Bremsstrahlung mechanism of heavy vector meson production

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Abstract

Vector mesons as φ, ω, J/Ψ, with $J^{PC} = 1^{--}$ are produced at colliding electron-positron high energy beams through the annihilation and the scattering channels. We consider here the scattering mechanism, namely the creation of a vector meson in the fragmentation region of one of initial leptons. It corresponds to the kinematics where the vector meson is emitted close to the direction of one of the colliding leptons, the direction of the other lepton keeping close to the initial one. The annihilation channel contribution enhanced by the initial hard photon emission mechanism occurs in the kinematical region where the final particles are emitted at large angles and plays the role of background. Differential distributions of the energy fraction and of the transversal component of the vector meson are calculated. The relevant formalism and the numerical estimations are presented.

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I. INTRODUCTION

The production of a pair of charged particles in high energy electron-positron collision was intensively investigated in the 70's [1]. Both cases of production of a fermion and a boson pair were considered. The main attention was devoted to the two-photon mechanism of pair creation. In peripheral kinematics, the pair is emitted in the 'pionization region' with small energies and large emission angles. On the opposite, in the case of heavy quarkonia production, the main mechanism is the annihilation of e^+e^- to a virtual photon and subsequent conversion to a vector bound state of quarks (quarkonia). Radiative return mechanism contributes as well.

We consider here the process of vector meson production in peripheral kinematics, in the fragmentation region of electron or positron. At our knowledge, such mechanism of heavy vector meson production was not considered in the literature (we are grateful to Dr. V. Serbo for pointing out this to us). Let us remind that the production of a vector state is forbidden through the two photon mechanism, by Landau-Yang selection rules. In this paper we calculate this process and suggest an experimental program to investigate such mechanism, for different vector mesons: $e^+ + e^- \to e^+ + e^- + \rho$, $e^+ + e^- \to e^+ + e^- + \omega$, $e^+ + e^- \to e^+ + e^- + \varphi$, $e^+ + e^- \to e^+ + e^- + J/\Psi$.

Let us underline that as a general feature of peripherical kinematics, the total differential cross sections of the considered mechanism do not depend on the total center of mass energy $\sqrt{s}$ and are of the order of $B_{Ve^+e^-}\alpha/M_V^2$, where $B_{Ve^+e^-}$ is the branching ratio for the decay of vector particle to lepton pair. An interesting kinematical situation appears in such mechanism of heavy particle production, let's say for definiteness, in the electron fragmentation region: the electron reflection, as pointed out years ago by I.B. Kriplovich [2]. This effect consists in reversing the direction of the initial electron. Such effect, totally free from any background, can in principle be measured and provide an independent test of this mechanism, which requires the measurement of the invariant mass of vector meson decay products.

II. FORMALISM

We consider vector meson production in electron positron collisions :

$$e^-(p_-) + e^+(p_+) \to e^-(q_-) + e^+(q_+) + V_i(k),\ V_i = \rho, \omega, \varphi, J/\Psi, \tag{1}$$

Peripherical kinematics is characterized by :

$$s = 2p_-p_+ \gg M^2, \quad p_\pm^2 = q_\pm^2 = m^2, \quad k^2 = M^2, \tag{2}$$

The cross section of peripherical processes does not depend on the energy $E = \sqrt{s}/2 =$ of the initial particles in the center of mass frame and grows logarithmically with respect to the contribution of the annihilation channel cross sections, which decreases with energy as $1/s$. Such mechanism is important for the experimental study of vector mesons as ρ, ω, φ, J/Ψ. To describe peripherical dynamics let us introduce first two light-like four vectors. They are linear combination of the initial fermion momenta

$$p_-^T = (p_-^0, p_-^z, p_-^x, p_-^y)^T = E(1,1,0,0), \quad p_+^T = E(1,-1,0,0), \tag{3}$$

which are the linear combination of the initial particles four-momenta

$$p_\pm^T = p_\pm - p_\mp \frac{m^2}{s}, \quad (p_\pm^T)^2 = O(m^6/s^2). \tag{4}$$

where m is the mass of the fermion (the lepton). Any four-vector can be decomposed as the sum of components parallel and transversal to these light-like vectors (Sudakov's representation) [3]:

$$a = \alpha_a p_+^T + \beta_a p_-^T + a_\perp, \quad a_\perp p_\pm^T = 0, a_\perp^2 = -\vec{a}^2 < 0.$$

Peripherical kinematics is characterized by the creation of two jets moving in the direction close to the direction of the initial particles. In the case considered below, the jet from the vector meson as well as the final positron move in the direction of the initial electron, whereas the final electron moves in the opposite direction.

$$q_- = \alpha_- p_+^T + x_- p_-^T + q_{-\perp}, \quad k = \alpha_k p_+^T + x_v p_-^T + k_\perp, \quad q_+ = \alpha_+ p_+^T + \beta_+ p_-^T + q_{+\perp} \tag{5}$$

The transferred four-momentum $q = p_+ - q_+$ is expressed in terms of the Sudakov variables as:

$$q = \alpha p_+^T + \beta p_-^T + q_\perp. \tag{6}$$

The conservation law and the on mass shell conditions of the final particles can be expressed in the form:

$$x_- + x_v = 1, \quad \vec{k} + \vec{q}_- = \vec{q}, \quad \alpha = \alpha_- + \alpha_k,$$
$$\alpha_- = \frac{1}{sx_-}[\vec{q}_-^2 + m^2] \approx \frac{\vec{q}_-^2}{sx_-}, \quad \alpha_k = \frac{1}{sx_v}[\vec{k}^2 + M^2],$$
$$q^2 = -\frac{1}{1-\alpha}[\vec{q}^2 + m^2\alpha^2] \approx -[\vec{q}^2 + m^2\frac{s_1^2}{s^2}], \quad s_1 = s\alpha, \tag{7}$$

where M is the mass of the vector meson.

The phase volume of the final particles can be expressed in terms of Sudakov variables. For this aim we introduce the additional (unity) operator $d^4q\delta^4(p_+ - q - q_+)$ and use the relation

$$\frac{d^3q_i}{2E_i} = d^4q_i\delta(q_i^2 - m_i^2) = \frac{s}{2}d\alpha_i d\beta_i d^2\vec{q}_i\delta(s\alpha_i\beta_i - \vec{q}_i^2 - m_i^2).$$

As a result we obtain:

$$d\Gamma_3 = \frac{(2\pi)^4}{(2\pi)^9}\frac{d^3q_-}{2E_-}\frac{d^3q_+}{2E_+}\frac{d^3k}{2\omega}\delta^4(p_- + p_+ - q_- - q_+ - k) = \frac{d^2\vec{q}}{\pi}\frac{d^2\vec{k}}{\pi}\frac{dx_v}{x_v(1-x_v)}\frac{1}{128\pi^3 s}. \tag{8}$$

The matrix element for the vector meson production process in Born approximation has the form

$$M^{e_-e_+\to(ve_-)e_+} = \frac{4\pi\alpha\sqrt{g(k^2)}}{q^2}g^{\mu\nu}\bar{v}(p_+)\gamma_\nu v(q_+)J_\mu, \; J_\mu = \bar{u}(q_-)O_{\nu\rho}u(p_-)\epsilon^{\lambda\rho}(k),$$
$$O_{\nu\rho} = \gamma_\nu\frac{\hat{q}_- - \hat{q} + m}{D'}\gamma_\rho + \gamma_\rho\frac{\hat{p}_- + \hat{q} + m}{D}\gamma_\nu, \tag{9}$$

with $D = (p_- + q)^2 - m^2$, $D' = (q_- - q)^2 - m^2$ and $\epsilon_\mu^\lambda(k)$ is the polarization vector of vector meson:

$$\sum_\lambda \epsilon_\mu^\lambda\epsilon_\nu^\lambda = -g_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}, \quad \epsilon^\lambda(k)k = 0 \tag{10}$$

$\sqrt{g(k^2)}$ is the coupling constant of vector meson with lepton, will be specified below. We use further the Gribov's parameterization of the metric tensor

$$g_{\mu\nu} = g_{\mu\nu\perp} + \frac{2}{s}\left[p_{+\nu}^T p_{-\mu}^T + p_{+\mu}^T p_{-\nu}^T\right] \approx \frac{2}{s}p_{-\nu}^T p_{+\mu}^T,$$

where we omit the contributions of the kind $O\left(\frac{M^2}{s}\right)$, compared with those of order of unity. With these substitutions the matrix element can be written in the form:

$$M^{e_-e_+\to(ve_-)e_+} = \frac{8\pi\alpha s\sqrt{g(k^2)}}{q^2}N_+N_-, \; N_+ = \frac{1}{s}\bar{v}\hat{p}_- v(q_+), \; N_- = \frac{1}{s}p_+^\mu\bar{u}(q_-)O_{\mu\rho}\epsilon^\rho(k). \tag{11}$$

Both quantities N_+, N_- do not depend on s for sufficiently large values s:

$$\sum |N_+|^2 = 2. \tag{12}$$

Using the current conservation condition $q^\mu J_\mu = 0; J_\mu = \bar{u}(q_-)O_{\mu\nu}u(p_-)\epsilon^\nu(k)$ and the Sudakov representation $q \approx \alpha T_+ + q_\perp$, we obtain

$$N_- = \frac{|\vec{q}|}{s_1}(\vec{n}\vec{J}), \quad \vec{n} = \frac{\vec{q}}{|\vec{q}|}. \tag{13}$$

It is convenient, nevertheless, to give the direct calculation. For this aim let present the expression for N_- as:

$$\begin{aligned} N_- &= \frac{1}{s}\bar{u}(q_-)[\frac{1}{D'}\hat{p}_+(\hat{q}_- - \hat{q} + m)\hat{\epsilon} + \frac{1}{D}\hat{\epsilon}(\hat{p}_- + \hat{q} + m)\hat{p}_+]u(p_-) \\ &= \bar{u}(q_-)[A\hat{\epsilon} - \frac{1}{sD'}\hat{p}_+\hat{q}_\perp\hat{\epsilon} + \frac{1}{sD}\hat{\epsilon}\hat{q}_\perp\hat{p}_+]u(p_-), \end{aligned} \tag{14}$$

with $A = (x_-/D') + (1/D) \approx -2\vec{q}\vec{q}_-/(x_- s_1^2)$. Here and further we extract only the terms, proportional to the first power of $\vec{q}$, which allow to obtain the cross section in the leading logarithmic approximation, the Weiszaecker-Williams (WW) approximation:

$$\int \frac{d\vec{q}^2 d\varphi_q}{\pi} \frac{\vec{q}^2}{(\vec{q}^2 + m^2(s_1/s)^2)^2} f(\vec{n}) = (L-1)\bar{f}(\vec{k}^2, x_v),$$

$$L = \ln\frac{Q^2 s^2}{m^2 s_1^2} \approx \ln\frac{s^2}{m^2 M^2}, \quad \bar{f}(\vec{k}^2, x_v) = \frac{1}{\pi}\int_0^{2\pi} f(\vec{n}, x_v) d\varphi_q, \tag{15}$$

where $Q^2 \sim M^2$ is the effective value of the momentum transfer square. The explicit calculation leads to

$$\begin{aligned} \sum \frac{\vec{q}^2}{s_1^2}(\vec{n}\vec{J})^2 &= \frac{8\vec{q}^2}{s_1^2 x_-^3}[\frac{1}{2}x_-^2(1+x_-^2) + x_v x_- \frac{\vec{k}^2}{s_1} + (\frac{\vec{k}^2}{s_1})^2], \\ s_1 &= \frac{d}{x_- x_v}, \quad d = x_- M^2 + \vec{k}^2, \; x_- + x_v = 1. \end{aligned} \tag{16}$$

Let us introduce the resonance factor, $g(k^2)$, which has the form:

$$g(k^2) = \frac{12\pi\Gamma_{ee}\Gamma_f M^2}{(k^2 - M^2)^2 + M^2\Gamma^2}, \tag{17}$$

where Γ_{ee}, Γ_f, Γ are the partial widths and the total width of the vector meson and k^2 is the invariant mass squared of its decay products. The vector meson V of momentum k decays into different particles a_i of momentum k_i.

$$V(k) = a_1(k_1) + ...a_n(k_n), \; k^2 = (k_1 + ...k_n)^2, \tag{18}$$

Due to energy and momentum conservation laws, $x_v = \sum(x_i)$ where x_i is the energy fraction of each vector meson decay product, and $\vec{k} = \sum(\vec{k}_i)$, where $\vec{k}$ is the relevant transversal momentum. To give an example, the cross section σ_B^{ee} of the process $e^-e^+ \to J/\Psi \to e^-e^+$ is

$$\sigma_B^{ee}(s) = \frac{12\pi}{s} \frac{\Gamma_{ee}^2 M^2}{(s-M^2)^2 + M^2\Gamma^2} = \frac{1}{s} g(k^2). \tag{19}$$

The final expression for the double differential cross section in WW approximation is

$$\begin{aligned} \frac{d\sigma}{dxdk^2}(e^-e^+ \to (e^-V)e^+) &= \frac{\alpha^2(L-1)g(k^2)}{\pi}\frac{(1-x_-)}{d^2} \\ &\times \left[\frac{1}{2}(1+x_-^2) + (1-x_-)^2\frac{\vec{k}^2}{d} + (1-x_-)^2(\frac{\vec{k}^2}{d})^2\right], \end{aligned}$$

where x_- is the energy fraction of the electron accompanied by the vector meson, $x_v = 1-x_-$.

The accuracy of WW approximation is of the order several percent $\sim 1 + O(\frac{1}{L})$.

Let us give as well the expression for one-variable distributions. The distribution on the energy fraction of electron $x_- = x$ reads as

$$\begin{aligned} \frac{d\sigma}{dx} &= \sigma_0 f(x), \quad \sigma_0 = \frac{\alpha^2 g(\bar{k^2})(L-1)}{\pi M^2}, \\ f(x) &= \frac{1-x}{3x}(4-5x+4x^2), \quad x_0 < x < 1 - \frac{2M}{\sqrt{s}}, \end{aligned} \tag{20}$$

and $x_0 = 2E_{min}/\sqrt{s}$, E_{min} is the experimental threshold of the scattered electron detection. The estimation of total cross-section is given it Table I for energies of initial particles available at BES-III.

The distribution on the transverse momentum square of the electron is

$$\frac{d\sigma}{d\vec{q}_-^2} = \frac{\sigma_0}{M^2}F(\eta), \quad \eta = \frac{\vec{q}_-^2}{M^2}, \tag{21}$$

$$F(\eta) = I_{20} - I_{21} + I_{22} - I_{23} + 2\eta[I_{30} - 3I_{31} + 3I_{32} - I_{33}] + 2\eta^2[I_{40} - 3I_{41} + 3I_{42} - I_{43]}],$$

where the integrals $I_{mn} = \int\limits_0^1 x^n(\eta+x)^{-m}$ read:

$$\begin{aligned} I_{20} &= \frac{1}{\eta(1+\eta)}, \quad I_{21} = l - \eta I_{20}, \; I_{22} = 1 - 2\eta l + \eta^2 I_{20} \\ I_{23} &= \frac{1}{2} - 2\eta + 3\eta^2 l - \eta^3 I_{20}, \quad I_{30} = \frac{1}{2}\left[\frac{1}{\eta^2} - \frac{1}{(1+\eta)^2}\right], \quad I_{31} = I_{20} - \eta I_{30}, \\ I_{32} &= l - 2\eta I_{20} + \eta^2 I_{30}, \quad I_{33} = 1 - 3\eta l + 3\eta^2 I_{20} - \eta^3 I_{30}, \quad I_{40} = \frac{1}{3}\left[\frac{1}{\eta^3} - \frac{1}{(1+\eta)^3}\right], \\ I_{41} &= I_{30} - \eta I_{40}, \quad I_{42} = I_{20} - 2\eta I_{30} + \eta^2 I_{40}, \quad I_{43} = l - 3\eta I_{20} + 3\eta^2 I_{30} - \eta^3 I_{40}, \end{aligned} \tag{22}$$

and $l = \ln \dfrac{1+\eta}{\eta}$.

The functions $f(x), F(\eta)$ are shown in Figs. 1 and 2 respectively. The main background process is the returning to resonance mechanism (emission of a hard photon by the initial electron or positron) with the subsequent annihilation of electron and positron to the vector meson [4]:

$$d\sigma_{ret} = dW(x)\left[d\sigma_B^{e^+e^-\to V}(p_-(1-x), p_+) + d\sigma_B^{e^+e^-\to V}(p_,(1-x)p_+)\right], \tag{23}$$

with

$$dW(x) = \frac{\alpha dx}{2\pi x}\left[(1+(1-x)^2)(L_s-1)+x^2\right], \quad L_s = \ln\frac{s}{m_e^2}, \quad x = \frac{\omega}{E}. \tag{24}$$

Compared with the bremsstrahlung mechanism considered above, the decay products of the vector meson are emitted at large angles. Except for hard photon energy fractions far from unity, the cross section of the bremsstrahlung mechanism considerably exceeds the background cross section:

$$\frac{d\sigma_{ret}}{d\sigma_{Br}} = O\left(\frac{M^2}{sx_v(1-x_v)}\right) \ll 1. \tag{25}$$

It is interesting to note that in bremsstrahlung kinematics the electron "accompanied" by the heavy photon moves in opposite direction (effectively along the initial positron direction) when the energy fraction of the vector meson is sufficiently large. Really such a mechanism takes place when the component of the scattered electron momentum along the positron direction exceeds the component along the electron $\vec{k}^2/(s(1-x_v)) > (1-x_v)$.

meson	ρ	ω	φ
$\frac{\sigma_{tot}}{g(k^2)}$ [μb]	0.26	0.25	0.13

Tab. I: Estimation of total cross-section (integrated over lepton energy fraction, see Eq. (20)) for ρ, ω, φ meson production at initial energy $\sqrt{s} = 4$ GeV, $E_{min} = 0.1$ GeV.

III. CONCLUSION

In this paper we consider the bremsstrahlung process of vector meson production in electron-positron collision. The main characteristic feature of such a mechanism which

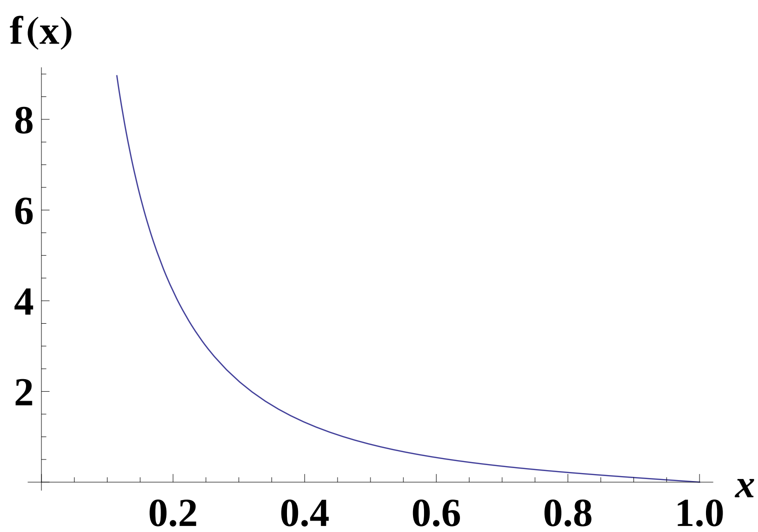

Fig. 1: Distribution on energy fraction of electron (see Eq. 20)

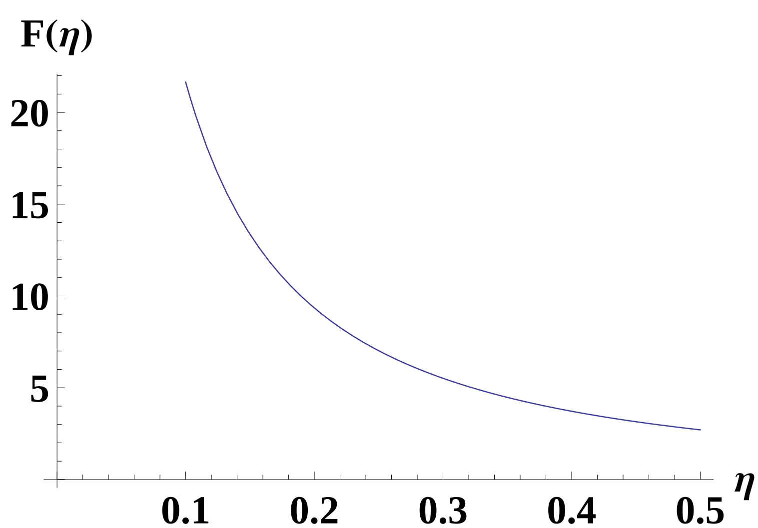

Fig. 2: Distribution on transversal momentum square of electron (see Eq. 21).

occurs in the fragmentation region of the initial leptons is that the cross section does not decrease with energy. One of the signatures of this mechanism is the effect of the total reflection of the parent lepton discussed in this paper.

Therefore such mechanism can be used to measure the leptonic widths of vector meson decays.

The expressions for the total and differential cross sections are derived and quantitative estimations at BEPC-II energies are given.

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