerable isotopic asymmetries in the initial reaction phase, much larger than what is expected according to the models.

During the multifragmentation reaction, small and intermediate-mass nuclei are produced which are surrounded by other fragments and by a nucleon gas. The

structure of these highly excited fragments, at the chemical freeze-out point, may be considerably different from that of stable nuclei. The combined effects of the interaction with their neighbours and of their own expanded structure provide sufficient reason for a strong reduction of the symmetry term at the time of chemical freeze-out. It would also not invalidate previous results obtained with the statistical multifragmentation model and standard parameters. The global predictions for the partitions and the relation of temperature and excitation energy are only mildly affected by even drastic changes of γ. It is clear, however, that any conclusion in this direction depends crucially on the isotopic evolution of the multi-fragmenting system as it approaches the chemical freeze-out. This question requires further attention.

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