FIRST DESIGN FOR THE OPTICS OF THE DECAY RING FOR THE BETA-BEAMS

A. Chancé and J. Payet CEA/DAPNIA/SACM

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1. INTRODUCTION

The aim of the "beta-beams" is to produce pure electronic neutrino and anti-neutrino highly energetic beams, coming from β radioactive disintegration of the $^{18}Ne^{10+}$ and $^{6}He^{2+}$, directed to experiment situated in the Fréjus tunnel. The high ion intensities are stored in a ring, until the ions decay [1], [2]. Consequently, all the injected particles will be lost anywhere in the ring and we have to take into account a high level of losses.

Only the disintegrations of the ion beam moving to the detector direction are useful. We have to maximize this. Then, the shape of the decay ring is a racetrack. The two long straight sections are aligned with the experiment detector.

The losses due to the decay of the radioactive ions are compensated with regular injections. These should be done in presence of the circulating beam. The new ions are injected at a different energy from the stored beam energy, then the old and the new ions are merged with a RF specific program [3], [4]. The design of the ring must enable this type of injection and accept the injected and stored beams. In this note, we will focus on the study of the design of such a ring at the first and second orders.

2. OPTICS OF THE DECAY RING

2.1. Decay ring scheme

The ring circumference is equal to the SPS circumference, 6911 m for synchronism reasons. The straight sections must be as long as possible in order to maximize the useful neutrino flux. To keep reasonable bend field in the arcs, the straight section length was chosen to be about 2500 m, 35% of the circumference. Then each arc is about 1000 m long.

The periodicity of the decay ring is 2 and each period can be cut in 5 operational parts:

- A long straight section aligned with the detector
- A regular FODO lattice in the arc
- A matching section to adapt the optical functions of the straight section with the FODO lattices
- An insertion for the injection in the arc
- A matching section to adapt the FODO lattices with the insertion

The arc is a 2π phase advance insertion, which improves the optical properties (dynamic aperture and momentum acceptance) and makes easy the determination of the working point by the optics of the straight sections. The arc is designed for $^{6}He^{2+}$ and $^{18}Ne^{10+}$ at $\gamma = 100$.

2.2. Straight section optics

The straight sections are FODO lattices. Their length is 2462 m. The distance between two successive quadripoles is around 85 m, and there are 14 periods. Since one of the two straight sections is the neutrino source for the detector which is 130 km far away; the emission angle has to be quite small. In other terms, the maximum angle for the ions in the straight section (given by the transverse emittance and the Twiss parameters γ_x and γ_z) has to be negligible compared to the emission cone angle of the neutrinos given by $1/\gamma$ with γ the relativistic factor. The maximum value for γ_x and γ_z is $1.3.10^{-2}$ with this optics (Figure 1). For a $\varepsilon = 9\ \pi$ mm.mrad emittance at $9\ \sigma$, the maximum angle of an ion of the transported beam by this optics is then equal to 0.35 mrad. It is less than 4% times the angle of the neutrino emission cone equal to 10 mrad. The chosen optics corresponds to the quadripoles given in the Table 1.

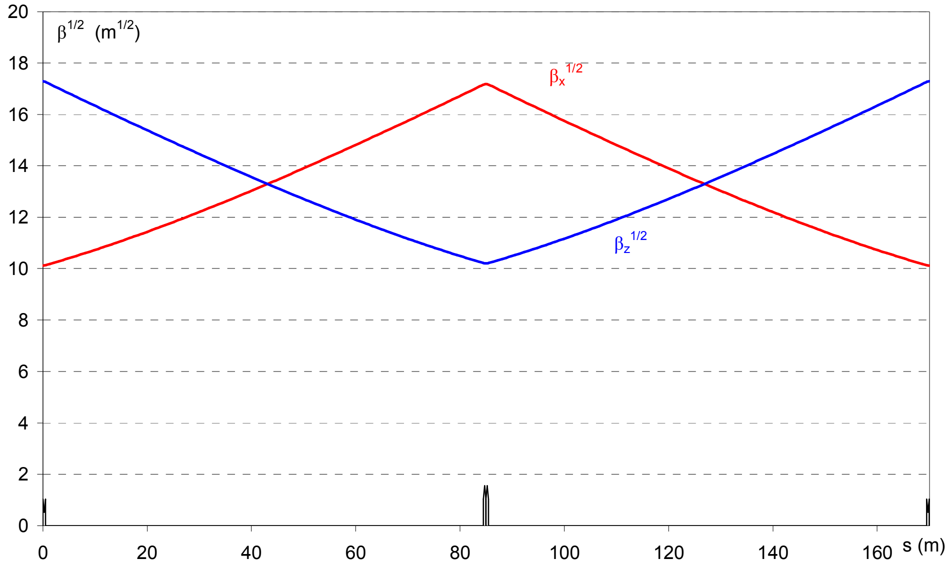


Figure 1 : Optical functions in the straight sections

QUADRIPOLES			
name	length (m)	strength (m^{-2})	gradient (T/m)
DQP1	1	$-1.11\ 10^{-2}$	-10.4
DQP2	1	$1.12\ 10^{-2}$	10.5

Table 1 : Straight section quadripoles

2.3. Regular FODO lattice in the arc

To transport the beam along the arc, we use FODO lattices excepted in the section dedicated to the injection. There are 10 38.7 m long regular FODO periods in each arc. To limit the apertures needed for the dipoles, the dispersion and the optical function β_x have to be quite small. To have a compact ring, a small bend radius is needed. The dipole is split in two pieces to enable the insertion of a beam stopper between, which will absorb the decay products [5]. The parameters of the magnetic elements are given in the Table 2, and the optical functions of the arc are presented in Figure 2. The total arc length is then 994 m.

QUADRIPOLES			
name	length (m)	strength (m^{-2})	gradient (T/m)
QP1	2	-3.16 10^{-2}	-29.5
QP2	2	4.85 10^{-2}	45.3

DIPOLE			
angle (rad)	radius (m)	length (m)	field (T)
π/86	155.67	5.69	5.99

Table 2 : Regular arc magnetic elements

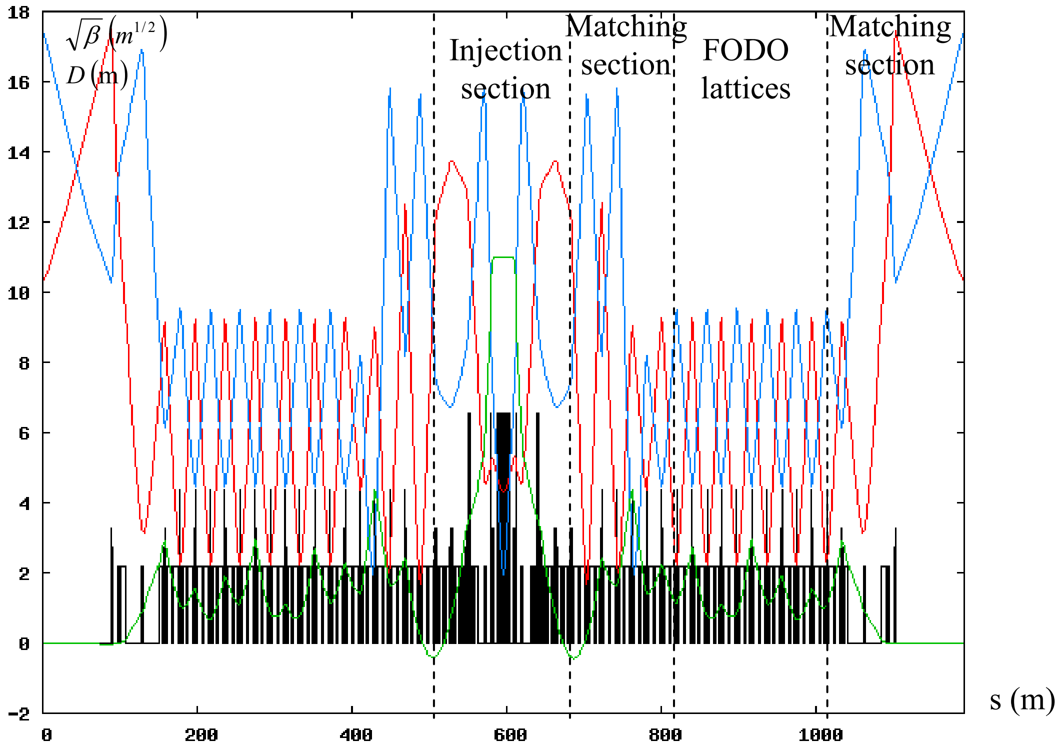


Figure 2 : Optical functions in the arc

In red β function in the horizontal plane. In blue in the vertical one. In green horizontal dispersion

2.4. Matching section between the arc and the straight section

The relative difference of the magnetic rigidity between an ion and its decay product is high: it is -1/3 for $^{6}He^{2+}$ (which gives $^{6}Li^{3+}$) and 1/9 for $^{18}Ne^{10+}$ (which gives $^{18}F^{9+}$). The focusing is smooth in the whole section. Consequently, wherever an ion decays in the straight section, its decay product will be transported until the first dipole in the arc and be lost behind. This represents 35% of the total amount of disintegration. The deposited power per meter in the chamber at the arc entry is then huge. To avoid that, the matching section is specially designed to allow extracting the decay products $^{6}Li^{3+}$ and $^{18}F^{9+}$ from the ring. It has to begin with a dipole to separate the decay products from the reference beam. The dipole aperture fixes its length (the decay products must pass the bend without hitting the walls [5]). Behind, a long drift is needed to have a better separation. If we assume the magnet transverse half size is 0.3 m, an extraction septum will be necessary to extract the Fluorine contrary to the Lithium (Figure 3).

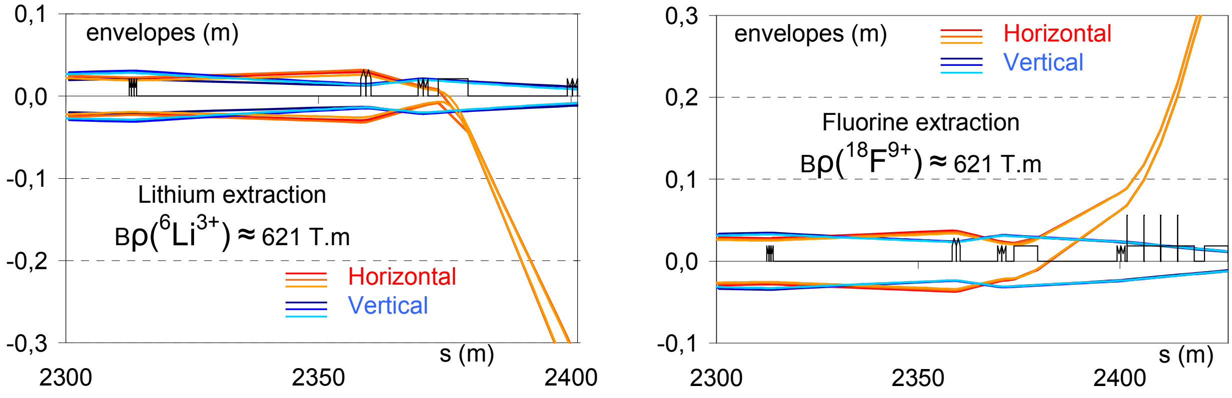


Figure 3 : Beam sizes for Li^{3+} (left) and F^{9+} (right) after crossing a straight section. A 0.6 T septum is used to extract the Fluorine beam

The values for the magnetic elements in that section are given in the Table 3. We use the same dipoles as in the arc.

QUADRIPOLES			
name	length (m)	strength (m^{-2})	gradient (T/m)
MDQP0	2	1.83 10^{-2}	17.1
MDQP1	2	-0.81 10^{-2}	-7.57
MDQP2	2	-.1.45 10^{-3}	-13.5
MDQP3	2	4.07 10^{-2}	38
MDQP4	2	-2.66 10^{-2}	-24.8

Table 3 : Matching section quadripoles

2.5. Low β, high D_x insertion for the injection

The injection is « off-momentum » [6]: the injected beam energy (E+ΔE) is different from the stored beam one (E). The injection principle appears in the Figure 4:

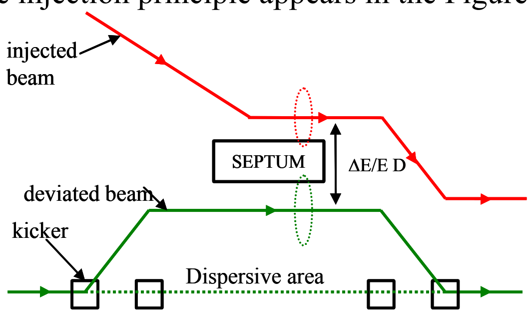


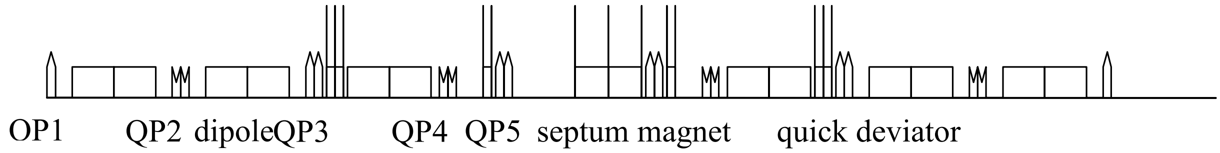
Figure 4 : Injection layout

The injection is realized in a high horizontal dispersion (D_x) area. In the following, we will use the index i to refer to the injected beam and m to refer to the stored beam. The relation between the relative beam energy difference Δ, the momentum spread δ_m, the total transverse emittances ε_m and ε_i, the optical functions β_m and β_i, the dispersion function D_x and the septum blade thickness e_S is:

$$\Delta.D = \sqrt{\beta_m \varepsilon_m + (\delta_m.D_x)^2} + e_S + \delta_m.D_x + \sqrt{\beta_i \varepsilon_i}$$

If we want to minimize the value of Δ necessary for the injection, we need to have β_m as low and D_x as high as possible. The injection section is thus an insertion at low β and high D_x.

At the injection point, we have to focus β_x and β_z to obtain a cross over here. Before, there is a doublet of quadripoles to focus in the both planes. The insertion structure is of this type:



The parameters for the magnetic elements are given in the Table 4.

QUADRIPOLES			
name	length (m)	strength (m^{-2})	gradient (T/m)
QP1	3	1.48 10^{-2}	13.8
QP2	3	-0.30 10^{-2}	-2.8
QP3	3	0.89 10^{-2}	8.31
QP4	3	-2.65 10^{-2}	-24.8
QP5	3	1.74 10^{-2}	16.25

DIPOLE			
angle (rad)	radius (m)	length (m)	field (T)
π/86	155.67	5.69	5,99

Table 4 : Injection magnetic elements

The dipoles are split in two parts to decrease the deposition of decay products there [5]. The dispersion at the injection point is 10.9 m and β_x is equal to 23 m. The momentum spread of the stored beam is 0.1%. The rms transverse horizontal emittance is $\varepsilon_x = 0.06\pi$ mm.mrad and the rms transverse vertical emittance is $\varepsilon_y = 0.11\pi$ mm.mrad. We preserve respectively 9 σ and 7 σ for the stored and injected beams. We use an 18 m long 1 T septum magnet, inserted in a 25 m long drift. We assume that the blade is 16 mm thick and we have added a 1 mm guard to each side of the septum magnet blade to avoid the effects due to the fringe fields. Then, the calculation gives $\Delta \approx 5.10^{-3}$ [6]. With an arc designed as a 2π insertion, which determines the matching section, we obtain the optical functions of the Figure 5.

2.6. Beam sizes

We use the parameters given in the previous paragraph for the calculation of the beam sizes in the ring (Figure 6). We can see that the injected beam determines the needed magnetic apertures in the arc. A 6 cm half-aperture is necessary for the dipoles. However, to avoid the deposition of the decay products from $^{18}Ne^{10+}$ on the dipole walls, an 8 cm half-aperture would be better [5].

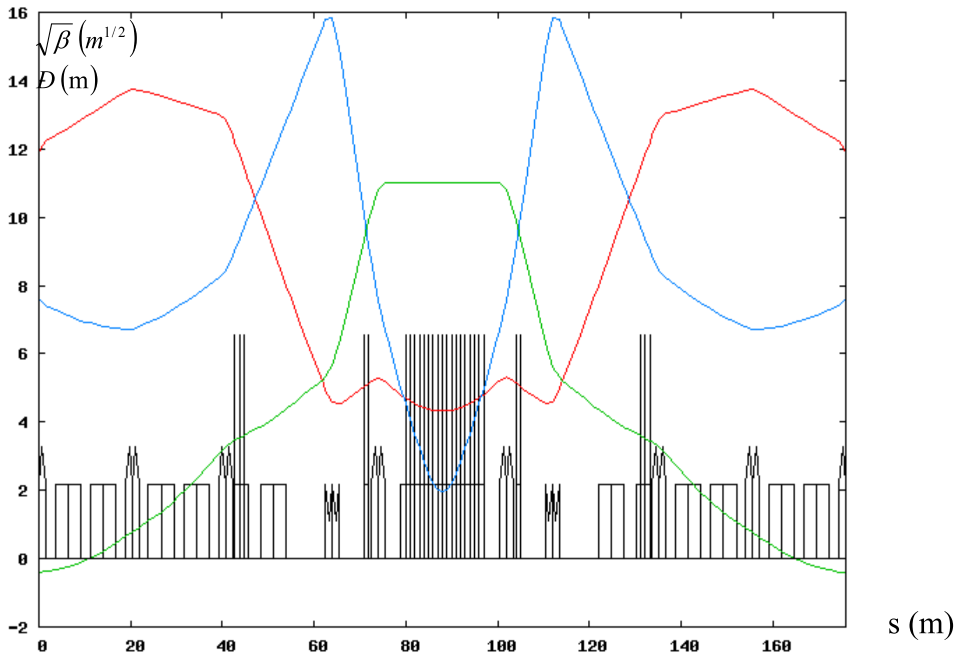


Figure 5 : Optical functions in the injection section

In red β function in the horizontal plane. In blue in the vertical one. In green horizontal dispersion

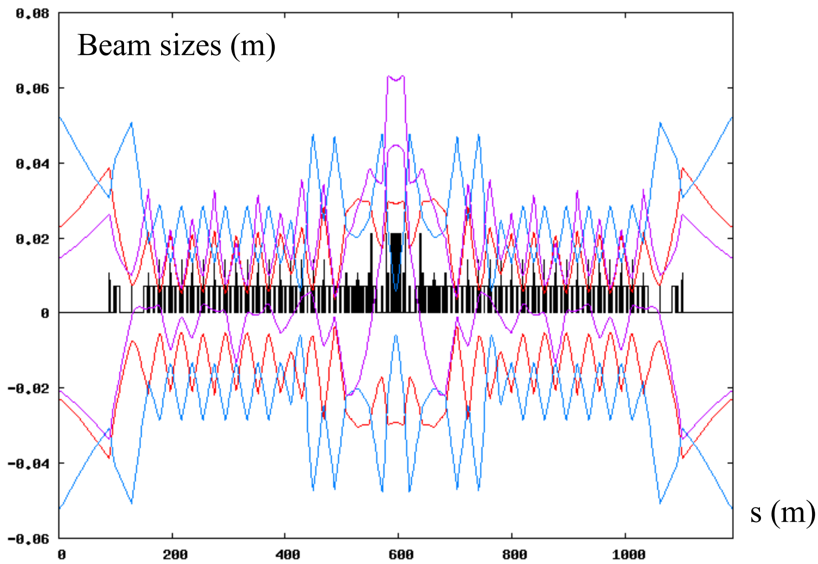


Figure 6 : Sizes of the stored beam (red) at 9 σ and of the injected beam (purple) at 7 σ
In red envelope of the stored beam in the horizontal plane, in blue envelope in the vertical plane, in purple envelope of the injected beam in the horizontal plane
The calculation is at the second order

2.7. Second order studies

The natural normalized chromaticities ($\xi = \frac{1}{\nu}\frac{\partial \nu}{\partial \delta}$) in the ring are -1.67 for the horizontal plane and -2.17 and the vertical one. With $\upsilon_x \sim 20$ and $\upsilon_z \sim 12$, the tune shift for a 5.10^{-3} injected off momentum beam is then -0.17 and -0.13 in each plane, which makes the chromaticity correction necessary. To minimize the effect of the third order resonances, we have chosen the working point $\upsilon_x = 18.225$ and $\upsilon_z = 10.16$ (Figure 7). Normally, we need only two sextupole families situated in the FODO lattices to correct the chromaticity but we will see that it is not sufficient. We will have to add then other sextupoles.

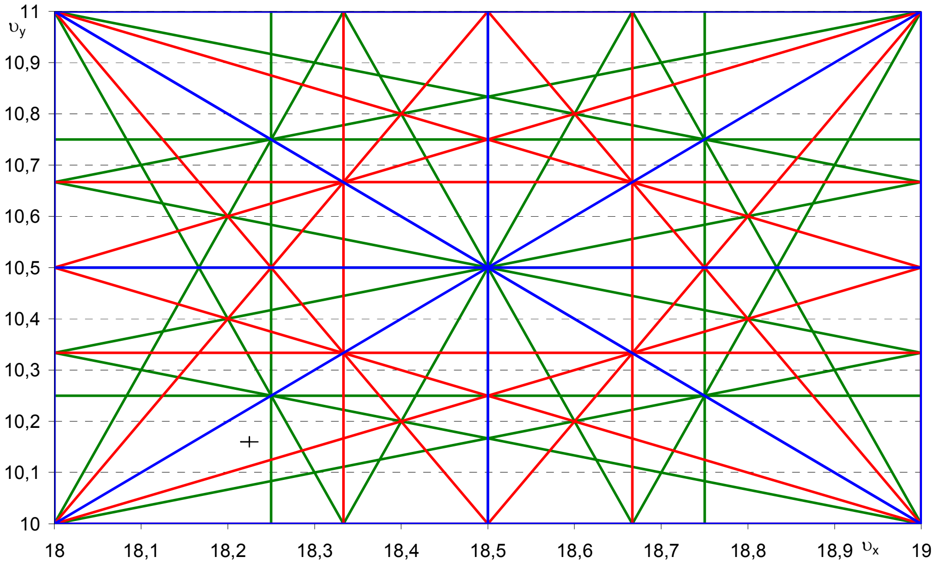


Figure 7 : Tune diagram
In blue, 2nd order resonances, in red 3rd order resonances and in green, 4th order resonances
The chosen working point is in black

2.7.1 Correction of the chromaticity

On Figure 8, if we track the ions without an initial vertical component, the injected beam has not an elliptical shape in the phase space anymore. It is due to the excitation of the third order resonances $\upsilon_x = 20$ and $3\upsilon_x = 60$. The beam is deformed but still accepted by the decay ring. However, if we consider now the coupling between the horizontal and vertical planes, we are likely to excite the coupling resonances $\upsilon_x+2\upsilon_z = 44$ and $\upsilon_x-2\upsilon_z = -4$. The injected beam is then unstable in the horizontal plane if we do the tracking (Figure 9) without an initial horizontal component. The vertical emittance is equal to 9 π mm.mrad.

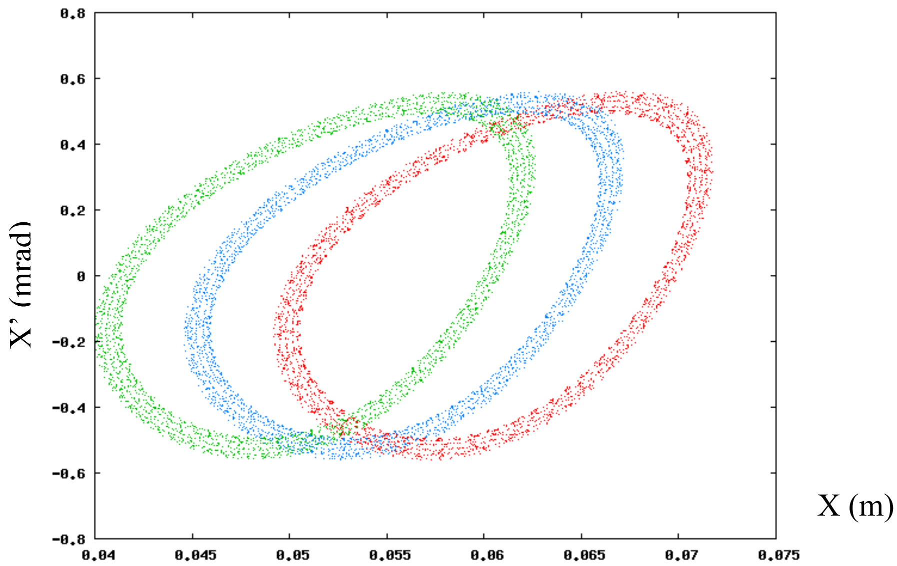


Figure 8 : Tracking in the horizontal phase space of the beam envelope during 100 turns
The initial beam is assumed without a vertical component
In red, beam at $\Delta p/p$=0.54%, in blue beam at $\Delta p/p$=0.5 %, in green beam at $\Delta p/p$= 0.46 %

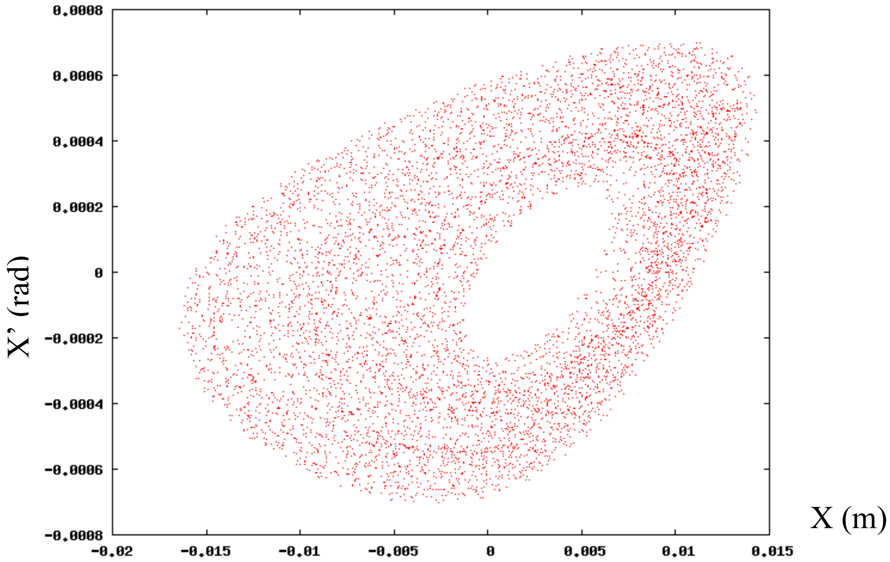


Figure 9 : Representation in the horizontal phase space of the stored beam at 9 σ during 100 turns
The initial beam is assumed without a horizontal component

We clearly see that we have a transfer and a blow up in the horizontal plane. It is due to coupling resonances. To identify the resonance type, we have plotted on Figure 10 the variation of the tune with the position of the ion. We can conclude that we excite two coupling resonances during the transport: $\upsilon_x - 2\upsilon_z = -4$ and $\upsilon_x - \upsilon_z = 8$. The dynamic aperture (Figure 14) is not large enough to accept the whole beam: we have to compensate these resonances.

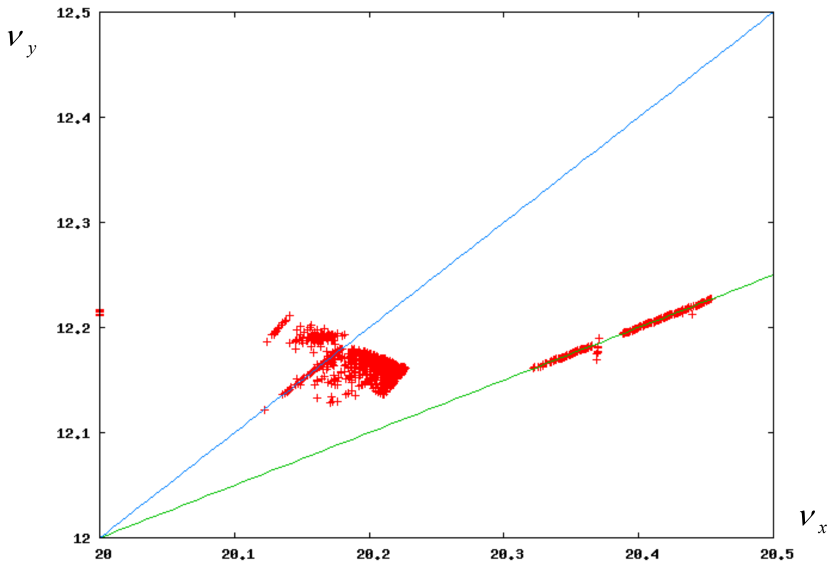


Figure 10 Working points crossed by the beam ions
The horizontal position is varied from -10 to 10 mm and the vertical one form -15 mm to 15 mm
In blue, line corresponding to the resonance υ_x-υ_y=8, in green, line corresponding to υ_x-2υ_y=-4

2.7.2 Correction of the chromaticity and of resonances

We have put other sextupoles in the matching section between the insertion for the injection and the FODO lattices to compensate the sextupolar effects. We have minimized the contribution of the third order resonances near from the working point. Moreover, we have put constraints on the values of the derivative of the tune as a function of the amplitude. The results are that the injected beam has now an elliptical shape in the horizontal phase space (Figure 11). It is now stable for ions at 9 σ in the vertical and horizontal planes (Figure 11). We do not cross the coupling second order resonance anymore (Figure 13) but we still cross the resonance $\nu_x - 2\nu_y = -4$. However, its effect has been minimized. The dynamic aperture has increased sufficiently to accept the whole injected beam (Figure 14). Moreover, the tune variations with momentum are quite small (Figure 15): the dynamic aperture stays the same for the injected beam. However, whereas the blow up of the injected and stored beams due to the sextupole effects is now small, we are going to see that the space charge effect is to take into account.

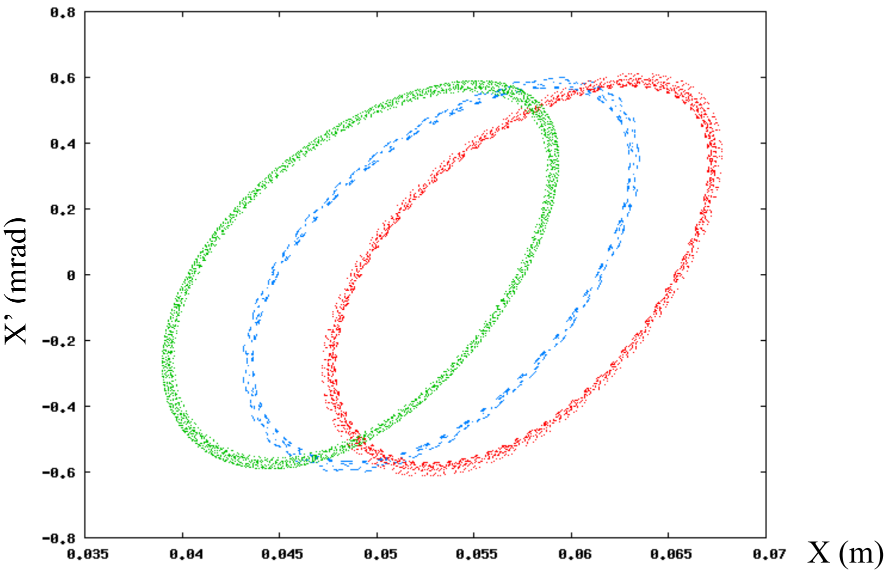


Figure 11 : Tracking in the horizontal phase space of the beam envelope during 100 turns
The initial beam is assumed without a vertical component
In red, beam at Δp/p=0.54%, in blue beam at Δp/p=0.5 %, in green beam at Δp/p= 0.46 %

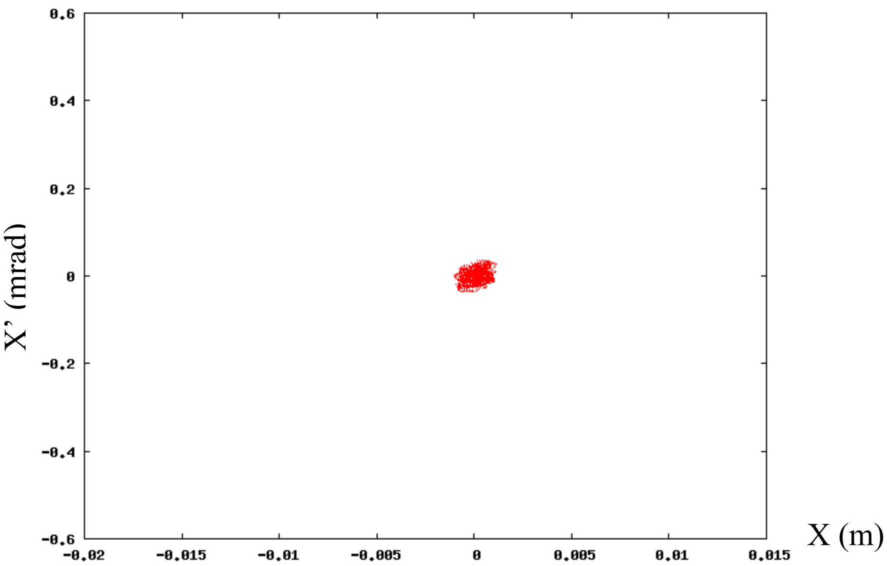


Figure 12 : Representation in the horizontal phase space of the stored beam at 9 σ during 100 turns
The initial beam is assumed without a horizontal component

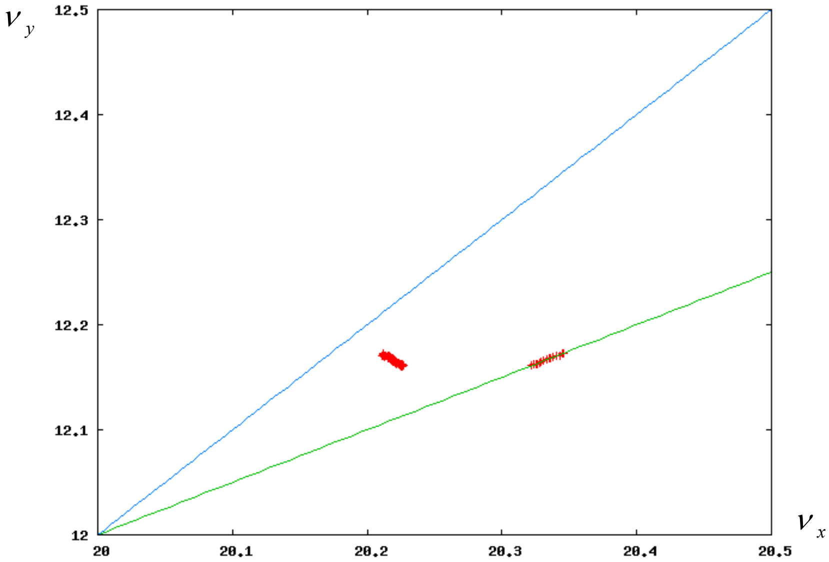


Figure 13 Working points crossed by the beam ions
The horizontal position is varied from -10 to 10 mm and the vertical one form -15 mm to 15 mm
In blue, line corresponding to the resonance υ_x-υ_y=8, in green, line corresponding to υ_x-2υ_y=-4

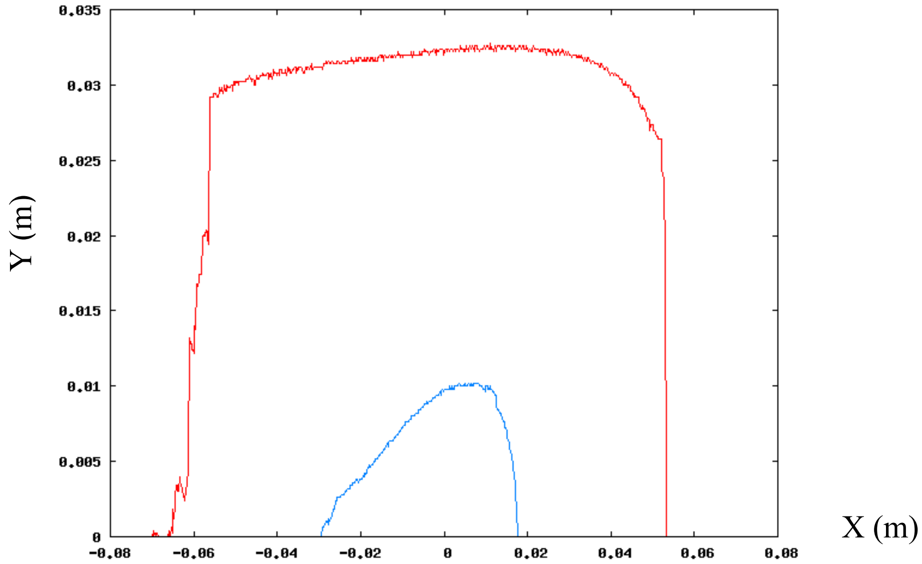
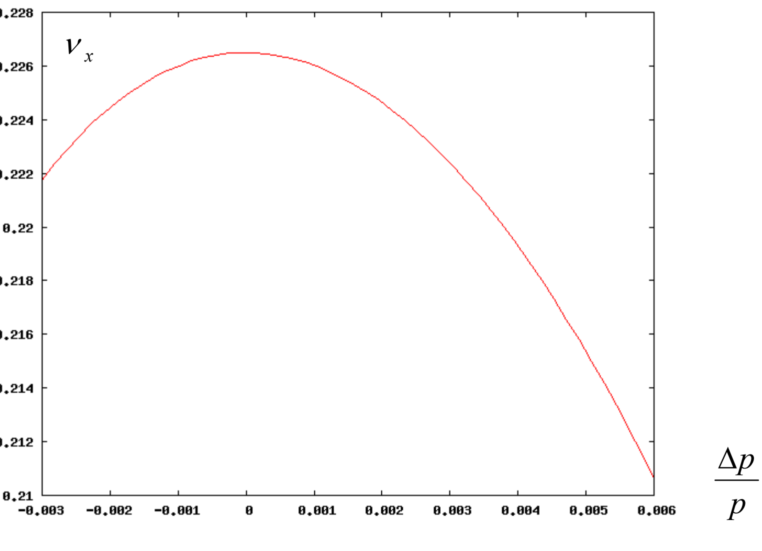


Figure 14 : Dynamic aperture at the injection point,
In blue, chromaticity correction only in red chromaticity correction and resonance compensation



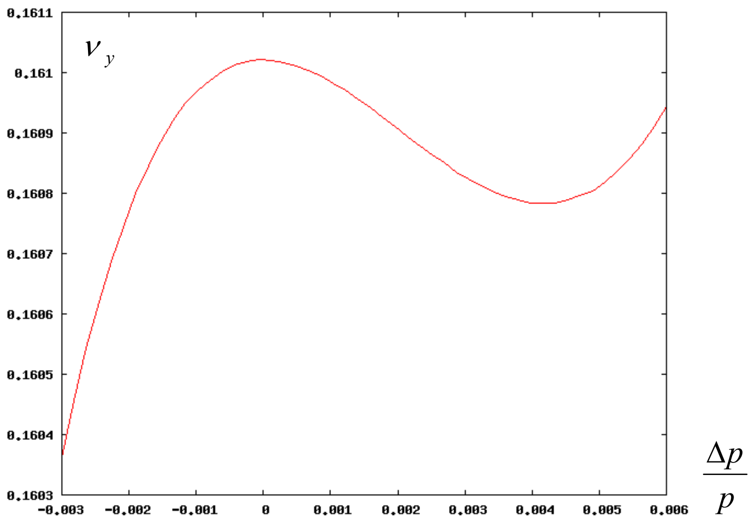


Figure 15 : Representation of fractional part the horizontal tune (above) and of the vertical tune (below) as a function of Δp/p

2.8. *Space charge effect*

To have a high neutrino flux means to store high ion intensities in the decay ring. Since we want to have a duty factor as small as possible, we use compact bunches. The question is then to know the effects of the space charge on the tune. In a top-down approach for the intensities, if we want to reach the neutrino flux given in [1], we have the Table 5.

	He^6	Ne^{18}	LHC
γ	100	100	7461
T/ion (GeV)	555	1660	7000
$\tau_{repetition}$ (s)	6	3.6	
Number of injected ions	$9.05\ 10^{12}$	$4.26\ 10^{12}$	$3.2\ 10^{14}$
Injected beam energy (MJ)	0.82	1.13	23.3
Number of stored ions	$9.71\ 10^{13}$	$7.4\ 10^{13}$	
Stored beam energy (MJ)	8.8	19.7	362

Table 5 Stored and injected beam intensities in the decay ring

The Laslett tune shift, which gives the tune shift due to the space charge, is given by:

$$\Delta\nu = -\frac{3}{4}\frac{Z^2}{A}\frac{r}{\beta^2\gamma^3}\frac{R}{L_b}\frac{N}{\varepsilon/\pi}$$

where Z is the charge number of the ion
A is the mass number of the ion
r is the proton radius
β is equal to v/c
R is the average radius of the ring
L_b is the bunch length in meter

N is the number of ions in the bunch
ε is the rms transverse emittance of the beam

We assume that the stored intensity is distributed in 20 bunches. The calculation gives for the decay ring, with $\varepsilon = 0.06\ \pi$ mm.mrad, $L_b = 1.65$ m and R = 1100 m: -0.04 and -0.26 respectively for $^{6}He^{2+}$ for $^{18}Ne^{10+}$. Therefore, the space charge effects are not negligible in spite of the fact that we use ions and that we are in high energies. We will have to take them into account if we keep these intensities in the decay ring. Moreover, the tune shift for $^{18}Ne^{10+}$ is so high that we will cross half-integer resonances which will make the beam unstable. To face that, we may have to increase the number of bunches in a batch, which decreases the stored intensity in each bunch, or to increase the emittance.

3. CONCLUSIONS

A first design for the decay ring has been realized with BETA [7] at the first and second orders. We have reached the constraint on the dispersion in the injection section: a horizontal dispersion superior to 10 m with $\beta_x = 20$ m. We have put sextupoles in the arcs to correct the chromaticity. In the same time, we have compensated the third order resonances to have a large enough dynamic aperture. So, the decay ring accepts the injected and stored beams. In a top-down approach, the high stored intensities impose to take into account the space charge effects. However, due to the merging, the beam blows up after each injection in the longitudinal space charge, which imposes to include a momentum collimation section in the decay ring. We will have to do the design of such an insertion and study its effect on the machine optics.

4. ACKNOWLEDGEMENTS

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