# DESIGN OF SLUG TUNERS FOR THE SPIRAL2 RFQ 

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## Abstract

Tuner parameters: number (or separation distance), diameter and position range, are determined in order to fit two main requirements: (1) compensation of construction errors specified between given bounds, and (2) compatibility with magnetic-field bead-pull measurements. Tuner slopes possibly derived from 2D or 3D simulations are compared. RFQ 4-wire transmission line model is used to calculate tuner position range required to compensate for given capacitance relative errors. The position of the bead guiding-wire is deduced from 3D field maps and magnetic-field-to-voltage calibration accuracy requirement.

## INTRODUCTION

The SPIRAL 2 RFQ is designed to accelerate at 88 MHz two kinds of charge-over-mass ratio ( $\mathrm{Q} / \mathrm{A}$ ) particles. The proposed injector can accelerate a 5 mA deuteron beam $(\mathrm{Q} / \mathrm{A}=1 / 2)$ or a 1 mA ion beam with $\mathrm{Q} / \mathrm{A}=1 / 3$ up to 0.75 $\mathrm{MeV} / \mathrm{A}$. It is a CW machine which has to show stable operation, provide the requested availability, have the minimum losses in order to minimize the activation and show the best performance/cost ratio.


Figure 1: Location of tuner slugs and loop port in one 1meter long RFQ module (final design).

It will be a 4 vane RFQ type, mechanically assembled (Fig. 1), the global goal being to build an RFQ without any brazing step.

## SLUG TUNERS 3D MODELING

The SPIRAL2 RFQ design was initially based on a distribution of three tuner planes per meter, with 130 mm diameter tuners in planes \#1-4, \#6-10 and \#12-15, and 90 mm -diameter tuners in planes \#5 (on each side of vacuum port) and \#11 (on each side of feeder loop), thus amounting to a total number of 68 tuners $(52 \times 130 \mathrm{~mm}$
and $16 \times 90 \mathrm{~mm}$ ). Tuner inductance slopes were derived from 2D SuperFish simulations.

Recently, 3D simulations of slug tuners have been done with SOPRANO, and also with COMSOL. The results are summarized in Table 1. For these simulations, a short section of RFQ is meshed (Fig. 2) with electrical-parallel, magnetic-normal end boundary conditions, which act as mirror boundary conditions. Thus the simulations represent infinite periodic RFQ's. One half 130 mm diameter slug is located at each extremity of the model (Fig. 3).


Figure 2: One-quarter of RFQ cross section with 2 half slugs meshed with I-DEAS and solved with SOPRANO (a) Magnetic field, (b) Electric field

Table 1: Short RFQ segment 3D simulation with tuners.

| SOFTWARE | $\begin{array}{c}\text { LENGTH } \\ (\mathrm{mm})\end{array}$ |  | $\begin{array}{c}\text { TUNER POS. } \\ \# 1\end{array}$ |  | $\begin{array}{c}\mathrm{F} \\ (\mathrm{MHz})\end{array}$ | $\begin{array}{c}\mathrm{F}-\mathrm{F}_{2 \mathrm{D}} \\ (\mathrm{MHz})\end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | \(\left.\begin{array}{l}\mathrm{h}_{qq0} <br>

(\mathrm{mm})\end{array}\right]\)


Figure 3: SOPRANO models corresponding to different tuner positions $[\# 1, \# 2]$ : (a) $[-30,+50]$, (b) $[+0,+0]$, (c) $[+50,+0]$, (d) $[+130,+0]$.

The tuner position function $\mathrm{h}_{\mathrm{QQ}}(\mathrm{z})=\mathrm{h}_{\mathrm{QQ} 0}(\mathrm{z})+\mathrm{h}_{\mathrm{QQ1}}(\mathrm{z})$ is broken down into its even $\mathrm{h}_{\mathrm{QQ} 0}$ and $\mathrm{h}_{\mathrm{QQ} 1}$ parts, where the former induces frequency and even voltage perturbations, and the latter odd voltage perturbation only. Different slug tuner positions have been examined, from -30 mm to +130 mm ; the resulting linear frequency shift vs. $\mathrm{h}_{\mathrm{QQ} 0}$ (Fig. 4) shows that Slater's small perturbation theory may be applied safely in this case. Note that the "flush" position ( $\mathrm{h}_{\mathrm{QQ} 0}=0$ ) has been defined on the slug center line, leading to a -6.22 mm maximum recession at slug periphery, and a -274 kHz difference with respect to the resonance frequency of the 2D geometry.


Figure 4: 3D frequency shift ( kHz ) vs. tuner position even part $\mathrm{h}_{\mathrm{QQ} 0}(\mathrm{~mm}) .0 \mathrm{kHz}$ reference: 2D simulation.

Slater's formula may also be used to derive the estimate of the 2 D slope: $\partial \mathrm{f} / \mathrm{dh}=(4 / \pi)\left(\mathrm{z}_{\mathrm{t}} / \phi_{\mathrm{t}}\right)(\Delta \mathrm{f} / \Delta \mathrm{h})$, where $\mathrm{z}_{\mathrm{t}}$ is tuner axes spacing, $\phi_{t}$ is tuner diameter, and the $4 / \pi$ factor accounts for the circular shape. The 3D simulations give $\partial \mathrm{f} / \mathrm{dh}=+127.5 \mathrm{kHz} / \mathrm{mm}$, while 2D SuperFish simulations led to $81 \mathrm{kHz} / \mathrm{mm}$ only for some unclear reason.

## MINIMUM TUNER SPACING

Tuner spacing is directly related to the voltage tuning process: bead-pull measurements sense longitudinal magnetic field $H_{z}\left(x_{0}, y_{0}, z\right)$ vs. abscissa $z$ at some transverse location $\left\{\mathrm{x}_{0}, \mathrm{y}_{0}\right\}$ in RFQ quadrants, and a conversion factor $\kappa(\mathrm{z})=\mathrm{V}(\mathrm{z}) / \mathrm{H}_{\mathrm{z}}\left(\mathrm{x}_{0}, \mathrm{y}_{0}, \mathrm{z}\right)$ is applied to
recover the value of inter-vane voltage $\mathrm{V}(\mathrm{z})$. This procedure remains valid provided that some regions do exist where $H_{z}$ is not perturbed by the tuners themselves. The relative variation $\left[\kappa(z)-\kappa^{0}\right] / \kappa^{0}$, where $\kappa^{0}$ is the value of the conversion factor for the pure 2D geometry, is plotted at Fig. 5 for various bead position radii. The local perturbation due to the tuner is clearly visible up to about 230 mm from tuner axis; hence tuner separation should be at least 460 mm , leading to a new design with 40 identical 130 mm -diameter tuners ( 2 per quadrant and per meter). Note that the residual value of $\left[\kappa(z)-\kappa^{0}\right] / \kappa^{0}$ at large $z$ determines a lower bound on tuning accuracy ( $\sim 1 \%$ ).


Figure 5: $\left[\kappa(\mathrm{z})-\kappa^{0}\right] / \kappa^{0}$ vs. z for various bead position radii (color). Position of tuner located at $\mathrm{z}=0$ is +50 mm .

## TUNER POSITION RANGE

## Method

The loaded lossless 4-wire transmission line model [1] is used to consistently relate inter-vane voltages to originating perturbations, such mechanical tolerances and tuners. Tolerance bounds are then easily transformed into tuner position range requirements. Inter-vane voltages $\left|\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{u}_{4}\right|$, expressed in $\{\mathrm{Q}, \mathrm{S}, \mathrm{T}\}$ basis: $\mathrm{u}_{1}=+\mathrm{U}_{\mathrm{Q}}+\mathrm{U}_{\mathrm{S}}$, $u_{2}=-U_{Q}+U_{T}, u_{3}=+U_{Q}-U_{S}, u_{4}=-U_{Q}-U_{T}$, are solution of the differential problem

$$
\begin{equation*}
\left\{-\partial^{2} U / \partial z^{2}+A U=\lambda U\right\}+\text { boundary conditions } \tag{1}
\end{equation*}
$$

where A contains line inductance and capacitance data (derived from 2D simulations) vs. abscissa z , and $\lambda=(\omega / \mathrm{c})^{2}$. Mechanical tolerances are modelled by the perturbations of inter-vane capacitances

$$
\begin{array}{ll}
\Delta \mathrm{C}_{1}=\Delta \mathrm{C}_{\mathrm{QQ}}+\Delta \mathrm{C}_{\mathrm{SQ}}, & \Delta \mathrm{C}_{2}=\Delta \mathrm{C}_{\mathrm{QQ}}-\Delta \mathrm{C}_{\mathrm{TQ}}, \\
\Delta \mathrm{C}_{3}=\Delta \mathrm{C}_{\mathrm{QQ}}-\Delta \mathrm{C}_{\mathrm{SQ}}, \quad \Delta \mathrm{C}_{4}=\Delta \mathrm{C}_{\mathrm{QQ}}+\Delta \mathrm{C}_{\mathrm{TQ}} .
\end{array}
$$

The resulting perturbation of the accelerating mode $\mathrm{Q}_{\mathrm{n}}$ is

$$
\Delta \mathrm{V}_{\mathrm{Qn}}=\sum_{\delta \neq \mathrm{n}} \mathrm{q}_{\mathrm{Q} \delta} \mathrm{~V}_{\mathrm{Q} \delta}+\sum_{\alpha} \mathrm{q}_{\mathrm{S} \alpha} \mathrm{~V}_{\mathrm{S} \alpha}+\sum_{\beta} \mathrm{q}_{\mathrm{T} \beta} \mathrm{~V}_{\mathrm{T} \beta}
$$

where $\mathrm{V}_{\mathrm{Q} \delta}, \mathrm{V}_{\mathrm{S} \alpha}, \mathrm{V}_{\mathrm{T} \beta}$ are the eigen-functions of Eq. 1. First order analysis yields the expansions

$$
\Delta \mathrm{C}_{\mathrm{XQ}}(\mathrm{z})=\sum_{\mathrm{i}} \Delta \mathrm{p}_{\mathrm{XQi}_{\mathrm{i}}} \mathrm{C}_{\mathrm{XQi}}(\mathrm{z}), \quad \mathrm{X} \in\{\mathrm{Q}, \mathrm{~S}, \mathrm{~T}\}
$$

in the $\left\{\mathrm{C}_{\mathrm{XQi}}\right\}$ dual bases, and the linear relations
$\Delta \lambda_{\mathrm{Qn}}=\Delta \mathrm{p}_{\mathrm{QQn}}, \quad\left(\lambda_{\mathrm{Qn}}-\lambda_{\mathrm{Xi}}\right) \mathrm{q}_{\mathrm{Xi}}=\Delta \mathrm{p}_{\mathrm{XQi}}, \mathrm{X} \in\{\mathrm{Q}, \mathrm{S}, \mathrm{T}\}$, (2) where the $\lambda_{\mathrm{Xi}}$ are the eigen-values of Eq. 1. In the same way, positions of the 4 tuners in plane $\# t$ are expressed as $\mathrm{h}_{1 \mathrm{t}}=\mathrm{h}_{\mathrm{QQt}}+\mathrm{h}_{\mathrm{SQt}}, \mathrm{h}_{2 \mathrm{t}}=\mathrm{h}_{\mathrm{QQt}}-\mathrm{h}_{\mathrm{TQt}}, \mathrm{h}_{3 \mathrm{t}}=\mathrm{h}_{\mathrm{QQt}}-\mathrm{h}_{\mathrm{SQt}}, \mathrm{h}_{4 \mathrm{t}}=\mathrm{h}_{\mathrm{QQt}}+\mathrm{h}_{\mathrm{TQt}}$.

First order analysis yields the expansions

$$
\mathrm{h}_{\mathrm{XQt}}=\sum_{\mathrm{i}} \Delta \mathrm{p}_{\mathrm{XQi}} \xi_{\mathrm{XQti}}, \quad \mathrm{X} \in\{\mathrm{Q}, \mathrm{~S}, \mathrm{~T}\},
$$

in the $\left\{\xi_{\mathrm{XQti}}\right\}$ dual bases, and the same linear relations given by Eq. 2. The tuner position limits in plane $\# t$ required to compensate for a maximum $\mathrm{C}_{\mathrm{XQ}}$ capacitance relative error $\rho$ are given by the solutions of the problems
find min and max of $-\mathrm{h}_{\mathrm{XQt}}$, given the constraints

$$
-\rho \leq \sum_{\mathrm{i}} \Delta \mathrm{p}_{\mathrm{XQi}_{\mathrm{i}}} \mathrm{C}_{\mathrm{XQi}}\left(\mathrm{z}_{\mathrm{s}}\right) / \mathrm{C}\left(\mathrm{z}_{\mathrm{s}}\right) \leq+\rho, \mathrm{s}=1 \ldots \mathrm{~S},
$$

where $\left\{z_{s}\right\}$ is an adequate abscissa sampling set. This linear programming problem (LP) is easily solved by Dantzig's simplex algorithm [2], repeatedly for all $\mathrm{X} \in\{\mathrm{Q}, \mathrm{S}, \mathrm{T}\}$ and all tuner planes t . There is a total of 6 T problems ( $\mathrm{T}=$ number of tuner planes; $\times 2$ for min and max; $\times 3$ for $\mathrm{Q}, \mathrm{S}, \mathrm{T}$ components). Each LP problem has M decision variables ( $M=$ number of modes used in the expansions) and 2 S single-sided constraints.

## Application

Inter-vane capacitances are assumed to be roughly inversely proportional to their separating gaps. The minimum value of gap $g=7.2 \mathrm{~mm}$ in the SPIRAL2 RFQ and a tolerance $\Delta \mathrm{g}=50 \mu \mathrm{~m}$ yield
$\left|\Delta \mathrm{C}_{\mathrm{i}} / \mathrm{C}\right| \#\left|\Delta \mathrm{~g}_{\mathrm{i}} / \mathrm{g}_{\mathrm{i}}\right| \leq \rho=0.05 / 7.2=0.007, \mathrm{i}=1 \ldots 4$. Using simple geometrical considerations, it is possible to show that the points $\left\{\Delta \mathrm{C}_{\mathrm{QQ}}, \Delta \mathrm{C}_{\mathrm{SQ}}, \Delta \mathrm{C}_{\mathrm{TQ}}\right\}$ define an octahedron (Fig. 6).


Figure 6: Inter-vane capacitance errors envelope.
The above method has been applied to the various cuts labeled I, II and III in Fig. 6, and has been found to yield identical results in the three cases. Cut II main results are summarized in Table 2. The required tuning frequency is used to determine the value of $\Delta \lambda_{\mathrm{Qn}}$ in Eq. 2. The untuned frequency, corresponding to the no-tuner geometry, would be in fact obtained with a 7.5 mm tuner position, as shown in Fig. 4. Thus 7.5 mm should be added to the tuner position values in Table 2. The realized constraints are quite close to the specified ones; this accuracy could be improved by increasing the set size $S$.

Detailed results for the maximum tuner position QQcomponents in Cut II are displayed in Fig. 7. The maximum position of each tuner (consistently identified by color code) is shown in Fig. 7(d); the corresponding
relative capacitance perturbation is shown in Fig 7(a). The relative voltage error resulting from this perturbation (prior to tuning) is shown in Fig. 7(b), and its spectral coefficients $\mathrm{q}_{\mathrm{Q} \delta}$ in Fig. 7(c).

Table 2. Results for Cut II.

| resonance frequency, un-tuned | 87.47 MHz |
| :--- | :---: |
| resonance frequency, tuned | 88.05 MHz |
| number of modes in $\mathrm{Q}, \mathrm{S}, \mathrm{T}$ expansions | 10 |
| size of abscissa sampling set $\left\{\mathrm{z}_{\mathrm{s}}\right\}$ | 100 |
| $\left\|\Delta \mathrm{C}_{\mathrm{QQ}} / \mathrm{C}\right\|,\left\|\Delta \mathrm{C}_{\mathrm{SQ}} / \mathrm{C}\right\|,\left\|\Delta \mathrm{C}_{\mathrm{TQ}} / \mathrm{C}\right\| \leq \rho / 2=$ | $3.510^{-3}$ |
| minimum tuner position | -0.26 mm |
| maximum tuner position | +44.52 mm |
| max number of simplex iterations | 10 |
| realized min constraint | $-3.50110^{-3}$ |
| realized max constraint | $+3.50810^{-3}$ |



Figure 7: Maximum of $-\mathrm{h}_{\mathrm{QQt}}$ found by the simplex algorithm for Cut II.

## CONCLUSION

The tuner position limits given in Table 2 are linear functions of the tolerance $\rho$, and can be easily extended to other values of $\rho$. The recommended specification is $-40 \mathrm{~mm} \sim+70 \mathrm{~mm}$, obtained by considering (i) the 7.5 mm trim already mentioned, (ii) the reduced efficiency of tuners when recessed from the cavity wall, and (iii) comfortable safety margins.

## REFERENCES

[1] A. France, F. Simoens, "Theoretical Analysis of a Real-Life RFQ Using 4-Wire Line Model and the Theory of Differential Operators", EPAC 2002.
[2] J.W. Chinneck, "Practical Optimization: a Gentle Introduction", Carleton University, Ottawa, Canada, http://www.sce.carleton.ca/faculty/chinneck/po.html.

