STATISTICAL ANALYSIS OF THE QUENCH FIELDS IN SCRF CAVITIES

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Abstract :

For a given defect , the quench field level in superconducting radiofrequency (SCRF) cavities is strongly decreasing with frequency. That would indicate that lower frequencies are recommended for high fields applications. On the other hand, the lower the frequency, the larger the cavity size. Hence, if defects are randomly distributed, one would assume that larger defects are expected at lower frequencies. That would favor the use of higher frequencies, counteracting the previous statement. Is there an optimal frequency resulting? The aim of this paper is to evaluate the strength of these two opposite effects, calculating the expected average quench field as a function of frequency including both the thermal analysis and the statistical one. That integrated statistical analysis might also help for predicting the percentage of cavities under a given specified field that would be rejected in a production series.

Introduction :

It is generally assumed that the quench field level in SCRF cavities is due to defects lying on the inner surface. That idea is supported by the fact that thermal instability analysis [**1**] show that the maximum magnetic field on defect-free surfaces should be higher than 150 mT. This uniform case assumes no defects and the use of a high RRR niobium (RRR > 200). But most experimental results on actual cavities exhibit quenches around 80 mT, with very localized heating (detected by thermometry). The same thermal analysis including small micron-size defects might explain the non-homogeneous behavior leading to a local thermal runaway. Although a lot of effort has been devoted these last years in most laboratories to try to identify these defects, it seems that their nature can be still considered as unknown. (Due probably to their small sizes, they are very difficult to detect by standard techniques. As an example, a local increase of the impurities contents in the niobium might be considered as a “defect”). Therefore, one might consider a statistical analysis where the defects are randomly distributed in the material, simulating the experimental distribution of quench fields on actual cavities.

Statistics :

Let us assume that any kind of defect may exist on all cavities with a given probability. For simplicity, only one parameter will characterize each "typical" defect, namely its size Φ. This parameter is chosen because it has been shown to be one of the most sensitive with regard to the quench field value [**1**]. Any other type of defect will then be assumed to be equivalent to our "standard" defect provided it has been given the appropriate size (even though it is not its actual size).

If p (Φ,S) denotes the probability function that all defects on a given area S must have a size smaller than Φ, this function will have the shape drawn on figure 1 as a function of Φ.

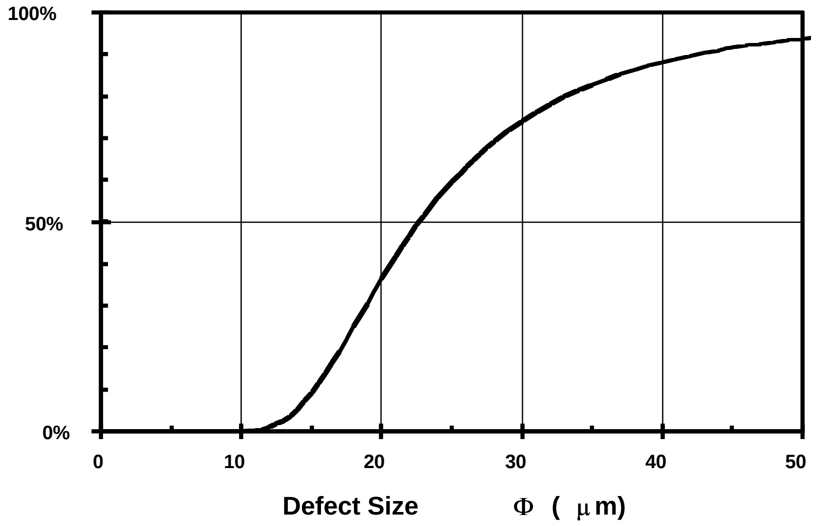

Figure 1 - Typical probability function variation with defect size.

For larger defect sizes, the probability should approach unity as the number of defects should decrease with size (most probably, all defects on a given area are smaller than some large size). *A contrario*, for very small defects, p would be close to zero, indicating that tiny defects are much more numerous (and hard to get rid of, too !).

For a surface S' = 2.S, basic probability will impose that :

$$p\left(\Phi\ ,\ S' = 2S\right) \quad = \quad \left[p\left(\Phi\ ,\ S\right)\right]^2$$

which straightforwardly implies that the probability function p should be an exponential function of the surface S. Namely,

$$p\left(\Phi, S\right) \quad = \quad e^{-\frac{S}{S_a(\Phi)}} \qquad\qquad (1)$$

(the constant is determined because p (Φ, S=0) = 1). This simply reflects the fact that the probability to be free of defects on a small surface (i.e. at higher frequency) is greater than on a large area (at lower frequency).

Relation between size and quench field : The quench field value B_q can be related to the defect size Φ, using the thermal model [**1**]. If we assume that our "typical" defect is a normal conducting disk with a given electrical conductivity σ, one can plot the quench field B_q as a function of Φ. B_q is clearly a strongly decreasing function of Φ (figure 2) that might be approximated by :

$$B_q \approx \frac{B_a}{\sqrt{1+\left(\frac{\Phi}{\Phi_0}\right)^2}} \qquad (2)$$

where $\Phi_0 = 20\ \mu m$ with $\sigma = 10^7\ \Omega^{-1}\ m^{-1}$.

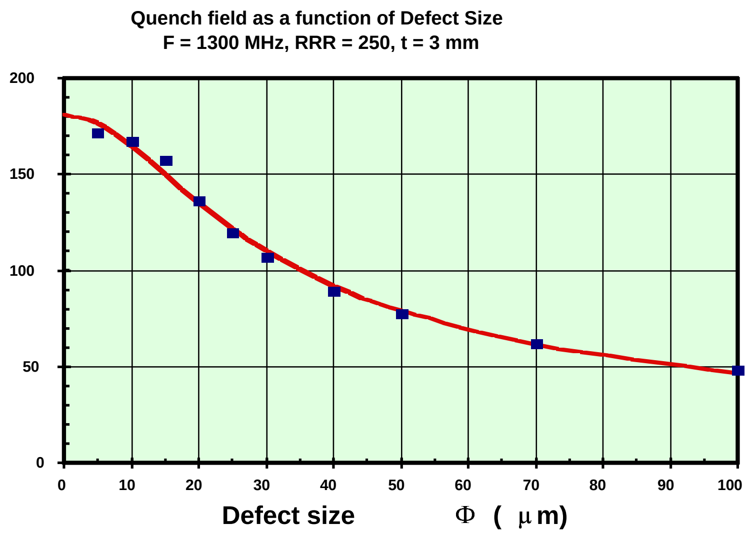

Figure 2 - The magnetic quench field decreases with the defect size. The points are calculated using thermal analysis . The line is the approximate expression used in this paper.

B_a is the quench field value varying with both frequency and bath temperature. Again, a complete thermal analysis is needed to evaluate the exact frequency behavior. But, for a given defect, it can be shown [**2**] that it is similar to the above one. A good approximation of B_a is given by :

$$B_a \approx \frac{B_c(T)}{\sqrt{1+\left(\frac{F}{F_0}\right)^2}} \approx B_0 \frac{\left(1-\left(\frac{T}{T_c}\right)^2\right)}{\sqrt{1+\left(\frac{F}{F_0}\right)^2}} \qquad (3)$$

with $B_0 = B_c$ (T=0) = 198 mT and T_c = 9.25 K for niobium and F_0 = 2.4 GHz for a superfluid helium bath.

<u>Probability function :</u>

Coming back to equation (1), some hypothesis on the S_a function has to be made in order to explicitly determine the probability function p. The simplest hypothesis (taking into account that $S_a(\Phi=0)$ should be zero, and that S_a should be an increasing function of Φ) is to assume a power law function :

$$S_a(\Phi) = S_0 . \left(\frac{\Phi}{\Phi_0}\right)^m \qquad (4)$$

where S_0 is a constant area and Φ_0 the same constant size used above. The only free parameter here is the power m which will determine the width of the distribution as it will be seen later on. Making this assumption results in the probability function being :

$$p(\Phi, S) = e^{-\left(\frac{S}{S_0}\right) \frac{1}{\left[\left(B_a/B\right)^2 - 1\right]^{m/2}}} \qquad (5)$$

In order to get the probability for having a quench field lying between B and (B+dB), the derivatives of the above function is :

$$\frac{dp}{dB} = \left(\frac{S}{S_0}\right) \frac{m\left(B_a/B\right)^3}{\left[\left(B_a/B\right)^2 - 1\right]^{m/2+1}} \frac{p}{B_a} \qquad (6)$$

This probability function can be checked as having real probability characteristics, in particular, one gets the following standard relationships in statistics :

$$\begin{cases} \int_0^{B_0} \left(\frac{dp}{dB}\right).dB = 1 \\ \int_0^{B_0} \left(\frac{dp}{dB}\right).B.dB = \overline{B} \\ \int_0^{B_0} \left(\frac{dp}{dB}\right).\left(B - \overline{B}\right)^2.dB = \sigma_B^2 \end{cases}$$

where $\overline{B}$ is the mean statistical value for the quench field and σ_B the quadratic standard deviation. Using equation (6), one can calculate the statistical distribution of the quench field values for a given frequency and area and deduce the expected average quench field value $\overline{B}$.

Application for (β = 1) cavities at F = 1300-1500 MHz :

Single-cell cavities :

Let us apply the above analysis to a single-cell cavity at a frequency of F = 1300 MHz. The quench distribution computed from equation (6) is shown in Figure 3 assuming the following parameters S0 = 0.1 m2 and m = 5. The average quench field will be directly proportional to B0 whereas the value of the exponent m will determine the width of the distribution (the higher m, the narrower the distribution). The case of non-heat treated cavities have been addressed by specifying a lower value for B0, namely B0 = 150 mT, whereas the theoretical critical field value (198 mT) have been taken in the case of heat treated cavities. The value of m has been chosen to fit the experimental distribution obtained on real cavities at 1300 MHz and 1500 MHz. The value of S0 is arbitrary and has been chosen for simplicity to be approximately equal to the area of one single-cell cavity at the frequency of F = 1300 MHz.

Multi-cell cavities :

When going from a single-cell to a multi-cell cavity, the RF area is multiplied by the number of cells. Therefore, one expects to obtain an average quench field that decreases with the number of cells. From equation (6), a variation in the form of $\left(\frac{S}{S_a}\right) e^{-\left(\frac{S}{S_a}\right)}$ is expected. This is shown in Figure 4 where the average computed quench field of the above distribution is decreasing with the number of cells. This is fully in accordance with what is indeed experimentally observed and legitimitizes the statistical analysis here exposed. For example, a nine-cell cavity would exhibit a 25% lower quench field as compared to a single-cell.

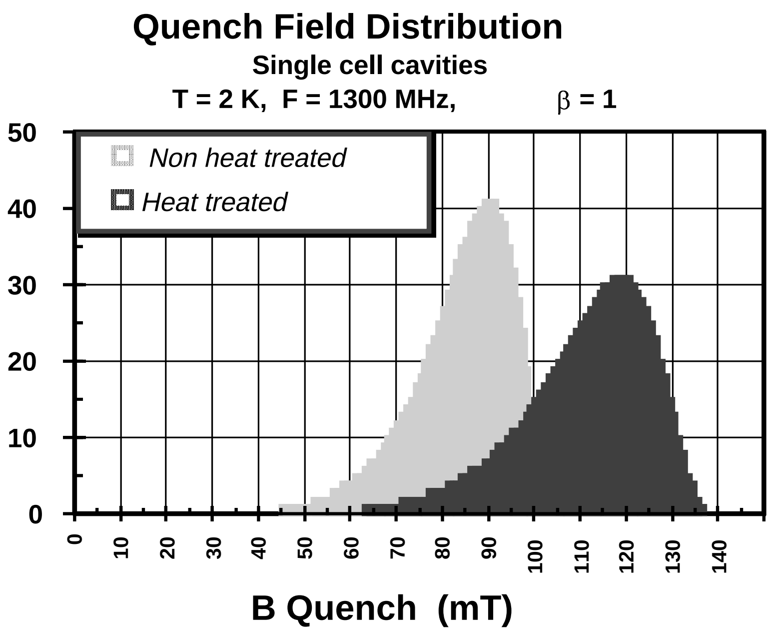

Figure 3 - Statistical calculated distributions of quench fields for non-heat treated and heat treated SCRF cavities.

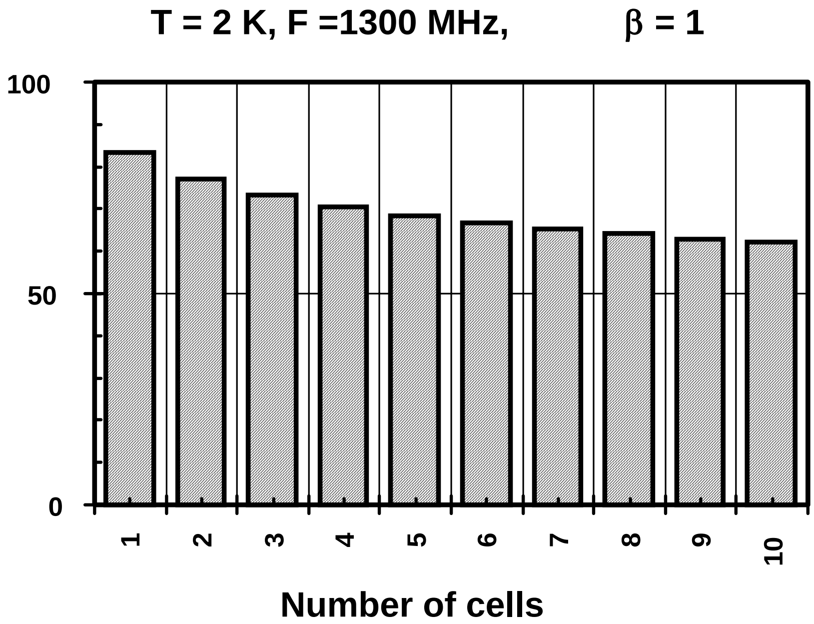

Figure 4 - The average quench field obtained decreases with the number of cells in a cavity as its surface increases. The quench field of a 9-cell cavity would be 25% lower when compared to a single-cell.

Application for (β = 0.64) cavities at F = 700 MHz :

The same analysis can be applied to superconducting cavities for proton accelerators which are generally running at lower frequencies (700 MHz is taken as an example). Because they are designed for non relativistic particles, the cavity shapes are rather squeezed. Consequently, one would expect a slightly higher average quench field value $\overline{B}$ (although the resulting accelerating field might end up being lower due to a high (B_{peak}/E_{acc}) ratio). In Figure 5, the computed quench field distribution is plotted for non heat treated single-cell and five cell cavities. The average fields are $\overline{B}$ = 87.5 mT for single-cells and $\overline{B}$ = 71.2 mT for 5-cells. The estimated rejection rate would be less than 2% if the specification acceptance were to be fixed at B_{spec} = 40 mT.

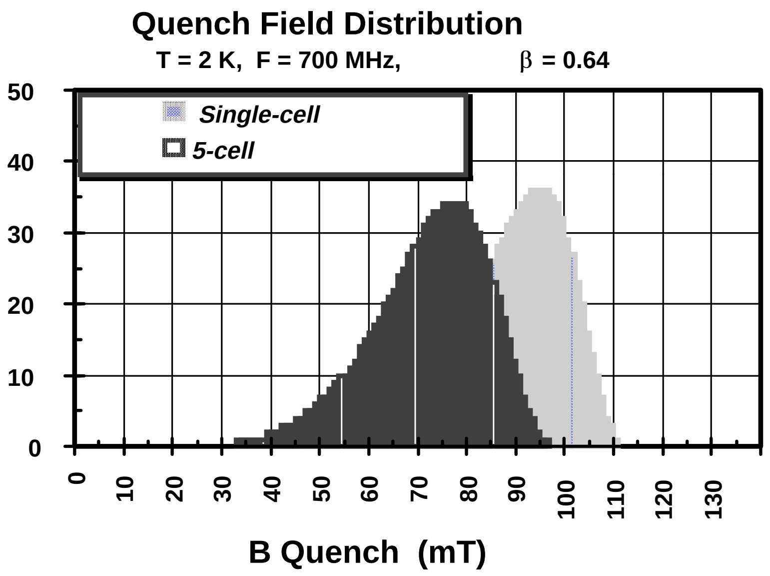

Figure 5 - Quench field distribution for non heat treated single and 5-cell proton cavities (β = 0.64). The average expected quench field is slightly higher than for electron cavities at 1300 MHz.

<u>Quench field vs Frequency :</u>

Equation (3) reflects the fact that the quench field given by the thermal analysis is a decreasing function of frequency. This is true even in the uniform case (ideal case with no defects) where the thermal instability starts to show up above F_0= 2.4 GHz [**2**,**3**,**4**,**5**]. This can be easily understood since the BCS surface resistance is increasing roughly as the square of the frequency (F^2). Below F_0, the breakdown is mainly a magnetic transition from the superconducting to the normal state, while above F_0, the thermal runaway will dominate. So, for a given defect size, the quench field given by (2) will steadily decrease with frequency. On the other hand, while the frequency is increased, the cavity surface S will decrease accordingly (an estimated area, in m^2, would be $S = 0.17\, n\beta^2 / F^2$, with F in GHz and n the number of cells). Therefore, the probability function p will increase (equation 5). These two opposite effects are counterbalancing in equation (6) where B_a is decreasing while p is increasing with frequency. The final balance exhibits a rather low dependence on frequency as can be seen in figure 6. While the overall result is still in favor of low frequencies, the statistical effect is strongly reducing the benefit of the single-defect thermal analysis. The large area involved at low frequency is wiping out the gain resulting from the lower surface resistance.

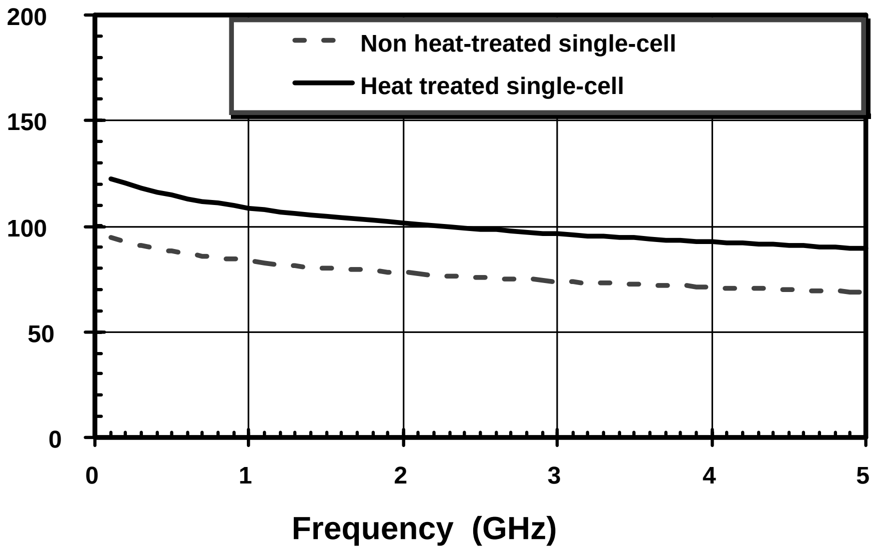

Figure 6 - Calculated quench field as a function of frequency taking in account both the thermal and the statistical analysis. The result show a rather low dependence on frequency, the two effects almost counterbalancing.

This statistical analysis holds as long as the defects are assumed to be randomly distributed. If, in the future, some progress were to be made on the understanding or the control of these defects, the scope might change. Niobium sheets could then be obtained with no defects greater than a given size. That would shift the probability function p to lower defect sizes by reducing Φ_0. The overall balance would consequently definitely lean towards favoring lower frequencies as the thermal model analysis indicates.

Conclusion :

In conclusion, an integrated picture including both the thermal analysis and the statistical distribution of defects in a SCRF cavity has been developed. This statistical tool allows prediction of the expected quench field distribution for a set of cavities that might be useful in a production scheme. While the thermal analysis for a single defect is clearly in favor of low frequencies, the statistical effect seems to almost counterbalance that advantage. The final average quench field resulting shows a rather smooth variation with frequency. In the long run, while improvement in the high RRR niobium sheets production is foreseen, with regard to the quench field performance (and disregarding other constraints), the choice of lower frequencies (500-1000 MHz) would be recommended.

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5 R. Roth, Proc. of EPAC, Berlin (1992)