

Strong pickup-channel coupling effects in proton scattering: the case of $p + {}^{10}\text{Be}$

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Abstract

The dynamic polarization potential (DPP) contribution to the effective proton-nucleus interaction, that is due to the coupling of deuteron channels, is evaluated by applying $S_{lj} \rightarrow V(r)$ inversion to the elastic channel S -matrix from coupled reaction channel calculations of proton elastic scattering. This was done for protons scattering from ${}^{10}\text{Be}$ at 12, 13, 14, 15, and 16 MeV; non-orthogonality corrections were included. We find a consistent pattern of a repulsive real and an absorptive imaginary DPP, with the absorption shifted to a larger radius. This is consistent with what has been found for proton scattering from the neutron skin nucleus ${}^8\text{He}$. The DPP is not of a form that can be represented by a renormalization of the bare potential, and has properties suggesting an underlying non-local process. We conclude that deuteron channels cannot be omitted from a full theoretical description of the proton-nucleus interaction (optical potential).

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I. INTRODUCTION

It is many years since coupled reaction channel (CRC) calculations first suggested that the coupling to deuteron channels makes a significant contribution to proton scattering, and in fact significantly improves the fit to certain data [1–4]. The development of inverse scattering techniques, whereby the potential that yields a specific S-matrix S_{ij} can reliably be calculated [5], has made it possible to represent the coupling effects as a contribution to the dynamic polarization potential (DPP). A consistent finding [6, 7] has been that the deuteron channels contribute a significant *repulsive* component to the nucleon-nucleus interaction, in addition to the expected additional absorptive component.

The CRC calculations cited above all omit the non-orthogonality corrections [8, 9]. It has recently become possible to include these terms and their importance has been verified in a recent study [10] of the contribution of the ${}^8\text{He}(p,d){}^7\text{He}$ pickup reaction to $p + {}^8\text{He}$ elastic scattering at a bombarding energy of 15.7 A MeV. Very strong coupling effects were found. Although the non-orthogonality terms were found to modify the details of the DPP quite significantly, the qualitative features were consistent with earlier findings. Nevertheless, ${}^8\text{He}$ is a far from typical nucleus, the ${}^8\text{He}(p,d){}^7\text{He}$ reaction having a combination of almost perfect Q-matching ($Q = -0.36$ MeV) and a large spectroscopic factor [10, 14] leading to the large DPP that was found. The present work sets out to demonstrate that the properties of the DPP found for ${}^8\text{He}$ are more generally true. It is a first step in tracing out the systematics of the evolution of the effect with incident energy, Q-value and spectroscopic factor.

Since the nucleon-nucleus interaction is fundamental to nuclear physics, the nature of the pickup DPP should be verified, especially in view of the following findings [10]: (i) the form of the DPP makes it unrealistic to correct a folding model potential with real and imaginary normalization factors, as is commonly done; (ii) there is a local emissive region in the imaginary part of the DPP which suggests that the local potential is representing non-local effects (the reference here is not to exchange non-locality) that would not arise in folding models based on a local density approximation; (iii) the repulsive nature of the real part of the DPP is at variance with the findings of dispersive optical models in which that part of the optical potential, often identified as the DPP, that is added to the Hartree-Fock component, is attractive [11–13].

II. CRC CALCULATIONS FOR $P + {}^{10}\text{Be}$

We present here a series of CRC calculations evaluating the contribution of ${}^{10}\text{Be}(p,d){}^9\text{Be}$ pickup coupling to proton scattering for which a set of ${}^{10}\text{Be} + p$ elastic scattering data at incident proton energies of 12, 13, 14, 15 and 16 MeV is available in the literature [15]. Fitting this data helps to ensure that the calculations are realistic. The ${}^{10}\text{Be}$ nucleus provides an excellent opportunity for comparison with ${}^8\text{He}$, as the number of neutrons is unchanged. The addition of two protons presents us with a case with a much more negative Q-value for the (p,d) pickup reaction for ${}^{10}\text{Be}$, -4.6 MeV, raising the possibility that the influence of Q-value on the DPP can be studied.

Although there are unfortunately at present no data for the ${}^{10}\text{Be}(p,d)$ pickup, there are data for the ${}^9\text{Be}(d,p){}^{10}\text{Be}$ stripping reaction at several energies which enable the ${}^{10}\text{Be}/{}^9\text{Be}$ spectroscopic factor to be fixed empirically. The shell model calculation of Cohen and Kurath [16] gives a value of $C^2S = 2.36$, as does that reported in [17], although the empirical value obtained from the systematic DWBA analysis of [17] is significantly lower, 1.58 ± 0.15 . In this study we have taken the latter value to give a conservative estimate of the coupling effect of the ${}^{10}\text{Be}(p,d)$ pickup reaction.

The CRC calculations were performed with the code FRESKO [9] and included the complex remnant term and non-orthogonality correction. We took the global nucleon potential for 1p-shell nuclei of [18] as a starting point for the entrance channel potentials. The neutron-proton overlap was calculated using the Reid soft-core potential [19] and included the small D-state component of the deuteron ground state. The $n+{}^9\text{Be}$ binding potential was of Woods-Saxon form, with a central part of radius $r_0 = 1.25$ fm and diffuseness $a = 0.65$ fm and a spin-orbit component of the same geometry and fixed depth of 6 MeV, the depth of the central part being adjusted to obtain the correct binding energy.

For the $d + {}^9\text{Be}$ exit channel, continuum discretized coupled channels (CDCC) calculations, similar to those described in [20] but without the (d,p) coupling, were carried out to fit the appropriate ${}^9\text{Be}(d,d)$ data [21–23] taking the deuteron breakup effect explicitly into account. The $n,p + {}^9\text{Be}$ potentials required as input were based on the global potential of Ref. [24]. In order that the CDCC calculations fit the data, concentrating on the forward scattering angles, the potentials of Ref. [24] were renormalized as follows: the real parts were increased by 30-50 % and the imaginary parts were decreased by about 20 % except for

the ${}^9\text{Be}(d,d)$ data [21] at the appropriate energy for the 12 MeV ${}^{10}\text{Be}(p,d){}^9\text{Be}$ calculation where an increase of the imaginary part by a factor of two was necessary, probably because of the rather low equivalent deuteron energy. The n-p continuum was discretized into bins of width $\Delta k = 0.1 \text{ fm}^{-1}$ up to $k_{\text{max}} = 0.4 \text{ fm}^{-1}$. The resulting CDCC input parameters were then incorporated into the ${}^{10}\text{Be}(p,d){}^9\text{Be}$ CRC calculations, yielding a combined CRC/CDCC calculation similar to that carried out for the ${}^8\text{He}(p,d){}^7\text{He}$ reaction [10]. Transfers to the $L = 0$ and $L = 2$ unbound states of the n-p system were included. Couplings to the $L = 1$ and $L = 3$ states were completely omitted as they are found to have a small effect [25] on deuteron elastic scattering. Indeed, test calculations in which the breakup couplings in general were omitted showed that they had minimal effect in this case, at least on the proton elastic scattering.

The entrance channel potentials were then re-tuned to obtain the best fit to the ${}^{10}\text{Be}(p,p)$ elastic scattering data. In practice, it was found that a single potential geometry, slightly modified from that of the initial global optical potential of [18], was able to provide good fits to all the data with minor adjustments to the real and imaginary potential depths. The final “bare” potential parameters are given in Table I, along with the χ^2/N values obtained from the full calculations. The parameters of the spin-orbit potential were fixed: $V_{\text{so}} = 5.5$ MeV, $r_{\text{so}} = 1.14$ fm and $a_{\text{so}} = 0.57$ fm, as there are no polarization data available. From the χ^2/N values in the last column, it can be seen that the fit becomes relatively poor at 12 MeV; the change seen in the data beyond 130° for this 1 MeV step is either a resonance-like effect or a problem with the data itself. In neither case would we expect the data to be fitted within the optical model framework. For the purpose of calculating the DPP at 12 MeV, it is reasonable to use the potential that fits the data for $\theta < 130^\circ$ and has the same geometry as used at higher energies.

The calculations are compared to the data in Figure 1 where we also plot the “bare” no-coupling results. The good agreement with the data is borne out by the χ^2/N values given in Table I. With a final total of four adjustable parameters (real and imaginary potential depths in the entrance channel and real and imaginary potential normalization parameters in the exit channel) this is as it should be, although the real and imaginary potential normalization factors for the ${}^9\text{Be}(d,d)$ CDCC calculations in the exit channels were determined separately against the appropriate elastic scattering data and then held fixed. However, the motivation of the current work was to determine the DPP due to the (p,d) pickup as accurately as

TABLE I: “Bare” $p + {}^{10}\text{Be}$ potential parameters and χ^2/N values for the full CRC calculations. The real and imaginary potentials are of volume and surface Woods-Saxon form, respectively.

Energy (MeV)	V (MeV)	r_V (fm)	a_V (fm)	W_D (MeV)	r_D (fm)	a_D (fm)	χ^2/N
16	65.7	1.137	0.514	7.15	1.068	0.497	3.46
15	65.7	1.137	0.514	7.15	1.068	0.497	2.38
14	65.7	1.137	0.514	6.00	1.068	0.497	1.93
13	65.0	1.137	0.514	6.70	1.068	0.497	1.00
12	65.0	1.137	0.514	6.00	1.068	0.497	8.52

possible, hence the need for the best possible fit to the data.

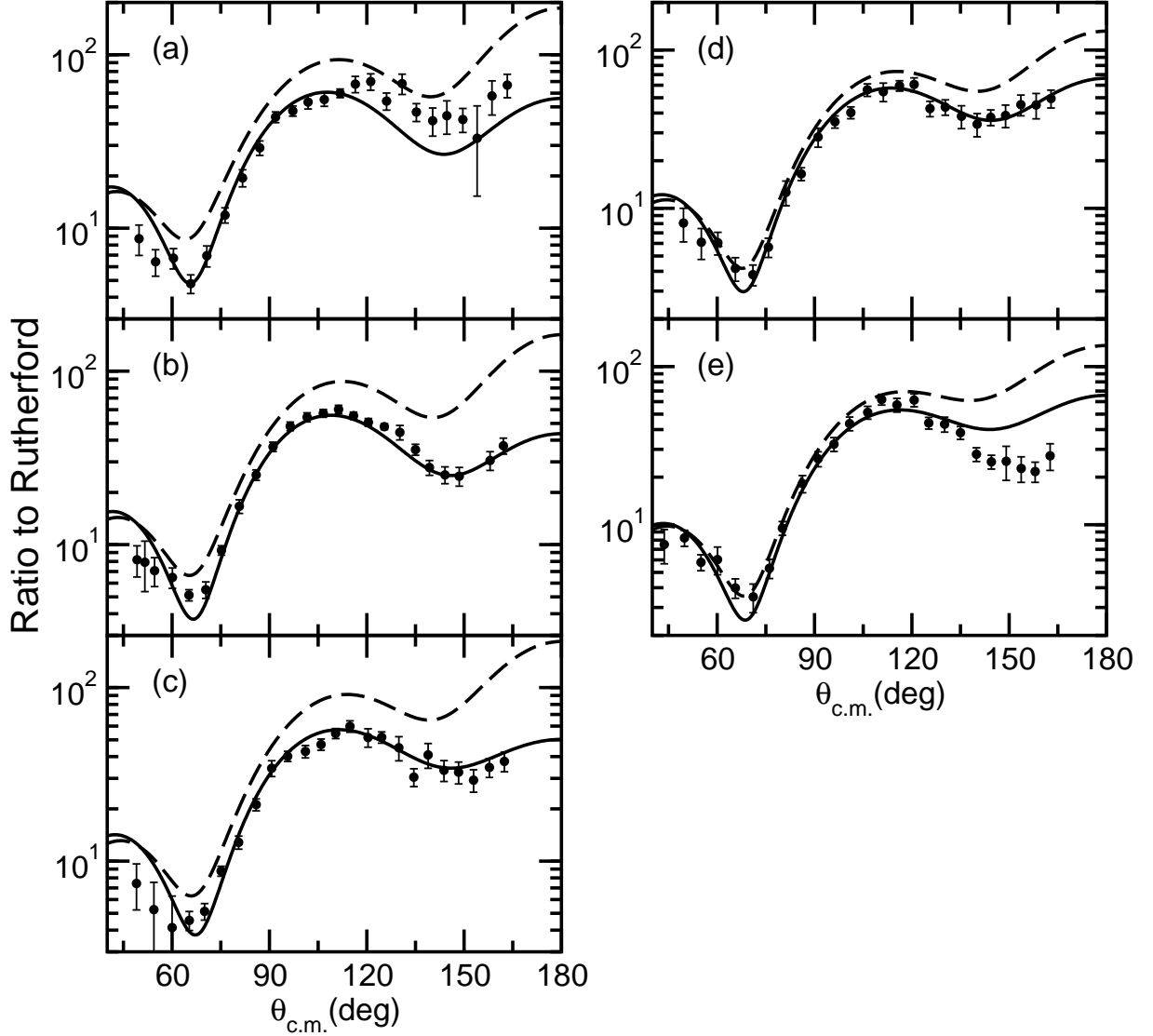
It is apparent from a comparison of the full and dashed curves in Fig. 1 that the effect of the (p,d) pickup coupling remains important for the ${}^{10}\text{Be}(p,p)$ elastic scattering at these energies, although considerably less than was found for the ${}^8\text{He}(p,p)$ elastic scattering. In the next section we describe the effect upon the DPP of increasing the spectroscopic factor to the value used in the ${}^8\text{He} + p$ scattering analysis. It is also apparent from Fig. 1 that the coupling effect at 12 and 13 MeV is significantly smaller than at the slightly higher energies.

III. CALCULATION OF THE DPP

We apply $S_{lj} \rightarrow V(r)$ inversion [5] to the diagonal (elastic scattering) part of the S -matrix produced in the CRC calculations described above to yield a potential $V_{\text{crc}}(r)$. The inversion is carried out using the iterative-perturbative (IP) method [5, 26, 27]. The local potential found by inversion would, if inserted into an optical model (single channel) code, precisely reproduce the theoretical elastic scattering from the CRC calculations. The potential $V_{\text{crc}}(r)$ is, of course, complex and contains a complex spin-orbit term. The difference $V_{\text{dpp}}(r) = V_{\text{crc}}(r) - V_{\text{bare}}(r)$, between $V_{\text{crc}}(r)$ and the bare potential $V_{\text{bare}}(r)$, is a local and L -independent representation of the contribution to the dynamic polarization part of the proton-nucleus potential that is generated by the coupling to the pickup channels.

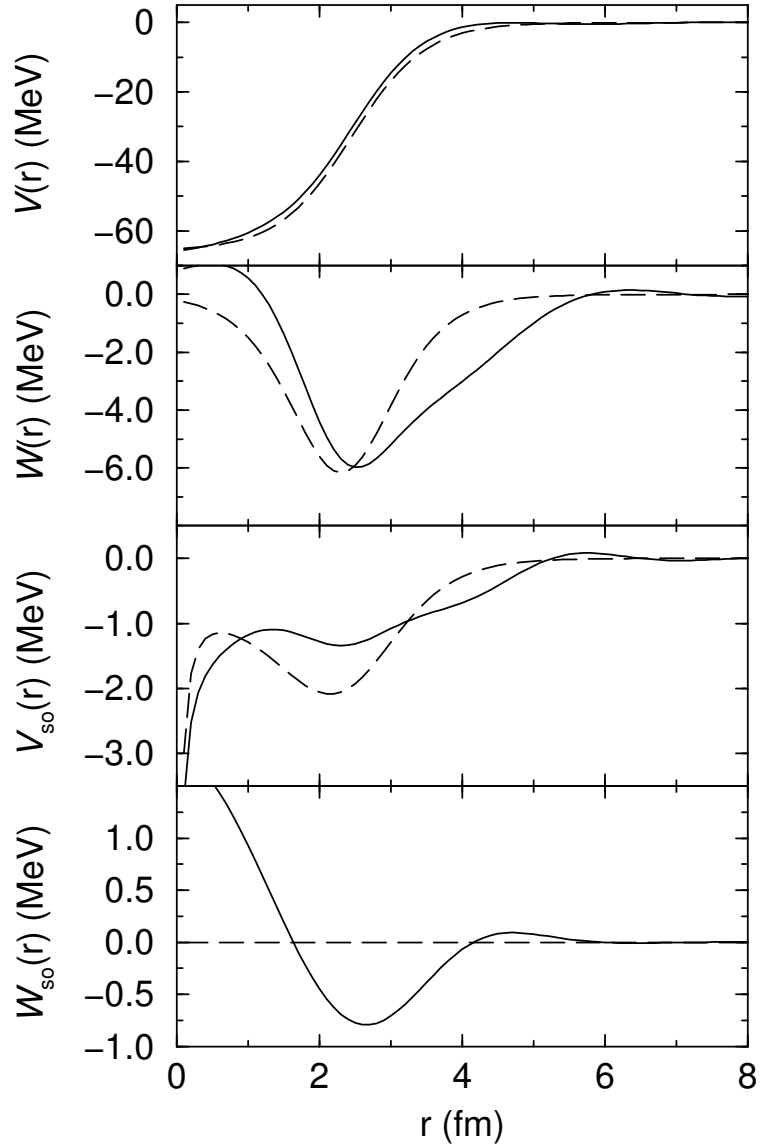
In Figure 2 we compare the bare potentials $V_{\text{bare}}(r)$ (dashed curve) with $V_{\text{crc}}(r)$ (solid curve) at 14 MeV. The effect is qualitatively the same at all five energies, although the magnitude of the effect falls off at the lowest energies, as might be expected by comparing

FIG. 1: Data for $^{10}\text{Be}(p,p)$ elastic scattering [15] compared with the full CRC calculations (full curves) and the no-coupling calculations (dashed curves). (a) 16 MeV, (b) 15 MeV, (c) 14 MeV, (d) 13 MeV, (e) 12 MeV; plotted as ratio to Rutherford in each case.



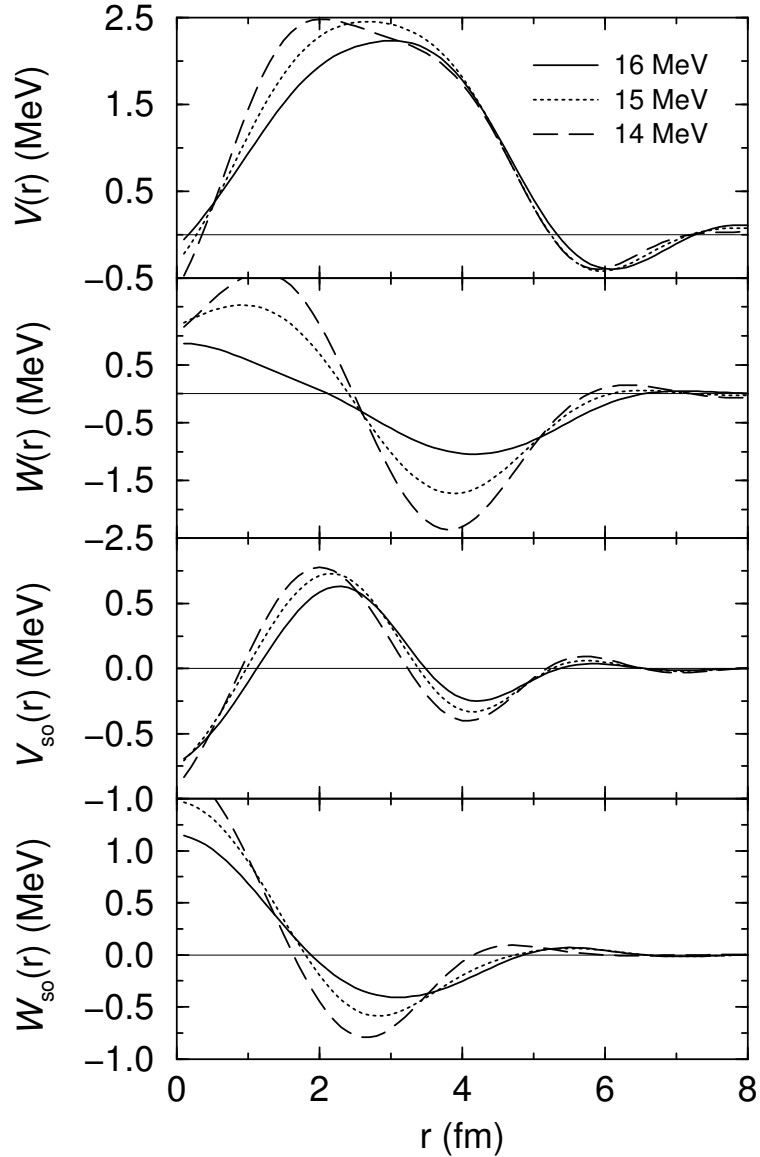
the 12 MeV fit with those at 14 – 16 MeV in Figure 1. The predominant effect on the real part at each energy is repulsive and is not negligible, amounting, as quantified below, to an almost 10 % effect. The effect on the imaginary part is quite large, in each case moving the absorptive region significantly outwards to a larger radius. The modification of the potentials can certainly not be represented as a multiplicative factor times the bare potential.

FIG. 2: Potentials for 14 MeV $^{10}\text{Be}(p,p)$: inverted potential (full curve) and bare potential (dashed curve).



The contributions from the coupled reaction channels are more clearly seen by examining the DPP itself, $V_{\text{dpp}}(r)$, presented for each energy in Figures 3 and 4; the 14 MeV DPP is given in each to facilitate comparisons. The imaginary part of the central DPP is particularly large, showing a clear emissive region at smaller radii. This does not, of course, correspond to any overall emissive regions in the potential, nor any unitarity breaking, but is significant. Such emissive features are commonly found in DPPs that are generated by coupled channels, and probably relate to the fact that the DPP, as presented here, is a

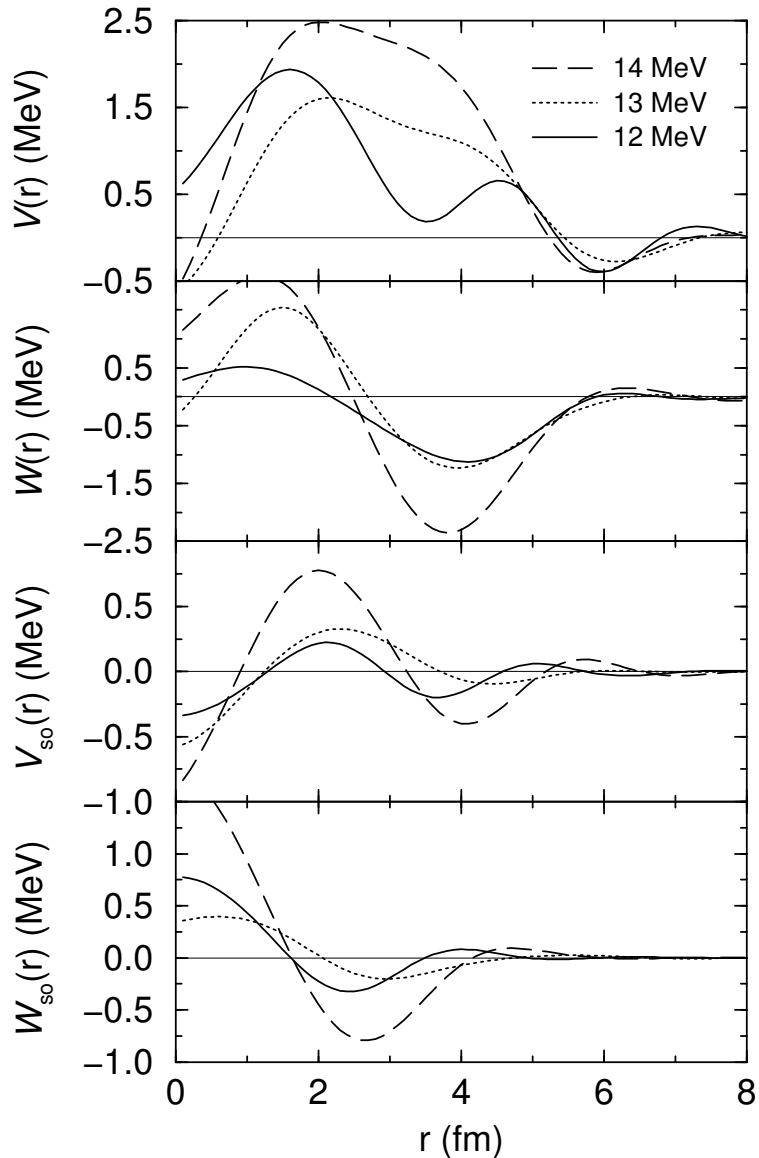
FIG. 3: Dynamic polarization potential, DPP, for 14, 15 and 16 MeV $^{10}\text{Be}(p,p)$ elastic scattering.



local and L -independent representation of a term that, in a full Green's function treatment, would be non-local and L -dependent. This non-locality is quite distinct from that arising from exchange, and the L -dependence referred to is distinct from the parity dependence arising from heavy particle stripping, see [28].

The pickup contributions can usefully be quantified using the conventionally defined [8] real and imaginary volume integrals, J_R and J_I and rms radii, $R_R = (\sqrt{\langle r^2 \rangle})_R$ and $R_I = (\sqrt{\langle r^2 \rangle})_I$. The three most conspicuous changes are a decrease in J_R (a repulsive effect) an

FIG. 4: Dynamic polarization potential, DPP, for 12, 13 and 14 MeV $^{10}\text{Be}(p,p)$ elastic scattering.



increase in J_I (absorption) and an increase in R_I , representing the net shift of the absorptive part of the potential to a larger radius. Table II presents the changes induced by coupling to these three quantities for each energy. For comparison, the bare potential at 15 and 16 MeV has $J_R = 576.1$ (MeV fm^3) and $J_I = 108.8$ (MeV fm^3), so the changes are of the order of 10% for the real part and 50 % for the imaginary part.

Apart from the 12 MeV case, the results are rather consistent. It should be noted that all the quantities given in this table are relatively small differences between pairs of quantities,

E_{lab} (MeV)	ΔJ_{R} (MeV fm ³)	ΔJ_{I} (MeV fm ³)	ΔR_{I} (fm)
16	-66.9	57.3	0.623
15	-68.8	67.9	0.626
14	-67.9	77.2	0.694
13	-44.9	47.2	0.699
12	-24.4	50.1	0.613

TABLE II: The changes in selected characteristics of the proton-nucleus interactions induced by pickup coupling.

one of each pair being subject to uncertainties that are hard to evaluate precisely. At lower energies, the number of active partial waves falls, and the linear equations upon which the IP inversion procedure centers become less definitely over-determined. As a result the IP inversion procedure yields potentials that may be less well-determined; in particular they may have small wiggles in the surface that can contribute disproportionately to the volume integrals and rms radii. For this reason, it is the qualitative properties of the DPP rather than point-by-point values that should be considered well-determined at 12 and 13 MeV.

Nevertheless, the general properties of the DPP can be considered to be well established. In particular, the smaller magnitude of ΔJ_{I} at 13 and especially 12 MeV appears to be a dynamic effect and not an artifact of the inversion. This is supported by the fact that the difference between the solid and dashed lines in the 12 MeV and 13 MeV cases in Figure 1, over the mid-angle range, is substantially less (noting that these are logarithmic plots) than for the other three figures corresponding to 14 — 16 MeV. These figures suggest that the CRC model itself entails a smaller DPP as the energy falls below 14 MeV; the small changes in the bare potentials given in Table I are not expected to have a dramatic effect on the DPPs. Note that the outward shift in the imaginary potentials is as large at 12 MeV as it is at the higher energies.

The general properties of the DPP are similar to those found [10] in the scattering of 15.7 MeV protons on ⁸He. In that case $\Delta J_{\text{R}} = -51.2$ (MeV fm³), $\Delta J_{\text{I}} = 261.41$ (MeV fm³) and (not reported in [10]) $\Delta R_{\text{I}} = 0.827$ fm. The much larger value of ΔJ_{I} in that case might be attributed to the near zero Q-value and much larger spectroscopic factor (2.9, compared to 1.58 for the ¹⁰Be case). In addition, the bare proton potential in the ⁸He case had a

very small imaginary part, $J_I = 33$ (MeV fm³), the deuteron channel contributing a large proportion of the reaction cross section. In a model calculation for $p + {}^{10}\text{Be}$ at 13 MeV, in which we increased the spectroscopic factor from 1.58 to 2.9, the ${}^8\text{He}$ value, we found the following: $\Delta J_R = -73.0$ (MeV fm³), $\Delta J_I = 100.2$ (MeV fm³) and $\Delta R_I = 1.004$ fm. In Section II we noted that we had chosen the spectroscopic factor to give a conservative estimate of pickup coupling contributions.

All the above results were obtained using bare potentials that had the same geometry, but which had minor adjustments to the depths to give the best CRC fits to the elastic scattering. The optimum parameters for such potentials can not be determined uniquely with data of the range and precision seen in Figure 1. The question then arises as to how the results might depend upon the bare potential and we performed a small number of calculations with different potentials. The general result is that the qualitative results do not depend upon the specific potential, but the specific magnitudes of the effects do depend somewhat upon the details of the bare potential. Further explorations of this matter will hopefully lead to an understanding of such questions as to why the real DPP is repulsive. See Reference [27] for an earlier discussion of this but in the context of zero-range CRC without non-orthogonality corrections.

IV. CONCLUSIONS: IMPLICATIONS FOR UNDERSTANDING NUCLEON-NUCLEUS INTERACTIONS

We have presented a local and L -independent potential, generated by the coupling to deuteron channels, for the case of protons scattering from ${}^{10}\text{Be}$. The overall properties of the DPP were qualitatively similar to what was found in the case of scattering from neutron skin nucleus ${}^8\text{He}$ [10], although the contribution to the overall absorptive potential was less than for ${}^8\text{He}$. We therefore propose that the effects found there were not peculiar to proton scattering from ${}^8\text{He}$, and the overall repulsive/absorptive effect is an example of a more general phenomenon. As in all cases studied previously, prior to the inclusion of non-orthogonality corrections, the real part had an overall repulsive character. A small imaginary spin-orbit interaction was generated.

The complex DPP that we found was not at all of a form that could be represented by renormalizing a folding model potential. We therefore conclude that the determination of

such normalization factors is not a satisfactory way of evaluating folding models. It would seem preferable to use model-independent fitting to determine an additive component to a folding model potential. Ideally, this would fully exploit the information content of the data to yield empirical DPPs that could be compared with those calculated in studies such as this.

Some basic problems must be acknowledged. There exists no numerical implementation of a fully rigorous reaction theory, certainly not one based on realistic nucleon-nucleon interactions. As a result there are inevitable uncertainties in the interpretation of our results. For example, the deuteron channel states are not orthogonal to particle-hole states. Such particle-hole states enter into any local density model devised to handle realistic nucleon-nucleon interactions. Moreover, strictly speaking, the concept of the DPP corresponding to specific channels rests on the orthogonality of the coupled channels [29]. There is therefore an unresolved double counting problem. Nevertheless, the CRC calculation does provide a representation of processes that would not be present in a local-density folding model. This is suggested, for example, by the form of the imaginary DPP with its emissive feature at small r . The underlying non-locality that this suggests probably corresponds to the fact that the deuteron in the intermediate state is propagating in a potential with a strong gradient, something local-density models do not encompass.

The nature of the DPP presented here does not fit naturally into the generally accepted understanding of nucleon-nucleus scattering and it is natural to ask for supporting evidence. The present rather good fits to the elastic scattering data for protons on ^{10}Be can equally well be reproduced by a local single-channel optical model. However, other cases do exist [2], albeit studied prior to the inclusion of non-orthogonality terms, in which data could be fitted when the coupling to pickup channels was included that persistently resisted fitting with conventional optical potentials (at least smooth, L -independent potentials); we intend to pursue such cases in the future. Further in support of our conclusions, we note that the general CRC formalism used here, as applied to the elastic scattering of heavy ions, has very successfully explained such phenomena as the threshold anomaly in a range of systems, e.g. [30, 31]. We therefore feel confident that we have demonstrated that *the coupling to deuteron channels must be included in a full account of nucleon scattering from nuclei*. It is possible that local density models include some of the effect *in an average way*, in which case the challenge will be to relate the specific pickup channels for specific nuclei to irregularities

in the N and Z dependence in elastic scattering, something that is essential before the interaction between nucleons and nuclei can be said to be understood.

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