Two-component model for the axial form factor of the nucleon

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Abstract

We suggest a simple phenomenological parametrization for the axial nucleon form factors, and show that a good fit on the available data, with a minimal number of parameters, can be obtained. The present description of the nucleon structure is based on a compact core, surrounded by an axial meson cloud.

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I. INTRODUCTION

Besides the electromagnetic form factors (FFs), which describe the distribution of charge and magnetization in the hadrons, the nucleon is characterized by the axial form factor, $G_a(Q^2)$, which is related to weak neutral currents. FFs are function of one kinematical variable, the momentum transfer squared $Q^2 = -t$, and are real in the space-like (SL) region of momentum transfer. The axial FF has been measured directly by neutrino scattering, $\nu_\mu + p \to \mu^+ + n$, or indirectly, by near threshold charged pion electroproduction in SL region. Parity violating terms in electron hadron interaction contain also information on weak neutral currents.

The interest in parity-odd (P-odd) terms in electron-hadron interaction, suggested by Zeldovich in 1959 [1], has been recently renewed due to the possibility of very precise polarization measurements in electron proton scattering. The asymmetry in *ep* elastic scattering, generated by a longitudinally polarized beam on an unpolarized target, which should vanish in Born one photon approximation, has been measured to be different from zero at 10^{-6} level [2]. The origin of such asymmetry has been attributed to the interference of $1\gamma^* \bigotimes 1Z$ boson exchanges. This asymmetry can be related to the axial, pseudoscalar, strange nucleon FFs, when the electromagnetic FFs are known. [3]

Following Lorentz, parity and time invariance, the vector electromagnetic (EM) hadronic current (which describes the vertex of a virtual photon with a hadron), the vector neutral and axial hadronic currents, (which describe the vertex of a virtual Z-boson with a hadron) can be written in a general form as:

$$\begin{aligned}
\langle N(p')|J_\mu^{EM}|N(p)\rangle &= \bar{u}(p')\left[F_1(Q^2)\gamma_\mu + i\frac{F_2(Q^2)}{2m_N}\sigma_{\mu\nu}q^\nu\right]u(p)\\
\langle N(p')|J_\mu^{NC,V}|N(p)\rangle &= \bar{u}(p')\left[\tilde{F}_1(Q^2)\gamma_\mu + i\frac{\tilde{F}_2(Q^2)}{2m_N}\sigma_{\mu\nu}q^\nu\right]u(p)\\
\langle N(p')|J_{\mu 5}^{NC,A}|N(p)\rangle &= \bar{u}(p')\left[\tilde{G}_A(Q^2)\gamma_\mu + \frac{\tilde{G}_P(Q^2)}{m_N}q_\mu\right]\gamma_5 u(p), \qquad (1)
\end{aligned}$$

where p, p' are the four momenta of incoming and outgoing nucleon, $q = p' - p$ is the four momentum transfer, F_1, F_2 are the usual nucleon EM Dirac and Pauli form factors, $\tilde{F}_1$, $\tilde{F}_2$ are the corresponding neutral weak (vector) FFs and $\tilde{G}_A$, $\tilde{G}_P$ are the nucleon axial and pseudoscalar form factors. Similarly to the definition of EM Sachs form factors, the neutral weak vector FFs can be defined as:

$$\tilde{G}_E(Q^2) = \tilde{F}_1(Q^2) - \tau \tilde{F}_2(Q^2)\,, \quad \tilde{G}_M(Q^2) = \tilde{F}_1(Q^2) + \tilde{F}_2(Q^2). \tag{2}$$

Let us discuss the measurement of asymmetry in parity violating electron-proton (*ep*) elastic scattering. It is assumed that the underlying mechanism is the exchange of a virtual photon and a virtual Z^0 boson. In principle *ep* scattering experiments probe both the electromagnetic and weak neutral currents. However the electromagnetic interaction is several order of magnitudes stronger than the weak interaction for $Q^2 \ll M_Z^2$. In order to detect the very small weak neutral current contribution to *ep* scattering one has to use an experimental observable which is due to parity violating weak interaction. The difference between the differential cross sections for the scattering of longitudinally polarized electrons with spin parallel (+) and antiparallel (−) to their momenta, vanishes in Born approximation. A nonzero value signs the interference term between one photon and one boson exchange and is implies parity violation.

For elastic scattering on a spin 1/2 target, the PV asymmetry depends, in principle, on all FFs appearing in Eq. (1). However, if one assumes SU(3) symmetry, i.e., the dominance of light quarks u, d, s in the nucleon, FFs can be expressed in terms of the quark content. Furthermore, extracting the charges of the quarks, the vector current is the same for EM and weak interaction and the weak FFs can be expressed as a linear combination of EM isoscalar, isovector and strange FFs. If one further assumes charge symmetry (i.e., correspondence of the u and d quarks wave functions in the proton and in the neutron, respectively) PV asymmetry in elastic *ep* scattering depends on three form factors $G_E^{(s)}$, $G_M^{(s)}$, G_A.

Together with experimental data on other observables measured in processes mentioned at the beginning of this section, PV asymmetry can be used to separate data on these form factors. In the present asymmetry measurements information on the strange form factors is extracted assuming that the axial form factor does not contribute in the conditions where most of the experiments are performed. G0 plans dedicated runs at backward angles in order to extract information on the axial coupling of the photon with the nucleon. SAMPLE gave values for the axial form factor, combining the results on the proton and deuteron targets.

Nucleon pseudoscalar form factor can be extracted from ordinary muon capture on liquid hydrogen target and on light nuclei at $Q^2 = 0.88 M_\mu^2$. We will not discuss this further, as it is usually neglected in the analysis of the data we consider below.

The existing experimental information on axial form factor is available directly through the reaction $\nu_\mu + p \to \mu^+ + n$, or indirectly, through near threshold charged pion electroproduction, in SL region. In our analysis we consider data from pion electroproduction only. The axial form factor is related to the slope of differential cross section as a function of ε near threshold. Low energy theorems calculate electric dipole amplitude at threshold in case of soft pions. In order to compare with real data, model dependent corrections must be introduced to take into account finite pion mass. The data and their corrections will be discussed below. A good parametrization is considered a dipole fit [17]:

$$G_A^D(Q^2) = \frac{g_a(0)}{(1 + Q^2/M_A^2)^2} \tag{3}$$

where $M_A = 1.069 \pm 0.018$ and $g_A(0)$ is the axial-vector coupling constant $g_A(0) = 1.2673 \pm 0.0035$.

In neutrino scattering experiments the dipole approximation is assumed a priori and the axial meson mass is extracted from the data. The corresponding value, $M_A = 1.026 \pm 0.021$ GeV is somehow inconsistent with the best fit value from electroproduction experiments. It has been shown in [17] that an agreement can be found between these two values, if one takes into account corrections due to finite pion mass, in baryon chiral perturbation theory.

The dipole parametrization for form factors had been considered a very reasonable approximation for long time not only for axial FF, but also for magnetic FF of proton and neutron (in all kinematical range of Q^2 in SL region) and for the electric proton FF. The electric neutron form factor was assumed to be zero or very small following the Galster approximation [4]. But recently it has been shown that the electric distribution in the proton is different from the magnetic one and that the ration $\mu G_E^p/G_M^p$ linearly deviates from from unity with increasing Q^2 down to a value 0.24 at $Q^2 = 5.5$ GeV^2 [5]. This result was obtained in a series of polarization measurements at Jefferson Laboratory based on an idea firstly suggested by A.I. Akhiezer and M.P. Rekalo [6].

The model Iachello, Jackson, Landé (IJL) [7] predicted such behavior for the electric proton form factor long before the data appeared. Such model is based on a two component picture of the nucleon, where a hard core of radius $r = 0.34$ fm is surrounded by a meson cloud. It has shown to be very successful in the description of the four nucleon electromagnetic FFs in SL and TL regions [7], of the strange nucleon FF [10] and recently applied to deuteron [11].

The purpose of this paper is to extend IJL model to axial form factors. An interesting property of this model is that it can be analytically extended to time-like (TL) region. The axial FF has not yet been measured in TL region. Suggestions for its determination can be found in [12–14], through the reaction $N\bar{p} \to \gamma^* N\pi$ and the crossed channels.

II. FORMALISM

Following the IJL model [7], the axial FF can be parametrized as:

$$G_A(Q^2) = g(Q^2)\left[1-\alpha+\alpha\frac{m_A^2}{m_A^2+Q^2}\right] g_A(0);\ g(Q^2) = \left(1+\gamma Q^2\right)^{-2}, \tag{4}$$

where $Q^2 > 0$ in the SL region, α is a fitting parameter which corresponds to the coupling of the photon with an axial meson, $m_A = 1.170$ GeV is the mass of the lightest axial meson $h_1(1170)$, with quantum numbers $I^G(J^{PC}) = 1^+(1^{+-})$. The function $g(Q^2)$ describes the internal core of the nucleon, with $\gamma \simeq 0.25$ GeV^{-2}, as derived from the fit of nucleon electromagnetic form factors. We will keep γ as a fixed parameter. Let us note however that this value is not good from a t channel point of view, because it gives a pole in the physical region, $t_0 = \frac{1}{\gamma}=4$ GeV$^{-2} > 4m^2 = 3.52$ GeV^{-2}, the corresponding threshold.

This parametrization can give a zero in the SL region, for $Q^2 = Q_0^2 = m_A^2/(\alpha-1)$, if $\alpha > 1$. In principle the mass m_A can be considered a fitted parameter, also.

The asymptotic behavior of this parametrization is driven by:

$$G_A(Q^2) = \frac{(1-\alpha)g_A(0)}{(\gamma Q^2)^2} \tag{5}$$

with a negative value for $\alpha > 1$.

The extension to TL region of presented model can be done by analytical continuation, similarly to the models of nucleon EM FFs. It can be summarized in following steps:

- The sign of kinematical variable should be changed: $Q^2 \to -t$;
- A complex phase δ, similarly as for IJL model [10], is introduced in the internal core term (4);
- VMD term corresponding to exchange of axial meson should be substituted by a Breit-Wigner formula due to the considerable width of the axial meson.

These modifications lead to the following parametrization of the axial FF in TL region:

$$G_A(t) = g(t)\left[1-\alpha+\alpha\frac{m_A^2\left(m_A^2-t+im_A\Gamma_A\right)}{\left(m_A^2-t\right)^2+\left(m_A\Gamma_A\right)^2}\right]g_A(0), \qquad (6)$$

where

$$g(t) = \left(1-e^{i\delta}\gamma t\right)^{-2}.$$

III. ANALYSIS OF THE DATA

The considered set of data includes all points measured from pion electroproduction on the nucleon. A compilation can be found in Ref. [17].

The Q^2-dependence of the nucleon axial form factor $G_A(Q^2)$, was measured in several pion electroproduction experiments at threshold since a few decades. The slope of the total unpolarized differential cross section at threshold, contains information on $G_A(Q^2)$, but the numerical value of this FF is highly model dependent. In general, four different approaches were used to extract the values of the axial form factor of the nucleon. Soft pion approximation (SP) [18], partially conserved axial current approximation (PCAC) [19], Fulran approximation (FPV) [20] (enhanced soft pion production) and Dombey and Read approximation (DR) [21]. As a consequence of these competing approaches, up to four experimental values may be extracted from a single measurement (at fixed Q^2). All together 77 experimental points are available, corresponding to 32 measurements. Data from Ref. [16] were considered separately, as they correspond to Δ excitation in final state. In order to evaluate the systematic error, the data were therefore separated in 4(5) groups according to used approach (measured processes). The data from [22] were not considered in the fit, following Ref. [17] as they are systematically larger, as well as data from [18].

The data, normalized to one, are plotted in Fig. 1. Different symbols correspond to different models used for the extraction of the data but may correspond to the same experiment.

Individual fits to the 4(5) data sets, according to Eq. 4, were performed, as well as a general one parameter fit. The results are shown in Table I and in Fig. 1. The global fit gives $\alpha = 1.46 \pm 0.04$, with $\chi^2/n.d.f. = 81.47/48 = 1.70$. Such fit does not correspond to the smaller χ^2, due to the dispersion of the data, but the error associated to the parameter is smaller, due to the larger number of points. This value of α, which can be considered as an average to the different corrections, will be used in the following analysis. The associated systematic error, which takes into account the dispersion of the model analysis, can be evaluated from the results of the individual fits to be $< |0.33|$.

The parametrization provides a reasonable description of data, in the limit of the fact that the error associated to the data itself should be considered a 'corridor' which includes not only statistical error of the data, but especially the systematic errors related to the model dependence of the data extraction.

Model	DR	FPV	SP	PCAC	LAMBDA	all
α	1.29 ± 0.08	1.74 ± 0.13	1.08 ± 0.06	1.66 ± 0.05	1.13 ± 0.07	1.46 ± 0.04
$\chi^2/n.d.f.$	1.38	0.80	3.75	0.76	0.45	1.70

TABLE I: Fitted α parameter for different assumed models of extracting data on axial FF.

Once the parameter α has been fixed on SL data, the TL behavior of nucleon axial FF can be calculated, according to Eq. (6) and it is shown in Fig. 2. The magnitude of the axial FF is significantly higher than in SL region. The position and the shape of the peak is determined by the values of γ and δ in the internal core term. The validity of such TL behavior is based on the analytical continuation from the SL region, but can not be tested on experiment as no data on the nucleon axial FF are available in TL region.

IV. LOW Q^2 LIMIT

The slope of the axial form factor at $t \to 0$ is related to the axial radius by:

$$G_A(t) = g_A(0)\left(1 + \frac{1}{6} < r_A^2 > t + \mathcal{O}(t^2)\right). \tag{7}$$

Let us compare the low t limits of the present parametrization for $G_A(t)$:

$$G_A(t) \underset{t\to 0}{\longrightarrow} g_A(0)(1 + (2\gamma + \frac{\alpha}{m_A^2})t),$$

and of the dipole parametrization $G_A^D(t)$:

$$G_A^D(t) \underset{t\to 0}{\longrightarrow} g_A(0)(1 + \frac{2}{M_A^2}t),$$

where m_A is mass of lightest axial meson and M_A is mass of an 'effective' axial meson fitted in dipole approximation.

Equalizing the above expressions, one can extract a value of the coupling constant α

$$\alpha = 2m_A^2 \left(\frac{1}{M_A^2} - \gamma \right) = 1.71.$$

This value, if inserted in parametrization (4) brings compatibility for the two models at low Q^2.

One can, alternatively find express the axial radius in terms of α and of the masses:

$$< r_A^2 >= \frac{6}{g_A(O)} \frac{dG_A(t)}{dt} \bigg|_{t=0} = \frac{12}{M_A^2} \tag{8}$$

In case of dipole parametrization one finds $\sqrt{< r_A^2 >} = 0.638$ fm to be compared, in case of the present parametrization, with

$$\sqrt{< r_A^2 >} = \left[\frac{6}{g_A(O)} \left(2\gamma + \frac{\alpha}{m_A^2}) \right) \right]^{1/2} = 0.6039 \text{ fm}. \tag{9}$$

As mentioned in the Introduction, the axial radius extracted from neutrino scattering is larger, $\sqrt{< r_A^2 >} = 0.67 \pm 0.01$ fm and corrections in frame of chiral perturbation theory may bring this and the dipole radius in agreement [17].

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V. CONCLUSIONS

A parametrization of the axial nucleon FF has been proposed, following a model where a compact core is surrounded by an axial meson cloud. This parametrization satisfies the analytical properties of FFs, and can be extended to the whole kinematical region of momentum transfer squared. It satisfies asymptotic properties and has similar behavior as the dipole parametrization at small q^2, but the present parameters suggest a smaller axial radius.

Experimental data are model dependent in SL region and inexistent in TL region. A possible way to access the axial FF in time-like region and in the unphysical region (below the reaction threshold) has been suggested through the reactions $N\bar{p} \to \gamma^* N\pi$ and the crossed channels [12–14]. The cross section related to these processes is large and such experiments may be planned in future colliders.

Possible improvements of the present parametrization, which will be required in case of new, more precise data, can be foreseen in two directions. Firstly, as the width of any axial meson is large, even in comparison with the ρ meson, in principle one can modify the corresponding propagator in a complicated form, similarly to what was done for the ρ meson [7]. Moreover, one can include the contribution of two axial mesons, with different masses.

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[1] Ya.B. Zeldovich, JETP **9**, 682 (1959).

[2] F. E. Maas *et al.*, Phys. Rev. Lett. **94**, 152001 (2005); VF. E. Maas *et al.* [A4 Collaboration], Phys. Rev. Lett. **93**, 022002 (2004). D. T. Spayde *et al.* [SAMPLE Collaboration], Phys. Lett. B **583**, 79 (2004); T. M. Ito *et al.* [SAMPLE Collaboration], Phys. Rev. Lett. **92**, 102003 (2004); A. Acha *et al.* [HAPPEX collaboration], Phys. Rev. Lett. **98**, 032301 (2007) D. S. Armstrong *et al.* [G0 Collaboration], Phys. Rev. Lett. **95**, 092001 (2005).

[3] M. J. Musolf, T. W. Donnelly, J. Dubach, S. J. . Pollock, S. Kowalski and E. J. Beise, Phys. Rept. **239** (1994) 1.

[4] S. Galster, H. Klein, J. Moritz, K. H. Schmidt, D. Wegener and J. Bleckwenn, Nucl. Phys. B **32**, 221 (1971).

[5] M. K. Jones *et al.*, Phys. Rev. Lett. 84 (2000) 1398; O. Gayou *et al.*, Phys. Rev. Lett. 88 (2002) 092301; V. Punjabi *et al.*, Phys. Rev. C 71 (2005) 055202. [Erratum-ibid. C **71** (2005) 069902]

[6] A. Akhiezer and M. P. Rekalo, Dokl. Akad. Nauk USSR, **180**, 1081 (1968); Sov. J. Part. Nucl. **4**, 277 (1974).

[7] F. Iachello, A.D. Jackson, A. Landé, Phys. Lett. 43B, 171 (1973).

[8] R. Bijker and F. Iachello, Phys. Rev. C **69**, 068201 (2004); Q. Wan and F. Iachello, Int. J. Mod. Phys. A **20**, 1846 (2005).

[9] F. Iachello, Eur. Phys. J. A **19** (2004) SUPPL129 [FizikaB **13** (2004) 13].

[10] R. Bijker, J. Phys. G **32** (2006) L49 [arXiv:nucl-th/0511060].

[11] E. Tomasi-Gustafsson, G. I. Gakh and C. Adamuscin, Phys. Rev. C **73** (2006) 045204 [arXiv:nucl-th/0512039].

[12] M. P. Rekalo, Sov. J. Nucl. Phys. **1**, 760 (1965).

[13] A. Z. Dubnickova, S. Dubnicka and M. P. Rekalo, Z. Phys. C **70**, 473 (1996).

[14] C. Adamuscin, E. A. Kuraev, E. Tomasi-Gustafsson and F. E. Maas, arXiv:hep-ph/0610429, to be published in Phys. Rev. C.

[15] An International Accelerator Facility for Beams of Ions and Antiprotons, *Conceptual Design Report*, http://www.gsi.de.

[16] P. Joos *et al.*, Phys. Lett. B **62**, 230 (1976).

[17] V. Bernard, L. Elouadrhiri and U. G. Meissner, J. Phys. G **28**, R1 (2002).

[18] Y. Nambu and M. Yoshimura, "Axial vector form-factor on nucleon determined from threshold electropion Phys. Rev. Lett. **24** (1970) 25.

[19] G. Benfatto, F. Nicolo and G. C. Rossi, Nuovo Cim. A **14** (1973) 425.

[20] G. Furlan, N. Paver and C. Verzegnassi, Nuovo Cim. A **70** (1970) 247.

[21] N. Dombey and B. J. Read, Nucl. Phys. B **60** (1973) 65.

[22] E. D. Bloom *et al.*, "Measurements Of Inelastic Electron Scattering Cross-Sections Near One Pion Phys. Rev. Lett. **30** (1973) 1186.

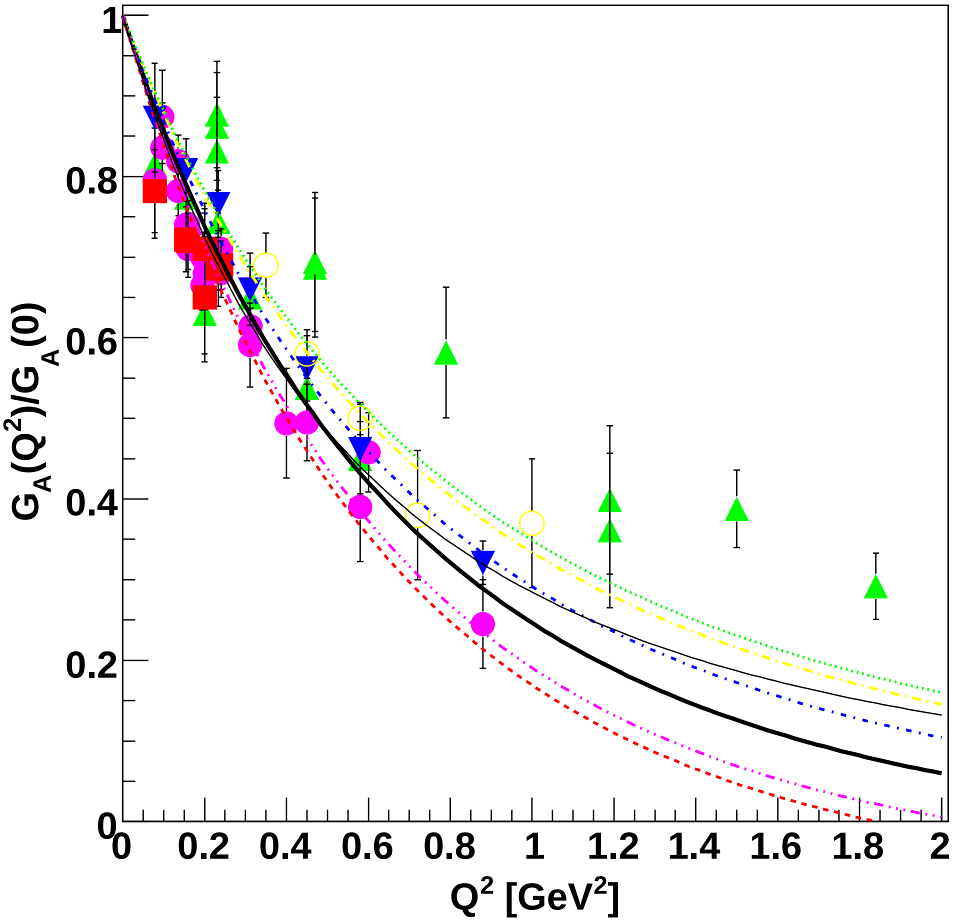

FIG. 1: Normalized axial form factors as a function of Q^2. The solid thick line correspond to the global fit. Different symbols and fits correspond to data extracted according different models: partially conserved axial current approximation (PCAC) [19] (pink) : solid circles; dashed-triple dotted line; Fulran approximation (FPV) (red) [20]: solid squares and dotted line; soft pion approximation (SP) (green)[18] :solid triangles and dashed line; Dombey and Read approximation (DR) (blue) [21]: trianglesdown and dash-dotted line; data corresponding to delta excitation (Delta) (yellow) [16]: open circles and dash-short dotted line. The solid thin (black) line corresponds to the dipole parametrization, Eq. (3).

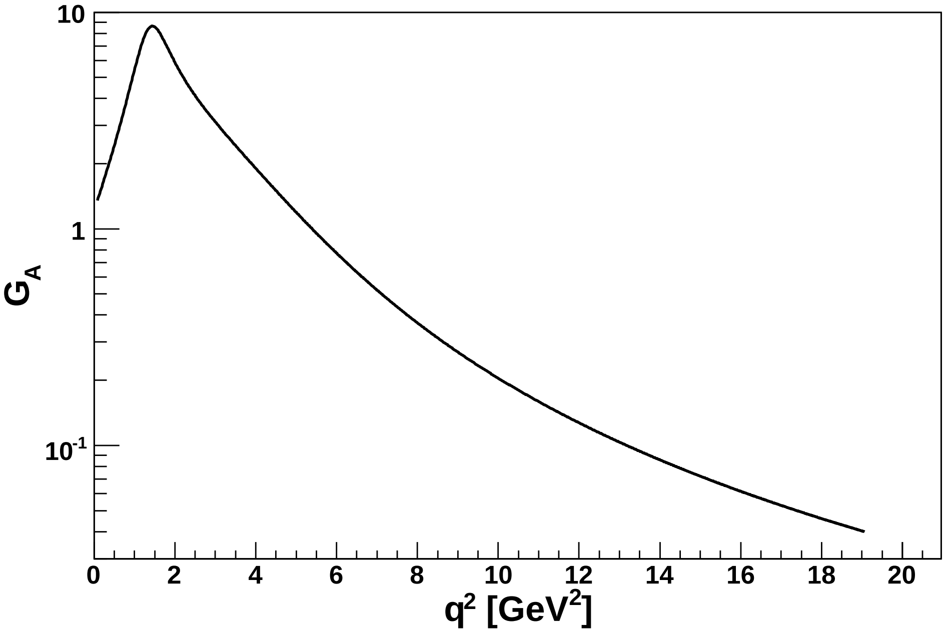

FIG. 2: Nucleon axial form factors TL behavior.