

Binding of Hypernuclei in the Latest Quark–Meson Coupling Model

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Abstract

The most recent development of the quark-meson coupling (QMC) model, in which the effect of the mean scalar field in-medium on the hyperfine interaction is also included self-consistently, is used to compute the properties of hypernuclei. The calculations for Λ and Ξ hypernuclei are of comparable quality to earlier QMC results without the additional parameter needed there. Even more significantly, the additional repulsion associated with the increased hyperfine interaction in-medium completely changes the predictions for Σ hypernuclei. Whereas in the earlier work they were bound by an amount similar to Λ hypernuclei, here they are unbound, in qualitative agreement with the experimental absence of such states. The equivalent non-relativistic potential felt by the Σ is repulsive inside the nuclear interior and weakly attractive in the nuclear surface, as suggested by the analysis of Σ -atoms.

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I. INTRODUCTION

The study of Λ hypernuclei has a long and impressive history, with the shell structure mapped out across the periodic table [1, 2, 3, 4, 5]. It is known that the single particle spin-orbit force is very small [6] and systematic studies of the energy levels of light Λ hypernuclei have enabled the extraction of considerable detail concerning the effective Λ -N interaction. Current studies of electroproduction at JLab will provide important new information in this area [5, 7].

When one turns to Σ and Ξ hypernuclei, the situation is quite different. The special case of ${}^4\text{He}$ aside, there is no experimental evidence for any Σ hypernuclei [8, 9, 10], despite extensive searches. Indeed, it seems likely that the Σ -nucleus interaction is somewhat repulsive and that there are no bound Σ hypernuclei beyond $A=4$. In the case of the Ξ , the experimental situation is very challenging, with just a handful of observations of doubly strange nuclei. While we eagerly await studies of Ξ hypernuclei with new facilities at J-PARC and GSI-FAIR, it seems likely that these hypernuclei may exist [11, 12] and it would be helpful to have a range of predictions from various theoretical models.

There is currently considerable interest in the equation of state of dense nuclear matter, especially in connection with the calculation of neutron star properties [13, 14]. The density at which hyperons appear and particularly their order, plays a crucial role in the equation of state (EoS) – for example, the inclusion of hyperons in the QMC model seems essential if the direct URCA process is to play a role in cooling for observed neutron stars [15]. For matter in β -equilibrium, negatively charged hyperons are favoured a priori and in many models Σ^- hyperons enter soon after the Λ . Other models suggest that the Ξ^- should appear after the Λ (followed by the Ξ^0), with the Σ playing no role. Whichever scenario is ultimately correct, hyperon physics will play a critical role at $2 - 3 \rho_0$ and a relativistic treatment is essential [17].

In this work we apply the latest development of the quark-meson coupling (QMC) model [15] to calculate the properties of hypernuclei. The major improvement in the model that we use here is the inclusion of the effect of the medium on the hyperfine interaction. This has the effect of increasing the splitting between the Λ and Σ masses as the density rises. This is the prime reason why we find that Σ hypernuclei are unbound. On the other hand, the model produces quite good binding energies for Λ hypernuclei and predicts modest

binding of Ξ hypernuclei.

II. RECENT IMPROVEMENT IN THE QMC MODEL

The QMC model was created to provide insight into the structure of nuclear matter, starting at the quark level [18, 19, 20]. Nucleon internal structure was modeled using the MIT bag, while the binding was described by the self-consistent coupling of the confined quarks to σ and ω meson fields generated by the confined quarks in the other “nucleons” in the medium. While the use of effective scalar and vector fields to carry the forces is very similar to QHD, the explicit treatment of the N internal structure represents an important departure from that classical approach. The self-consistent response of the bound quarks to the mean σ field leads to a novel saturation mechanism for nuclear matter, with the enhancement of the lower components of the valence Dirac wave functions, as the density increases, reducing the effective σ -N coupling. This effect is naturally represented by a single new parameter, d , the scalar polarizability, with the nucleon effective mass taking the form:

$$M_N^* = M_N - g_{\sigma N}\sigma + \frac{d}{2}(g_{\sigma N}\sigma)^2. \quad (1)$$

The QMC model has been used to study the binding of ω , η , η' and D nuclei [21, 22, 23], as well as the effect of the medium on K^\pm and J/Ψ production [24]. It leads to a very natural explanation of the small spin orbit force in Λ hypernuclei [25, 26]. A recent extension of the same physical ideas to a confining version of the NJL model [27] has led to a quantitative description of the EMC effect across the periodic table, with a remarkable prediction of a sizeable enhancement of the nuclear modification of spin structure functions [28, 29].

For the present purpose, the most significant recent development is the inclusion of the self-consistent effect of the mean scalar field on the familiar one-gluon-exchange hyperfine interaction that in free space leads to the N- Δ and Σ - Λ mass splitting [15]. The original QMC model treated this term as a small correction and ignored its medium modification. However, the term “hyperfine” is misleading, as the N- Δ splitting is 30% of the nucleon mass. One-gluon-exchange in the bag model is essentially determined by the magnetic moments of the confined quarks [30] and, in first approximation, the hyperfine splitting is

$$\delta_N = \delta_\Lambda = -3C\mu_0^2, \quad \delta_\Delta = +3C\mu_0^2, \quad \delta_\Sigma = +C\mu_0(\mu_0 - 4\mu_s) \quad (2)$$

	m_s	Λ	Σ	Ξ	Σ^*	Ξ^*	Ω
$F_s = 1$	0.341	1.135	1.176	1.355	1.416	1.599	1.784
$F_s = 0.726$	0.297	1.107	1.189	1.325	1.368	1.507	1.654
Exp.		1.116	1.195	1.315	1.385	1.533	1.672

TABLE I: Octet and decuplet masses in GeV. The parameter F_s and the strange mass m_s are fitted to the Λ, Σ, Ξ masses. The last three columns are predictions for the decuplet masses.

where C is a constant, essentially independent of the quark masses, and μ_0, μ_s are, respectively, the magnetic moments of the confined light and strange quarks. From the experimental value in free space $(\delta_\Sigma - \delta_\Lambda)/(\delta_\Delta - \delta_N) = 80/300$ we get $\mu_s/\mu_0 \sim 0.6$. This decrease of the magnetic moment with the increase of the quark mass is a relativistic effect similar in nature to the saturation mechanism of the QMC model. In the medium the light quark interacts with the nuclear σ field and this amounts to decreasing its mass. This in turn reduces its eigenenergy (which is roughly the equivalent of the constituent quark mass) and as a result the magnetic moment of the light quarks in medium changes to $(1 + \epsilon)\mu_0$ with $\epsilon > 0$. An explicit calculation predicts $\epsilon \sim 7\%$ at nuclear matter density. Assuming that the strange quark is not coupled to the nuclear field one thus finds that the relative change of $\delta_\Sigma - \delta_\Lambda$, because of the σ field, is

$$\frac{(\delta_\Sigma - \delta_\Lambda)_\sigma}{(\delta_\Sigma - \delta_\Lambda)} = \frac{(1 + \epsilon)[\mu_0(1 + \epsilon) - \mu_s]}{\mu_0 - \mu_s} \sim 1 + 3.5\epsilon, \quad (3)$$

where we have used $\mu_s/\mu_0 \sim 0.6$. This rough calculation thus implies that the Σ will be less bound than the Λ by $3.5\epsilon(\delta_\Sigma - \delta_\Lambda) \sim 20\text{MeV}$, which is consistent with the exact calculation described below. Our motivation for applying the model to finite hypernuclei is that this effect has the right sign to help solve the major problem found in the original QMC model, namely that Σ hypernuclei were not much less bound than Λ hypernuclei.

In view of the physical importance of the hyperfine interaction, as we just explained, we decided that it would be worthwhile to improve on the MIT bag model, which traditionally yields only 50 MeV of the $\Sigma - \Lambda$ mass difference, instead of the experimental value of 80 MeV. The change which we introduce is motivated by the work of Barnes [31], who explained the relatively large value of the effective strong coupling constant, α_s , required in the naive bag model. That model ignored the enhancement of the relative q-q wave function at short

distance caused by the attractive color Coulomb force. The short distance enhancement of the relative wave function is expected to be less effective for heavier quarks. As a phenomenological way of implementing these effects, we multiply the hyperfine interaction in the usual MIT bag model by a factor F_s^n , with n the number of strange quarks participating. We fix the bag model parameters in two steps. First, the masses of the nucleon and Δ , together with their stability conditions, are used to determine the vacuum pressure, B , zero point energy, Z_0 , and the strong coupling, α_s , for a given value of the nucleon radius, which is set to 0.8fm in practice. This step is independent of the quark mass and F_s . The nucleon and Δ masses are exactly reproduced with $B = 0.5541 \text{ fm}^{-3}$, $Z_0 = 2.6422$ and $\alpha_s = 0.4477$. Then the strange mass, m_s , and the parameter F_s are chosen to give the best fit for the Λ , Σ and Ξ masses. As shown in Table I, where we also display the masses corresponding to $F_s = 1$, we obtain an excellent fit with $F_s = 0.726$ and $m_s = 0.297 \text{ GeV}$. In particular, the $\Lambda - \Sigma$ splitting is well reproduced. For completeness, we show in the last 3 columns of Table I the good agreement obtained for the predicted masses for the rest of the baryon decuplet (Σ^* , Ξ^* and Ω).

III. APPLICATION TO THE ENERGY LEVELS OF HYPERNUCLEI

In order to calculate the properties of finite hypernuclei, we construct a simple shell model, with the nucleon core calculated in a combination of self-consistent mean scalar and vector mean fields. The determination of the scalar coupling to the octet baryons requires a sophisticated self-consistency calculation. However, within the Born-Oppenheimer approximation, which requires that the internal structure of the nucleon has time to adjust to the local environment (an approximation estimated in Ref. [19] to be good at the level of

3% or better in finite nuclei), the result can be parametrized in a very practical form [15]:

$$M_N(\sigma) = M_N - g_\sigma \sigma + \left[0.002143 + 0.10562 R_N^{free} - 0.01791 \left(R_N^{free} \right)^2 \right] (g_\sigma \sigma)^2 \quad (4)$$

$$M_\Lambda(\sigma) = M_\Lambda - \left[0.6672 + 0.04638 R_N^{free} - 0.0022 \left(R_N^{free} \right)^2 \right] g_\sigma \sigma + \left[0.00146 + 0.0691 R_N^{free} - 0.00862 \left(R_N^{free} \right)^2 \right] (g_\sigma \sigma)^2 \quad (5)$$

$$M_\Sigma(\sigma) = M_\Sigma - \left[0.6653 - 0.08244 R_N^{free} + 0.00193 \left(R_N^{free} \right)^2 \right] g_\sigma \sigma + \left[0.00064 + 0.07869 R_N^{free} - 0.0179 \left(R_N^{free} \right)^2 \right] (g_\sigma \sigma)^2 \quad (6)$$

$$M_\Xi(\sigma) = M_\Xi - \left[0.3331 + 0.00985 R_N^{free} - 0.00287 \left(R_N^{free} \right)^2 \right] g_\sigma \sigma + \left[-0.00032 + 0.0388 R_N^{free} - 0.0054 \left(R_N^{free} \right)^2 \right] (g_\sigma \sigma)^2 . \quad (7)$$

We take the bag radius, R_N^{free} , to be 0.8fm but note that (c.f. Fig. 1 of Ref. [15]) the results are quite insensitive to this parameter. Note that the coefficients in Eqs. (4) to (7) differ slightly from those in Ref. [15] because of the improvements in the MIT bag model explained in Sect. II. The only significant change is for the Σ hyperon and this is a result of ensuring the correct hyperfine splitting from the Λ . It is worth recalling that one underlying hypothesis of the model is that the (σ, ω, ρ) mesons do not couple to the strange quark, which is justified by the OZI rule since the interaction cannot proceed by quark exchange. Moreover, this hypothesis directly leads to the absence of spin-orbit splitting for Λ hypernuclei, which is consistent with experimental data. In this framework, the ω -baryon coupling goes like the number of non-strange quarks and thus follows the constraints of SU(6) symmetry. We note that, for example, QCD sum-rule investigations [16] have suggested that the ω -baryon couplings may differ from the SU(6) values. On the other hand, the success of the QMC model in leading to very realistic effective NN forces [17] gives us some confidence, *a posteriori*, for the hypothesis.

To calculate the hyperon levels, we use a relativistic shell model. In principle it would be preferable to generate the shell model core, consisting only of nucleons, using the Hartree-Fock approximation. However, in practice it is much easier to use Hartree approximation, which should produce very similar results, because the coupling constants are adjusted in each case to reproduce the properties of nuclear matter. Thus, we use the simpler Hartree

approximation, with free space meson nucleon coupling constants $g_\sigma^2 = 8.79m_\sigma^2$, $g_\omega^2 = 4.49m_\omega^2$ and $g_\rho^2 = 3.86m_\rho^2$, with $m_\sigma = 700$ MeV, $m_\omega = 770$ MeV and $m_\rho = 780$ MeV [15]. Once we have the shell model core wave functions there is, however, no practical reason for not using the more sophisticated Hartree-Fock couplings for the hyperon. In a previous study of high central density neutron stars [15], where the hyperon population is large enough that their exchange terms matter, we found that the Hartree-Fock couplings, $g_\sigma^2 = 11.33m_\sigma^2$, $g_\omega^2 = 7.27m_\omega^2$ and $g_\rho^2 = 4.56m_\rho^2$, gave a satisfactory phenomenology. So, for the hyperons we use these couplings (in Eqs.(4-7)).

In the $1s_{1/2}$ level of ^{208}Pb , this yields 26.9 MeV binding for a Λ and a mere 3.3 MeV binding for a Σ^0 . The reduction of more than 23 MeV binding for the Σ^0 relative to the Λ is a tremendous improvement over the original QMC model and this may be traced directly to the inclusion of the effect of the medium in enhancing the hyperfine splitting of the Σ and Λ . It may be of interest to note that, if this hyperfine effect, which is an essential feature of the improved QMC model, were to be neglected, one would need, for example, to increase the $\omega - \Sigma$ coupling with respect to $\omega - \Lambda$ by about 20%.

The remarkable agreement between the calculated and the experimental binding energy of the Λ in the $1s_{1/2}$ level of ^{208}Pb is another major success. In our earlier work the Λ was overbound by 12 MeV and we needed to add a phenomenological correction which we attributed to the Pauli effect. This correction is not needed when we use Hartree-Fock, rather than Hartree, coupling constants.

TABLE II: Single-particle energies (in MeV) for $^{17}_Y\text{O}$, $^{41}_Y\text{Ca}$ and $^{49}_Y\text{Ca}$ hypernuclei. The experimental data are taken from Ref. [5] (Table 11) for ^{16}O and from Ref. [33] for ^{40}Ca .

	$^{16}_\Lambda\text{O}$ (Expt.)	$^{17}_\Lambda\text{O}$	$^{17}_{\Xi^0}\text{O}$	$^{40}_\Lambda\text{Ca}$ (Expt.)	$^{41}_\Lambda\text{Ca}$	$^{41}_{\Xi^0}\text{Ca}$	$^{49}_\Lambda\text{Ca}$	$^{49}_{\Xi^0}\text{Ca}$
$1s_{1/2}$	-12.42 ±0.05 ± 0.36	-16.2	-5.3	-18.7 ±1.1	-20.6	-5.5	-21.9	-9.4
$1p_{3/2}$		-6.4	—		-13.9	-1.6	-15.4	-5.3
$1p_{1/2}$	-1.85±0.06±0.36	-6.4	—		-13.9	-1.9	-15.4	-5.6
$1d_{5/2}$					-5.5	—	-7.4	—
$2s_{1/2}$					-1.0	—	-3.1	—
$1d_{3/2}$					-5.5	—	-7.3	—

Already at this stage the binding of the Σ^0 in the $1s_{1/2}$ level of ^{208}Pb is just a few MeV – a major improvement over the earlier QMC results. However, as pointed out in that work, there is an additional piece of physics which really should be included and which goes beyond the naive description of the intermediate range attraction in terms of σ exchange. In particular, the energy released in the two-pion exchange process, $N \Sigma \rightarrow N \Lambda \rightarrow N \Sigma$, because of the Σ - Λ mass difference, reduces the intermediate range attraction felt by the Σ hyperon. In Ref. [25] this was modeled by introducing an additional vector repulsion for a Σ hyperon. Following the same procedure, we replace $g_\omega^\Sigma \omega(r)$ by $g_\omega^\Sigma \omega(r) + \lambda_\Sigma \rho_B$, with $\lambda_\Sigma = 50.3 \text{ MeV}\cdot\text{fm}^3$, as determined in Ref. [25] by comparison with the more microscopic model of the Jülich group [32]. We are comfortable using this earlier estimate of the effect of the coupled $\Lambda - N$ channel, even though there is a more recent potential from the Jülich group [34], because the latter tends to overbind hypernuclei [35] and the sign of the correction for coupling to a lower mass channel is in any case model independent.

Our results are presented in Tables II and III. The overall agreement with the experimental energy levels of Λ hypernuclei across the periodic table is quite good. The discrepancies which remain may well be resolved by small effective hyperon-nucleon interactions which go beyond the simple, single-particle shell model. Once again, we stress the very small spin-orbit force experienced by the Λ , which is a natural property of the QMC model [26].

There are no entries for the Σ -hyperon because neither the Σ^+ nor the Σ^0 is bound to a finite nucleus. This absence of bound Σ -hypernuclei constitutes a major advance over earlier work. We stress that this is a direct consequence of the enhancement of the hyperfine interaction (that splits the masses of the Σ and Λ hyperons) by the mean scalar field in-medium. It is especially interesting to examine the effective non-relativistic potential felt by the Σ^0 in a finite nucleus. This is shown in Fig.1 for Calcium and Lead. In the central region the vector interaction dominates over the scalar one leading to a repulsive effective potential which reaches respectively 30 MeV and 12 MeV at the center. It is only at the surface that the scalar attraction becomes dominant. While the exact numerical values depend on the mass taken for the σ meson, we stress the similarity to the phenomenological form found by Batty *et al.* [36]. For a recent review see [37]. It will clearly be very interesting to pursue the application of the current theoretical formulation to Σ^- -atoms.

We also note that this model supports the existence of a variety of bound Ξ -hypernuclei. For the Ξ^0 the binding of the 1s level varies from 5 MeV in $^{17}_{\Xi^0}\text{O}$ to 15 MeV in $^{209}_{\Xi^0}\text{Pb}$. The

TABLE III: Same as table II but for ${}_{\Lambda}^{91}\text{Zr}$ and ${}_{\Lambda}^{208}\text{Pb}$ hypernuclei. The experimental data are taken from Ref. [5] (Table 13).

	${}_{\Lambda}^{89}\text{Yb}$ (Expt.)	${}_{\Lambda}^{91}\text{Zr}$	${}_{\Xi^0}^{91}\text{Zr}$	${}_{\Lambda}^{208}\text{Pb}$ (Expt.)	${}_{\Lambda}^{209}\text{Pb}$	${}_{\Xi^0}^{209}\text{Pb}$
$1s_{1/2}$	-23.1 ± 0.5	-24.0	-9.9	-26.3 ± 0.8	-26.9	-15.0
$1p_{3/2}$		-19.4	-7.0		-24.0	-12.6
$1p_{1/2}$	-16.5 ± 4.1 (1p)	-19.4	-7.2	-21.9 ± 0.6 (1p)	-24.0	-12.7
$1d_{5/2}$		-13.4	-3.1	—	-20.1	-9.6
$2s_{1/2}$		-9.1	—	—	-17.1	-8.2
$1d_{3/2}$	-9.1 ± 1.3 (1d)	-13.4	-3.4	-16.8 ± 0.7 (1d)	-20.1	-9.8
$1f_{7/2}$		-6.5	—	—	-15.4	-6.2
$2p_{3/2}$		-1.7	—	—	-11.4	-4.2
$1f_{5/2}$	-2.3 ± 1.2 (1f)	-6.4	—	-11.7 ± 0.6 (1f)	-15.4	-6.5
$2p_{1/2}$		-1.6	—	—	-11.4	-4.3
$1g_{9/2}$		—	—	—	-10.1	-2.3
$1g_{7/2}$		—	—	-6.6 ± 0.6 (1g)	-10.1	-2.7
$1h_{11/2}$		—	—	—	-4.3	—
$2d_{5/2}$		—	—	—	-5.3	—
$2d_{3/2}$		—	—	—	-5.3	—
$1h_{9/2}$		—	—	—	-4.3	—
$3s_{1/2}$		—	—	—	-3.5	—

experimental search for such states at facilities such as J-PARC and GSI-FAIR will be very important.

IV. CONCLUDING REMARKS

In conclusion, we stress that including the effect of the medium on the hyperfine interaction between quarks within the quark-meson coupling model has led to some important advances. The agreement between the parameter free calculations and the experimental

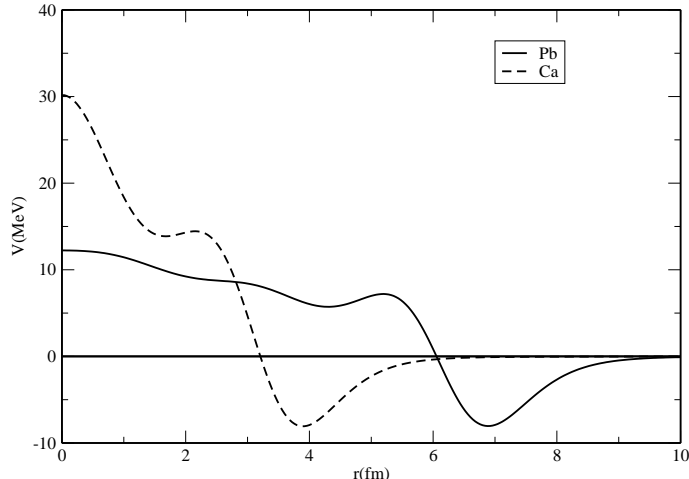


FIG. 1: The effective potential for Σ^0 in Calcium and Lead

ground state levels for Λ -hypernuclei from Calcium to Lead is impressive. In Oxygen the agreement is not as good and we take it as a reminder that the QMC model may be better at describing heavy nuclei, for which the constant density region dominates over the surface. For the d- and f-wave levels shown in Table III, there is a tendency for the model to overbind by several MeV. Whether this is a consequence of the use of an extreme single particle shell model for the core, the omission of residual $\Lambda - N$ interactions or an aspect of the current implementation of QMC that requires improvement remains to be seen. Nevertheless, we find these initial results, obtained with no adjustment of parameters to hyperon data, very encouraging.

A number of Ξ -hypernuclei are predicted to be bound, although not as deeply as in the Λ case. On the other hand, the additional repulsion arising from the enhancement of the hyperfine repulsion in the Σ -hyperon in-medium, together with the effect of the $\Sigma N - \Lambda N$ channel coupling on the intermediate range scalar attraction, means that no Σ -hypernuclei are predicted to be bound. This encouraging picture of finite hypernuclei, suggests that the underlying model, which is fully relativistic and incorporates the quark substructure of the baryons, is ideally suited for application to the properties of dense matter and neutron stars.

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