Nucleon removal from unstable nuclei investigated via intra-nuclear cascade

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An intra-nuclear cascade model for one-nucleon removal cross sections at intermediate energies beyond the sudden and inert-core assumptions is presented. Indirect processes such as inelastic excitations, evaporation and multiple nucleon-nucleon collisions are taken into account in the reaction mechanism via a Monte-Carlo based approach. The formalism is applied to a set of weakly-bound nuclei for which data are available. Results are in correct agreement with experiment including the case of deeply-bound nucleon removal. A comparison to eikonal predictions is presented.

Nucleons in atomic nuclei experience a shell structure that evolves within the nuclear landscape [1], revealing a new picture of shell closures for some exotic nuclei compared to the sequence first established by Goeppert-Mayer and Jensen [2]. Our understanding of nuclear structure far from stability relies on experiments based on the use of radioactive beams and nuclear reactions in inverse kinematics. Shell occupancies in ground-state wave functions can be revealed by analyzing cross sections of direct stripping reactions such as low-energy transfer or knockout reactions. The latter process at intermediate energies has been shown to be a particularly powerful tool to investigate unstable nuclei available with rather low intensities [3]. Recent experimental one-nucleon removal cross sections obtained at ∼50-120 MeV/nucleon, mainly from *sp* and *sd* shell nuclei on a ^{9}Be target, lead in specific cases to a strong disagreement with predictions based on shell-model calculations and single-particle cross sections from eikonal reaction models based on the sudden and inert-core approximations [4, 5]. The predicted cross sections are found to be systematically too high for deeply-bound nucleons by a factor of 3-4. A. Gade *et al.* [4, 5] reported a significant correlation between the ratio of experimental to calculated cross sections, $\sigma_{exp}/\sigma_{theo}$, and the asymmetry of the system defined as $\Delta S = \epsilon(S_n - S_p)$ where $\epsilon = +1(-1)$ for neutron (proton) removal reactions and $S_{n(p)}$ is the neutron (proton) separation energy of the initial nucleus. This disagreement could be interpreted either in terms of nuclear-structure effects or reaction-mechanism processes that are not considered in current models. Up to now, no quantitative explanation has been provided for the observed discrepancy.

It was first suggested that deeply-bound nucleons experience additional correlations, not taken into account in mean-field or shell-model calculations, which may lead to an unpredicted strong shell depletion [4], but modern Green's function calculations predict only a moderate increase of short range correlations for deeply-bound nucleons [6]. This discrepancy between theory and experiment for deeply-bound nucleon stripping has been investigated via low-energy transfer reactions in the Argon isotopic chain from ^{46}Ar to ^{34}Ar: a weak dependance of correlations on neutron-proton asymmetry has been suggested [7].

The question of indirect processes during nucleon knockout at relativistic energies has been extensively discussed for quasielastic $(e, e'p)$ and $(p, 2p)$ studies from stable nuclei [8, 9]. The quasielastic assumption is found to be relatively valid with *structureless* probes such as electrons or protons when restricted to certain kinematical regions. In the case of inclusive heavy-ion induced nucleon-removal reactions, multiple re-interaction of nucleons inside the projectile or excitations of the residual nucleus during the reaction may blur the simple picture of a single nucleon ejected from a frozen nucleus, with no other collision [10]. Rather weak beyond-eikonal effects in loosely-bound nucleon removal reactions have already been successfully reproduced via a coupled discretized continuum channels description of the elastic breakup [11]. In this article, we present new calculations of inclusive nucleon-removal reaction cross section based on an intra-nuclear cascade model and, within this formalism, study the effects of possible indirect processes during one-nucleon removal reactions from very asymmetric nuclei with ΔS ∼20 MeV.

The present formalism is based on the INCL4 cascade model [12], the latest development of the initial Liège Intra Nuclear Cascade (INC) code [13]. In this approach, the nuclear collision is treated as successive relativistic binary nucleon-nucleon collisions separated in time. Positions and moments of nucleons are followed as time evolves. Cross sections are determined from a set of collision events taken at different impact parameters and for which nucleon positions and impulsions are initially sampled for each participant nucleus. The calculation is performed in the reference frame of the nucleus from which a nucleon is removed. Since we are interested here in describing one-nucleon removal from unstable nuclei on a light heavy-ion target such as ^{9}Be or ^{12}C, we consider the following prescription: ^{9}Be or ^{12}C will be referred to the *target* nucleus and the unstable nucleus to the *projectile* nucleus. Deviations from the sudden and inert-core approximations are taken into account by introducing the intrinsic momenta of the nucleons inside the nucleus and their evolution in time during the collision. The momentum of each nucleon is taken randomly along a flat distribution from 0 to a maximum value P_F chosen according to the Hartree-Fock (HF) potential well depth V_0 and the HF nucleon separation energy $S_{n(p)}$ in the case of neutron (proton) removal, following the relationship

$$V_0 = S_n + T_F \quad with \quad T_F = \sqrt{P_F^2 c^2 + m^2 c^4} - m^2 c^2 \quad (1)$$

where mc^2=938.28 MeV is the nucleonic mass. In a first approximation, the momentum distribution is taken to be the same for protons and neutrons. In the present work, the momentum value sampled for each nucleon determines the maximum radius R_M it can experience. The initial position $\overrightarrow{r}$ is taken randomly in a sphere of radius R_M. In the case of ^{9}Be (^{12}C), the neutron and proton densities are taken as normalized gaussian densities with mean square radius of $r = 2.36(2.44)$ fm.

In previous studies using INC approaches, the authors were interested in the spallation or fragmentation production cross sections from stable nuclei. A refined microscopic description of the projectile density was not necessary since almost all reaction channels were not direct and not sensitive to the detail of the density. In these studies, the neutron and proton densities were taken as Woods-Saxon profiles with parameters set to reproduce the nuclear radius following a $A^{1/3}$ law. On the contrary, one-nucleon removal reactions from unstable nuclei are expected to be particularly sensitive to the relative density of neutrons and protons at the surface of the nucleus. In the present work, the projectile neutron and proton densities are therefore taken from microscopic single-particle wave functions from Hartree-Fock calculations with the HFBRAD code [14] and the Skyrme Sly4 interaction [15]. Each event is fired at a given impact parameter b, ranging from 0 to a distance larger than the interaction distance of the two heavy ions, typically 10 fm for the nuclei studied in the present work. The nucleons that are in the overlap region of the projectile and target densities are considered as participants. If two nucleons approach each other at a distance lower than a minimum distance r_m, they interact. r_m is calculated from an energy-dependent parameterization of the NN interaction cross section for pn, pp and nn collisions [16]. The parameterization was fitted on experimental total NN interaction cross sections [17] and gives very good agreement from about 85 MeV/nucleon to several GeV. For lower energies, the free NN interaction cross sections of S.K. Charagi and S.K. Gupta [18] was used. During collisions, the Pauli blocking is taken into account in a statistical way [12]. In our case, the cascade process ends after a fixed time Δt=50 fm/c has elapsed. The convergence of the results has been checked with several values of Δt ranging from 50 to 100 fm/c. The excitation energy of the residual nuclei is then dissipated by nucleon and ion emission based on experimental separation energies, using the evaporation code ABLA [19]. In this work, the complete history of each event, including the location of each NN interactions and the kinematics of all final nuclei or emitted particles, is recorded.

The events leading to the one-nucleon removal reaction channel can be divided into three types: *(1)* one target-projectile interaction occurs and results in the removal of one nucleon at the end of the cascade (type (a) event of Fig. 1), *(2)* several nucleon-nucleon interactions occur during the cascade phase (type (b) event), *(3)* nucleons were evaporated after the cascade phase (type (c) event). Only the first type (a) of events is assumed to be direct. Note that here our semi-classical treatment of the nuclear structure and the fact that we do not consider any microscopic re-arangement during the collision leads to an impossibility of determining which final state is populated in the case of several bound excited states in the residual nucleus of interest. In the following, we focus on reactions where the populated unstable nucleus has only one bound state. In the Monte-Carlo calculations, each direct event i of type (a), is weighted by a ratio w_i

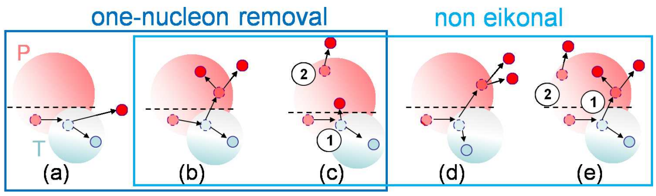

FIG. 1: (Color online) Schematic description of events that contribute to the one-nucleon removal cross section (from the projectile) and events that, due to beyond-the-Glauber-approximation processes, do not contribute to the one-nucleon removal cross section : (a) knockout, (b) multiple scattering, (c) core excitation followed by evaporation, (d),(e) re-interactions in the medium that lead to a reduction of the one-nucleon removal cross section. Nucleons from the projectile P (target T) are in red (blue).

$$w_i = \frac{|\phi_\alpha(r_i)|^2}{\rho_{n(p)}(r_i)}, \quad (2)$$

where r_i is the radial distance in the projectile where the nucleon removal take place, $\rho_{n(p)}$ is the neutron (proton) density in the case of neutron (proton) removal, and $|\phi(r_i)|^2$ is the single-particle density probability for a nucleon initially in the state of interest labeled with the quantum number α. The one-nucleon removal cross section is then obtained from the sum of direct and indirect components, following the relation

$$\sigma = (\frac{A}{A-1})^\eta \;\; C^2S \;\; (\sum_i w_i)\sigma_{casc}^{NN=1} \;\; + \sigma_{casc}^{NN>1} \;\; + \sigma_{evap} \quad (3)$$

where i runs over direct events of type (a). In the following, we consider spectroscopic factors from the USD interaction [20] for sd-shell nuclei and the WBT interaction [21] for sp-shell nuclei, and a center-of-mass motion correction (with $\eta = 0, 1, 2...$ the major oscillator quantum number). To determine the influence of indirect components in the reaction process discribed in the present framework, we compare our predictions to eikonal predictions with the same neutron and proton densities and the same HF single-particle wave function as used in our calculation. The present eikonal calculations slightly differ from published ones [22] where the single-particle wave functions of the removed nucleons were calculated in a Woods-Saxon potential whose parameters are tuned to reproduce the experimental separation energy and, for example in reference [5], the mean square radius of the HF wave functions. In cases for which HF calculations give a larger binding energy for the Fermi level nucleons, the calculated single-particle cross section is generally too low, a feature already discussed in [5, 23].

As mentioned earlier, we focus in this first study on one-nucleon removal reactions from very asymmetric nuclei with ΔS∼20 MeV and no bound excited states. The cases for which experimental data are available are ^{14}O at 57 MeV/u [24], ^{24}Si at 85 MeV/u, ^{28}S at 80 MeV/u [5], ^{32}Ar at 65 MeV/u [4] and ^{24}O at 920 MeV/nucleon [25]. In the specific case of neutron knockout from ^{24}O, the first excited state of the residual nucleus ^{23}O has been reported recently to be unbound [25, 26]. Fig. 2 shows the interaction cross section obtained with our approach and with the eikonal approximation at energies for which one-nucleon removal cross sections have been measured. Calculations at 300 MeV/nucleon are also shown to illustrate the energy dependance of the cross section in both the intra-nuclear cascade and eikonal approaches. Both formalisms give comparable values within 15%, as one could expect since the same nuclear densities have been considered. Only one exeption, ^{14}O, shows a lower reaction cross section for the intra-nuclear approach at 57 MeV/nucleon. At energies below 100 MeV/nucleon, our approach gives lower reaction cross sections, a feature that has already been reported [12]. This means that the nuclear transparency is slightly larger in this work than in the eikonal approach.

The present study gives reasonable agreement with experimental data for both weakly-bound and deeply-bound one-nucleon removal cross sections. The deviation to experiment $\Delta = |\sigma - \sigma_{exp}|/\sigma_{exp}$ is lower than 40% in all cases, except for ^{32}Ar (channel −1n) where $\Delta = 0.8$, as illustrated in Fig. 3. The agreement for deeply-bound nucleon removal is remarkable and represents the first microscopic calculation that reproduces these low cross sections. This agreement may still be fortuitous in these cases as our work gives lower cross sections than the eikonal theory in most of studied reactions. Nevertheless, in the case of loosely-bound nucleons, the present approach still gives reasonable values compared to experimental data. All results are summarize in Tab. I. In order to test the energy dependance of the divergence between our approach and the eikonal predictions, nucleon-removal cross sections at 300 MeV/nucleon (with no available experimental data) are also shown in Fig. 3. The disagreement for deeply-bound nucleon removal remains the same qualitatively, whereas predictions for loosely-bound nucleon removal seem to show a slightly better agreement with eikonal predictions compared to lower incident energies. Within the present formalism, the disagreement with data of Glauber-approximation-based predictions for deeply-bound removal cross sections is expected to remain at higher incident energies of several hundreds of MeV/nucleon.

At this stage of the discussion, we would like to stress that the study presented here shows a rather good predictive power, based on the assumption of localized nucleon-nucleon interactions at energies as low as 50 MeV/nucleon. Regarding the reliability of such assumptions, one could argue that the de Broglie wave length of the incident nucleons is smaller than the nucleus size.

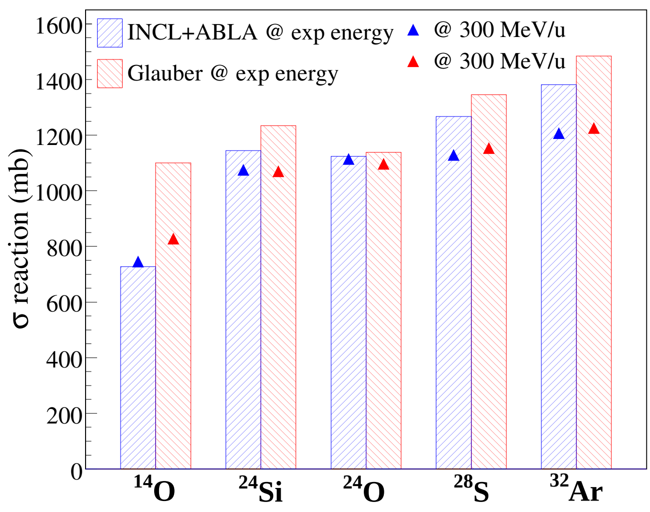

FIG. 2: (Color online) Reaction cross sections for ^{14}O, ^{24}Si, ^{28}S and ^{32}Ar on a ^{9}Be target at energies of 57, 85, 80 and 65 MeV/nucleon, respectively, and for ^{24}O on ^{12}C at 920 MeV/nucleon. The energies considered are those for which one-nucleon removal cross sections have been measured. Predictions from the present work (blue bars) are compared to eikonal predictions (red bars). The energy dependance of the two formalisms is illustrated by calculations for an incident energy of 300 MeV/nucleon (blue triangles for INCL+ABLA and red triangles for eikonal).

TABLE I: Inclusive one-nucleon removal cross section from ^{14}O, ^{24}Si, ^{24}O, ^{28}S, ^{32}Ar at 57, 85, 920, 80 and 65 MeV/nucleon, respectively. A ^{9}Be target was used in all cases except for the one-nucleon removal from ^{24}O which was performed with a ^{12}C target. Data are taken from refs [4, 5, 24, 25]. Experimental cross sections (σ_{exp}) are compared to our predictions ($\sigma = \sigma_{casc} + \sigma_{evap}$) and eikonal calculations (σ_{eik}). The ratio $\delta = \sigma/\sigma_{eik}$ of intra-nuclear to eikonal predictions is also reported.

Proj.	channel	ℓj	σ_{exp} (mb)	σ_{casc}	σ_{evap} (mb)	σ	σ_{eik} (mb)	δ
^{14}O	-n	$p_{3/2}$	13.4±1.4	11.6	4.2	15.8	50	0.32
	-p	$p_{1/2}$	67±6	22.5	31.4	53.9	41.2	1.31
^{24}Si	-n	$d_{5/2}$	9.8±1.0	9.7	2.6	12.3	23.3	0.53
	-p	$d_{5/2}$	67.3±3.5	24.8	19.7	44.5	65.5	0.68
^{24}O	-n	$s_{1/2}$	63±7	34.3	4.2	38.5	51.2	0.75
^{28}S	-n	$d_{5/2}$	11.9±1.2	12.6	3.2	15.8	33.2	0.48
^{32}Ar	-n	$d_{5/2}$	10.4±1.3	11.2	7.1	18.3	34.6	0.53

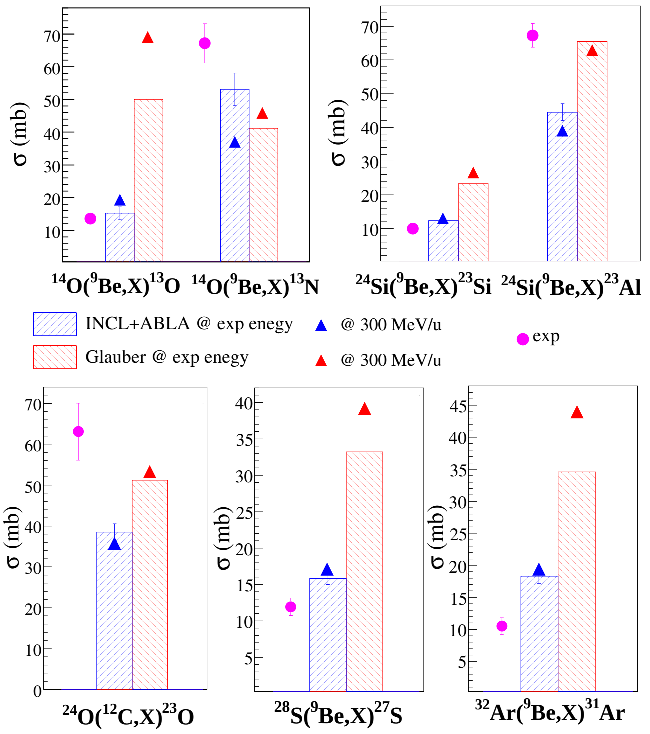

FIG. 3: (Color online) One-neutron and one-proton removal cross sections for the very asymmetric nuclei ^{14}O, ^{24}Si, ^{24}O, ^{28}S and ^{32}Ar. Experimental data (pink dots) are compared to the present work (blue bars) and eikonal predictions (red bars). The energy dependance of the two formalisms is illustrated by calculations for an incident energy of 300 MeV/nucleon (triangles).

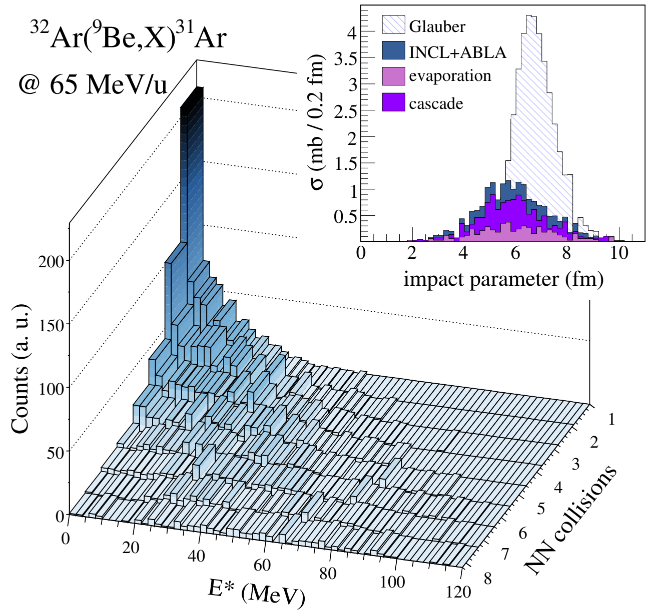

FIG. 4: (Color online) Excitation function restricted to cascade events where ^{31}Ar is formed from ^{32}Ar+^{9}Be at 65 MeV/nucleon as a function of the number of nucleon-nucleon collisions. Excitation-energy bins are 2 MeV. (insert) Contribution of the different impact parameters to the one-neutron removal ^{32}Ar(^{9}Be,X)^{31}Ar at 65MeV/nucleon. The INC (blue) and eikonal approaches (blue strips) are compared. The population of ^{31}Ar from the cascade step (light purple) and evaporation (deep purple) are also indicated.

This warning concerns applications of both cascade and Glauber models at low energy. Note that INCL has already been shown to work surprisingly well down to incident energy of the order of 50 MeV [27]. We nevertheless acknowledge that breakup reactions at these energies can also be correctly described assuming a softer process such as in the *towing-mode* description [28]. The wave-particle nature of nucleons is still an interesting question with respect to the treatment of collisions at energies of about 50 MeV/nucleon.

To further investigate the origin of the small values calculated for deeply-bound removal cross sections with the present approach compared to eikonal predictions, we study the reaction processes that take place in the one-nucleon removal events. As detailed in Eq. 3, the one-nucleon removal has a contribution from evaporation of the cascade step. In the case of deeply-bound nucleons, their separation energy prevents them being evaporated in favor of the other nucleon species with a much smaller separation energy as discussed in [29]. This is visible in the decomposition of the cross section into direct, multiple scattering and evaporation components (see Tab. I): the evaporation contribution is small. By way of illustration, we focus in the following on the ^{32}Ar(^{9}Be,X)^{31}Ar case, but similar conclusions can be drawn for all the other deeply-bound removal cases studied. As a first insight into the reaction mechanism, we note that our model predicts, as expected, that the one-nucleon removal events occur at the surface of the nucleus. A comparison of the impact-parameter contribution with the eikonal approach is illustrated in the insert of Fig. 4. The impact parameters that contribute to the cross section are smaller than in the Glauber-approximation-based calculation, as also observed in other cases of this work. This feature may be related to a larger nuclear transparency encountered in the INC approach. The contribution to the one-nucleon removal channel is found to be small in our work due to the low proton separation energy of ^{31}Ar (S_p=440 keV): after the cascade phase, a large part of the ^{31}Ar formed evaporates protons due to the excitation energy gained in indirect processes during the collision (see Fig. 4). This effect may explain a part of the observed disagreement between eikonal calculations and experiment for deeply-bound systems [4, 5]. The measurement of the production of ^{30}Cl from ^{32}Ar(^{9}Be,X)^{30}Cl in this particular exemple, and its comparison to model predictions could help testing this hypothesis.

To summarize, we performed new calculations of one-nucleon removal reactions at intermediate energies from a light heavy-ion target, ^{9}Be or ^{12}C, based on an intra-nuclear cascade and evaporation approach. Microscopic Hartree-Fock neutron and proton densities were employed. Predictions for the one-proton and one-neutron removal in very asymmetric systems assuming shell-model nucleonic occupancies are in satisfactory agreement with experiment, with typically $|\sigma - \sigma_{exp}|/\sigma_{exp} < 40\%$ in the examined data set. In particular, the cross sections for deeply-bound nucleons from asymmetric nuclei are fairly well predicted, at variance with eikonal-model predictions that significantly overestimate these cross sections. In our approach, core excitation during the collision plays a role in the reduction of deeply-bound nucleon removal cross sections. From this study we expect that the recently observed disagreement between experiment and predictions based on shell-model calculations and the eikonal reaction-mechanism model should persist at energies of several hundreds of MeV/nucleon. This first detailed intra-nuclear-cascade application to one-nucleon removal from instable nuclei suggests that the incorporation of indirect terms and core excitation in the reaction mechanism could be of importance for a correct modeling of well bound nucleon removal reactions. Nevertheless, additional work is needed to reach a higher level of predictivity: (i) a more realistic intrinsic-momentum distribution for nucleons has to be implemented. (ii) The trajectory of the projectile for a given impact parameter may be affected by Coulomb and nuclear interaction with the target. (iii) More fundamentally, the interplay between the reaction process and the shell structure of the projectile, when restricted to inverse kinematics, still has to be introduced in a proper way in the formalism. Indeed, in cases where a small amount of energy is transfered to a nucleon, the assumption of interacting "ball-like" nucleons is obviously no longer correct. A time-dependent treatment of a spatially-extended wave function would be for example a better framework to describe microscopically the resulting nuclear rearrangement. Exclusive data of nucleon removal reactions from unstable nuclei (along the line followed by D. Bazin *et al.* [30]) are called for and represent an essential step in establishing the relevant processes to include in the models.

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[1] O. Sorlin and M. G. Porquet, Prog. in Part. and Nucl. Phys. **61**, 602 (2008).
[2] M. G. Mayer and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure (Wiley, New York, 1955).
[3] P. G. Hansen and J. Tostevin, Annu. Rev. Nucl. Part. Sci. **53**, 221 (2003).
[4] A. Gade *et al.*, Phys. Rev. Lett. **93**, 042501 (2004).
[5] A. Gade *et al.*, Phys. Rev. C **77**, 044306 (2008).
[6] C. Barbieri, Phys. Rev. Lett. **103**, 202502 (2009).
[7] J. Lee *et al.* , Phys. Rev. Lett. **104**, 112701 (2010).
[8] C. Barbieri and L. Lapikas, Phys. Rev. C **70**, 054612 (2004).
[9] C. Samanta *et al.*, Phys. Rev. C **34**, 1610 (1986).
[10] P.E. Hodgson, Nuclear Heavy Ion Reaction (Clarendon Press, Oxford, 1978).
[11] J. Tostevin *et al.*, Phys. Rev. C **66**, 024607 (2002).
[12] A. Boudard, J. Cugnon, S. Leray and C. Volant, Phys. Rev. C **66**, 044615 (2002).
[13] J. Cugnon, C. Volant, and S. Vuillier, Nucl. Phys. A **620**, 475 (1997).
[14] K. Bennaceur and J. Dobaczewski, Comp. Phys. Commun. **168**, 96 (2005).
[15] E. Chabanat *et al.*, Nucl. Phys. A **627**, 710 (1997).
[16] J. Cugnon, D. L'Hôte and J. Vandermeulen, Nucl. Instr. Meth. B **111**, 215 (1996).
[17] A. Baldini *et al.*, New series, Group 1, Vol. 12 (Springer, Berlin, 1988).
[18] S.K. Charagi and S.K. Gupta, Phys. Rev. C **41**, 1610 (1990).
[19] J. Benlliure *et al.*, Nucl. Phys. A **628**, 458 (1998).
[20] B. A. Brown and B. H. Wildenthal, Annu. Rev. Nucl. Part. Sci. **38**, 29 (1988).
[21] E. K. Warburton and B. A. Brown, Phys. Rev. C **46**, 923 (1992).
[22] F. Flavigny, A. Obertelli, and I. Vidana, Phys. Rev. C **79**, 064617 (2009).
[23] J. Lee *et al.*, Phys. Rev. C **73**, 044608 (2006).
[24] F. Flavigny *et al.*, preliminary data (2010).
[25] R. Kanungo *et al.*, Phys. Rev. Lett. **102**, 152501 (2009).
[26] A. Schiller *et al.*, Phys. Rev. Lett. **99**, 112501 (2007).
[27] J. Cugnon and P. Henrotte, Eur. Phys. J. A **16**, 393 (2003).
[28] D. Lacroix, J. A. Scarpaci and Ph. Chomaz, Nucl. Phys. A **658**, 273 (1999).
[29] D. Bazin *et al.*, Phys. Rev. Lett. **91**, 012501 (2003).
[30] D. Bazin *et al.*, Phys. Rev. Lett. **102**, 232501 (2009).