Model independent analysis of polarization effects in elastic proton-electron scattering

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Abstract

The experimental observables for the elastic reaction induced by protons on electrons are calculated in the Born approximation. Model independent expressions are derived for the differential cross section and polarization observables. Numerical estimations are given for spin correlation coefficients, polarization transfer coefficients and depolarization coefficients, in a wide kinematical range. Specific attention is given to the kinematical conditions, i.e., to the specific range of incident energy and transferred momentum.

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I. INTRODUCTION

The polarized and unpolarized scattering of electrons by protons has been widely studied, as it is considered the simpler way to access information on proton structure. Model independent expressions, which relate the cross section and polarization observables to the proton electromagnetic form factors were firstly derived in Ref. [1]. The scattering of proton on electrons at rest (inverse kinematics) is more complicated, in principle, because approximations, such as neglecting the electron mass, do not hold anymore. Liquid hydrogen targets are considered as proton targets, but any reaction on such targets involves also reactions with atomic electrons, which we will assume to be at rest.

Large interest in inverse kinematics (proton projectile on electron target) aroused, due to two possible applications: - the possibility to build beam polarimeters, for high energy polarized proton beams, in the RHIC energy range [2], - the possibility to build polarized antiprotons beams [3], which would open a wide domain of polarization studies at the FAIR facility [4, 5]. Indeed, assuming C-invariance in electromagnetic interaction, the (elastic and inelastic) reactions $p + e^-$ and $\bar{p} + e^+$ are strictly equivalent.

Concerning the polarimetry of the high energy proton beams, in Ref. [2] analyzing powers corresponding to polarized proton beam and electron target were numerically calculated for the elastic proton-electron scattering, assuming one photon exchange mechanism and dipole approximation for the proton form factors. It was shown that the analyzing powers, as functions of the proton beam energy E, reach a maximum for forward scattering at $E = 50$ GeV, where the cross section is small. The authors concluded that the concept of such polarimeter is realistic for longitudinal as well as transverse proton beam polarizations. On the other hand, in that paper, explicit expressions for the analyzing powers were not given.

The possibility of polarizing a proton beam in a storage ring by circulating through a polarized hydrogen target was reported in Ref. [6]. Possible explanations of the polarizing mechanisms were published in a number of papers [7–9], and more recently in Refs. [10, 11].

In this work, we derive the cross section and the polarization observables for proton electron elastic scattering, in a relativistic approach assuming Born approximation. We derive relations connecting kinematical variables in direct and inverse kinematics. Three types of polarization effects are studied: - the spin correlation, due to the polarization of the proton beam and of the electron target, - the polarization transfer from the polarized electron target to the scattered proton, - and the depolarization coefficients which describe the polarization of the scattered proton, depending on the polarization of the proton beam. Numerical estimations of the polarization observables have been performed in a wide range of the proton beam energy and for different values of the scattering angle.

We discuss model independent properties of the observables for proton–electron elastic scattering and compare to the recent theoretical and experimental ongoing work related to the production and the properties of high energy polarized (anti)proton beams.

II. FORMALISM

Let us consider the reaction (Fig. 1):

$$p(p_1) + e(k_1) \to p(p_2) + e(k_2), \tag{1}$$

where the particle momenta are indicated in parenthesis, and $k = k_1 - k_2 = p_2 - p_1$ is the four momentum of the virtual photon. In the one photon exchange approximation, the matrix element $\mathcal{M}$ of the reaction (1) can be written as:

$$\mathcal{M} = \frac{e^2}{k^2} j_\mu J_\mu, \tag{2}$$

where $j_\mu (J_\mu)$ is the leptonic (hadronic) electromagnetic current:

$$\begin{aligned} j_\mu &= \bar{u}(k_2)\gamma_\mu u(k_1), \\ J_\mu &= \bar{u}(p_2)\left[F_1(k^2)\gamma_\mu - \frac{1}{2M}F_2(k^2)\sigma_{\mu\nu}k_\nu\right]u(p_1) \\ &= \bar{u}(p_2)\left[G_M(k^2)\gamma_\mu - F_2(k^2)P_\mu\right]u(p_1). \end{aligned} \tag{3}$$

Here $F_1(k^2)$ and $F_2(k^2)$ are the Dirac and Pauli proton electromagnetic form factors (FFs), $G_M(k^2) = F_1(k^2) + F_2(k^2)$ is the Sachs proton magnetic FF, M is the proton mass, and $P_\mu = (p_1 + p_2)_\mu/(2M)$. The matrix element squared is:

$$|\mathcal{M}|^2 = 16\pi^2\frac{\alpha^2}{k^4}L_{\mu\nu}W_{\mu\nu}, \text{ with } L_{\mu\nu} = j_\mu j_\nu^*, \ W_{\mu\nu} = J_\mu J_\nu^*, \tag{4}$$

where $\alpha = 1/137$ is the electromagnetic fine structure constant. The leptonic tensor, $L_{\mu\nu}^{(0)}$, for unpolarized initial and final electrons (averaging over the initial electron spin) has the form:

$$L_{\mu\nu}^{(0)} = k^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}). \tag{5}$$

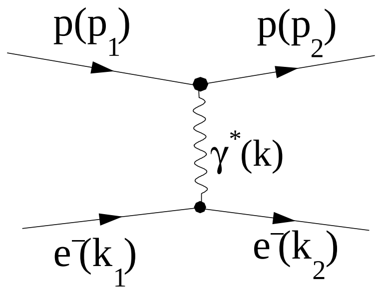

FIG. 1: Feynman diagram for the reaction $p(p_1)+e(k_1)\to p(p_2)+e(k_2)$. The transfer momentum of the virtual photon is $k=k_1-k_2=p_2-p_1$.

The contribution to the electron tensor corresponding to a polarized electron target is

$$L_{\mu\nu}^{(p)}=2im\epsilon_{\mu\nu\alpha\beta}k_\alpha S_\beta, \tag{6}$$

where S_β is the initial electron polarization four vector and m is the electron mass.

The hadronic tensor, $W_{\mu\nu}^{(0)}$, for unpolarized initial and final protons can be written in the standard form, through two unpolarized structure functions:

$$W_{\mu\nu}^{(0)}=\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{k^2}\right)W_1(k^2)+P_\mu P_\nu W_2(k^2). \tag{7}$$

Averaging over the initial proton spin, the structure functions W_i, $i=1,2$, can be expressed in terms of the nucleon electromagnetic FFs as:

$$\begin{aligned} W_1(k^2) &= -k^2G_M^2(k^2),\\ W_2(k^2) &= 4M^2\frac{G_E^2(k^2)+\tau G_M^2(k^2)}{1+\tau}, \end{aligned} \tag{8}$$

where $G_E(k^2)=F_1(k^2)-\tau F_2(k^2)$ is the proton electric FF and $\tau=-k^2/4M^2$.

The differential cross section is related to the matrix element squared (4) by

$$d\sigma=\frac{(2\pi)^4\overline{|\mathcal{M}|^2}}{4\sqrt{(k_1\cdot p_1)^2-m^2M^2}}\frac{d^3\vec{k}_2}{(2\pi)^3 2\epsilon_2}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\delta^4(k_1+p_1-k_2-p_2), \tag{9}$$

where $p_2(E_2)$ is the momentum (energy) of the final proton, ϵ_2 is the energy of the scattered electron. From this point, formulas will differ from the elastic electron-proton scattering, as we introduce a reference system where the electron is at rest. In this system, the differential cross section can be written as:

$$\frac{d\sigma}{d\epsilon_2}=\frac{1}{32\pi}\frac{\overline{|\mathcal{M}|^2}}{m\vec{p}^2}, \tag{10}$$

where $\vec{p}$ is the momentum of the proton beam. The average over the spins of the initial particles has been included in the leptonic and hadronic tensors. Using the relation

$$k^2 = 2m(m - \epsilon_2) \tag{11}$$

one can write

$$\frac{d\sigma}{dk^2} = \frac{1}{64\pi} \frac{\overline{|\mathcal{M}|^2}}{m^2 \vec{p}^2}. \tag{12}$$

The differential cross section over the solid angle can be written as:

$$\frac{d\sigma}{d\Omega_e} = \frac{1}{32\pi^2} \frac{1}{mp} \frac{\vec{k}_2^3}{-k^2} \frac{\overline{|\mathcal{M}|^2}}{E+m}, \tag{13}$$

where E is the proton beam energy and $d\Omega_e = 2\pi d\cos\theta$ (due to azimuthal symmetry). We used the relation

$$d\epsilon_2 = \frac{p}{E+m} \frac{\vec{k}_2^3}{m(\epsilon_2 - m)} \frac{d\Omega_e}{2\pi}. \tag{14}$$

Let us focus here on three types of polarization observables, for elastic proton-electron scattering

1. The polarization transfer coefficients which describe the polarization transfer from the polarized electron target to the scattered proton, $p + \vec{e} \to \vec{p} + e$;

2. The spin correlation coefficients when both initial particles have arbitrary polarization, $\vec{p} + \vec{e} \to p + e$;

3. The depolarization coefficients which define the dependence of the scattered proton polarization on the polarization of the proton beam, $\vec{p} + e \to \vec{p} + e$. In our knowledge, this case was not previously considered in the literature.

The first case is the object of a number of recent papers [3] in connection with the possibility to polarize proton (antiproton) beams. The second case was considered in Ref. [2], in view of using the polarized proton-electron scattering for the measurement of the longitudinal and transverse polarizations of a high energy proton beams.

Let us calculate the hadronic tensor, when the initial or final proton is polarized. The contribution of the proton polarization to the hadronic tensor is:

$$W_{\mu\nu}(\eta_j) = -2iG_M(k^2)\left[MG_M(k^2)\epsilon_{\mu\nu\alpha\beta}k_\alpha\eta_{j\beta} + F_2(k^2)(P_\mu\epsilon_{\nu\alpha\beta\gamma} - P_\nu\epsilon_{\mu\alpha\beta\gamma})p_{1\alpha}p_{2\beta}\eta_{j\gamma}\right], \tag{15}$$

where the four vector η_j ($j = 1, 2$) stands for initial (final) proton polarization. One can see that all the correlation coefficients in $\vec{p}\vec{e}$ collisions are proportional to the proton magnetic FF. This is a well known fact for $\vec{e}\vec{p}$ scattering [12]. The dependence of the different polarization observables, namely, the spin correlation (the polarization transfer) coefficients on the polarization four vector of the initial (scattered) proton is completely determined by the spin dependent part of the hadronic tensor $W_{\mu\nu}(\eta_j)$, $j = 1$ ($j = 2$).

The expression of the differential cross section for unpolarized proton-electron scattering, in the coordinate system where the electron is at rest can be written as:

$$\frac{d\sigma}{dk^2} = \frac{\pi\alpha^2}{2m^2\vec{p}^2}\frac{\mathcal{D}}{k^4}, \tag{16}$$

with

$$\mathcal{D} = k^2(k^2 + 2m^2)G_M^2(k^2) + 2\left[k^2M^2 + 2mE\left(2mE + k^2\right)\right]\left[F_1^2(k^2) + \tau F_2^2(k^2)\right]. \tag{17}$$

It can be written in terms of the Sachs FFs as:

$$\mathcal{D} = k^2(k^2 + 2m^2)G_M^2(k^2) + 2\left[k^2M^2 + \frac{1}{1+\tau}\left(2mE + \frac{k^2}{2}\right)^2\right]\left[G_E^2(k^2) + \tau G_M^2(k^2)\right]. \tag{18}$$

This expression is consistent with Ref. [2].

Note that the differential cross section diverges as k^4 when $k^2 \to 0$. This is expected from the one photon exchange mechanism.

A. Polarization transfer coefficients, T_{ij}, in the $p + \vec{e} \to \vec{p} + e$ reaction

These polarization observables describe the polarization transfer from the polarized target to the ejectile. The transfer coefficients are also called T_{i00j} in the notations from [13]. Here the four subscripts denote, in the order, ejectile, recoil, projectile, target. The indexes i, j correspond to n, t, ℓ, according to the direction of the polarization vectors of each particle.

The dependence of the scattered proton polarization on the polarization state of the initial electron is obtained by contraction of the spin-dependent leptonic tensor $L_{\mu\nu}^{(p)}$, Eq. (6), and the spin-dependent hadronic tensor $W_{\mu\nu}(\eta_2)$, Eq. (15). The following formula hold in any reference system and can be used to obtain the polarization transfer coefficients :

$$\mathcal{D}T(S, \eta_2) = 4mMG_M(k^2)\left[G_E(k^2)(k\cdot Sk\cdot\eta_2 - k^2S\cdot\eta_2) - k^2F_2(k^2)P\cdot SP\cdot\eta_2\right]. \tag{19}$$

In the frame where the initial electron is at rest, the polarization four vectors of the electron S_μ and of the scattered proton $\eta_{2\mu}$ have the following components:

$$S \equiv (0, \vec{\xi}), \ \eta_2 \equiv \left(\frac{1}{M} \vec{p}_2 \cdot \vec{S}_2, \vec{S}_2 + \frac{\vec{p}_2(\vec{p}_2 \cdot \vec{S}_2)}{M(E_2 + M)} \right), \tag{20}$$

where $\vec{\xi}$ and $\vec{S}_2$ are the unit three-vectors of the initial electron and scattered proton polarizations in their rest systems, respectively; $\vec{p}_2(E_2)$ is the momentum (energy) of the final proton. In the laboratory system (inverse kinematics) one can write $\vec{p} = \vec{k}_2 + \vec{p}_2$, $m + E = E_2 + \epsilon_2$, where $\vec{k}_2(\epsilon_2)$ is the momentum (energy) of the scattered electron.

Using the P-invariance of the hadron electromagnetic interaction, one can parametrize the dependence of the differential cross section on the polarizations of the electron target and of the scattered proton as follows:

$$\frac{d\sigma}{dk^2}(\vec{\xi}, \vec{S}_2) = \left(\frac{d\sigma}{dk^2} \right)_{un} [1 + T_{\ell\ell}\xi_\ell S_{2\ell} + T_{nn}\xi_n S_{2n} + T_{tt}\xi_t S_{2t} + T_{\ell t}\xi_\ell S_{2t} + T_{t\ell}\xi_t S_{2\ell}] \,, \tag{21}$$

where T_{ik}, $i, k = \ell, t, n$ are the corresponding polarization transfer coefficients, with the following notations: ℓ is the component of the polarization vector along the momentum of the initial proton, n is the component which is orthogonal to the momenta of the initial proton and of the scattered electron, i.e., orthogonal to the scattering plane, and t is the component which is orthogonal to the initial proton momentum and lies in the scattering plane.

At high energy, the polarization transfer coefficients depend essentially on the direction of the scattered proton polarization. Let us choose an orthogonal system with the z axis directed along $\vec{p}$, $\vec{k}_2$ lies in the xz plane (θ is the angle between the initial proton and the final electron momenta) and the y axis is directed along the vector $\vec{p} \times \vec{k}_2$. Therefore, in this system $\ell \parallel z$, $t \parallel x$ and $n \parallel y$. The explicit expressions for the polarization transfer coefficients are given in Appendix A.

B. Polarization correlation coefficients, C_{ij}, in the $\vec{p} + \vec{e} \to p + e$ reaction

In the reaction involving polarized proton beam and polarized electron target, one can derive explicit expressions for the spin correlation coefficients. These coefficients are also called double analyzing powers and denoted A_{00ij} in the notations from Ref. [13].

The contraction of the spin dependent leptonic $L_{\mu\nu}^{(p)}$ and hadronic $W_{\mu\nu}(\eta_1)$ tensors, in an arbitrary reference frame, gives:

$$\mathcal{D}C(S,\eta_1) = 8mMG_M(k^2)\left[(k\cdot Sk\cdot\eta_1 - k^2 S\cdot\eta_1)G_E(k^2) + \tau k\cdot\eta_1(k\cdot S + 2p_1\cdot S)F_2(k^2)\right]. \tag{22}$$

All spin correlation coefficients for the elastic $\vec{p}\vec{e}$ collisions can be obtained from this expression and they are, therefore, proportional to the proton magnetic FF.

In the considered frame, where the target electron is at rest, the polarization four vector of the initial proton has the following components

$$\eta_1 = \left(\frac{\vec{p}\cdot\vec{S}_1}{M}, \vec{S}_1 + \frac{\vec{p}(\vec{p}\cdot\vec{S}_1)}{M(E+M)}\right), \tag{23}$$

where $\vec{S}_1$ is the unit vector describing the polarization of the initial proton in its rest system.

Applying the P-invariance of the hadron electromagnetic interaction, one can write the following expression for the dependence of the differential cross section on the polarization of the initial particles:

$$\frac{d\sigma}{dk^2}(\vec{\xi},\vec{S}_1) = \left(\frac{d\sigma}{dk^2}\right)_{un}\left[1 + C_{\ell\ell}\xi_\ell S_{1\ell} + C_{tt}\xi_t S_{1t} + C_{nn}\xi_n S_{1n} + C_{\ell t}\xi_\ell S_{1t} + C_{t\ell}\xi_t S_{1\ell}\right], \tag{24}$$

where C_{ik}, $i,k=\ell,t,n$ are the corresponding spin correlation coefficients which characterize $\vec{p}\vec{e}$ scattering. Here also one expects large sensitivity of these observables to the direction of the proton beam polarization. Small coefficients (in absolute value) are expected for the transversal component of the beam polarization at high energies. This can be seen from the expression of the components of the proton beam polarization four vector at large energies, $E \gg M$:

$$\eta_{1\mu} = (0,\vec{S}_{1t}) + S_{1\ell}\left(\frac{p}{M}, \frac{\vec{p}}{M}\frac{E}{p}\right) \sim S_{1\ell}\frac{p_{1\mu}}{M}. \tag{25}$$

The effect of the transversal beam polarization appears to be smaller by a factor $1/\gamma$, $\gamma = E/M \gg 1$. This is a consequence of the relativistic description of the spin of the fermion at large energies.

The explicit expressions of the spin correlation coefficients, are given in Appendix B. One can see that $C_{nn} = T_{nn}$.

C. Depolarization coefficients, D_{ij}, in the $\vec{p}+e\to\vec{p}+e$ reaction

In this section explicit expressions for the depolarization coefficients, (also denoted D_{i0j0} in the notations from Ref. [13]), which define the polarization transfer from the initial to the final proton, are derived for the reaction $\vec{p}+e\to\vec{p}+e$.

The part of the hadronic tensor, $W_{\mu\nu}(\eta_1,\eta_2)$, which corresponds to polarized protons in initial and final states can be written as:

$$W_{\mu\nu}(\eta_1,\eta_2)=A_1\widetilde{g}_{\mu\nu}+A_2P_\mu P_\nu+A_3(\widetilde{\eta}_{1\mu}\widetilde{\eta}_{2\nu}+\widetilde{\eta}_{1\nu}\widetilde{\eta}_{2\mu})+A_4(P_\mu\widetilde{\eta}_{1\nu}+P_\nu\widetilde{\eta}_{1\mu})+A_5(P_\mu\widetilde{\eta}_{2\nu}+P_\nu\widetilde{\eta}_{2\mu}), \tag{26}$$

where

$$\widetilde{g}_{\mu\nu}=g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2},\ \widetilde{\eta}_{i\mu}=\eta_{i\mu}-\frac{k\cdot\eta_i}{k^2}k_\mu,\ i=1,2,$$

and

$$\begin{aligned}
A_1 &= \frac{G_M^2}{2}(2k\cdot\eta_1 k\cdot\eta_2-k^2\eta_1\cdot\eta_2),\ A_2=-\eta_1\cdot\eta_2\frac{2M^2}{1+\tau}(G_E^2(k^2)+\tau G_M^2(k^2)),\\
A_3 &= G_M^2(k^2)\frac{k^2}{2},\ A_4=-MG_M(k^2)\frac{G_E(k^2)+\tau G_M(k^2)}{1+\tau}k\cdot\eta_2,\\
A_5 &= MG_M(k^2)\frac{G_E(k^2)+\tau G_M(k^2)}{1+\tau}k\cdot\eta_1.
\end{aligned}$$

The dependence of the polarization of the scattered proton on the polarization state of the proton beam is obtained by contraction of the spin independent leptonic tensor (not averaged over the spin of the initial electron), i.e., $2L^{(0)}_{\mu\nu}$, Eq. (6), and the spin-dependent hadronic tensor $W_{\mu\nu}(\eta_1,\eta_2)$, Eq. (26).

One obtains the following formula which holds in any reference system:

$$\begin{aligned}
\mathcal{D}D(\eta_1,\eta_2) &= 2(1+\tau)^{-1}\Big\{k\cdot\eta_1 k\cdot\eta_2 G_M(k^2)\left[k^2\left(G_M(k^2)-G_E(k^2)\right)+2m^2(1+\tau)G_M(k^2)\right]\\
&\quad +k^2(1+\tau)G_M^2(k^2)(2k_1\cdot\eta_2 k_2\cdot\eta_1-m^2\eta_1\cdot\eta_2)\\
&\quad +4G_M(k^2)(k\cdot\eta_1 k_1\cdot\eta_2-k\cdot\eta_2 k_1\cdot\eta_1)\left[M^2\tau\left(G_E(k^2)-G_M(k^2)\right)\right.\\
&\quad \left.+mE\left(G_E(k^2)+\tau G_M(k^2)\right)\right]\\
&\quad -\eta_1\cdot\eta_2\left(G_E^2(k^2)+\tau G_M^2(k^2)\right)\left[k^2(M^2-2mE)+4m^2E^2\right]\Big\}.
\end{aligned} \tag{27}$$

Applying the P-invariance of the hadron electromagnetic interaction, one can write the following expression for the dependence of the differential cross section on the polarization of the incident and scattered protons which participate in the reaction as:

$$\frac{d\sigma}{dk^2}(\eta_1,\eta_2) = \left(\frac{d\sigma}{dk^2}\right)_{un} \left[1 + D_{tt}S_{1t}S_{2t} + D_{nn}S_{1n}S_{2n} + D_{\ell\ell}S_{1\ell}S_{2\ell} + D_{t\ell}S_{1t}S_{2\ell} + D_{\ell t}S_{1\ell}S_{2t}\right], \tag{28}$$

where D_{ik}, $i, k = \ell, t, n$ are the corresponding spin depolarization coefficients which characterize $\vec{p} + e \to \vec{p} + e$ scattering. The explicit expressions of the depolarization coefficients, are given in Appendix C, in terms of the hadron form factors.

D. Kinematics

A general characteristic of all reactions of elastic and inelastic hadron scattering by atomic electrons (which can be considered at rest) is the small value of the transfer momentum squared, even for relatively large energies of the colliding hadrons. Let us give details of the order of magnitude and the range which is accessible to the kinematic variables, as they are very specific for this reaction. The following formulas can be partly found in Ref. [12].

One can show that, for a given energy of the proton beam, the maximum value of the four momentum transfer squared, in the scattering on the electron at rest, is (Fig. 2):

$$(-k^2)_{max} = \frac{4m^2\vec{p}^2}{M^2 + 2mE + m^2}. \tag{29}$$

Being proportional to the electron mass squared, the four momentum transfer squared is restricted to very small values, where the proton can be considered point-like. Comparing the expressions for the total energies in two reactions: $s^I = m^2 + M^2 + 2mE$, where E is the proton energy in the elastic proton electron scattering, and $s^D = m^2 + M^2 + 2M\epsilon$, where ϵ is the electron beam energy in the electron proton elastic scattering, one finds the following relation between the proton beam energy and the electron beam energy, in order to reach the same total energy $s^I = s^D$

$$E = \frac{M}{m}\epsilon \sim 2000\ \epsilon. \tag{30}$$

The four momentum transfer squared is expressed as a function of the energy of the scattered electron, ϵ_2, as:

$$k^2 = (k_1 - k_2)^2 = 2m(m - \epsilon_2), \tag{31}$$

where

$$\epsilon_2 = m\frac{(E+m)^2 + p^2\cos^2\theta}{(E+m)^2 - p^2\cos^2\theta}, \tag{32}$$

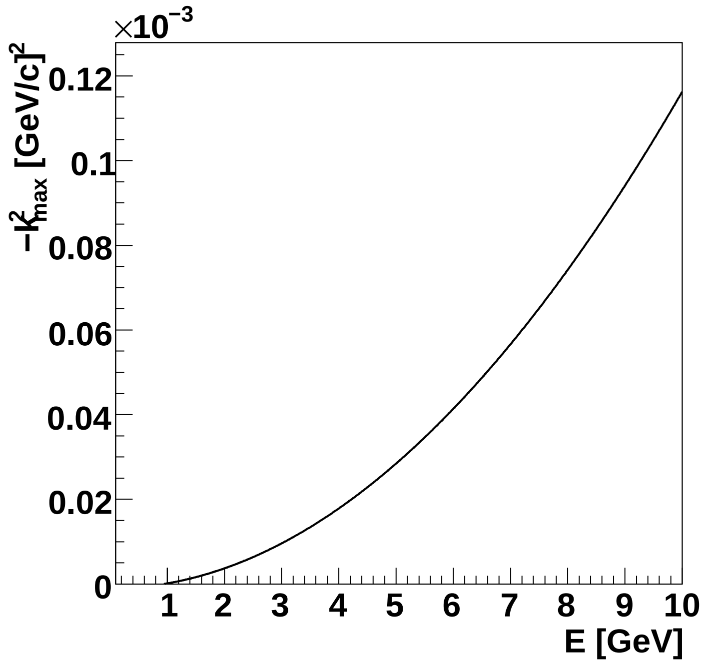

FIG. 2: Maximum four momentum transfer squared as a function of the proton beam energy.

and θ is the angle between the proton beam and the scattered electron momenta.

From energy and momentum conservation, one finds the following relation between the angle and the energy of the scattered electron:

$$\cos\theta = \frac{(E+m)(\epsilon_2 - m)}{|\vec{p}|\sqrt{(\epsilon_2^2 - m^2)}}, \tag{33}$$

which shows that $\cos\theta \geq 0$. One can see from Eq. (32) that in the inverse kinematics, the available kinematical region is reduced to small values of ϵ_2:

$$\epsilon_{2,max} = m\frac{2E(E+m) + m^2 - M^2}{M^2 + 2mE + m^2}. \tag{34}$$

From momentum conservation, on can find the following relation between the energy and the angle of the scattered proton E_2 and θ_p:

$$E_2 = \frac{(E+m)(M^2+mE) \pm M|\vec{p}|^2\cos\theta_p\sqrt{\frac{m^2}{M^2} - \sin^2\theta_p}}{(E+m)^2 - |\vec{p}|^2\cos^2\theta_p}. \tag{35}$$

Let us introduce the invariant

$$\nu = k\cdot p_1 = E(m-\epsilon_2) + |\vec{k}_2||\vec{p}|\cos\theta = \frac{k^2}{2m}\left(E - |\vec{p}|\cos\theta\sqrt{1 - 4\frac{m^2}{k^2}}\right). \tag{36}$$

The following relation holds: $k^2 + 2\nu = 0$.

For the angle between the initial and final hadron, it exists a maximum value which is determined by the ratio of the electron and scattered hadron masses, $\sin\theta_{h,max} = m/M$. One concludes that hadrons are scattered on atomic electrons at very small angles, and that the largest is the hadron mass, the smaller is the available angular range for the scattered hadron.

III. NUMERICAL RESULTS

A. Experimental observables

For a given proton beam energy E the observables are functions of only one kinematical variable, that we chose as k^2, as it is a kinematical invariant. Transformation to the scattering electron angle are straightforward. The proton structure is taken into account through the parametrization of FFs. We took the dipole parametrization:

$$G_E(k^2) = G_M(k^2)/\mu_p = [1 - k^2/0.71]^{-2}, \tag{37}$$

where μ_p is the proton magnetic moment, and k^2 is expressed in GeV^2. The normalization to the static point is $G_E(0) = 1$ and $G_M(0) = \mu_p$. The standard dipole parametrization coincides with more recent descriptions for $-k^2 < 1$ GeV^2. At higher k^2, different choices may affect the cross section and at a lesser extent, the polarization observables, but as we showed above, the maximum value of k^2 which can be achieved in inverse kinematics, justifies the choice of dipole parametrization, and even of constant FFs, where the constants correspond to the static values.

The differential cross section, Eq. (16), is plotted as a function of $(-k)^2$ in Fig. 3. One can see that it is monotonically decreasing as a function of k^2 up to a value of k^2_{max} according to Eq. (29).

The polarization transfer coefficients, Eq. (A1), are shown in Fig. 6 as a function of the incident energy for $\theta = 0$ (black solid line), 10 mrad (red dashed line), 30 mrad (green dash-dotted line), 50 mrad (blue dotted line). The spin correlation coefficients Eq. (B1) are shown in Fig. 7. The spin depolarization coefficients, Eq. (C1), are shown in Fig. 8.

One can see that in collinear kinematics, in general, either polarization observables take the maximal values or they vanish. An interesting kinematical region appears at $E = 20$

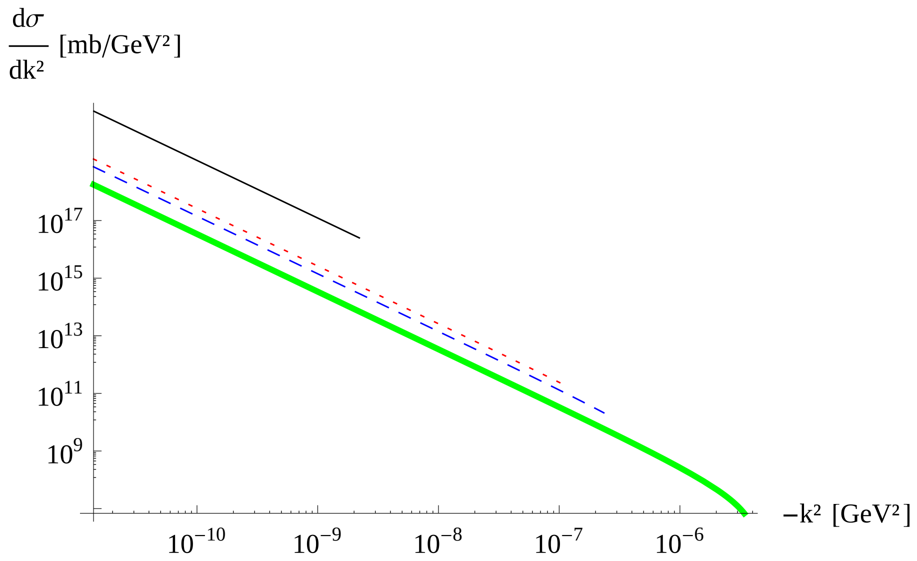

FIG. 3: Differential cross section as a function of $-k^2$ for different incident energies: E=1 MeV (black solid line), E=50 MeV (red dotted line), E=100 MeV (blue dashed line), E=1 GeV (green thick line).

GeV, where a structure is present in agreement with the results of Ref. [2].

As shown in Section II, Eq. (16), the cross section diverges for $k^2 \to 0$. This condition is obtained when the scattering angle is small (high energies, and large impact parameters), or when the energy is small.

In the first case, one introduces a minimum scattering angle, which is related to the impact parameter, which classical (c) and quantum expressions (q), are given by [14]:

$$\theta_{min}^{(c)} = \frac{2e^2}{p\beta b},\ \theta_{min}^{(q)} = \frac{\hbar}{pb}, \tag{38}$$

where b is the impact parameter and β is the relative velocity. Let us take as characteristic impact parameter, the Bohr radius, $b = 0.519 \cdot 10^5$ fm. We have shown above that there is a maximum scattering angle for the proton, which does not depend on the energy, and a corresponding maximum value for the transferred momentum k^2. The condition $k_{min} < k_{max}$ from Eqs. (38), is obtained for $E \geq 1$ MeV. When the relative energy is very low, the electron and proton may be trapped in a bound system, and the present description based on one photon exchange is not valid. The Born approximation corresponds to the first term of an expansion in the parameter α/v which should be lower than unity. The condition $\alpha/v = 0.1c$ is satisfied for $E > 2.5$ MeV.

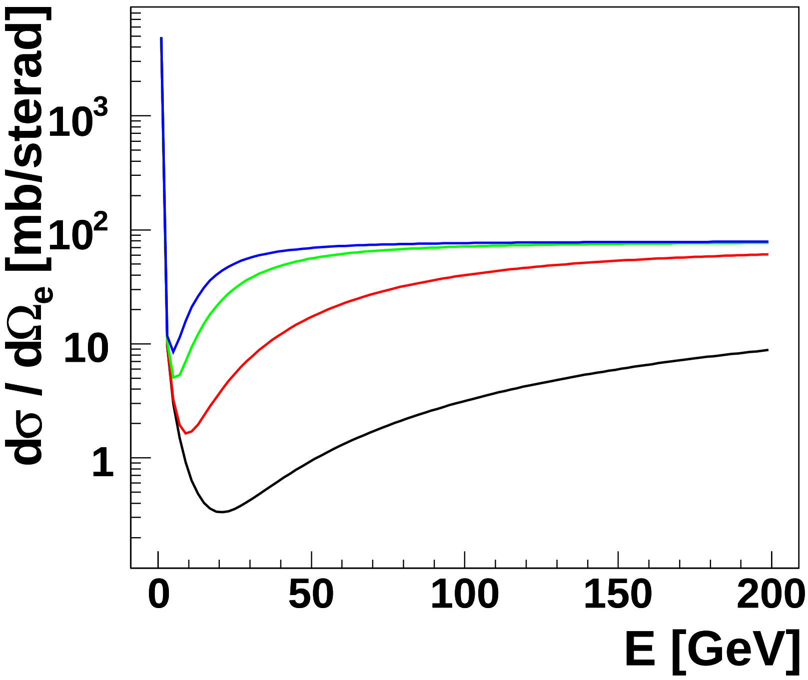

FIG. 4: Differential cross section as a function of the incident energy E for different angles: $\theta = 0$ (black solid line), 10 mrad (red dashed line), 30 mrad (green dotted line), 50 mrad (blue dashed-dotted line).

The description of Coulomb effects at low energies require approximations and it is outside the purpose of this paper. We will apply the present calculation for $E \geq 3$ MeV.

Al low energy, screening effects are introduced multiplying the cross section by the factor

$$\chi = \frac{\chi_b}{e^{\chi_b} + 1}, \ \chi_b = -2\pi\frac{\alpha}{\beta}. \tag{39}$$

Such factor is attractive for opposite charges and increases the cross section for the reaction of interest here.

The total cross section has been calculated by integration from a value of k^2_{min} extracted from Eqs. (31,32), and it is given as a function of the incident proton kinetic energy $T = E - M$ in Fig. 5, for values of the proton kinetic energy in the MeV range.

Let us calculate the cross section for a non polarized proton beam colliding with a polarized target:

$$\sigma_{ij} = \int N\mathcal{D}T_{ij}P_iP_j \, \mathrm{d}k^2, \ N = \frac{\pi\alpha^2}{2m^2p^2k^4}. \tag{40}$$

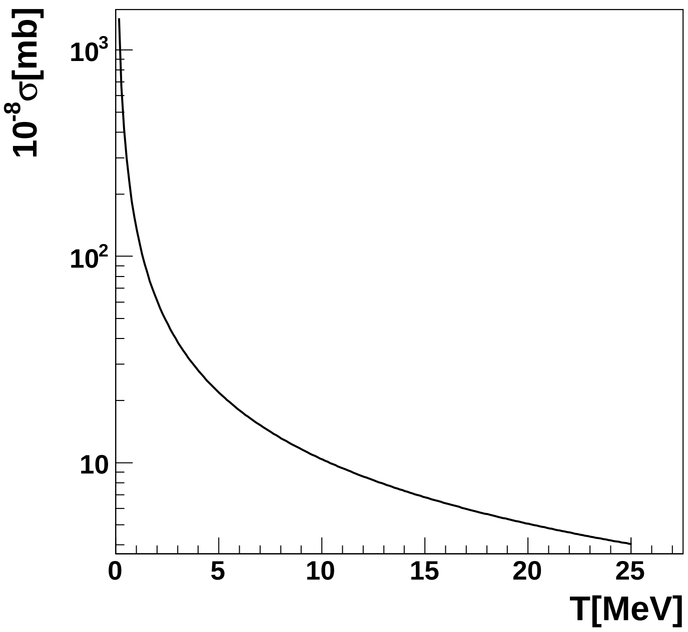

FIG. 5: Total unpolarized cross section as a function of the incident proton kinetic energy T.

Assuming $P_i = P_j = 1$, the values for different incident energies are reported in Table I for the total polarized and unpolarized cross sections and in Table II for the corresponding integrated polarization coefficients.

T [GeV]	σ_{unp} [mb]	$\sigma_{t\ell}$ [mb]	$\sigma_{\ell t}$ [mb]	$\sigma_{\ell\ell}$ [mb]	σ_{tt} [mb]	σ_{nn} [mb]
$23 \cdot 10^{-3}$	$4.4 \cdot 10^8$	26	26.7	-125.3	-16.9	-139.3
$50 \cdot 10^{-3}$	$2 \cdot 10^8$	11.5	12.2	-62.8	-7.4	-67
1	$2.5 \cdot 10^7$	0.4	0.8	-5.6	-0.2	-2.9
10	$1.9 \cdot 10^7$	$9.1 \cdot 10^{-3}$	$10.6 \cdot 10^{-2}$	-1.01	$-0.6 \cdot 10^{-2}$	-0.09
50	$1.8 \cdot 10^7$	$0.4 \cdot 10^{-3}$	$2.3 \cdot 10^{-2}$	-0.2	$-0.3 \cdot 10^{-3}$	$-0.5 \cdot 10^{-2}$

TABLE I: Unpolarized cross section and polarized transfer cross sections (in mb) for different incident energies.

The spin transfer cross section σ_{nn} and $\sigma_\ell = (\sigma_{\ell\ell} + \sigma_{\ell t})/2$, are illustrated in Fig. 9 in the MeV range.

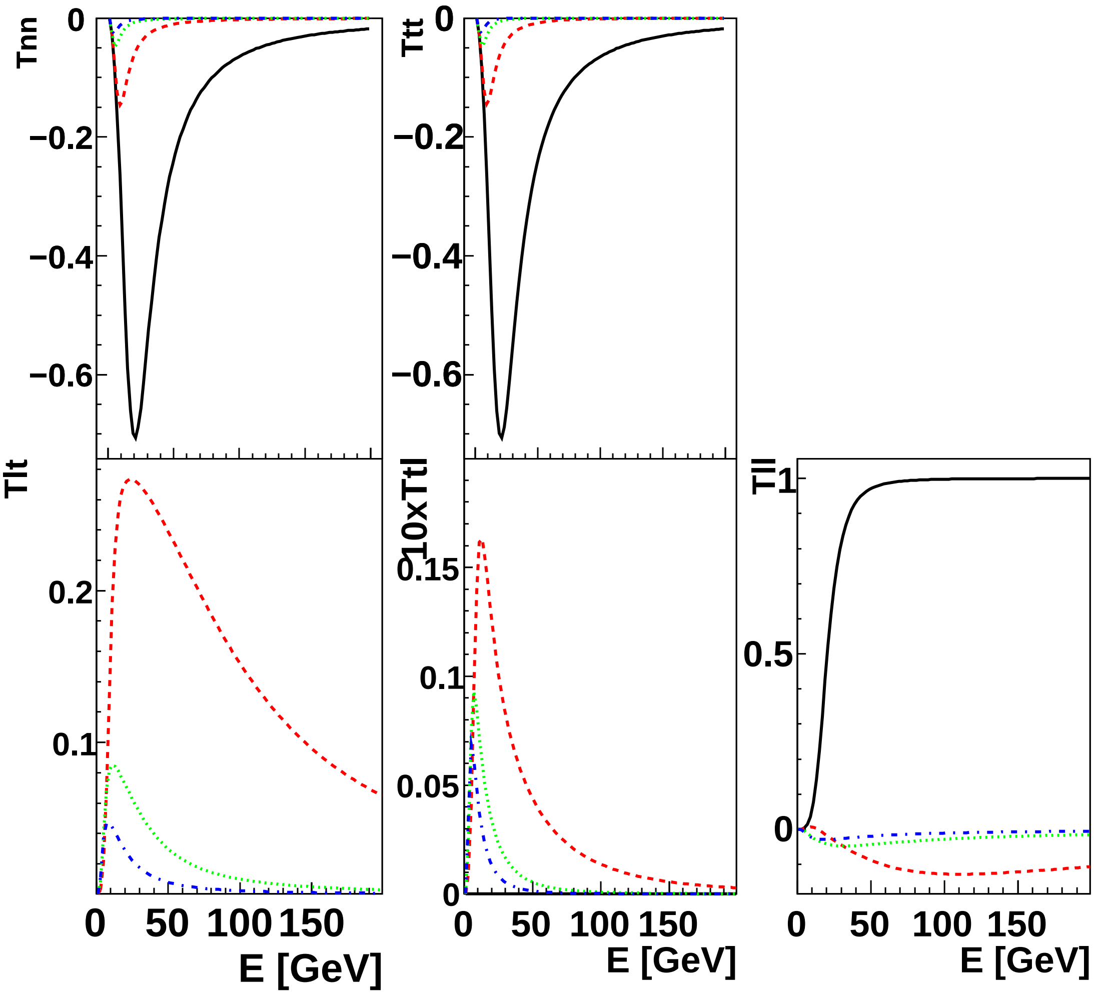

FIG. 6: Polarization transfer coefficients as a function of E for different angles. Notations as in Fig. 4.

These values are very sensitive to the incident energy, and they are consistent with the findings of Refs. [6, 7, 16]. Although they cannot be compared directly with the previous calculations, as our formalism is derived in the laboratory system, they allow a more direct comparison to the experiment.

B. High energy polarimetry

From Figs. 6, 7, 8 it appears that polarization coefficients are in general quite large, except at low energy. Proton electron scattering can be used, in principle, to measure the

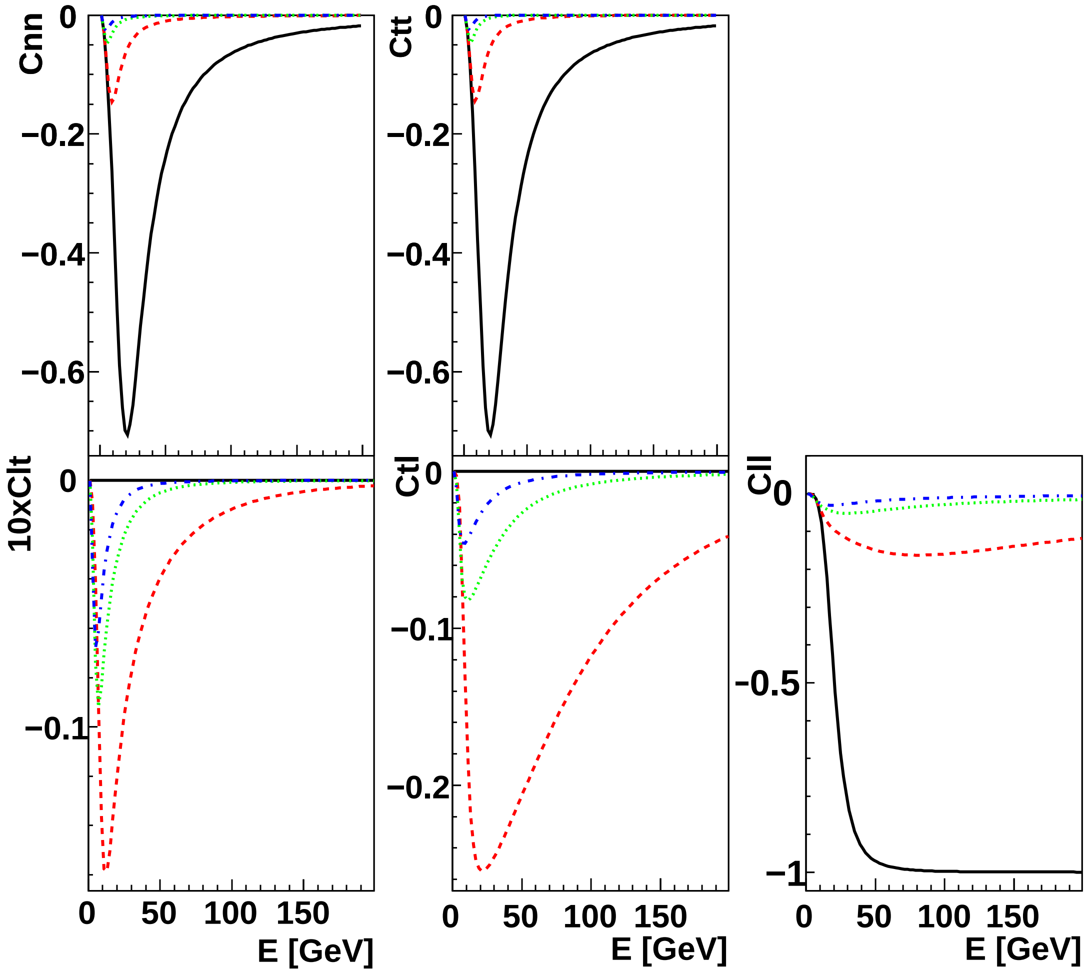

FIG. 7: Same as Fig. 6 for the spin correlation coefficients.

polarization of high energy beams. The idea to use pe elastic scattering for beam polarimetry has already been suggested in Refs. [16]. Let us calculate the figure of merit, for measuring the polarization of a secondary proton beam, after scattering on atomic electrons.

The differential figure of merit is defined as

$$\mathcal{F}^2(\theta_p) = \epsilon(\theta_p) A_{ij}^2(\theta_p),$$

where A_{ij} stands for a generic polarization coefficient and $\epsilon(\theta_p) = N_f(\theta_p)/N_i$ is the number of useful events over the number of the incident events in an interval $\Delta\theta_p$ around θ_p. This quantity is useful, as it is related to the inverse of the statistical error on the polarization

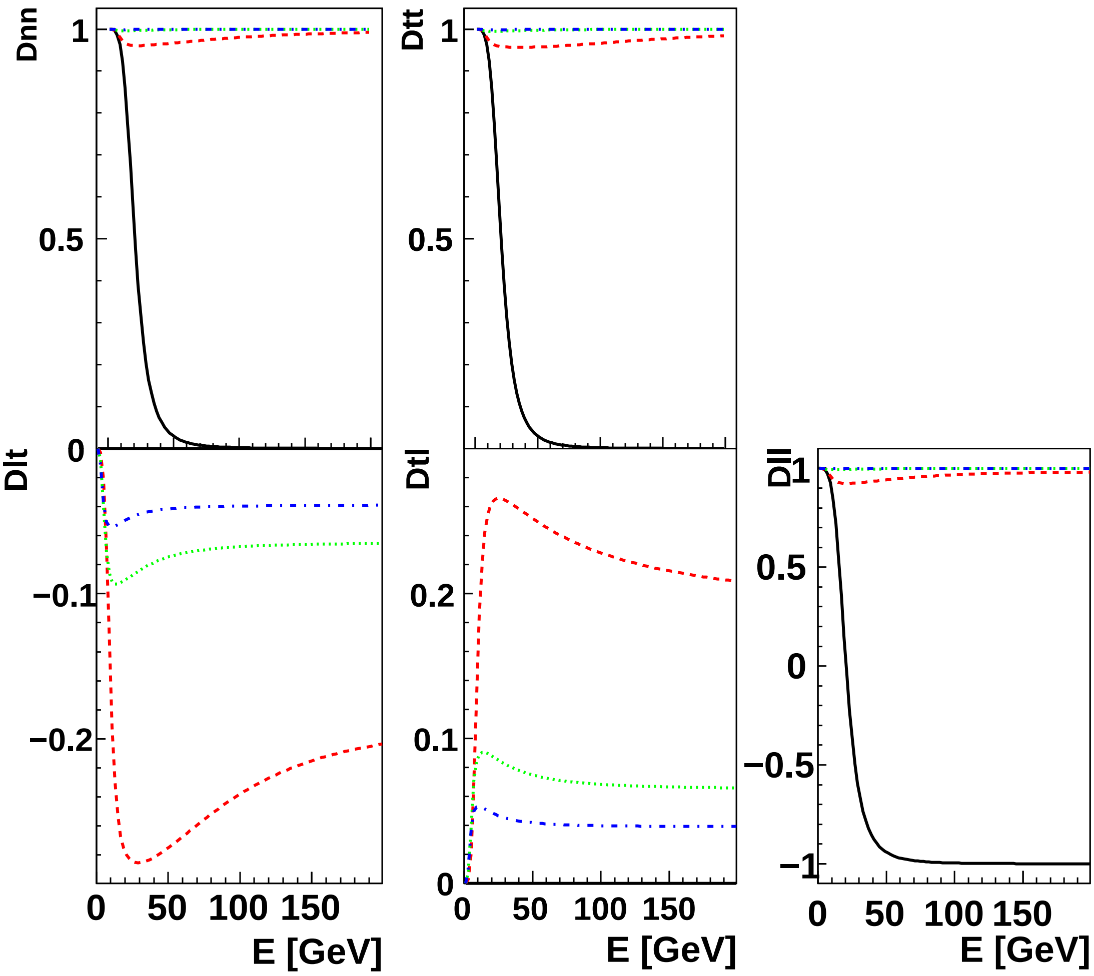

FIG. 8: Same as Fig. 6 for the spin depolarization coefficients.

measurement, for a proton with degree of polarization P:

$$\left(\frac{\Delta P(\theta_p)}{P}\right)^2 = \frac{2}{N_i(\theta_p)\mathcal{F}^2(\theta_p)P^2} = \frac{2}{Lt_m(d\sigma/d\Omega)d\Omega A_{ij}^2(\theta_p)P^2}, \tag{41}$$

t_m is the time of measurement. The correlation coefficient squared, weighted by the differential cross section, $A_{t\ell}^2(k^2)(d\sigma/dk^2)$ and $A_{\ell\ell}^2(k^2)(d\sigma/dk^2)$ are shown in Fig. 10 for different electron angles.

The integrated figure of merit

$$F^2 = \int \frac{d\sigma}{dk^2} A_{ij}^2(k^2)\mathrm{d}k^2 \tag{42}$$

as a function of the incident energy is shown in Fig. 11. In Refs. [17] it was suggested

T [GeV]	$T_{t\ell}$	$T_{\ell t}$	$T_{\ell\ell}$	T_{tt}	T_{nn}
$23 \cdot 10^{-3}$	$1.5 \cdot 10^{-12}$	$1.5 \cdot 10^{-12}$	$-1.3 \cdot 10^{-12}$	$-2.6 \cdot 10^{-12}$	$-3.8 \cdot 10^{-12}$
$50 \cdot 10^{-3}$	$7.2 \cdot 10^{-12}$	$7.5 \cdot 10^{-12}$	$-6.3 \cdot 10^{-12}$	$-1.2 \cdot 10^{-11}$	$-1.8 \cdot 10^{-11}$
1	$3.3 \cdot 10^{-9}$	$6.8 \cdot 10^{-9}$	$-4.8 \cdot 10^{-9}$	$-6.8 \cdot 10^{-9}$	$-9.2 \cdot 10^{-9}$
10	$3.5 \cdot 10^{-7}$	$3.9 \cdot 10^{-6}$	$-1.4 \cdot 10^{-6}$	$-1.1 \cdot 10^{-6}$	$-1.2 \cdot 10^{-6}$
50	$5.9 \cdot 10^{-6}$	$0.3 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$	$-1.4 \cdot 10^{-5}$	$-0.2 \cdot 10^{-4}$

TABLE II: Polarization coefficients for different incident energies.

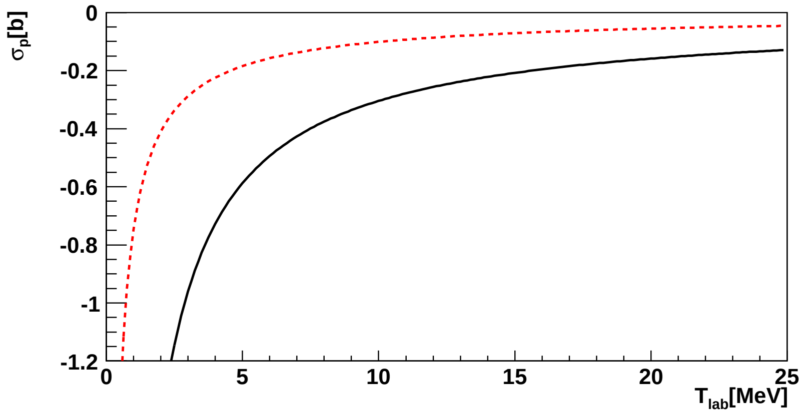

FIG. 9: Spin transfer cross section $\sigma_\ell = (\sigma_{\ell\ell} + \sigma_{\ell t})/2$ (black solid line) and σ_{nn} (red dashed line) as a function of the proton beam energy.

to use the scattering of a transversally polarized proton beam on a longitudinally polarized electron target. From Fig. 11, one can see that the figure of merit takes its maximum value for $T \simeq 10$ GeV. Assuming a luminosity of 10^{32} cm^{-2} s^{-1}, for an ideal detector with acceptance and efficiency of 100%, one could measure the beam polarization with an error of 1% in a time interval of 3 min.

If one detects the outgoing proton, which seems more challenging as its kinematical characteristics are close to those of the beam, one could in principle build a polarimeter based on the scattering of the polarized beam (the polarization of which should be known) on an unpolarized target. In this case, from the azimuthal distribution, one can reconstruct the components of the polarization which are normal to the scattering plane.

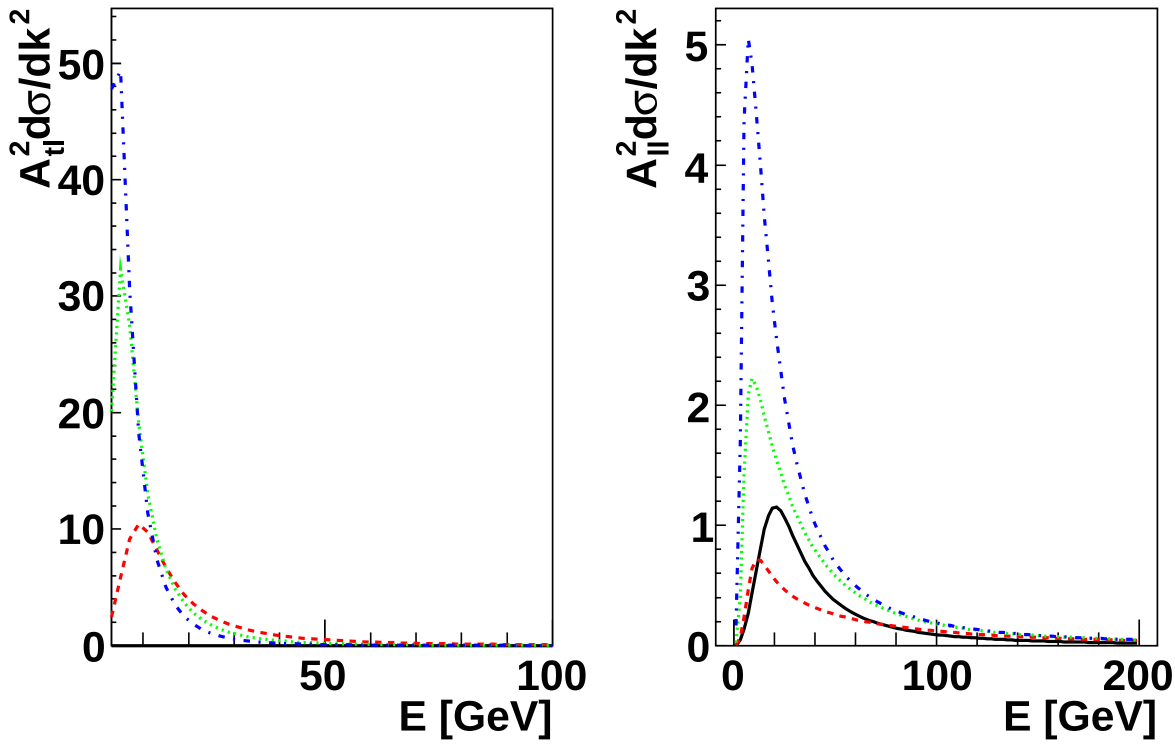

FIG. 10: Variation of the differential quantities $A^2_{t\ell}(k^2)(d\sigma/dk^2)$ (left) and $A^2_{\ell\ell}(k^2)(d\sigma/dk^2)$ (right) as a function of the incident energy for a polarized proton beam on a polarized electron target $\vec{p} + \vec{e} \to p + e$, at different angles. Notations as in Fig. 4.

IV. CONCLUSIONS

The elastic scattering of protons on electrons at rest was investigated in a relativistic approach in the one photon exchange (Born) approximation. This reaction, where the target is three order of magnitude lighter than the projectile, has specific kinematical features due to the 'inverse kinematics', i.e., the projectile is heavier than the target. For example, the

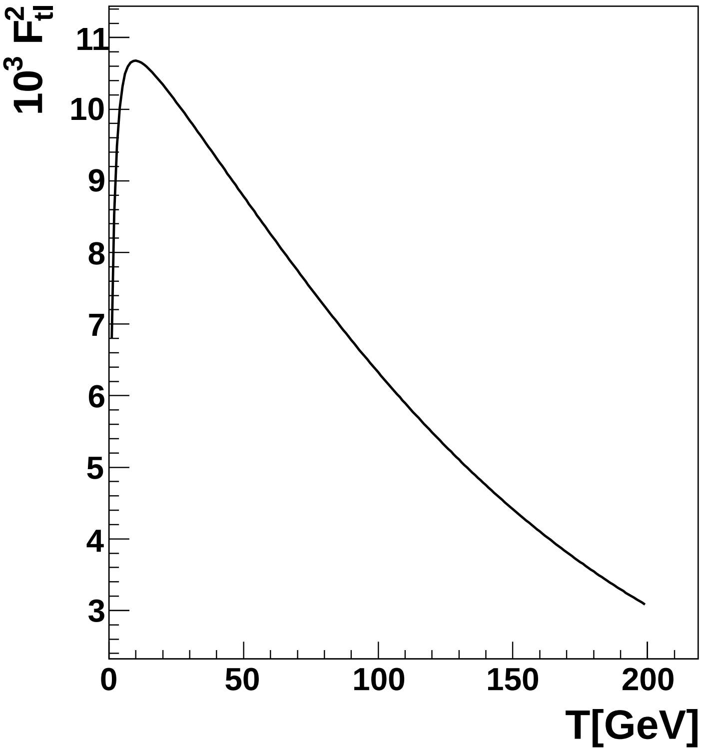

FIG. 11: Variation of the quantity F^2 as a function of the kinetic proton beam energy for a transversally polarized proton beam on a longitudinally polarized electron target $\vec{p} + \vec{e} \to p + e$.

proton is scattered at very small angles and the allowed momentum transfer are in the MeV^2 scale, even when the proton incident energy is of the order of GeV. The differential cross section and various double spin polarization observables have been calculated in terms of the nucleon electromagnetic FFs. Note that at the values of transferred momentum involved, any parametrization of FFs is equivalent and it is very near to the static values. The spin transfer coefficients to a polarized scattered proton were calculated for two cases: when the proton beam is polarized or the electron target is polarized. The correlation spin coefficients when the proton beam and the electron target are both polarized were also calculated. Note that the expressions for the polarization observables in the considered reaction, in a relativistic approach, are absent in the literature (at our knowledge). Numerical estimations showed that polarization effects may be sizable in the GeV range, and that the polarization transfer coefficients for $\vec{p}+e \to \vec{p}+e$ could be used to measure the polarization of high energy proton beams. This result confirms previous estimations from [16]. The calculated values of the scattered proton polarization in the reaction $p+\vec{e} \to \vec{p}+e$ at energies of the proton beam lower then few tens of MeV, show that it is not possible to obtain sizable polarization of the antiproton beam in an experimental set up where antiprotons and electrons collide with small relative velocities. The present results confirm that the polarization of the scattered proton has large values at high energies of the proton beam (in the GeV range). So, one could consider an experimental set up were high energy protons collide with a polarized electron target at rest. The low values of momentum transfer which are involved, insure that the cross section is sizable.

V. ACKNOWLEDGMENTS

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APPENDIX A: POLARIZATION TRANSFER COEFFICIENTS

The explicit expressions for the polarization transfer coefficients for $p+\vec{e}\to\vec{p}+e$ are:

$$
\begin{aligned}
\mathcal{D}T_{nn} &= 4mMk^2 G_E(k^2)G_M(k^2),\\
\mathcal{D}T_{tt} &= 4mMk^2\frac{G_M(k^2)}{1+\tau}\Bigg\{(1+\tau)G_E(k^2)-\left(E+M+\frac{k^2}{2m}\right)^{-1}\left(1-\frac{k^2}{k^2_{max}}\right)\\
&\quad \big[(E+M+2E\tau)G_E(k^2)-\tau(E+M+2M\tau)G_M(k^2)\big]\Bigg\},\\
\mathcal{D}T_{t\ell} &= -2mpk^2\frac{G_M(k^2)}{1+\tau}\left(E+M+\frac{k^2}{2m}\right)^{-1}\left[-k^2\left(1-\frac{k^2}{k^2_{max}}\right)\right]^{1/2}\\
&\quad\Bigg\{\frac{M}{m}\frac{m+M}{E-M}\big[(1+2\tau)G_E(k^2)-\tau G_M(k^2)\big]+\\
&\quad\left(1-2m\frac{E+m}{s}\frac{k^2}{k^2_{max}}\right)\big[G_E(k^2)+\tau G_M(k^2)\big]\Bigg\},\\
\mathcal{D}T_{\ell t} &= -4mpk^2\frac{G_M(k^2)}{1+\tau}\left(E+M+\frac{k^2}{2m}\right)^{-1}\left[-k^2\left(1-\frac{k^2}{k^2_{max}}\right)\right]^{1/2}\\
&\quad\Bigg\{(1+\tau)G_M(k^2)+\frac{E-M}{2M}\big[G_M(k^2)-G_E(k^2)\big]\\
&\quad-m\frac{E+m}{s}\frac{1}{k^2_{max}}\Big[k^2\big(G_E(k^2)+\tau G_M(k^2)\big)+\\
&\quad 2M(E+M)\big(G_E(k^2)(1+2\tau)-\tau G_M(k^2)\big)\Big]\Bigg\},\\
\mathcal{D}T_{\ell\ell} &= 4mMk^2\frac{G_M(k^2)}{1+\tau}\Bigg\{(1+\tau)\Bigg[\frac{E}{M}+\frac{xk^2}{2m}(E+M+2m)-\\
&\quad\frac{(E+m)^2}{s}\frac{k^2}{k^2_{max}}\left(1+\frac{xk^2}{2m}(m-M)\right)+\frac{1}{s}(m+M)(E+m)xp^2\frac{k^2}{k^2_{max}}\Bigg]G_E(k^2)\\
&\quad+\tau\Bigg[xp^2\frac{M+m}{m}\left(1-3\frac{m(E+m)}{s}\frac{k^2}{k^2_{max}}\right)+\\
&\quad\frac{(E+m)^2}{s}\frac{k^2}{k^2_{max}}\left(1-\frac{xk^2}{2m}(m+M)\right)-\frac{E+m}{m}\Bigg]\big[G_M(k^2)-G_E(k^2)\big]\Bigg\},
\end{aligned}
\tag{A1}
$$

with $x^{-1}=M(E+M+\frac{k^2}{2m})$, and $s=m^2+M^2+2mE$ is the total energy in the proton electron elastic scattering.

APPENDIX B: POLARIZATION CORRELATION COEFFICIENTS

The explicit expressions of the spin correlation coefficients, as a function of the Sachs FFs can be written as:

$$\begin{aligned}
\mathcal{D}C_{nn} &= 4mMk^2 G_E(k^2)G_M(k^2), \\
\mathcal{D}C_{tt} &= 4mM\tau k^2 \frac{G_M(k^2)}{1+\tau}\left[(1-\frac{4M^2}{k^2_{max}})G_E(k^2)+(\frac{k^2}{k^2_{max}}-1)G_M(k^2)\right], \\
\mathcal{D}C_{t\ell} &= 8mMp\left[-k^2\left(1-\frac{k^2}{k^2_{max}}\right)\right]^{1/2}\frac{G_M(k^2)}{1+\tau}\Big\{\tau\left[G_M(k^2)-G_E(k^2)\right] \\
&\quad -\frac{k^2}{k^2_{max}}\frac{m(E+m)}{s}\left[\tau G_M(k^2)+G_E(k^2)\right]\Big\}, \\
\mathcal{D}C_{\ell t} &= -2mM\frac{k^2}{p}\left(\frac{E}{M}-\frac{M}{m}\right)\left[-k^2\left(1-\frac{k^2}{k^2_{max}}\right)\right]^{1/2}\frac{G_M(k^2)}{1+\tau}\left[\tau G_M(k^2)+G_E(k^2)\right], \\
\mathcal{D}C_{\ell\ell} &= 4k^2\frac{G_M(k^2)}{1+\tau}\Big\{(mE-\tau M^2)G_E(k^2)+\tau(M^2+mE)G_M(k^2) \\
&\quad -(M^2+mE)\frac{k^2}{k^2_{max}}\frac{m(E+m)}{s}\left[\tau G_M(k^2)+G_E(k^2)\right]\Big\}.
\end{aligned} \tag{B1}$$

APPENDIX C: DEPOLARIZATION COEFFICIENTS FOR $\vec{p}+e\to\vec{p}+e$

The depolarization coefficients from the polarized beam to the ejectile for $\vec{p}+e\to\vec{p}+e$, are expressed in terms of the hadron form factors, as:

$$\begin{aligned}
\mathcal{D}D_{tt} &= -R_1-k^2\left(1-\frac{k^2}{k^2_{max}}\right)\left\{\frac{m}{M}\left(R_3-R_4\right)-xR_1+\left(1-xk^2\frac{m+M}{2m}\right)R_2\right\},\\
\mathcal{D}D_{nn} &= -R_1,\\
\mathcal{D}D_{\ell\ell} &= \frac{R_1}{M}\left\{\frac{p}{M}\left(p+\frac{k^2}{2m}\frac{E+m}{p}\right)-E\left[1+x\left(p+\frac{k^2}{2m}\frac{E+m}{p}\right)^2\right]\right\}+\\
&\quad R_4\frac{m}{M}\frac{k^2}{2m}\left\{\frac{1}{M}\left(p-E\frac{E+m}{p}\right)\left(p+\frac{k^2}{2m}\frac{E+m}{p}\right)+E+m-\right.\\
&\quad \left.p\left(p+\frac{k^2}{2m}\frac{E+m}{p}\right)\left[\frac{1}{M}-x\left(E-m+\frac{k^2}{2m}\right)\right]\right\}+R_2\frac{1}{M}\left(\frac{k^2}{2m}\right)^2\left(p-E\frac{E+m}{p}\right)\\
&\quad \left\{\left(p+\frac{k^2}{2m}\frac{E+m}{p}\right)\left[\frac{1}{M}-x\left(E-m+\frac{k^2}{2m}\right)\right]-\frac{E+m}{p}\right\}+\\
&\quad R_3\frac{m}{M}p\left\{\frac{k^2}{2m}\frac{E+m}{p}+\left(p+\frac{k^2}{2m}\frac{E+m}{p}\right)\left[\frac{m}{M}-\frac{1}{M}\frac{k^2}{2m}+x\frac{k^2}{2m}\left(E-m+\frac{k^2}{2m}\right)\right]\right\}+\\
&\quad R_3\frac{m}{M^2}\left(p+\frac{k^2}{2m}\frac{E+m}{p}\right)\left[mp+\frac{1}{p}\frac{k^2}{2m}(M^2+mE)\right],\\
\mathcal{D}D_{t\ell} &= \frac{1}{p}\left[-k^2\left(1-\frac{k^2}{k^2_{max}}\right)\right]^{1/2}\left\{x\left[p^2+\frac{k^2}{2m}(E+m)\right]\right.\\
&\quad \left.\left[R_1+\frac{k^2}{2m}(m+M)R_2+\frac{m}{Mx}(R_4-R_3)\right]-\frac{k^2}{2m}(E+m)R_2\right\},\\
\mathcal{D}D_{\ell t} &= \frac{1}{M^2p}\left[-k^2\left(1-\frac{k^2}{k^2_{max}}\right)\right]^{1/2}\left\{-mp^2\left[MR_4+(M+2m)R_3\right]+\right.\\
&\quad \frac{k^2}{2}(M^2+mE)\left(R_4-R_3-\frac{M}{m}R_2\right)+\\
&\quad \frac{xk^2}{2m}M(m+M)\left[mp^2(R_3+R_4)+(M^2+mE)\frac{k^2}{2m}R_2\right]+\\
&\quad \left.xMR_1\left[\frac{k^2}{2m}(M^2+mE)-Mp^2\right]\right\},
\end{aligned}\tag{C1}$$

where

$$\begin{aligned}
R_1 &= -2\left[m^2k^2G_M^2+\frac{G_E^2+\tau G_M^2}{1+\tau}(M^2k^2+2mEk^2+4m^2E^2)\right],\\
R_2 &= 2\frac{G_M}{1+\tau}\left[2m^2(1+\tau)G_M+k^2(G_M-G_E)\right],\\
R_3 &= 2k^2G_M^2,\\
R_4 &= 2(k^2+4mE)G_M\frac{G_E+\tau G_M}{1+\tau}.
\end{aligned}\tag{C2}$$

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