AN APPLICATION OF THE EXTREME VALUE THEORY TO BEAM LOSS ESTIMATES IN THE SPIRAL2 LINAC BASED ON LARGE SCALE MONTE CARLO COMPUTATIONS

R. Duperrier, D. Uriot
Laboratoire d'Étude et de Développement pour les Accélérateurs
CEN Saclay 91191 Gif sur Yvette.

Abstract

The influence of random perturbations of high intensity accelerator elements on the beam losses is considered. This paper presents the error sensitivity study which has been performed for the SPIRAL2 linac in order to define the tolerances for the construction. The proposed driver aims to accelerate a 5 mA deuteron beam up to 20 A.MeV and a 1 mA ion beam for q/A = 1/3 up to 14.5 A.MeV. It is a CW linac, designed for a maximum efficiency in the transmission of intense beams and a tunable energy. The Extreme Value Theory is used to estimate the expected beam losses. The described method couples large scale computations to obtain probability distribution functions. The bootstrap technique is used to provide confidence intervals associated to the beam loss predictions. With such a method, it is possible to measure the risk to loose a few watts in this high power linac (up to 200 kW).

INTRODUCTION

Once the reference design for the accelerator with perfect elements respects the requirements, it is necessary to evaluate the effects of imperfect elements. This evaluation permits to define tolerances for the construction of the linac and to test the robustness of the achieved architecture. To correct such errors, a correction scheme based on correctors and diagnostics has to be designed taking into account that the diagnostics are also imperfect (misalignments, measurement,...).
Several authors studied the effects of imperfect ion linacs on the beam [1, 2, 3, 4, 5, 6]. In the references [2, 3], the effect of non linear space charge force is not treated. The halo induced by these effects is then underestimated and the loss prediction becomes distorted. The approach in [1] is helpful if the Coulomb force is negligible but is inaccurate for high power linac at low energy. To tend to "realistic" simulation of a high intensity linac, it is necessary to perfom start to end transport to be capable to estimate the impact of halo produced at low energy on the beam losses at the high energy part of the accelerator. The references [4, 5, 6] detail start to end simulations to take into account this point. In these references, the main mechanisms to produce the beam halo are the space charge and/or the non linear external fields. These studies used macroparticles to estimate the beam distribution and to record the losses at the beam pipe. The discrete recorded losses at different locations in the linac allow to build Cumulative Density Function (CDF) to provide a probability to deposite more than a certain fraction of beam. But the discrete form of this CDF induces that the probability to loose more than the more extreme recorded loss becomes null. We are not capable then to predict very extreme events.
The Extreme Value Theory provides a firm theoretical foundation to perform such a goal (Fisher and Tipett (1928) and Gnedenko (1943)). Combining this theory with the bootstrap technique, we propose in this paper to detail a procedure to compute average probability of occurence of extreme events such a very low beam loss (10^{-5}) including a confidence interval (error bar) associated to this evaluation. To illustrate the method, the SPIRAL2 linac is used.

THE REFERENCE SIMULATION WITHOUT ERROR

To compare with the results including the element errors, this paragraph shows a simulation of the reference design. This design has been presented at the EPAC 2004 conference [7]. A 1,300,000 macroparticle $4 \times \sigma$ gaussian distribution is used at the input of the LEBT line. The transverse rms normalized emittance used is 0.2 π.mm.mrad. The beam current is 5 mA. A deuteron beam is considered to estimate the most critical beam losses. Multiparticle simulations are performed from the Low Energy Beam Transport (LEBT) line to the target through the radio frequency quadrupole (RFQ), the Medium Energy Beam Transport (MEBT), the super-conducting linac (SCL) and the High Energy Beam Transport (HEBT) line. The transport of the beam through the RFQ is computed with the code TOUTATIS [8]. The rest of the linac is simulated with the TraceWin/PARTRAN package[9]. To manage the necessary huge number of runs for the Monte Carlo study, we implemented in Tracewin a software package that permits to pilot a heterogenous collection of PCs [9]. The package is based on a client/server architecture to distribute the different independent runs. This is a multiparameter scheme and not a paralell scheme which is less optimal as each run can be performed by a single PC (less communication between each node). The figure 1 shows the beam density projection per plane in the linac.

The figure 2 shows the losses which occur in the structure. Three main peak losses are observed. They correspond to the different scrapers.

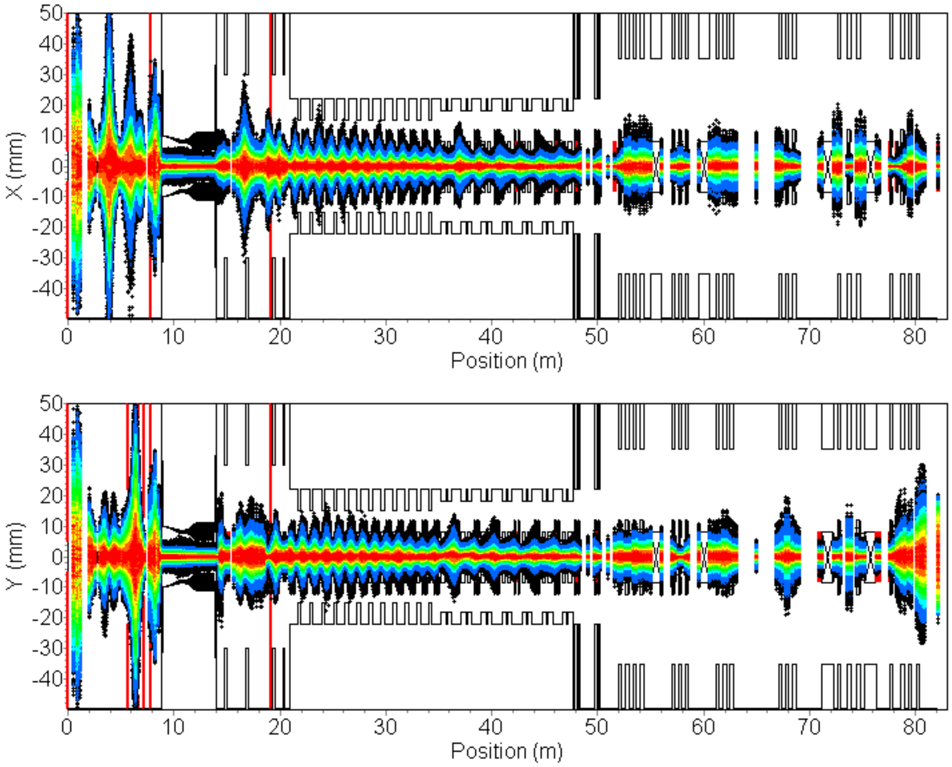

Figure 1: The deuteron density projection in the transverse plane in the SPIRAL2 linac.

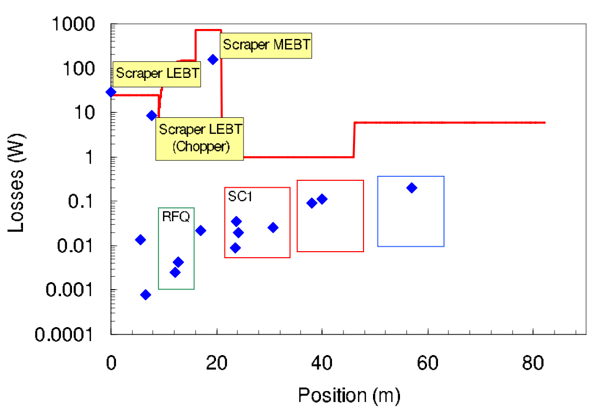

Figure 2: Losses (W) along the structure, the loss limit requirements are the red line.

SENSITIVITY OF THE LINAC TO ELEMENT ERRORS

Before detailing the different types of error, it is important to remark that two families of errors have to be coped for:

The "corrected" errors: These errors are applied before the tuning of the linac. They are, for instance, the cavity and quadrupole misalignments or the field errors. The strategy of the correction scheme is established to minimize the effects of these static errors.

The uncorrected errors: These errors are applied after the correction procedure. They represent the dynamic fluctuations of the RF field or mechanical vibrations from the environment. Fortunately, these errors have usually lower amplitude but their frequencies may be problematic.

Depending on the linac section, errors with different amplitudes have been used. For an error of amplitude A, the value has a uniform probability to be between $-A$ and $+A$. Usually, first, each defect is studied separately and is amplified until an unacceptable threshold is reached. Second, the defaults are combined and amplified until the threshold is reached again. The weighting for the combination has to take into account the relative sensitivity and the capacity to respect the induced tolerances. The main threshold for the SPIRAL2 project is to avoid losses in the superconducting section above 1 W per cavity. As this threshold is exceeded without error, the beam dump of the MEBT is also used as a scraper to control the loss level in the SC linac. Once the errors are included in the simulations, the losses can be still kept below one watt per cavity. The amplitudes of errors have been chosen after iterating with the engineering teams and the background from previous studies on high intensity linacs [11]. The transport of 13.000 macroparticles has been simulated for each linac of a set of 100 different linacs in order to get a convergence for the average losses. These average values will help us to select the acceptable

tolerances for the SPIRAL2 driver. To compute the linear density of the deposited power, the total power in a section is divided by the total length of the section.
The figure 3 shows the losses for the different amplitudes of errors (all plots show the mean values of each set of 100 linacs). A power of 40 W is dissipated in the scrapers of the LEBT. The collimators are excluded. It appears that the losses in the the vacuum chamber of the LEBT don't significantly increase with the errors. The main reason is that the cumulative effect of unperfections is weak (beginning of the linac). Comparing to the figure 3, the RFQ of the SPIRAL2 project appears to have a large acceptance. The losses are always kept below 1 W/m. For the MEBT line, the losses are always lower than the radioprotection threshold. The losses in the cavities for the first family of the SCL are always lower than 1 W. The main dissipated power is located in the first quadrupole. In the second SC family, the 1 W threshold is reached at $\sim 150\%$. Acceptable mean peak losses lower than 6 W are recorded if the amplitude of combined errors are lower than 140%.
All these results show that SPIRAL2 requirements are respected if the amplitudes of errors are lower than 140% if we consider mean values. A safer approach would be to choose an amplitude equal to 100% as a good compromise to minimize constraint for a possible upgrade to 100 MeV/u. The following section shows detailed results for this case.

APPLICATION OF THE EXTREME THEORY FOR THE LOSS ESTIMATE

Introduction

To study more precisely the losses occurring in the linac, the number of particles per run has been increased to 1,300,000 in order to reach the required resolution (less than 1 W for a 200 kW beam) and the number of run has been decreased to 341 runs.
This set of simulations provides data which can be used to build statistical models describing the extreme events. Extreme value theory (EVT) provides a firm theoretical foundation to perform such a goal [12, 13, 14]. This paper won't detailed this theory. See the reference [14] which reviews the basics and illustrates EVT with examples. By "extreme events", in our case, we mean that we want to be able to provide the probability to loose more than 1 Watt or 10 Watt, and so on and so forth with a confidence interval. To model the tails of our deposited beam power in the SPIRAL2 linac, we will apply the following method:

- first, scan the mean deposited power for each element of the accelerator to detect the most critical components.
- second, fit the data with the Generalized Extreme Value (GEV) distribution.
- Third, estimate confidence intervals for value of interest with the bootstrap method.

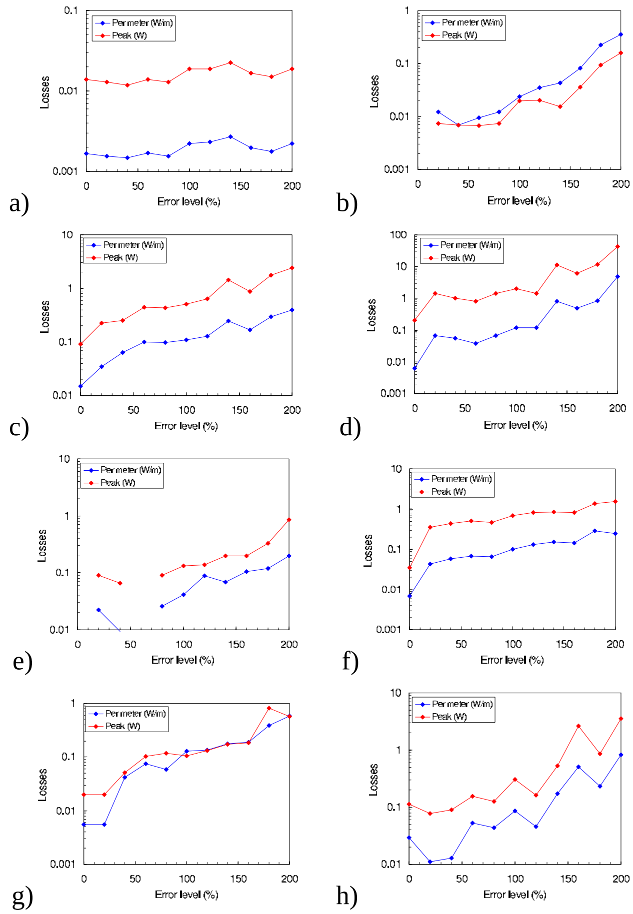

Figure 3: The losses in the deuteron LEBT (a), the RFQ (b), the MEBT (c), the HEBT (d), the SCL warm part (first family in e, and second in f), and the SCL cold part (first family in g, and second in h), in respect to the error level.

Figure 4 shows the mean losses repartition along the structure for the 341 linacs and the corresponding dissipated power. These last data allow us to select the most critical component in a particular section. It is assumed that elements with a high standard deviation have also a high mean value. If we focus on the results for the SCL, we can observe two critical elements. The first one is the first quadrupole of the first super-conducting section and the second one is the first cavity of the $\beta = 0.12$ section.

First quadrupole of the $\beta = 0.07$ section

Figure 5 shows the recorded loss distribution at the first quadrupole of the first super-conducting section. This represents the unnormalized probability density function (PDF) computed with the results of the 341 linacs with $1,000,000$ simulated macro-particles per linac. With this number of macro-particles, one particle represents ~ 8 mW at this location of the linac. Using this unnormalized PDF, we can build a Cumulative Distribution Function (CDF) which will be our reference data to fit with the GEV

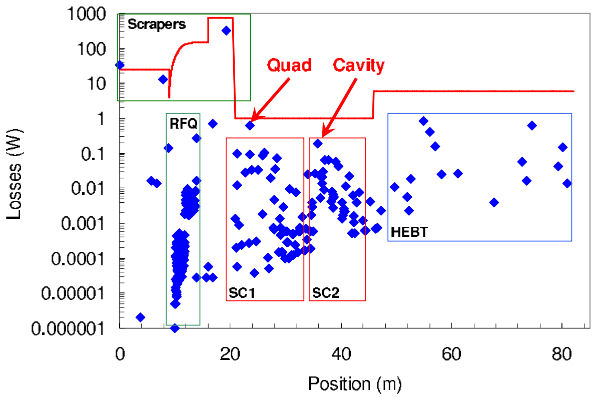

Figure 4: Average loss repartition along the structure. The most critical components are pointed with red arrows.

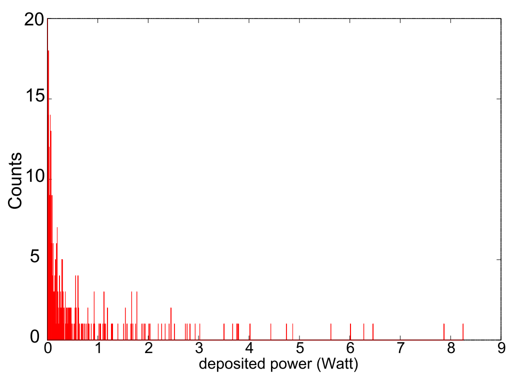

Figure 5: Unormalized probability density function for the losses at the first quadrupole of the first section. The deposited beam power (W) forms the abscissa and the number of counts the ordinate.

function of the lost power p:

$$H_{\xi\sigma\mu}(p) = exp\left(-\left(1+\xi\frac{p-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right) \tag{1}$$

with μ, the location parameter, σ, the scale parameter and ξ, the Jenkinson and von Mises parameter. To build the CDF, we used the following formula:

$$F_n\left(x_i^n\right) = \frac{i}{n} \quad \text{for} \quad i = 1, ..., n \tag{2}$$

which is the sample distribution function for a set of n observations, given in increasing order $x_1^n \leq ... \leq x_i^n$. For our case, n is equal to 341. The GEV fitted with these data is plotted in the figure 6. At this location of the linac, the requirements assume that less than 4Watt should be deposited on the pipe. With the fitted GEV, we can estimate that the probability to loose less than 4 Watt is 0.97 which is very confortable. The fitted parameters are $\hat{\xi} = 0.223$, $\hat{\sigma} = 0.89$ and $\hat{\mu} = -0.86$. To see how sensible is this result in respect to the achieved statistics, we can calculate a confidence interval at 95%. The bootstrap method is a helpful technique to construct such confidence interval. We resampled 1000 times the recorded PDF and recomputes the expected return power level for a probability of 0.97. The figure 7 shows the empirical bootstrap distribution for the return level for this probability. The confidence interval at 95% is then $[2.3; 5.9]$ Watt. This indicates that the recorded losses are sufficiently numerous to estimate that, with a good accuracy, we kept the beam losses at an acceptable level. If we need to estimate probability for very high loss level, the same procedure has to be repeated. For instance, with the same set of events, we can estimate that for a probability of occurence of 10^{-4} the mean deposited power is 36 Watt with a confidence interval at 95% which is $[20; 52]$ Watt. It indicates that more recorded losses are required if we need to shrink the confidence interval around this mean value of 36 Watt.

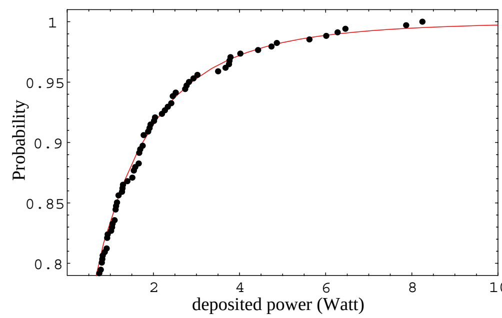

Figure 6: GEV fitted with the recorded losses for the quadrupole. The deposited beam power (W) forms the abscissa and the CDF the ordinate.

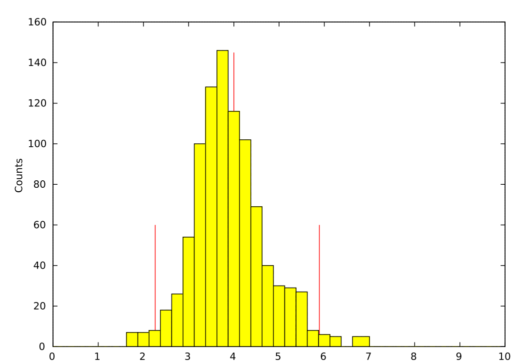

Figure 7: Empirical bootstrap distribution for the return level with a probability of 0.97. The two small red marks indicate the $\pm 2\sigma$ interval, the big red mark indicates the return level obtained with a direct estimate from the recorded losses.

First cavity of the $\beta = 0.12$ section

With the same procedure, we can construct a GEV function fitted with the recorded losses at the cavity location. The figure 8 shows the fitted GEV with the recorded losses

Table 1: Beam loss estimates (PE) and 95% bootstrap confidence intervals.

	CDF @ PE	Lower bound	Point estimate	Upper bound
Quad (W)	0.97	2.3	4	5.9
Cavity (W)	0.99	0.44	1	1.33

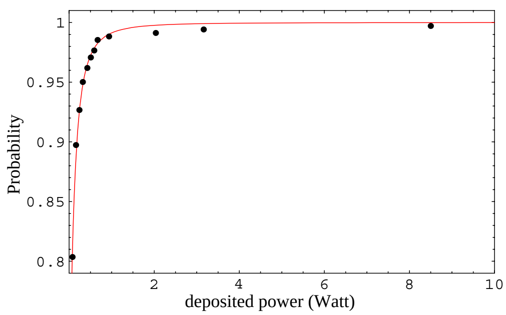

Figure 8: GEV fitted with the recorded losses for the most critical cavity. The deposited beam power (W) forms the abscissa and the CDF the ordinate.

at the cavity location. The fitted parameters are $\hat{\xi} = 0.465$, $\hat{\sigma} = 0.062$ and $\hat{\mu} = -0.061$. The probability to loose less than one watt is 0.99. With the bootstrap method, we can estimate a confidence interval for this probability. It is $[0.44; 1.33]$ Watt. The figure 9 illustrates the empirical bootstrap distribution for the return level for this probability. The table 1 gives a summary of the results for the most lossy quadrupole and cavity. To give an other example of the main interest to use EVT, we are capable to estimate that the probability to loose more than 10 Watt in this cavity is 8.10^{-5}.

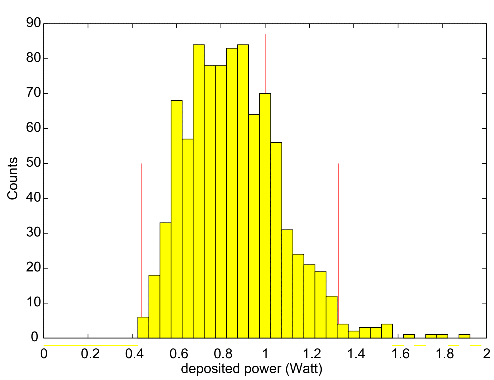

Figure 9: Empirical bootstrap distribution for the return level with a probability of 0.99. The two small red marks indicate the $\pm 2\sigma$ interval, the big red mark indicates the return level obtained with a direct estimate from the recorded losses.

CONCLUSIONS

This application of the Extreme Value Theory to beam loss estimates in the SPIRAL2 linac based on large scale Monte Carlo computations allowed us to provide loss probability for this linac. The probability to loose more than one watt in a superconducting cavity predicted with the GEV is less than 10^{-2}. Such an event will happen on average one linac over one hundred built linacs. The bootstrap technique has been used to estimate the precision of this prediction. A $\pm 2\sigma$ confidence interval equal to $[0.44; 1.33]$ Watt has been calculated for this probability To go further to "realistic" estimates of the beam loss, a more faithful modelisation of the linac is required. For instance, the output beam distribution of the ECR source is necessary to enhance the start to end modelisations and the beam interaction with the residual gas (neutralisation) has to be taken into account to simulate more accurately the space charge force especially at low energy.

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