

Model-independent analysis of polarization effects in elastic electron-deuteron scattering in presence of two-photon exchange

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Abstract

The general spin structure of the matrix element, taking into account the two-photon exchange contribution, for the elastic electron (positron)–deuteron scattering has been derived using general symmetry properties of the hadron electromagnetic interaction, such as P-, C- and T-invariances as well as lepton helicity conservation in QED at high energy. Taking into account also crossing symmetry, the amplitudes of $e^\mp d$ -scattering can be parametrized in terms of fifteen real functions. The expressions for the differential cross section and for all polarization observables are given in terms of these functions. We consider the case of an arbitrary polarized deuteron target and polarized electron beam (both longitudinal and transverse). The transverse polarization of the electron beam induces a single-spin asymmetry which is non-zero in presence of two-photon exchange. It is shown that elastic deuteron electromagnetic form factors can still be extracted in presence of two photon exchange, from the measurements of the differential cross section and of one polarization observable (for example, the tensor asymmetry) for electron and positron deuteron elastic scattering, in the same kinematical conditions.

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I. INTRODUCTION

The study of the structure of hadrons and nuclei with electromagnetic probes is based on the validity of the one-photon exchange mechanism for elastic and inelastic electron-hadron scattering. This approach is valid when the possible two-photon exchange (TPE) contribution is small.

Recently it has been possible to apply the polarization method [1] for the measurements of the electromagnetic nucleon form factors (FFs) at high transfer momentum squared q^2 [2]. Very precise results were obtained for the ratio of the proton electric and magnetic FFs which differ from unpolarized cross section measurements (Rosenbluth fit) [3]. This discrepancy increases when q^2 values increase.

In a recent experiment at Jefferson Laboratory [4] a precise Rosenbluth extraction of the proton FFs, detecting the recoil proton, confirms this discrepancy. It was suggested that the presence of TPE contribution as large as 5% could solve this problem [5]. Model calculations may give TPE contribution to the cross section of ep elastic scattering of the order of few percent [6]. A parton model calculation [7] leads to a quantitative agreement between the standard Rosenbluth fit and the polarization transfer measurements. Recently the TPE contribution has been studied for the case of the inelastic electron-nucleon scattering, $eN \rightarrow e\Delta(1232)$, with the aim of a precision study of the ratios of electric quadrupole ($E2$) and Coulomb quadrupole ($C2$) to the magnetic dipole ($M1$) $\gamma^*N\Delta$ transitions [8]. Inelastic intermediate state as Δ has been calculated in [9, 10], and it was found to have opposite sign than the proton intermediate state. It has also been argued that elastic and inelastic contributions eventually cancel [10].

An exact calculation has been done in frame of QED and shows that the TPE contribution does not exceed 1% [11]. A recent calculation of the box diagram in ep elastic scattering [12] also shows that the contribution of this diagram is very small. Higher order radiative corrections, based on the structure function method, when applied to the unpolarized cross section, can bring the results in quantitative agreement [10], while the corrections on the polarization ratio are small as well as the TPE contribution.

Note also that the search of the deviation from linearity of the Rosenbluth fit to the differential cross section, using the most recent data on elastic electron-proton scattering,

does not show evidence for such deviation [13].

A model independent study of the TPE mechanism in the elastic electron–nucleon scattering and its consequences on the experimental observables, has been carried on in [14–16], and in the crossed channels: proton–antiproton annihilation into lepton pair [17] and annihilation of e^+e^- –pair into nucleon–antinucleon [18].

The fact that the TPE mechanism, where the momentum transfer is equally shared between the two virtual photons, can become important when q^2 increases, was already indicated more than thirty years ago [19–21].

Estimates of the TPE contribution to the elastic electron–deuteron scattering were made in Refs. [19, 20] within the framework of the Glauber theory. It was shown [19] that this contribution decreases very slowly with increase of the momentum transfer squared q^2 and may dominate in the cross section at high q^2 values. Since the TPE amplitude is essentially imaginary in this model, the difference between positron and electron scattering cross sections depends upon the small real part of the TPE amplitude [19]. Recoil polarization effects may be substantial, in the region where the one– and two–photon exchange contributions are comparable. If the TPE mechanism become sizable, the extraction of the nucleon or deuteron electromagnetic FFs from the experimental data would only be possible after the determination of several (polarization) observables.

It is known that double scattering dominates in collisions of high–energy hadrons with deuterons at high q^2 values, and it was predicted that the TPE contribution in the elastic electron–deuteron scattering contributes for 10 % at $|q^2| \cong 1.3 \text{ GeV}^2$ [20]. At the same time the importance of the TPE mechanism was considered in Ref. [21].

Using high precision data on the elastic electron–deuteron scattering, recently obtained at the Jefferson Laboratory, the authors of the paper [22] looked for a possible contribution of the TPE mechanism at relatively high momentum transfer. While they did not find a definite evidence for the presence of the TPE contribution in the elastic electron–deuteron scattering, it was the first attempt to obtain a quantitative upper limit of a possible TPE contribution using a parametrization of the TPE term and the existing experimental data. The discrepancy among the set of data [23, 24] was therefore attributed to systematic effects.

Tests of the limits of the validity of the one–photon approximation have been done in the past, using different methods, but no effect has been found within the accuracy of the performed experiments. At first the TPE contribution was experimentally observed in

the domain of very small energies in atomic physics [25] and later the measurements of the beam asymmetry in the scattering of transversally polarized electrons on unpolarized protons, performed at MIT (Bates) [26] and at MAMI (Mainz) [27], gave small but non-zero values for this observable, contrary to what is expected in the one-photon-exchange (Born) approximation. This asymmetry is related to the imaginary part of the interference between one- and two-photon exchange amplitudes and can be connected only indirectly with the real part of this interference which contribute to the differential cross section or to the T-even polarization observables of the elastic electron-hadron scattering.

The recoil-deuteron vector polarization for the elastic scattering of electrons from an unpolarized deuteron target, which vanishes in the Born approximation, was measured in an experiment which aimed to test time-reversal invariance in electromagnetic interaction at high momentum transfer. The value obtained for the vector polarization was $|P| = 0.075 \pm 0.088$ [28], and the precision of the experiment did not allow to find evidence for the TPE contribution.

In this paper we analyze, in a model-independent way, the influence of the TPE contribution on the differential cross section and various polarization observables in the elastic electron (positron)-deuteron scattering

$$e^\mp + d \rightarrow e^\mp + d. \quad (1)$$

Our approach is similar to the one used earlier for the analysis of the TPE mechanism in the elastic electron-nucleon scattering [14]. The situation with elastic $e^\mp d$ -scattering is more complicated in comparison with the elastic eN -scattering since the deuteron has spin one. In this case the spin structure of the matrix elements of the reactions (1) are completely determined by fifteen real functions: six complex amplitudes (in comparison with three complex amplitudes for the case of elastic eN -scattering) depending on two variables and three deuteron electromagnetic FFs (two nucleon FFs for elastic eN -scattering) which are real functions of one variable, Q^2 .

The purpose of this paper is to derive general expressions for the differential cross section and various polarization observables in elastic electron (positron)-deuteron scattering and to suggest model independent methods to extract deuteron electromagnetic FFs also in presence of the TPE contribution, without any underlying assumptions.

Note that the presence of the TPE contribution in $e^\mp + d \rightarrow e^\mp + d$ results in nonlocal spin

structure of the matrix element. The standard analysis of the polarization effects, which is known for the one-photon-exchange mechanism, does not apply anymore. The extraction of form factors can be done after a more complicated derivation involving additional polarization observables.

II. MATRIX ELEMENT AND SYMMETRY RELATIONS

The starting point of our analysis is the following general parametrization of the spin structure of the matrix element for elastic electron-deuteron scattering, which can be obtained from the non-spin-flip part of the amplitude of the elastic nucleon-deuteron scattering [29]

$$M = \frac{e^2}{Q^2} l_\mu J_\mu. \quad (2)$$

The leptonic and hadronic currents have the form

$$l_\mu = \bar{u}(k_2) \gamma_\mu u(k_1), \quad (3)$$

$$\begin{aligned} J_\mu = (p_1 + p_2)_\mu \left[-G_1(s, Q^2) U_1 \cdot U_2^* + \frac{1}{M^2} G_3(s, Q^2) (U_1 \cdot q U_2^* \cdot q - \frac{q^2}{2} U_1 \cdot U_2^*) \right] + \\ + G_2(s, Q^2) (U_{1\mu} U_2^* \cdot q - U_{2\mu}^* U_1 \cdot q) + \frac{1}{M^2} (p_1 + p_2)_\mu \left[G_4(s, Q^2) U_1 \cdot k U_2^* \cdot k + \right. \\ \left. + G_5(s, Q^2) (U_1 \cdot q U_2^* \cdot k - U_1 \cdot k U_2^* \cdot q) \right] + G_6(s, Q^2) (U_{1\mu} U_2^* \cdot k + U_{2\mu}^* U_1 \cdot k), \quad (4) \end{aligned}$$

where k_1 (k_2) and p_1 (p_2) are the four-momenta of the initial (scattered) electron and initial (scattered) deuteron, respectively; $k = k_1 + k_2$, $q = k_1 - k_2 = p_2 - p_1$, $Q^2 = -q^2$, M is the deuteron mass, $U_{1\mu}$ ($U_{2\mu}$) is the initial (final) deuteron polarization four-vector.

The six complex amplitudes, $G_i(s, Q^2)$, $i = 1 - 6$, which are generally functions of two independent kinematical variables, Q^2 and $s = (k_1 + p_1)^2$ (s is the square of the total energy of the colliding particles), fully describe the spin structure of the matrix element for the elastic electron-deuteron scattering - for any number of exchanged virtual photons.

In the Born (one-photon-exchange) approximation these amplitudes reduce to three:

$$G_1^{Born}(s, Q^2) = G_1(Q^2), \quad G_2^{Born}(s, Q^2) = G_2(Q^2), \quad G_3^{Born}(s, Q^2) = G_3(Q^2), \quad (5)$$

$$G_i^{Born}(s, Q^2) = 0, \quad i = 4, 5, 6,$$

where $G_i(Q^2)$, ($i = 1, 2, 3$), are the deuteron electromagnetic FFs depending only on the virtual photon four-momentum squared. Due to the current hermiticity, FFs $G_i(Q^2)$ are real functions in the region of the space-like momentum transfer. The same FFs describe also the one-photon-exchange mechanism for elastic positron-deuteron scattering.

These FFs are related to the standard deuteron FFs: $G_C(Q^2)$ (the charge monopole), $G_M(Q^2)$ (the magnetic dipole) and $G_Q(Q^2)$ (the charge quadrupole). These relations are

$$\begin{aligned} G_M(Q^2) &= -G_2(Q^2), \quad G_Q(Q^2) = G_1(Q^2) + G_2(Q^2) + 2G_3(Q^2), \\ G_C(Q^2) &= \frac{2}{3}\tau[G_2(Q^2) - G_3(Q^2)] + (1 + \frac{2}{3}\tau)G_1(Q^2), \end{aligned} \quad (6)$$

with

$$\tau = \frac{Q^2}{4M^2}.$$

The standard FFs have the following normalizations:

$$G_C(0) = 1, \quad G_M(0) = (M/m)\mu_d, \quad G_Q(0) = M^2Q_d, \quad (7)$$

where m is the nucleon mass, $\mu_d = 0.857(Q_d = 0.2859)$ is deuteron magnetic (quadrupole) moment. The numerical values are taken from [30, 31].

The spin structure of the matrix element for the elastic electron-deuteron scattering can be established in analogy with the elastic nucleon-deuteron scattering [29], using the general properties of the electron-hadron interaction, such as the Lorentz invariance and P-invariance. Taking into account the identity of the initial and final states and the T-invariance of the electromagnetic interaction, the reactions $e^\mp + d \rightarrow e^\mp + d$, where a spin 1/2 particle is scattered by a spin 1 particle, are described by twelve independent complex amplitudes. So, the model-independent parametrization of the corresponding matrix element can be done (in many different but equivalent forms) in terms of twelve invariant complex amplitudes, $G_i(s, Q^2)$, $i = 1 - 12$.

At high energies we can neglect the contributions which are proportional to the electron mass. In this limit, any Feynman diagram in QED is invariant under the chirality operation $u(p) \rightarrow \gamma_5 u(p)$. This invariance implies that invariant structures in the matrix element which change their sign under this transformation can be neglected since they are proportional to the electron mass. So, the structures as $\bar{u}(k_2)u(k_1)$ and $\bar{u}(k_2)\gamma_\mu\gamma_\nu u(k_1)$ can be neglected.

As a result, we can reduce the number of invariant amplitudes for elastic electron–deuteron scattering from twelve amplitudes to six ones.

Let us stress that in the general case

- The amplitudes $G_i(s, Q^2)$, $i = 1 - 6$, are the complex functions of two independent variables, Q^2 and s .

- The connection of these amplitudes with the deuteron electromagnetic FFs is non-trivial since the amplitudes are related to the amplitudes of the virtual Compton scattering process which are presently unknown.

- The set of the amplitudes $G_i^{(-)}(s, Q^2)$ for the reaction $e^- + d \rightarrow e^- + d$ is different from the corresponding set of the amplitudes $G_i^{(+)}(s, Q^2)$ describing the charge conjugated reaction $e^+ + d \rightarrow e^+ + d$. This means that the properties of the positron–deuteron scattering cannot be derived from the $G_i^{(-)}(s, Q^2)$ amplitudes. However, prescriptions based on C–invariance help to derive expressions which rely real FFs, which are functions of Q^2 , to experimental observables. The strategy for their determination in presence of TPE will be detailed below.

Let us introduce another set of variables: ϵ and Q^2 , which is equivalent to s and Q^2 (in Lab system):

$$\epsilon^{-1} = 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2},$$

where θ is the electron scattering angle in the Lab system. These variables ϵ (the degree of the linear polarization of the virtual photon) and Q^2 are well suited for the description of the electron–hadron elastic scattering in the one–photon–exchange approximation, since only the Q^2 –dependence of FFs has a dynamical origin, whereas the linear ϵ –dependence of the differential cross section is a consequence of the one–photon mechanism. The variables s and Q^2 are more convenient for the annihilation channel and for the analysis of the consequences of the crossing symmetry.

To separate the effects caused by the Born (one–photon exchange) and TPE contributions, let us single out the dominant contribution and define the following decompositions of the amplitudes (taking into account the C–invariance of the electromagnetic interaction of hadrons)

$$G_i^{(\mp)}(Q^2, \epsilon) = \mp G_i(Q^2) + \Delta G_i(Q^2, \epsilon), \quad i = 1, 2, 3, \quad (8)$$

$$G_i^{(-)}(Q^2, \epsilon) = G_i^{(+)}(Q^2, \epsilon) = G_i(Q^2, \epsilon), \quad i = 4, 5, 6, \quad (9)$$

where $\Delta G_{1,2,3}$ and $G_{4,5,6}$ describe the TPE contribution only.

The order of magnitude of these quantities is $\Delta G_i(Q^2, \epsilon)$, ($i = 1, 2, 3$), and $G_i(Q^2, \epsilon)$, ($i = 4, 5, 6$), $\sim \alpha$, and $G_i(Q^2)$, ($i = 1, 2, 3$), $\sim \alpha^0$. Since the terms ΔG_i , ($i = 1, 2, 3$), and G_i , ($i = 4, 5, 6$), are small in comparison with the dominant ones, we neglect in following by the bilinear combinations of these small terms.

Therefore the reactions $e^\mp + d \rightarrow e^\mp + d$ are described by fifteen different real functions:

- three real FFs $G_i(Q^2)$ ($i = 1 - 3$), which are functions of one variable only. This holds in the space-like region since in the time-like region these FFs became complex functions due to the strong interaction in the final state as in the case of $e^+ + e^- \rightarrow \rho^+ + \rho^-$, $\bar{d} + d$ reactions [32].

- six functions: $\Delta G_{1,2,3}(Q^2, \epsilon)$ and $G_{4,5,6}(Q^2, \epsilon)$, which are, in the general case, complex functions of two variables, Q^2 and ϵ .

We will use the following notations

$$\begin{aligned} G_M^{(\mp)}(Q^2, \epsilon) &= -G_2^{(\mp)}(Q^2, \epsilon), \quad G_Q^{(\mp)}(Q^2, \epsilon) = G_1^{(\mp)}(Q^2, \epsilon) + G_2^{(\mp)}(Q^2, \epsilon) + 2G_3^{(\mp)}(Q^2, \epsilon), \\ G_C^{(\mp)}(Q^2, \epsilon) &= \frac{2}{3}\tau[G_2^{(\mp)}(Q^2, \epsilon) - G_3^{(\mp)}(Q^2, \epsilon)] + (1 + \frac{2}{3}\tau)G_1^{(\mp)}(Q^2, \epsilon). \end{aligned} \quad (10)$$

So, the quantities $G_i^{(\mp)}(Q^2, \epsilon)$, $i = M, Q, C$ can be considered as generalized magnetic, quadrupole and charge FFs.

We can separate the Born and TPE contributions, in these generalized FFs, in the following way

$$G_i^{(\mp)}(Q^2, \epsilon) = \mp G_i(Q^2) + \Delta G_i(Q^2, \epsilon), \quad i = M, Q, C, \quad (11)$$

where $\Delta G_i(Q^2, \epsilon)$ contain the TPE contribution.

III. GENERAL ANALYSIS

In the Laboratory (Lab) system, including the contribution of the TPE mechanism, the unpolarized differential cross section for elastic $e^\mp d$ - scattering can be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4M^2Q^4} \frac{E'}{E} \left(1 + 2\frac{E}{M} \sin^2 \frac{\theta}{2}\right)^{-1} L_{\mu\nu} H_{\mu\nu}, \quad (12)$$

with

$$L_{\mu\nu} = l_\mu l_\nu^*, \quad H_{\mu\nu} = J_\mu J_\nu^*,$$

where $E(E')$ is the energy of the initial (scattered) electron or positron. Here and below we neglect the electron mass where it is possible.

The leptonic tensor, for the case of polarized electron or positron beam, has the form

$$L_{\mu\nu} = q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}) + 2im_e \langle \mu\nu q s_e \rangle, \quad (13)$$

where $\langle \mu\nu ab \rangle = \varepsilon_{\mu\nu\rho\sigma} a_\rho b_\sigma$, m_e is the electron mass and $s_{e\mu}$ is the polarization four-vector of the initial electron or positron.

Since we consider only the case of the polarized target, the hadronic tensor can be expanded according to the polarization state of the initial deuteron as follows:

$$H_{\mu\nu} = H_{\mu\nu}(0) + H_{\mu\nu}(s) + H_{\mu\nu}(Q), \quad (14)$$

where the tensor $H_{\mu\nu}(0)$ corresponds to the unpolarized target, the tensor $H_{\mu\nu}(s)$ ($H_{\mu\nu}(Q)$) describes the case when deuteron target has vector (tensor) polarization.

The spin-density matrix of the initial (polarized) and recoil (unpolarized) deuterons can be written as

$$\begin{aligned} \rho_{i\mu\nu} &= U_{1\mu} U_{1\nu}^* = -\frac{1}{3} \left(g_{\mu\nu} - \frac{p_{1\mu} p_{1\nu}}{M^2} \right) + \frac{i}{2M} \langle \mu\nu s p_1 \rangle + Q_{\mu\nu}, \\ \rho_{f\mu\nu} &= U_{2\mu} U_{2\nu}^* = - \left(g_{\mu\nu} - \frac{p_{2\mu} p_{2\nu}}{M^2} \right), \end{aligned} \quad (15)$$

where s_μ is the polarization four-vector describing the vector polarization of the target ($p_1 \cdot s = 0$, $s^2 = -1$) and $Q_{\mu\nu}$ is the tensor describing the tensor (quadrupole) polarization of the target ($Q_{\mu\nu} = Q_{\nu\mu}$, $Q_{\mu\mu} = 0$, $p_{1\mu} Q_{\mu\nu} = 0$). In Lab system all time components of the tensor $Q_{\mu\nu}$ are zero and the tensor polarization of the target is described by five independent space components ($Q_{ij} = Q_{ji}$, $Q_{ii} = 0$, $i, j = x, y, z$).

In the hadronic current J_μ , the presence of the TPE contribution leads to the terms which contain the momenta from the leptonic vertex. The general structure of the tensor $H_{\mu\nu}(0)$ becomes more complicated: four structure functions are present instead of the two standard structure functions $A(Q^2)$ and $B(Q^2)$. The general structure of this tensor can be written as

$$H_{\mu\nu}(0) = H_1 \tilde{g}_{\mu\nu} + H_2 p_\mu p_\nu + H_3 (k_\mu p_\nu + k_\nu p_\mu) + iH_4 (k_\mu p_\nu - k_\nu p_\mu), \quad (16)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$, $p = p_1 + p_2$. One can get the following expressions for these

structure functions when the hadronic current is given by Eq. (4):

$$\begin{aligned}
H_1 &= \frac{2}{3}q^2 [(1 + \tau)|G_2|^2 - a\text{Re}G_2G_6^*], \\
H_2 &= (1 + 2\tau)|G_1|^2 - \frac{8}{3}\tau\text{Re}(G_4 + 2aG_5)G_1^* + \\
&\quad + \frac{2}{3}(2\tau - 1)\text{Re}(\tau G_1 + 2\tau G_3 - a^2G_4 - 4\tau aG_5 - aG_6)G_1^* + \\
&\quad + \frac{2}{3}\tau\text{Re}[(1 + 2\tau)(2G_1 + G_2 - 4aG_5) - 2a(aG_4 + G_6)]G_2^* - \frac{4}{3}\tau a\text{Re}(aG_4 + G_6)G_3^* + \\
&\quad + 4\tau^2 \left[|G_3|^2 + \frac{2}{3}\text{Re}(2G_4 + G_2 - 2aG_5)G_3^* \right], \\
H_3 &= \frac{2}{3}\text{Re}(\tau G_2 - G_1 + 2\tau G_3)G_6^* + \frac{4}{3}\tau\text{Re}[aG_4 + 2(1 + \tau)G_5]G_2^*, \\
H_4 &= \frac{2}{3}\text{Im}(\tau G_2 + G_1 - 2\tau G_3)G_6^* - \frac{4}{3}\tau\text{Im}[aG_4 + 2(1 + \tau)G_5]G_2^*, \tag{17}
\end{aligned}$$

where $a = k \cdot p_1/M^2$. One can see that the structure functions H_3 and H_4 are completely determined by the TPE contribution. We recover the standard tensor structure for $H_{\mu\nu}(0)$ tensor, if the TPE contribution is absent.

Let us consider the part of the hadronic tensor that corresponds to the vector-polarized deuteron target. It can be represented as the sum of a symmetrical and antisymmetrical tensors (with respect to the indexes μ and ν):

$$H_{\mu\nu}(s) = iA_{\mu\nu}(s) + S_{\mu\nu}(s), \tag{18}$$

where the antisymmetrical tensor $A_{\mu\nu}(s)$ can be written as (we neglect the terms proportional to q_μ or q_ν since the leptonic tensor is conserved, so these terms do not contribute to the observables)

$$\begin{aligned}
A_{\mu\nu}(s) &= A_1 \langle \mu\nu s p_1 \rangle + A_2(p_{1\mu}p_{2\nu} - p_{1\nu}p_{2\mu}) + A_3(p_\mu k_\nu - p_\nu k_\mu) + A_4(A_\mu k_\nu - A_\nu k_\mu) + \\
&\quad + A_5(p_\mu A_\nu - p_\nu A_\mu) + A_6(p_{2\mu}A_\nu - p_{2\nu}A_\mu) + A_7(p_\mu B_\nu - p_\nu B_\mu) + \\
&\quad + A_8(p_{2\mu}B_\nu - p_{2\nu}B_\mu), \tag{19}
\end{aligned}$$

with $A_\mu = \langle \mu p_1 p_2 s \rangle$, $B_\mu = \langle \mu p_1 k s \rangle$, and the structure functions A_{1-8} can be written as

$$\begin{aligned}
A_1 &= 2M\tau [(1 + \tau)|G_2|^2 - a\text{Re}G_2G_6^*], \\
A_2 &= \frac{b}{2M^3}\text{Re}[(2\tau G_5 + aG_4)G_2^* - G_1G_6^*], \\
A_3 &= -\frac{b}{2M^3}\text{Re}G_2G_4^*, \quad A_4 = \frac{1}{2M}\text{Re}G_2G_6^*, \\
A_5 &= \frac{1}{2M}\text{Re}[-2(1 + \tau)G_1G_2^* + 2a\tau G_2G_5^* + aG_1G_6^*], \\
A_6 &= -\frac{1}{2M}(2\tau|G_2|^2 - a\text{Re}G_2G_6^*), \\
A_7 &= \frac{\tau}{M}\text{Re}[aG_4 + 2(1 + \tau)G_5]G_2^*, \\
A_8 &= -\frac{a}{2M}\text{Re}G_2G_6^*, \quad b = \langle skp_1p_2 \rangle. \tag{20}
\end{aligned}$$

The symmetrical tensor $S_{\mu\nu}(s)$ can be represented as (neglecting again the terms proportional to q_μ or q_ν)

$$\begin{aligned}
S_{\mu\nu}(s) &= B_1g_{\mu\nu} + B_2p_\mu p_\nu + B_3p_{2\mu}p_{2\nu} + B_4(p_\mu p_{2\nu} + p_\nu p_{2\mu}) + \\
&+ B_5(p_\mu k_\nu + p_\nu k_\mu) + B_6(A_\mu k_\nu + A_\nu k_\mu) + B_7(A_\mu p_\nu + A_\nu p_\mu) + \\
&+ B_8(A_\mu p_{2\nu} + A_\nu p_{2\mu}) + B_9(p_\mu B_\nu + p_\nu B_\mu) + B_{10}(p_{2\mu}B_\nu + p_{2\nu}B_\mu). \tag{21}
\end{aligned}$$

The structure functions B_{1-10} can be written as

$$\begin{aligned}
B_1 &= -\frac{b}{M}\text{Im}G_2G_6^*, \quad B_2 = \frac{b}{M^3}\text{Im}G_1[aG_4 + 2(1 + \tau)G_5]^*, \quad B_3 = \frac{b}{M^3}\text{Im}G_2G_6^*, \\
B_4 &= \frac{b}{2M^3}\text{Im}[G_1G_6^* - 2\tau G_5G_2^* - aG_4G_2^*], \quad B_5 = \frac{b}{2M^3}\text{Im}G_4G_2^*, \quad B_6 = \frac{1}{2M}\text{Im}G_2G_6^*, \\
B_7 &= -\frac{a}{2M}\text{Im}G_1G_6^* + \frac{1}{M}\text{Im}[(1 + \tau)G_1 - a\tau G_5]G_2^*, \quad B_8 = \frac{a}{2M}\text{Im}G_6G_2^*, \\
B_9 &= -\frac{\tau}{M}\text{Im}[aG_4 + 2(1 + \tau)G_5]G_2^*, \quad B_{10} = \tau\frac{a}{M}\text{Im}G_6G_2^*. \tag{22}
\end{aligned}$$

Let us note that symmetric tensor is completely determined by the TPE terms for the case of the space-like region of the momentum transfer squared (where all deuteron electromagnetic FFs are real functions). So, this tensor vanishes in the Born approximation. As it is determined by the product $\text{Im}(G_C + \tau/3G_Q)G_M^*$, it may be non-zero in time-like region, where deuteron FFs are complex functions.

Let us consider the part of the hadronic tensor that corresponds to a tensor-polarized deuteron target. It can be also written as the sum of a symmetrical and an antisymmetrical tensors:

$$H_{\mu\nu}(Q) = S_{\mu\nu}(Q) + iA_{\mu\nu}(Q). \tag{23}$$

The symmetrical tensor $S_{\mu\nu}(Q)$ can be written as (we neglect here the terms proportional to q_μ or q_ν)

$$\begin{aligned}
S_{\mu\nu}(Q) = & R_1 Q_{\mu\nu} + R_2 \left(g_{\mu\nu} - \frac{p_{2\mu} p_{2\nu}}{M^2} \right) + R_3 p_\mu p_\nu + R_4 (p_\mu k_\nu + p_\nu k_\mu) + \\
& + R_5 (p_\mu p_{2\nu} + p_\nu p_{2\mu}) + R_6 (p_\mu Q_{1\nu} + p_\nu Q_{1\mu}) + R_7 (k_\mu Q_{1\nu} + k_\nu Q_{1\mu}) + \\
& + R_8 (Q_{1\mu} p_{2\nu} + Q_{1\nu} p_{2\mu}) + R_9 (p_\mu Q_{2\nu} + p_\nu Q_{2\mu}) + R_{10} (p_{2\mu} Q_{2\nu} + p_{2\nu} Q_{2\mu}), \quad (24)
\end{aligned}$$

where $Q_{1\mu} = Q_{\mu\nu} q_\nu$, $Q_{2\mu} = Q_{\mu\nu} k_\nu$, and the structure functions R_{1-10} are

$$\begin{aligned}
R_1 &= 4M^2 \tau [(1 + \tau) |G_2|^2 - a \text{Re} G_2 G_6^*], \quad R_2 = -Q_1 |G_2|^2 + 2Q_{12} \text{Re} G_2 G_6^*, \\
R_3 &= \frac{Q_1}{M^2} \text{Re} (G_1 + 2G_3 - 2aG_5) G_1^* + 2 \frac{Q_{11}}{M^2} \text{Re} (G_1 - 2\tau G_3) G_4^* - 2 \frac{Q_{12}}{M^2} \text{Re} (aG_1 G_4^* + \\
& + 2\tau G_1 G_5^* + 4\tau G_3 G_5^*), \quad R_4 = \frac{1}{M^2} \text{Re} (Q_1 G_5 + Q_{12} G_4) G_2^*, \\
R_5 &= \frac{1}{M^2} \text{Re} (Q_1 G_2 - Q_{12} G_6) G_1^* - \frac{1}{M^2} \text{Re} [aQ_{12} G_4 + (aQ_1 + 2\tau Q_{12}) G_5] G_2^*, \\
R_6 &= \text{Re} (2\tau G_2 - aG_6) G_1^* + 2\tau \text{Re} (2G_3 - aG_5) G_2^*, \quad R_7 = \text{Re} G_2 G_6^*, \\
R_8 &= 2\tau |G_2|^2 - a \text{Re} G_2 G_6^*, \quad R_9 = 2\text{Re} (G_1 - 2\tau G_3) G_6^* - 2\tau \text{Re} [aG_4 + 2\tau(1 + \tau) G_5] G_2^*, \\
R_{10} &= -2\tau \text{Re} G_2 G_6^*, \quad (25)
\end{aligned}$$

with $Q_1 = Q_{\mu\nu} q_\mu q_\nu$, $Q_{12} = Q_{\mu\nu} q_\mu k_\nu$, and $Q_{11} = Q_{\mu\nu} k_\mu k_\nu$. The antisymmetrical tensor $A_{\mu\nu}(Q)$ has the form (also neglecting the terms proportional to q_μ or q_ν)

$$\begin{aligned}
A_{\mu\nu}(Q) = & W_1 (p_\mu k_\nu - p_\nu k_\mu) + W_2 (p_\mu p_{2\nu} - p_\nu p_{2\mu}) + W_3 (p_\mu Q_{1\nu} - p_\nu Q_{1\mu}) + \\
& + W_4 (k_\mu Q_{1\nu} - k_\nu Q_{1\mu}) + W_5 (p_{2\mu} Q_{1\nu} - p_{2\nu} Q_{1\mu}) + W_6 (p_\mu Q_{2\nu} - p_\nu Q_{2\mu}) + \\
& + W_7 (p_{2\mu} Q_{2\nu} - p_{2\nu} Q_{2\mu}), \quad (26)
\end{aligned}$$

where the structure functions W_{1-7} are

$$\begin{aligned}
W_1 &= \frac{1}{M^2} \text{Im} (Q_1 G_5 + Q_{12} G_4) G_2^*, \\
W_2 &= -\frac{1}{M^2} \text{Im} G_1 (Q_1 G_2 + Q_{12} G_6)^* + \frac{1}{M^2} \text{Im} G_2 [aQ_{12} G_4 + (aQ_1 + 2\tau Q_{12}) G_5]^*, \\
W_3 &= \text{Im} G_1 (2\tau G_2 - aG_6)^* + 2\tau \text{Im} (2G_3 - aG_5) G_2^*, \\
W_4 &= \text{Im} G_2 G_6^*, \quad W_5 = -a \text{Im} G_2 G_6^*, \\
W_6 &= 2\text{Im} (G_1 - 2\tau G_3) G_6^* - 2\tau \text{Im} [aG_4 + 2(1 + \tau) G_5] G_2^*, \\
W_7 &= 2\tau \text{Im} G_2 G_6^*. \quad (27)
\end{aligned}$$

For simplicity, we omitted in the hadronic structure functions the upper indexes (\mp) referring to electron- or positron-scattering. The expressions for all hadronic structure functions hold in both cases and the expression for the amplitudes should be understood as: $G_i = G_i^{(\mp)}(Q^2, \epsilon)$.

IV. T-EVEN POLARIZATION OBSERVABLES

Let us specify the coordinate frame in the Lab system: the z axis is directed along the momentum transfer \vec{q} and the momenta of the initial and scattered electrons lie in the xz plane. The y axis is directed along the direction of the vector $\vec{q} \times \vec{k}_1$.

The following general formula holds for the differential cross section of the elastic scattering of an unpolarized electron (positron) beam by an unpolarized deuteron target (taking into account the TPE contribution at the level of its interference with the Born term):

$$\begin{aligned} \frac{d\sigma_{un}^{(\mp)}}{d\Omega} &= \sigma_0 N^{(\mp)}(Q^2, \epsilon), \quad N^{(\mp)}(Q^2, \epsilon) = A^{(\mp)}(Q^2, \epsilon) + B^{(\mp)}(Q^2, \epsilon) \tan^2 \frac{\theta}{2}, \\ \sigma_0 &= \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \left(1 + 2 \frac{E}{M} \sin^2 \frac{\theta}{2} \right)^{-1}. \end{aligned} \quad (28)$$

The functions $A^{(\mp)}(Q^2, \epsilon)$ and $B^{(\mp)}(Q^2, \epsilon)$ contain the TPE contribution and they have the following form (the signs (\mp) correspond to the $e^{(\mp)}d$ -scattering)

$$A^{(\mp)}(Q^2, \epsilon) = A(Q^2) \mp \Delta A(Q^2, \epsilon), \quad B^{(\mp)}(Q^2, \epsilon) = B(Q^2) \mp \Delta B(Q^2, \epsilon),$$

where the structure functions $A(Q^2)$ and $B(Q^2)$ are the standard real functions of a single variable Q^2 describing the elastic ed -scattering in the Born approximation. They are quadratic combinations of the deuteron electromagnetic FFs

$$A(Q^2) = G_C^2(Q^2) + \frac{2}{3}\tau G_M^2(Q^2) + \frac{8}{9}\tau^2 G_Q^2(Q^2), \quad B(Q^2) = \frac{4}{3}\tau(1 + \tau)G_M^2(Q^2).$$

The additional terms $\Delta A(Q^2, \epsilon)$ and $\Delta B(Q^2, \epsilon)$ are due to the TPE contribution and they

can be written as

$$\begin{aligned}
\Delta A(Q^2, \epsilon) &= 2G_C(Q^2)Re\Delta G_C(Q^2, \epsilon) + \frac{4}{3}\tau G_M(Q^2)Re\Delta G_M(Q^2, \epsilon) + \\
&+ \frac{16}{9}\tau^2 G_Q(Q^2)Re\Delta G_Q(Q^2, \epsilon) + \\
&+ \frac{8}{3}\tau \left\{ \frac{2}{3}\tau G_Q(Q^2) - G_C(Q^2) + c^2 \left[(1-\tau)G_C(Q^2) + 2\tau G_M(Q^2) - \frac{2}{3}\tau^2 G_Q(Q^2) \right] \right\} \\
&ReG_4(Q^2, \epsilon) + \frac{16}{3}c\tau\sqrt{\tau(1+\tau)}(G_M(Q^2) - G_C(Q^2) - \frac{4}{3}\tau G_Q(Q^2))ReG_5(Q^2, \epsilon) + \\
&+ \frac{4}{3}c\sqrt{\frac{\tau}{1+\tau}} \left[(1-\tau)G_C(Q^2) + 2\tau G_M(Q^2) - \frac{2}{3}\tau(1+2\tau)G_Q(Q^2) \right] ReG_6(Q^2, \epsilon), \\
\Delta B(Q^2, \epsilon) &= \frac{8}{3}\tau G_M(Q^2) \left[(1+\tau)Re\Delta G_M(Q^2, \epsilon) + c\sqrt{\tau(1+\tau)}ReG_6(Q^2, \epsilon) \right], \quad (29)
\end{aligned}$$

where

$$c = \sqrt{\frac{1+\epsilon}{1-\epsilon}} = \sqrt{1 + \frac{\cot^2 \frac{\theta}{2}}{1+\tau}}.$$

Note that these formulas were obtained neglecting the terms of the order of α^2 compared to the dominant (Born approximation) terms. In the Born approximation these expressions reduce to the well known result for the differential cross section of elastic ed -scattering (see, for example, [33]).

The structure function $B^{(\mp)}(Q^2, \epsilon)$, which is determined in the Born approximation by the magnetic FF only, acquires two additional terms proportional to $Re\Delta G_M(Q^2, \epsilon)$ and $ReG_6(Q^2, \epsilon)$.

The real parts of all six complex TPE amplitudes contribute to the structure function $A^{(\mp)}(Q^2, \epsilon)$, which is determined in the Born approximation by all three deuteron FFs.

One can see that the sum of the differential cross sections for the $e^\mp d$ -scatterings has precisely the Rosenbluth form, in terms of the standard deuteron electromagnetic FFs, since the TPE contribution is canceled out

$$\Sigma = \frac{1}{2} \left(\frac{d\sigma_{un}^{(-)}}{d\Omega} + \frac{d\sigma_{un}^{(+)}}{d\Omega} \right) = \sigma_0 \left[A(Q^2) + B(Q^2) \tan^2 \frac{\theta}{2} \right]. \quad (30)$$

This quantity allows to separate the magnetic FF $G_M(Q^2)$ and the following combination of the charge and quadrupole FFs: $G(Q^2) = G_C^2(Q^2) + \frac{8}{9}\tau^2 G_Q^2(Q^2)$, in presence of the TPE contribution. It is a model independent statement, taking into account the interference of the one- and two-photon exchange amplitudes.

On the contrary, the difference of the differential cross sections for the $e^\mp d$ -scatterings is completely determined by the interference of the one- and two-photon exchange amplitudes and it can be written as

$$\frac{1}{2} \left(\frac{d\sigma_{un}^{(+)}}{d\Omega} - \frac{d\sigma_{un}^{(-)}}{d\Omega} \right) = \sigma_0 \left[\Delta A(Q^2, \epsilon) + \Delta B(Q^2, \epsilon) \tan^2 \frac{\theta}{2} \right]. \quad (31)$$

This quantity contains information about the size of the TPE term and its dependence on the variables Q^2 and ϵ . We can see if there is a relative increase of this contribution, in comparison with the Born mechanism, when the variable Q^2 increases.

Let us consider the asymmetries arising from the tensor polarization of the deuteron target. The differential cross section can be written in this case as

$$\frac{d\sigma^{(\mp)}}{d\Omega} = \frac{d\sigma_{un}^{(\mp)}}{d\Omega} \left(1 + A_{zz}^{(\mp)}(Q^2, \epsilon) Q_{zz} + A_{xz}^{(\mp)}(Q^2, \epsilon) Q_{xz} + A_{xx}^{(\mp)}(Q^2, \epsilon) (Q_{xx} - Q_{yy}) \right), \quad (32)$$

with the following decomposition of the asymmetries $A_{ij}^{(\mp)}(Q^2, \epsilon)$, ($ij = xx, xz, zz$):

$$N^{(\mp)}(Q^2, \epsilon) A_{zz}^{(\mp)}(Q^2, \epsilon) = A_{zz}(Q^2, \epsilon) \mp \Delta A_{zz}(Q^2, \epsilon). \quad (33)$$

The explicit expressions of the asymmetries as a function of FFs are:

- for the zz -component

$$\begin{aligned} A_{zz}(Q^2, \epsilon) &= 4\tau G_Q(Q^2) [G_C(Q^2) + \frac{\tau}{3} G_Q(Q^2)] + \frac{\tau}{2} [1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}] G_M^2(Q^2), \\ \Delta A_{zz}(Q^2, \epsilon) &= \tau [1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}] G_M(Q^2) \text{Re} \Delta G_M(Q^2, \epsilon) + 4\tau [G_C(Q^2) \text{Re} \Delta G_Q(Q^2, \epsilon) + \\ &\quad + G_Q(Q^2) \text{Re}(\Delta G_C(Q^2, \epsilon) + \frac{2}{3} \tau \Delta G_Q(Q^2, \epsilon))] + 4\tau [(1 - c^2 - 2c^2 \tau) G_C(Q^2) + \\ &\quad + c^2 \tau G_M(Q^2) - \frac{2}{3} \tau (1 - c^2 + 4c^2 \tau) G_Q(Q^2)] \text{Re} G_4(Q^2, \epsilon) + \\ &\quad + 4c\tau \sqrt{\tau(1 + \tau)} \left[G_M(Q^2) - 4(G_C(Q^2) + \frac{4}{3} \tau G_Q(Q^2)) \right] \text{Re} G_5(Q^2, \epsilon) - \\ &\quad - 2c \sqrt{\tau(1 + \tau)} \tan^2 \frac{\theta}{2} [\tau(\tau - c^2) G_M(Q^2) + \\ &\quad + (c^2 - 1)(1 + 2\tau) \left(G_C(Q^2) - \frac{2}{3} \tau G_Q(Q^2) \right)] \text{Re} G_6(Q^2, \epsilon), \end{aligned} \quad (34)$$

with

$$N^{(\mp)}(Q^2, \epsilon) A_{xz}^{(\mp)}(Q^2, \epsilon) = A_{xz}(Q^2, \epsilon) \mp \Delta A_{xz}(Q^2, \epsilon), \quad (35)$$

- for the xz -component

$$\begin{aligned}
A_{xz}(Q^2, \epsilon) &= -4\tau \sec \frac{\theta}{2} \sqrt{\tau(1 + \tau \sin^2 \frac{\theta}{2})} G_M(Q^2) G_Q(Q^2), \\
\Delta A_{xz}(Q^2, \epsilon) &= -4\tau \sec \frac{\theta}{2} \sqrt{\tau(1 + \tau \sin^2 \frac{\theta}{2})} \{ G_Q(Q^2) \text{Re} \Delta G_M(Q^2, \epsilon) + \\
&\quad + G_M(Q^2) \text{Re} \Delta G_Q(Q^2, \epsilon) + 2[\tau(c^2 - 1)G_Q(Q^2) - (c^2 - 1 + c^2\tau)G_M(Q^2)] \\
&\quad \text{Re} G_4(Q^2, \epsilon) - 4c\sqrt{\tau(1 + \tau)} \left[G_M(Q^2) + \frac{(1 - c^2)}{c^2} G_Q(Q^2) \right] \text{Re} G_5(Q^2, \epsilon) - \\
&\quad - \frac{1}{2c\sqrt{\tau(1 + \tau)}} [(1 - \tau + 2c^2 + c^2\tau)G_M(Q^2) + \\
&\quad + 2\tau(2c^2 - 1)(G_M(Q^2) - G_Q(Q^2))] \text{Re} G_6(Q^2, \epsilon) \}, \tag{36}
\end{aligned}$$

with

$$N^{(\mp)}(Q^2, \epsilon) A_{xx}^{(\mp)}(Q^2, \epsilon) = A_{xx}(Q^2) \mp \Delta A_{xx}(Q^2, \epsilon), \tag{37}$$

- and for the xx -component

$$\begin{aligned}
A_{xx}(Q^2) &= \frac{\tau}{2} G_M^2(Q^2), \\
\Delta A_{xx}(Q^2, \epsilon) &= \tau G_M(Q^2) \text{Re} \Delta G_M(Q^2, \epsilon) + 4\tau [c^2 \tau G_M(Q^2) + (c^2 - 1)(G_C(Q^2) - \\
&\quad - \frac{2}{3} \tau G_Q(Q^2))] \text{Re} G_4(Q^2, \epsilon) + 4c\tau \sqrt{\tau(1 + \tau)} G_M(Q^2) \text{Re} G_5(Q^2, \epsilon) + \\
&\quad + 2c \sqrt{\frac{\tau}{(1 + \tau)}} [\tau G_M(Q^2) + G_C(Q^2) - \frac{2}{3} \tau G_Q(Q^2)] \text{Re} G_6(Q^2, \epsilon). \tag{38}
\end{aligned}$$

Note that the terms $A_{zz}(Q^2, \epsilon)$, $A_{xz}(Q^2, \epsilon)$ and $A_{xx}(Q^2)$ are the asymmetries in the Born approximation and they coincide with the corresponding results of Ref. [34].

Due to the symmetry properties following from C-invariance, it is also interesting to build the sum and difference of the corresponding expressions for the electron and positron asymmetries due to the tensor-polarized deuteron target:

$$\begin{aligned}
\mathcal{A}_{zz}^\pm(Q^2, \epsilon) &= \frac{1}{2} [A_{zz}^{(-)}(Q^2, \epsilon) \pm A_{zz}^{(+)}(Q^2, \epsilon)], \\
\mathcal{A}_{xz}^\pm(Q^2, \epsilon) &= \frac{1}{2} [A_{xz}^{(-)}(Q^2, \epsilon) \pm A_{xz}^{(+)}(Q^2, \epsilon)], \\
\mathcal{A}_{xx}^\pm(Q^2, \epsilon) &= \frac{1}{2} [A_{xx}^{(-)}(Q^2, \epsilon) \pm A_{xx}^{(+)}(Q^2, \epsilon)]. \tag{39}
\end{aligned}$$

One can see that the quantities $\mathcal{A}_{ij}^\pm(Q^2, \epsilon)$, ($ij = zz, xz, xx$) do not contain the TPE contribution, neglecting terms containing the square of the TPE amplitudes. In this approximation

we have

$$\begin{aligned} N\mathcal{A}_{zz}^+ &= A_{zz}, \quad N\mathcal{A}_{xz}^+ = A_{xz}, \quad N\mathcal{A}_{xx}^+ = A_{xx}, \\ N &= \frac{1}{2}(N^{(+)} + N^{(-)}) = A(Q^2) + B(Q^2) \tan^2 \frac{\theta}{2}, \end{aligned} \quad (40)$$

coinciding with the asymmetries obtained in the Born approximation.

The standard procedure for the determination of deuteron electromagnetic FFs consists in measuring the unpolarized differential cross section (at various electron scattering angles but at the same Q^2 value) and one additional polarization observable (it is usually the asymmetry A_{zz} due to the tensor polarization of the deuteron target or t_{20} , the tensor polarization of the recoil deuteron, with unpolarized electron beam). The measurement of the quantity Σ , Eq. (30), and of \mathcal{A}_{zz}^+ (or \mathcal{A}_{xz}^+), Eq. (39), can be considered as the generalization of the standard procedure for extracting electromagnetic FFs, in presence of the TPE mechanism.

On the contrary, the differences of the tensor asymmetries in the elastic electron– or positron–deuteron scatterings, due to the tensor polarization of the target, are proportional to the interference of the Born amplitude and real part of the TPE contribution. In the same approximation we have

$$\begin{aligned} N\mathcal{A}_{zz}^- &= rA_{zz} - \Delta A_{zz}, \quad N\mathcal{A}_{xz}^- = rA_{xz} - \Delta A_{xz}, \quad N\mathcal{A}_{xx}^- = rA_{xx} - \Delta A_{xx}, \\ r &= \frac{1}{N}(\Delta A + \Delta B \tan^2 \frac{\theta}{2}). \end{aligned} \quad (41)$$

The measurement of these quantities (T–even polarization observables) is sensitive to the relative contribution of the real part of TPE term with respect to the Born approximation.

Let us consider the double–spin asymmetries due to the longitudinal polarization of the electron beam and the vector polarization of the deuteron target (the transverse components of the electron spin lead to the asymmetries suppressed by a factor (m_e/M) and they are considered below). So, the longitudinal polarization of the electron beam leads to two asymmetries which can be written as

$$N^{(\mp)}(Q^2, \epsilon)A_x^{(\mp)}(Q^2, \epsilon) = A_x(Q^2, \epsilon) \mp \Delta A_x(Q^2, \epsilon), \quad (42)$$

with

$$\begin{aligned}
A_x(Q^2, \epsilon) &= -2\sqrt{\tau(1+\tau)} \tan \frac{\theta}{2} G_M(Q^2) \left[G_C(Q^2) + \frac{1}{3}\tau G_Q(Q^2) \right], \\
\Delta A_x(Q^2, \epsilon) &= -2\sqrt{\tau(1+\tau)} \tan \frac{\theta}{2} \left\{ G_M(Q^2) \text{Re}[\Delta G_C(Q^2, \epsilon) + \frac{1}{2}(1+2\tau)\Delta G_M(Q^2, \epsilon) + \right. \\
&\quad \left. + \frac{1}{3}\tau\Delta G_Q(Q^2, \epsilon)] - \left[G_C(Q^2) + \frac{1}{3}\tau G_Q(Q^2) \right] \text{Re}\Delta G_M(Q^2, \epsilon) - \right. \\
&\quad \left. - 2c^2\tau^2 G_M(Q^2) \text{Re}G_4(Q^2, \epsilon) - 4c\tau\sqrt{\tau(1+\tau)} G_M(Q^2) \text{Re}G_5(Q^2, \epsilon) - \right. \\
&\quad \left. - \frac{c}{2}\sqrt{\frac{\tau}{1+\tau}} \left[(1+\tau - c\sqrt{\tau(1+\tau)}) G_M(Q^2) - \right. \right. \\
&\quad \left. \left. - 2 \left(G_C(Q^2) + \frac{1}{3}\tau G_Q(Q^2) \right) \right] \text{Re}G_6(Q^2, \epsilon) \right\}, \tag{43}
\end{aligned}$$

and

$$N^{(\mp)}(Q^2, \epsilon) A_z^{(\mp)}(Q^2, \epsilon) = A_z(Q^2, \epsilon) \mp \Delta A_z(Q^2, \epsilon), \tag{44}$$

with

$$\begin{aligned}
A_z(Q^2, \epsilon) &= -\tau\sqrt{(1+\tau)(1+\tau\sin^2\frac{\theta}{2})} \tan \frac{\theta}{2} \sec \frac{\theta}{2} G_M^2(Q^2), \\
\Delta A_z(Q^2, \epsilon) &= -\tau\sqrt{(1+\tau)(1+\tau\sin^2\frac{\theta}{2})} \tan \frac{\theta}{2} \sec \frac{\theta}{2} G_M(Q^2) \left\{ 2\text{Re}\Delta G_M(Q^2, \epsilon) + \right. \\
&\quad \left. + \left[2c\sqrt{\frac{\tau}{1+\tau}} - 1 - c^2 \right] \text{Re}G_6(Q^2, \epsilon) + \right. \\
&\quad \left. + 4\tau(1+c^2) \left[\text{Re}G_4(Q^2, \epsilon) + \frac{1}{c}\sqrt{\frac{1+\tau}{\tau}} \text{Re}G_5(Q^2, \epsilon) \right] \right\}. \tag{45}
\end{aligned}$$

In the Born approximation these expressions coincide with the results of Ref. [34] except the general sign, since in that paper another sign of the vector part of the deuteron spin-density matrix was taken.

Note that we can also remove or extract the TPE contribution in these double-spin asymmetries in a similar way as it was done for the differential cross section and the tensor asymmetries.

V. T-ODD POLARIZATION OBSERVABLES

Let us consider the single-spin asymmetry induced by the transverse polarization of the electron or positron beam. The expressions for the spin-dependent leptonic tensor and for

the hadronic tensor, for the case of the unpolarized final state, show that the single-spin asymmetry is proportional to the TPE term and suppressed by the factor (m_e/M) .

The measurement of this small asymmetry is planned in next future [35]. As mentioned in the Introduction, in spite of the suppression factor, recent measurements of the asymmetry in the scattering of transversely polarized electrons on unpolarized protons found values different from zero, contrary to what is expected in the Born approximation [26, 27], and only one experiment measured a single-spin observable, the recoil-deuteron vector polarization for the elastic scattering of unpolarized electrons by unpolarized deuteron target [28].

To calculate the beam asymmetry, it is necessary to take into account also the small amplitudes (neglected earlier, since they give a small contribution to the other observables) which are proportional to the electron mass (the so-called helicity flip amplitudes). The part of the matrix element of the reaction $e^- + d \rightarrow e^- + d$, containing the helicity flip amplitudes, can be established in analogy with the elastic nucleon-deuteron scattering [29], using the general properties of the electron-hadron interaction, such as the Lorentz invariance and P-invariance. It can be written as follows

$$\begin{aligned}
M^{(flip)} = & \frac{m_e}{M} \frac{e^2}{Q^2} \bar{u}(k_2) \left[MG_7(s, Q^2) U_1 \cdot U_2^* + \frac{1}{M} G_8(s, Q^2) U_1 \cdot k U_2^* \cdot k + \right. \\
& + \frac{1}{M} G_9(s, Q^2) U_1 \cdot p U_2^* \cdot p + \frac{1}{M} G_{10}(s, Q^2) (U_1 \cdot k U_2^* \cdot p + U_2^* \cdot k U_1 \cdot p) + \\
& + \frac{1}{M} G_{11}(s, Q^2) (U_1 \cdot p \hat{U}_2^* \hat{p} - U_2^* \cdot p \hat{U}_1 \hat{p}) + \frac{1}{M} G_{12}(s, Q^2) (U_1 \cdot k \hat{U}_2^* \hat{p} - \\
& \left. - U_2^* \cdot k \hat{U}_1 \hat{p} - U_1 \cdot k U_2^* \cdot p + U_2^* \cdot k U_1 \cdot p) \right] u(k_1), \tag{46}
\end{aligned}$$

where all these amplitudes $G_i(s, Q^2)$ ($i = 7 - 12$) are, in general case, complex functions of two variables and vanish in the Born approximation $G_i^{(Born)}(s, Q^2) = 0$, $i = 7 - 12$.

The corresponding asymmetry can be written as

$$\begin{aligned}
N^{(\mp)}(Q^2, \epsilon) A_e^{(\mp)}(Q^2, \epsilon) = & \\
& \mp \frac{4}{3} \frac{m_e}{M} \tan \frac{\theta}{2} s_y^{(\mp)} \left\{ -\tau G_M(Q^2) [ImG_6(Q^2, \epsilon) + 4(1 + \tau)ImG_5(Q^2, \epsilon) + \right. \\
& + 2c\sqrt{\tau(1 + \tau)} (2ImG_4(Q^2, \epsilon) + ImG_{10}(Q^2, \epsilon) - \\
& - 2(1 + \tau)ImG_{11}(Q^2, \epsilon)) + 2c^2 (\tau ImG_8(Q^2, \epsilon) - \\
& - (1 + \tau)(2\tau - 1)ImG_{12}(Q^2, \epsilon))] + G_C(Q^2)(ImG_6(Q^2, \epsilon) + \\
& + \frac{3}{4}ImG_7(Q^2, \epsilon)) - \frac{2}{3}\tau G_Q(Q^2) [ImG_6(Q^2, \epsilon) + 3\tau(1 - c^2)ImG_8(Q^2, \epsilon)] + \\
& + \tau(G_C(Q^2) + \frac{4}{3}\tau G_Q(Q^2)) \left[\frac{1}{2}ImG_7(Q^2, \epsilon) + (1 + \tau)(ImG_9(Q^2, \epsilon) + \right. \\
& + 2c^2ImG_{12}(Q^2, \epsilon)) + (1 - c^2 + c^2\tau)ImG_8(Q^2, \epsilon) + \\
& \left. \left. + 2c\sqrt{\tau(1 + \tau)} \left(ImG_{10}(Q^2, \epsilon) + \frac{1 + \tau}{\tau}ImG_{11}(Q^2, \epsilon) \right) \right] \right\}, \quad (47)
\end{aligned}$$

where $A_e^{(-)}$ ($A_e^{(+)}$) is the single-spin asymmetry (the so-called beam asymmetry) in the scattering of transversely polarized electron (positron) beam by unpolarized deuteron target, and $\vec{s}^{(-)}$ ($\vec{s}^{(+)}$) is the unit vector describing the polarization of the electron (positron) beam in its rest frame. One can see that

- $A_e^{(\pm)}$ is proportional to the electron mass and it is determined by the electron or positron spin component perpendicular to the reaction plane.

- $A_e^{(\pm)}$ is a T-odd observable and it vanishes in the Born approximation as it is determined by the imaginary part of the interference between the one- and two-photon exchange amplitudes. Thus, the asymmetry A_e is determined by the three real electromagnetic form factors $G_M(Q^2), G_C(Q^2), G_Q(Q^2)$ as well as by the complex TPE amplitudes: $G_4(Q^2, \epsilon), G_5(Q^2, \epsilon)$ and $G_6(Q^2, \epsilon)$ (helicity conserving) and $G_i(Q^2, \epsilon)$, ($i = 7 - 12$) (helicity non conserving). Therefore, this observable contains all amplitudes on equal footing, i.e., here the helicity flip amplitudes are not suppressed in comparison with the helicity conserving ones. Measurement of this asymmetry in the case of elastic electron-deuteron scattering may be a more difficult task than for the case of elastic electron-nucleon scattering due mainly to the fact that deuteron FFs are much smaller than nucleon FFs.

- $A_e^{(\pm)}$ vanishes, for $\theta = 0^0$ and 180^0 , as it is determined by the product $(\vec{q} \times \vec{k}_1) \cdot \vec{s}_e$, and in this case $\vec{q} \parallel \vec{k}_1$.

Let us consider now the single-spin asymmetry due to the vector-polarized deuteron target (the so-called target normal-spin asymmetry). The corresponding asymmetry $A_y^{(\mp)}$

can be written as

$$\begin{aligned}
N^{(\mp)}(Q^2, \epsilon) A_y^{(\mp)}(Q^2, \epsilon) = & \pm \tan \frac{\theta}{2} s_y \left\{ 2c \sqrt{\tau(1+\tau)} [G_M(Q^2) \text{Im} \Delta G(Q^2, \epsilon) + \right. \\
& + G(Q^2) \text{Im} \Delta G_M(Q^2, \epsilon)] + 2c\tau \sqrt{\tau(1+\tau)} [(1-\tau)(2c^2-1)G_M(Q^2) - \\
& - 4(c^2-1)G(Q^2)] \text{Im} G_4(Q^2, \epsilon) - 2\tau(1+\tau) [\tau(4c^2-1)G_M(Q^2) + \\
& + 4(c^2-1)G(Q^2)] \text{Im} G_5(Q^2, \epsilon) + \tau [2c\tau \sqrt{\tau(1+\tau)} G_M(Q^2) + \\
& + (4c^2-1)(G(Q^2) + G_M(Q^2))] + \\
& \left. + 2(1-\tau)((1+\tau)c^2-1)G_M(Q^2)] \text{Im} G_6(Q^2, \epsilon) \right\}, \tag{48}
\end{aligned}$$

where $G(Q^2, \epsilon) = G_C(Q^2, \epsilon) + \tau/3G_Q(Q^2, \epsilon)$, \vec{s} is the deuteron unit polarization vector describing the vector polarization of the target.

One can see that

- $A_y^{(\mp)}$ is determined by the component of the deuteron polarization vector perpendicular to the reaction plane, i.e., by the following product $(\vec{k}_1 \times \vec{k}_2) \cdot \vec{s}$.
- $A_y^{(\mp)}$ vanishes when $\vec{k}_2 \parallel \vec{k}_1$, i.e., for forward and backward scatterings.
- $A_y^{(\mp)}$ is a T-odd observable and it is zero in the Born approximation. It is determined by the interference of the one- and two-photon exchange amplitudes (by the imaginary parts of all six complex TPE helicity conserving amplitudes).

At last let us consider the scattering of the longitudinally polarized electron or positron beam by the tensor-polarized deuteron target. In this case, the asymmetries can be written as follows:

$$\begin{aligned}
N^{(\mp)}(Q^2, \epsilon) A_{xy}^{(\mp)}(Q^2, \epsilon) = & \mp 2 \sqrt{\frac{\tau}{1+\tau}} \left\{ \left[\tau G_M(Q^2) - 2G_C(Q^2) + \frac{4}{3}\tau G_Q(Q^2) \right] \text{Im} G_6(Q^2, \epsilon) + \right. \\
& \left. + 4\tau G_M(Q^2) \text{Im} \left[c \sqrt{\tau(1+\tau)} G_4(Q^2, \epsilon) + (1+\tau) G_5(Q^2, \epsilon) \right] \right\}, \\
N^{(\mp)}(Q^2, \epsilon) A_{yz}^{(\mp)}(Q^2, \epsilon) = & \mp 2\tau \sqrt{\frac{\tau}{1+\tau}} \tan \frac{\theta}{2} \left\{ 2(1+\tau) [G_Q(Q^2) \text{Im} \Delta G_M(Q^2, \epsilon) - \right. \\
& - G_M(Q^2) \text{Im} \Delta G_Q(Q^2, \epsilon)] + \\
& + 4c(1+\tau) \sqrt{\tau} G_M(Q^2) \text{Im} \left[c \sqrt{\tau} G_4(Q^2, \epsilon) + 2\sqrt{(1+\tau)} G_5(Q^2, \epsilon) \right] + \\
& \left. + c \sqrt{\frac{1+\tau}{\tau}} [2\tau G_Q(Q^2) + (2\tau-1)G_M(Q^2)] \text{Im} G_6(Q^2, \epsilon) \right\}. \tag{49}
\end{aligned}$$

Note that the asymmetry A_{xy} is determined by the imaginary parts of the amplitudes $G_{4,5,6}$, which differ in spin structure from the Born spin structure. Both asymmetries are zero

in the Born approximation since they are determined by the interference of the one- and two-photon exchange mechanisms.

The polarized deuteron targets, generally used in high-energy experiments, have zero Q_{xy} and Q_{yz} parameters, since the polarization state is determined by the population numbers $n_{\pm,0}$, i.e., by the diagonal elements of the spin-density matrix of the deuteron target. The determination of these asymmetries requires a polarized deuteron targets with non-zero Q_{xy} and Q_{yz} parameters or the measurement of the corresponding components of the tensor polarization of the recoil deuteron (the target in this case is unpolarized).

In polarization experiments it is possible to prepare the deuteron target with polarization along (opposite) a definite direction. In our case the natural direction is the virtual photon momentum (or z axis). Similar polarization effects were considered in Ref. [36]: longitudinal and transverse polarizations of the recoil deuteron in the elastic electron-deuteron scattering.

Let us consider the case when the spin of the deuteron target has definite projection on z axis. It is convenient in this case to write the contraction of the leptonic and hadronic tensors in the following general form

$$S = L_{\mu\nu}H_{\mu\nu} = S_{\mu\nu}U_{\mu}U_{\nu}^*, \quad (50)$$

where U_{μ} is the polarization four-vector of the deuteron target. Then, with unpolarized electron beam, the $S_{\mu\nu}$ tensor can be written as

$$S_{\mu\nu} = S_1g_{\mu\nu} + S_2q_{\mu}q_{\nu} + S_3k_{\mu}k_{\nu} + S_4(q_{\mu}k_{\nu} + q_{\nu}k_{\mu}) + iS_5(q_{\mu}k_{\nu} - q_{\nu}k_{\mu}), \quad (51)$$

where the structure functions S_i , ($i = 1 - 5$), can be expressed in terms of the generalized

form factors $G_i(Q^2, \epsilon)$, $i = M, C, Q$, and of the amplitudes $G_i(Q^2, \epsilon)$, $i = 4, 5, 6$, as follows:

$$\begin{aligned}
S_1 &= -q^4 \left[(1 + \tau) |G_M|^2 + \frac{1}{\tau} \cot^2 \frac{\theta}{2} |G_C - \frac{2}{3} \tau G_Q|^2 + 2c \sqrt{\tau(1 + \tau)} \text{Re} G_M G_6^* \right], \\
S_2 &= -q^2 \left(1 + \tau + \cot^2 \frac{\theta}{2} \right) \left[|G_M|^2 + 4 \frac{\tau}{1 + \tau} \text{Re} G_M G_Q^* \right] - 4 \frac{q^2}{1 + \tau} \cot^2 \frac{\theta}{2} \left[\frac{\tau}{3} |G_Q|^2 + \right. \\
&\quad \left. + \text{Re} G_C G_Q^* \right] + 4cq^2 \sqrt{\frac{\tau}{1 + \tau}} \text{Re} \left\{ G_M [2(1 + \tau)(1 - c^2 + \tau c^2) G_5 + G_6]^* + \right. \\
&\quad \left. + G_C \left[4 \cot^2 \frac{\theta}{2} G_5 - (2 + \tau) G_6 \right]^* + \frac{\tau}{3} G_Q \left[4 \cot^2 \frac{\theta}{2} G_5 + (1 + 2\tau) G_6 \right]^* \right\}, \\
S_3 &= 4M^2 \tau (1 + \tau) \text{Re} G_M \left[G_M + 8(\tau G_4 + c \sqrt{\tau(1 + \tau)} G_5) \right]^* + \\
&\quad + 16M^2 \text{Re} \left(G_C - \frac{2}{3} \tau G_Q + \tau G_M \right) \left[2\tau \cot^2 \frac{\theta}{2} G_4 + c \sqrt{\tau(1 + \tau)} G_6 \right]^*, \\
S_4 &= cq^2 \sqrt{\tau(1 + \tau)} \left[|G_M|^2 + 2 \text{Re} G_M G_Q^* \right] + \\
&\quad + q^2 \text{Re} \left\{ G_M \left[-4c \sqrt{\tau(1 + \tau)} \left(3\tau + \frac{1 - \tau}{1 + \tau} \cot^2 \frac{\theta}{2} \right) G_4 + 8\tau \left(1 + \tau + \cot^2 \frac{\theta}{2} \right) G_5 + \right. \right. \\
&\quad \left. \left. + \left(2\tau - 3 - 2 \frac{1 - \tau}{1 + \tau} \cot^2 \frac{\theta}{2} \right) G_6 \right]^* + 2 \left(G_C - \frac{2}{3} \tau G_Q \right) \left[4c \sqrt{\frac{\tau}{1 + \tau}} \cot^2 \frac{\theta}{2} G_4 + \right. \right. \\
&\quad \left. \left. + \left(2 - \tau + \frac{2}{1 + \tau} \cot^2 \frac{\theta}{2} \right) G_6 \right]^* + 2\tau G_Q \left[4c \sqrt{\frac{\tau}{1 + \tau}} \cot^2 \frac{\theta}{2} G_4 + \right. \right. \\
&\quad \left. \left. + 4 \cot^2 \frac{\theta}{2} G_5 + \left(1 + \frac{2}{1 + \tau} \cot^2 \frac{\theta}{2} \right) G_6 \right]^* \right\}, \\
S_5 &= 8cM^2 \sqrt{\tau(1 + \tau)} \text{Im} G_M (G_C + \frac{\tau}{3} G_Q)^* + \\
&\quad + q^2 \text{Im} \left\{ 4 \cot^2 \frac{\theta}{2} \left(G_C + \frac{\tau}{3} G_Q \right) \left[2G_5 + \frac{1}{1 + \tau} \left(2c \sqrt{\tau(1 + \tau)} G_4 + G_6 \right) \right]^* + \right. \\
&\quad \left. + 2\tau \left[\left(1 + \frac{2}{3} \tau \right) G_Q - G_C \right] G_6^* + G_M \left[-12c\tau \sqrt{\tau(1 + \tau)} G_4 + (2\tau - 3) G_6 - \right. \right. \\
&\quad \left. \left. - 2 \frac{1 - \tau}{1 + \tau} \cot^2 \frac{\theta}{2} (2c \sqrt{\tau(1 + \tau)} G_4 + G_6) \right]^* \right\}. \tag{52}
\end{aligned}$$

The contraction of the leptonic and hadronic tensors in the case when the polarization of the deuteron target has definite projection on the z axis, $m = \pm 1, 0$, can be written as

$$S^{(m)} = S_{\mu\nu} U_\mu^{(m)} U_\nu^{(m)*}, \tag{53}$$

where $U_\mu^{(m)}$ is the deuteron polarization four-vector with projection m

$$U_\mu^{(\pm)} = \mp \frac{1}{\sqrt{2}} (0, 1, \pm i, 0), \quad U_\mu^{(0)} = (0, 0, 0, 1).$$

After straightforward calculations we obtain

$$S^{(+)} = S^{(-)} = -S_1 + \frac{1}{2} \frac{Q^2}{1 + \tau} \cot^2 \frac{\theta}{2} S_3, \tag{54}$$

$$S^{(0)} = -S_1 + (1 + \tau)Q^2 S_2 + Q^2(1 + \tau + \cot^2 \frac{\theta}{2}) \frac{\tau}{1 + \tau} S_3 + 2c\sqrt{\tau(1 + \tau)}Q^2 S_4.$$

It follows that the cross sections of the elastic electron–deuteron scattering can be written in the familiar form

$$\frac{d\sigma^{(m)}}{d\Omega} = \sigma_0 \left[A^{(m)} + B^{(m)} \tan^2 \frac{\theta}{2} \right]. \quad (55)$$

Let us separate the dominant (Born) and TPE contributions to these structure functions and define

$$A^{(m)} = A_B^{(m)} + \Delta A^{(m)}, \quad B^{(m)} = B_B^{(m)} + \Delta B^{(m)}, \quad (56)$$

where the index B indicates the Born contribution. The Born terms can be expressed in terms of the deuteron FFs as:

$$\begin{aligned} A_B^{(+)} &= A_B^{(-)} = \frac{\tau}{2} G_M^2(Q^2) + \left[G_C(Q^2) - \frac{2\tau}{3} G_Q(Q^2) \right]^2, \\ B_B^{(+)} &= B_B^{(-)} = \tau(1 + \tau) G_M^2(Q^2), \\ A_B^{(0)} &= \tau G_M^2(Q^2) + G_C^2(Q^2) + \frac{8}{3} \tau G_C(Q^2) G_Q(Q^2) + \frac{16}{9} \tau^2 G_Q^2(Q^2), \\ B_B^{(0)} &= 2\tau(1 + \tau) G_M^2(Q^2). \end{aligned} \quad (57)$$

Summing the structure functions over the index m (i.e., over all possible deuteron spin projections) and dividing by three (the averaging over the deuteron spin) we obtain the usual structure functions $A(Q^2)$ and $B(Q^2)$.

Gourdin and Piketty calculated the longitudinal and two transverse polarizations of the recoil deuteron in the elastic electron–deuteron scattering [36]. The two transverse polarizations are orthogonal to the recoil deuteron momentum (it is \vec{q} in lab. system), one in the scattering plane (along x axis in our case) and the other one normal to this plane (along y axis). The quantities $S^{(x,y)}$, corresponding to the deuteron target polarized along x and y directions, are

$$S^{(x)} = -S_1 + \frac{Q^2}{1 + \tau} \cot^2 \frac{\theta}{2} S_3, \quad S^{(y)} = -S_1, \quad (58)$$

and the corresponding structure functions $A^{(x,y)}$ and $B^{(x,y)}$ can be written as (in the Born approximation)

$$\begin{aligned} A_B^{(x)} &= \tau G_M^2(Q^2) + \left[G_C(Q^2) - \frac{2\tau}{3} G_Q(Q^2) \right]^2, \quad B_B^{(x)} = \tau(1 + \tau) G_M^2(Q^2), \\ A_B^{(y)} &= (G_C(Q^2) - \frac{2\tau}{3} G_Q(Q^2))^2, \quad B_B^{(y)} = \tau(1 + \tau) G_M^2(Q^2). \end{aligned} \quad (59)$$

This result for $A_B^{(i)}$, $B_B^{(i)}$, $i = x, y$ coincides with the results obtained in Ref. [36]. Following Ref. [36], it is convenient to introduce a total transverse differential cross section σ_T and calculate the difference $D_T = (\sigma_T^{(+)} + \sigma_T^{(-)} - \sigma_T^{(0)})$. An interesting result is the independence of D_T from the the deuteron magnetic FF G_M and consequently with respect to the scattering angle

$$\frac{1}{\sigma_0}(\sigma^{(+)} + \sigma^{(-)} - \sigma^{(0)}) = G_C^2(Q^2) - \frac{16}{3}\tau G_C(Q^2)G_Q(Q^2) - \frac{8}{9}\tau^2 G_Q^2(Q^2). \quad (60)$$

This expression (valid in the Born approximation) coincides also with the result of Ref. [36].

The explicit expressions of the corrections $\Delta A^{(m)}$ and $\Delta B^{(m)}$ due to the TPE contributions are

$$\begin{aligned} \Delta A^{(+)} &= \Delta A^{(-)} = \mp \tau G_M(Q^2) \text{Re} \left\{ \Delta G_M(Q^2, \epsilon) + 4\tau c^2 G_4(Q^2, \epsilon) + \right. \\ &\quad \left. + 2c\sqrt{\tau(1+\tau)} \left[2G_5(Q^2, \epsilon) + \frac{1}{1+\tau} G_6(Q^2, \epsilon) \right] \right\} \mp 2 \left[G_C(Q^2) - \frac{2}{3}\tau G_Q(Q^2) \right] \\ &\quad \text{Re} \left[\Delta G_C(Q^2, \epsilon) - \frac{2}{3}\tau \Delta G_Q(Q^2, \epsilon) + 2\tau(c^2 - 1)G_4(Q^2, \epsilon) + c\sqrt{\frac{\tau}{1+\tau}} G_6(Q^2, \epsilon) \right], \\ \Delta B^{(+)} &= \Delta B^{(-)} = \mp 2\tau\sqrt{1+\tau} G_M(Q^2) \text{Re} \left[\sqrt{1+\tau} \Delta G_M(Q^2, \epsilon) + c\sqrt{\tau} G_6(Q^2, \epsilon) \right], \\ \Delta A^{(0)} &= \mp \left\{ 2\tau G_M(Q^2) \text{Re} \Delta G_M(Q^2, \epsilon) + 2G_C(Q^2) \text{Re} \left[\Delta G_C(Q^2, \epsilon) + \frac{4}{3}\tau \Delta G_Q(Q^2, \epsilon) \right] + \right. \\ &\quad \left. + \frac{8}{3}\tau G_Q(Q^2) \text{Re} \left[\Delta G_C(Q^2, \epsilon) + \frac{2}{3}\tau \Delta G_Q(Q^2, \epsilon) \right] \right\} \mp \\ &\quad \mp 4\tau G_M(Q^2) \text{Re} \left\{ 2\tau \left(1 + 4\tau + \frac{2}{1+\tau} \cot^2 \frac{\theta}{2} \right) G_4(Q^2, \epsilon) + \right. \\ &\quad \left. + c\sqrt{\tau(1+\tau)} \left[2(1+2\tau)G_5(Q^2, \epsilon) + \frac{1}{1+\tau} G_6(Q^2, \epsilon) \right] \right\} \pm \\ &\quad \pm 4\tau \left[G_C(Q^2) + \frac{4}{3}\tau G_Q(Q^2) \right] \text{Re} \left\{ 2\tau \left(1 + \frac{1}{1+\tau} \cot^2 \frac{\theta}{2} \right) G_4(Q^2, \epsilon) + \right. \\ &\quad \left. + c\sqrt{\tau(1+\tau)} \left[4G_5(Q^2, \epsilon) + \frac{1}{1+\tau} G_6(Q^2, \epsilon) \right] \right\}, \\ \Delta B^{(0)} &= \mp 4\tau(1+\tau) G_M(Q^2) \text{Re} \left[\Delta G_M(Q^2, \epsilon) - \right. \\ &\quad \left. - 2c(1+\tau)\sqrt{\tau(1+\tau)} G_5(Q^2, \epsilon) + 8\tau^2 G_4(Q^2, \epsilon) \right] \mp \\ &\quad \mp 4\tau c\sqrt{\tau(1+\tau)} \left[G_M(Q^2) + (1+2\tau)(G_C(Q^2) - \frac{2}{3}\tau G_Q(Q^2)) \right] \text{Re} G_6(Q^2, \epsilon), \quad (61) \end{aligned}$$

where the signs (\mp) indicate the scattering of the electron (positron) by a polarized deuteron target.

VI. CONCLUSION

Precise measurements of various observables in the elastic electron–proton scattering arose the question of the importance of the TPE mechanism. In this work, the study of TPE contribution and its consequences on the extraction of hadron electromagnetic FFs has been extended to electron–deuteron elastic scattering. The determination of the deuteron electromagnetic FFs from the measurement of the differential cross section and one polarization observable in the elastic electron–deuteron scattering is valid only in Born approximation. In case of deuteron and light nuclei the relative contribution of TPE term with respect to the main term, the 1γ exchange, is expected to be larger at the same momentum transfer squared, due to the steeper decrease of the FFs. Therefore, it can be detectable at smaller Q^2 values than in the case of the elastic electron–proton scattering.

A model–independent analysis of the influence of the two–photon exchange mechanism on the differential cross section and on various polarization observables has been performed for the elastic electron (positron)–deuteron scattering. General symmetry properties of the electromagnetic lepton–hadron interaction (as the lepton helicity conservation in QED at high energies, the C–invariance and crossing symmetry) were used in this analysis. These properties allows to parametrize the amplitudes of $e^\pm d$ –scattering in terms of fifteen real functions, in presence of the TPE mechanism: three standard electromagnetic deuteron FFs, which are the functions of one variable Q^2 , and six complex functions that depend on two variables, Q^2 and ϵ . The expressions for the differential cross section and all polarization observables have been given in terms of these functions. We have considered the case of an arbitrary polarized deuteron target and polarized electron beam (both longitudinal and transverse). The transverse polarization of the electron beam leads to a single–spin asymmetry which is non–zero in presence of the two–photon exchange contribution but it is suppressed by the factor (m_e/M) . Let us note that this factor is appreciably larger in the case of muon–deuteron scattering.

It was shown that the measurements of the differential cross section and one polarization observable (for example, the tensor asymmetry) for electron and positron deuteron elastic scattering, in the same kinematical conditions, allows to extract the deuteron electromagnetic form factors.

All the results derived in this work hold when the terms proportional to the square of the two-photon exchange amplitudes are neglected.

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