Comments on ep and $e\mu$ elastic scattering

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Abstract

In this note we prove that the contribution of the box diagram calculated for electron muon elastic scattering can be considered an upper limit to electron proton scattering. As an exact QED calculation can be performed, this statement is useful for constraining model calculations involving the proton structure.

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The problem of the two photon exchange amplitude (TPE) contribution to elastic electron-proton scattering amplitude has been widely discussed in the past. This amplitude has in principle a complexe nature. Experimentally its real part can be obtained from electron proton and positron proton scattering in the same kinematical conditions. A similar information in the annihilation channel (electron-positron annihilation into proton-antiproton and in the reversal process) can be obtained from the measurement of forward-backward asymmetry of the angular distribution of the emitted hadron in the reaction center of mass (CMS) system.

Recently, a lot of attention was devoted to the two photon exchange amplitude (TPE) exchange amplitude in electron proton elastic scattering as a possible solution to a discrepancy between polarized and unpolarized measurements devoted to the determination of the proton form factors [1]. The theoretical description of TPE amplitude is strongly model dependent, as it involves the modelisation of the proton and of its excited states.

However, it is still possible to derive rigorous results and predict exact properties of the two photon box. Model independent statements based on symmetry properties of the strong and electromagnetic interaction have been suggested in in Ref. [2, 3]. It has been proved that, due to C-parity conservation, the amplitude for $e^+ + e^- \to p + \bar{p}$, taking into account the interference between one and two photon exchange, should be an odd function of $\cos\theta$, where θ is the angle of the emitted proton in the CMS of the reaction. This is equivalent, in the scattering channel, to destroy the linearity of the Rosenbluth fit, i.e., the (reduced) differential cross section as a function of $\epsilon = [1 + 2(1+\tau)\tan^2(\theta_e/2)]^{-1}$, where θ_e is the electron scattering angle, for a fixed value of the momentum transfer squared, Q^2, between the incident and the outgoing electron. This property must be satisfied by all model calculations.

A second possibility is to do an exact calculation of the box diagram, which is possible for electron electron and electron muon scattering, and in the crossed channel (i.e., replacing the proton with a lepton) [4]. The electron can be considered as a massless, point-like proton. The muon can be considered a structureless proton. Even if such calculation can not be considered a realistic model for the interaction on proton, the interest of a pure QED calculation is that the results can be considered as an upper limit for any calculation involving protons.

The purpose of this note is to prove that, modelling the proton by a Q^2 decreasing form factor, must lead to a smaller contribution of the box diagram, compared to the QED case. We will prove this for the imaginary part of the box diagram, and the validity for the full amplitude can be inferred through dispersion relations.

Let us consider the cases where the target is a proton (Fig. 1a) and a muon (Fig. 1b) with the following convention for the particle four momenta:

$$e(p_1) + p(p) \to e(p_1'') + p(p'') \to e(p_1') + p(p') \tag{1}$$

where q_1 and q_2 are the momenta carried by the virtual photons,

$$e(p_1) + \mu(p) \to e(p_1 - k) + \mu(p+k) \to e(p_1') + \mu(p') \tag{2}$$

where k, and $q-k$ are the momenta carried by the virtual photons.

The following kinematical relations hold in the center of mass frame:

$p_1 + p = p_1' + p'$, $q = p_1 - p_1'$, $p_1' + p' = p_1'' + p''$, $q_1 = p_1 - p_1''$, $q_2 = p_1'' - p_1'$,

$Q_1^2 = -q_1^2 = -(p_1 - p_1'')^2 = 2(\vec{p})^2(1 - c_1)$,

$Q_2^2 = -q_2^2 = -(p_1'' - p_1')^2 = 2(\vec{p})^2(1 - c_2)$,

$Q^2 = -q^2 = -(p_1 - p_1'')^2 = 2(\vec{p})^2(1 - c)$,

where $c_1 = \cos\theta_1$, $c_2 = \cos\theta_2$, $c = \cos\theta$, and $\theta_1 = \widehat{\vec{p}_1\vec{p}_1''}$, $\theta_2 = \widehat{\vec{p}_1''\vec{p}_1'}$, and $\theta = \widehat{\vec{p}_1\vec{p}_1'}$.

The contribution to the Feynman amplitude corresponding to the diagram of Fig. 1a can be written as

$$\mathcal{M} = \frac{1}{(2\pi)^2}\int \frac{d\Gamma}{(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)}, \tag{3}$$

where λ is a fictitous photon mass and $d\Gamma$ is the phase volume of the loop intermediate state. Taking into account the fact that the intermediate particles are on shell, one can write for the proton case:

$$\begin{aligned} d\Gamma &= d^4p_1''\delta(p_1''^2 - m^2)\delta(p''^2 - M^2)d^4p''\delta^4(p_1 + p - p_1'' - p'') \\ &= \frac{d^3p_1''}{2\epsilon_1''}\frac{d^3p''}{2\epsilon''}\delta^4(p_1 + p - p_1'' - p'') = \frac{d^3p_1''}{4\epsilon_1''\epsilon''}\delta(\sqrt{s} - \epsilon_1'' - \epsilon''), \\ \epsilon_1'' &= \frac{s - M^2}{2\sqrt{s}},\ \epsilon'' = \frac{s+M^2}{2\sqrt{s}}, \end{aligned} \tag{4}$$

Finally

$$d\Gamma = \frac{s - M^2}{8s}dO_1'' \tag{5}$$

dO_1'' is the solid angle of the electron in the intermediate state, which can be expressed as a function of the angles defined above as:

$$dO_1'' = \frac{2dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1 Q_0^2}},\ \mathcal{D}_1 = 2(Q_1^2+Q_2^2)Q^2Q_0^2 - 2Q^2Q_1^2Q_2^2 - (Q_1^2-Q_2^2)Q_0^2 - (Q^2)^2Q_0^2 \tag{6}$$

with the relation $Q_0^2 = 2\vec{p}^2 = (s-M^2)^2/(2s)$. The positivity of the function $\mathcal{D}$ defines the solid angle kinematically available for the reaction.

Therefore one can write the contributions corresponding to the 'QED' diagram in Fig. 1b as:

$$\mathcal{M}_{1a} = \frac{1}{\sqrt{8s}}\int \frac{dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)} \tag{7}$$

Introducing a generalized form factor for the proton, one finds for the 'QCD' diagram of Fig. 1a :

$$\mathcal{M}_{1b} = \frac{1}{\sqrt{8s}}\int \frac{dQ_1^2 dQ_2^2 F(Q_1^2)F(Q_2^2)}{\sqrt{\mathcal{D}_1}(Q_1^2+\lambda^2)(Q_2^2+\lambda^2)} \tag{8}$$

Therefore the condition $F(Q_1^2)F(Q_2^2) < 1$ is equivalent to the statement that the value of the electron-muon scattering amplitude can be considered an upper estimation of the amplitude for electron-proton scattering.

Nucleon form factors are functions which are rapidly decreasing with Q^2. The Pauli and Dirac form factors, F_1 and F_2, are related to the Sachs form factors by :

$$F_1(Q^2) = \frac{\tau G_M(Q^2) + G_E(Q^2)}{\tau+1};\ F_2(Q^2) = \frac{G_M(Q^2) - G_E(Q^2)}{\tau+1},\ \tau = \frac{Q^2}{4M^2}; \tag{9}$$

with the following normalization: $F_1(0) = 1$, $F_2(0) = \mu_p - 1 = 1.79$, where μ_p is the magnetic moment of the proton in units of Born magneton.

Let us consider the dipole approximation as a good approximation at least for the magnetic proton form factor G_M, although it has been shown that the electric form factor G_E deviates from the dipole form. In any case, any parametrization closer to the data will give even lower values as compared to the dipole form. In this approximation, we have:

$$F_1^D(Q^2) = \frac{(\tau\mu_p + 1)G_D(Q^2)}{\tau+1};\ F_2(Q^2) = \frac{(\mu_p - 1)G_D(Q^2)}{\tau+1},\ G_D(Q^2) = [1 + Q^2(\mathrm{GeV})^2/0.71]^2 \tag{10}$$

In Fig. 2 we show $F_1(Q^2)$ (dashed line), $F_2(Q^2)$ (dotted line) and the product $F_1(Q^2)F_2(Q^2)$ (solid line), which are smaller than unity practically overall the Q^2 range. The product $F_1(Q_1^2)F_1(Q_2^2)$ is shown in Fig. 3 as a bidimensional plot, and in Fig 4, as a projection on the Q_1^2 axis for $Q_2^2 = 0.05$ GeV^2 (solid line), $Q_2^2 = 1.2$ GeV^2 (dashed line), $Q_2^2 = 2$ GeV^2 (dotted line).

One can see that the condition $F(Q_1^2)F(Q_2^2) < 1$ is satisfied, starting from very low values of Q^2. Let us stress that $F_1(Q^2)$ is normalized to 1 and it decreases with Q^2, being therefore smaller than unity; in the expression of the hadronic current, $F_2(Q^2)$ is multiplied by q_μ, which lowers its contribution at small Q^2, whereas at larger Q^2 it does not compensate the steep Q^{-6} behavior of this form factor, as expected from quark counting rules [6].

Therefore all model calculations for *ep* elastic scattering as [5] should result in smaller contribution of the two photon amplitude, as compared to QED calculations [4].

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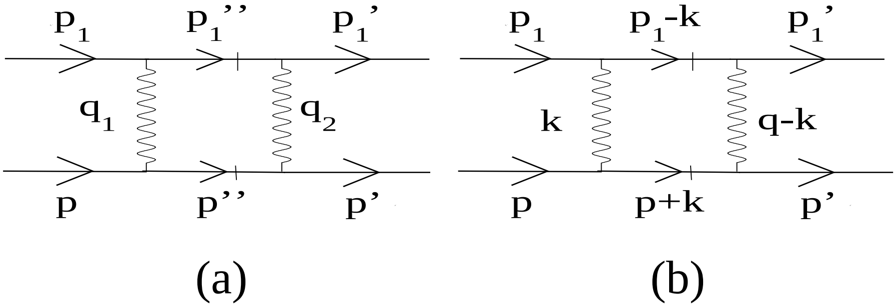

FIG. 1: Feynman Box Diagram (a) for ep and (b) for $e\mu$ scattering.

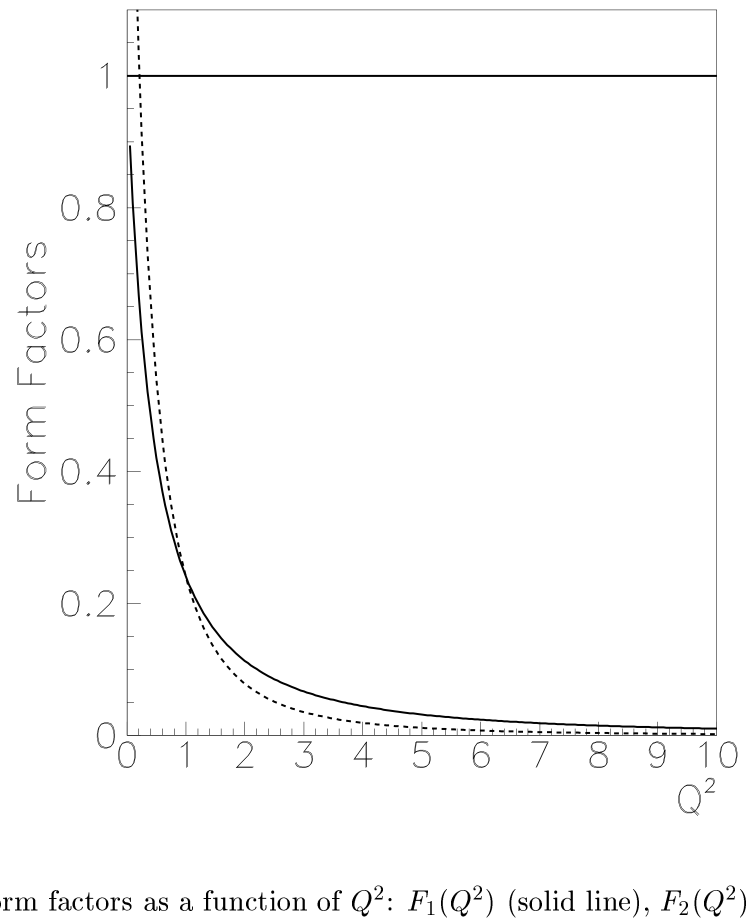

FIG. 2: Form factors as a function of Q^2: $F_1(Q^2)$ (solid line), $F_2(Q^2)$ (dashed line).

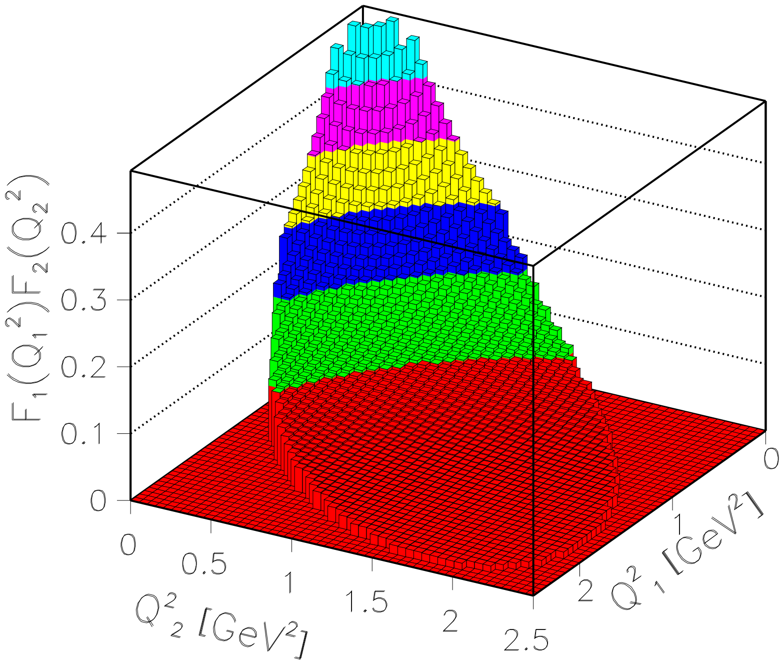

FIG. 3: Bidimensional plot of $F_1(Q_1^2)F_1(Q_2^2)$ as function of Q_1^2 and Q_2^2.

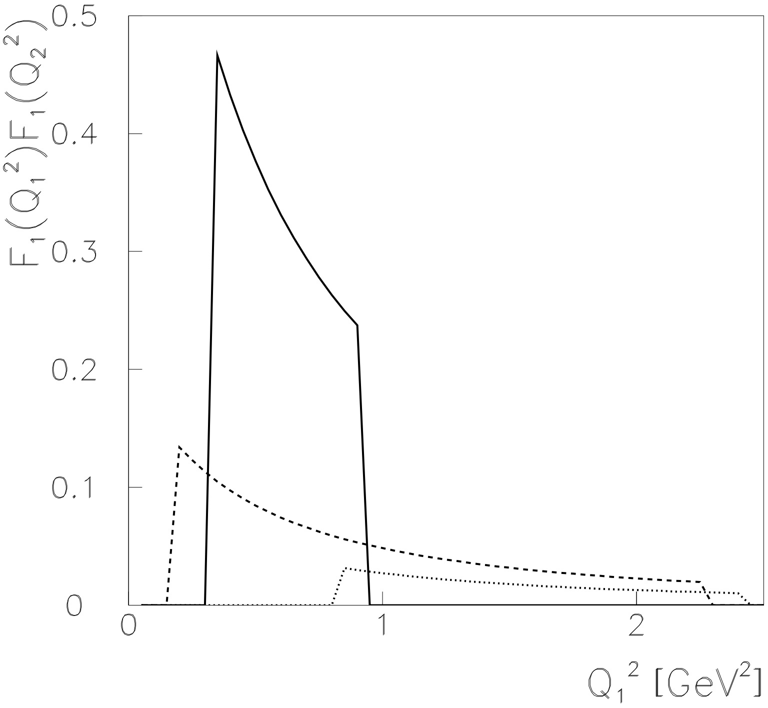

FIG. 4: Projection on $F_1(Q_1^2)F_1(Q_2^2)$ on the Q_1^2 axis for $Q_2^2 = 0.05$ GeV^2 (solid line), $Q_2^2 = 1.2$ GeV^2 (dashed line), $Q_2^2 = 2$ GeV^2 (dotted line).