POLARIZATION PHENOMENA IN ANNIHILATION AND SCATTERING

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Abstract
Model independent properties of the matrix element and polarization observables are derived for electron positron annihilation in different channels involving spin one particles. The general spin structure depends on the reaction mechanism, here assumed one photon exchange. The expression of the observables are derived as functions of the hadron electromagnetic form factors, and are given here, as an example, for the reaction $e^+ + e^- \to a_1 + \pi$. Hadron form factors should obey specific analytical properties, in space and time-like regions. Possible parametrizations and experimental constraints are illustrated.

1 Introduction

The electron positron annihilation into hadrons constitutes an important source of information on the internal structure of the mesons: the light quarks and their interactions as well as the spectroscopy of their bound states. The experimental data about these reactions in the low–energy region are also relevant to the determination of the strong interaction contribution to the anomalous magnetic moment of the muon and to the test of standard model predictions for the hadronic tau–lepton decay, which is related by the conservation of vector currents.

Recently, the construction of the new detectors with a large solid angle, which can operate at new colliders with high luminosity, opened new possibilities for the investigation of the reactions $e^+ + e^- \to$ multihadrons [1]. Not only the statistic is highly increased, but also the possibility to detect charged as well as neutral pions allows to draw conclusions on the nature of the intermediate states. In the energy region $1 \leq W \leq 2.5$ GeV (W is the total energy of the colliding beams) the process of four pion production is one of the dominant processes of the reaction $e^+ + e^- \to$hadrons. Its cross section is larger than 2π production and comparable to $e^+ + e^- \to \mu^+ + \mu^-$.

The process of e^+e^- annihilation into four pions was firstly detected in Frascati [2] and later on in Novosibirsk [3]. Through a simultaneous analysis of the differential distributions in two final channels: $2\pi^+2\pi^-$ and $\pi^+\pi^-2\pi^0$, it was shown in [1] that the reaction predominantly occurs through the $a_1(1260)\pi$ and $\omega\pi^0$ intermediate states in the energy range 1.05–1.38 GeV. It was also found that the relative fraction of the $a_1(1260)\pi$ state increases with the beam energy. The measurement of the $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ cross section was extended to lower energies. Data obtained with larger statistical and systematic precision [4] confirmed that the dominant production mechanism is consistent with the $a_1(1260)\pi$ intermediate state. The process of multihadron production at large energies was also investigated with the BABAR detector at the PEP–II asymmetric electron–positron storage ring using the initial–state radiation [5]. In particular, the cross section for the process $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ was measured for center–of–mass (CMS) energies from 0.6 to 4.5 GeV, providing evidence of a resonant

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structure, with preferred quasi–two–body production of $a_1(1260)\pi$. A detailed understanding of the four–pion final state requires also information from final states such as $\pi^+\pi^-\pi^0\pi^0$, to which the $\rho^+\rho^-$ intermediate state can contribute.

The high degree of precision of the current experiments require to devote special attention to radiative corrections. The lepton structure function method [6] provides a very convenient formalism, at thousandth precision, which can be conveniently included in Monte Carlo generators [7].

2 The $e^- + e^+$ annihilation into spin one particles

The differential (and total) cross section and the elements of the spin–density matrix of the a_1-meson are calculated in terms of the electromagnetic form factors (FFs) of the corresponding $\gamma^* a_1 \pi$ current. A model independent formalism, derived in [8] for spin one particles, and applied to the process $e^+ + e^- \to \rho^+ + \rho^-$ in [9], allows to express the experimental observables (differential cross section, polarization observables, elements of the density matrix..) in terms of hadron FFs. In annihilation reactions, these FFs should be known, or extrapolated from the space-like region into the time-like (TL) region, on the basis of analytical arguments. The expression of the experimental observables in terms of (complex) amplitude is model independent, but these amplitudes or form factors are built in frame of hadron models.

We illustrate here the application of this formalism to the reaction $e^+ + e^- \to a_1 + \pi$. In the one-photon approximation, the differential cross section in terms of the hadronic, $W_{\mu\nu}$, and leptonic, $L_{\mu\nu}$, tensors, neglecting the electron mass, is written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{q^6}\frac{p}{2W}L_{\mu\nu}W_{\mu\nu}, \tag{1}$$

where $\alpha = 1/137$ is the electromagnetic constant, $p = \sqrt{(q^2 + m^2 - M^2)^2 - 4m^2q^2}/2W$ is the final–particle momentum in the reaction CMS, m and M are the masses of the pion and of the a_1 meson, respectively. The four momentum of the virtual photon is $q = k_1 + k_2 = p_1 + p_2$, with $q^2 = W^2$, and W is the total energy of the initial beams (note that the cross section is not averaged over the spins of the initial beams).

The leptonic tensor (for the case of longitudinally polarized electron beam) is

$$L_{\mu\nu} = -q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}) + 2i\lambda\varepsilon_{\mu\nu\sigma\rho}k_{1\sigma}k_{2\rho}\ , \tag{2}$$

where λ is the degree of the electron beam polarization (further we assume that the electron beam is completely polarized and consequently $\lambda = 1$).

The hadronic tensor $W_{\mu\nu}$ is defined as bilinear combination of the electromagnetic current J_μ, describing the transition $\gamma^* \to \pi a_1$, as follows:

$$W_{\mu\nu} = J_\mu J_\nu^*. \tag{3}$$

J_μ depends on two FFs, $f_i(q^2)$ $i = 1, 2$, which are complex functions of q^2 (the mass of the virtual exchanged photon) in the region of the TL momentum transfer ($q^2 > 0$). Assuming the P– and C–invariance of the hadron electromagnetic interaction this current can be written as [10]

$$J_\mu = f_1(q^2)(q^2 U_\mu^* - q\cdot U^* q_\mu) + f_2(q^2)(q\cdot p_2 U_\mu^* - q\cdot U^* p_{2\mu}), \tag{4}$$

where U_μ is the polarization four-vector describing the spin one a_1–meson.

In case of real photon, f_1 does not contribute, and the value $f_2(0)$ can be obtained from the experimental data on the decay width $\Gamma(a_1 \to \pi\gamma)$. For unstable particles, the vector and tensor polarizations are directly related to the angular distribution of their decay products; the angular distribution can be expressed in terms of the spin–density matrix. The complete derivation of the observables can be found in Ref. [11]. Here we give the expressions for the differential cross section and for the single spin polarization observables.

In the reaction CMS the unpolarized differential cross section, assuming one photon exchange, can be written as

$$\frac{d\sigma^{un}}{d\Omega} = \frac{\alpha^2}{2q^4}\frac{p}{W}(A + B\sin^2\theta),$$

$$A = |q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2|^2, B = 2\tau p^2[q^2|f_1 + f_2|^2 - M^2|f_2|^2], \quad (5)$$

where θ is the angle between the momenta of the axial–meson ($\vec{p}$) and of the electron beam ($\vec{k}$), $\tau = q^2/4M^2$. Integrating this expression with respect to the axial–meson angular variables one obtains the following formula for the total cross section:

$$\sigma_{tot}(e^+e^- \to \pi a_1) = \frac{2\pi\alpha^2}{3q^4}\frac{p}{W}\Big[3|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2|^2 + +4\tau p^2[q^2|f_1 + f_2|^2 - M^2|f_2|^2]\Big]. \quad (6)$$

Let us define an angular asymmetry, R, with respect to the differential cross section, $\sigma_{\pi/2}$, measured at $\theta = \pi/2$,

$$\frac{d\sigma^{un}}{d\Omega} = \sigma_{\pi/2}(1 + R\cos^2\theta), R = -B/(A + B). \quad (7)$$

As it was previously shown in the case of $e^+ + e^- \to d + \bar{d}$ [8], this observable is very sensitive to the different underlying assumptions on the axial–meson FFs and does not require polarization measurements.

The vector polarization of the outgoing axial–meson, P_y, which does not require polarization in the initial state is

$$P_y = \frac{1}{8}\frac{\sqrt{\tau}}{\sigma_0}\left[(q^2 + M^2 - m^2)^2 - 4M^2q^2\right]\sin(2\theta)\mathrm{Im}f_1 f_2^*, \quad (8)$$

where $\sigma_0 = A + B\sin^2\theta$. One can see that this polarization is determined by non–zero phase difference of the complex FFs f_1 and f_2.

Let us consider now the case of a longitudinally polarized electron beam. The other two components of the axial–meson vector polarization (P_x, P_z) require the initial particle polarization and are

$$\begin{aligned} P_x &= -\frac{1}{4}\frac{\sqrt{\tau}}{\sigma_0}\sin\theta\Big\{2q^2(q^2 + M^2 - m^2)|f_1|^2 + \left[(q^2 - M^2)^2 - m^4\right]|f_2|^2 + \\ &\quad + \left[q^2(3q^2 - 2M^2 - 2m^2) - (M^2 - m^2)^2\right]\mathrm{Re}f_1 f_2^*\Big\}, \\ P_z &= \frac{1}{2}\frac{1}{\sigma_0}\cos\theta\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2\right|^2. \end{aligned} \quad (9)$$

The axial–meson FFs are complex functions in the TL region. So, for the complete determination of FFs it is necessary to measure three quantities: two moduli of FFs and their

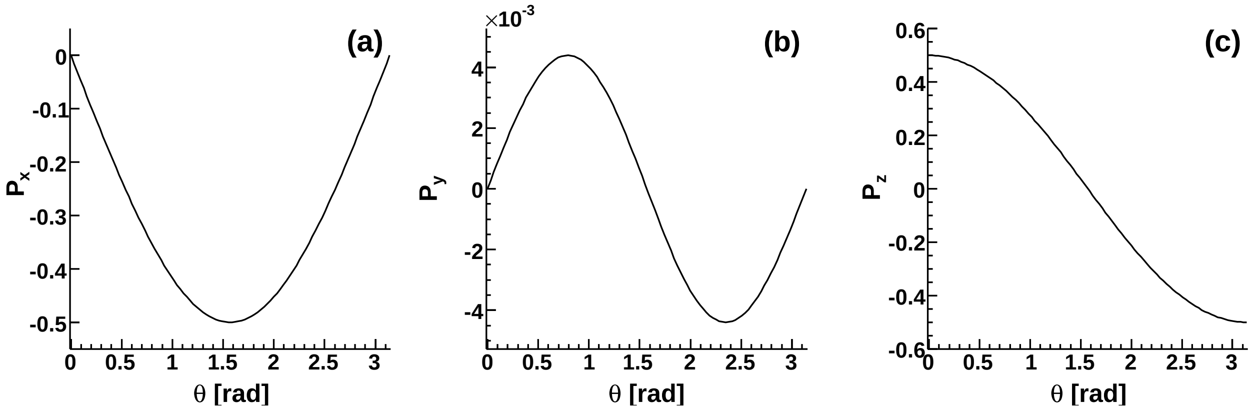

Figure 1: Angular dependence of the vector polarization observables P_x, P_y, P_z at $q^2 = 2$ GeV^2, Eqs. (8,9).

phase difference. Therefore, the measurement of the unpolarized differential cross section does not allow to determine completely FFs. For this aim, it is also necessary to determine the spin–density matrix elements of the produced axial meson measuring the angular distribution of its decay products.

3 Parametrizations for a_1 form factors

Vector Meson Dominance (VMD) inspired models have proved to be very successful in describing the structure of hadrons. Such models contain a small number of parameters, with transparent physical meaning, and can be analytically extended to the full region of momentum transfer squared. The proton electromagnetic FFs in space-like region were successfully reproduced by a parametrization based on two components in the hadron structure: an intrinsic structure, very compact, characterized by a dipole q^2 dependence and a meson cloud [12]. A generalization of this picture to the deuteron [8] gives a very good description of all known data on the three deuteron electromagnetic FFs, with as few as six free parameters, including evident physical constraints [8].

In order to predict the behavior of the experimental observables, we suggest a simple model for the a_1 transition FFs, in TL region. We used a simple VMD-based parametrization saturated by vector mesons. The contribution of one vector meson is given by the Breit-Wigner form

$$\Delta f_i = \frac{C_{v,i} M_v}{M_v^2 - q^2 + i M_v \Gamma_v}, \ i = 1, 2, \tag{10}$$

where M_v and Γ_v are the mass and the width of a vector meson carrying the interaction. In general one should introduce all allowed vector mesons, but, as shown in Refs. [13, 14], the largest contribution to the cross section is given by the $\rho(770)$, $\rho'(1450)$, and, at higher energies, ρ"(1700). We consider only data for the total cross section at energies above the $a_1\pi$ kinematical threshold, which have been compiled from Refs. [15, 5]. The experimental data [5] show a clear contribution from J/ψ around $\sqrt{s} = 3.01$ GeV. We excluded the corresponding (four) data points from the fit. The final form of our parametrization has, therefore, in total six parameters: three normalization constants, $C_{\rho,i}$, $C_{\rho',i}$, $C_{\rho'',i}$ for each of the two FFs. The $\gamma^*\pi a_1$ transition form factors (moduli) are presented in the Fig.2. Peaks can be seen in correspondence with the masses of the chosen vector mesons. For $|f_2|$, one can see a bump around $q^2 = 4$ GeV^2, which results from the interference of the different terms.

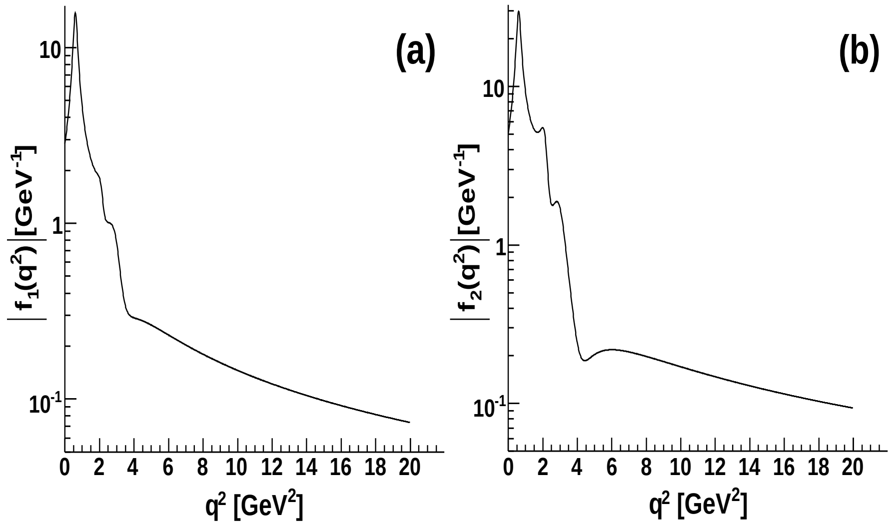

Figure 2: q^2 dependence of the $\gamma^*\pi a_1$ transition form factors f_1, f_2.

Such fit assumes that the intermediate state $a_1\pi$ saturates the cross section [4]. If other intermediate channels contribute to this yield, and only a fraction of the cross section is due to the $a_1\pi$ intermediate state, then (assuming no dependence on q^2) the normalization parameters should be rescaled by the square root of that fraction.

Fig. 1 shows the vector polarization observables. One should note here that the vector polarization of vector mesons can not be measured through their decays which are driven by strong and electromagnetic interaction with conservation of P-parity [16]. In general polarization observables can be quite large and show a particular behavior, which can be experimentally verified.

4 Conclusions

A model independent formalism, developed for scattering and annihilation involving spin 1/2 particles, has been extended to spin one particles. In particular it has been recently applied to $e^+ + e^-$ annihilation into $d + \bar{d}$ and $\rho\rho$ in final state.

Here we have shown results concerning the reaction $e^+ + e^- \to a_1 + \pi$. Substantial progress has been recently done, in the detection of multipion production in e^+e^- annihilation, and in the identification of the intermediate states. The expression of the cross section and of polarization observables is given in a general form in terms of transition FFs. FFs are complex in the time-like region and have been parametrized according to a VMD inspired q^2 dependence, saturated by vector mesons. The parameters have been fitted to the available data. Polarization effects either vanish or are large and measurable. Through the comparison with the existing data, this formalism gives constraints for the $\gamma^* \to a_1(1260)\pi$ transition FFs.

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