Summary: This note presents the calculations and discussions leading to the choice of the external quality factor “Q_{ext}” between the power couplers and the HWR cavities for IFMIF/EVEDA.

NOTE

External Quality Factor « Q_{ext} » for IFMIF/EVEDA

WBS : 4.7

	Name	Role	Institute	Visa
Author	J. PLOUIN		CEA	
Approvers	F. ORSINI	ASG Coordinator	CEA	
	P. BOSLAND	Scientific Responsible	CEA	

1. Expression of the optimal Q_{ext} for each cavity

For an accelerating cavity, the coupling between the resonant structure and the power transferred through the input coupler P_{ext} is described by the external quality factor Q_{ext}:

$$Q_{ext} = \frac{\omega . W}{P_{ext}}$$

W is the energy stored inside the cavity and ω the resonant frequency.
The power provided by the generator, P_g, is partially transferred to the cavity, and partially reflected (P_r). The power transferred to the cavity is either absorbed by the beam (P_b) or dissipated in the cavity walls (P_{cav}):

$$P_g = P_r + P_b + P_{cav}$$

Since the cavities are superconducting, the power dissipated in the walls can be neglected, thus:

$$P_g = P_r + P_b$$

The beam power is given by: $P_b = V_{cav} . I_b . \cos(\phi_s)$, Φ_s being the synchronous phase, I_b the beam current and V_{cav} the cavity voltage (including the Transit Time Factor).

For a single cavity, the optimal Q_{ext}, designated by Q_{opt}, corresponds to the case where whole of the power from the generator is transferred to the beam, ie $P_r = 0$ and $P_g = P_b$.
In this case, $P_{ext} = P_g$ and:

$$Q_{opt} = \frac{\omega . W}{P_g} = \frac{\omega . W}{P_b} = \frac{\omega . W}{V_{cav} I_b \cos(\phi_s)}$$

This expression is being transformed by using:

$$\frac{r}{Q} = \frac{V_{cav}^{\ 2}}{\omega . W} \qquad (1)$$

which leads to :

$$\boxed{Q_{opt} = \frac{V_{cav}}{(r/Q)\, I_b \cos(\phi_s)}}$$

The values for V_{cav} and Φ_s have been calculated according to the beam dynamics, and are given in columns #2 and #3 of Table 1, for the 8 cavities of the first cryomodule of IFMIF/EVEDA. Their values correspond to an amount of $W = 4\ J$ for the stored energy. The values of r/Q given in the column #4 of Table 1 have been calculated with equation (1). V_{cav} and r/Q increase along the cavities with the increasing of $\beta = v/c$. At the design value $\beta = 0.094$, r/Q is equal to 142 Ω, which corresponds to cavity n°7.

Column #5 of Table 1gives the values for the optimal external coupling of each cavity. The last column gives the beam power, calculated with the design value I_b = 125 mA.

Coupler	V_{cav} (MV)	Phis (deg)	r/Q (Ω)	Q_{opt}	P_b (kW)
1	0,66	- 51,090	99	8,49E+04	51,82
2	0,70	- 50,430	110	7,95E+04	55,34
3	0,72	- 49,800	119	7,53E+04	58,41
4	0,75	- 49,210	127	7,20E+04	61,08
5	0,77	- 48,640	133	6,95E+04	63,27
6	0,78	- 48,080	138	6,75E+04	65,14
7	0,79	- 47,540	142	6,60E+04	66,66
8	0,80	- 47,000	144	6,47E+04	67,94

Table 1

Since the same coupler, with a given Q_{ext}, is used for all the cavities of this cryomodule, the coupling between the loaded cavities and the generator will not be optimized for all the cavities. We thus have to estimate the value of Q_{ext} which is the most appropriate for the set of cavities.

2. Reflected power for a non optimal Q_{ext}

The expression of the power generated by the source is taken from reference (1), in the case where the cavity losses are neglected:

$$P_g = \frac{V_{cav}^{\;2}}{4\frac{r}{Q}\cdot Q_{ext}}\left[\left(1+\frac{\frac{r}{Q}\cdot Q_{ext}\cdot I_b\cdot}{V_{cav}}\cos\varphi_s\right)^2+\left(\frac{\Delta f}{f_{1/2}}+\frac{\frac{r}{Q}\cdot Q_{ext}\cdot I_b\cdot}{V_{cav}}\sin\varphi_s\right)^2\right]$$

The quantity $f_{1/2} = \frac{f}{2.Q_{ext}}$ is the bandwidth of the cavity (when Q_0>>Q_{ext}). The parameter Δf is the detuning of the cavity from the resonance, which can be adjusted by the user. The tuning angle

ψ defined by $\tan\Psi = 2Q_{ext}\frac{\Delta f}{f}$ is then introduced in the expression of P_g. Moreover, $\frac{V_{cav}^{\;2}}{r/Q}$ is equal to $\omega.W$, which can be replaced by $P_b.Q_{opt}$.

The expression of P_g is then transformed into:

$$P_g = \frac{P_b.Q_{opt}}{4.Q_{ext}}\left[\left(1+\frac{Q_{ext}}{Q_{opt}}\right)^2+\left(\tan\psi+\frac{Q_{ext}}{Q_{opt}}\tan\varphi_s\right)^2\right] \quad (2)$$

The reflected power P_r is then given by $P_g - P_b$.

The reflected power, which is an additional amount of provided power, is minimized for each cavity by:

- ✓ Fixing to zero the last term of expression (2), by choosing the tuning angle ψ_{opt}
  $\tan \psi_{opt} = -\frac{Q_{ext}}{Q_{opt}} \tan \varphi_s$
- ✓ Minimizing the quantity $\frac{P_b.Q_{opt}}{4.Q_{ext}}\left[\left(1+\frac{Q_{ext}}{Q_{opt}}\right)^2\right]$ by choosing $Q_{ext} = Q_{opt}$

In that case, the coupling between the source and the beam is perfectly adapted, which corresponds to the case where $P_g = P_b$.

3. Choice of the Q_{ext} for the HWR couplers

The Q_{ext} value is fixed by the distance between the tip of the antenna and the cavity axis (see Figure 1) according to a relation plotted in Figure 2.

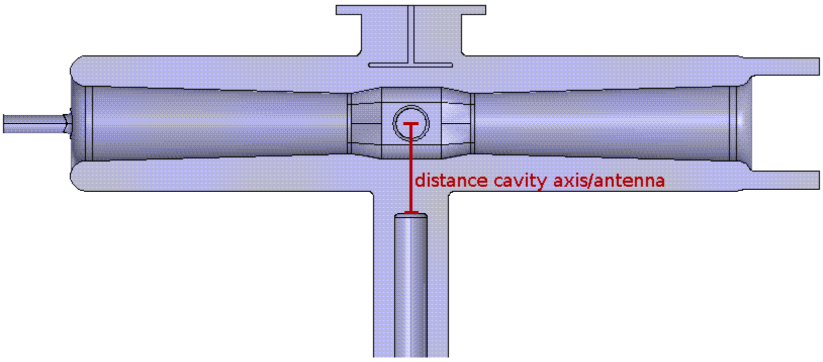

Figure 1: Scheme of the coupling between coupler and cavity

The total amount of reflected power for the 8 cavities has been calculated as a function of the Q_{ext} fixed by the coupler design. For this purpose, Equation (2) has been used with the second term equal to zero, which means that the tuning angle is optimized for each cavity. The result is plotted in Figure 2. The total reflected power is minimized to a value of 1 kW for a Q_{ext} around $7.2*10^4$. The power reflected in the last coupler (#8) is also shown in Figure 2. Since this coupler is submitted to the highest amount of power, it is safer to limit the reflected power on it. With a Q_{ext} equal to $6.5*10^4$ the reflected power on this coupler is cancelled, and the total reflected power is 2 kW.

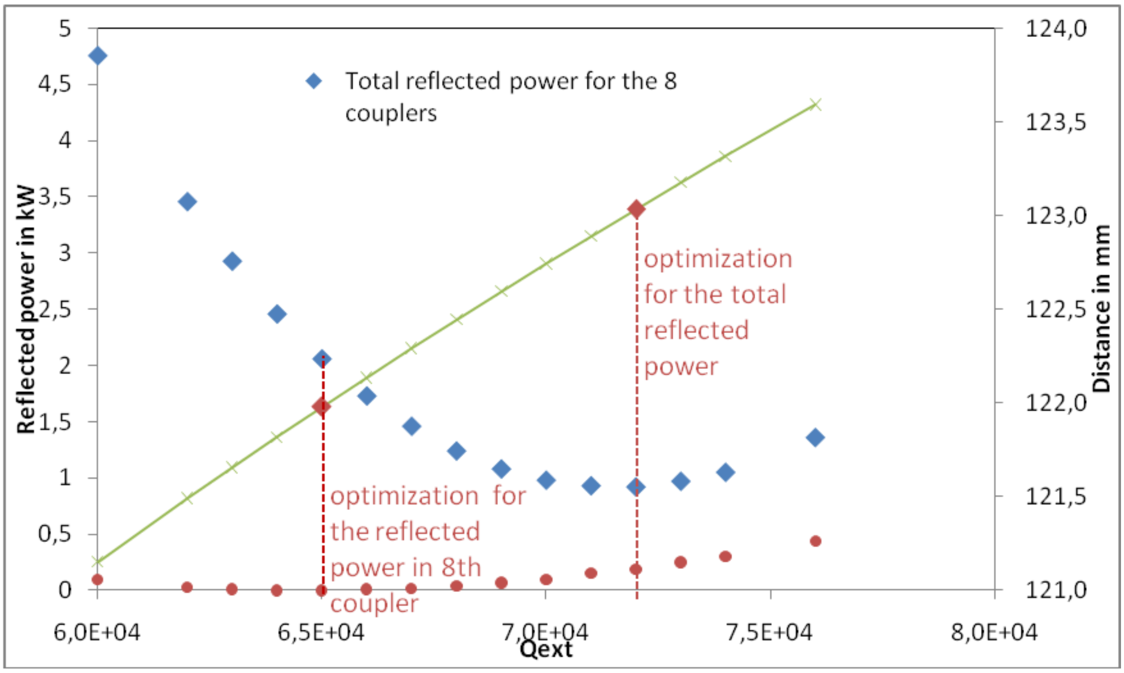

Figure 2: Reflected power and distance cavity axis/antenna vs Q_{ext}.

Finally, there is not a single possibility for the choice of the external coupling factor Q_{ext}. In particular, there are two main options:

- ✓ Decreasing the overall reflected power. ⇨ **Minimizes the overall power provided by the sources.**
- ✓ Decreasing the reflected power in coupler #8 which receives the highest amount of power. ⇨ **Minimizes the risk of coupler failure**.

The choice made for the IFMIF/EVEDA couplers corresponds to the second option. This is generally the solution chosen for high power couplers.
A further argument for this choice comes from the calculation of the maximum voltage in the couplers, given by

$$V_{max} = \sqrt{2.Z_0.P_g} + \sqrt{2.Z_0.P_r}$$

Z_0 being the characteristic impedance of the coaxial part of the coupler, in this case 50 Ω.
For the first option, this voltage increases from 2.47 kV in coupler #1 to 2.75 kV in coupler #8, while it stays at 2.6 kV in all the couplers for the second option. Thus by minimizing the reflected power in coupler #8, we also minimize the maximum voltage.

Consequently, the external coupling factor for the IFMIF/EVEDA couplers is fixed at $6.5*10^4$, which corresponds to a distance between the tip of the antenna and the axis of the cavity of 122 mm.

Bibliography

1. **Schilder, T.** *Vector Sum Control of Pulsed Accelerating Fields in Lorentz Force Detuned Superconducting Cavities.* s.l. : Tesla Collaboration, 1998.